12-14 Aprile 2023

θ-angle physics of 2-color QCD

Fixed baryon charge and Near Conformal Dynamics

Based on [1] and [2]
in collaboration with J. Bersini, F. Sannino and M. Torres

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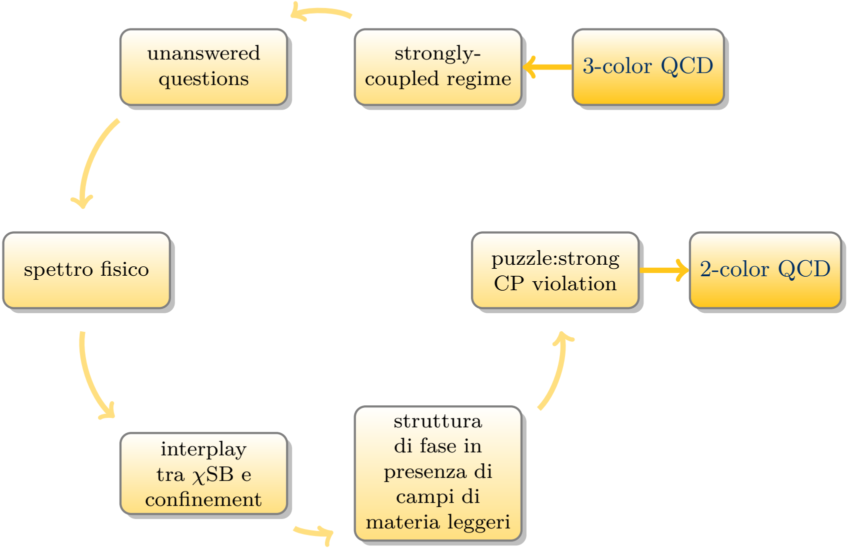
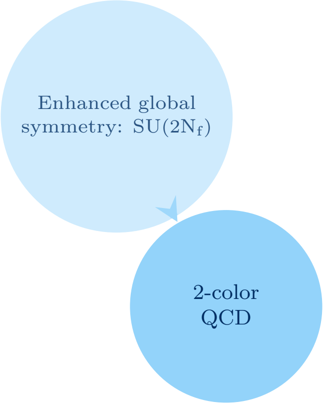
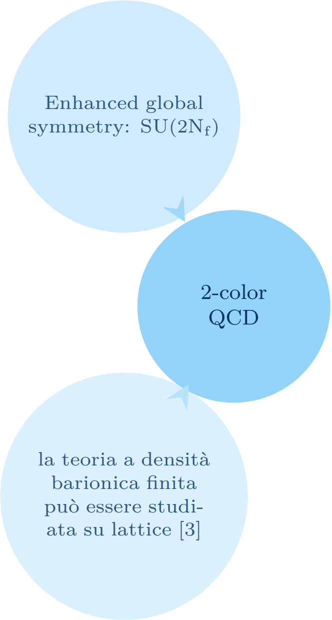
Università degli studi di Napoli "Federico II"

IFAE 2023 - Incontri di Fisica delle Alte Energie, Catania


unanswered questions
strongly-coupled regime
3-color QCD
spettro fisico
puzzle:strong CP violation
2-color QCD
interplay tra χSB e confinement
struttura di fase in presenza di campi di materia leggeri

2-color
QCD


Enhanced global symmetry: SU(2N$_f$)
2-color QCD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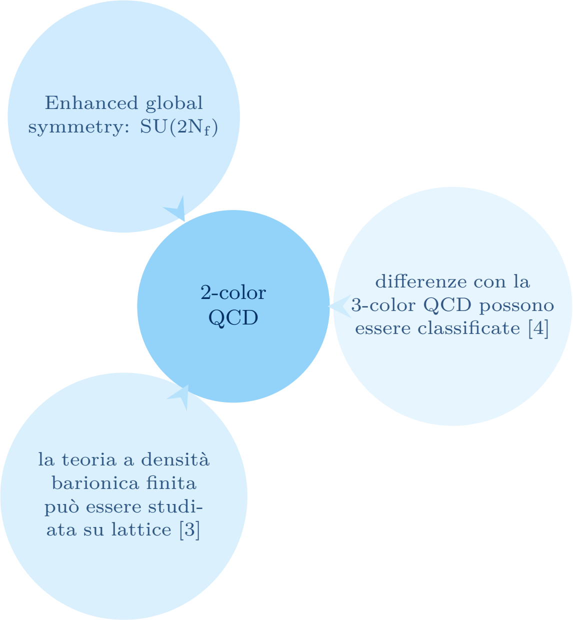
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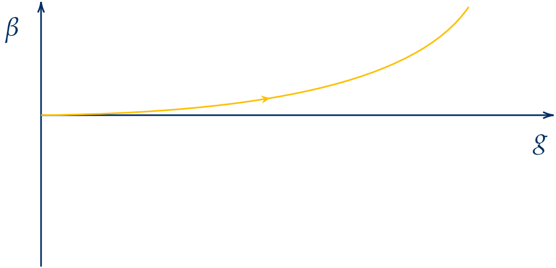
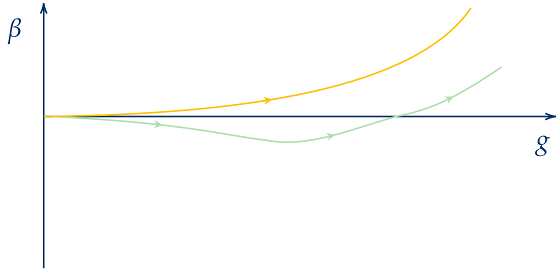
SU(2) [5]



SU(2) : conformal window [6]



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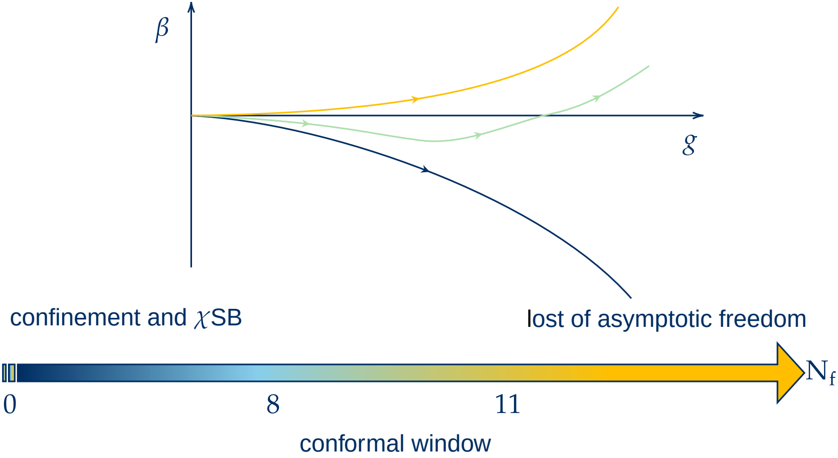

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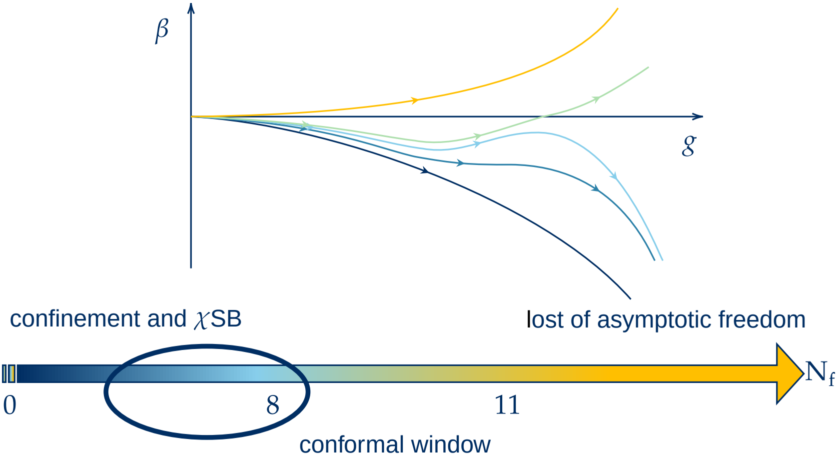

The θ-angle physics of two-color QCD at fixed baryon charge

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θ-dependence of the energy [1, 7, 8]

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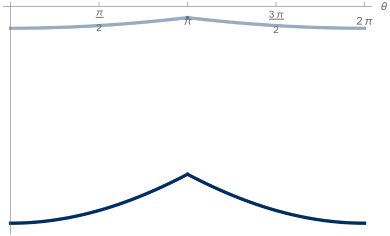

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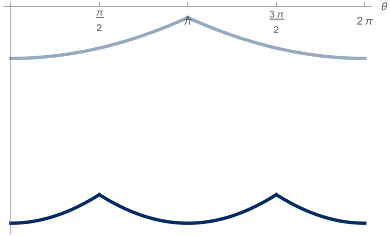

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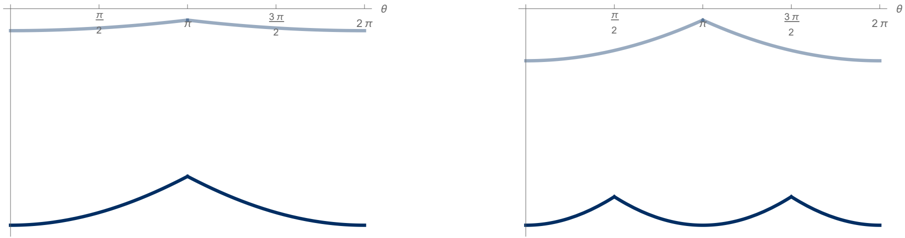

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$$0 \leq \cos\frac{\theta}{N_f} \leq \pi \leq \cos\frac{\theta+2\pi(N_f-1)}{N_f} \leq 2\pi$$

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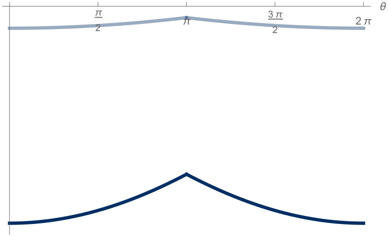

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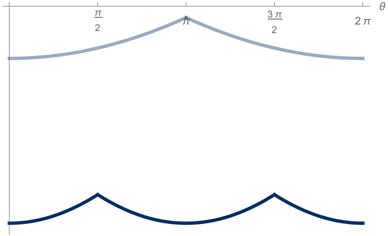

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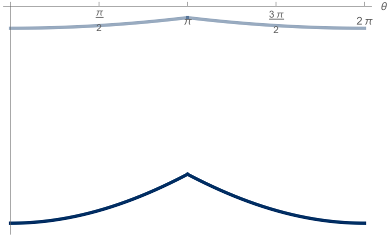

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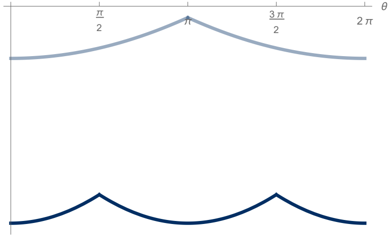

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Take home messages

EFT della 2-color QCD a carica barionica fissata e simmetria globale $SU(2N_f)$ e θ-angle

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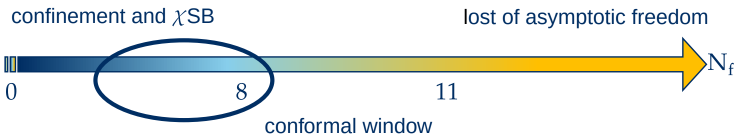
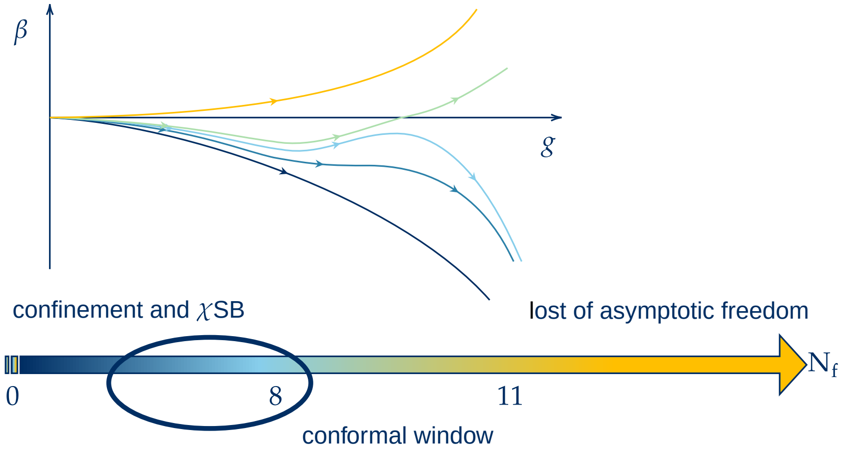


SU(2) : walking [6, 9]



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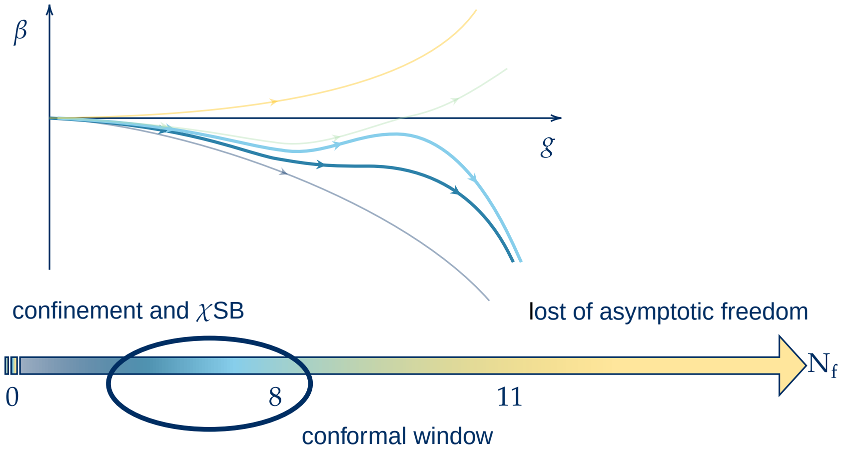

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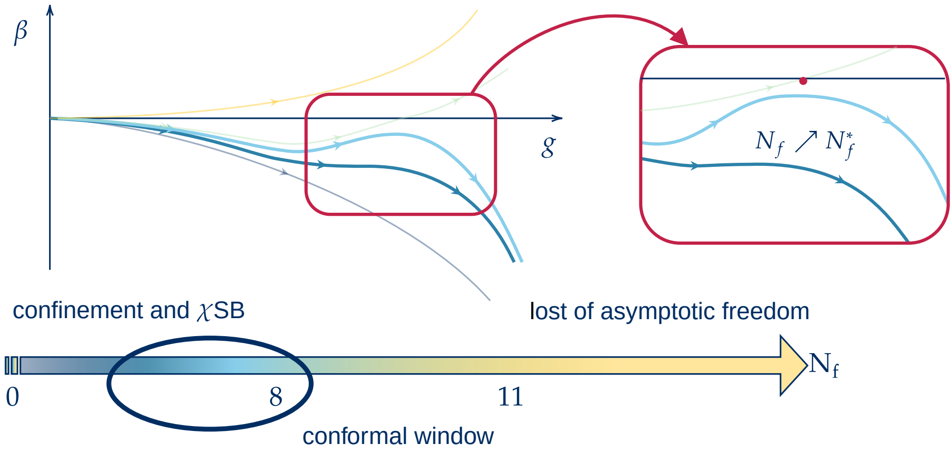

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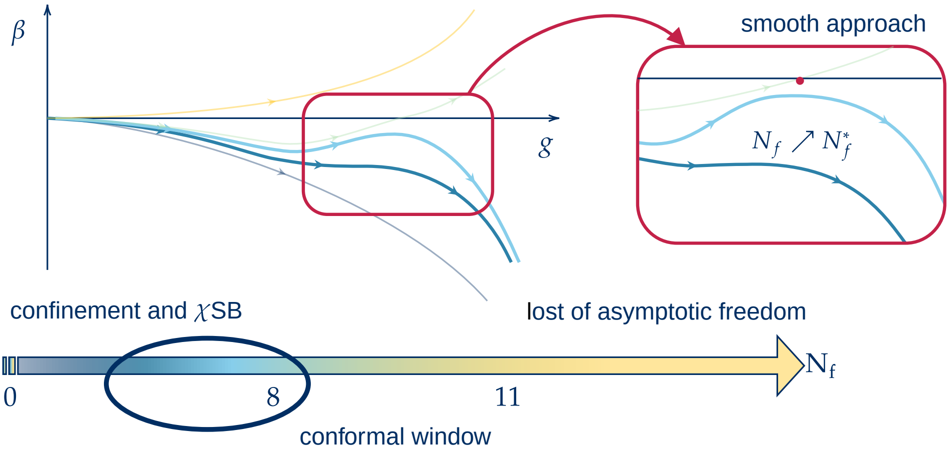

Dinamica quasi-conforme della teoria

Teorie QCD-like: presenza di un mesone scalare singoletto di flavour nello spettro [9]

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Dilatone: pNGB associato alla rottura spontanea dell'invarianza di scala [8, 10, 11]

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Charging the conformal window at nonzero θ-angle [2]

Dilaton-EFT della 2-color QCD con simmetria globale $\mathrm{SU}(2\mathrm{N_f})$ su background geometrico non banale

$$\tilde{\mathcal{L}} = \tilde{\mathcal{L}}_{\pi} + \tilde{\mathcal{L}}_{\mu} + \tilde{\mathcal{L}}_{\theta} + \mathrm{V}(\sigma) + \underline{\tilde{\mathcal{L}}_{\mathcal{M}}} \tag{3}$$

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$$\mathcal{M} = \mathbb{R} \times \mathcal{S}^3, \quad V(\sigma) = \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{m_\sigma^2}{16f^2}\left(4f\sigma + e^{-4f\sigma} - 1\right) [10], \quad \underline{\tilde{\mathcal{L}}_{\mathcal{M}}} = \Lambda_0 e^{-4f\sigma} - \frac{R^2}{12f^2} e^{-2f\sigma}$$

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studio dell'energia di vuoto della teoria nella fase superfluida con metodi semiclassici [11]

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stato nella fase superfluida sul cilindro ⟷ (state/operator approssimata; $E_Q\,\mathcal{R} = \Delta_Q$ [12]) ⟷ operatore con carica interna grande

$$E^{\gamma\ll 1} = \frac{c_{4/3}Q^{4/3}}{\tilde{V}^{1/3}} + Q^{2/3}\tilde{V}^{1/3}\Bigg\{c_{2/3}\tilde{R} - \frac{{X_{00}}^2}{4\pi^2 N_f^3 c_{4/3}^4}\left(\frac{9m_\pi^2}{32\nu}\right)^2\Bigg[1-\gamma\Bigg(\frac{2}{3}\log Q - \frac{X_{10}}{X_{00}} - \log\left(\frac{32N_f\nu^2\pi^2 c_{4/3}\tilde{V}^{2/3}}{3}\right)\Bigg)\Bigg]\Bigg\} - \tilde{V}\log Q\Bigg\{\frac{16\pi^2}{9}N_f c_{2/3}c_{4/3}\nu^2 m_\sigma^2 - \frac{\gamma}{3\pi^2 N_f^4 c_{4/3}^5}\left(\frac{9m_\pi^2}{32\nu}\right)^2 \cdot \Bigg[\frac{5}{8\pi^2 c_{4/3}^4 N_f^2}\left(\frac{9m_\pi^2}{32\nu}\right)^2 {X_{00}}^4 - c_{2/3}\tilde{R}N_f {X_{00}}^2 + \frac{9X_{00}X_{01}}{32c_{4/3}}\Bigg]\Bigg\} + (Q^0)$$

$$E^{1-\gamma\ll 1} = \frac{c_{4/3}Q^{4/3}}{\tilde{V}^{1/3}} + c_{2/3}Q^{2/3}\tilde{R}\tilde{V}^{1/3} - \frac{9(1-\gamma){X_{00}}^2 m_\pi^4\tilde{V}\log Q}{64c_{4/3}^3 N_f^2} - \frac{16}{9}\pi^2 m_\sigma^2 N_f c_{2/3}c_{4/3}\nu^2\tilde{V}\log Q + (Q^0) \;,$$

$$E^{\gamma\ll 1} = \frac{c_{4/3}Q^{4/3}}{\tilde{V}^{1/3}} + Q^{2/3}\tilde{V}^{1/3}\Bigg\{c_{2/3}\tilde{R} - \frac{{X_{00}}^2}{4\pi^2 N_f^3 c_{4/3}^4}\left(\frac{9m_\pi^2}{32\nu}\right)^2\Bigg[1-\gamma\Bigg(\frac{2}{3}\log Q - \frac{X_{10}}{X_{00}} - \log\left(\frac{32N_f\nu^2\pi^2 c_{4/3}\tilde{V}^{2/3}}{3}\right)\Bigg)\Bigg]\Bigg\} - \tilde{V}\log Q\Bigg\{\frac{16\pi^2}{9}N_f c_{2/3}c_{4/3}\nu^2 m_\sigma^2 - \frac{\gamma}{3\pi^2 N_f^4 c_{4/3}^5}\left(\frac{9m_\pi^2}{32\nu}\right)^2 \cdot \Bigg[\frac{5}{8\pi^2 c_{4/3}^4 N_f^2}\left(\frac{9m_\pi^2}{32\nu}\right)^2 {X_{00}}^4 - c_{2/3}\tilde{R}N_f {X_{00}}^2 + \frac{9X_{00}X_{01}}{32c_{4/3}}\Bigg]\Bigg\} + (Q^0)$$

$$E^{1-\gamma\ll 1} = \frac{c_{4/3}Q^{4/3}}{\tilde{V}^{1/3}} + c_{2/3}Q^{2/3}\tilde{R}\tilde{V}^{1/3} - \frac{9(1-\gamma){X_{00}}^2 m_\pi^4 \tilde{V}\log Q}{64c_{4/3}^3 N_f^2} - \frac{16}{9}\pi^2 m_\sigma^2 N_f c_{2/3}c_{4/3}\nu^2\tilde{V}\log Q + (Q^0) \ ,$$

dove

$$c_{4/3} = \frac{3}{8}\left(\frac{\Lambda^2}{\pi N_f\nu^2}\right)^{2/3}, \quad c_{2/3} = \frac{1}{4f^2}\left(\frac{\pi^2}{N_f\nu^2\Lambda^4}\right)^{1/3}, \quad \tilde{R} = \frac{R}{6} \quad \text{and} \quad \tilde{V} = \frac{V}{2\pi^2}\,, \qquad (4)$$

Take home messages

2-color QCD+non-zero baryon charge+ θ-angle

Take home messages

studio del vuoto della teoria in funzione del numero di flavour

2-color QCD+non-zero baryon charge+ θ-angle

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fasi nuove per flavour pari e dispari [1]

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Δ del lowest operator con carica Q [2]

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Grazie! ☺

Backup slides

Transformation properties of the fields

	[SU(2)]	$SU(N_f)_L$	$\times$	$SU(N_f)_R$	$\times$	$U(1)_V$	$\times$	$U(1)_A$
q_L	$\square$	$\square$		1		+1		+1
$i\sigma_2\tau_2 q_R^*$	$\square$	1		$\bar{\square}$		−1		+1

	[SU(2)]	$SU(2N_f)$	$\times$	$U(1)_A$
$\mathcal{Q}$	$\square$	$\square$		+1

Table: Transformation properties of q_L, $i\sigma_2\tau_2 q_R^*$ and $\mathcal{Q}$ under the action of the symmetry groups.

EOMs for the Witten variables

The equations of motion read

$$\sin\varphi\left(\mathrm{N_f}\cos\varphi - \frac{\mathrm{m}_\pi^2}{\mu^2}\mathrm{X}\right) = 0 \tag{5}$$

$$2\mathrm{m}_\pi^2 \sin\alpha_\mathrm{i}\cos\varphi = \mathrm{a}\bar{\theta}\,, \quad \mathrm{i} = 1,..,\mathrm{N_f} \tag{6}$$

and the energy of the system is

$$\mathrm{E} = -\nu^2\left[4\mathrm{m}_\pi^2\mathrm{X} - \mathrm{a}\bar{\theta}^2\right]\,, \qquad \text{normal phase } (\varphi = 0) \tag{7}$$

$$\mathrm{E} = -\nu^2\left[2\frac{\mathrm{N_f^2}\mu^4 + \mathrm{m}_\pi^4\mathrm{X}^2}{\mathrm{N_f}\mu^2} - \mathrm{a}\bar{\theta}^2\right]\,, \qquad \text{superfluid phase } \left(\cos\varphi = \frac{\mathrm{m}_\pi^2}{\mathrm{N_f}\mu^2}\mathrm{X}\right)\,. \tag{8}$$

In the normal phase, the Witten variables are related to θ by the well-known equation

$$2\mathrm{m}_\pi^2\sin\alpha_\mathrm{i} = \mathrm{a}\bar{\theta} = \mathrm{a}\left(\theta - \sum_\mathrm{i}^{\mathrm{N_f}}\alpha_\mathrm{i}\right)\,. \tag{9}$$

For the general solution we must have for any $\bar{\theta}$ fixed $\boxed{\sin\alpha_\mathrm{i} = \sin\alpha_\mathrm{j}}$.

To solve for the α_i we consider the expansion in the parameter $\frac{\mathrm{m}_\pi^2}{\mathrm{a}} \ll 1$.

EOMs for the Witten variables

At the leading order one needs to solve for $\bar{\theta} = 0$ and the angles α_i satisfy

$$\alpha_i = \begin{cases} \pi - \alpha, & i = 1, \ldots, n \\ \alpha, & i = n+1, \ldots, N_f \end{cases} \tag{10}$$

where α is the solution of the following modular equation

$$n(\pi - \alpha) + (N_f - n)\alpha = \theta \text{ Mod } 2\pi \ . \tag{11}$$

The modulo comes from the fact that if a solution $\{\alpha_i\}$ of eq.(9) is found, then it is possible to build another solution as follows

$$\alpha_1(\theta + 2\pi) = \alpha_1(\theta) + 2\pi \,, \qquad \alpha_i(\theta + 2\pi) = \alpha_i(\theta) \,, \quad i = 2, \ldots, N_f \,. \tag{12}$$

However, since the physics depends only on $e^{-i\alpha_i}$, the dynamics is invariant under $\theta \to \theta + 2\pi$. The solution of eq.(11) can be written as

$$\alpha = \frac{\theta + (2k - n)\pi}{(N_f - 2n)}, \quad k = 0, \ldots, N_f - 2n - 1 \,, \quad n = 0, ..., \left[\frac{N_f - 1}{2}\right] \,. \tag{13}$$

The range for k above emerges because for $k \geq N_f - 2n$ we repeat the solution for a given n.

Solutions of the EOM for Witten variables

One can ask when two different solutions of the equation of motion can have the same energy. This corresponds to requiring

$$\cos\left(\frac{\theta + 2\pi \mathrm{k}_1}{\mathrm{N_f}}\right) = \cos\left(\frac{\theta + 2\pi \mathrm{k}_2}{\mathrm{N_f}}\right) , \qquad \text{normal phase} \tag{14}$$

$$\cos^2\left(\frac{\theta + 2\pi \mathrm{k}_1}{\mathrm{N_f}}\right) = \cos^2\left(\frac{\theta + 2\pi \mathrm{k}_2}{\mathrm{N_f}}\right) , \qquad \text{superfluid phase .} \tag{15}$$

- Both conditions are satisfied when $\boxed{\mathrm{k}_1 = -\frac{\theta}{\pi} - \mathrm{k}_2 + \mathrm{N_f}}$.
- k_1 and k_2 are integers
- It is sufficient to consider the case $\mathrm{k}_1 = 0$ that for $[0, \pi]$ interval corresponds to the ground state energy, furthermore at $\theta = \pi$ it forces the second solution to be $\mathrm{k}_2 = \mathrm{N_f} - 1$

Superfluid N_f odd

We have the solution $\boxed{k_1 = -k_2 + \frac{N_f}{2} - \frac{\theta}{\pi}}$ which can be realized for

$\alpha = \frac{\theta}{N_f}$ $\quad$ $\alpha = \frac{\theta - \pi}{N_f} + \pi$ $\quad$ $\alpha = \frac{\theta - 2\pi}{N_f}$

CP breaking

- Note that when $n \neq 0$, the vacuum spontaneously breaks $Sp(2N_f)$ because of the different phases for each quark flavour.
- CP is preserved when $\bar{\theta} = 0$. For equal mass quarks as considered here, this happens when $m_\pi = 0$ or $\theta = 0$.
- For $\theta = \pi$ the Lagrangian possess CP symmetry but in the normal phase the latter is spontaneously broken by the vacuum [Dashen:1970et,DiVecchia:2013swa,Gaiotto:2017tne,DiVecchia:2017xpu] , leading to a strong θ-dependence near $\theta = \pi$.

CP breaking

Assuming that the ground state does not break $\mathrm{Sp}(2N_f)$ spontaneously (i.e. $n = 0$), the vacua lie at [7]

$$U(\alpha_i) = e^{i\frac{\theta+2\pi k}{N_f}} \mathbb{1}_{2N_f} \ . \tag{16}$$

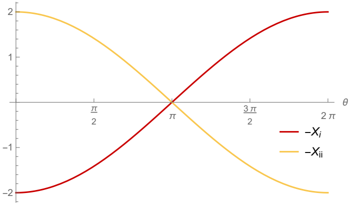
For $\theta = \pi$ one has $X = \cos\left(\frac{(2k+1)\pi}{N_f}\right)$, which is maximized when $k = 0$ and $k = N_f - 1$, that is

$$U(\alpha_i) = e^{\frac{i\pi}{N_f}} \mathbb{1}_{2N_f} \ , \qquad U(\alpha_i) = e^{-\frac{i\pi}{N_f}} \mathbb{1}_{2N_f} \ . \tag{17}$$

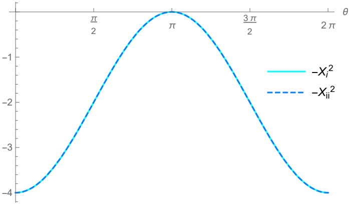
The two solutions are related by a CP transformation $U \to U^\dagger$ and thus CP is spontaneously broken.

CP breaking $N_f = 2$

For $N_f > 2$ the minima are separated by an energy barrier while for $N_f = 2$ the leading order quark-mass induced potential vanishes [Smilga:1998dh] , apparently leading to a paradoxical situation according to which one has massless pions and no explicit breaking of chiral symmetry.



(a) θ-dependence of the energy in the normal phase for $N_f = 2$.



(b) θ-dependence of the energy in the superfluid phase for $N_f = 2$.

Figure: θ-dependence of the energy for $N_f = 2$.

Symmetry breaking pattern & Spectrum

$SU(2N_f) \times U(1)_A \overset{2N_f^2 - N_f}{\rightsquigarrow} Sp(2N_f)$

$a,\ m$; μ

$SU(N_f)_V \times U(1)_B$

superfluid phase

$\frac{N_f^2 - N_f}{2}$

$Sp(N_f)_V$

$$\omega_1^2 = k^2 + \mu^2 \,, \qquad \frac{1}{2} N_f (N_f + 1)$$

$$\omega_2^2 = k^2 + \frac{m_\pi^4 X^2}{\mu^2 N_f^2} \,, \qquad \frac{1}{2} N_f (N_f - 1) - 1$$

$$\omega_3^2 = k^2 + \frac{2\left(\mu^4 N_f^2 + 3 m_\pi^4 X^2\right)}{N_f^2 \mu^2} + A \,, \qquad \bullet + \quad \frac{1}{2} N_f (N_f - 1)$$

$$\omega_4^2 = k^2 + \frac{2\left(\mu^4 N_f^2 + 3 m_\pi^4 X^2\right)}{N_f^2 \mu^2} - A \,, \qquad \bullet + \quad \frac{1}{2} N_f (N_f - 1)$$

$$\omega_5^2 = k^2 + M_S^2 \,, \qquad \bullet \quad 1$$

where

$$A = \frac{2}{N_f^2 \mu^2} \sqrt{\left(N_f^2 \mu^4 + 3 m_\pi^4 X^2\right)^2 + 4 N_f^2 \mu^2 m_\pi^4 k^2 X^2} \,, \tag{18}$$

$$M_S^2 = \frac{a \mu^4 N_f^3 + 2\mu^2 m_\pi^4 X^2}{2\mu^4 N_f^2 - 2 m_\pi^4 X^2} \left(1 - \frac{m_\pi^4 X^2}{\mu^2 N_f^2}\right) \tag{19}$$

Large charge setup

We will consider our system on a manifold $\mathcal{M}$ with volume V and curvature R such that the underlying new scale of the theory is

$$\Lambda_Q = (Q/V)^{1/3} \tag{20}$$

where Q is the fixed baryon charge.
Concretely, we will take our manifold to be

$$\mathcal{M} = \mathbb{R} \times S^{d-1} \tag{21}$$

such that we can consider an approximate state-operator correspondence that implies

$$\Delta_Q = \tilde{V}^{1/3} E_Q\,, \qquad E_Q = \mu Q - \mathcal{L} \tag{22}$$

where Δ_Q is the scaling dimension of the lowest-lying operator with baryon charge Q, E_Q is the ground state energy on $\mathbb{R} \times S^{d-1}$ at fixed charge, $\tilde{V}^{1/3}$ is the radius of S^{d-1}.

Large charge expansion of the θ-angle physics

We double-expanded X first in γ and then also in $1/Q$ as follows

$$X = X_0 + X_1\gamma + \left(\gamma^2\right) , \qquad X_k = X_{k0} + \frac{X_{k1}}{Q^{2/3}} + \left(Q^{-4/3}\right) , \qquad \text{for } \gamma \ll 1$$

$$X = X_0 + X_1(1-\gamma) + \left((1-\gamma)^2\right) , \qquad X_k = X_{k0} + \frac{X_{k1}}{Q^{4/3}} + \left(Q^{-2}\right) , \qquad \text{for } 1-\gamma \ll 1 .$$

where

$$X_{00} = N_f \cos\left(\frac{\theta + 2k\pi}{N_f}\right)$$

$$X_{01} = \frac{9m_\pi^4 \sin^2\left(\frac{\theta+2k\pi}{N_f}\right)\cos\left(\frac{\theta+2k\pi}{N_f}\right)}{8\ 2^{2/3}\pi^{4/3}a\ c_{4/3}^2}$$

$$X_{10} = 0$$

$$X_{11} = 0$$

$$\bar{\theta}_{00} = 0$$

$$\bar{\theta}_{01} = \frac{m_\pi^2 X_{00}\sin\left(\frac{\theta+2\pi k}{N_f}\right)}{aN_f}$$

$$\bar{\theta}_{10} = 0$$

$$\bar{\theta}_{11} = \frac{3m_\pi^2 \sin\left(\frac{2(\theta+2\pi k)}{N_f}\right)\log\left(\frac{8192\pi^2 c_{4/3}^3 N_f^3 v^6}{27Q^2}\right)}{32\ 2^{2/3}\pi^{4/3}a\ c_{4/3}^2}$$

EOMs

Evaluating the lagrangian (3) on the vacuum ansatz

$$\mathcal{L}_{\theta,\sigma}\left[\Sigma_0,\sigma_0\right] = -e^{-4f\sigma_0}\left(\Lambda^4 - \frac{m_\sigma^2}{16f^2}\right) - \frac{m_\sigma^2\left(4f\sigma_0 + e^{-4f\sigma_0} - 1\right)}{16f^2} - \frac{R\,e^{-2f\sigma}}{12f^2} + \\ + 4m_\pi^2\nu^2 X\cos\varphi\; e^{-f\sigma_0 y} + 2\mu^2 N_f\nu^2 e^{-2f\sigma_0}\sin^2\varphi - a\nu^2 e^{-4f\sigma_0}\bar{\theta}^2 \;, \qquad (23)$$

where

$$\bar{\theta} \equiv \theta - \sum_i^{N_f}\alpha_i\;, \qquad X \equiv \sum_i^{N_f}\cos\alpha_i\;, \qquad \Lambda^4 \equiv \Lambda_0^4 + \frac{m_\sigma^2}{16f^2}\;. \qquad (24)$$

The respective equations of motion are

$$N_f\mu^2 e^{-2f\sigma}\cos\varphi - m_\pi^2 X e^{-f\sigma y} = 0 \qquad (25)$$

$$a e^{-4f\sigma}\bar{\theta} - 2m_\pi^2\sin\alpha_i\cos\varphi e^{-f\sigma y} = 0\,, \qquad i = 1,..,N_f \qquad (26)$$

$$\frac{Re^{-2f\sigma}}{6f} + 4af\nu^2 e^{-4f\sigma}Y^2 + 4f\Lambda_0^4 e^{-4f\sigma} - \frac{m_\sigma^2\left(1 - e^{-4f\sigma}\right)}{4f} + \\ -4f\mu^2 N_f\nu^2 e^{-2f\sigma}\sin^2\varphi - 4fm_\pi^2\nu^2 yX\cos\varphi e^{-f\sigma y} = 0 \qquad (27)$$

$$4\mu N_f\nu^2 e^{-2f\sigma}\sin^2\varphi = \frac{Q}{V}\;. \qquad (28)$$

Δ_Q

- $\gamma \ll 1$

$$\frac{\Delta_Q}{\Delta_Q^*} = 1 - \left(\frac{9m_\pi^2}{32\pi\nu}\right)^2 \frac{1-\gamma\log\left(\frac{3\rho^{2/3}}{16(2\pi^2)^{1/3}c_{4/3}\nu^2 N_f}\right)}{4c_{4/3}^5 N_f}\cos^2\left(\frac{\theta+2\pi k}{N_f}\right)\left(\frac{1}{2\pi^2\rho}\right)^{2/3}$$
$$+\frac{\gamma}{c_{4/3}^6 N_f}\cos^2\left(\frac{\theta+2\pi k}{N_f}\right)\left(\frac{27m_\pi^4\,\sin^2\left(\frac{\theta+2\pi k}{N_f}\right)}{256\;2^{2/3}\pi^{4/3}a\;c_{4/3}^3 N_f^2}+\frac{5\left(\frac{9m_\pi^2}{64\pi\nu}\right)^2\cos^2\left(\frac{\theta+2\pi k}{N_f}\right)}{6c_{4/3}^4 N_f}-\frac{c_{2/3}}{2}\left(\frac{\rho}{2\pi^2 Q}\right)^{2/3}\right)$$
$$\times\left(\frac{9m_\pi^2}{32\pi\nu}\right)^2\left(\frac{1}{2\pi^2\rho}\right)^{4/3}\log Q-\frac{16}{9}\pi^2 c_{2/3}\nu^2 N_f m_\sigma^2\left(\frac{1}{2\pi^2\rho}\right)^{4/3}\log Q$$

- $(1-\gamma) \ll 1$

$$\frac{\Delta_Q}{\Delta_Q^*} = 1 - \left(\frac{9m_\pi^4}{64c_{4/3}^4}(1-\gamma)\cos^2\left(\frac{\theta+2\pi k}{N_f}\right)+\frac{16}{9}\pi^2 c_{2/3}\nu^2 N_f m_\sigma^2\right)\left(\frac{1}{2\pi^2\rho}\right)^{4/3}\log Q$$

Spectrum

$$\mathrm{SU(2N_f)\times U(1)_A} \overset{2N_f^2-N_f}{\rightsquigarrow} \mathrm{Sp(2N_f)} \longrightarrow \mathrm{SU(N_f)_V\times U(1)_B} \overset{\frac{N_f^2-N_f}{2}}{\rightsquigarrow} \mathrm{Sp(N_f)_V} \tag{29}$$

Having in mind the hierarchy of scales $m \ll \sqrt{a} \leq \mu \ll 4\pi\nu$, we focus on the spectrum of light modes

- $\frac{1}{2}N_f(N_f-1)$ massless Goldstones: $\begin{smallmatrix}\square\\\square\end{smallmatrix}+\bullet$ of $\mathrm{Sp(N_f)}$
- 1 pseudo-Goldstone $\bullet$ of $\mathrm{Sp(N_f)}$ with mass $\propto \sqrt{a}$

the spectrum changes when (near)conformal dynamics is realized through the dilaton dressing

we expand around the vacuum solution as follows

$$\Sigma = e^{i\Omega}\Sigma_0 e^{i\Omega^t} \quad \text{where} \quad \Omega = \begin{pmatrix} \pi & 0 \\ 0 & -\pi^t \end{pmatrix} + \tilde{\beta}S\begin{pmatrix} 1_{N_f} & 0 \\ 0 & 1_{N_f} \end{pmatrix}, \quad \tilde{\beta} \equiv \frac{1}{\sqrt{2N_f}}, \quad \pi = \sum_{a=0}^{\dim \frac{U(N_f)}{Sp(N_f)}} \pi^a T_a$$

Spectrum

$$\frac{\tilde{\mathcal{L}}}{4\nu^2 \sin^2\varphi\, e^{-2\sigma_0 f}} = \begin{pmatrix} \pi^0 & \hat{\sigma} & S \end{pmatrix} D^{-1} \begin{pmatrix} \pi^0 \\ \hat{\sigma} \\ S \end{pmatrix} + \sum_{a=1}^{\dim(\square)} \partial^\mu \pi^a \partial_\mu \pi^a \tag{30}$$

with the inverse propagator D^{-1} defined as

$$D^{-1} = \begin{pmatrix} \omega^2 - k^2 & i\omega\mu f\sqrt{2N_f} & 0 \\ -i\omega\mu f\sqrt{2N_f} & \frac{\omega^2-k^2}{8\nu^2\sin^2\varphi} - M_\sigma^2 & \frac{1}{2} I_{\hat{\sigma}S} \\ 0 & \frac{1}{2} I_{\hat{\sigma}S} & \frac{(\omega^2-k^2)}{\sin^2\varphi} - M_S^2 \end{pmatrix}, \qquad I_{\hat{\sigma}S} = \frac{\sqrt{2} f\mu^2 m_\pi^4 \sqrt{N_f} X y Z}{m_\pi^4 X^2 - \mu^4 N_f^2 e^{2f\sigma_0(y-2)}} \tag{31}$$

where $Z \equiv \sum_{i=1}^{N_f} \sin\alpha_i$ and the Lagrangian masses for the dilaton-field and the S mode are given by

$$M_\sigma^2 = -\frac{f^2\mu^2 N_f e^{-6f\sigma_0}\left(\nu^2 m_\pi^4 X^2 \left(y^2-2\right) e^{6f\sigma_0} + 2\mu^4\nu^2 N_f^2 e^{2f\sigma_0(y+1)} - 4\Lambda^4\mu^2 N_f e^{2f\sigma_0 y}\right)}{2\nu^2\left(\mu^4 N_f^2 e^{2f\sigma_0(y-2)} - m_\pi^4 X^2\right)} \tag{32}$$

$$M_S^2 = \frac{a\mu^4 N_f^3 e^{2f\sigma_0(y-1)} + 2\mu^2 m_\pi^4 X^2 e^{4f\sigma_0}}{2\mu^4 N_f^2 e^{2f\sigma_0 y} - 2m_\pi^4 X^2 e^{4f\sigma_0}}\,. \tag{33}$$

Spectrum

conformal invariance dictates the existence of a massless mode with speed $v_G = \frac{1}{\sqrt{d-1}} = \frac{1}{\sqrt{3}}$ [13] → mixing between the singlet mode π_0 with the dilaton → change π_0 velocity from $v_G = 1 \to v_G = \frac{1}{\sqrt{3}}$

In the large-charge limit, the above reduces to

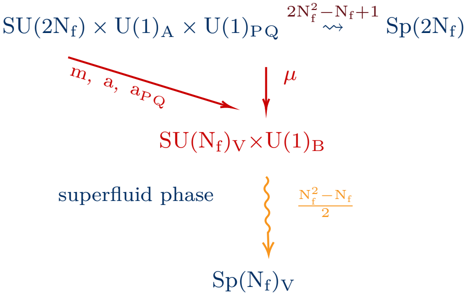
$$\gamma \ll 1 : \quad \omega_2 = k\left[\frac{1}{\sqrt{3}} + \frac{\sqrt{3}\,X_{00}{}^2}{(2\pi^2)^{2/3}\,c_{4/3}^5 N_f^3}\left(\frac{9m_\pi^2}{128\pi\nu}\right)^2\left(\frac{V}{Q}\right)^{2/3} + \dots\right] + \mathcal{O}(k^2)$$

$$(1-\gamma) \ll 1 : \quad \omega_2 = k\left[\frac{1}{\sqrt{3}} + 1\left(\frac{2^{5/3}c_{2/3}\nu^2 m_\sigma^2}{3\sqrt{3}\pi^{2/3}} + \frac{9\sqrt{3}m_\pi^4 X_{00}{}^2}{128\sqrt[3]{2}\pi^{8/3}c_{4/3}^4 N_f^2}\right)\left(\frac{V}{Q}\right)^{4/3} + \dots\right] + \mathcal{O}(k^2)$$

Axion

We denote by ν_{PQ} the scale of $\mathrm{U}(1)_{\mathrm{PQ}}$ spontaneous symmetry breaking and by a_{PQ} the coefficient of the $\mathrm{U}(1)_{\mathrm{PQ}}$ anomalous term.

$$\mathcal{L}_{\hat{a}} = \nu^2 \mathrm{Tr}\{\partial_\mu \Sigma \partial^\mu \Sigma^\dagger\} + \nu_{\mathrm{PQ}}^2 \partial_\mu \mathrm{N} \partial^\mu \mathrm{N}^\dagger + 4\mu\nu^2 \mathrm{Tr}\{\mathrm{B}\Sigma^\dagger \partial_0 \Sigma\} + \mathrm{m}_\pi^2 \nu^2 \mathrm{Tr}\{\mathrm{M}\Sigma + \mathrm{M}^\dagger \Sigma^\dagger\}$$
$$+ 2\mu^2\nu^2 \left[\mathrm{Tr}\{\Sigma \mathrm{B}^{\mathrm{T}} \Sigma^\dagger \mathrm{B}\} + \mathrm{Tr}\{\mathrm{BB}\}\right] - \mathrm{a}\nu^2 \left(\theta - \frac{\mathrm{i}}{4}\mathrm{Tr}\{\log\Sigma - \log\Sigma^\dagger\} - \frac{\mathrm{i}}{4}\mathrm{a}_{\mathrm{PQ}}(\log \mathrm{N} - \log \mathrm{N}^\dagger)\right)^2 . \tag{34}$$



$$\mathrm{D}^{-1} = \begin{pmatrix} \frac{\omega^2 - k^2}{\sin^2\varphi} - \mathrm{M_S^2} & -\frac{\mathrm{a}\sqrt{\mathrm{N_f}}\mathrm{a_{PQ}}}{4\sqrt{2}\nu_{\mathrm{PQ}}\sin^2\varphi} \\ -\frac{\mathrm{a}\sqrt{\mathrm{N_f}}\mathrm{a_{PQ}}}{4\sqrt{2}\nu_{\mathrm{PQ}}\sin^2\varphi} & \frac{\omega^2 - k^2}{4\nu^2\sin^2\varphi} - \mathrm{M}_{\hat{a}}^2 \end{pmatrix} , \tag{35}$$

where

$$\mathrm{M_S^2} = \frac{(\mathrm{a}\mu^4 \mathrm{N_f} + 2\mu^2 \mathrm{m}_\pi^4)}{2\mu^4 - 2\mathrm{m}_\pi^4} \tag{36}$$

$$\mathrm{M}_{\hat{a}}^2 = \frac{\mathrm{a}\mu^4 \mathrm{a_{PQ}^2}}{16\nu_{\mathrm{PQ}}^2(\mu^4 - \mathrm{m}_\pi^4)} . \tag{37}$$