Un nuovo approccio per lo studio dei decadimenti inclusivi del B su reticolo

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basato su *J. High Energ. Phys. 2022, 83 (2022), 2203.11762*

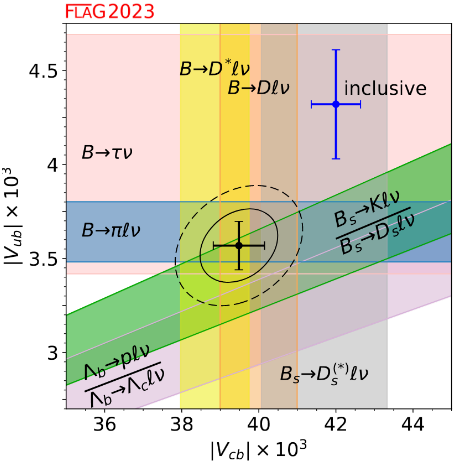
Incontri di Fisica delle Alte Energie
Catania
12 Aprile 2023

Vi è da tempo una tensione tra i valori degli elementi di matrice CKM $|V_{cb}|$ e $|V_{ub}|$ determinati da un'analisi **esclusiva** e gli stessi determinati da un'analisi **inclusiva**

	$\lvert V_{cb}\rvert$
Inclusivo	$(42.00 \pm 0.64) \cdot 10^{-3}$
Esclusivo	$(39.48 \pm 0.67) \cdot 10^{-3}$

	$\lvert V_{ub}\rvert$
Inclusivo	$(4.32 \pm 0.29) \cdot 10^{-3}$
Esclusivo	$(3.57 \pm 0.13) \cdot 10^{-3}$

[FLAG '23]



Calcolo inclusivo [Gambino, Hashimoto '20, **PRL** 2005.13730]

$$\frac{24\pi^3}{|\boldsymbol{q}|G_F^2|V_{cb}|^2}\frac{d\Gamma}{d\boldsymbol{q}^2} \propto Z^{(l)}(\boldsymbol{q}^2) \qquad \text{usando} \qquad \Theta^{(l)}(\omega) = \omega^l\theta(\omega)$$

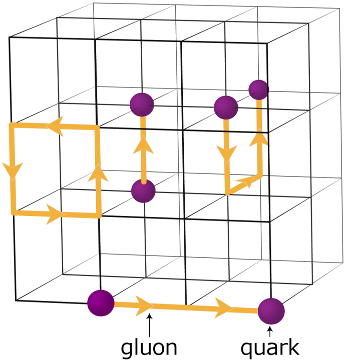
$$\boxed{Z^{(l)}(\boldsymbol{q}^2) = \int_0^\infty d\omega\ \Theta^{(l)}(\omega_{max}-\omega)\ W^{(l)}(\omega,\boldsymbol{q}^2)}$$

con:

$$W^{(0)} = W^{00} + \sum_{i,j=1}^{3}\frac{q^i}{\sqrt{\boldsymbol{q}^2}}\frac{q^j}{\sqrt{\boldsymbol{q}^2}}W^{ij} + \frac{q^i}{\sqrt{\boldsymbol{q}^2}}(W^{0i}+W^{i0})$$

Calcolo di $Z^{(l)}(\boldsymbol{q}^2)$ richiede metodi non-perturbativi

QCD su reticolo

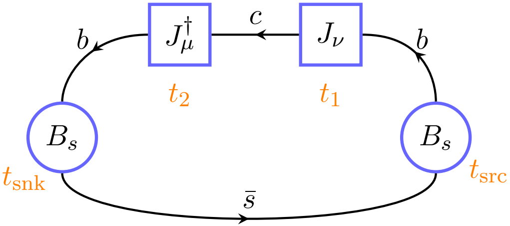


- permette calcoli non-pertutbativi partendo da principi primi
- azione Euclidea $\to$ peso statistico in una distribuzione di Boltzmann
- QCD viene discretizata su un reticolo con passo reticolare a
- spazio-tempo Euclideo in $4D$ a **volume finito** $L^3 \times T$

$$\langle \mathcal{O} \rangle = \mathcal{Z}^{-1} \int [DU] e^{-S_E[U]} \mathcal{O}[U] \approx \frac{1}{N} \sum_{i=1}^{N} \mathcal{O}[U_i]$$

[Hashimoto *PTEP* 2017, 1703.01881, Gambino-Hashimoto **PRL** 2020]

$$G_{\mu\nu}(t_2 - t_1; \boldsymbol{q}) = \lim_{\substack{t_{\rm snk}\to+\infty \\ t_{\rm src}\to-\infty}} \frac{C_{\mu\nu}(t_{\rm snk}, t_2, t_1, t_{\rm src})}{C(t_{\rm snk} - t_2)C(t_1 - t_{\rm src})}$$

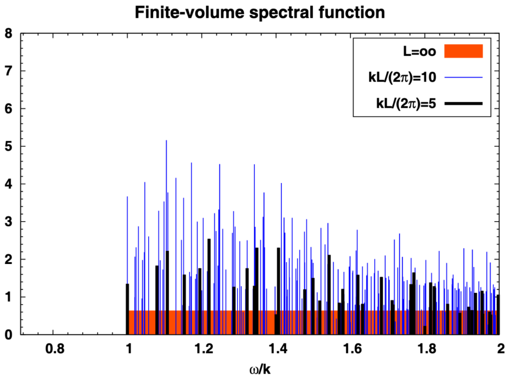


$C_{\mu\nu}(t_{\rm snk}, t_2, t_1, t_{\rm src})$

$$\boxed{G_{\mu\nu}(t; \boldsymbol{q}) = \int_0^\infty d\omega\, W^L_{\mu\nu}(\omega, \boldsymbol{q})\, e^{-\omega t}}$$

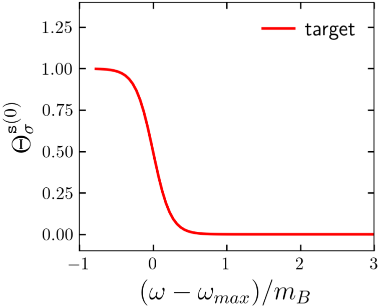
$$G_{\mu\nu}(t;\boldsymbol{q}) = \int_0^\infty d\omega\; W^L_{\mu\nu}(\omega,\boldsymbol{q})\; e^{-\omega t}$$

Per estrarre $W_{\mu\nu}(\omega, \boldsymbol{q}^2)$ occore risolvere un problema inverso mal condizionato



[H.B. Meyer, Eur.Phys.J.A47:86,2011]

$$\Theta_\sigma^{(l)}(\omega - \omega_{\mathrm{max}}) = \sum_{\tau}^{\tau_{\mathrm{max}}} g_\tau(\sigma, \omega) e^{-a\omega\tau}$$



smeared kernel

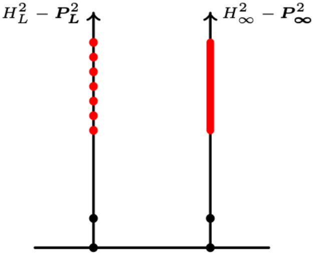
Ricordate

$$\boxed{Z^{(l)}(\boldsymbol{q}^2) = \int_0^\infty d\omega \; \Theta^{(l)}(\omega_{max} - \omega) \; W^{(l)}(\omega, \boldsymbol{q}^2)}$$

?

$$\Theta_\sigma^{(l)}(\omega - \omega_{\max}) = \sum_\tau^{\tau_{\max}} g_\tau(\sigma,\omega) e^{-a\omega\tau}$$

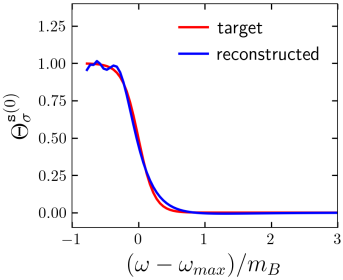
$$\begin{aligned}
\widehat{Z}_{\sigma,L}^{(l)}(\boldsymbol{q}^2) &= \int_0^\infty d\omega\ \Theta_\sigma(\omega-\omega_{\max}) W_L^{(l)}(\omega,\boldsymbol{q}^2) \\
&= \sum_\tau g_\tau(\sigma,\omega) \boxed{\int_0^\infty d\omega\ W_L^{(l)}(\omega,\boldsymbol{q}^2)\ e^{-a\omega\tau}} \\
&= \sum_\tau g_\tau(\sigma,\omega) G^{(l)}(a\tau,\boldsymbol{q}^2)
\end{aligned}$$

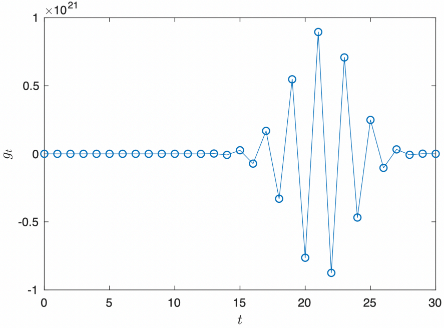


$$Z^{(l)}(\boldsymbol{q}^2) = \lim_{\sigma\to 0}\lim_{L\to\infty} \widehat{Z}_{\sigma,L}^{(l)}(\boldsymbol{q}^2, L)$$

Regolarizazione di Backus-Gilbert

Hansen,Lupo,Tantalo **PRD** '19 [1903.06476]



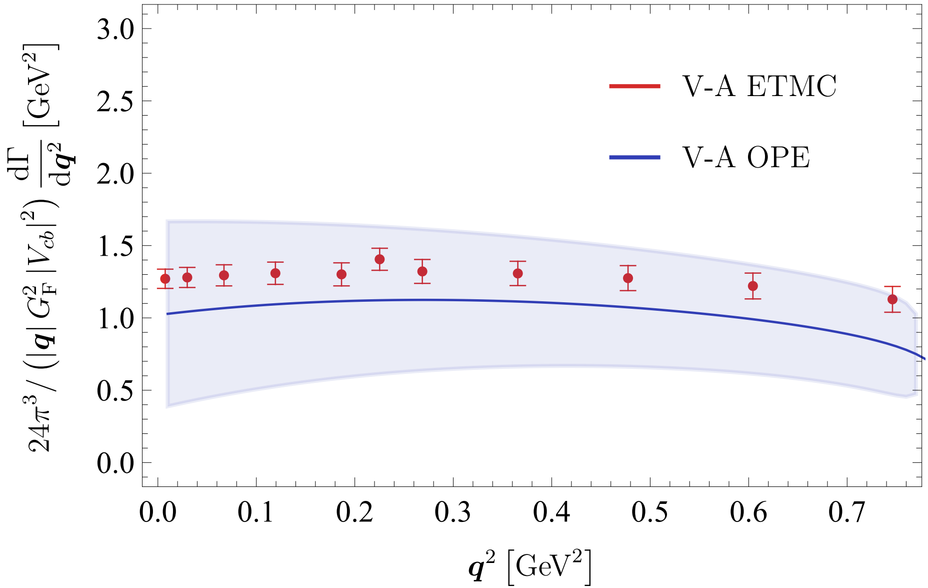


$$W_\lambda[g] = (1-\lambda)\frac{A[g]}{A[0]} + \lambda B[g]$$

$$B[g] = \sum_{\tau,\tau'=1}^{\tau_{max}} g_\tau g_{\tau'} \frac{\mathrm{Cov}[G^{(l)}(a\tau), G^{(l)}(a\tau')]}{[G^{(l)}(0)]^2}$$

$$A[g] = a\int_{E_0}^{\infty} dw \left\{ \Theta_\sigma^{(l)} - \sum_{\tau=1}^{\tau_{max}} g_\tau e^{-aw\tau} \right\}^2$$

Confronto con l'OPE



Conclusioni

- Abbiamo un nuovo metodo per calcolare i tassi di decadimento **inclusivi** dei processi semileptonici
- Il metodo si basa sulla ricostruzione numerica di kernel "smeared" e la regolarizzazione di un problema inverso mal condizionato
- Il lavoro presentato è esplorativo, occorre estendere l'analisi in modo da poter effettuare tutti i limiti previsti: $L \to \infty$, $a \to 0$
- Il metodo è generale e può essere applicato anche per altre quantità osservabili

Grazie per l'attenzione!

SLIDE DI BACKUP

correlatori

JLQCD

Möbius Domain Wall fermions

$N_f = 2 + 1,\ L \times T = 48^3 \times 96$

$a = 0.055\text{fm}, \quad M_\pi = 300 \text{ MeV}$

$m_b \simeq 2.44 m_c \to M_{B_s} = 3.45 \text{ GeV} < M_{B_s}^{phys}$

$t_{\text{src}} = 0,\ t_{\text{snk}} = 42a,\ t_2 = 26a,\ t_1 = 16a$

ETMC

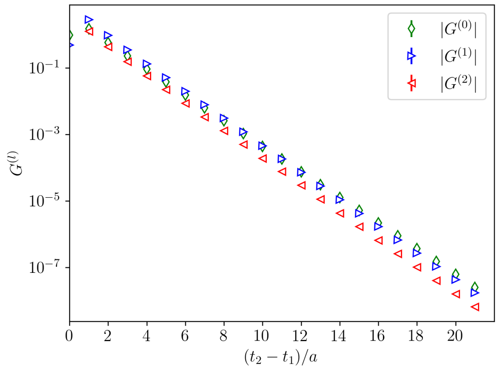
Twisted-Mass action (OS for s, c)

$N_f = 2 + 1 + 1,\ L \times T = 32^3 \times 64$

$a = 0.0815(30)\text{fm}, \quad M_\pi = 375(13) \text{ MeV}$

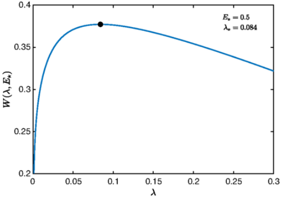
$m_b \simeq 2 m_c \to M_{B_s} = 3.08(11)\text{GeV} < M_{B_s}^{phys}$

$t_{\text{src}} = 0\ t_{\text{snk}} = 32a\ t_2 = 22a\ t_1 = 4a$



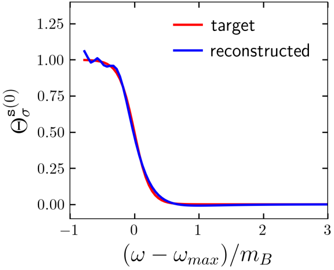
ETMC correlators

Trovare $\lambda_\star$

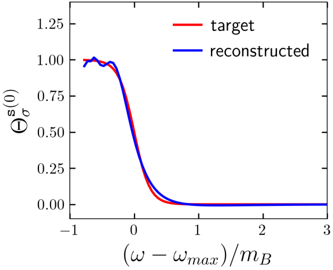


Hansen,Lupo,Tantalo **PRD** '19 [1903.06476]

$$\left.\frac{\partial W(\lambda)}{\partial \lambda}\right|_{\lambda=\lambda_\star} = 0$$

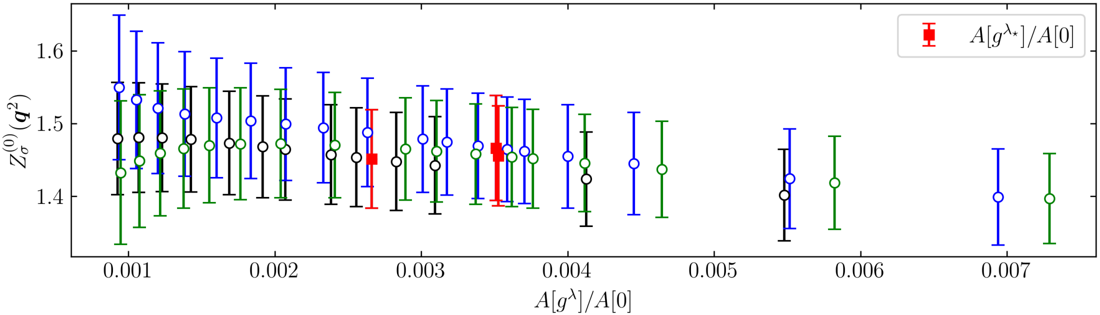


$\lambda < \lambda_\star$



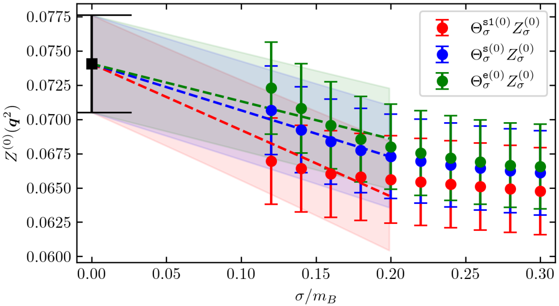
$\lambda = \lambda_\star$

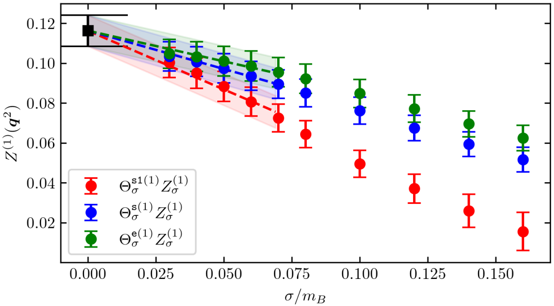
plot di stabilità [Bulava,Hansen M.T.,Hansen M.W.,Patella,Tantalo *JHEP* 2022 2111.12774]

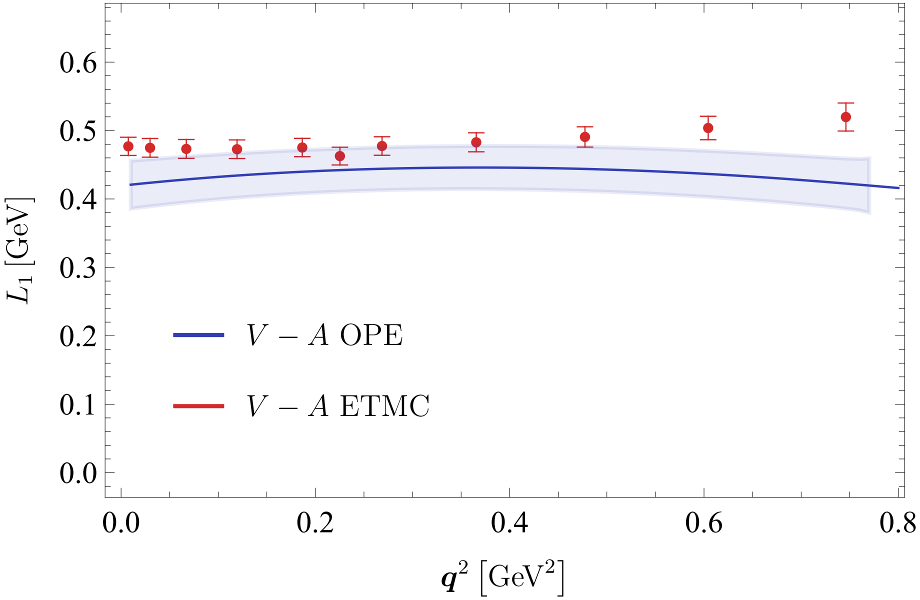


$$\theta^{\rm s}_\sigma(x) = \frac{1}{1+e^{-\frac{x}{\sigma}}}, \qquad \theta^{\rm s1}_\sigma(x) = \frac{1}{1+e^{-\sinh\left(\frac{x}{r^{\rm s1}\sigma}\right)}}, \qquad \theta^{\rm e}_\sigma(x) = \frac{1+\mathrm{erf}\left(\frac{x}{r^{\rm e}\sigma}\right)}{2}$$

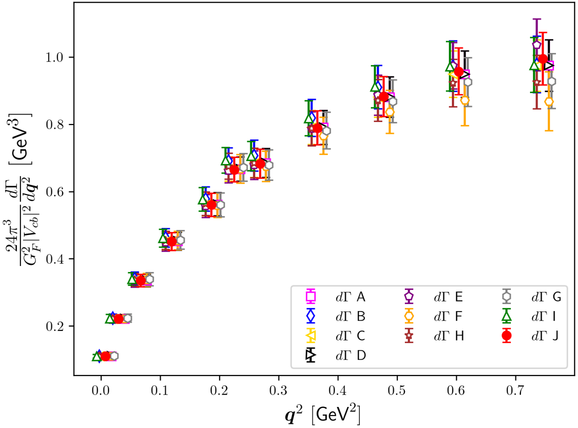
estrapolazioni $\sigma \to 0$






0.0
0.1
0.2
0.3
0.4
0.5
0.6
L_1 [GeV]
0.0
0.2
0.4
0.6
0.8
$\boldsymbol{q}^2$ [GeV^2]
$V - A$ OPE
$V - A$ ETMC

Systematics



(a) extrapolate $Z^{(l)}$ individually and then sum

(b) extrapolate with all values of σ

(c) choosing $\lambda < \lambda_\star$

(d) choosing $\lambda > \lambda_\star$

(e) sum and the extrapolate sum of $Z^{(l)}$

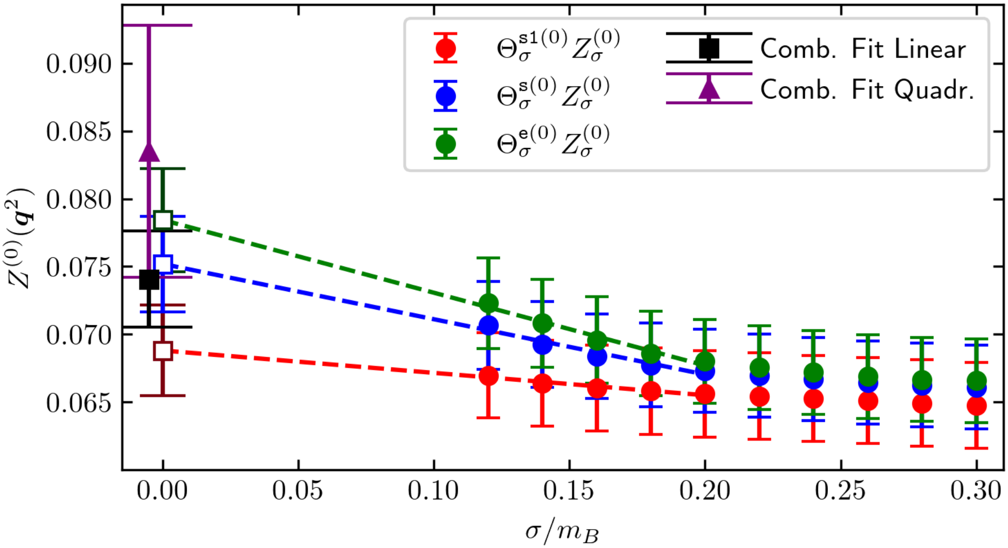
(f) $\tau_{\max} = 15$

(g) $\tau_{\max} = 16$

(h) $\tau_{\max} = 17$

(i) Bootstrap

Extrapolation



Tensor Decomposition

Accoding to Lorentz invariance and time-reversal symmetry, the Hadronic Tensor can be decomposed as follows

$$
\begin{aligned}
W^{\mu\nu}(p,q) = -g^{\mu\nu} W_1(w, \boldsymbol{q}^2) + \frac{p^\mu p^\nu}{m_{B_s}^2} W_2(w, \boldsymbol{q}^2) - \frac{i\varepsilon^{\mu\nu\alpha\beta} p_\alpha q_\beta}{m_{B_s}^2} W_3(w, \boldsymbol{q}^2) \\
+ \frac{q^\mu q^\nu}{m_{B_s}^2} W_4(w, \boldsymbol{q}^2) + \frac{p^\mu q^\nu + p^\nu q^\mu}{m_{B_s}^2} W_5(w, \boldsymbol{q}^2)
\end{aligned}
$$

La media PDG di $|V_{cb}|$ è: $(40.8 \pm 1.4) \cdot 10^{-3}$

Capire l'origine dell'incertezza su $|V_{cb}|$ è importante perché:

i. è un segnale che non abbiamo una comprensione totale dell'analisi esclusiva/inclusiva, con possibili implicazioni per $R(D^*)$

ii. una precisione limitata di $|V_{cb}|$ interessa gli studi delle correnti neutre (FCNC)

[Gambino,Jung,Shacht *Phy. Lett. B* '19, 1905.08209]