

# Axionic Strings, Domain Walls and Baryons via Holography

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Based on:

- F. Bigazzi, A. L. Cotrone, A.O. [JHEP 02, 194 \(2023\)](#).  
F. Bigazzi, A. L. Cotrone, A.O. [Phys. Rev. D 108, 026019 \(2023\)](#).  
F. Bigazzi, A. L. Cotrone, A.O. [ongoing work](#)

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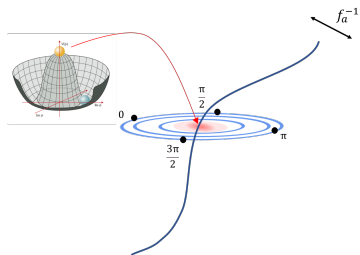


Istituto Nazionale di Fisica Nucleare  
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# Axionic Strings

## Strings & Topology

Non-simply connected vacuum manifold  $\mathcal{M}$  (Ex.  $\mathcal{M} = S^1$ ): existence of **strings**.



### Global Strings

Ex. Spontaneous Symmetry Breaking of Global  $U(1)$  at scale  $f_a$

Core size:  $\delta \sim f_a^{-1}$ ,

Tension:  $T \sim \pi f_a^2 \log(f_a R)$ ,  $R$  cutoff.

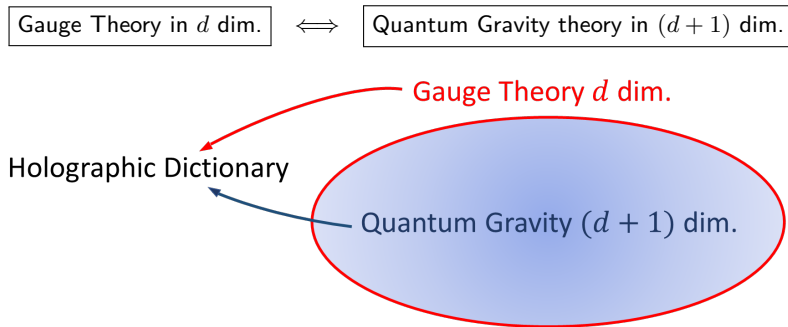
“Axionic Strings, Domain Walls and Baryons” PRD 108, 026019 (2023)

Composite Axion Model  $T \neq 0$ :  $SU(N)$  gauge group +  $N_f = 1$  extra-flavor.

Axionic string **effective action** from an UV complete theory (Holography!)

# Holographic Correspondence

J. M. Maldacena, Adv.Theor.Math.Phys. 2 (1998) 231-252.



## Weak/Strong Duality



# Holographic Yang-Mills at High Temperature

Witten, Adv. Theor. Math. Phys. 2, 505 (1998). Aharony et al., Annals Phys. 322, 1420 (2007).

4-dim.  $SU(N)$  YM  $N \gg 1$ ,  $\lambda = g_{YM}^2 N \gg 1$  + massive modes  $\Leftrightarrow$  Witten background

Witten Background at high temperature  $T > \Lambda$  (confining scale).



$$ds^2 = \left(\frac{u}{R}\right)^{3/2} [-f_T(u)dt^2 + dx^i dx^i + f_T(u)dx_4^2] + \left(\frac{R}{u}\right)^{3/2} \left[ \frac{du^2}{f_T(u)} + u^2 d\Omega_4^2 \right]$$

$f_T(u) = 1 - \frac{u_T^3}{u^3}$

Minkowski Space      Cylinder      Four-Sphere

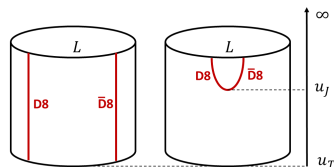
$R_{x_4} \sim M_{KK}^{-1}$   
 $u_T \sim T^2$  Black Hole Horizon

Confinement/Deconfinement Phase Transition  $T_c \sim M_{KK}$ .

# Flavors

Sakai, Sugimoto, Prog. Theor. Phys. 113, 843 (2005). Aharony et al., Annals Phys. 322, 1420 (2007). Bigazzi et al., JHEP 12, 056 (2019).

- $N_f = 1$  probe D8/ $\bar{D}8$  - branes (wrapped on the background four-sphere).
- $U(1)_L \otimes U(1)_R \rightarrow U(1)_V$ : **geometrical chiral symmetry breaking**.



- $L$  sets  $u_J \sim f_a \gg \Lambda$ .
- $T_c < T < f_a$ :  
**Model of Composite Axion!**

**$\chi$ -symmetry Broken/Preserved Phase Transition  $T_a \sim L^{-1}$ .**

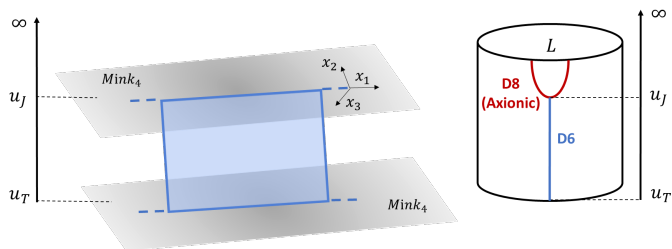
Theory on  $N_f = 1$  D8-brane world-volume:  
Maxwell-Chern-Simons theory in a **five-dimensional curved space-time**.

$$S \sim \lambda N \int_{M_5} \mathcal{F} \wedge \star d\mathcal{F} + N \int_{M_5} \mathcal{A} \wedge \mathcal{F} \wedge \mathcal{F}$$

# Holographic Axionic String

F. Bigazzi, A. L. Cotrone, A.O., Phys. Rev. D 108, 026019 (2023).

Axionic string as D6-brane wrapped on  $S^4$



D6-brane as a Magnetic Source

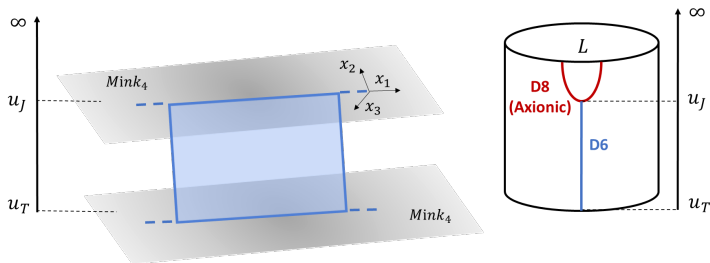
D6 brane is a source for the Maxwell-Chern-Simons theory on the D8-branes

$$d\mathcal{F} = \delta_3$$

Magnetic Source

# Holographic Axionic String

F. Bigazzi, A. L. Cotrone, A.O., Phys. Rev. D 108, 026019 (2023).



Maxwell-Bianchi system of equations from Maxwell-Chern-Simons theory:

$$d \star \mathcal{F} = a \mathcal{F} \wedge \mathcal{F},$$

$$d\mathcal{F} = -2\pi\delta(x_2)\delta(x_3)\delta(u - u_J)dx_2 \wedge dx_3 \wedge du.$$

Ansatz to solve the equations of motion:

$$\mathcal{F}_{x_2x_3} = l(u)\partial_u H, \quad \mathcal{F}_{x_2u} = m(u)\partial_{x_3} H, \quad \mathcal{F}_{x_3u} = n(u)\partial_{x_2} H.$$

# Holographic Axionic String

F. Bigazzi, A. L. Cotrone, A.O., Phys. Rev. D 108, 026019 (2023).

$$\text{Series Solution: } H(u, r) = \sum_{n=0}^{\infty} \zeta_n(u) Y_n(r), \quad r = \sqrt{x_2^2 + x_3^2}$$

Complete and orthonormal basis of eigenfunction  $\zeta_n(u)$  with eigenvalues  $\lambda_n$ .

$Y_0(r) \sim \log(T_a r)$ ,  $\lambda_0 = 0 \rightarrow$  **Axion**,  $Y_n(r) \sim K_0(T_a \sqrt{\lambda_n} r)$   $n \geq 1$ ,  $Y_n$  **mesonic modes**.

D8-brane action  $\Rightarrow$  Global string effective action **derived, not postulated!**

$$S = -\frac{1}{2} \sum_{n=0}^{\infty} \int d^4x [(\partial_r Y_n)^2 + T_a^2 \lambda_n Y_n^2].$$

**Tension and Core Thickness of the string:**

$$T \sim \lambda N T_c^2 \log\left(\frac{R}{\delta}\right), \quad \delta \sim (\lambda T^3 T_c)^{-1/2}.$$

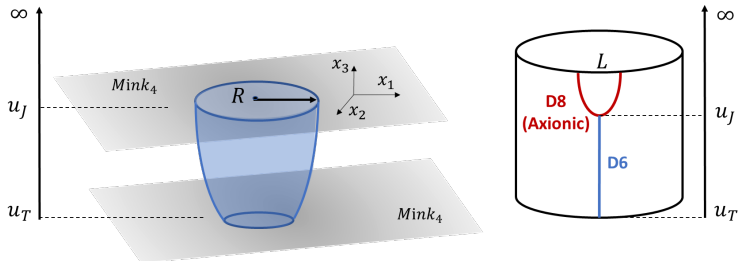
Near string divergence  $\rightarrow$  presence of a hard-core, i.e. the D6-brane.



# Holographic Vorton

Work in Progress.

$$\sqrt{x_1^2 + x_2^2} = \rho = \rho(u) \text{ non-trivial embedding function.}$$



$$d \star \mathcal{F} = a \mathcal{F} \wedge \mathcal{F},$$

$$d\mathcal{F} = -2\pi\delta(\rho(u) - R)\delta(x_3)\delta(u - u_J)d\rho \wedge dx_3 \wedge du.$$

Ansatz:

$$\mathcal{F}_{\rho x_3} = l(u)\partial_u H, \quad \mathcal{F}_{\rho u} = m(u)\partial_{x_3} H, \quad \mathcal{F}_{x_3 u} = n(u)\partial_\rho H.$$

# Holographic Vorton

Work in Progress.

Series Solution:

$$H(u, \rho, x_3; R) = \sum_{n=0}^{\infty} \zeta_n(u) Y_n(\rho, x_3; R), \quad Y_n \text{ mesonic modes (numerical solution).}$$

Effective action:

$$S = -\frac{1}{2} \sum_{n=0}^{\infty} \int d^4x \left[ (\partial_\rho Y_n)^2 + (\partial_3 Y_n)^2 + \left( \frac{1}{\rho^2} + T_a^2 \lambda_n \right) Y_n^2 \right]$$

Tension is not divergent at large distances!

**Vorton Stability** (to be studied quantitatively)

$\mathcal{A}_t \neq 0$ : Coulomb repulsion prevents the collapse.

(Qualitatively) Vorton Energy:  $E(R) \sim TR + A/R \Rightarrow R_{stable}$ .

$\mathcal{A}_t \iff J_t^B$ : Vorton stabilized by a conserved baryonic current.

# Holographic Domain Walls & A-Baryons

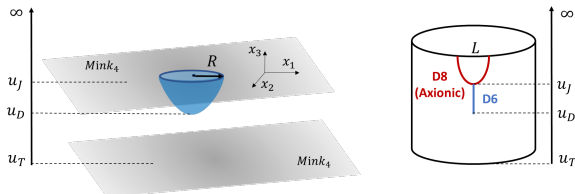
F. Bigazzi, A. L. Cotrone, A.O., JHEP 02, 194 (2023) + Work in Progress.

## String-Wall Network

$\mathcal{M}$  (vacuum manifold) with disconnected components: Domain Walls (DW) arise.  
DW attached to strings (Ex: axion mass switches on).

The holographic model allows us to study DW in the deconfined phase (unusual feature)!

Domain Wall as D6-brane wrapped on  $S^4$

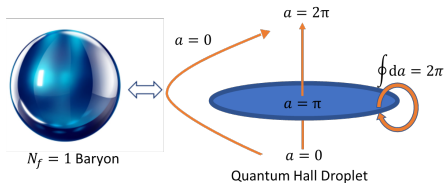


If charged  $\Rightarrow$  Axionic-Baryons in the holographic model!

# Holographic Domain Walls & A-Baryons

F. Bigazzi, A. L. Cotrone, A.O., JHEP 02, 194 (2023) + Work in Progress.

Low-energy description of  $N_f = 1$  baryons: the baryon number is associated to the current flowing along the boundary (Z. Komargodski, arXiv:1812.09253 [hep-th]).



DWs as D6-branes can be studied also in the confined phase:

- Infinite DW & DW bounded by a string (Bigazzi, Cotrone, A.O., JHEP 02, 194 (2023)).
- Charged DW (Work in Progress).

## Future Developments

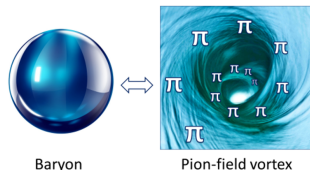
Phenomenological applications: Charged DWs & Vortons as Dark Matter candidates

Thank you for your time!

# Baryons in Low-Energy QCD

Z. Komargodski, arXiv:1812.09253 [hep-th].

Low-energy QCD  $\rightarrow$  Chiral Lagrangian (pionic degrees of freedom)



No low-energy description of  $N_f = 1$  Baryons

One-flavored baryons require an **alternative low-energy description**.

Chiral Lagrangian of  $N_f = 1$  QCD ( $\theta = 0$ ) in the large  $N$  limit

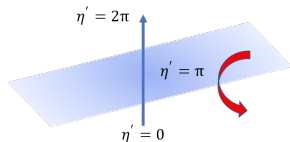
$\rightarrow$  Effective field theory for  $\eta'$  phase of the chiral condensate

$$\mathcal{L}_{\eta'} = \frac{1}{2}(\partial\eta')^2 - \frac{1}{2}m\Lambda \cos(\eta') - \frac{1}{2}m_{WV}^2 \min_{k \in \mathbb{Z}} (\eta' + 2\pi k)^2.$$

**Extended** excitation of the  $\eta'$  meson: **sheets**.

# Baryons as Quantum Hall droplets

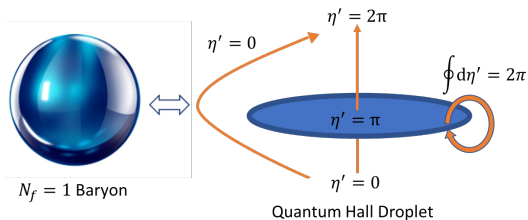
Z. Komargodski, arXiv:1812.09253 [hep-th].



Effective field theory on the sheet

$\eta' = \pi \rightarrow U(1)_N$  Chern-Simons (CS) theory

$$\frac{N}{4\pi} \int a \wedge da \Rightarrow \text{Fractional Quantum Hall effect!}$$



Chiral effective field theory is **Incomplete**  $\Rightarrow$  Holographic approach!

# Holographic Yang-Mills (Witten)

Witten, Adv. Theor. Math. Phys. 2, 505 (1998).

- (IIA Super String Theory)  $N$  **D4-branes** wrapped on a cycle with anti-periodic boundary conditions for fermions.
- Low-energy theory: four-dimensional Non-Supersymmetric  $SU(N)$  **Yang-Mills** + tower of massive Kaluza-Klein modes.
- Parameters:  $N \gg 1$ ,  $\lambda = g_{YM}^2 N \gg 1$ ,  $\Lambda_{QCD} \sim u_0$ .

$$ds^2 = \left(\frac{u}{R}\right)^{3/2} [\eta_{\mu\nu} dx^\mu dx^\nu + f(u) dx_4^2] + \left(\frac{R}{u}\right)^{3/2} \left[ \frac{du^2}{f(u)} + u^2 d\Omega_4^2 \right]$$

$f(u) = 1 - \frac{u_0^3}{u^3}$

Minkowski Space

Cigar

Four-Sphere

$u_0 \sim \Lambda_{QCD}$  Confinement scale

$R_{x_4} \sim M_{KK}^{-1}$

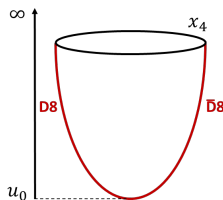
⇒ Confinement, Mass Gap.



## Holographic QCD (Sakai-Sugimoto)

Sakai, Sugimoto, Prog. Theor. Phys. 113, 843 (2005).

- $N_f$  probe D8/ $\overline{\text{D8}}$  - branes (wrapped on the background four-sphere).
- $U(N_f) \otimes U(N_f) \rightarrow U(N_f)$ : spontaneous symmetry **breaking of chiral symmetry** realized **geometrically**.
- Mesons arise from **fluctuations** of the D8-brane gauge field  $\mathcal{A}$ .



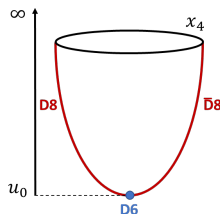
$$N_f = 1 \rightarrow S \sim \lambda N \int_{M_5} \mathcal{F} \wedge \star d\mathcal{F} + N \int_{M_5} \mathcal{A} \wedge \mathcal{F} \wedge \mathcal{F}.$$

Maxwell-Chern-Simons (**MCS**) theory in a **five-dimensional curved space-time**.

# Hall Droplet Sheets in Holographic QCD

F. Bigazzi, A. L. Cotrone, A.O., JHEP 02, 194 (2023).

Sheet  $\Leftrightarrow$  D6-brane wrapped on  $S^4$



Effective field theory on the Holo-sheet

$U(1)_N$  CS theory D6-brane world-volume.

$$\Rightarrow \frac{N}{4\pi} \int_{M_3} a \wedge da,$$

where  $M_3 = (t, x_1, x_2)$ .

D8-brane gauge field perspective

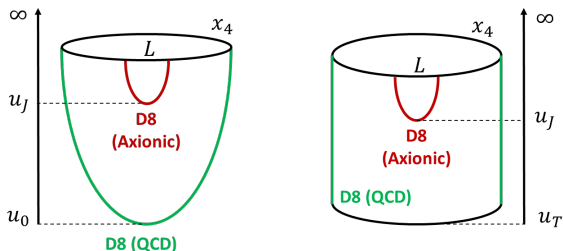
D6-brane is a source for the MCS theory  $\Rightarrow$  non-trivial  $\eta'$  profile.

D6-brane with a boundary in  $M_2 \rightarrow d\mathcal{F} = \delta_3$  **Magnetic charge source.**

## Holographic QCD Axion

Bigazzi, Caddeo, Cotrone, Di Vecchia, Marzolla, JHEP 12, 056 (2019).

- Non antipodal extra probe D8/ $\overline{D8}$  - branes.
- $U(1)_A$  anomalous and spontaneously broken (NJL-like term).
- Brane separation  $L$  sets  $f_a \sim u_J \gg u_0 \sim \Lambda_{QCD}$ .



- If  $T \neq 0$  and  $T_c < T < f_a$  axion survives  $\Rightarrow$  **Model of Composite Axion**.
- $m_a^2(T)$  grows with the temperature!