Shear viscosity to entropy density ratio in gravity analogue models

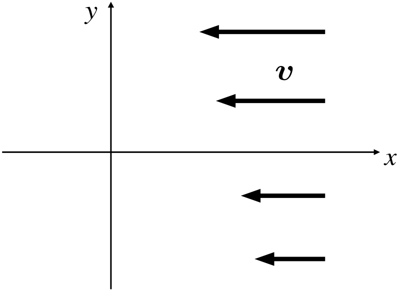
Silvia Trabucco

Cortona, September 29, 2023

silvia.trabucco@gssi.it

GRAN SASSO SCIENCE INSTITUTE

INFN Laboratori Nazionali del Gran Sasso



Efficiency of diffusion

$$\eta \sim np\lambda_{\mathrm{mfp}} \,,$$

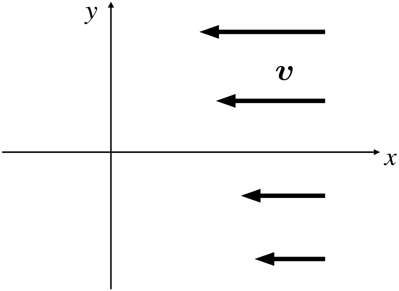
n number density,
p average momentum,
λ_{mfp} mean free path

$$p\lambda_{\mathrm{mfp}} \gtrsim \hbar \qquad \Rightarrow \qquad \frac{\eta}{n} \gtrsim \hbar$$

Switching to the entropy density $s \propto k_B n$, we expect

$$\frac{\eta}{s} \gtrsim \frac{\hbar}{k_{\mathrm{B}}}$$

Shear viscosity



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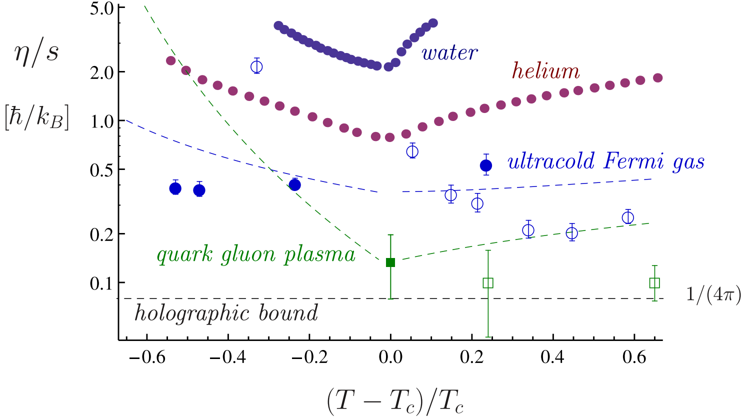
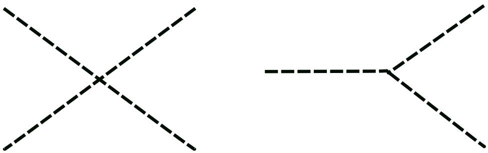
Adams *et al* 2012 *New J. Phys.* **14** 115009



$\frac{1}{4\pi}$ is the so-called KSS bound[1].

[1] Kovtun, Son, Starinets, PRL 2005

In a superfluid, viscosities are determined by processes like



and at $T \to 0$ are suppressed.

Gravity analogs and acoustic model

Inviscid, barotropic, irrotational fluid at zero temperature[2].

Scale separation $\theta(x) = \bar{\theta}(x) + \tilde{\theta}(x)$ and saddle point approximation

$$\mathcal{S}[\theta] = \mathcal{S}[\bar{\theta}] + \frac{1}{2}\int d^4x\sqrt{-g}g^{\mu\nu}\partial_\mu\tilde{\theta}\partial_\nu\tilde{\theta} + \dots$$

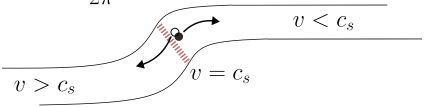
$$g^{\mu\nu} \propto \left(\eta^{\mu\nu} + (c_s^{-2} - 1)v^\mu v^\nu\right)$$

c_s adiabatic sound speed, $v^\mu \propto \partial^\mu\bar{\theta}$. We neglect any interactions for $\tilde{\theta}$ field.

[2] Unruh PRL 1981, Visser *et al* NJP 2010, Barceló *et al* Liv.Rev. 2011

KINETIC APPROACH

$$T_H = \frac{1}{2\pi}|\partial_x(v - c_s)|$$



We describe Hawking-like phonons with $f(x,p)$ covariant Bose Einstein distribution function

Thermodynamic quantities:

$$T^{\mu\nu}_{\text{ph}} = \int p^\mu p^\nu f(x,p) d\mathcal{P}$$

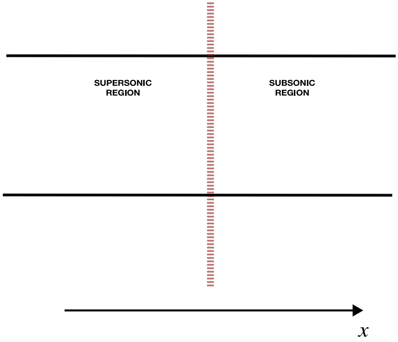
$$s^\alpha_{\text{ph}} = -\int p^\alpha [f \ln f - (1+f)\ln(1+f)] d\mathcal{P}$$

$$s^0_{\text{ph}} \propto T_H$$

$d\mathcal{P}$ appropriate covariant momentum measure [3]

[3] Mannarelli, Grasso, ST, Chiofalo PRD 2021

Dissipation at the acoustic horizon



Chiofalo, Grasso, Mannarelli, ST 2022

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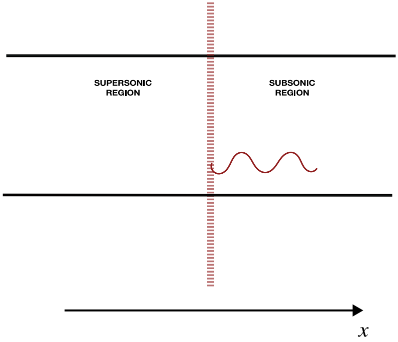

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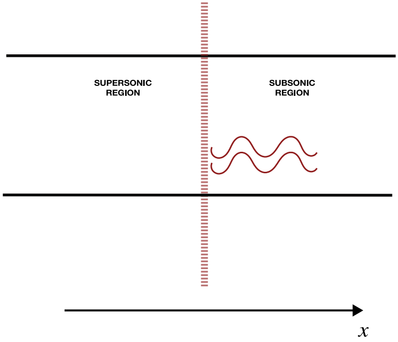

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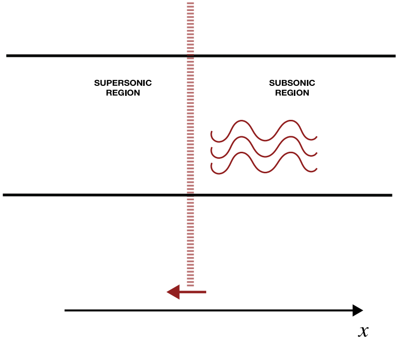

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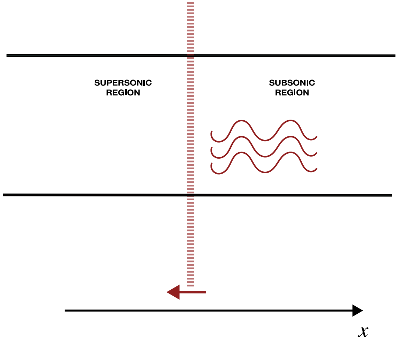
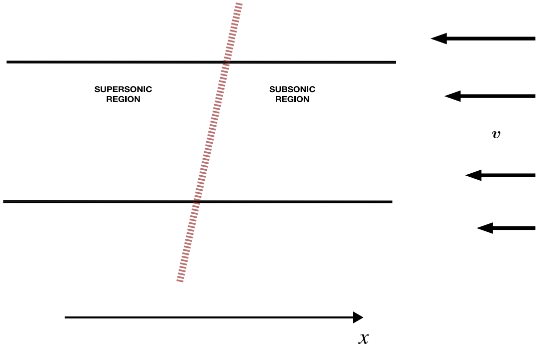


Effective bulk viscosity ζ

Chiofalo, Grasso, Mannarelli, ST 2022

Shear viscosity



Chiofalo, Grasso, Mannarelli, ST 2022

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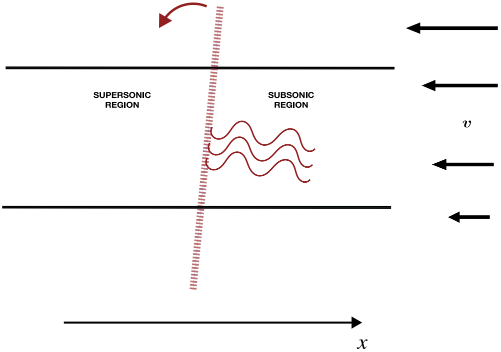
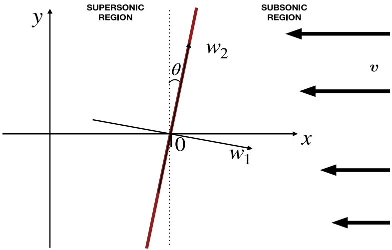


Effective shear viscosity η

Chiofalo, Grasso, Mannarelli, ST 2022

$v_x \simeq c_s - 2\pi T_H x + ky$



Viscous stress tensor $\sigma'_{ij} = \eta(\partial_i v_j + \partial_j v_i) + \zeta \delta_{ix}\delta_{jx} \nabla \cdot \mathbf{v}$

$$T^{\text{ph}}_{ij} = \sigma'_{ij} \qquad \text{yields} \qquad \frac{\zeta}{s_{\text{ph}}} = \frac{\eta}{s_{\text{ph}}} = \frac{1}{4\pi}$$

at leading order in k/T_H.

In general

$$2\pi T\zeta = P_{\rm ph}\,, \qquad Ts_{\rm ph} = \epsilon_{\rm ph} + P_{\rm ph} \qquad \Rightarrow \qquad \frac{\zeta}{s_{\rm ph}} = \frac{P_{\rm ph}}{2\pi(P_{\rm ph}+\epsilon_{\rm ph})}$$

- If $\epsilon_{\rm ph} = P_{\rm ph}$ we have saturation

$$\frac{\zeta}{s_{\rm ph}} = \frac{1}{4\pi}\,,$$

- if $\epsilon_{\rm ph} > P_{\rm ph}$ then

$$\frac{\zeta}{s_{\rm ph}} \leq \frac{1}{4\pi}$$

OUTLOOK: TWO SUPERFLUIDS

For two fluids A, B given $\mathbf{\Phi}^\dagger = (\Phi_A^*, \Phi_B^*)$

$$\mathcal{L} = \mathcal{L}_K(\mathbf{\Phi}) - \frac{1}{2}\mathbf{\Phi}^\dagger M \mathbf{\Phi} - V(\mathbf{\Phi}) \, ,$$

with $V(\mathbf{\Phi})$ preserving $U(1) \otimes U(1)$.

The covariant derivative includes the chemical potential

$$\mathcal{L}_K = (D_\nu \mathbf{\Phi})^\dagger (D^\nu \mathbf{\Phi}) \, ,$$

where

$$D_\nu = \mathcal{I}_2 \partial_\nu - i A_\nu \, , \qquad A_\nu = \mu \delta_\nu^0 \mathcal{I}_2 \, .$$

The mass matrix

$$M = \begin{pmatrix} m_A^2 & 2\lambda \\ 2\lambda & m_B^2 \end{pmatrix} = m^2 \mathcal{I}_2 + \Delta m^2 \, \sigma_3 + 2\lambda \, \sigma_1 \, ,$$

includes the explicit breaking term $\lambda(\Phi_A^* \Phi_B + \Phi_B^* \Phi_A)$ Rabi interaction.

Then we can have 1 NGB and 1 pseudo NGB.

The effective Lagrangian for the phonons is [4]

$$\mathcal{L}_{\text{eff}} = \frac{\sqrt{-g^{(I)}}}{2}\left(\eta^{\mu\nu} + \left(\frac{1}{c_{sI}^2} - 1\right) v^\mu v^\nu\right) \partial_\mu \tilde{\theta}_I \partial_\nu \tilde{\theta}_I + (I \rightarrow II) + \frac{1}{2}\tilde{m}_{II}^2 \tilde{\theta}_{II}^2$$

and

$$\tilde{m}_{II}^2 \propto \lambda \qquad \Rightarrow \tilde{\theta}_{II} \quad \text{pseudo-NGB}$$

[4] Liberati *et al* PRL 2006

Conclusions

In acoustic analog models, we reach the conjectured lower bound $1/4\pi$ for η/s.

Future perspectives:

- Compute η, ζ with Green-Kubo formulae in the two fluid model [5]
- Explore systems with two acoustic horizons

[5] Mannarelli, S.T. *in preparation.*

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Thanks for listening!

Backup slides

Kinetic approach

Covariant distribution function f: ansatz for bosonic particles

$$f(x,p) = \frac{1}{\exp(\beta_\mu p^\mu) - 1}$$

imposing that it is a solution of the Liouville flow

$$\nabla_\mu \beta_\nu + \nabla_\nu \beta_\mu = 0\,, \qquad \beta_\mu = (\beta, \mathbf{0})\,.$$

The covariant momentum measure

$$d\mathcal{P} = \sqrt{-g}\,2H(p)\delta(g_{\mu\nu}p^\mu p^\nu)\frac{dp^0 dp^3}{(2\pi)^3}\,,$$

with $g = \det g_{\mu\nu}$

Correlation functions for a single species

In LDA the action can be cast in the form $\Psi(x) = (\tilde{\rho}(x), \tilde{\theta}(x))^t$

$$\mathcal{S}[\Psi] = \int \frac{d^d p}{(2\pi)^d} \frac{1}{2} \Psi(-p)^t D^{-1}(p) \Psi(p)$$

where

$$D^{-1}(p) = \begin{pmatrix} p^\mu p_\mu - \tilde{m}^2 & iV^\mu p_\mu \\ -iV^\mu p_\mu & B^2 p^\mu p_\mu \end{pmatrix} ,$$

and its inverse gives the correlation function in momentum space

$$G(p) = \frac{1}{\det D^{-1}} \begin{pmatrix} B^2 p^\mu p_\mu & -iV^\mu p_\mu \\ iV^\mu p_\mu & p^\mu p_\mu - \tilde{m}^2 \end{pmatrix} .$$

The two fluid potential

One possibility is to choose a density-density interactions

$$V = U_{AA}(\mathbf{\Phi}^\dagger P_A \mathbf{\Phi})^2 + U_{BB}(\mathbf{\Phi}^\dagger P_B \mathbf{\Phi})^2 + U_{AB}(\mathbf{\Phi}^\dagger P_A \mathbf{\Phi})(\mathbf{\Phi}^\dagger P_B \mathbf{\Phi}) \,,$$

$$P_A = \frac{\mathcal{I}_2 + \sigma_3}{2} \quad \text{and} \quad P_B = \frac{\mathcal{I}_2 - \sigma_3}{2} \,,$$

Mescibility and stability require

$$U_{AA} > 0 \,, \quad U_{BB} > 0 \,, \quad |U_{AB}| < 2\sqrt{U_{AA} U_{BB}}$$

Petrov PRL 2015