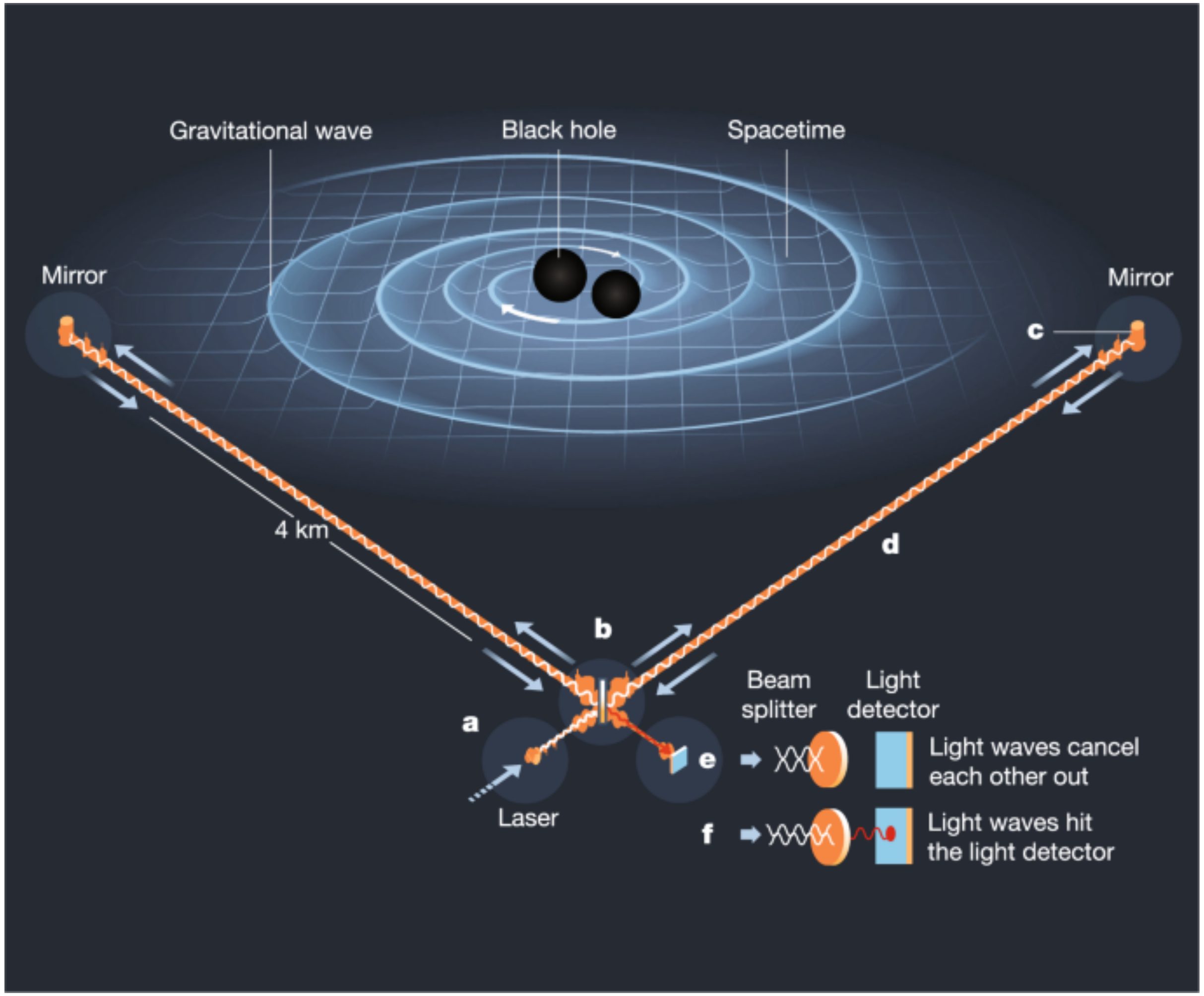
Primordial Black Holes in the era of Gravitational Wave Astronomy

New Frontiers in Theoretical Physics 2023

CONVEGNO NAZIONALE DI FISICA TEORICA

Antonio Riotto
University of Geneva

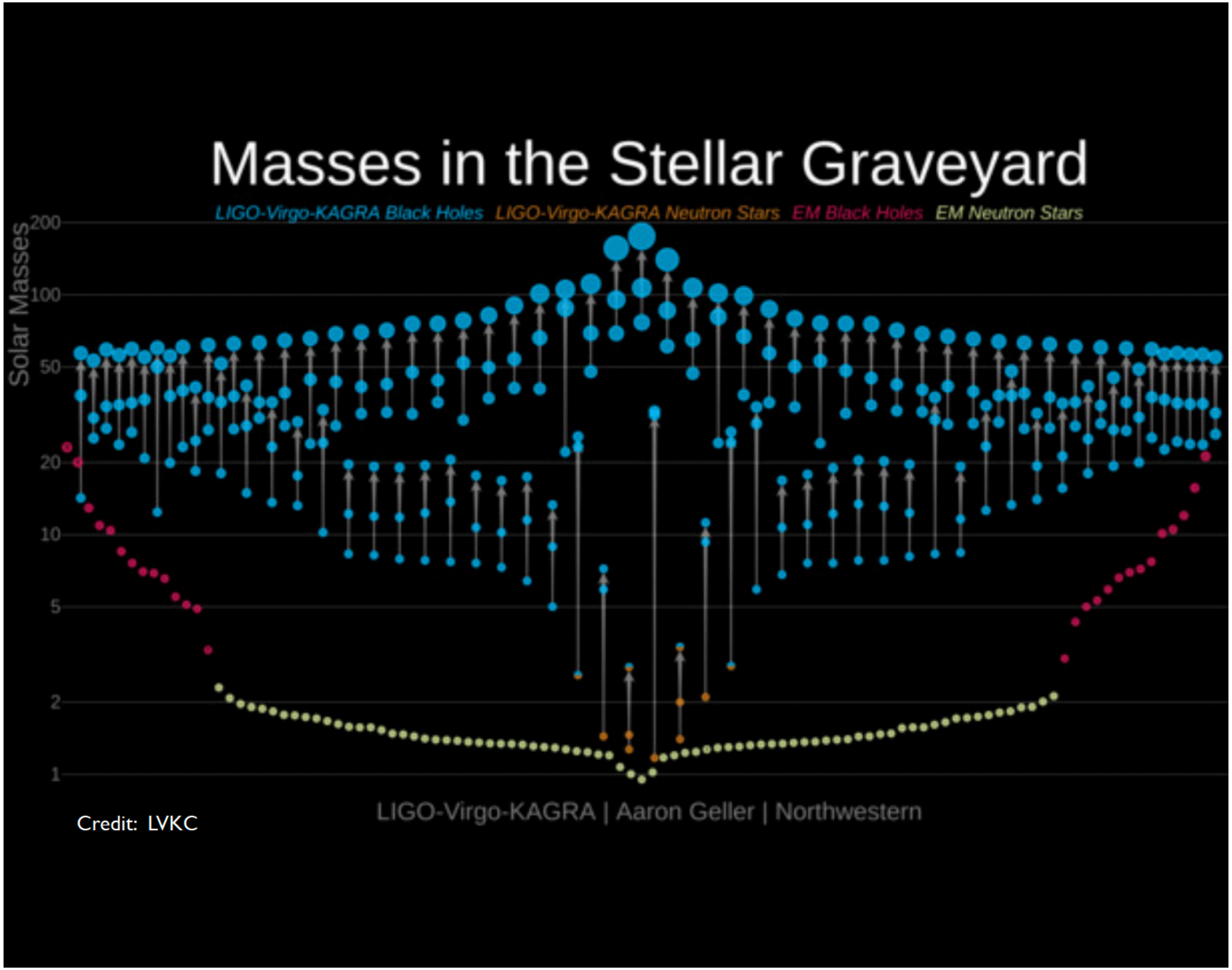
Gravitational waves and Black Holes are key predictions of General Relativity



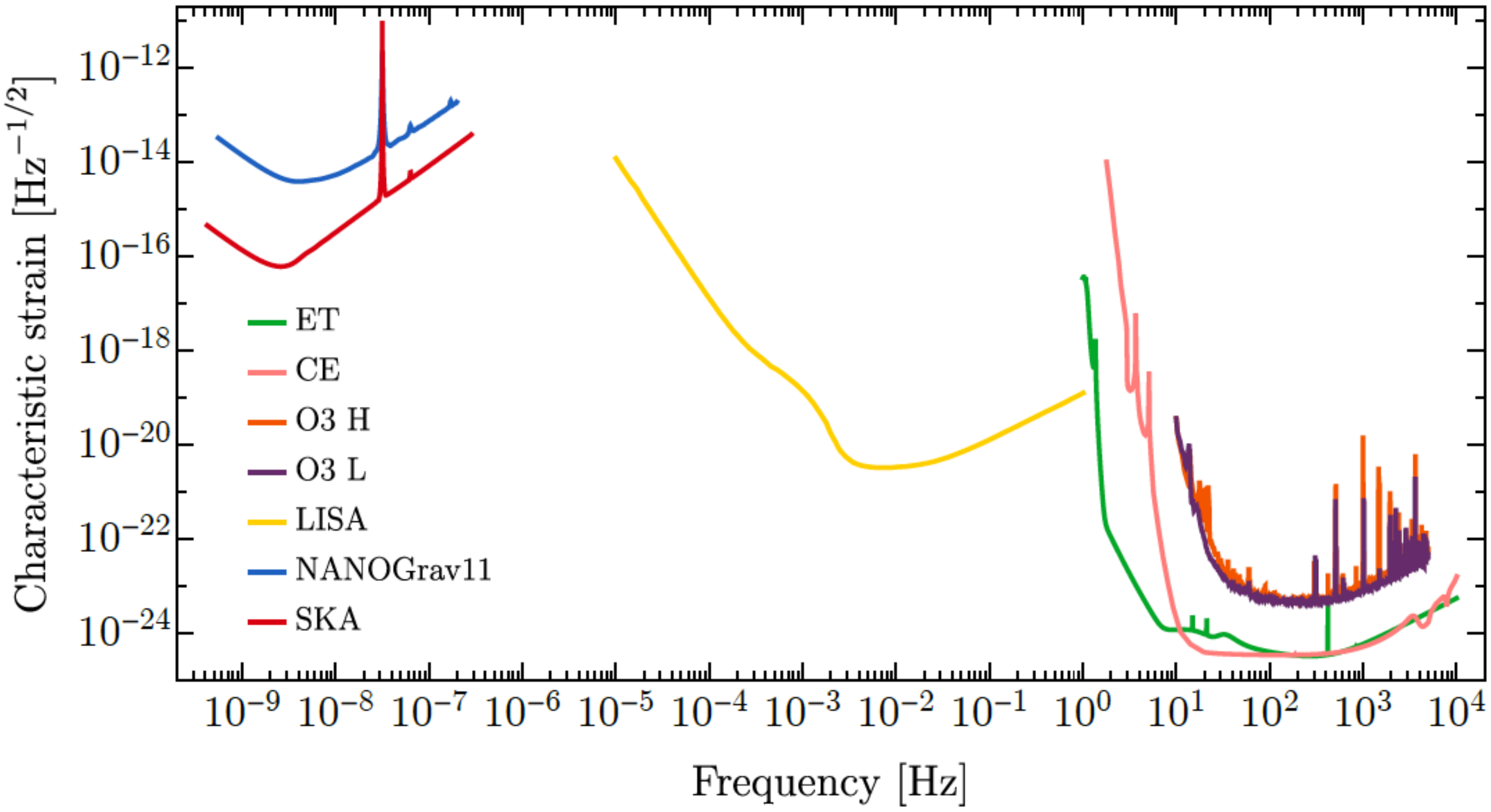
Nature 24/4/2019

The Era of Gravitational Wave Astronomy

Mass



Current and future sensitivities



Black Holes

- Astrophysical BHs forms from the gravitational collapse of a star. We know they exist. Their mass must be above the Chandrasekhar limit,

$$M > \mathcal{O}(1)\, M_\odot$$

- PBHs are formed in the early universe. Their mass can be small and they can still be around as long as they do not evaporate within the age of the universe

$$M > 10^{-18}\, M_\odot$$

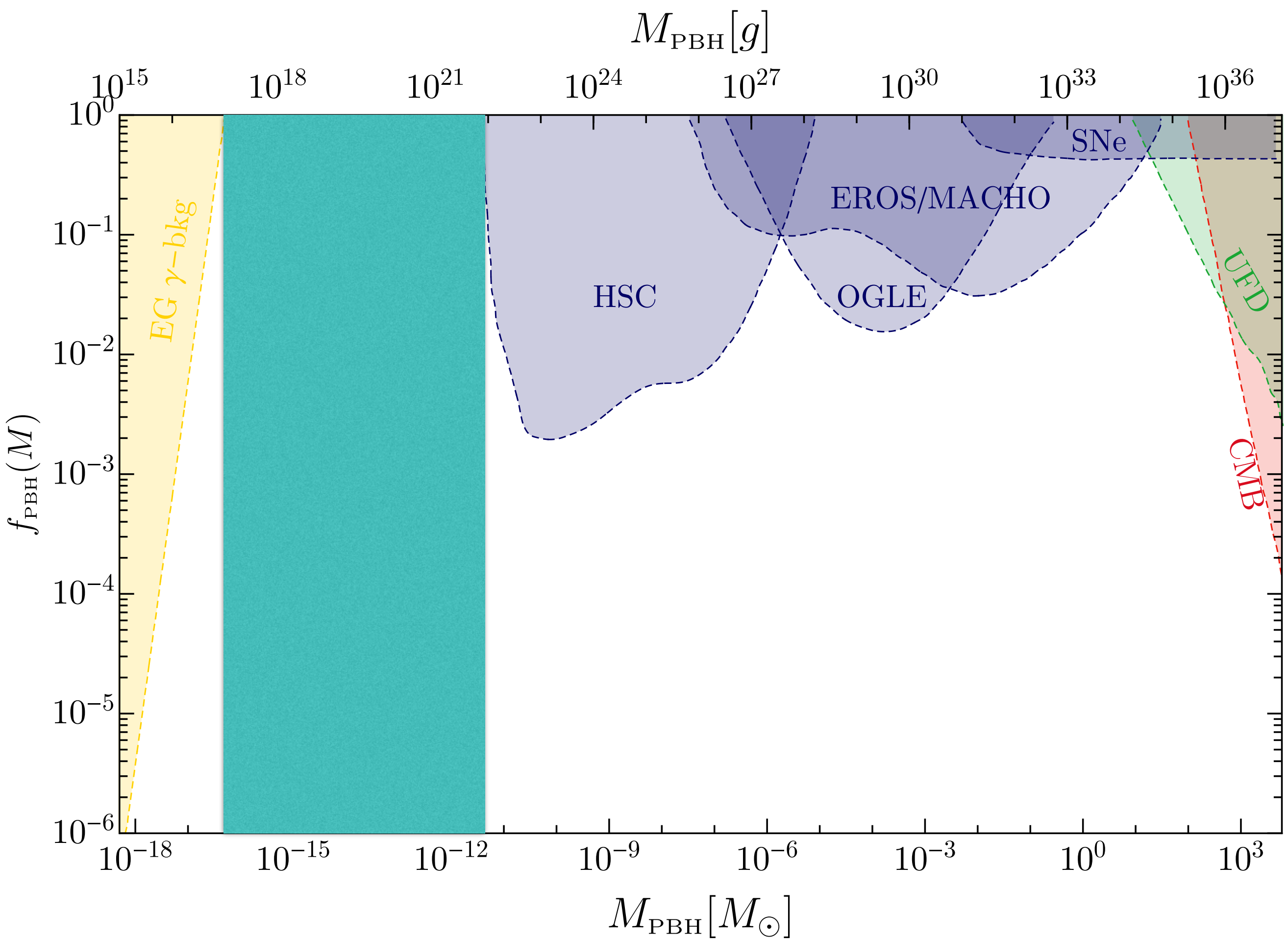
Key Questions on PBHs in the GW era

- Do (will) PBHs contribute to current (future) GW signals?
- What are the smoking-gun signals of PBHs and how to distinguish them from astrophysical sources?
- Can PBHs account for all the dark matter in the universe?

PBHs

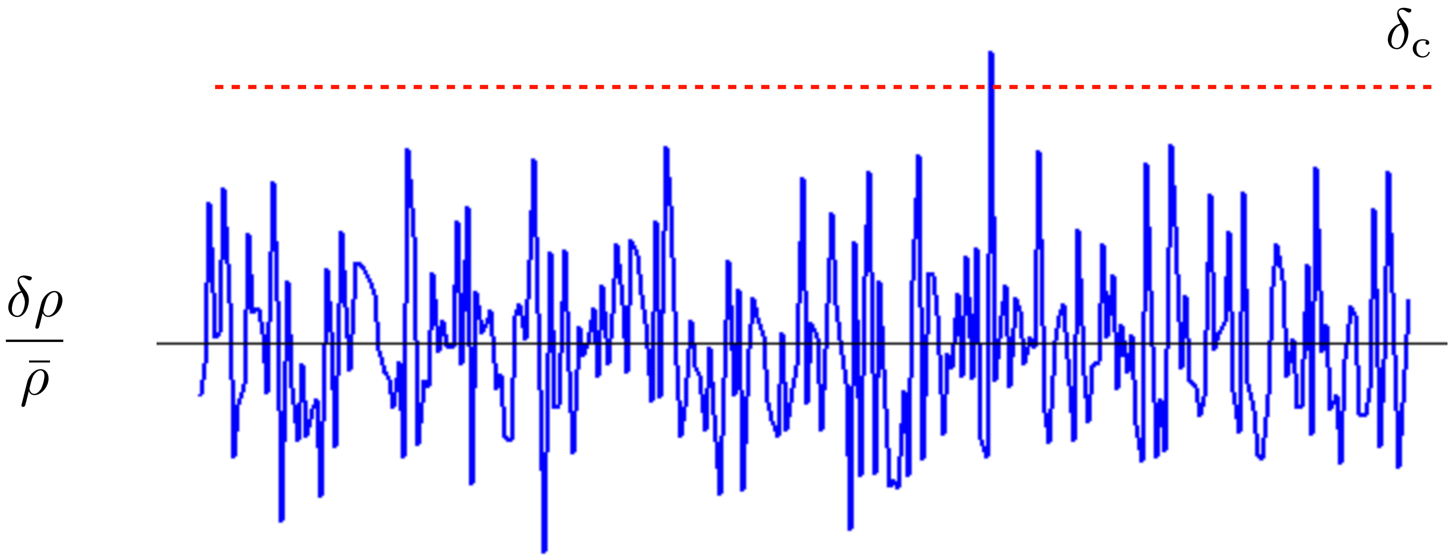
Primordial black holes can compose all the dark matter (or a fraction of it)

$$f_{\rm PBH} = \Omega_{\rm PBH}/\Omega_{\rm DM}$$



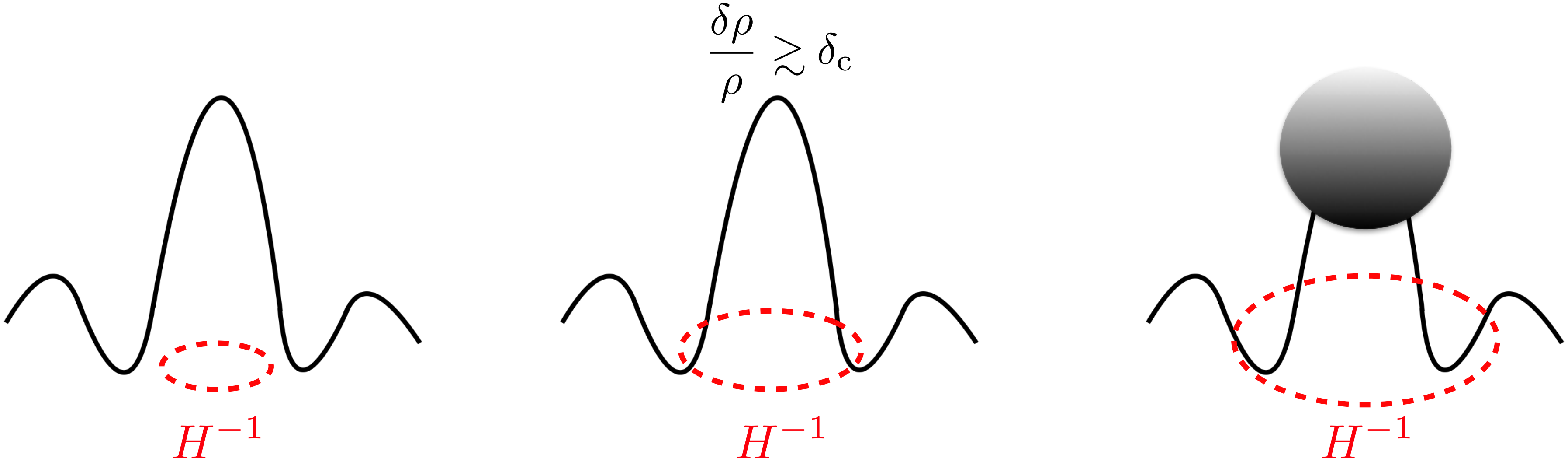
Where the PBHs may come from?

PBHs may be originated from peaks of the density perturbations generated in the early universe



Where the PBHs may come from?

PBHs may be originated from peaks of the density perturbations generated in the early universe

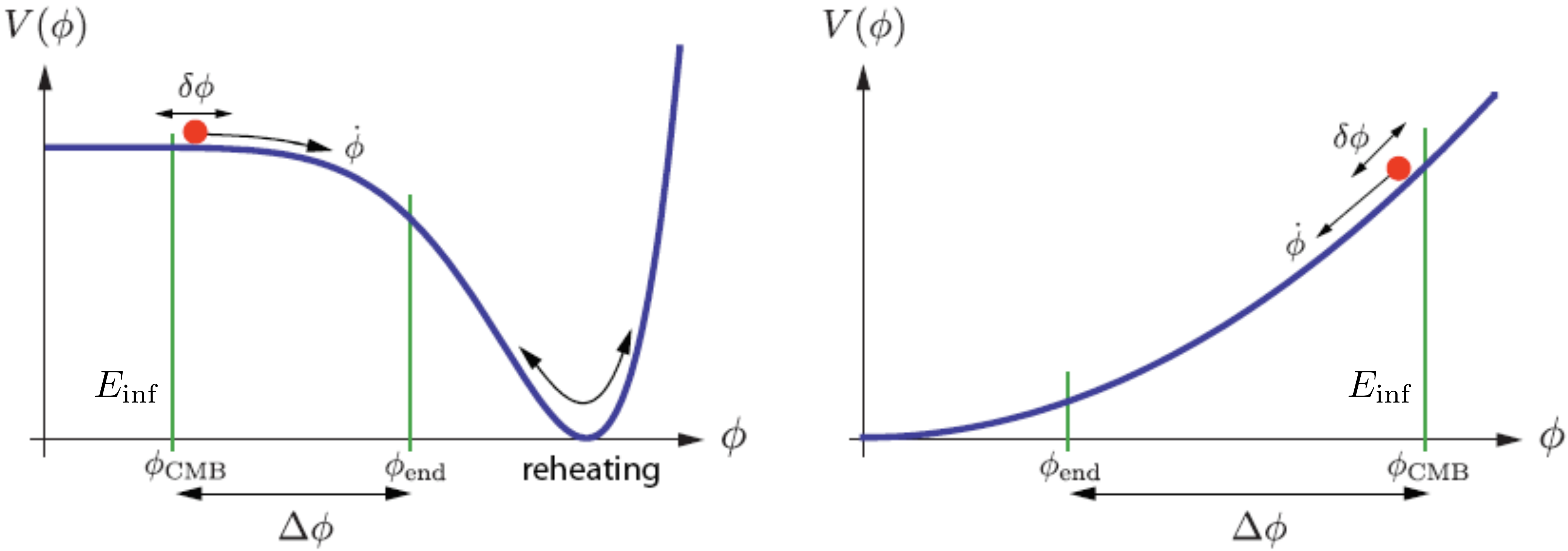


$$M_{\rm PBH} \sim M_{\rm H}$$

PBHs are rare events, tail of the distribution

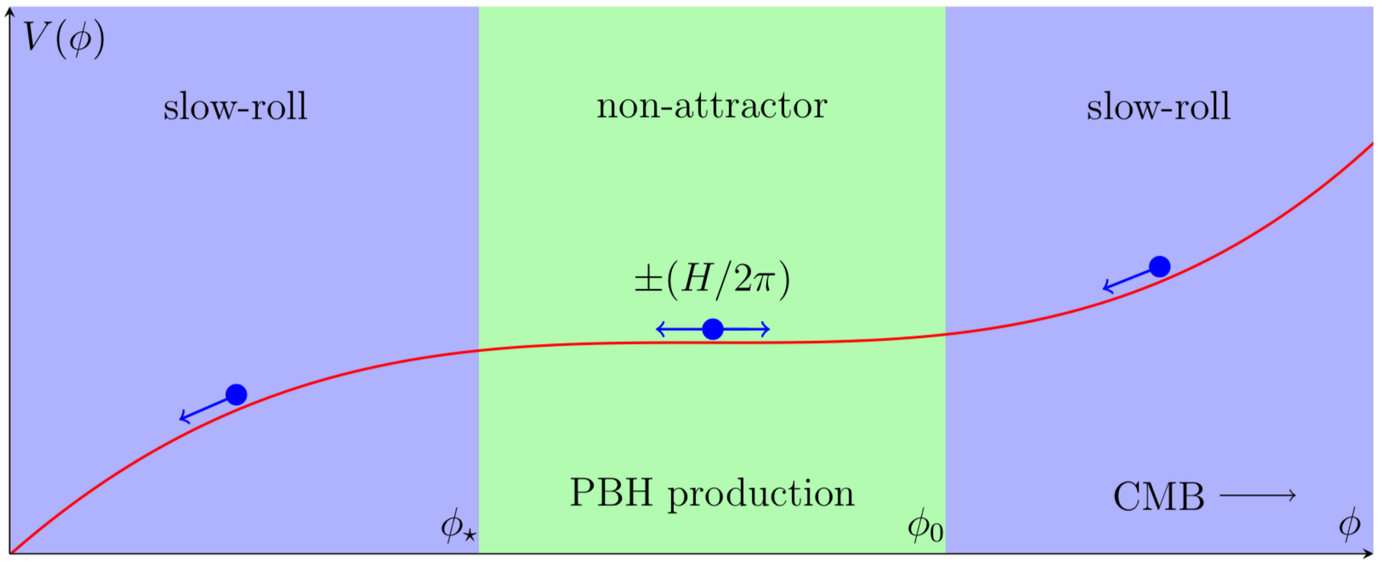
One possible mechanism: large fluctuations from inflation

Inflation



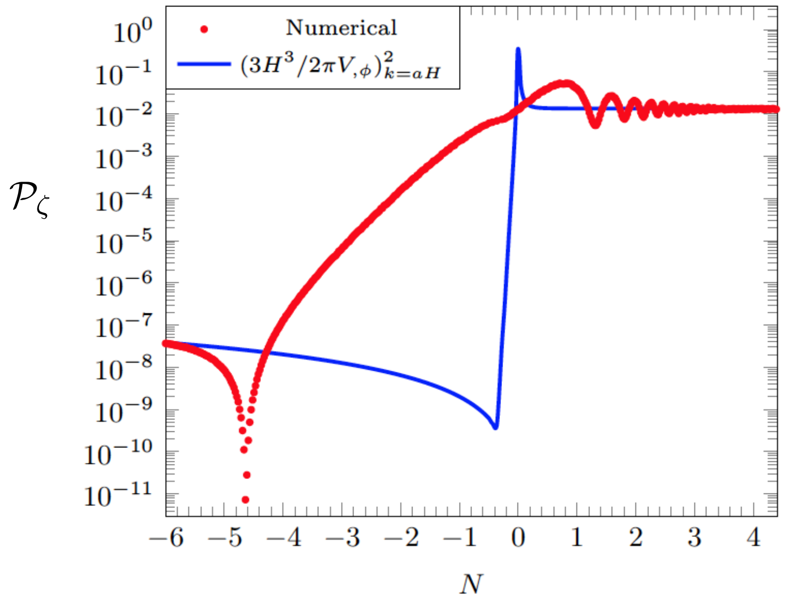
$$\left(\frac{\dot a}{a}\right)^2 = H^2 = \frac{\rho}{3m_{\rm P}^2} = \frac{E_{\rm inf}^4}{3m_{\rm P}^2} \Rightarrow a(t) \sim e^{Ht}$$

Ultra-slow-roll during inflation

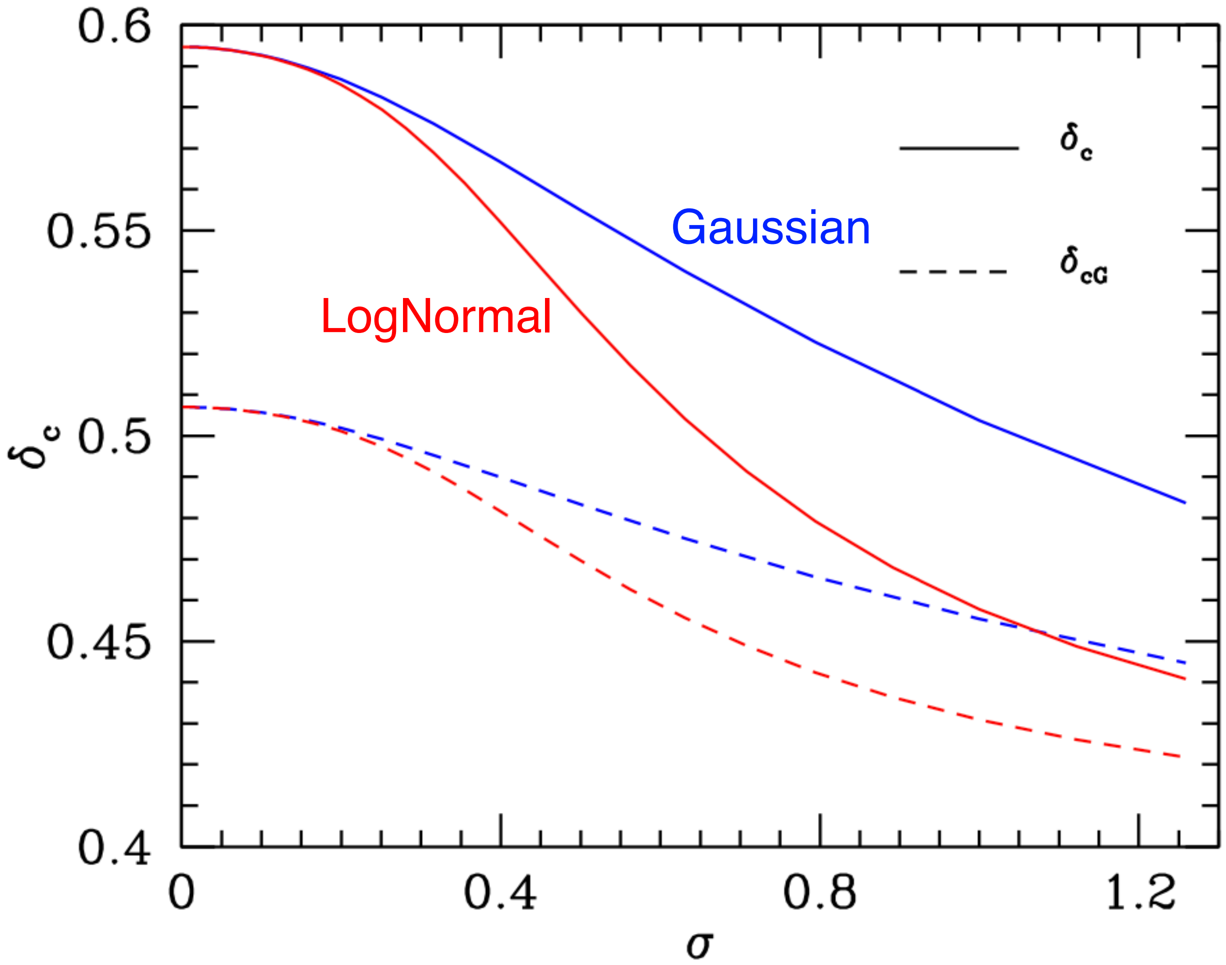


$$\mathcal{P}_\zeta^{1/2} = \frac{H^2}{2\pi|\dot{\phi}|}$$

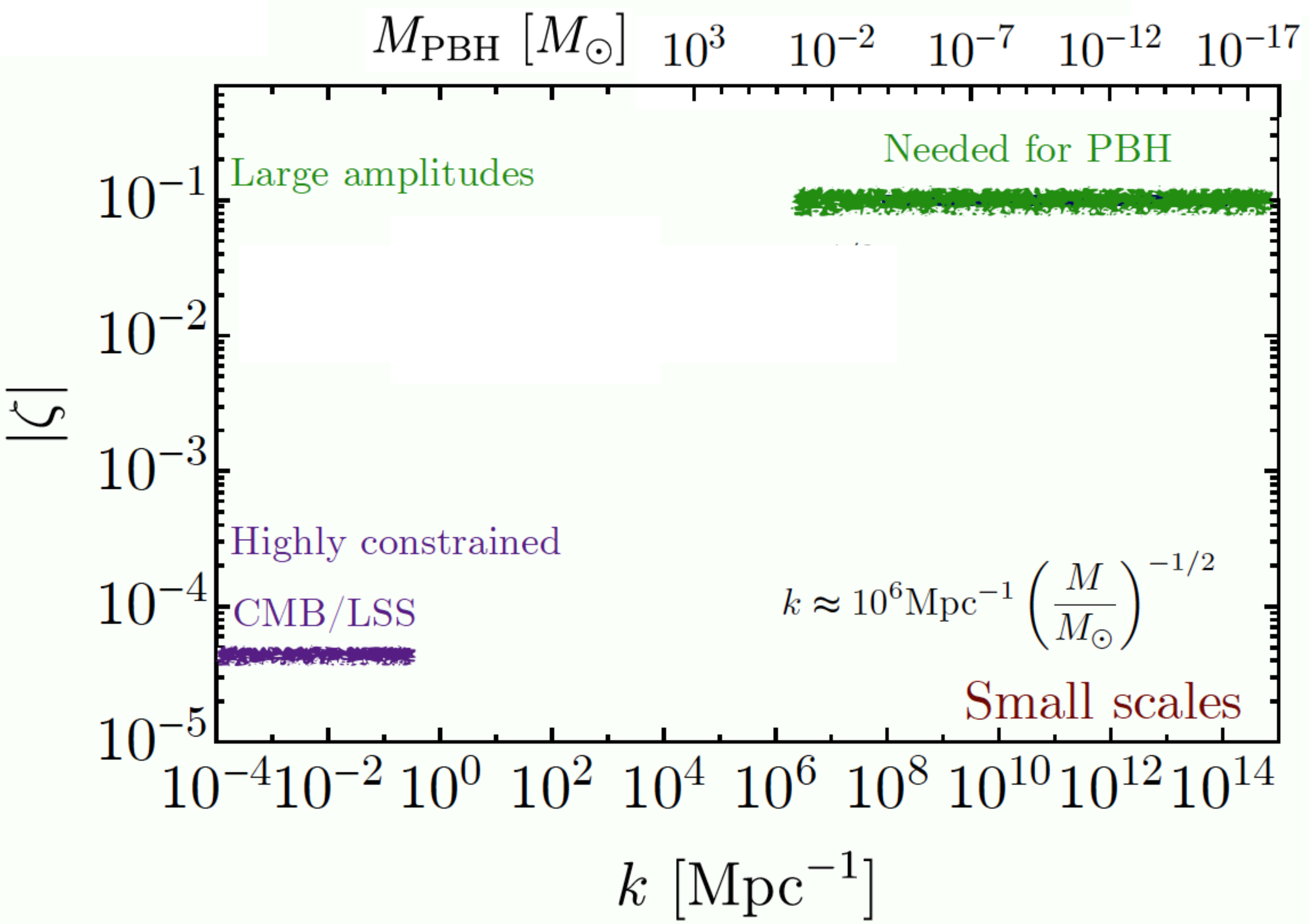
$$\frac{\mathrm{d}\phi}{\mathrm{d}N} \sim e^{-3N} \Rightarrow \mathcal{P}_\zeta^{1/2} \sim e^{3N}$$



The PBH threshold is NOT a universal number,
it does depend on the shape of the curvature perturbation



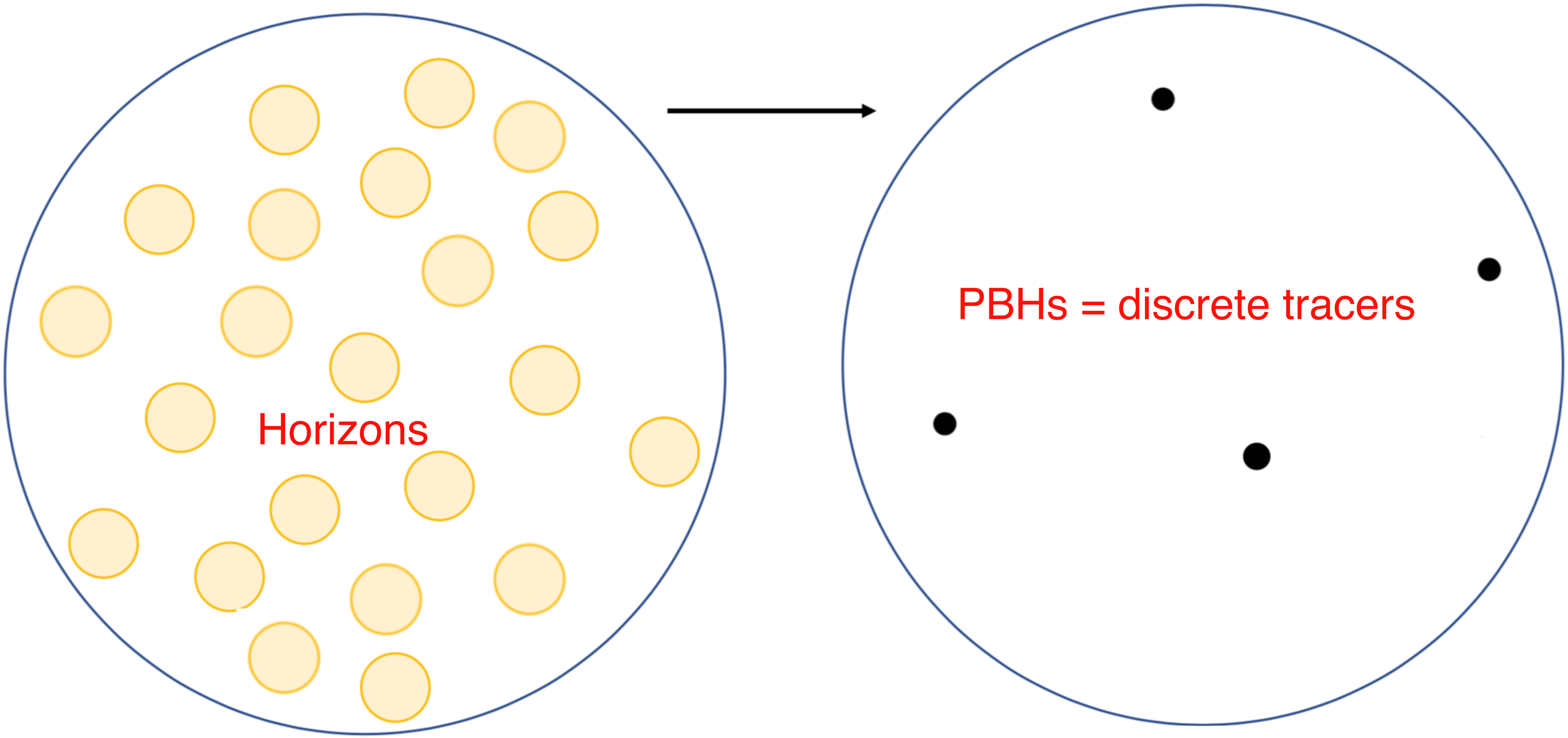
Musco et al. (2021)



$M_{\mathrm{PBH}}\ [M_\odot]$
10^{3}
10^{-2}
10^{-7}
10^{-12}
10^{-17}
Needed for PBH
Large amplitudes
Highly constrained
CMB/LSS
$k \approx 10^{6}\mathrm{Mpc}^{-1}\left(\frac{M}{M_\odot}\right)^{-1/2}$
Small scales
$|\zeta|$
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-4} 10^{-2} 10^{0} 10^{2} 10^{4} 10^{6} 10^{8} 10^{10} 10^{12} 10^{14}
$k\ [\mathrm{Mpc}^{-1}]$

Properties of PBHs at formation

PBHs are not clustered at formation in the absence of small-large scale correlation

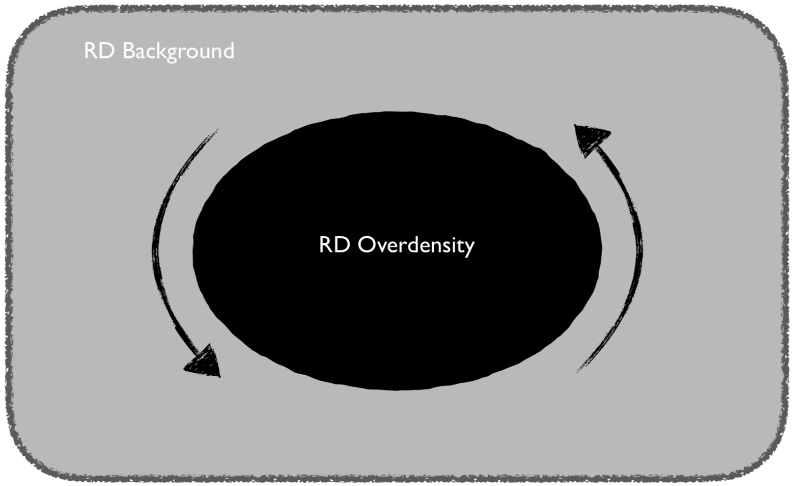


$$\left\langle \frac{\delta\rho_{\rm PBH}(\vec{x},z)}{\bar{\rho}_{\rm DM}} \frac{\delta\rho_{\rm PBH}(0,z)}{\bar{\rho}_{\rm DM}} \right\rangle = \frac{f_{\rm PBH}^2}{n_{\rm PBH}} \delta_{\rm D}(\vec{x}) + \xi(x,z)$$

with Desjaques (2018)

The spin of PBHs at formation is small

- PBHs originate from peaks, that is from *maxima* of the local density contrast.
- The spin results from the action of the torques generated by the gravitational tidal forces upon horizon crossing



$$\vec{\chi} = \vec{S}/G_N M_{\rm PBH}^2$$

$$\chi_{\rm i} \sim 10^{-2}\sqrt{1-\gamma^2}$$

Shape of the density power spectrum

De Luca et al. (2018)

The PBH mass function at formation

Mass distribution dependent on the overdensity perturbation spectrum and statistical properties

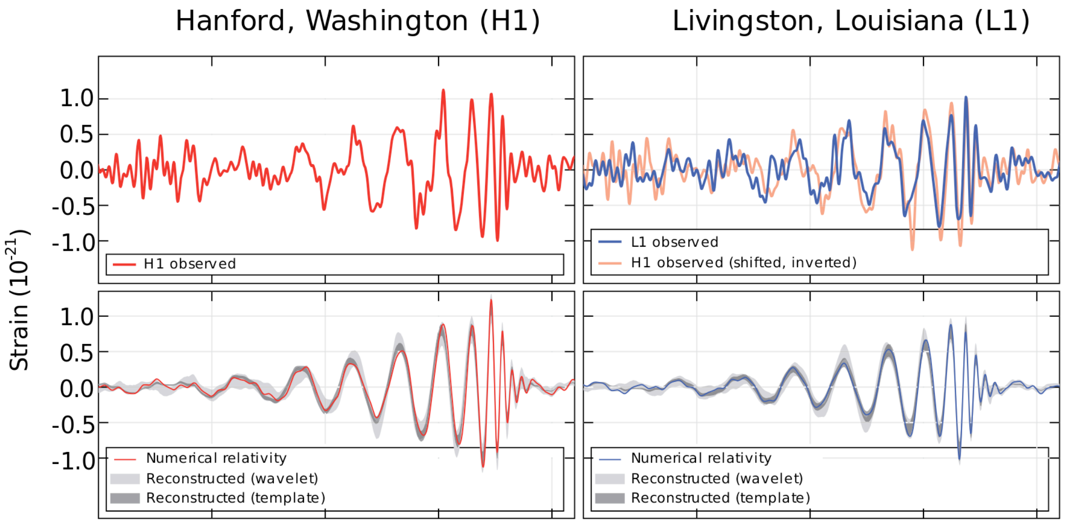
Standard parametrisation

$$\psi(M_{\rm PBH}) = \frac{1}{\sqrt{2\pi}M_{\rm PBH}} \exp\left(-\frac{\ln^2(M_{\rm PBH}/M_c)}{2\sigma^2}\right)$$

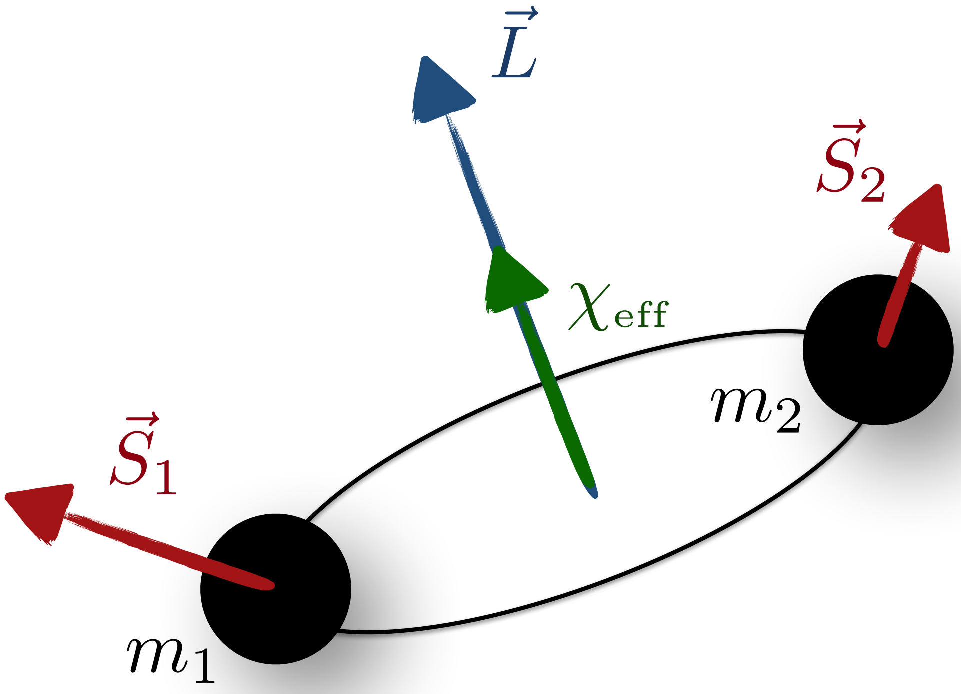
Key Questions on PBHs in the GW era

- Do (will) PBHs contribute to current (future) GW signals?
- What are the smoking-gun evidences for PBHs and how to distinguish them from astrophysical sources?
- Can PBHs account for all the dark matter in the universe?

BH binary



GW150914, LIGO (2016)



Waveforms dependent on the binary event parameters

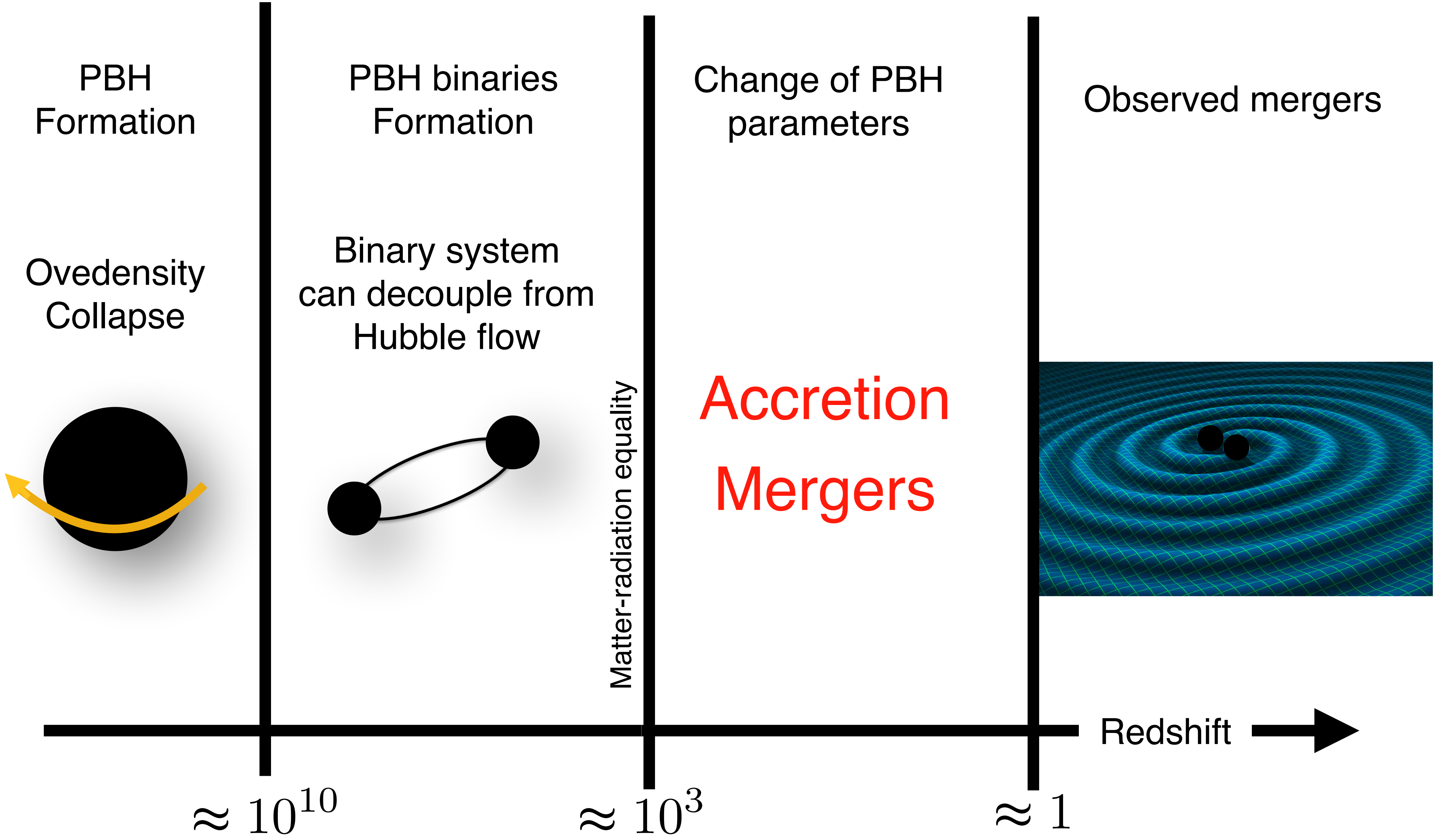
$$\mathcal{M} = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}}$$

$$q = m_2/m_1$$

$$\chi_{\rm eff} = \frac{\vec{S}_1/m_1 + \vec{S}_2/m_2}{m_1 + m_2} \cdot \hat{L}$$

$$\ldots$$

PBH evolution



Accretion onto isolated PBHs

For $f_{\rm PBH} < 1$ PBHs coexist with another DM component in the universe

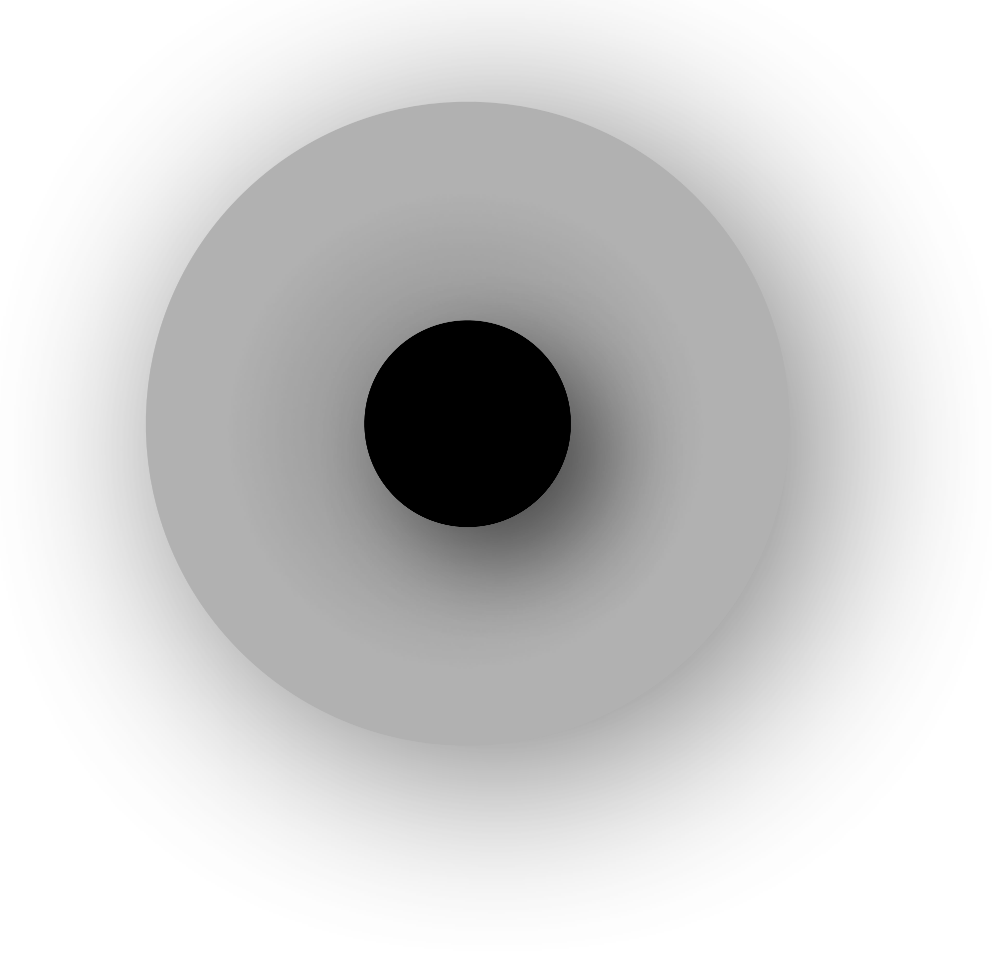
A DM halo builds up around the PBHs enhancing accretion

(larger gravitational potential well)

$$M_h(z) \approx 3 M_{\rm PBH} \left(\frac{1000}{1+z} \right)$$

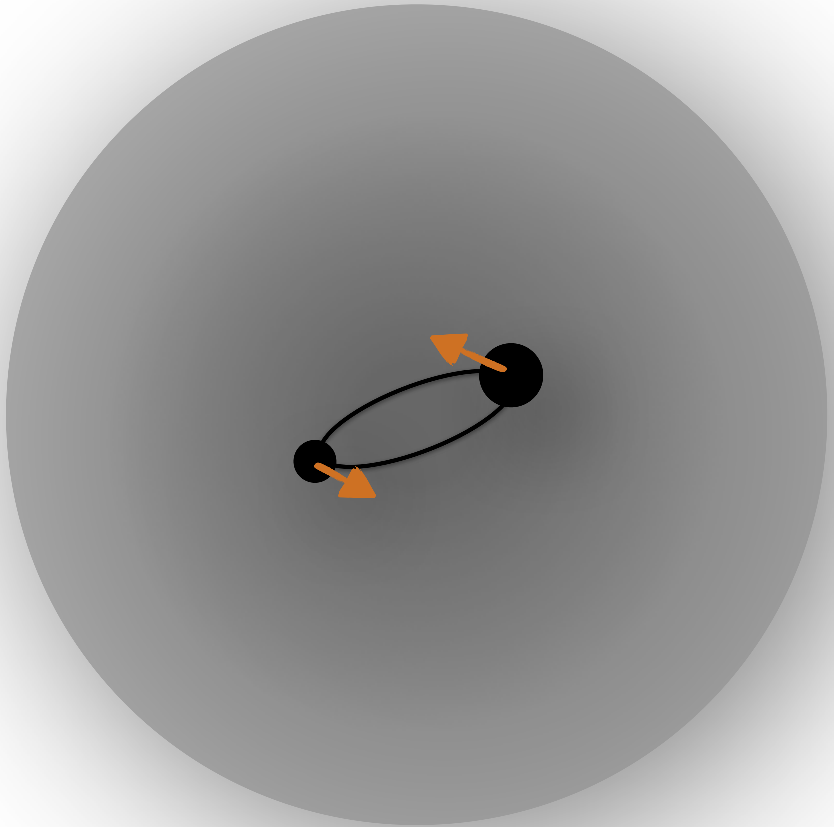
Bondi-Hoyle accretion from the surrounding baryonic fluid

$$\dot{M} = 4\pi \lambda m_H n_{\rm gas} v_{\rm eff}^{-3} M^2$$



Ricotti et al. (2005)

Accretion onto PBH binaries



Accretion on the system enhances the gas density around the PBH binary

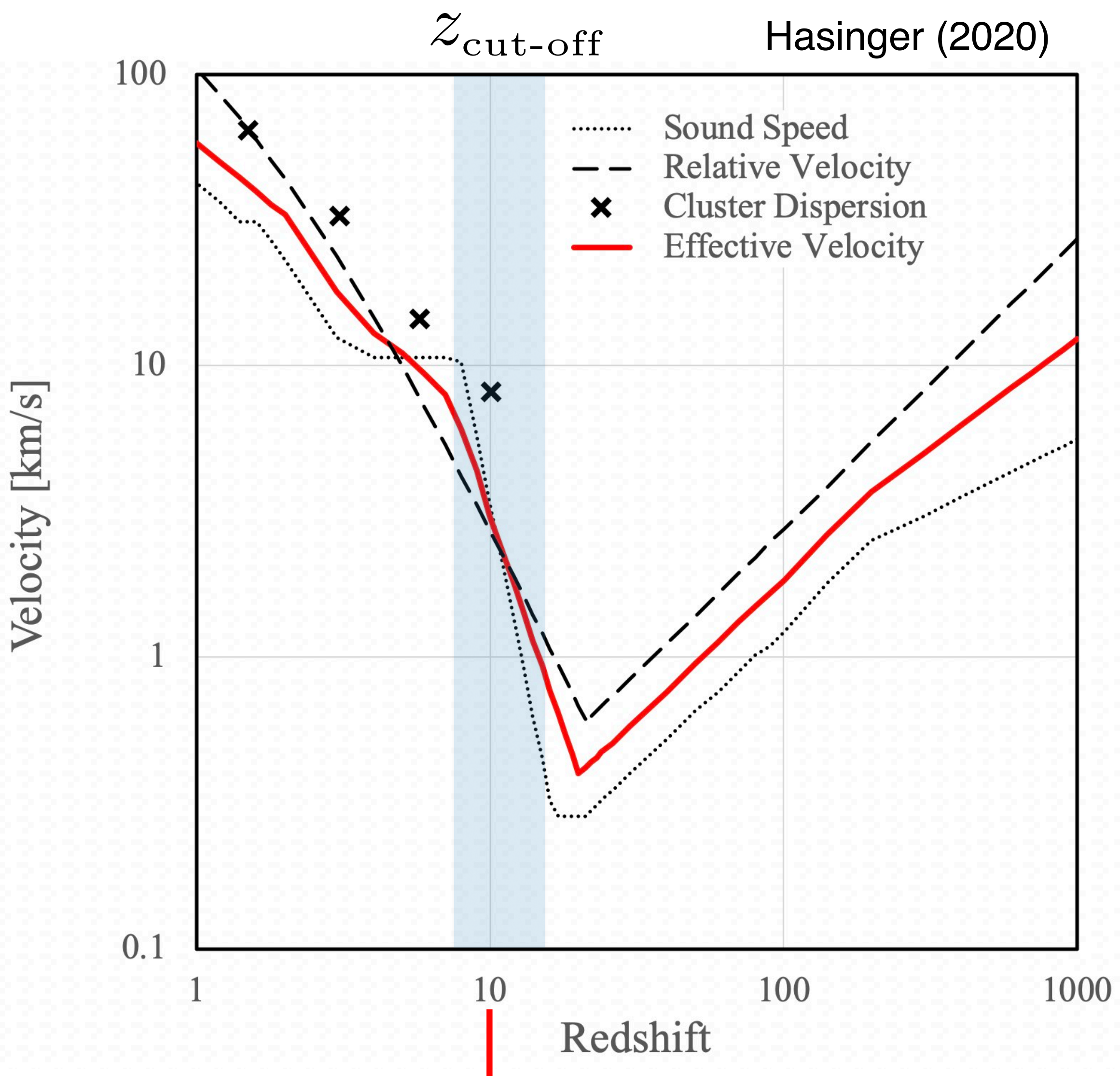
Accretion on the sigle PBH modulated by masses and orbital velocities

$$\dot{M}_1 = \dot{M} \frac{1}{\sqrt{2(1+q)}}$$

$$\dot{M}_2 = \dot{M} \sqrt{\frac{q}{2(1+q)}}$$

$q = 1$ fixed point

- The smaller PBH always experiences a larger relative accretion
- PBH can experience accretion for $M \gtrsim \mathcal{O}(10) M_\odot$



Structure formation
reionization epoch

- Virialised velocities
- Higher temperatures

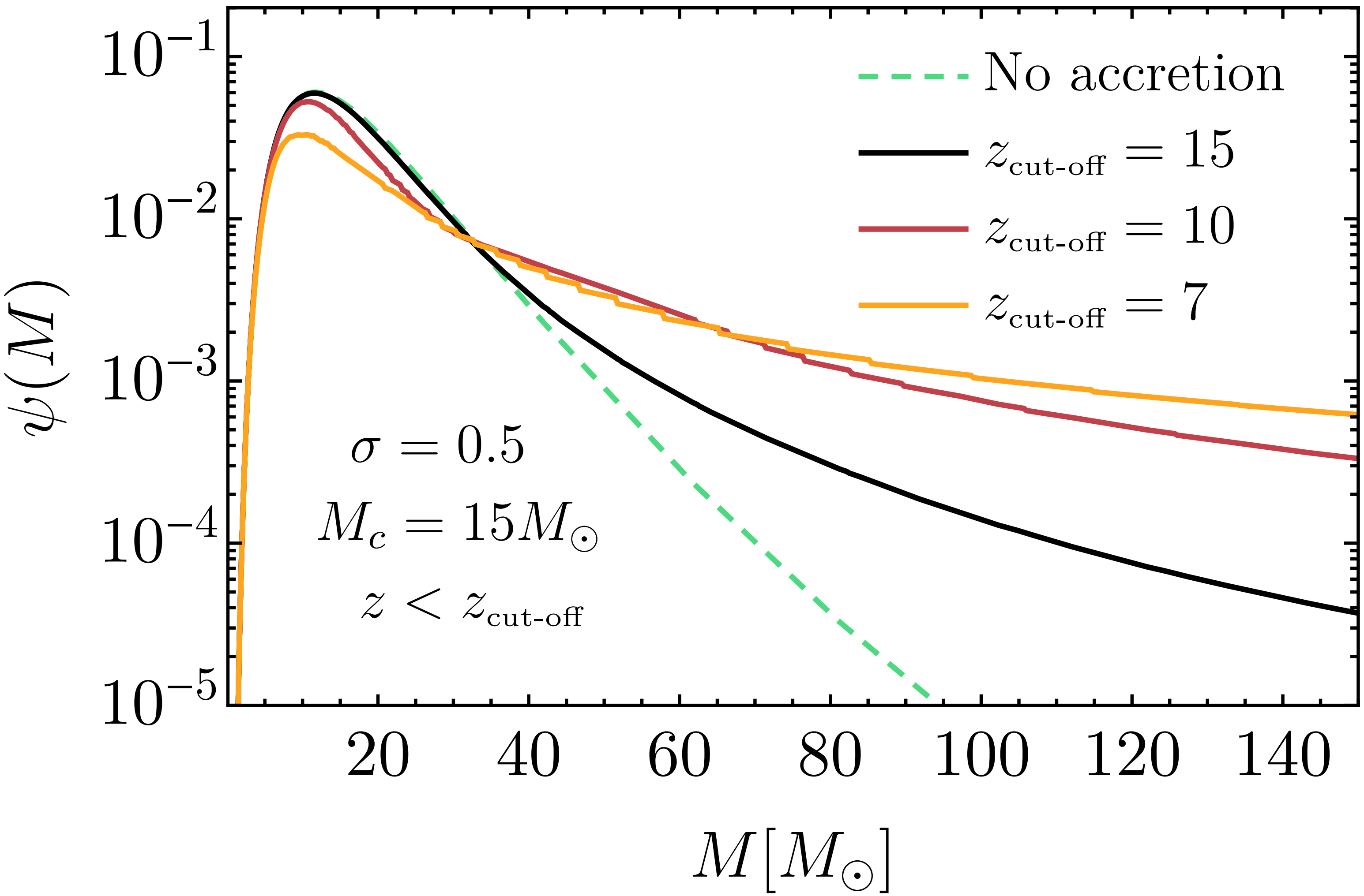
$$\dot{M} \approx \left(v_{\text{rel}}^2 + c_s^2\right)^{-3/2}$$

Strong suppression around

$z_{\text{cut-off}}$

Uncertainties
in the accretion model
accounted for by
varying the cut-off

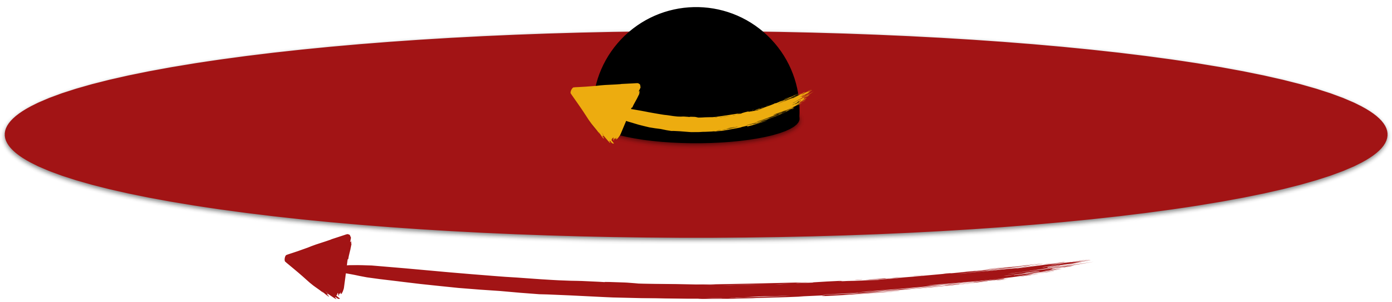
PBH mass function evolution



Non-linear mass evolution enhances large-mass tails

PBH spin evolution

If matter angular momentum is
large enough, an accreting disk forms,
leading to a spin growth



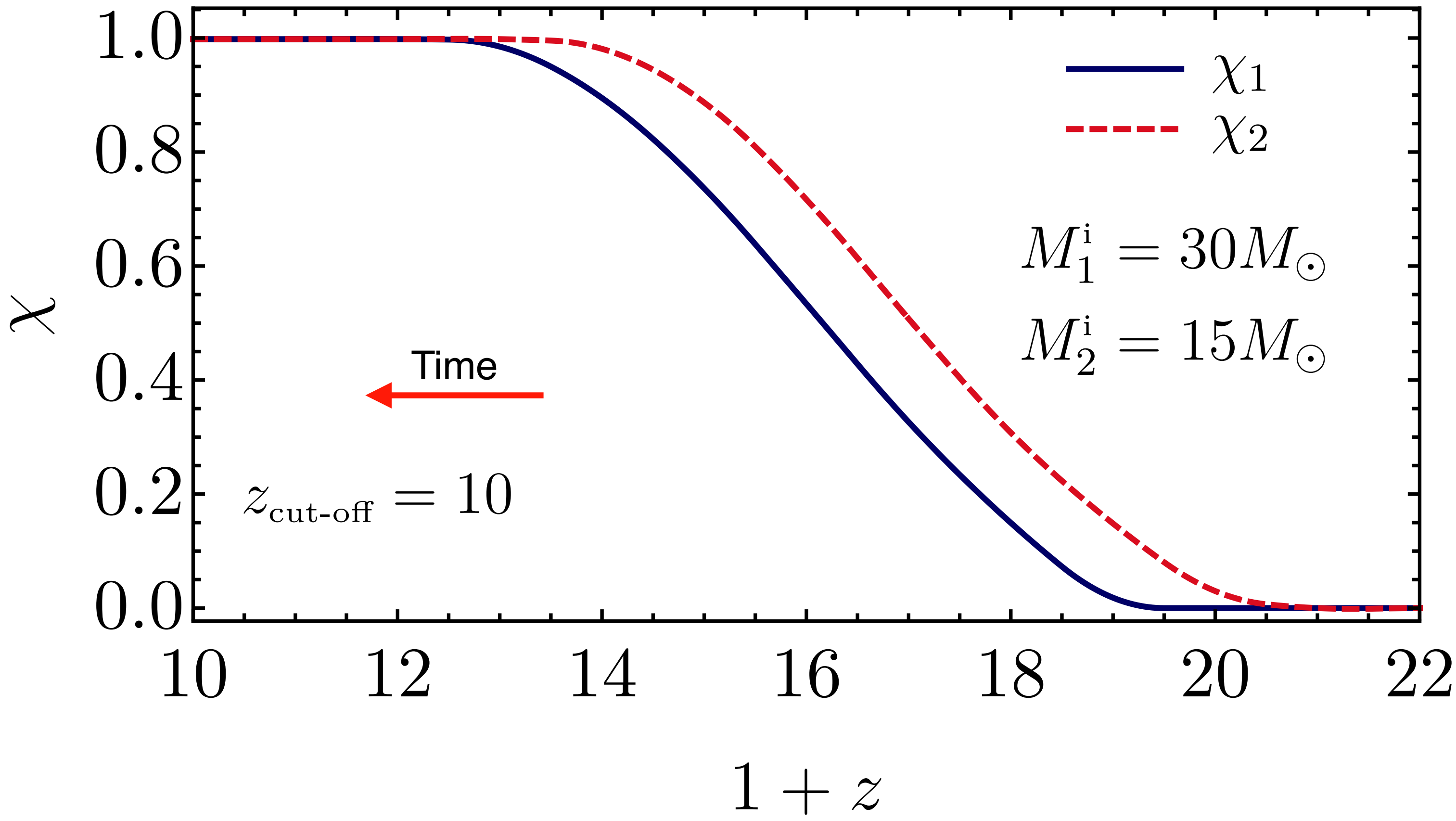
Angular momentum transfer
between gas and PBH

$$\dot{\chi} = g(\chi)\frac{\dot{M}}{M}$$

by solving the geodesic model of disk accretion

Bardeen et al. (1972)

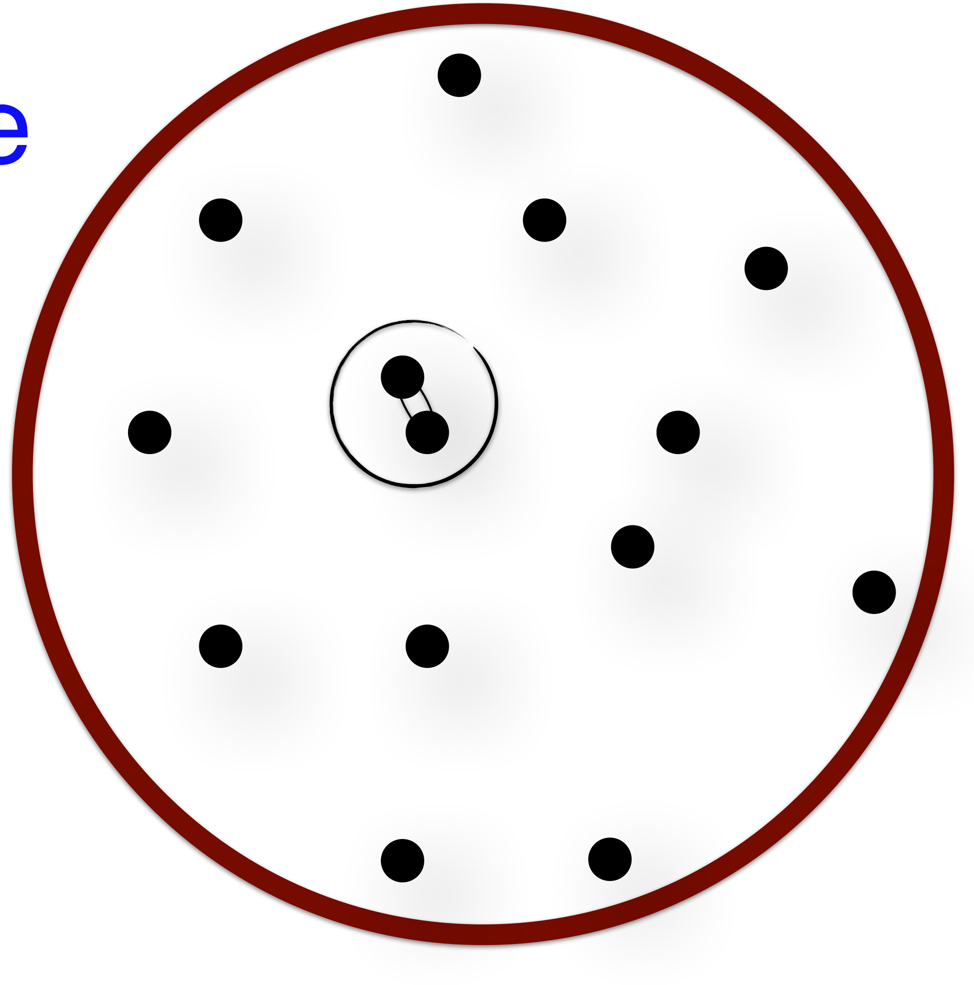
Spins pushed towards extremality



- Uncorrelated spin orientation
- Effective spin spreads around zero
- Accretion: low/large mass - low/large spin correlation

De Luca et al. (2020)

Merger rate



- Initial spatial Poisson distribution
- Random decoupling of binary systems

Compute probability of decoupling and the binary initial geometry

- Semi-major axis
- Eccentricity

Raidal et al (2018)

$$\frac{\mathrm{d}R}{\mathrm{d}m_1\mathrm{d}m_2} = \frac{1.6\times 10^6}{\mathrm{Gpc}^3\,\mathrm{yr}} f_{\mathrm{PBH}}^{\frac{53}{37}}\eta^{-\frac{34}{37}}\left(\frac{t}{t_0}\right)^{-\frac{34}{37}}\left(\frac{M_{\mathrm{tot}}}{M_\odot}\right)^{-\frac{32}{37}} S\left(M_{\mathrm{tot}}, f_{\mathrm{PBH}}\right)\mathcal{A}_{\mathrm{acc}}(m_j)\psi(m_1)\psi(m_2)$$

- Accretion hardens the binaries
- Larger masses leads to shorter mergers

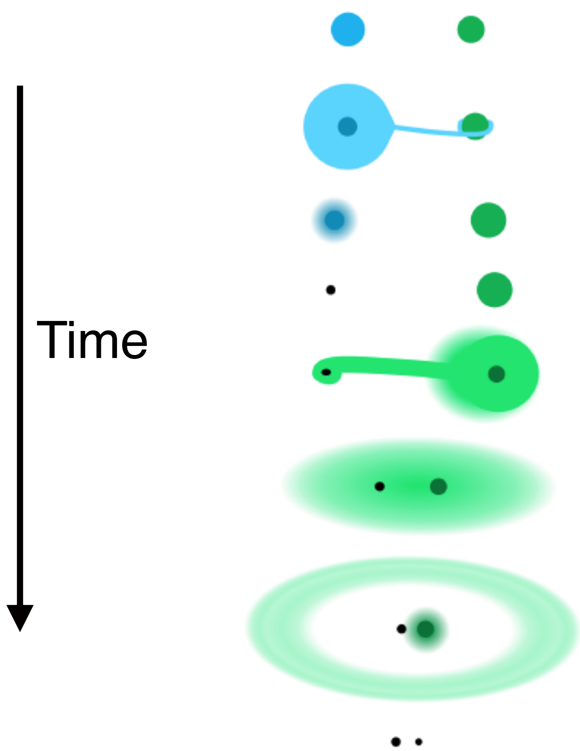
De Luca et al. (2020)

Astrophysical populations

Zevin et al. (2021)

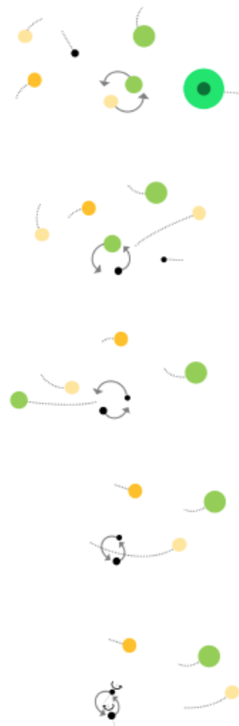
Isolated formation

Binary formation in galactic fields
through a Stable Mass Transfer (SMT)
or Common-Envelope (CE) phase



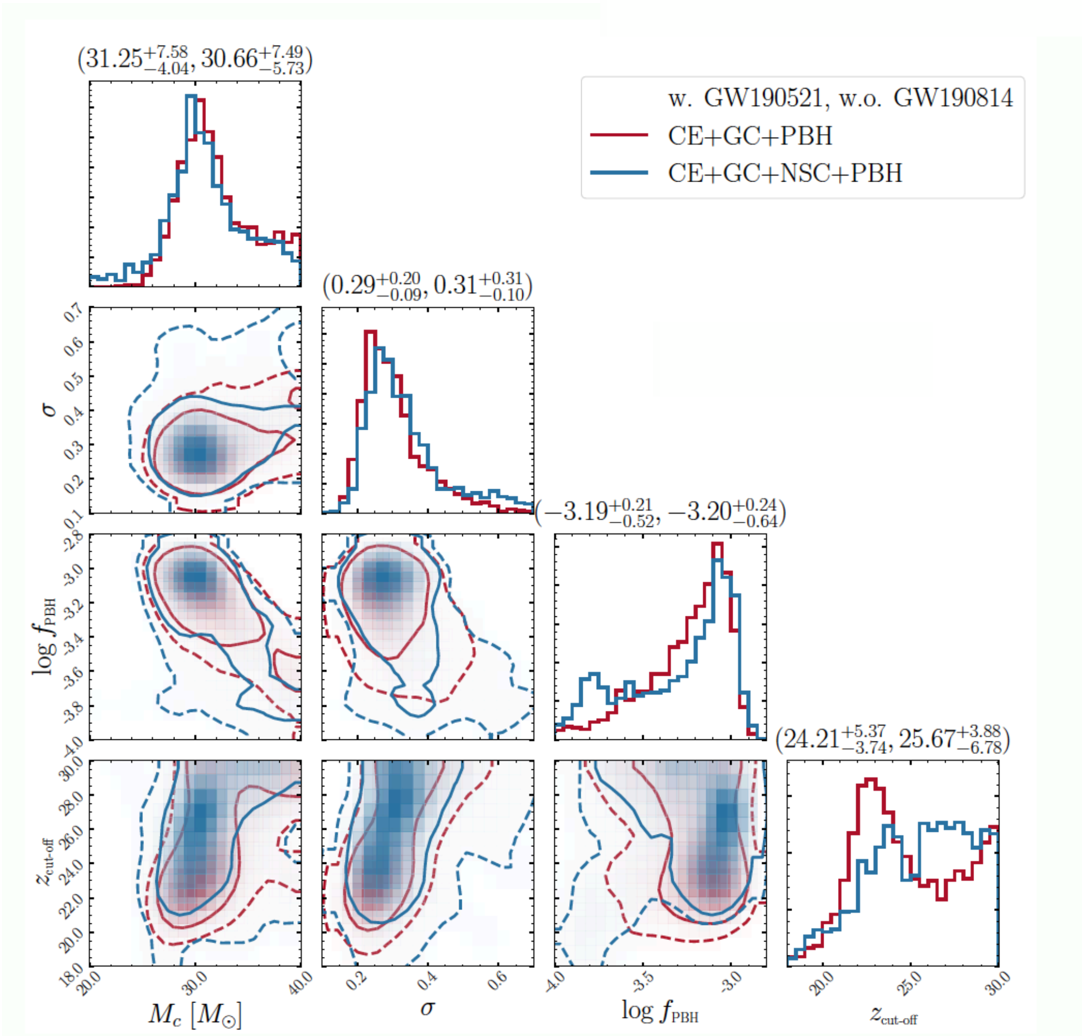
Dynamical formation

Binary formation in Globular Cluster (GC)
or Nuclear Star Clusters (NCS)
through encounters and GW captures



For a review, Mandel and Farmer (2018)

Population posterior distributions



$$M_c \simeq 30 M_\odot$$

$$\sigma \simeq 0.3$$

$$f_{\rm PBH} \simeq 6 \cdot 10^{-4}$$

$$z_{\rm cut\text{-}off} \simeq 25$$

PBH not the dark matter

Moderate accretion

Franciolini et al. (2021)

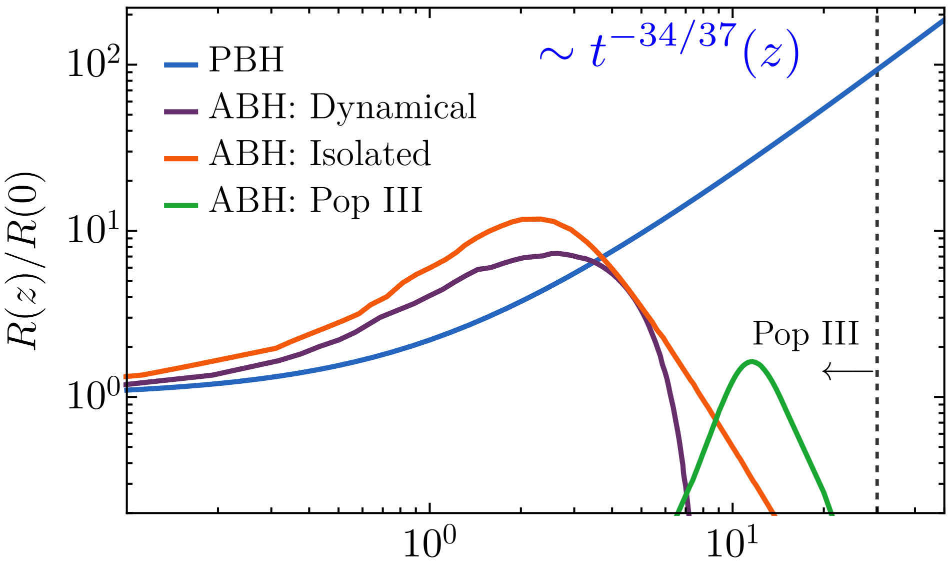
Key Questions on PBHs in the GW era

- Do (will) PBHs contribute to current (future) GW signals?
- What are the smoking-gun signals of PBHs and how to distinguish them from astrophysical sources?
- Can PBHs account for all the dark matter in the universe?

Smoking-gun signals of PBHs

- Merger rate time evolution at high redshifts
- Spin of PBHs (large spins for large masses)
- Stochastic GW background from PBHs at high redshifts

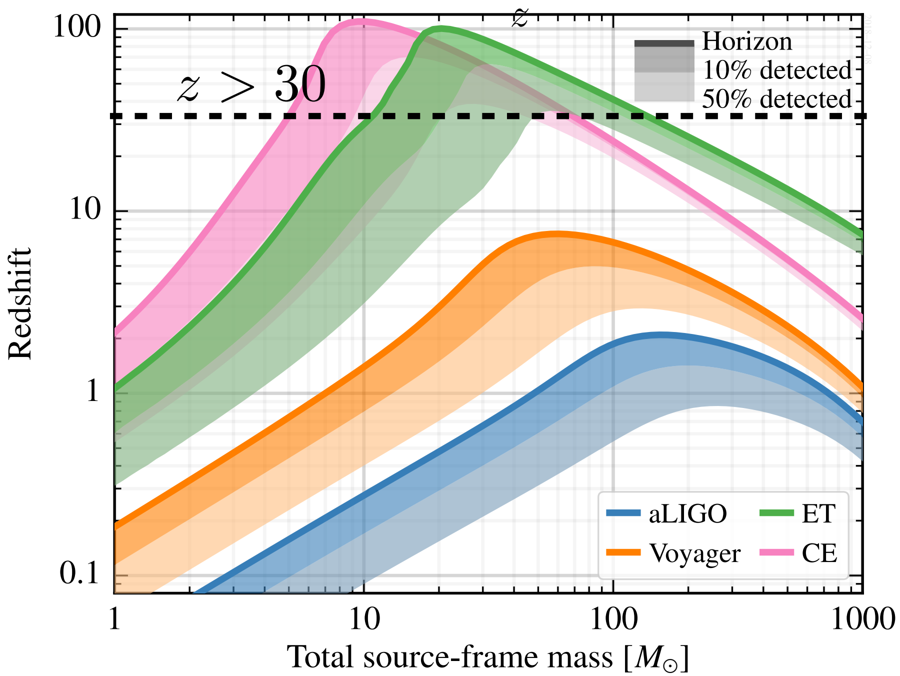
K. Ng et al. (2020)



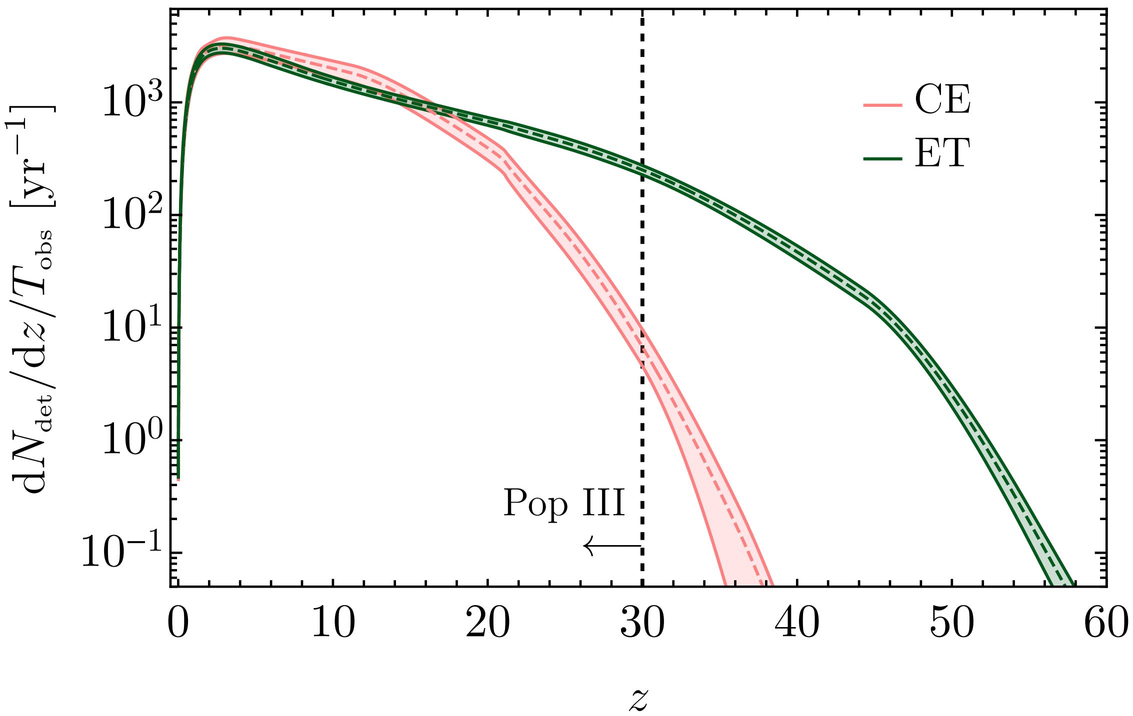
- The PBH population would imply high-redshift observations:

$$N_{\rm det}^{\rm ET}(z>30) = 1315^{+305}_{-168}\,/{\rm yr}$$

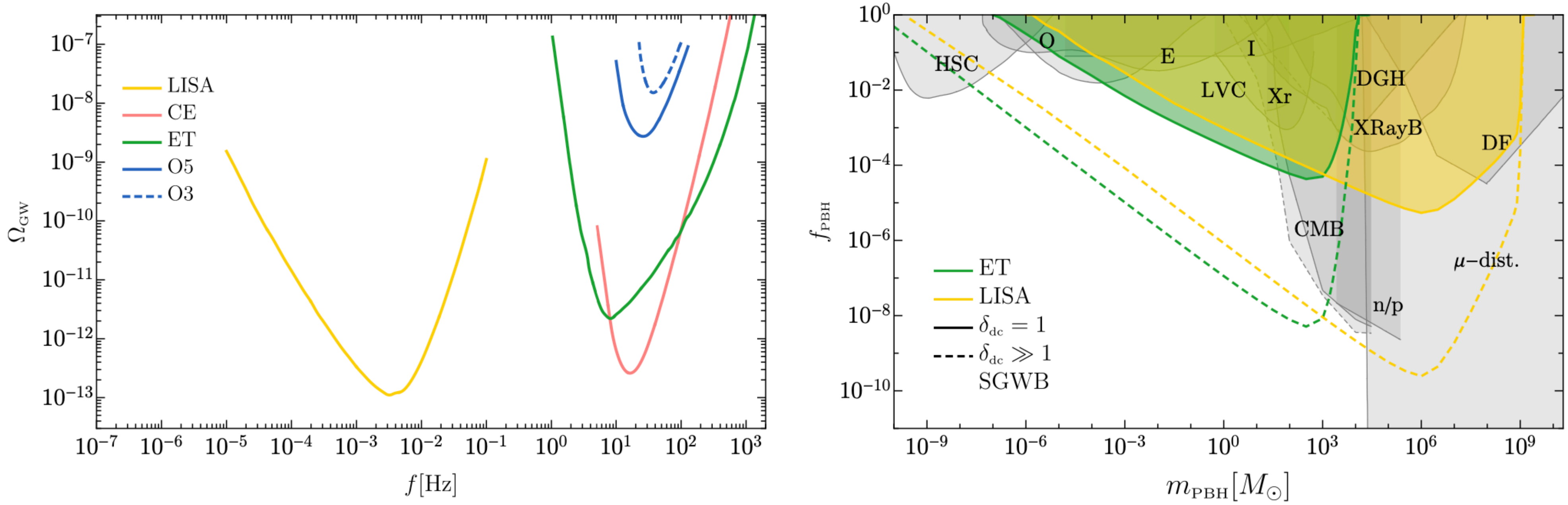
No astrophysical contamination



V. De Luca et al. (2021)

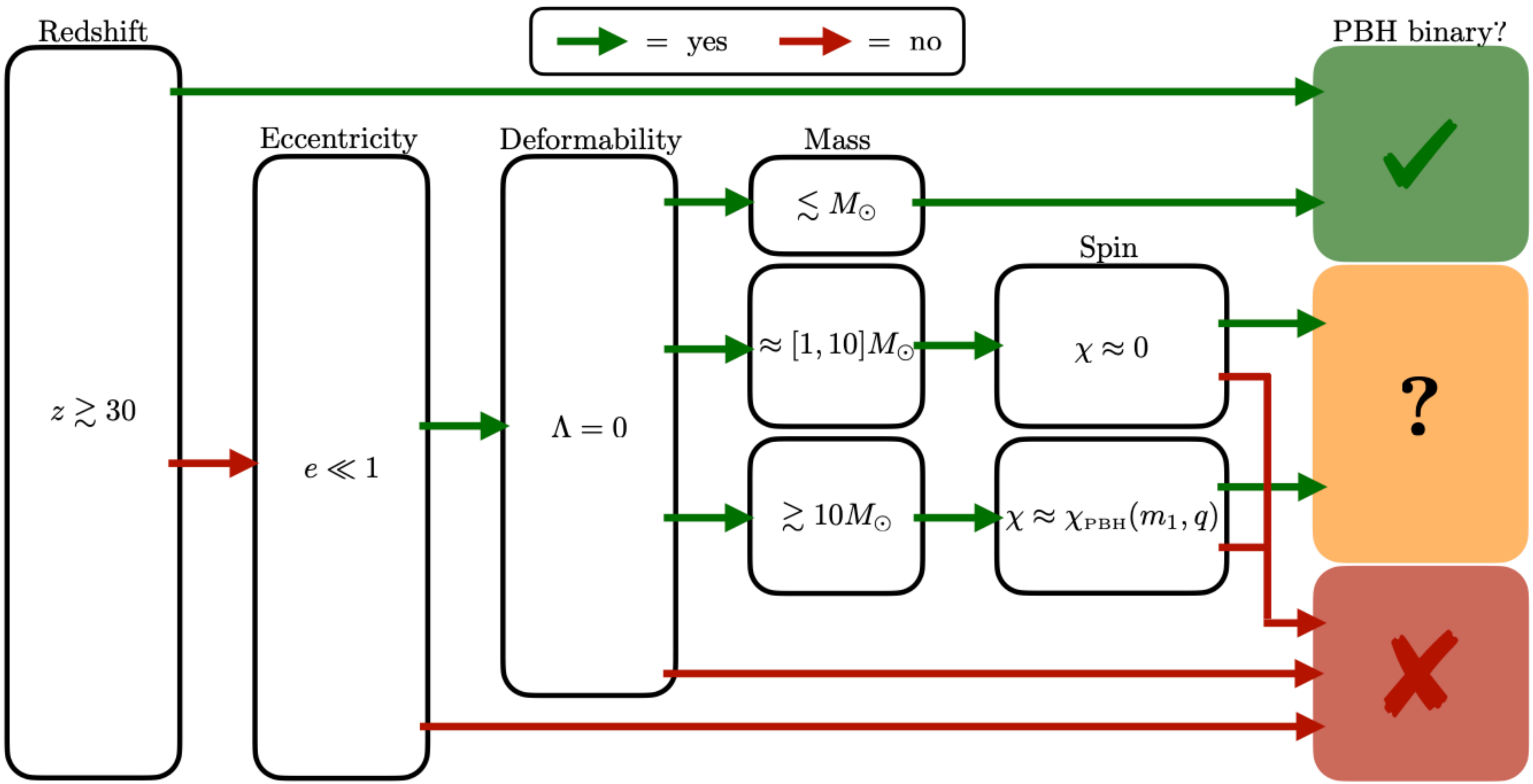


The minimum testable PBH abundance



De Luca et al. (2021)

Flowchart to assess PBHs



Franciolini et al. (2021)

Key Questions on PBHs in the GW era

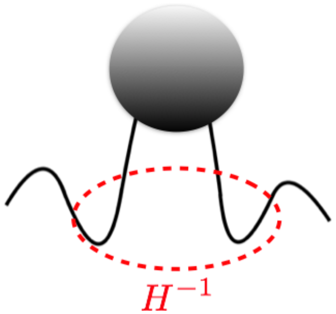
- Do (will) PBHs contribute to current (future) GW signals?
- What are the smoking-gun signals of PBHs and how to distinguish them from astrophysical sources?
- Can PBHs account for all the dark matter in the universe?

PBHs and the stochastic background of GWs

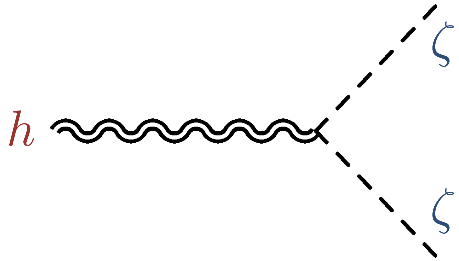
GWs from PBHs

The same curvature perturbations giving rise to PBHs are unavoidably a source for GWs at *second-order* in perturbation theory

$$\frac{\delta\rho}{\bar{\rho}} \sim \frac{\nabla^2\zeta}{a^2 H^2}$$



$$h''_{ij} + 2\mathcal{H}h'_{ij} - \nabla^2 h_{ij} = \mathcal{O}(\partial_i\zeta\partial_j\zeta)$$



Potentially observable at current and future GW observatories

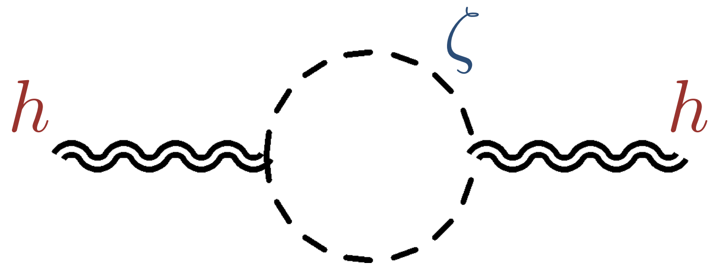
lisa

NANOGrav

GW Power Spectrum

Power spectrum of GWs:

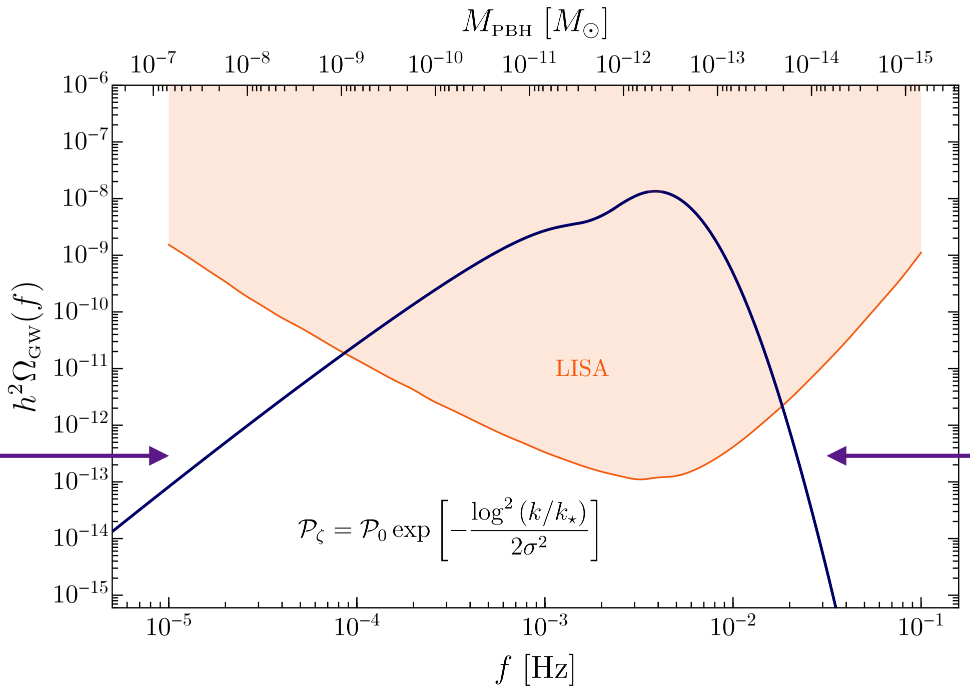
$$\left\langle h^{\lambda_1}(\eta,\vec{k}_1)h^{\lambda_2}(\eta,\vec{k}_2)\right\rangle' \approx \mathcal{P}_\zeta\mathcal{P}_\zeta$$



At second order in comoving curvature perturbation, after averaging over the fast oscillating pieces

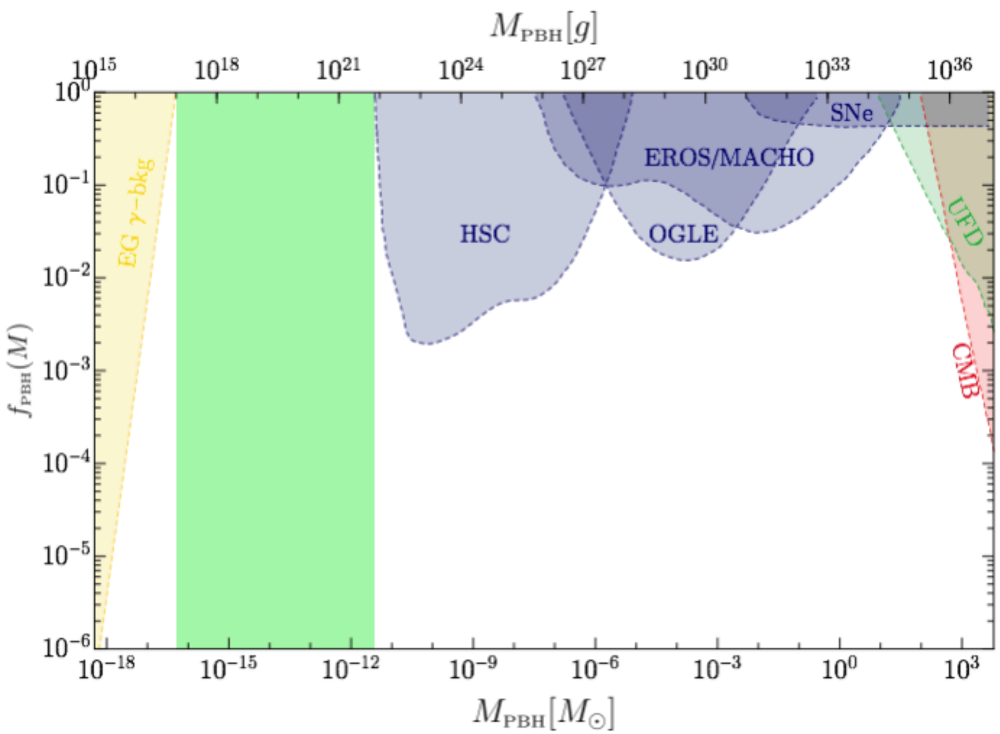
$$\Omega_{\rm GW}(\eta,k) = \frac{\pi^2}{243\mathcal{H}^2\eta^2}\int\frac{d^3p}{(2\pi)^3}\frac{p^4\left[1-\mu^2\right]^2}{p^3\,|\vec{k}-\vec{p}|^3}\,\mathcal{P}_\zeta(p)\,\mathcal{P}_\zeta\left(|\vec{k}-\vec{p}|\right)\,\mathcal{I}^2\left(\vec{k},\vec{p}\right)$$

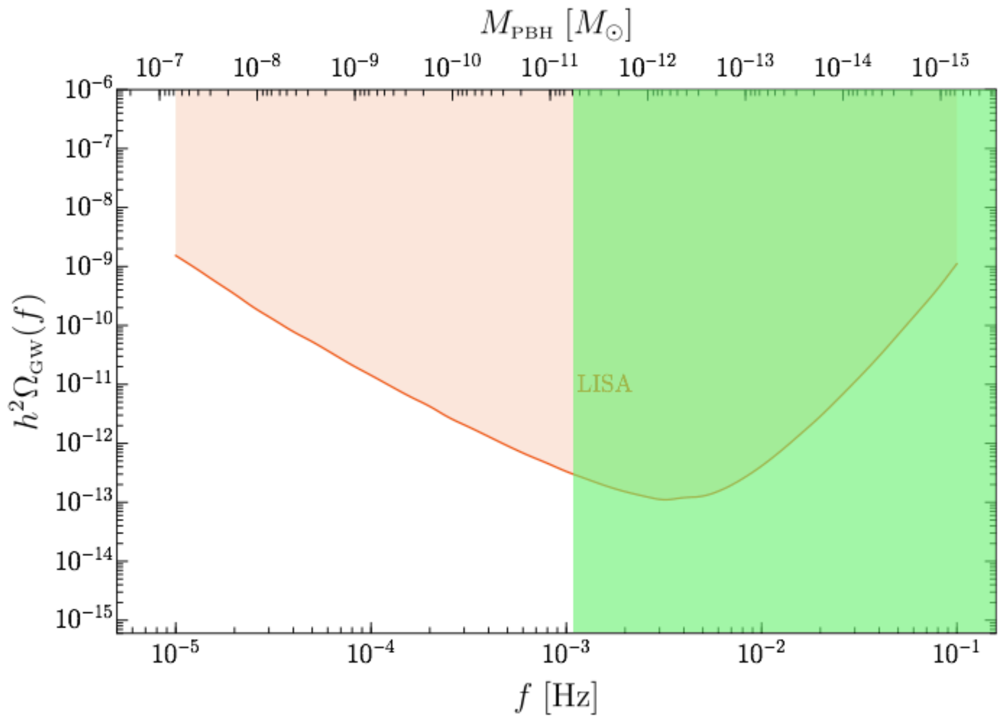
$\propto k^3$



$\propto \mathcal{P}_\zeta^2(k \gg k_*)$

The PBH dark matter-LISA serendipity





$$M \simeq 10^{-12} M_\odot \left(\frac{f_{\rm LISA}}{f}\right)^2$$

$$f_{\rm LISA} = 3.4\ {\rm mHz}$$

$$M \approx 10^{-12} M_\odot$$

Bartolo et al. PRL (2019)

Pulsar Timing Arrays

Millisecond pulsars whose signal sensitive to the stochastic GW background

Cross-correlation of timing residuals

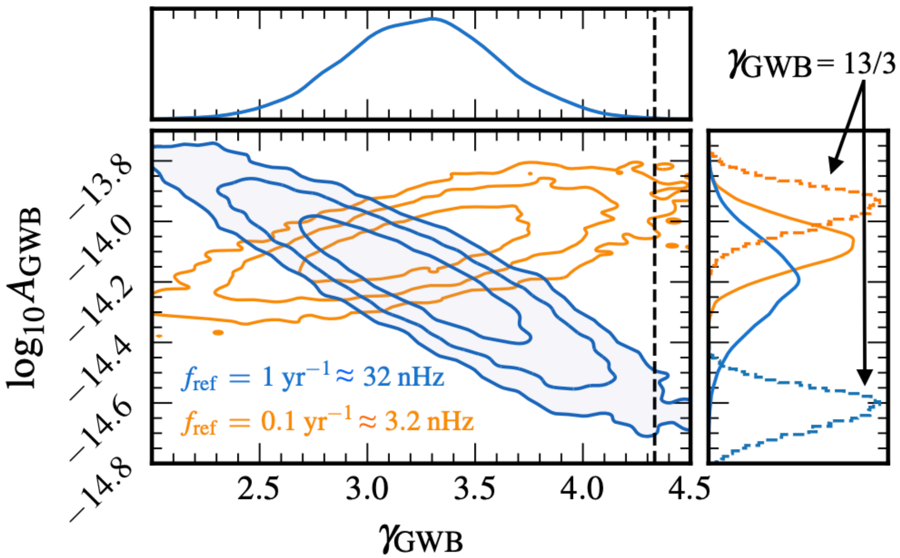
$$S_{ab} = \Gamma_{ab} \frac{h_c^2}{12\pi^2 f^3}$$

Pulsar Timing Arrays

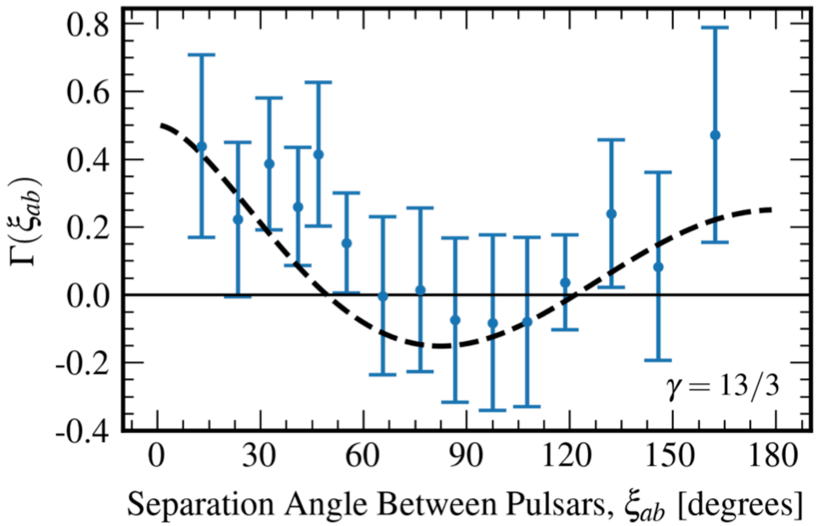
NANO-Grav-EPTA-PPTA-CPTA

Strong evidence for a SGWB

$$h_c = A_{\rm GWB}(f/{\rm yr}^{-1})^{\gamma_{\rm GWB}-5}$$



Hellings-Downs curve



Pulsar Timing Arrays and PBHs

$$\Omega_{\rm GWB} \sim \mathcal{P}_\zeta^2$$

$$f_{\rm PBH} \sim e^{-1/\mathcal{P}_\zeta}$$

PTA data require

$$\mathcal{P}_\zeta \gtrsim 6 \cdot 10^{-2}$$

giving

$$f_{\rm PBH} \gtrsim 1$$

see Iovino's talk

However, nonlinearities at horizon crossing reduce the PBH abundance

V. De Luca, A. Kehagias and A.R., 2307.13633

Conclusions

- The era of gravitational wave astronomy has begun opening a new window into fundamental physics and cosmology
- PBHs may exist and comprise the totality of the dark matter, future data will tell us