Flavour physics: status and prospects

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New Frontiers in Theoretical Physics

Cortona

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Motivation

Despite the SM successes,
there are open problems:

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Hierarchy problem

dark matter/dark energy

flavour hierarchies

neutrino masses

gravity

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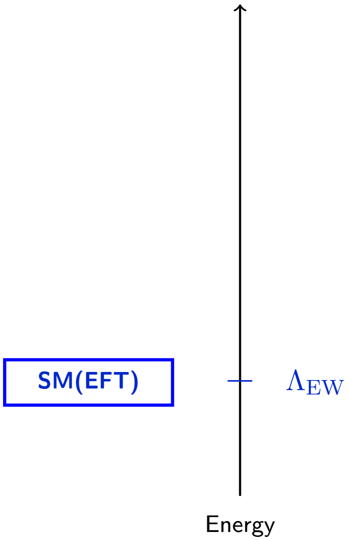

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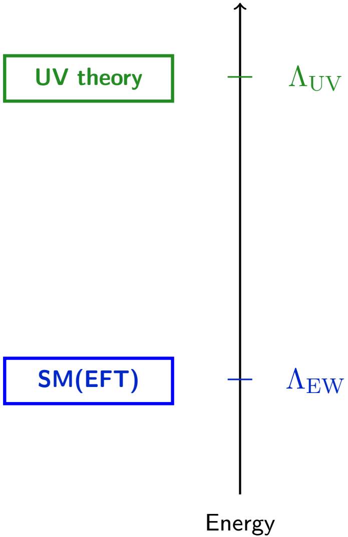

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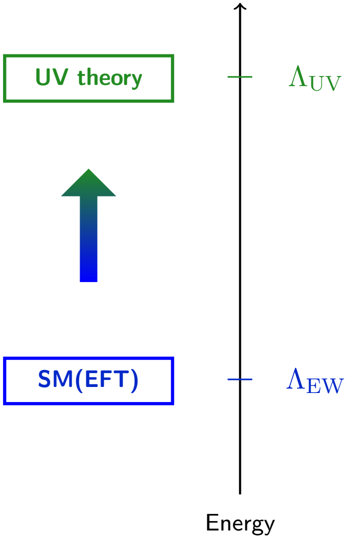
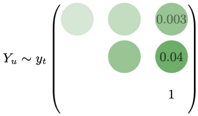


The (two) flavour problems

1. **The SM flavour problem**: The measured Yukawa pattern doesn't seem accidental

   ⇒ Is there any deeper reason for that?

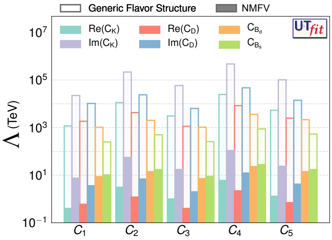


2. **The NP flavour problem**: If we regard the SM as an EFT valid below a certain energy cutoff Λ, why don't we see any deviations in flavour changing processes?

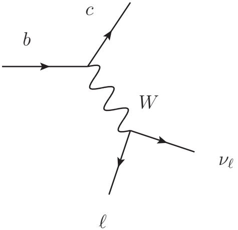
   ⇒ Which is the flavour structure of BSM physics?

   ⇒ Are accidental symmetries broken by NP?

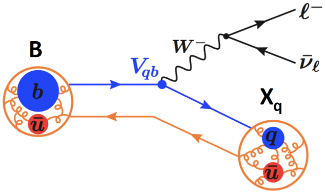
Pierini's EPS talk



Partonic vs Hadronic



$\mu_{\text{partonic}} = m_b$



$\mu_{\text{hadronic}} = \Lambda_{\text{QCD}}$

Fundamental challenge to match partonic and hadronic descriptions

What are the problems in flavour physics?

Kaon physics

Rare b-hadron decays

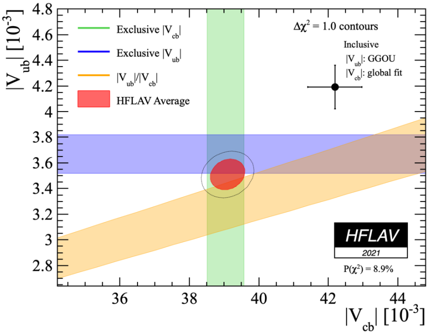
Higgs physics

τ decays

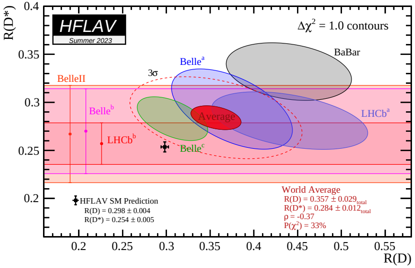
CP violation

Non leptonic b-hadron decays

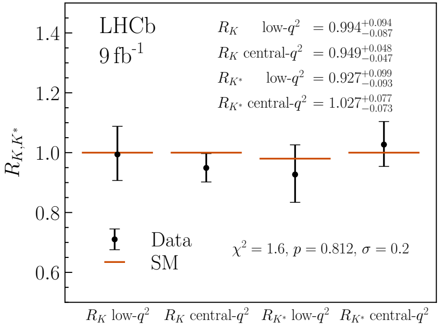
Semileptonic b-hadron decays



Exclusive $|V_{cb}|$
Exclusive $|V_{ub}|$
$|V_{ub}|/|V_{cb}|$
HFLAV Average
$\Delta\chi^2 = 1.0$ contours
Inclusive
$|V_{ub}|$: GGOU
$|V_{cb}|$: global fit
HFLAV
2021
$P(\chi^2) = 8.9\%$
$|V_{ub}|\ [10^{-3}]$
$|V_{cb}|\ [10^{-3}]$



HFLAV
Summer 2023
$\Delta\chi^2 = 1.0$ contours
BaBar
Belle[a]
3σ
BelleII
Belle[b]
Average
LHCb[a]
LHCb[b]
Belle[c]
HFLAV SM Prediction
R(D) = 0.298 ± 0.004
R(D*) = 0.254 ± 0.005
World Average
R(D) = 0.357 ± 0.029$_{total}$
R(D*) = 0.284 ± 0.012$_{total}$
ρ = -0.37
P(χ²) = 33%
R(D*)
R(D)



LHCb
9 fb^{-1}
R_K low-q^2 $= 0.994^{+0.094}_{-0.087}$
R_K central-q^2 $= 0.949^{+0.048}_{-0.047}$
R_{K^*} low-q^2 $= 0.927^{+0.099}_{-0.093}$
R_{K^*} central-q^2 $= 1.027^{+0.077}_{-0.073}$
Data
SM
$\chi^2 = 1.6,\ p = 0.812,\ \sigma = 0.2$
R_{K,K^*}
R_K low-q^2
R_K central-q^2
R_{K^*} low-q^2
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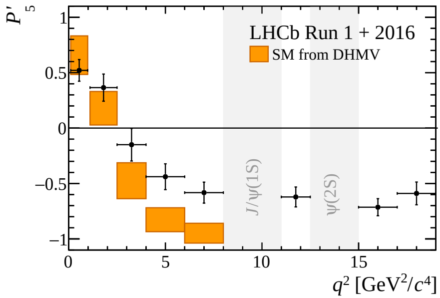
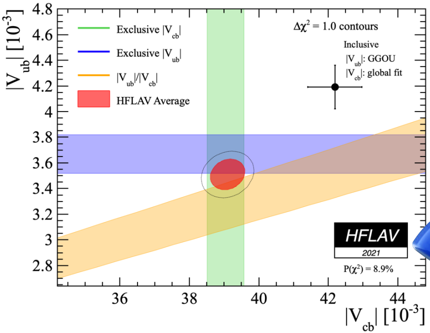


LHCb Run 1 + 2016
SM from DHMV
$J/\psi(1S)$
$\psi(2S)$
P'_5
$q^2\ [\mathrm{GeV}^2/c^4]$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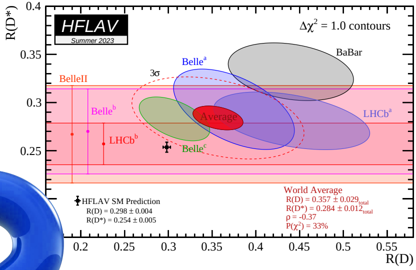
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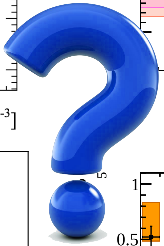

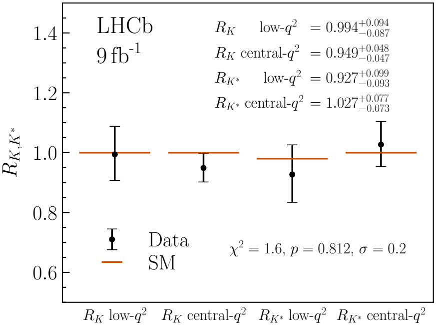
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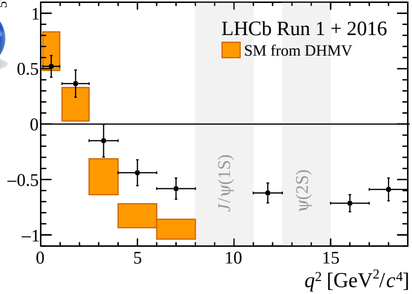
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$J/\psi(1S)$
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$q^2\ [\mathrm{GeV}^2/c^4]$

Semileptonic B-meson decays

Measuring V_{cb}

Interaction basis

$$-\mathcal{L}_{\rm Y} = Y_d^{ij}\bar{Q}_L^i H d_R^j + Y_u^{ij}\bar{Q}_L^i \tilde{H} u_R^j + \text{h.c.}$$

Non-diagonal Yukawa

Mass basis

$$\mathcal{L}_{cc} \propto \bar{u}_L^i \gamma^\mu d_L^j W_\mu^+ V_{ij}$$

CKM matrix

V_{cb} extraction

$$\mathcal{O}_{\rm theory}(V_{cb}, \vec{\mu}) = \mathcal{O}_{\rm exp}$$

theory inputs needed

Theory framework for $B \to X_c \ell \bar{\nu}$

Double expansion in $1/m$ and α_s

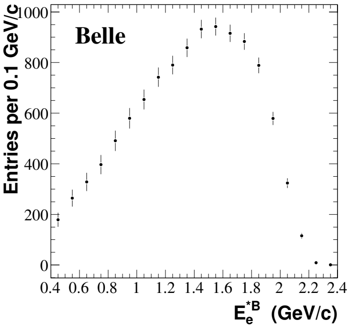
$$\Gamma_{sl} = \Gamma_0 f(\rho) \Big[1 + a_1 \left(\frac{\alpha_s}{\pi}\right) + a_2 \left(\frac{\alpha_s}{\pi}\right)^2 + a_3 \left(\frac{\alpha_s}{\pi}\right)^3 - \left(\frac{1}{2} - p_1 \left(\frac{\alpha_s}{\pi}\right)\right) \frac{\mu_\pi^2}{m_b^2} + \left(g_0 + g_1 \left(\frac{\alpha_s}{\pi}\right)\right) \frac{\mu_G^2(m_b)}{m_b^2} + d_0 \frac{\rho_D^3}{m_b^3} - g_0 \frac{\rho_{LS}^3}{m_b^3} + \ldots \Big]$$

- The coefficients are known
- $\mu_\pi^2(\mu) = \frac{1}{2m_B} \langle B | \bar{b}_v (i\vec{D})^2 b_v | B \rangle_\mu$ $\qquad$ $\mu_G^2(\mu) = \frac{1}{2m_B} \langle B | \bar{b}_v \frac{i}{2} \sigma_{\mu\nu} G^{\mu\nu} b_v | B \rangle_\mu$

  $\Rightarrow$ No Lattice QCD determinations are available yet
- Use for the first time of α_s^3 corrections **Fael, Schönwald, Steinhauser, '20**
- Ellipses stands for higher orders

  $\Rightarrow$ proliferation of terms and loss of predictivity

How do we constrain the hadronic parameters?



- Lepton energy and hadronic invariant mass distributions can be used to extract non perturbative information
- Moments of the kinematic distributions

$$\langle E_\ell^n \rangle = \frac{\int_{E_\ell > E_{\ell,\mathrm{cut}}} dE_\ell E_\ell^n \frac{d\Gamma}{dE_\ell}}{\Gamma_{E_\ell > E_{\ell,\mathrm{cut}}}}$$

$$R^* = \frac{\int_{E_\ell > E_{\ell,\mathrm{cut}}} dE_\ell \frac{d\Gamma}{dE_\ell}}{\int dE_\ell \frac{d\Gamma}{dE_\ell}}$$

- Similar definition for hadronic mass moments

- The moments give access to the distribution, but not to the normalisation
- They admit a $1/m$ and α_s expansion as the rate

The semileptonic fit

[MB, Capdevila, Gambino, '21]

m_b^{kin}	$\overline{m}_c(2\text{GeV})$	μ_π^2	ρ_D^3	$\mu_g(m_b)$	ρ_{LS}	$\text{BR}_{c\ell\nu}$	$10^3\|V_{cb}\|$
4.573	1.092	0.477	0.185	0.306	−0.130	10.66	42.16
0.012	0.008	0.056	0.031	0.050	0.092	0.15	0.51

- Constraints from FLAG $N_f = 2+1+1$: $\overline{m}_b(\overline{m}_b) = 4.198(12)\,\text{GeV}$ and $\overline{m}_c(\overline{m}_c) = 0.988(7)\,\text{GeV}$
- Experimental measurements from Babar, Belle
- The central value of V_{cb} is stable
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$$V_{cb} = 42.16(32)_{exp}(30)_{th}(25)_{\Gamma} \cdot 10^{-3}$$

Inclusive V_{cb} from q^2 moments

[Bernlochner et al., '22]

An alternative for the inclusive determination

- q^2 moments

$$R^* = \frac{\int_{q^2 > q^2_{\text{cut}}} dq^2 \, \frac{d\Gamma}{dq^2}}{\int_0 dq^2 \, \frac{d\Gamma}{dq^2}} \qquad \langle (q^2)^n \rangle = \frac{\int_{q^2 > q^2_{\text{cut}}} dq^2 (q^2)^n \, \frac{d\Gamma}{dq^2}}{\int_0 dq^2 \, \frac{d\Gamma}{dq^2}}$$

- Exploits HQE to reduce numbers of higher dimensional operators[Fael, Mannel, Vos, '18]

- Result:

$$|V_{cb}| = (41.69 \pm 0.63) \times 10^{-3}$$

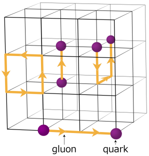
What's the issue with the previous determination?

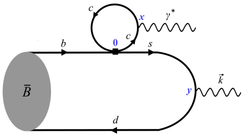
- The q^2 moments require a measurement of the branching ratio with a cut in q^2 which is not available yet
- By extrapolating from the current available measurements, the branching ratio is lower than what used
- If the same branching ratios is used, the two methods give the **same** result

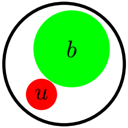
The results for inclusive V_{cb} are stable

Exclusive matrix elements

$$\langle H_c | J_\mu | H_b \rangle = \sum_i S_\mu^i \mathcal{F}_i$$







- Lattice QCD
- QCD SR, LCSR
- HQET (exploit $m_{b,c} \to \infty$ limit) + Data driven fits
- Dispersive analysis

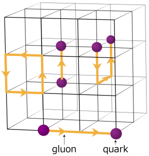
Exclusive matrix elements

$$\langle H_c | J_\mu | H_b \rangle = \sum_i S^i_\mu \mathcal{F}_i$$

← form factor

scale Λ_{QCD}

independent Lorentz structures



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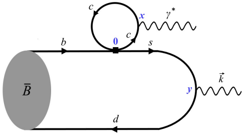

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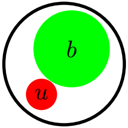
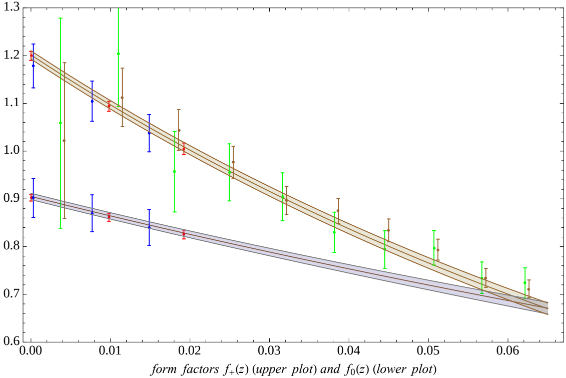


- HQET (exploit $m_{b,c} \to \infty$ limit) + Data driven fits
- Dispersive analysis

$B \to D$

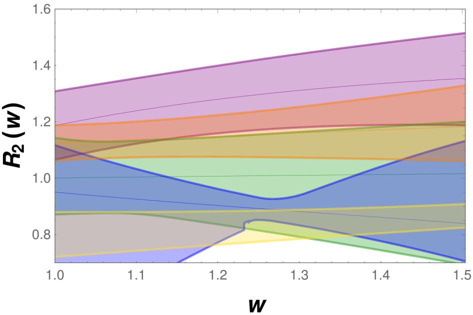
[Bigi, Gambino, '16]

- Belle+Babar data and HPQCD+FNAL/MILC Lattice points



$$|V_{cb}| = (40.49 \pm 0.97) \times 10^{-3}$$

$B \to D^*$



- FNAL/MILC '21
- HQE@$1/m_c^2$
- Exp data (BGL)
- JLQCD '23
- HPQCD '23

- Tensions between different lattice determinations, experimental data and non-lattice theory determination
- No consensus yet, ongoing checks
- New Belle/Belle II analysis available

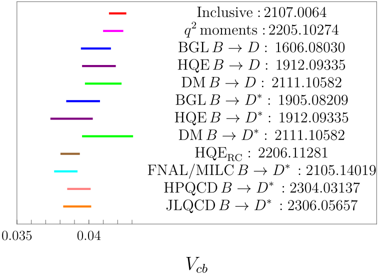
Pheno Status 1

- The inclusive determination is solid

- Only caveat: QED corrections for charged current decays are enhanced by the Coulomb factor (for neutral B mesons)

  ⇒ The impact has to be checked for each measurement

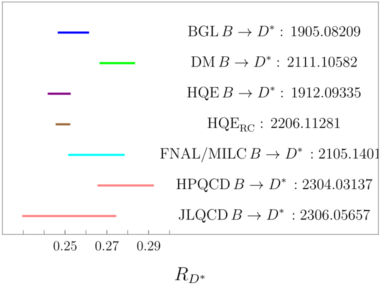
MB, Bigi, Gambino, Haisch, Piccione '23

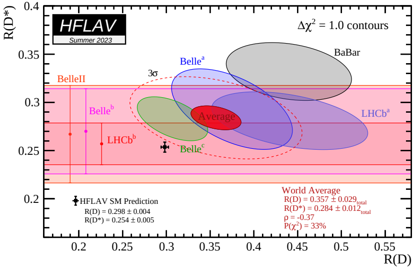


The exclusive determination depends on the dataset and hadronic form factor used

- No evident issues for $B \to D$
- Work in progress for the theory predictions of $B \to D^*$
- New experimental data are available and have to be still scrutinised

Pheno status 2





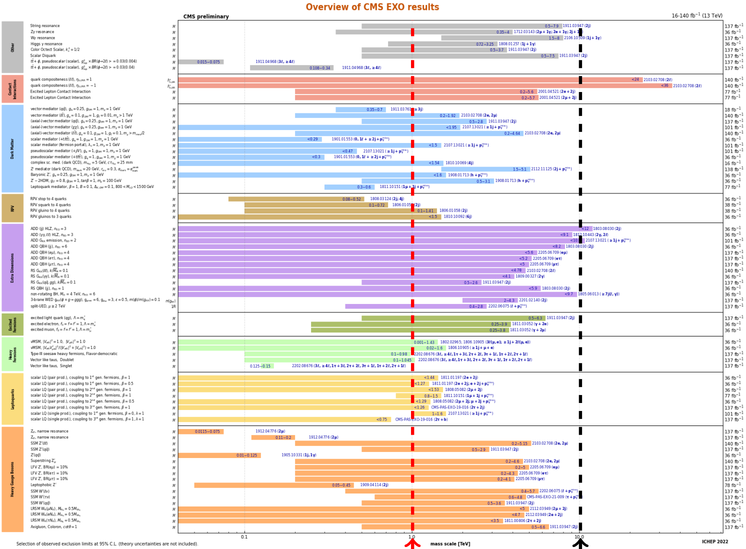
- Lepton Flavour Universality Ratios $R_{D^{(*)}} = \frac{\mathcal{B}(B \to D^{(*)}\tau\bar{\nu})}{\mathcal{B}(B \to D^{(*)}\ell\bar{\nu})}$
- Without LQCD prediction, the current combined tension is $\sim 3.3\,\sigma$
- Concerning R_D the situation is much stable because different LQCD collaborations agree with each other and experimental data

Conclusions

- Flavour physics is a powerful test for new physics living at different energy scales
- Providing precise predictions for hadronic decays is fundamental to test the Standard Model and ultimately the size of New Physics
- Efforts are ongoing in many directions to shed light on the current puzzles
- Synergy between the theory and experimental community is essential to achieve the ultimate precision

Appendix

Status of high energy bounds



3rd generation

universal new physics

The SM flavour problem

$$\mathcal{L}_{\text{Yukawa}} \supset Y_u^{ij} \bar{Q}_L^i H u_R^j$$

$$Y_u \sim y_t \begin{pmatrix} & & 0.003 \\ & & 0.04 \\ & & 1 \end{pmatrix}$$

The SM flavour problem

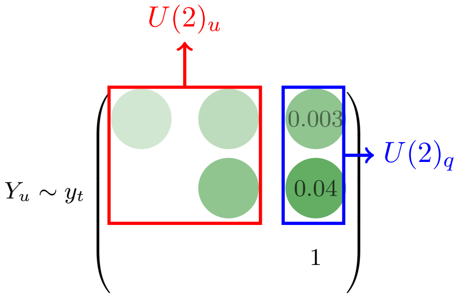
$$\mathcal{L}_{\text{Yukawa}} \supset Y_u^{ij} \bar{Q}_L^i H u_R^j$$

$$Y_u \sim y_t \begin{pmatrix} & & \\ & & \\ & & 1 \end{pmatrix}$$

Exact $U(2)^n$ limit

The SM flavour problem

$$\mathcal{L}_{\text{Yukawa}} \supset Y_u^{ij} \bar{Q}_L^i H u_R^j$$



An approximate $U(2)^n$ is acting on the light families!

The SM flavour problem

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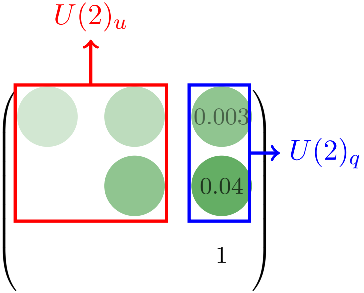

An approximate $U(2)^n$ is acting on the light families!

The NP flavour problem

$$\mathcal{L} = \boxed{\mathcal{L}_{\rm gauge}} + \boxed{\mathcal{L}_{\rm Higgs}}$$

Large Flavour symmetry

Three replica of the same fermion fields

$U(3)^5$ symmetry

Flavour degeneracy is broken

The breaking is peculiar

- In the SM: accidental $U(3)^5 \to$ approx $U(2)^n$

The NP flavour problem

$$\mathcal{L} = \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{Higgs}} + \sum_{d,i} \frac{c_i^{(d)}}{\Lambda^{d-4}} \mathcal{O}_i^d$$

Large Flavour symmetry

Three replica of the same fermion fields

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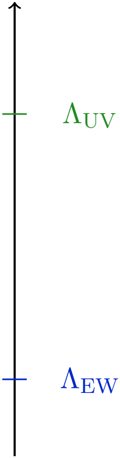
Flavour degeneracy is broken

The breaking is peculiar

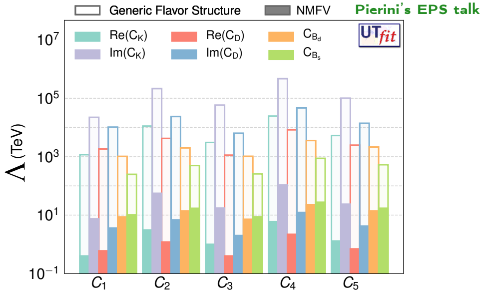
- In the SM: accidental $U(3)^5 \to$ approx $U(2)^n$
- **What happens when we switch on NP?**

The NP flavour problem

$$\mathcal{L} = \boxed{\mathcal{L}_{\text{gauge}}} + \boxed{\mathcal{L}_{\text{Higgs}}} + \boxed{\sum_{d,i} \frac{c_i^{(d)}}{\Lambda^{d-4}} \mathcal{O}_i^d}$$



- What is the energy scale of NP?
- Why haven't observed any violation of accidental symmetries yet?



no breaking of the $U(2)^n$ flavour symmetry at low energies

$B \to D^{(*)}$ form factors

- 7 (SM) + 3 (NP) form factors
- Lattice computation for $q^2 \neq q^2_{\text{max}}$ only for $B \to D$
- Calculation usually give only a few points
- q^2 dependence must be inferred
- Conformal variable z

$$z(q^2, t_0) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$

- $t_+ = (m_B + m_{D^{(*)}})^2$ pair production threshold
- $t_0 < t_+$ free parameter that can be used to minimise $|z_{\text{max}}|$
- $|z| \ll 1$, in the $B \to D$ case $|z| < 0.06$

Theory framework

$$\Gamma = \frac{1}{m_B} \mathrm{Im} \int d^4x \langle B(p)| T \left\{ \mathcal{H}_{\mathrm{eff}}^\dagger(x) \mathcal{H}_{\mathrm{eff}}(0) \right\} |B(p)\rangle$$

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$\uparrow$

$\sum_{n,i} \frac{1}{m_b^n} \mathcal{C}_{n,i} \mathcal{O}_{n+3,i}$

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$$\uparrow$$

$$\sum_{n,i} \frac{1}{m_b^n} \mathcal{C}_{n,i} \mathcal{O}_{n+3,i}$$

- The Wilson coefficients are calculated perturbatively
- The matrix elements $\langle B(p)|\mathcal{O}_{n+3,i}|B(p)\rangle$ are non perturbative
  - $\Rightarrow$ They need to be determined with non-perturbative methods, e.g. Lattice QCD
  - $\Rightarrow$ They can be extracted from data
  - $\Rightarrow$ With large n, large number of operators

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↑

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loss of predictivity

The HQE parametrisation 1

- Expansion of QCD Lagrangian in $1/m_{b,c} + \alpha_s$ corrections

  [Caprini, Lellouch, Neubert, '97]

- In the limit $m_{b,c} \to \infty$: all $B \to D^{(*)}$ form factors are given by a **single** Isgur-Wise function

$$F_i \sim \xi$$

- at higher orders the form factors are still related $\Rightarrow$ **reduction** of free parameters

$$F_i \sim \left(1 + \frac{\alpha_s}{\pi}\right)\xi + \frac{\Lambda_{\mathsf{QCD}}}{2m_b}\xi^i_{\mathsf{SL}} + \frac{\Lambda_{\mathsf{QCD}}}{2m_c}\xi^i_{\mathsf{SL}}$$

  - at this order 1 leading and 3 subleading functions enter
  - ξ^i are not predicted by HQE, they have to be determined using some other information

The HQE parametrisation 2

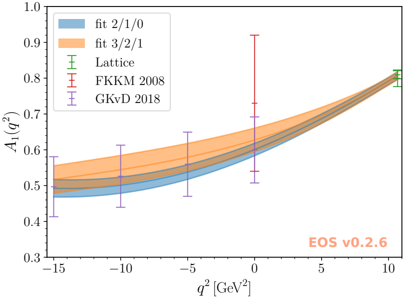
- Important point in the HQE expansion: $q^2 = q^2_{\text{max}}$
- At this point Luke's Theorem applies: the subleading corrections vanish for some form factors
- The leading Isgur-Wise function is normalised: $\xi(q^2 = q^2_{\text{max}}) = 1$
- **Problem**: contradiction with lattice data!
- $1/m_c^2$ corrections **have to be systematically included** **[Jung, Straub, '18, MB, M.Jung, D.van Dyk, '19]**
  - well motivated also since $\alpha_s/\pi \sim 1/m_b \sim 1/m_c^2$

The HQE results

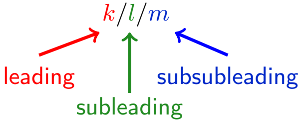
[MB, Jung, van Dyk, EPJC 80 (2020), MB, Gubernari, Jung, van Dyk, EPJC 80 (2020)]

Data points:

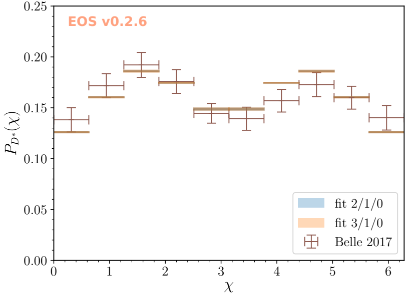
- theory inputs only (Lattice QCD, QCD Sum Rules, Light-cone Sum Rules, Dispersive Bounds)

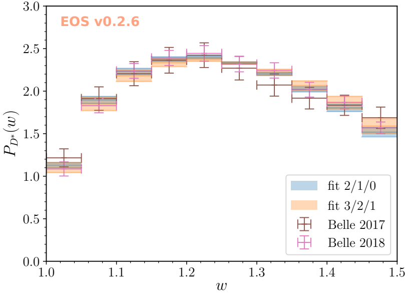


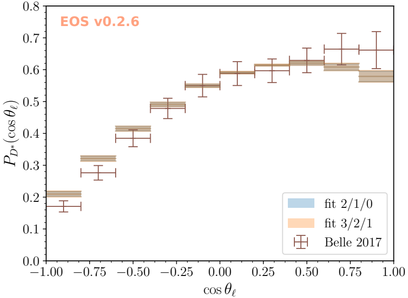
- Expansion in z up to order



Comparison with kinematical distributions



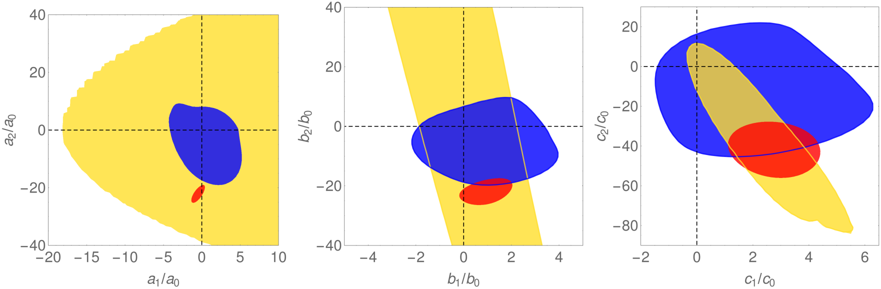




good agreement with kinematical distributions

Fit stability

- BGL fit to Belle 2017 and 2018 data (yellow)
- HQE fit 2/1/0 (red)
- HQE fit 3/2/1 (blue)



- compatibily of HQE fit with data driven one
- 2/1/0 underestimates massively uncertainties

$3/2/1$ is our nominal fit

Phenomenological results

- V_{cb} extraction

$$V_{cb}^{\text{average}} = (41.1 \pm 0.5) \times 10^{-3}$$

compatibility of 1.8σ between inclusive and exclusive

- Universality ratios

$$R_{D^*} = 0.2472 \pm 0.0050 \qquad R_{D_s^*} = 0.2472 \pm 0.0050$$

towards the combined 4σ discrepancy

- We observe no $SU(3)_F$ breaking

- Good compatibility with LHCb $\bar{B}_s \to D_s^{(*)}$ analysis in 2001.03225

HQET in a nutshell

- In HQET it is convenient to work with **velocities** instead of momenta
- Instead of q^2 we use the dimensionless variable $w = v_B \cdot v_{D^*}$
- When the $B(b)$ decays such that the $D^*(c)$ is at rest in the $B(b)$ frame

$$v_B = v_{D^*} \qquad \Rightarrow \qquad w = 1$$

- The brown muck doesn't realise that anything changed
- At zero recoil, the leading IW function is **normalized**

$$\xi(w = 1) = 1$$

V_{cb} and NP

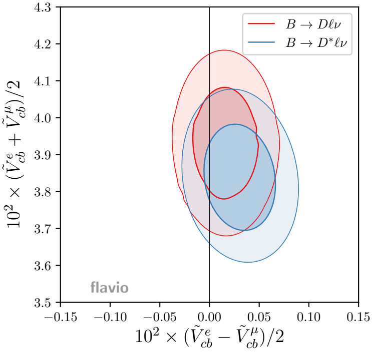
[Jung, Straub 2018]

- If we allow LFUV between μ and electrons

$$\tilde{V}_{cb}^{\ell} = V_{cb}(1 + C_{V_L}^{\ell})$$

- Fitting data from Babar and Belle

$$\frac{\tilde{V}_{cb}^{e}}{\tilde{V}_{cb}^{\mu}} = 1.011 \pm 0.012$$



$$\frac{1}{2}(\tilde{V}_{cb}^{e} + \tilde{V}_{cb}^{\mu}) = (3.87 \pm 0.09)\%$$
$$\frac{1}{2}(\tilde{V}_{cb}^{e} - \tilde{V}_{cb}^{\mu}) = (0.022 \pm 0.023)\%$$

Scheme conventions

The semileptonic width has a strong dependence on m_b: $\Gamma_0 \sim m_b^5$

Suitable choice for the mass scheme is needed:

- Pole mass scheme
  - $\Rightarrow$ Renormalon ambiguity
  - $\Rightarrow$ Perturbative series is factorially divergent

$$\Gamma_{sl} \sim \sum_k k! \left(\frac{\beta_0}{2} \frac{\alpha_s}{\pi} \right)^k$$

- We choose to use to b-quark mass and the non perturbative parameters in the kinetic scheme

[Bigi, Shifman,Uraltsev,Vainshtein]

$$m_b^{kin} \mu = m_b^{OS} - [\bar{\Lambda}(\mu)]_{\rm pert} - \frac{[\mu_\pi^2(\mu)]_{\rm pert}}{2 m_b^{kin}(\mu)}$$

$$\mu_\pi^2(0) = \mu_p i^2(\mu) - [\mu_\pi^2(\mu)]_{\rm pert}$$

$$\rho_D^3(0) = \rho_D^3(\mu) - [\rho_D^3(\mu)]_{\rm pert}$$

  - $\Rightarrow$ Wilsonian cutoff $\mu = 1\,\mathrm{GeV}$
  - $\Rightarrow$ Kinetic scheme tailored on the HQE

- We express the charm mass in the $\overline{\mathrm{MS}}$ scheme

Higher power corrections

- At $\mathcal{O}(1/m^4)$ the number of operators become large
  - $\Rightarrow$ 9 at dim 7
  - $\Rightarrow$ 18 at dim 8

Lowest Lying State Saturation Approximation: [Mannel, Turczyk, Uraltsev, '11]

$$\langle B|\mathcal{O}_1\mathcal{O}_2|B\rangle = \sum_n \langle B|\mathcal{O}_1|n\rangle\langle n|\mathcal{O}_2|B\rangle$$

↑ complete set of states

At dimension 6 the LLSA works well:

$$\rho_D^3 = \epsilon\mu_\pi^2 \qquad \rho_{LS}^3 = -\epsilon\mu_G^2 \qquad \epsilon \sim 0.4\,\mathrm{GeV}$$

- Large corrections to the LLSA are possible [Gambino, Mannel, Uraltsev, '12]
- 60% gaussian uncertainty on higher order parameters

$$V_{cb} = 42.00(53)\times 10^{-3}$$