Radiative corrections for the MUonE experiment

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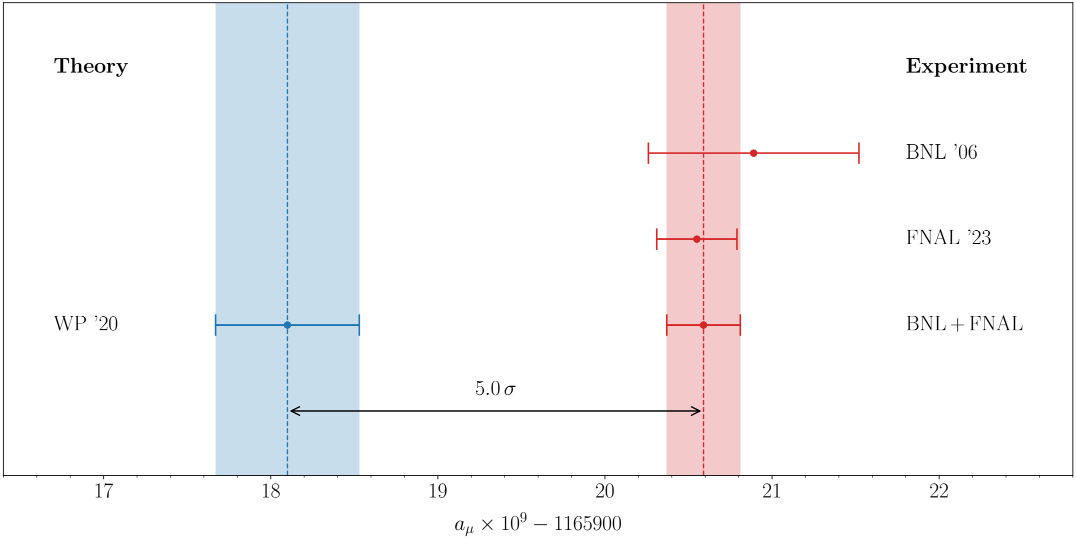
XXXVII Convegno Nazionale di Fisica Teorica

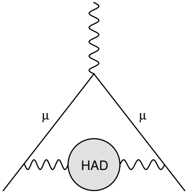
Cortona – 28 September 2023

The muon g−2 anomaly: theory vs. experiment



Time-like evaluation of the hadronic contribution

- The hadronic contribution to muon $g-2$ cannot be computed using perturbation theory!
- The evaluation requires **data-driven** methods or **lattice** simulations
- The **time-like** (or **dispersive**) approach employs $\sigma\left(\mathrm{e}^+\mathrm{e}^- \to \mathrm{hadrons}\right)$ data
- Large uncertainty arising from the several **resonances** in the low-energy cross section

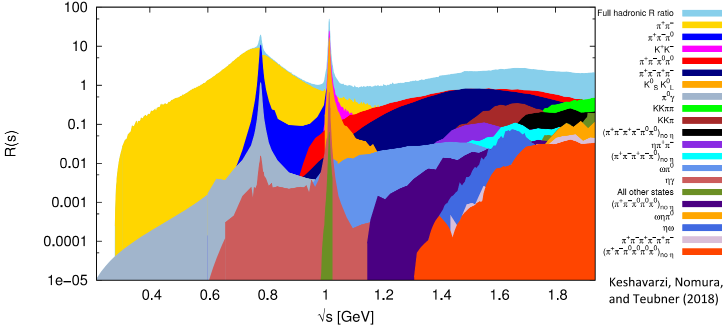


- Using the dispersive relations and the optical theorem we obtain:

$$a_\mu^{\mathrm{HLO}} \propto \int_{4m_\pi^2}^{\infty} \mathrm{d}s \, \frac{K(s)\, R(s)}{s^2}$$

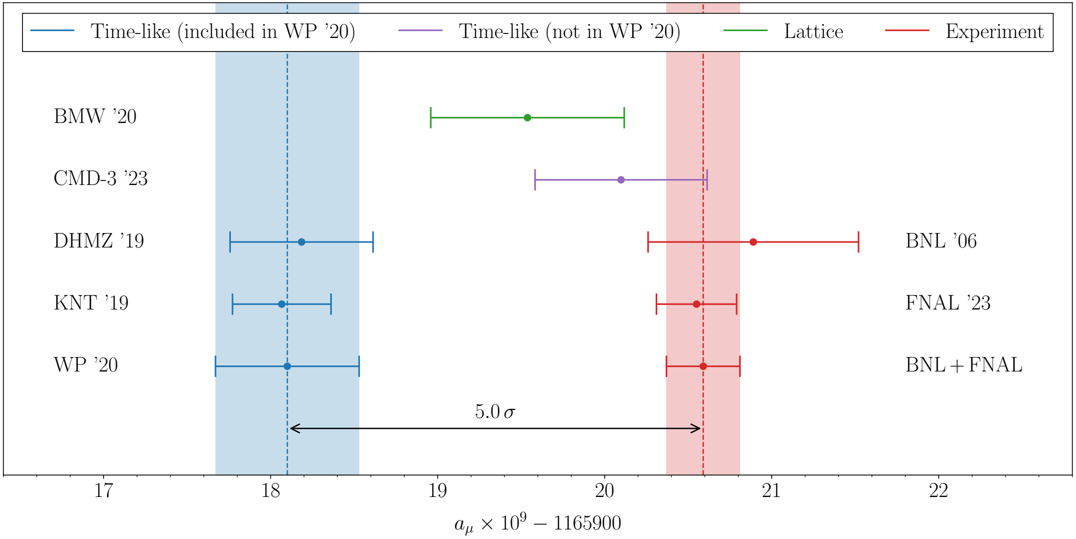
$$R(s) = \frac{\sigma(e^+e^- \to \mathrm{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

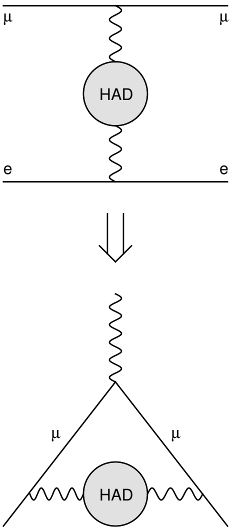
$$K(s) \sim 1/s \quad \to \quad \text{smooth}$$



Keshavarzi, Nomura, and Teubner (2018)

The hadronic contribution puzzle: time-like approach or lattice?





- The leading order hadronic contribution a_μ^{HLO} can be evaluated from the **running of the e.m. constant** $\alpha(t)$ in the **space-like region** $t < 0$ [Lautrup, et al., Phys. Rept. 3 (1972)]

$$a_\mu^{\mathrm{HLO}} = \frac{\alpha}{\pi} \int_0^1 \mathrm{d}x\,(1-x)\,\Delta\alpha_{\mathrm{had}}[t(x)] \qquad \alpha(t) = \frac{\alpha(0)}{1 - \Delta\alpha(t)}$$

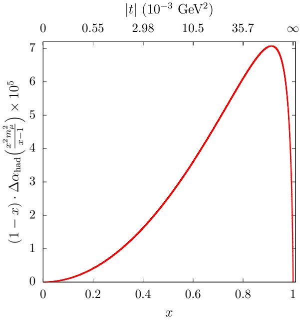
$$t(x) = \frac{x^2 m_\mu^2}{x-1} < 0 \quad \longrightarrow \quad \text{smooth!}$$

- $\Delta\alpha_{\mathrm{had}}(t)$ and a_μ^{HLO} can be measured directly in a single experiment involving a space-like process. [Carloni Calame, et al., Phys. Lett. B 746 (2015)]

- Ideal process: ***μe* elastic scattering**

  $\hookrightarrow$ only t-channel $\rightarrow d\sigma/dt \sim |\alpha(t)/\alpha(0)|^2$

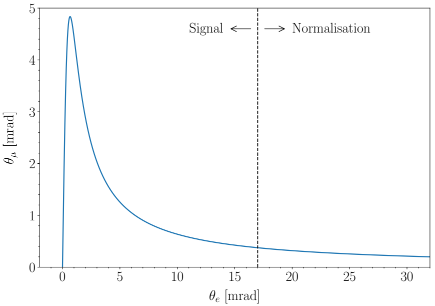
- $d\sigma$ must be measured very precisely to separate the **Hadronic Vacuum Polarisation** (HVP) contributions to $\alpha(t)$ from the leptonic ones



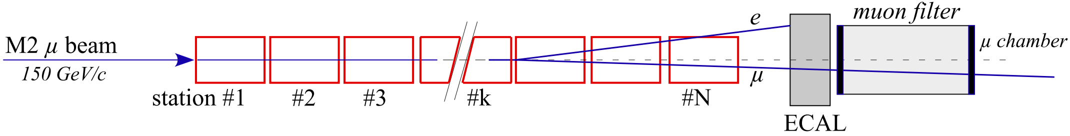
The MUonE proposal

- Independent evaluation of a_μ^{HLO} via $\mu^\pm e^- \to \mu^\pm e^-$ scattering
- M2 muon beam available at CERN North Area ($E_\mu \simeq 160$ GeV)
- Initial-state electrons at rest in a low-Z target (Be or C)
- Elastic scattering events selected by measuring θ_μ and θ_e
- Goal: accuracy of 0.3% on a_μ^{HLO} after 3 years of data taking
  $\hookrightarrow$ $d\sigma$ must be known with a precision of $\mathbf{10^{-5}}$

[Abbiendi et al., Eur. Phys. J. C 77(3) (2017) 139]
[LoI: The MUonE project, Tech. Rep. CERN-SPSC-2019-026]



$$t = \frac{(2\,m_e\,\beta\cos\theta_e)^2}{\cos^2\theta_e - 1} \qquad \beta = \frac{|p_\mu|}{E_\mu + m_e}$$



Theoretical work for MUonE

- Carloni Calame et al., PLB 746 (2015), 325
- Mastrolia et al., JHEP 11 (2017) 198
- Di Vita et al., JHEP 09 (2018) 016
- Alacevich et al., JHEP 02 (2019) 155
- Fael and Passera, PRL 122 (2019) 19, 192001
- Fael, JHEP 02 (2019) 027
- Carloni Calame et al., JHEP 11 (2020) 028
- Banerjee et al., SciPost Phys. 9 (2020), 027
- Banerjee et al., EPJC 80 (2020) 6, 591
- Budassi et al., JHEP 11 (2021) 098
- Balzani et al., PLB 834 (2022) 137462
- Bonciani et al., PRL 128 (2022) 2, 022002
- Budassi et al., PLB 829 (2022) 137138
- Broggio et al., JHEP 01 (2023) 112
- Badger at al., arXiv:2307.03098
- Ahmed et al., arXiv:2308.05028

- A lively theory community is active to provide state-of-the-art calculations to match the experimental accuracy.
  $\hookrightarrow$ 6 topical workshops in 6 years ($\sim$ 40 people involved)

- Two independent Monte Carlo codes are developed and cross-checked to validate the high-precision calculations:

  1) **MESMER**: **M**uon-**E**lectron **S**cattering with **M**ultiple **E**lectromagnetic **R**adiation
     $\hookrightarrow$ Fully exclusive MC event generator
     $\hookrightarrow$ `github.com/cm-cc/mesmer`

  2) **McMule**: **M**onte **C**arlo for **MU**ons and other **LE**ptons
     $\hookrightarrow$ MC integrator (generator under implementation)
     $\hookrightarrow$ `gitlab.com/mule-tools/mcmule`

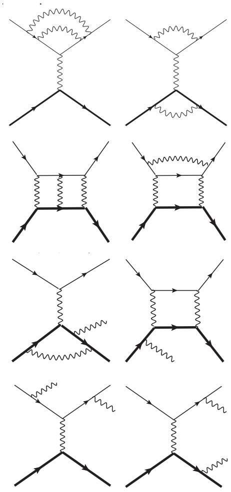
- All comparisons made so far have been successful ✓

Signal calculation

- Goal: theoretical accuracy of 10 ppm on the $\mu^\pm e^- \to \mu^\pm e^-$ differential cross section $d\sigma$
- How? **Perturbative expansion** of the cross section in power of the **QED** coupling α:

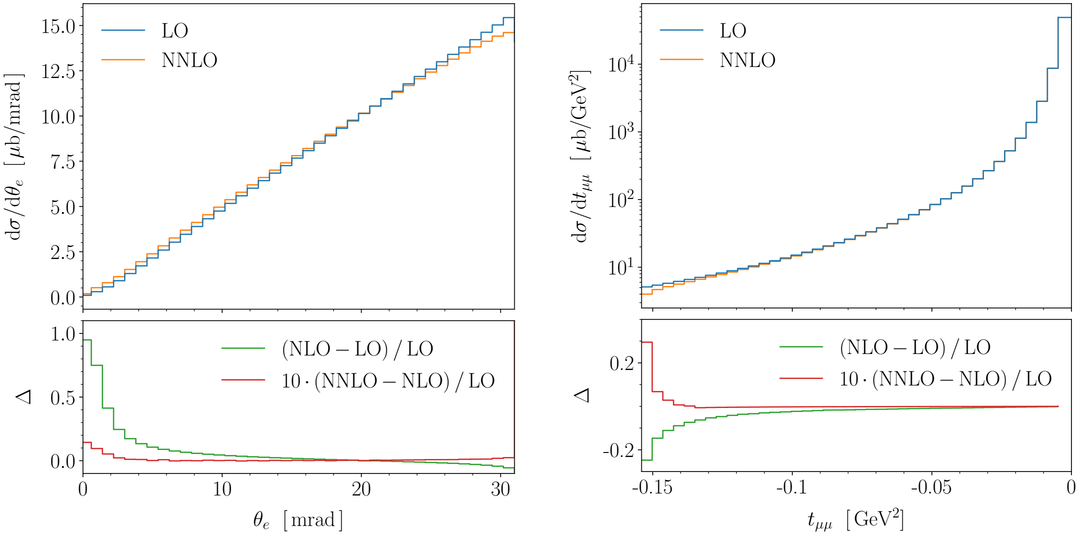
$$d\sigma = d\sigma_{\rm LO} + \left(\frac{\alpha}{\pi}\right) d\sigma_{\rm NLO} + \left(\frac{\alpha}{\pi}\right)^2 d\sigma_{\rm NNLO} + \dots$$

- What radiative corrections are needed to achieve such a precision?
  - At least next-to-next-to-leading order (**NNLO**) corrections with $m_e \neq 0$
  - Inlusion of large logarithms through the implementation of a **QED shower**
    - ↪ Emission of an arbitrary number of photons and e^+e^- pairs
    - ↪ Matching with fixed-order contributions to avoid double countings
  - Initial-state electrons are bound in atoms rather than free, what is the impact?
- The calculation must be implemented in a **Monte Carlo event generator** to be used in experimental simulations and analyses $\longrightarrow$ MESMER code developed in Pavia [JHEP 02 (2019) 155], [JHEP 11 (2020) 028], [JHEP 11 (2021) 098]



+ many more

Results for $\mu^+ e^- \to \mu^+ e^-$ at NNLO



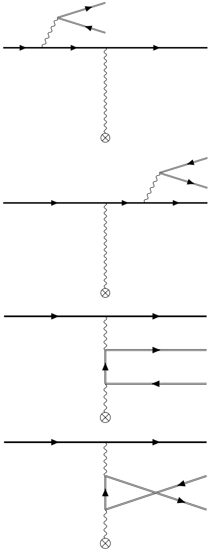
Background calculation

- The possible backgrounds must be implemented in the MC code for detailed simulations:
  - $\mu^\pm e^- \to \mu^\pm e^- \pi^0$ with $\pi^0 \to \gamma\gamma$ [PLB 829 (2022) 137138]
  - $\mu^\pm e^- \to \mu^\pm e^- \ell^+\ell^-$ with $\ell = e, \mu$ [JHEP 11 (2021) 098]
  - $\boldsymbol{\mu^\pm X \to \mu^\pm X\, \ell^+\ell^-}$ where X is a nucleus (WIP)
- Since the initial-state e^- are bound in a low-Z target (Be or C), the lepton pair production **muon-nucleus scattering** is expected to be the main source of experimental background
- It can resemble the signal if one particle is not reconstructed (**2 tracks events**)
- Model: scattering in external e.m. field with a **nuclear form factor** correction

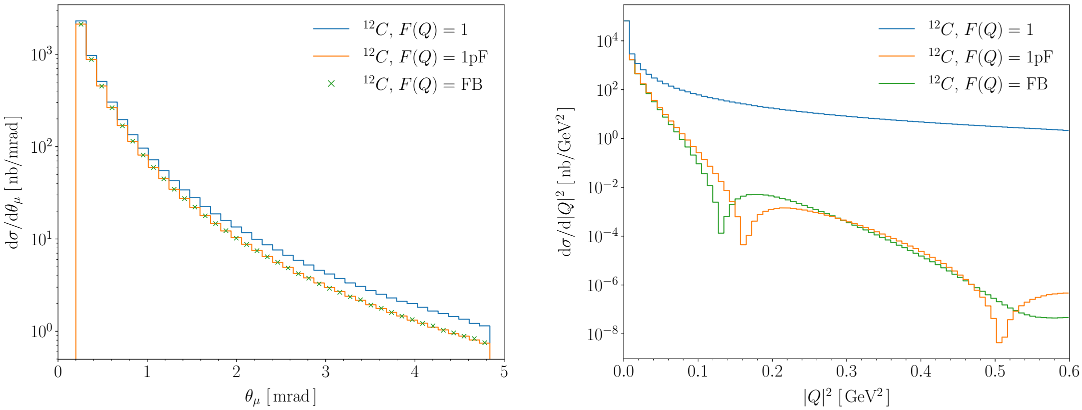
$$\mathrm{F(Q)} = \frac{1}{\mathrm{Ze}} \int_0^\infty \mathrm{dr\, r^2} \rho(\mathrm{r}) \frac{\sin(\mathrm{qr})}{\mathrm{qr}}$$

Q : momentum transferred to the nucleus, ρ : nuclear charge density

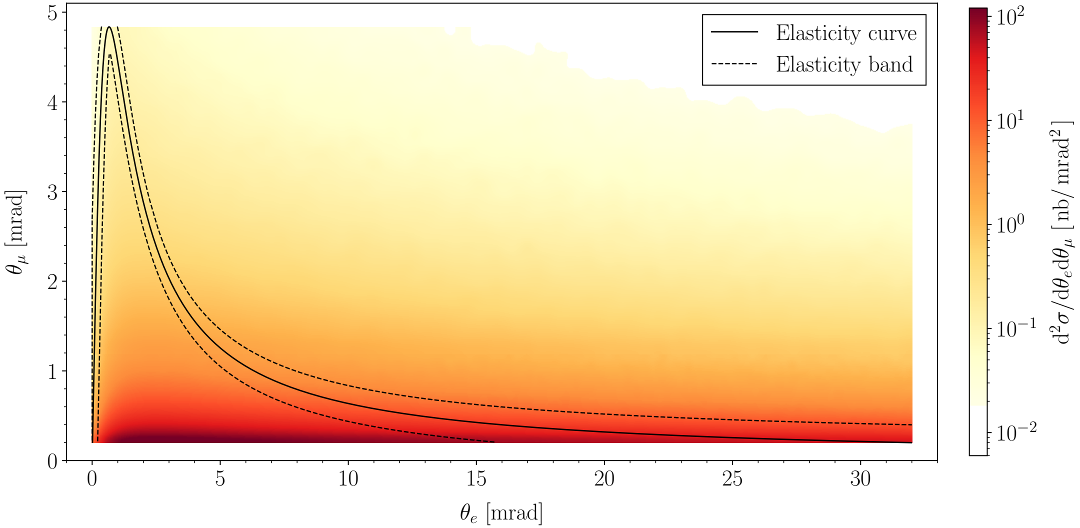
- Good approximation for small angles: $\theta_\mu < 5$ mrad $\longrightarrow$ $q^2 < 0.5$ GeV^2
- Different models for ρ to evaluate the **theoretical uncertainty** on the $\mathrm{F(Q)}$ model



Preliminary results for $\mu^+ C \to \mu^+ C\, e^+ e^-$



- Cuts: $E_e > 0.2$ GeV, $E_\mu > 10.2$ GeV, $\theta_e < 32$ mrad, $\theta_\mu < 4.8$ mrad, $\theta_\mu > 0.2$ mrad, $|Q|^2 < 0.6$ GeV^2
- Only the events with **one muon track** and **one electron track** in the acceptance region are selected
  $\hookrightarrow$ Only one $e^\pm$ is accepted, or both $e^\pm$ are accepted but their tracks overlap, i.e. $\theta_{ee} < 20\ \mu$rad


Elasticity curve
Elasticity band
θ_μ [mrad]
θ_e [mrad]
$d^2\sigma/d\theta_e d\theta_\mu$ [nb/mrad^2]
0
1
2
3
4
5
0
5
10
15
20
25
30
10^2
10^1
10^0
10^{-1}
10^{-2}

Conclusions

- The MUonE experiment will provide a new **independent** and **competitive** evaluation of the HVP contribution to the muon $g-2$, which will be helpful to understand the dispersive-lattice discrepancy

- The theory community is working to provide the high-precision calculations needed by the experiment:
  $\hookrightarrow$ $\mu^{\pm}e^{-} \to \mu^{\pm}e^{-}$ is required at the challenging accuracy of **10 ppm**
  $\hookrightarrow$ $\mu^{\pm}X \to \mu^{\pm}X\,\ell^{+}\ell^{-}$ and other backgrounds are needed for experimental simulations

- The calculations are under implementation in two independent MC codes (**MESMER** and **McMule**)

- The experimental proposal will be submitted to CERN in 2024, stay tuned!

Backup

- The **anomalous magnetic moment** of leptons is a fundamental observable in particle physics
  ↪ Precise comparison between theory and experiment to test the **Standard Model**

- The **gyromagnetic factor g** is the ratio between the magnetic moment μ and the spin S

$$\mu = g \, \frac{e}{2m} \, S$$

What is the value of g?

- Quantum Mechanics $\longrightarrow$ Solution of Dirac equation $\longrightarrow$ $\boldsymbol{g = 2}$

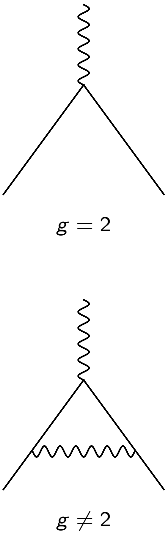
- Standard Model $\longrightarrow$ Interaction with virtual particles $\longrightarrow$ $\boldsymbol{g \neq 2}$

- For the electron the agreement between theory and experiment is at the level of 0.1 ppb

$$\text{Th:} \quad a_e = (g-2)/2 = 115965218.164(76) \cdot 10^{-11}$$

$$\text{Ex:} \quad a_e = (g-2)/2 = 115965218.073(28) \cdot 10^{-11}$$

- And for the muon?



Nuclear form factor models

Nuclear form factor: $$F(Q) = \frac{1}{Ze} \int_0^\infty dr\, r^2 \rho(r) \frac{\sin(qr)}{qr}$$

One-parameter Fermi model (1pF): $$\rho(r) = \frac{\rho_0}{1 + \exp(\frac{r-c}{z})} \qquad z \simeq 0.52 \text{ fm}$$

Modified harmonic oscillator model (MHO): $$\rho(r) = \rho_0 \left(1 + w\frac{r^2}{a^2}\right) \exp\left(-\frac{r^2}{a^2}\right)$$

Fourier-Bessel expansion (FB): $$\rho(r) = \begin{cases} \sum_k^n a_k\, j_0\left(\frac{k\pi r}{R}\right) & r \le R \\ 0 & r > R \end{cases}$$

References: [Phys. Rev. D 105 (5) (2022) 053006], [Nucl. Phys. A 188 (1972) 337–352], [Phys. Rev. C 44 (1991) 1096–1117]