LePDF: Standard Model PDFs for High-Energy Lepton Colliders

[FG, David Marzocca, Sokratis Trifinopoulos]
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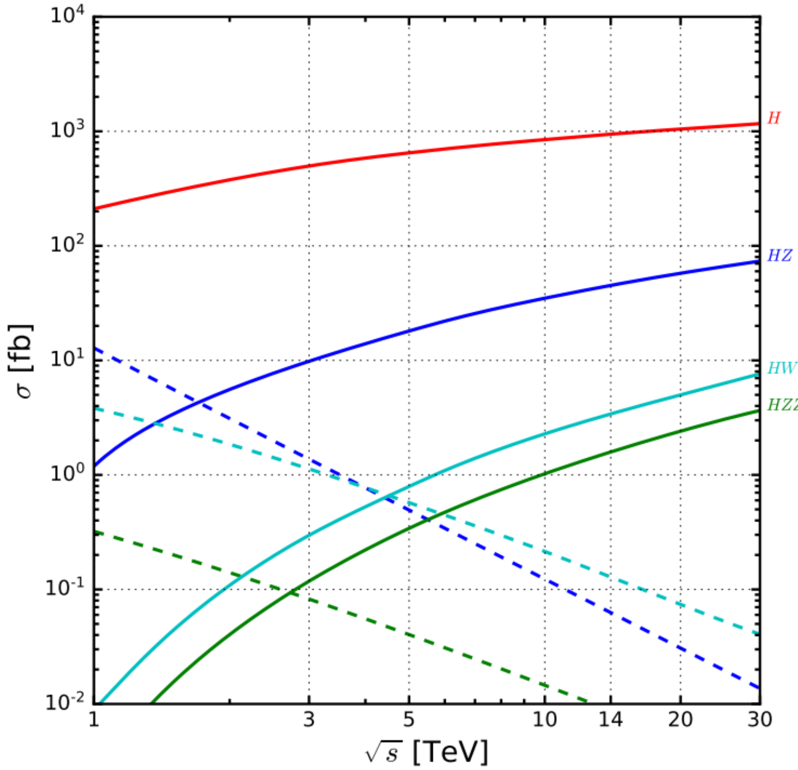
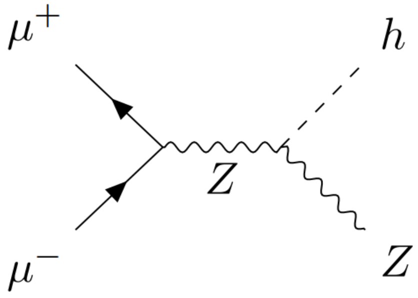
[2303.16964]

Francesco Garosi
28 September 2023 - Cortona

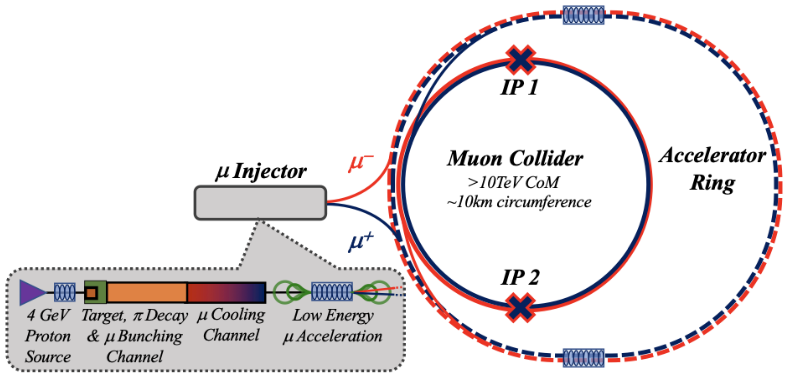
Vector boson fusion at Muon Colliders

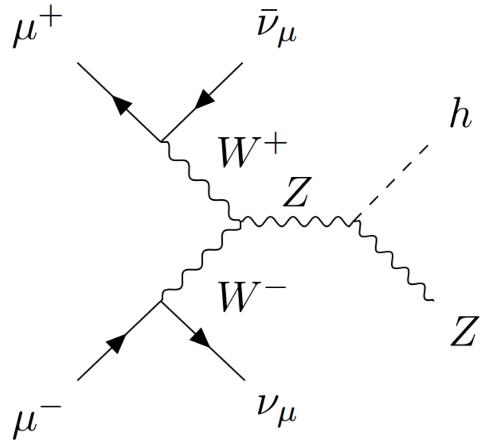


[Costantini, De Lillo, Maltoni, Mantani, Mattelaer, Ruiz, Zhao] 2005.10289



$$\sigma \propto \frac{\alpha^2}{s}$$





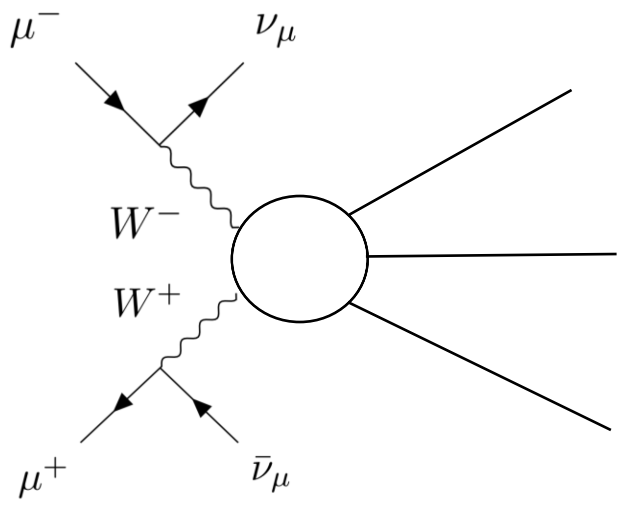
Dominates above ~ 2 TeV.

$$\frac{d\sigma}{dM_{hZ}^2} \propto \frac{\alpha^4}{M_{hZ}^2} \log\left(\frac{M_{hZ}^2}{M_W^2}\right) \log\left(\frac{s}{M_{hZ}^2}\right)$$

Muon Collider is a **Vector Boson Collider**.

Parton model for leptonic collisions

The radiation emitted by the muon is **mostly collinear** and it can be **factorized** from the hard scattering process (see 1911.12366 for the proof!), in the same way as in the **parton model for protons**.



$$\sigma_{\mu^+\mu^-\to X} = \sum_{i,j} \int_0^{\sqrt{s_0}} dm \frac{2m}{s_0} \mathcal{L}_{ij}\left(\frac{m^2}{s_0}\right) \sigma_{ij\to X}(m)$$

$$\mathcal{L}_{ij}(\tau) = \int_\tau^1 \frac{dx}{x} f_i(x,m) f_j\left(\frac{\tau}{x}, m\right)$$

Parton Distribution Functions

This works when the splittings happen at low energy w.r.t. the hard scattering: $p_T, M_{EW} \ll E$

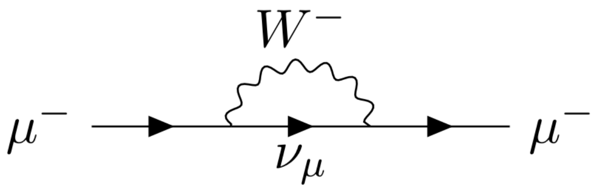
DGLAP equations

$$Q^2 \frac{df_B(x,Q^2)}{dQ^2} = P_B^v \, f_B(x,Q^2) + \sum_{A,C} \frac{\alpha_{ABC}}{2\pi} \widetilde{P}_{BA}^C \otimes f_A + \frac{v^2}{16\pi^2 Q^2} \sum_{A,C} \widetilde{U}_{BA}^C \otimes f_A$$

Virtual corrections

Massless terms

Ultra-collinear terms



$$|\mathcal{M}|^2 \propto p_T^2$$

$$|\mathcal{M}|^2 \propto v^2$$

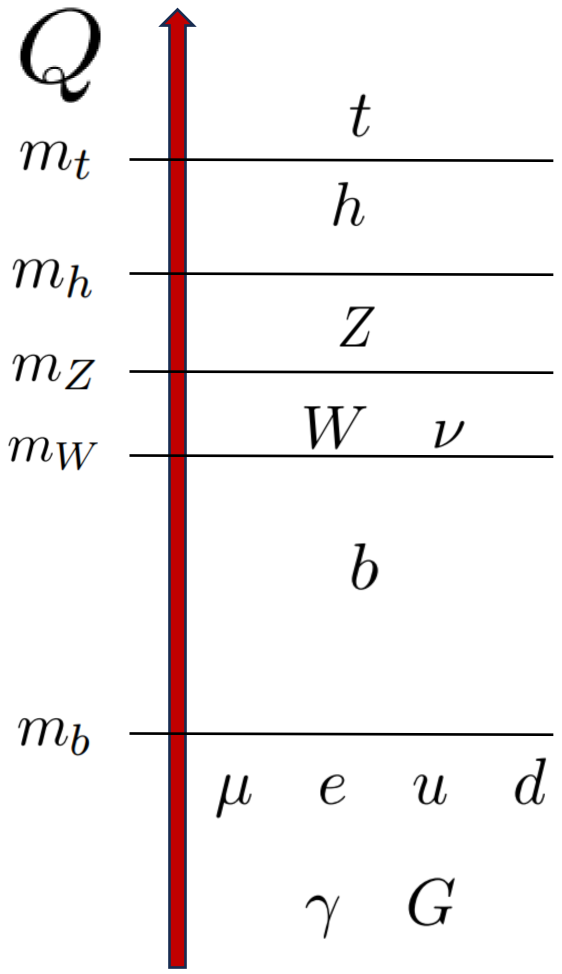
Resummation of the Leading Logarithms

The calculation is done from **first principles**, starting with $f_\mu(x, m_\mu) = \delta(1-x)$.

The strategy

1. Discretize the equations with a **grid in "x"**;
2. Use the **rectangles method** to perform the integrals;
3. Solve the coupled ODEs using a **Runge-Kutta algorithm**.

$$P \otimes f \equiv \int_x^1 \frac{dz}{z} P\left(\frac{x}{z}\right) f(z)$$



Leptons	μ_L	μ_R	e_L	e_R	ν_μ	ν_e	$\bar{\ell}_L$	$\bar{\ell}_R$	$\bar{\nu}_\ell$
Quarks	u_L	d_L	u_R	d_R	t_L	t_R	b_L	b_R	$+$ h.c.
Gauge Bosons	$\gamma_\pm$	$Z_\pm$	$Z\gamma_\pm$	$W_\pm^\pm$	$G_\pm$				
Scalars	h	Z_L	hZ_L	$W_L^\pm$					

Same equations + same i.c. = same PDF: **42** degrees of freedom, with **thresholds** at the various mass scales.

The Effective Vector Approximation

The EVA consists in taking into account just the splitting of a muon.

1. Only for $\mu, \nu_\mu, W^-, Z, \gamma, Z\gamma$

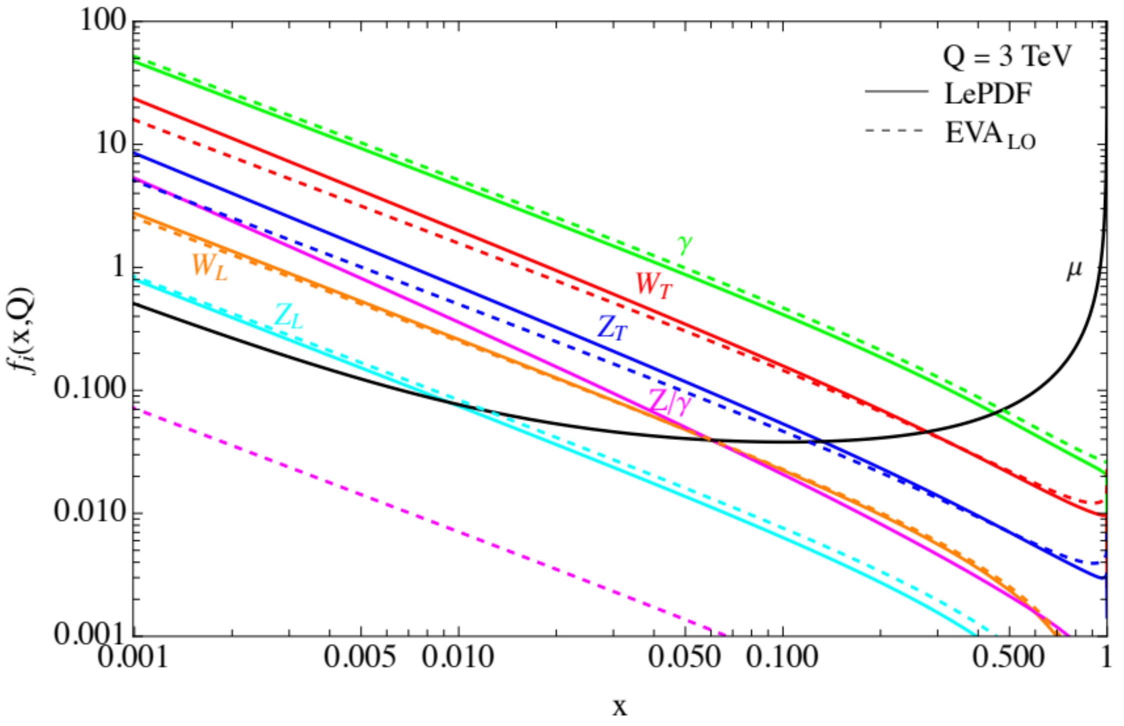
2. It does not take into account **double logarithms**.

$$f^{(\alpha)}_{W^-_\pm}(x,Q) = \frac{\alpha_2}{8\pi} P_{V^f_\pm f_L}(x) \left(\log \frac{Q^2 + (1-x)m_W^2}{m_\mu^2 + (1-x)m_W^2} - \frac{Q^2}{Q^2 + (1-x)m_W^2} \right)$$

Main **discrepancies**:

- transverse W and Z, up to O(50%) at small x and high scales;
- mixed Z-photon, $O(10^2)$.

$$\alpha_2 \log^2 \left(\frac{Q^2}{m_W^2} \right) \sim 1.1 \quad \text{for} \quad Q = 1.5\,\text{TeV}$$



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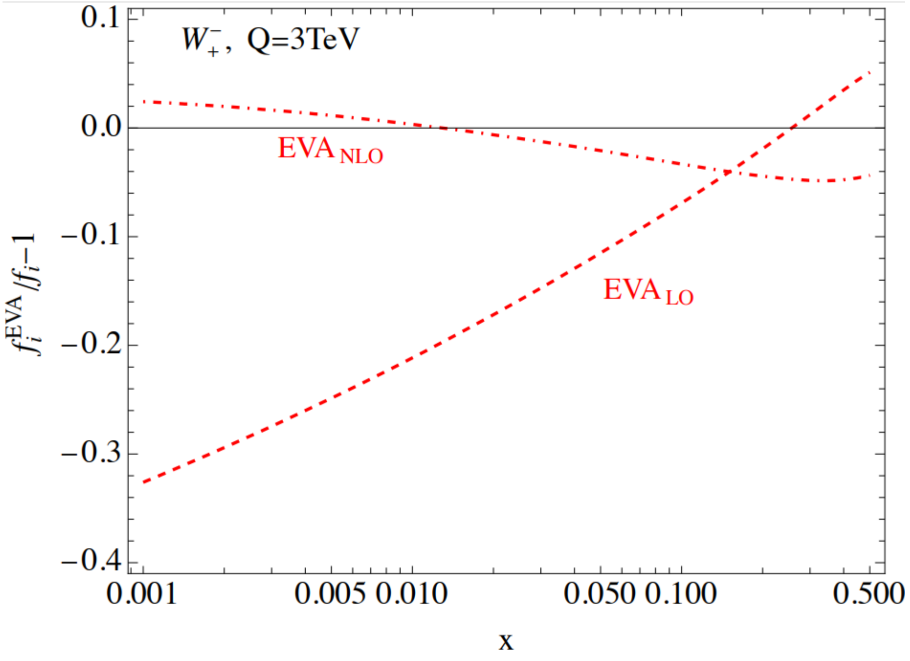
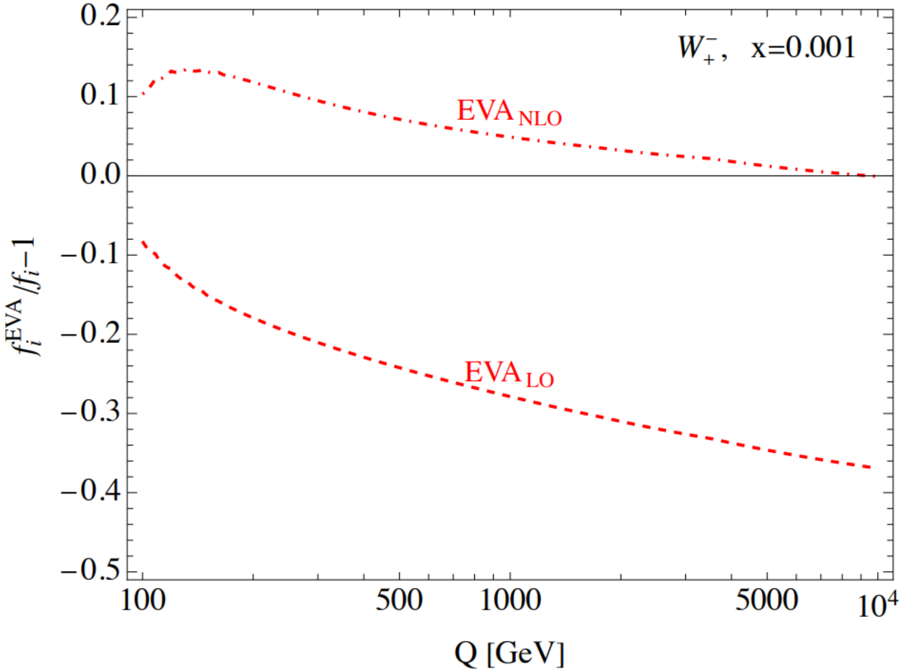
THEORETICAL PARTICLE PHYSICS

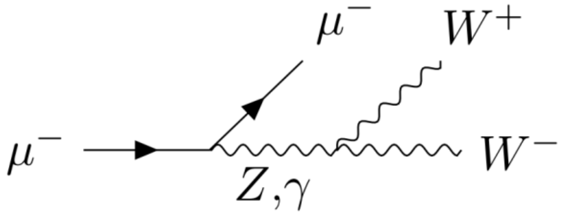
Failure of EVA (1) - EW gauge bosons

The EVA fails because we miss the three-vector vertices, which introduce DL: LO EVA gets worse at small x and large Q.





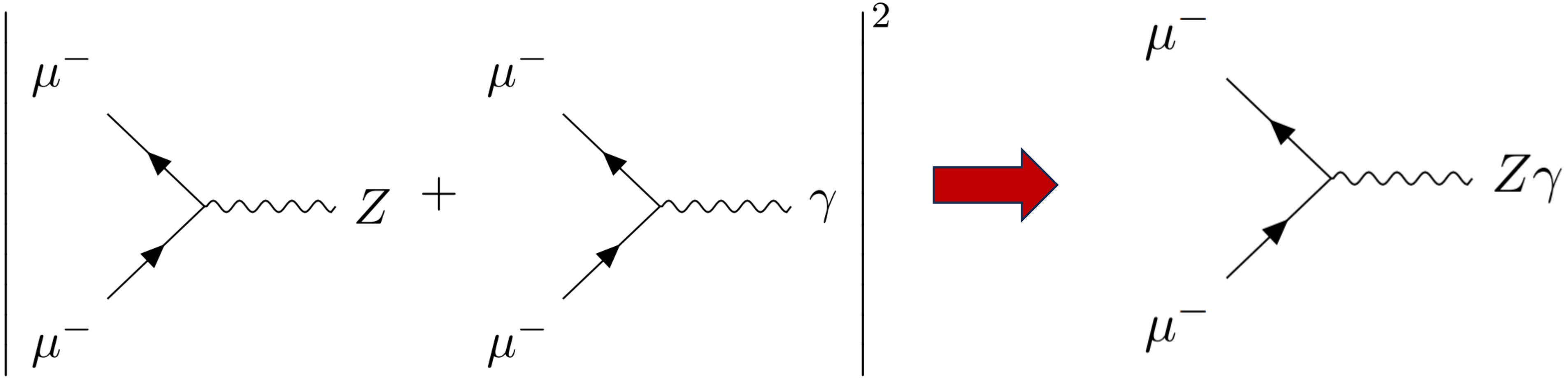
At NLO, add double splittings:



Other check: running the code without the triple gauge vertices we found a better agreement with LO EVA.

Failure of EVA (2) - Z-photon mixing

Factorization occurs at the level of the **scattering amplitude**: Z-photon PDF comes from **interference**.



$$f_{Z\gamma}^{(\alpha)} \propto \underbrace{Q^Z_{\mu_L} + Q^Z_{\mu_R}} = -\frac{1}{2} + 2s_W^2 \ll 1$$

→ **Accidental suppression**.

In the real case this sum is not there: **polarization** effects make left- and right-handed muons contribute differently.

What is on the market now?

Unfortunately the computational tools available DO NOT support the full set of LePDF.
For instance in **MadGraph5_aMC@NLO** it is only implemented an approximated version of LO EVA for W, Z and the photon.

$$f^{(\alpha)}_{W^-_\pm}(x,Q) = \frac{\alpha_2}{8\pi} P_{V^f_\pm f_L}(x)\left(\log\frac{Q^2+(1-x)m_W^2}{m_\mu^2+(1-x)m_W^2} - \frac{Q^2}{Q^2+(1-x)m_W^2}\right)$$

$$Q^2 \gg m_W^2$$

$$f^{(\alpha)}_{W^-_\pm} \sim \frac{\alpha_2}{8\pi} P^f_{V_\pm f_L} \log\left(\frac{Q^2}{(1-x)m_W^2}\right)$$

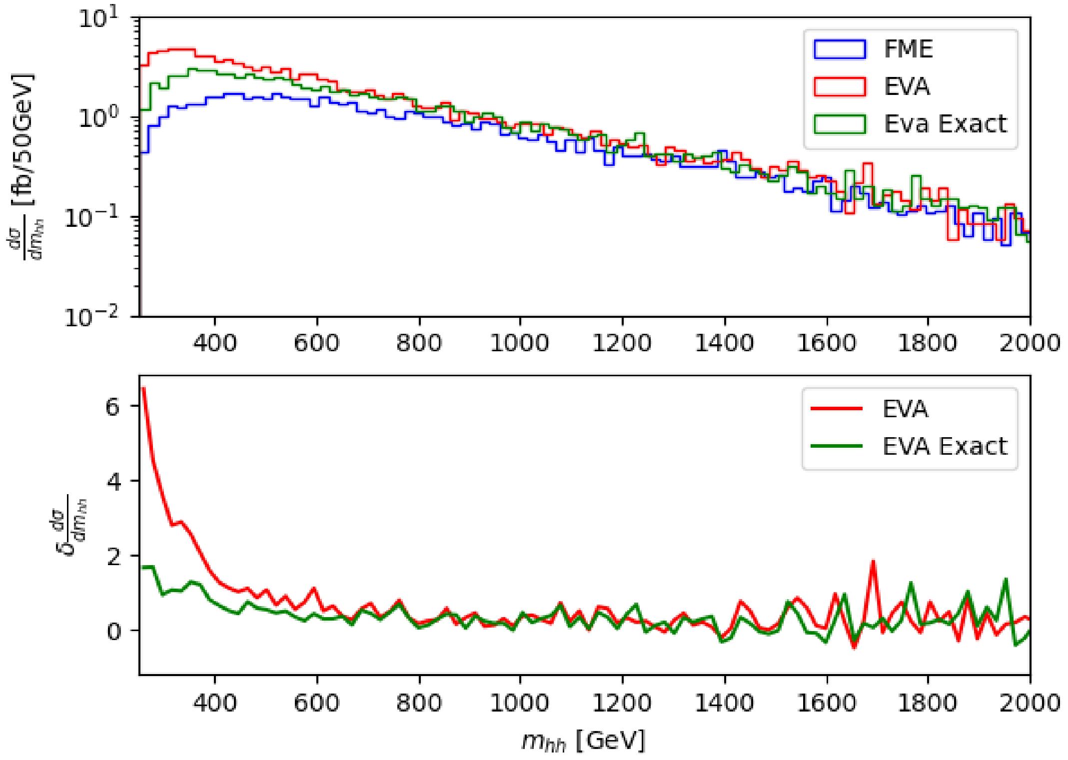
[Costantini, Maltoni, Mattelaer, Ruiz]
2111.02442

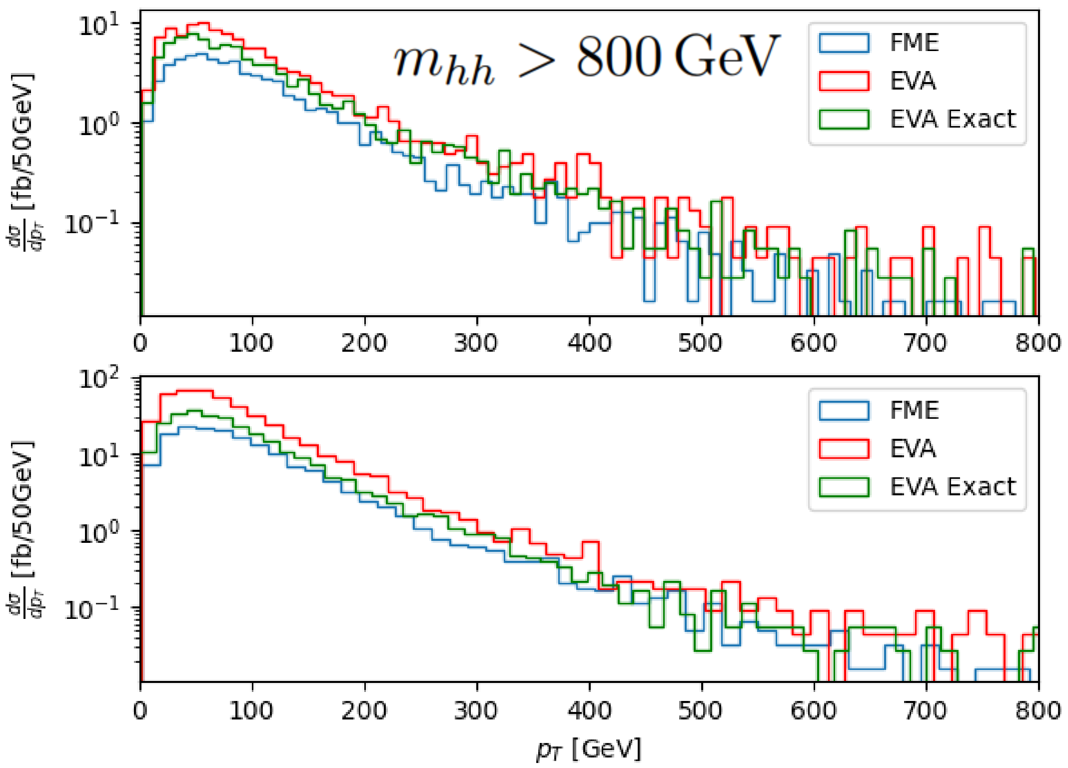
```
#*********************************************************************
# Collider type and energy                                           *
# lpp: 0=No PDF, 1=proton, -1=antiproton,                            *
#               2=elastic photon of proton/ion beam                  *
#            +/-3=PDF of electron/positron beam                      *
#            +/-4=PDF of muon/antimuon beam                          *
#*********************************************************************
     -4        = lpp1    ! beam 1 type
     4        = lpp2    ! beam 2 type
     10000.0     = ebeam1  ! beam 1 total energy in GeV
     10000.0     = ebeam2  ! beam 2 total energy in GeV
#*********************************************************************
# Beam polarization from -100 (left-handed) to 100 (right-handed)    *
#*********************************************************************
     0.0      = polbeam1 ! beam polarization for beam 1
     0.0      = polbeam2 ! beam polarization for beam 2


#*********************************************************************
# PDF CHOICE: this automatically fixes alpha_s and its evol.         *
# pdlabel: lhapdf=LHAPDF (installation needed) [1412.7420]           *
#          iww=Improved Weizsaecker-Williams Approx.[hep-ph/9310350] *
#          eva=Effective W/Z/A Approx.       [2111.02442]            *
#          edff=EDFF in gamma-UPC            [eq.(11) in 2207.03012] *
#          chff=ChFF in gamma-UPC            [eq.(13) in 2207.03012] *
#          none=No PDF, same as lhapdf with lppx=0                   *
#*********************************************************************
     eva     = pdlabel     ! PDF set
     230000     = lhaid     ! if pdlabel=lhapdf, this is the lhapdf number
```

EVA vs full matrix elements

We can compare the EVA with the full fixed-order matrix element to look at regimes of validity.

$$\mu^+ \mu^- \to \nu_\mu \bar{\nu}_\mu h\, h$$





Conclusions and outlooks

- We computed from first principles the **PDFs** for leptons. They are now public and accessible on github and can be used to study processes at future lepton colliders (e.g. for **sensitivity studies**).

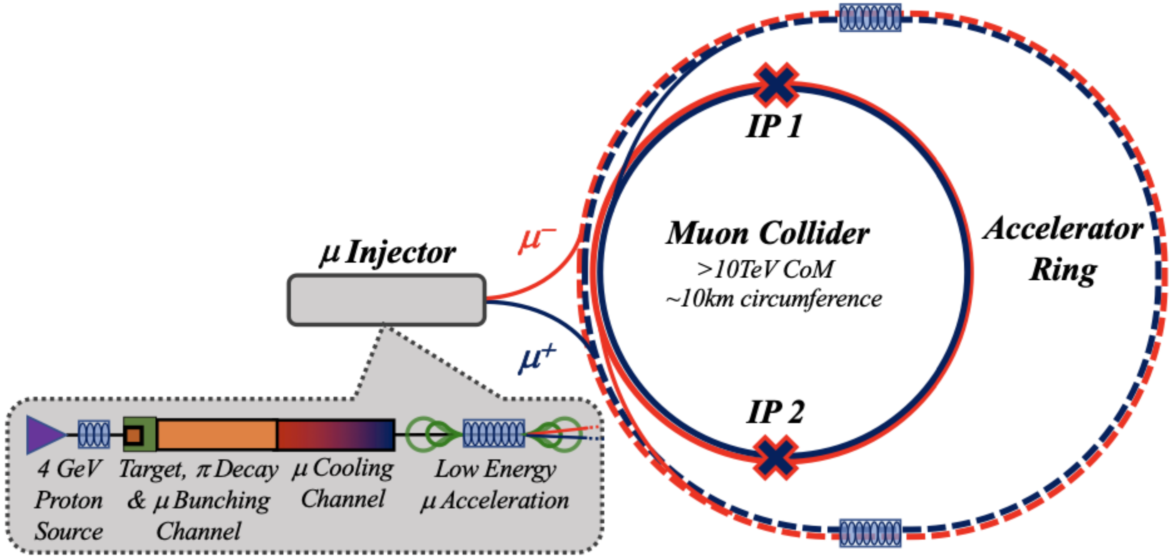
- The PDFs show significant deviations with EVA, up to O(50%) for transverse W and Z and O(10^2) for Z-photon, showing the limits of such an approximation.

- Unfortunately, at the moment only EVA is implemented in MadGraph and comparing it with fixed order full matrix elements we saw that it can be used only in certain regimes. One of the main goals for the future is then implementing LePDFs in MadGraph, maybe improved to NLL.

THANKS FOR THE ATTENTION!

Backup Slides

The Muon Collider



- $\mu^+\mu^-$ circular collider
- Could start around **2045**
- Collider Rings:
  - **3 TeV ~ 4.5 km** circumference
  - **10 TeV ~10 km** circumference

[Reports] 2201.07895, 2203.08033, 2203.07224, 2203.07256, 2203.07261

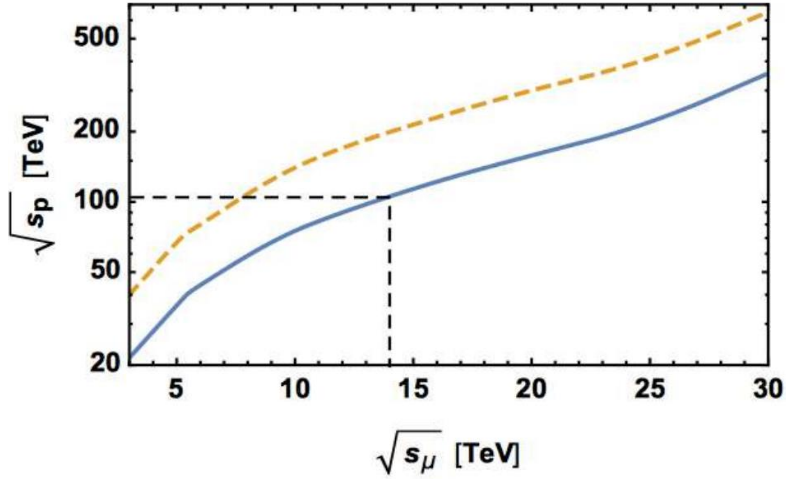
See also GGI Tea Break on MuC: https://youtu.be/17JoTcuIs6k

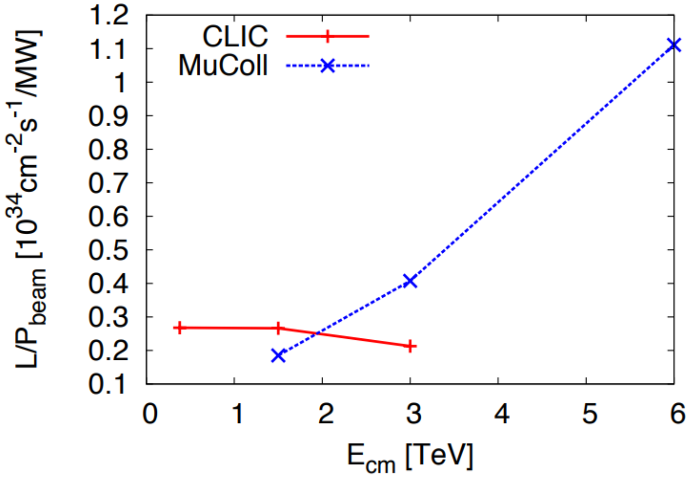
International MUON Collider Collaboration

Why Muon Colliders?

Muon colliders combine the advantages of both proton-proton (high-energy) and electron-positron colliders (precision):

- high energy reach (not limited by synchrotron radiation)
- high precision measurements (low QCD background & clean initial state)
- Luminosity / Beam power increases with energy.
- all beam energy available in $\mu+\mu-$ collisions.





General formalism

DGLAP: $$Q^2 \frac{df_B(x,Q^2)}{dQ^2} = P_B^v(x,Q^2) f_B(x,Q^2) + \sum_{A,C} \int_x^{z_{\max}^{ABC}} \frac{dz}{z} Q^2 \frac{d\mathcal{P}_{A\to B+C}}{dz dp_T^2}(z,Q^2)\, f_A\left(\frac{x}{z}, Q^2\right)$$

$$\frac{d\mathcal{P}_{A\to B+C}}{dz dp_T^2}(z,p_T^2) = \frac{1}{16\pi^2 \tilde{p}_T^4} z\bar{z} \left|\mathcal{M}_{A\to B+C}\right|^2$$

$$\tilde{p}_T^2 \equiv \bar{z}(m_B^2 - p_B^2) = p_T^2 + z m_C^2 + \bar{z} m_B^2 - z\bar{z} m_A^2 + \mathcal{O}\left(\frac{m^2}{E^2}, \frac{p_T^2}{E^2}\right)$$

Massless splitting functions

$$\left|\mathcal{M}_{A\to B+C}\right|^2 \equiv 8\pi\alpha_{ABC} \frac{p_T^2}{z\bar{z}} P_{BA}^C(z)$$

Ultra-collinear splitting functions

$$\left|\mathcal{M}_{A\to B+C}\right|^2 = \frac{v^2}{z\bar{z}} U_{BA}^C$$

$$\bar{z} = 1 - z$$

$$\tilde{P}_{BA}^C(z,p_T^2) = \left(\frac{p_T^2}{\tilde{p}_T^2}\right)^2 P_{BA}^C(z)$$

Rescaling to account for masses in the propagators.

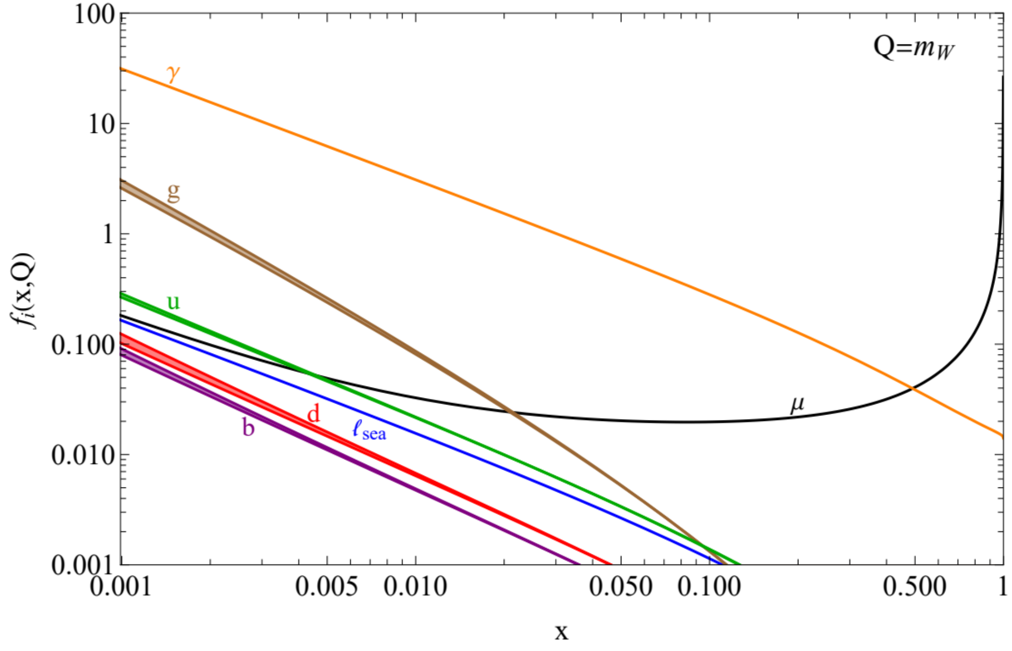
DGLAP evolution below EW scale: QED and QCD

Below the EW scale only QED and QCD contribute: in the first phase we evolve only leptons, quarks, gluon and photon with QED and QCD.

$$f_{\ell_{\rm sea}} = f_e = f_\tau = f_{\bar e} = f_{\bar\mu} = f_{\bar\tau} \,,$$
$$f_{q^u} = f_u = f_{\bar u} = f_c = f_{\bar c} \,,$$
$$f_{q^d} = f_d = f_{\bar d} = f_s = f_{\bar s} \,,$$
$$f_b = f_{\bar b} \,.$$

$$t \equiv \log\left(Q^2/m_\mu^2\right)$$

$$\frac{df_\ell}{dt} = \frac{\alpha_\gamma(t)}{2\pi}\left[P_f^v f_\ell + P_{ff}^V \otimes f_\ell + P_{fV}^f \otimes f_\gamma\right] \,,$$
$$\frac{df_{q^u}}{dt} = \frac{\alpha_\gamma(t)}{2\pi} Q_u^2 \left[P_f^v f_{q^u} + P_{ff}^V \otimes f_{q^u} + N_c P_{fV}^f \otimes f_\gamma\right] + \frac{\alpha_3(t)}{2\pi}\left[C_F P_f^v f_{q^u} + C_F P_{ff}^V \otimes f_{q^u} + T_F P_{fV}^f \otimes f_g\right] \,,$$
$$\frac{df_{q^d,b}}{dt} = \frac{\alpha_\gamma(t)}{2\pi} Q_d^2 \left[P_f^v f_{q^d,b} + P_{ff}^V \otimes f_{q^d,b} + N_c P_{fV}^f \otimes f_\gamma\right] + \frac{\alpha_3(t)}{2\pi}\left[C_F P_f^v f_{q^d,b} + C_F P_{ff}^V \otimes f_{q^d,b} + T_F P_{fV}^f \otimes f_g\right] \,,$$
$$\frac{df_\gamma}{dt} = \frac{\alpha_\gamma(t)}{2\pi}\left[P_\gamma^v f_\gamma + \sum_f Q_f^2 P_{Vf}^f \otimes (f_f + f_{\bar f})\right] \,,$$
$$\frac{df_g}{dt} = \frac{\alpha_3(t)}{2\pi}\left[C_A P_g^v f_g + C_A P_{VV} \otimes f_g + C_F P_{Vf}^f \otimes \sum_q (f_q + f_{\bar q})\right]$$



QED iterative solution

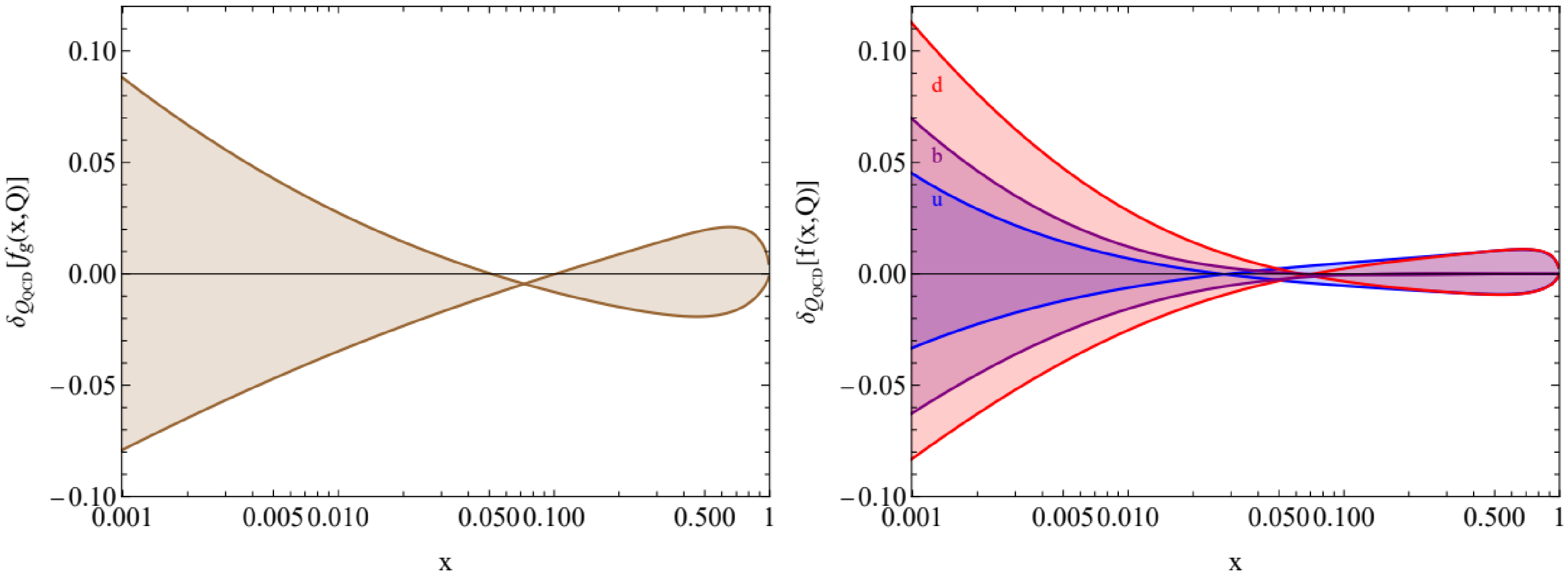
We can solve DGLAP equations for QED iteratively: at first order we use the initial conditions inside the integral and then we plug again the solution to find the higher orders.

$$f_{\mu}^{(\alpha^2)}(x,t) = \delta(1-x) + \frac{\alpha_\gamma}{2\pi} t \left(P_f^v \delta(1-x) + P_{ff}^V(x)\right)$$
$$+ \frac{1}{2}\left(\frac{\alpha_\gamma}{2\pi} t\right)^2 \left[\left(P_f^v\right)^2 \delta(1-x) + 2 P_f^v P_{ff}^V(x) + I_{fVVf}(x) + I_{ffff}(x)\right] ,$$

$$f_{\ell_{\text{sea}}}^{(\alpha^2)}(x,t) = \frac{1}{2}\left(\frac{\alpha_\gamma}{2\pi} t\right)^2 I_{fVVf}(x) ,$$

$$f_{\gamma}^{(\alpha^2)}(x,t) = \frac{\alpha_\gamma}{2\pi} t\, P_{Vf}^f(x) + \frac{1}{2}\left(\frac{\alpha_\gamma}{2\pi} t\right)^2 \left[\left(P_f^v + P_\gamma^v\right) P_{Vf}^f(x) + I_{Vfff}(x)\right]$$

DGLAP evolution below EW scale: QCD scale effects



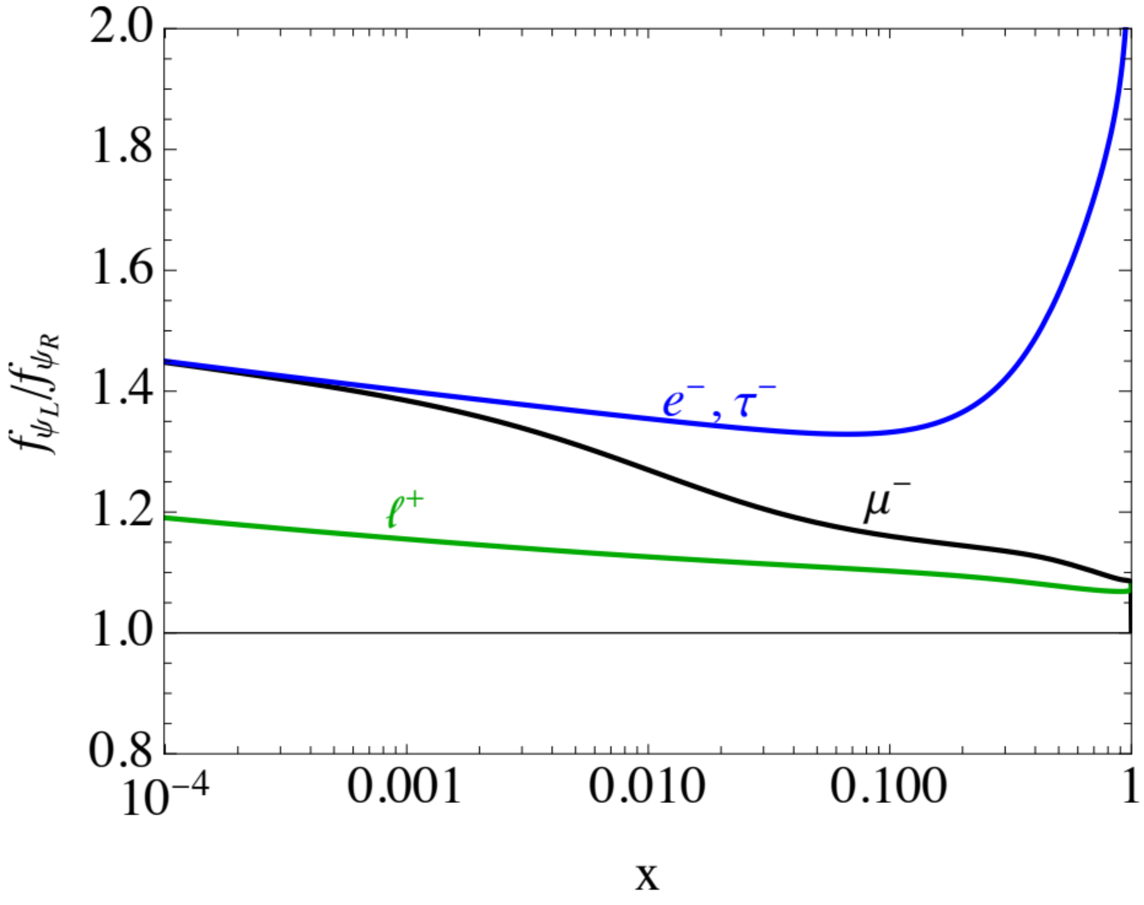
We turn on QCD interactions above a scale Q_{QCD}: this scale is not precisely determined, we choose it to be 0.7 GeV and study the effects on PDFs when we vary it.

$$\delta_{Q_{QCD}}\left[f_A(x,Q)\right] = \frac{f_A(x,Q)|_{Q_{QCD}} - f_A(x,Q)|_{0.7\ \mathrm{GeV}}}{f_A(x,Q)|_{0.7\ \mathrm{GeV}}}, \quad Q_{QCD} = \{0.52\ \mathrm{GeV}, 1\ \mathrm{GeV}\}$$

Polarization in the PDFs

EW interactions are **chiral** and induce **polarization** effects in the PDFs:

- the splitting off a W boson makes left handed fermion PDFs greater than the corresponding right handed ones;
- the effect can be of O(1).



The suppression in the Z-photon PDF never happens, because left and right muon PDFs are different and we do not get the sum of Z charges.