Top-quark loops for precision Higgs physics

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New Frontiers in Theoretical Physics
Cortona
Sep 28 2023

Outline

1. Precision Higgs Physics at the LHC
2. Example: $gg \to XY$ @ NLO QCD
3. Top-quark loops via pT expansion

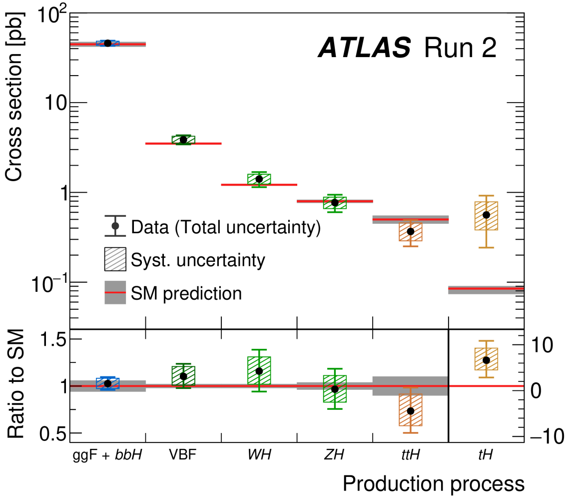
Work in collaboration with
L. Alasfar, L. Bellafronte, G. Degrassi, P.P. Giardino,
R. Gröber, X. Zhao

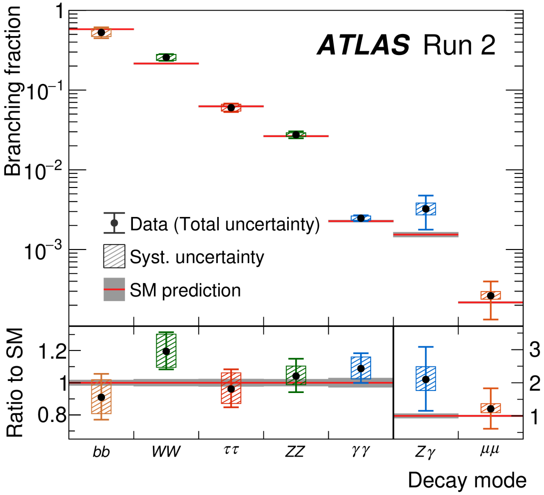
Higgs Physics at the LHC

Does the discovered Higgs boson behave as the SM predicts?

What we know after Run2 $(139\,\mathrm{fb}^{-1})$

- CP-even scalar
- Mass measured with **permille** precision
- Production and decay channels all compatible with SM predictions
- Experimental uncertainties in the 10-20% range

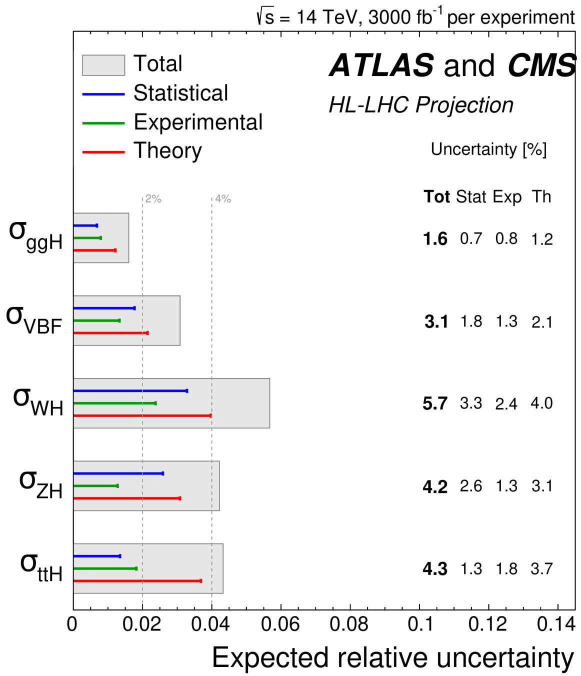


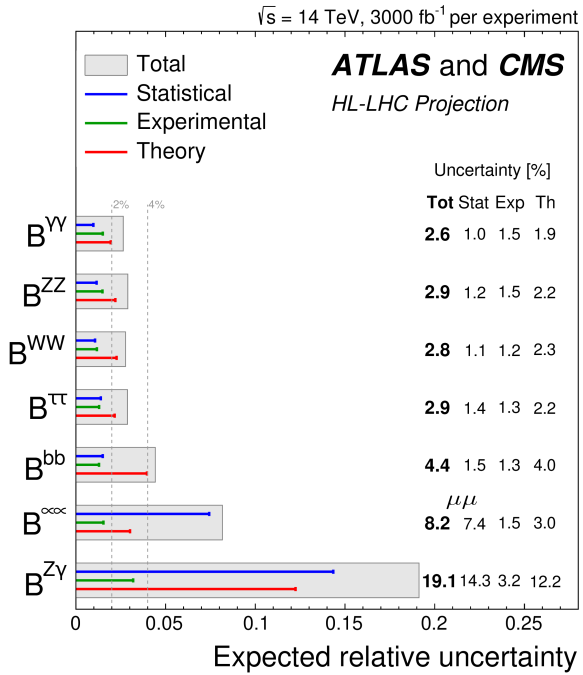


[ATLAS-2207.00092]

What next? Projections for High-Luminosity LHC

- Systematic uncertainties will play important role





[Cepeda et al. - 1902.00134]

Theory uncertainties need to be reduced

GOAL : **percent accuracy**

[THIS TALK]
Missing higher orders in perturbative calculations

(multi-)loop Feynman diagrams

Other theory uncertainties

- Parametric uncertainties
- PDF determination
- Matching with parton showers

Where to look for improvements?

- Les Houches **precision wishlist** [Huss et al. - 2207.02122]

Table 1. Precision wish list: Higgs boson final states. $\mathrm{N}^x\mathrm{LO}^{(\mathrm{VBF}*)}_{\mathrm{QCD}}$ means a calculation using the structure function approximation. $V = W, Z$.

Process	Known	Desired
$pp \to H$	$\mathrm{N^3LO_{HTL}}$ $\mathrm{NNLO}^{(t)}_{\mathrm{QCD}}$ $\mathrm{N^{(1,1)}LO^{(HTL)}_{QCD\otimes EW}}$ $\mathrm{NLO_{QCD}}$	$\mathrm{N^4LO_{HTL}}$ (incl.) $\mathrm{NNLO}^{(b,c)}_{\mathrm{QCD}}$
$pp \to H + j$	$\mathrm{NNLO_{HTL}}$ $\mathrm{NLO_{QCD}}$ $\mathrm{N^{(1,1)}LO_{QCD\otimes EW}}$	$\mathrm{NNLO_{HTL}} \otimes \mathrm{NLO_{QCD}} + \mathrm{NLO_{EW}}$
$pp \to H + 2j$	$\mathrm{NLO_{HTL}} \otimes \mathrm{LO_{QCD}}$ $\mathrm{N^3LO^{(VBF*)}_{QCD}}$ (incl.) $\mathrm{NNLO^{(VBF*)}_{QCD}}$ $\mathrm{NLO^{(VBF)}_{EW}}$	$\mathrm{NNLO_{HTL}} \otimes \mathrm{NLO_{QCD}} + \mathrm{NLO_{EW}}$ $\mathrm{N^3LO^{(VBF*)}_{QCD}}$ $\mathrm{NNLO^{(VBF)}_{QCD}}$
$pp \to H + 3j$	$\mathrm{NLO_{HTL}}$ $\mathrm{NLO^{(VBF)}_{QCD}}$	$\mathrm{NLO_{QCD}} + \mathrm{NLO_{EW}}$
$pp \to VH$	$\mathrm{NNLO_{QCD}} + \mathrm{NLO_{EW}}$ $\mathrm{NLO}^{(t,b)}_{gg \to HZ}$	
$pp \to VH + j$	$\mathrm{NNLO_{QCD}}$ $\mathrm{NLO_{QCD}} + \mathrm{NLO_{EW}}$	$\mathrm{NNLO_{QCD}} + \mathrm{NLO_{EW}}$
$pp \to HH$	$\mathrm{N^3LO_{HTL}} \otimes \mathrm{NLO_{QCD}}$	$\mathrm{NLO_{EW}}$

Table 3. Precision wish list: vector boson final states. $V = W, Z$ and $V', V'' = W, Z, \gamma$. Full leptonic decays are understood if not stated otherwise.

Process	Known	Desired
$pp \to V$	$\mathrm{N^3LO_{QCD}}$ $\mathrm{N^{(1,1)}LO_{QCD\otimes EW}}$ $\mathrm{NLO_{EW}}$	$\mathrm{N^3LO_{QCD}} + \mathrm{N^{(1,1)}LO_{QCD\otimes EW}}$ $\mathrm{N^2LO_{EW}}$
$pp \to VV'$	$\mathrm{NNLO_{QCD}} + \mathrm{NLO_{EW}}$ $+ \mathrm{NLO_{QCD}}$ (gg channel)	$\mathrm{NLO_{QCD}}$ (gg channel, w/ massive loops) $\mathrm{N^{(1,1)}LO_{QCD\otimes EW}}$
$pp \to V + j$	$\mathrm{NNLO_{QCD}} + \mathrm{NLO_{EW}}$	hadronic decays
$pp \to V + 2j$	$\mathrm{NLO_{QCD}} + \mathrm{NLO_{EW}}$ (QCD component) $\mathrm{NLO_{QCD}} + \mathrm{NLO_{EW}}$ (EW component)	$\mathrm{NNLO_{QCD}}$

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$pp \to H + 2j$	$NLO_{HTL} \otimes LO_{QCD}$ $N^3LO^{(VBF*)}_{QCD}$ (incl.) $NNLO^{(VBF*)}_{QCD}$ $NLO^{(VBF)}_{EW}$	$NNLO_{HTL} \otimes NLO_{QCD} + NLO_{EW}$ $N^3LO^{(VBF*)}_{QCD}$ $NNLO^{(VBF)}_{QCD}$
$pp \to H + 3j$	NLO_{HTL} $NLO^{(VBF)}_{QCD}$	$NLO_{QCD} + NLO_{EW}$
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$pp \to HH$	$N^3LO_{HTL} \otimes NLO_{QCD}$	NLO_{EW}

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$pp \to V + 2j$	$NLO_{QCD} + NLO_{EW}$ (QCD component) $NLO_{QCD} + NLO_{EW}$ (EW component)	$NNLO_{QCD}$

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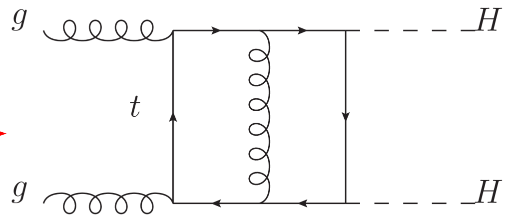

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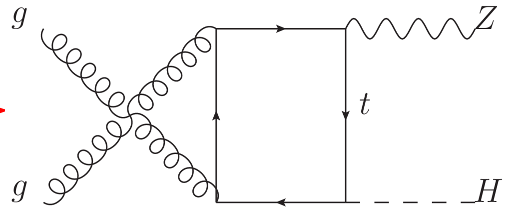
$pp \to VH$	$\mathrm{NNLO_{QCD}} + \mathrm{NLO_{EW}}$ $\mathrm{NLO}^{(t,b)}_{gg \to HZ}$	



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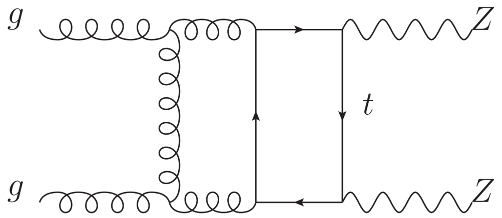
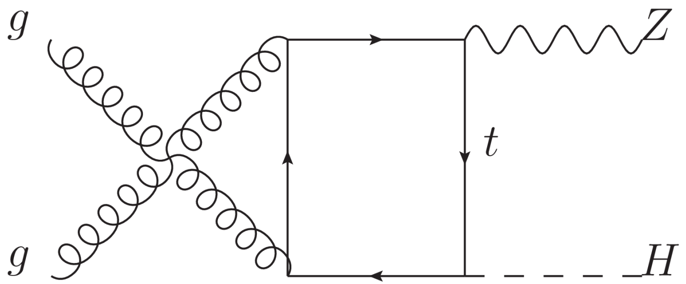


Gluon-initiated 2→2 processes
Two-loop diagrams with **massive** internal lines

Main problem in the NLO calculation
Multi-scale (m_Z, m_H, m_t, s, t) two-loop integrals
No full analytic results

$gg \to ZH$



Solutions

Numerical Evaluation [Chen, Heinrich, Jones, Kerner, Klappert, Schlenk - 2011.12325]

- (+) Exact results
- (−) Demanding in terms of computing resources and time
- (−) Issues with flexibility

- **Analytic Approximations:** exploit **hierarchies** of masses/kinematic invariants

- (+) Reduce the number of scales in Feynman integrals
- (−) Proliferation of integrals
- (−) Restricted to specific phase-space regions

- Limit $m_t \to \infty$
  [Altenkamp, Dittmaier, Harlander, Rzehak, Zirke - 1211.50]

- Large mass expansion: add finite top-mass effects
  [Hasselhuhn, Luthe, Steinhauser - 1611.05881]

- High-energy expansion: $m_Z^2, m_H^2 \ll m_t^2 \ll \hat{s}, \hat{t}$
  [Davies, Mishima, Steinhauser - 2011.12314]

- Small-mass expansion: $m_Z, m_H \to 0$
  [Wang, Xu, Xu, Yang - 2107.08206]

- pT expansion: $m_Z^2, m_H^2, p_T^2 \ll m_t^2, \hat{s}$
  [Alasfar, Degrassi, Giardino Groeber, MV – 2103.06225]
  [Bonciani, Degrassi, Giardino, Groeber - 1806.11564]

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pT Expansion: calculation overview

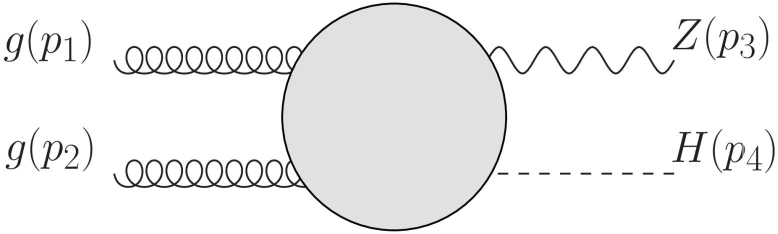
1. Generation of Feynman diagrams - O(100 diags) (FeynArts [Hahn - 0012260])
2. Lorentz decomposition of the amplitude: **projectors** and **scalar form factors** (FeynCalc [Mertig et al. ('91) ; Shtabovenko et al. - 1601.01167]): contractions, Dirac traces...

$$\mathcal{A}_{\mu\nu\rho} = \sum_{i=1}^{6} \mathcal{P}^{(i)}_{\mu\nu\rho} F^{(i)} \qquad\qquad F^{(i)} = \sum_{i=1}^{n} C^{(i)}\; I^{(i)}(\hat{s}, \hat{t}, m_Z^2, m_H^2, m_t^2)$$

3. Expansion of the form factors in the limit of small pT
4. Decomposition of scalar integrals using integration-by-parts (IBP) identities (LiteRed [Lee - 1310.1145])
5. Evaluation of master integrals

Steps implemented in **Mathematica** code on a **desktop machine**

pT Expansion



- We assume the limit of a **forward kinematics**

$$(p_1+p_3)^2 \to 0 \Leftrightarrow \hat{t} \to 0 \Rightarrow p_T \to 0$$

- Then Taylor-expand the form factors in the ratios

$$\frac{m_H^2}{\hat{s}}, \frac{m_Z^2}{\hat{s}}, \frac{p_T^2}{\hat{s}} \ll 1 \qquad \frac{p_T^2}{4m_t^2} \ll 1$$

Expansion at integrand level

- Now scalar loop integrals depend on fewer scales

$$I(\hat{s},\hat{t},m_Z^2,m_H^2,m_t^2) \to I'(\hat{s},\hat{t},m_t^2)$$

- The new scalar integrals are decomposed in MIs using IBP relations
- The MIs depend on the ratio $\hat{s}/m_t^2 \Rightarrow$ **only one scale**

$$I(\hat{s},\hat{t},m_Z^2,m_H^2,m_t^2) \to I'(\hat{s},\hat{t},m_t^2) \to \mathrm{MI}(\hat{s}/m_t^2)$$

- 52 MIs already known in the literature + SAME MIs FOR $gg \to HH$, $gg \to ZH$, $gg \to ZZ$

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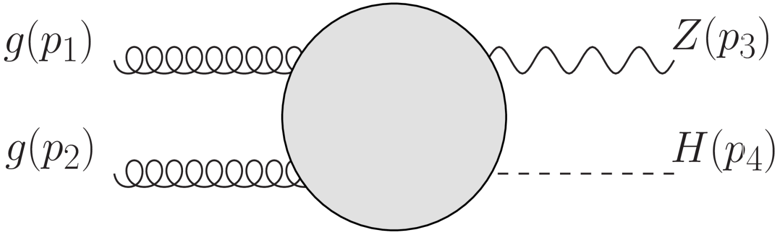

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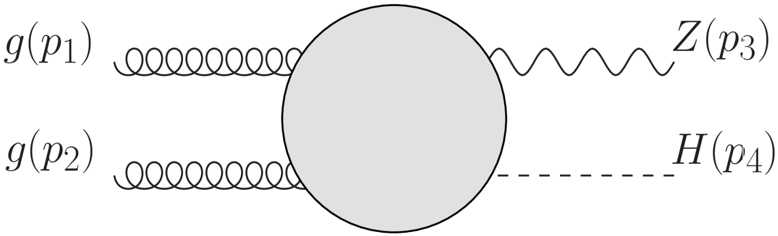

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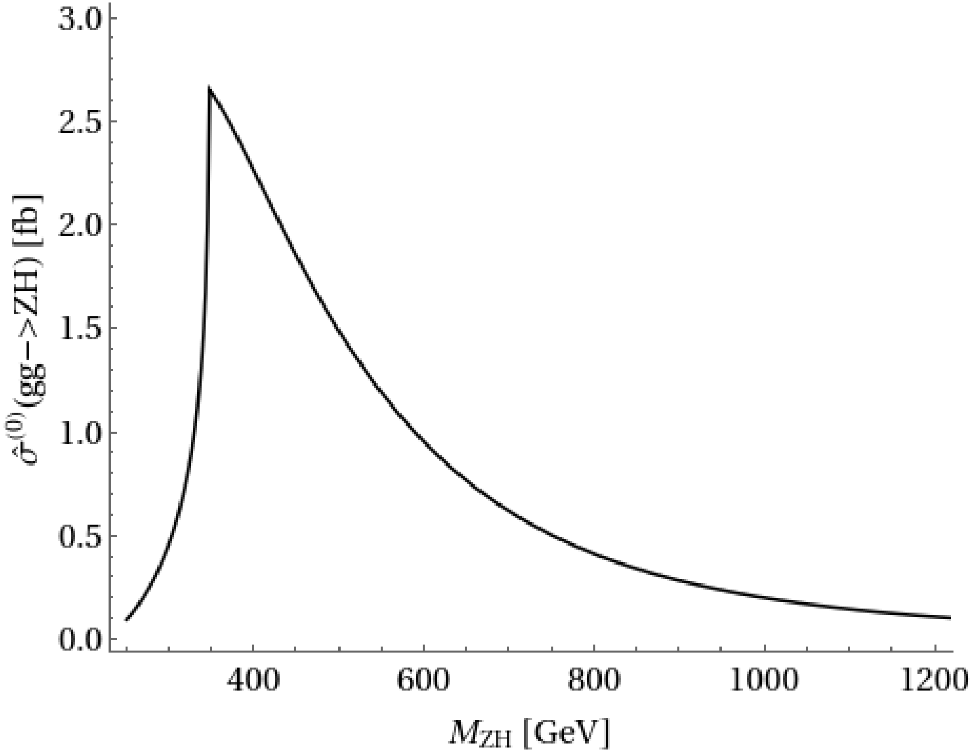
Comparing Validity Ranges



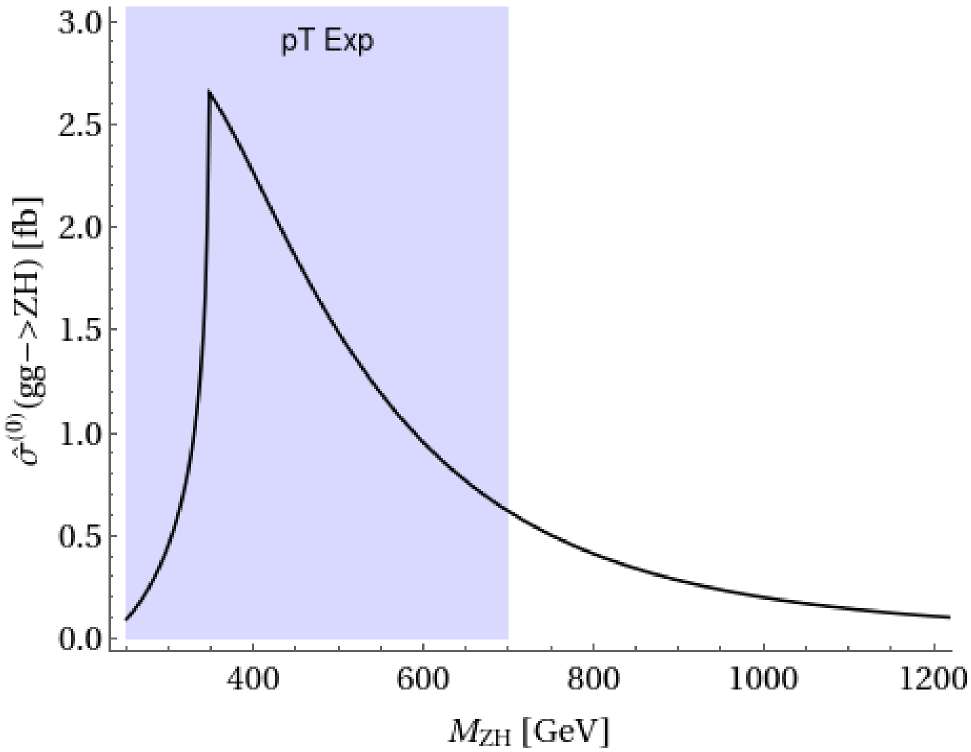
Comparing Validity Ranges

- p_T exp

$p_T^2 \lesssim 4m_t^2$

or

$\hat{t} \lesssim 4m_t^2$



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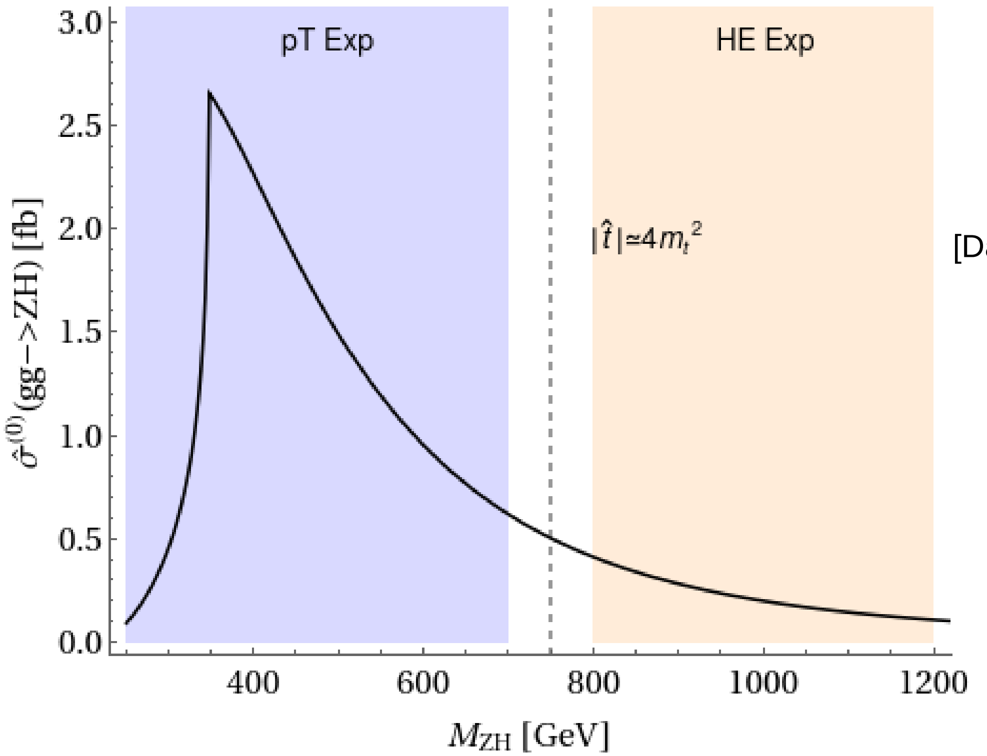


[Davies, Mishima, Steinhauser - 2011.12314]

- High-Energy exp

$\hat{t} \gtrsim 4m_t^2$

The two expansions can be combined!!

[Bellafronte, Degrassi, Giardino, Gröber, MV -2103.06225]

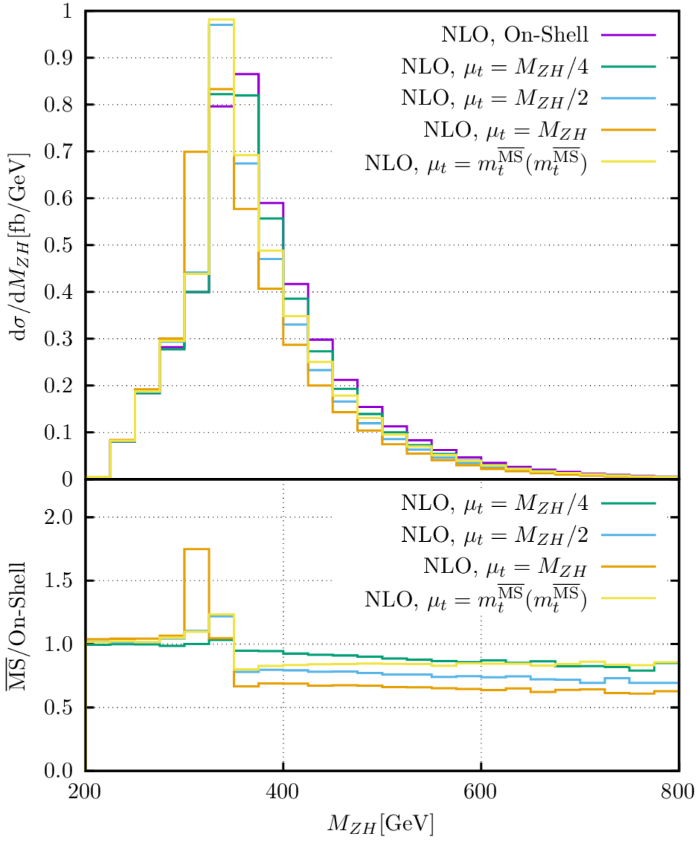
- Accuracy below the percent level $\Rightarrow$ OK for phenomenology
- Evaluation time for a phase-space point below 0.1 *s* $\Rightarrow$ suitable for Monte Carlo

Full NLO QCD Results

Inclusive cross section

Top-mass scheme	LO [fb]	$\sigma_{LO}/\sigma_{LO}^{OS}$	NLO [fb]	$\sigma_{NLO}/\sigma_{NLO}^{OS}$	$K=\sigma_{NLO}/\sigma_{LO}$
On-Shell	$64.01^{+27.2\%}_{-20.3\%}$	—	$118.6^{+16.7\%}_{-14.1\%}$	—	1.85
$\overline{\rm MS}, \mu_t = M_{ZH}/4$	$59.40^{+27.1\%}_{-20.2\%}$	0.928	$113.3^{+17.4\%}_{-14.5\%}$	0.955	1.91
$\overline{\rm MS}, \mu_t = m_t^{\overline{\rm MS}}(m_t^{\overline{\rm MS}})$	$57.95^{+26.9\%}_{-20.1\%}$	0.905	$111.7^{+17.7\%}_{-14.6\%}$	0.942	1.93
$\overline{\rm MS}, \mu_t = M_{ZH}/2$	$54.22^{+26.8\%}_{-20.0\%}$	0.847	$107.9^{+18.4\%}_{-15.0\%}$	0.910	1.99
$\overline{\rm MS}, \mu_t = M_{ZH}$	$49.23^{+26.6\%}_{-19.9\%}$	0.769	$103.3^{+19.6\%}_{-15.6\%}$	0.871	2.10

- NLO corrections are the same size as LO (*K*~2)
- Scale uncertainties reduced by 2/3 wrt LO
- Agreement with independent calculations
  [Wang et al. - 2107.08206] [Chen et al. - 2204.05225]



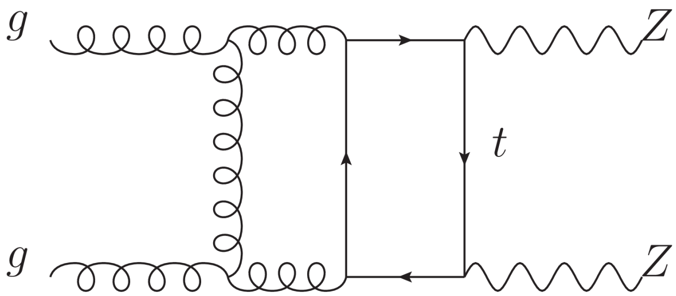
[Degrassi, Gröber, MV, Zhao - 2205.02769]

Top mass scheme uncertainty

- Take deviations of $\overline{\rm MS}$ scheme wrt OS result as top mass scheme uncertainty (used for HH production in [Baglio et al. - 1811.05692, 2003.03227])
- Analytic results → change of top mass scheme is straightforward

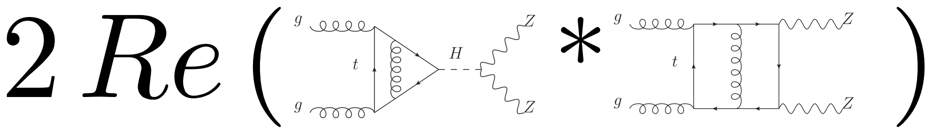
$$F_i^{NLO,\overline{\rm MS}} = F_i^{NLO,\rm OS} - \frac{1}{4}\frac{\partial F_i^{LO}}{\partial m_t^2}\Delta_{m_t^2} \qquad \Delta_{m_t^2} = 2m_t^2 C_F\left[-4+3\log\left(\frac{m_t^2}{\mu^2}\right)\right]$$

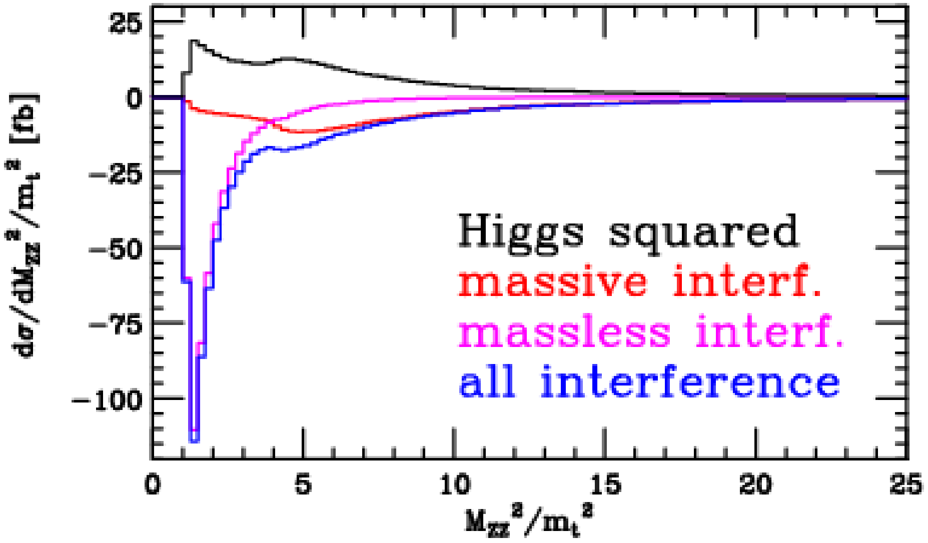
$gg \rightarrow ZZ$



$gg \to ZZ$ and Higgs physics

- Destructive interference between $gg \to H^* \to ZZ$ and $gg \to ZZ$ in the off-shell region
- Relevant for indirect measurements of Higgs total width
  [Kauer, Passarino – 1206.4803]
  [Caola, Melnikov – 1307.4935]
  [Campbell. Ellis, Williams - 1311.3589]
- Top loops are dominant in off-shell region
- Use pT expansion for the two-loop box diagrams
  [Degrassi, Gröber, MV – in preparation]



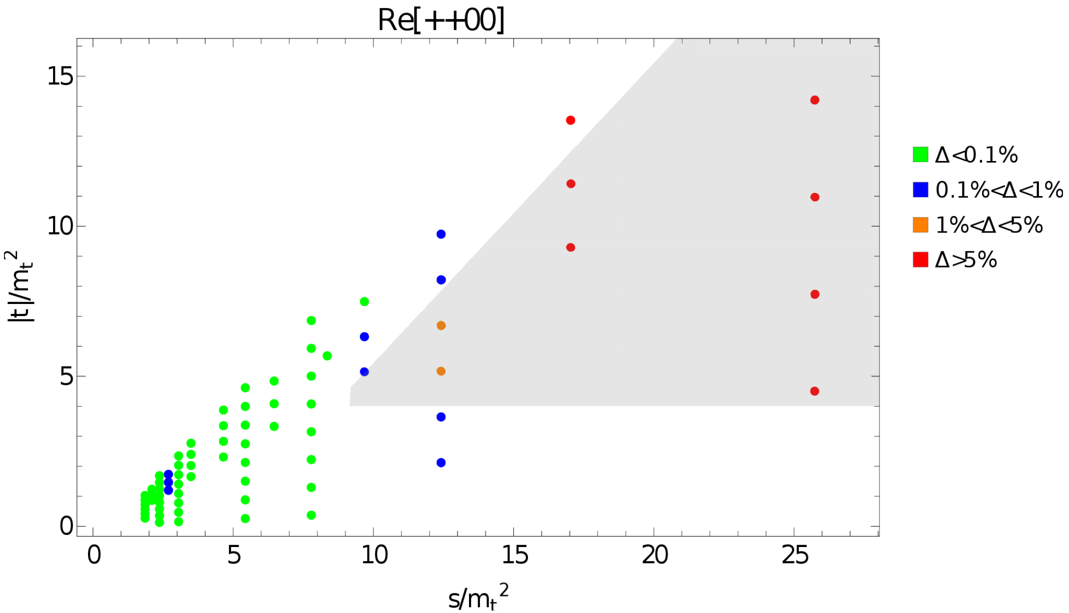


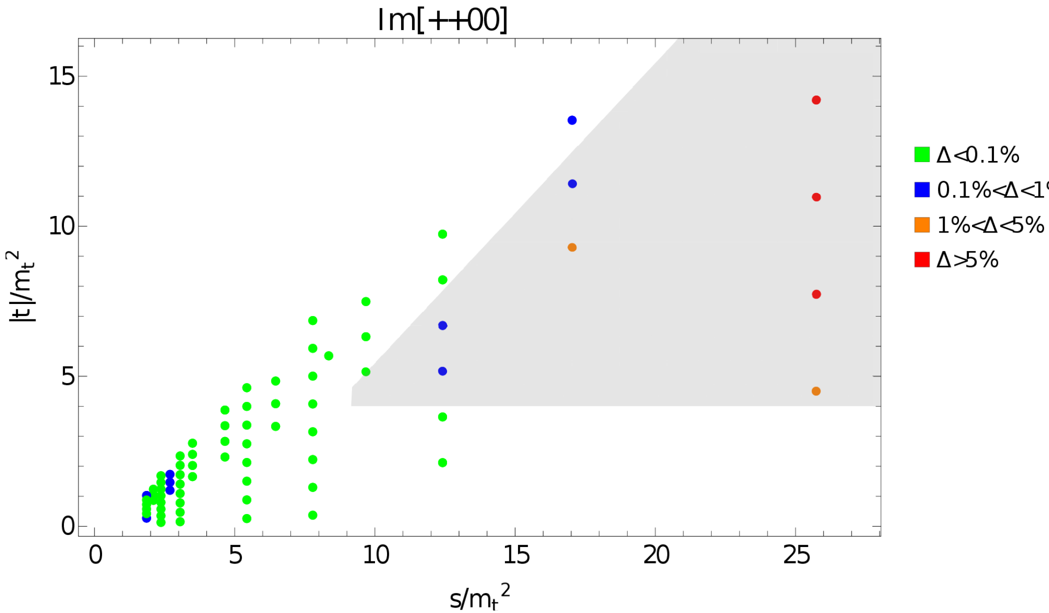
[Campbell et al. - 1605.01380]

Helicity amplitudes at NLO

$$\mathcal{M}^{\text{fin}}_{\lambda_1,\lambda_2,\lambda_3,\lambda_4} = \left(\frac{\alpha_s}{2\pi}\right)\mathcal{M}^{(1)}_{\lambda_1,\lambda_2,\lambda_3,\lambda_4} + \left(\frac{\alpha_s}{2\pi}\right)^2 \boxed{\mathcal{M}^{(2)}_{\lambda_1,\lambda_2,\lambda_3,\lambda_4}} + \mathcal{O}(\alpha_s^3)$$

[Agarwal, Jones, von Manteuffel - 2011.15113]





[PRELIMINARY]

Next step → combine pT and HE expansions

Conclusions & outlook

- Higgs precision measurements call for improved theoretical predictions
- $2 \to 2$ processes with **massive** loops are hard to compute
- Analytic approximations are useful: flexibility and efficiency
- pT and high-energy expansions can be combined

  see also [Davies, Mishima, Schönwald, Steinhauser - 2302.01356]

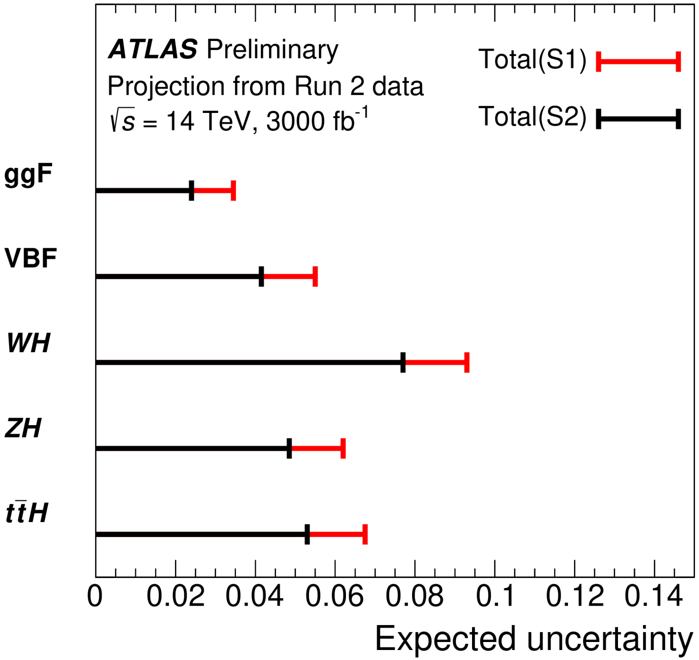
- $gg \to WW$? How to deal with both heavy and light quarks running in the loops
- EW corrections to $2 \to 2$ processes? Possibly different master integrals

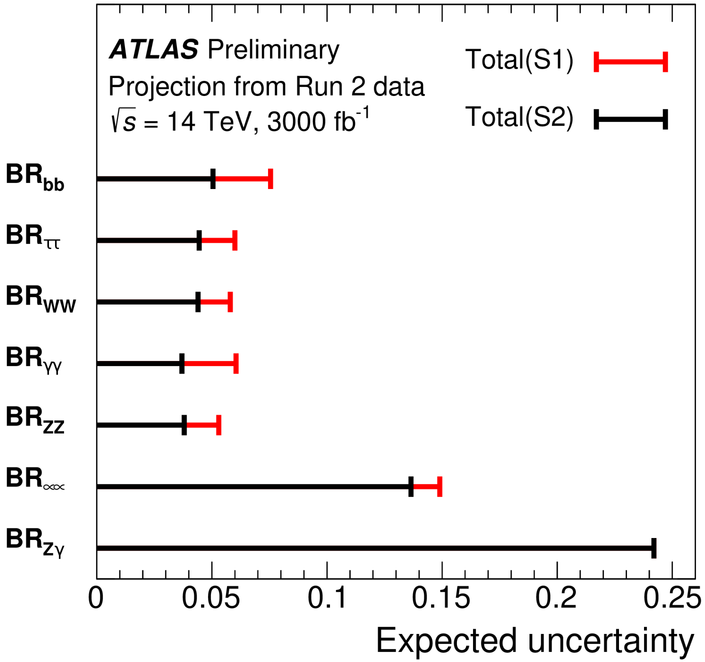
Thank you for your attention

Backup

What next? Projections for High-Luminsoity LHC

- Systematic uncertainties will begin to dominate





[ATLAS-PHYS-PUB-2018-0548-054]

- Scenario 1: systematics as in Run2 (conservative)
- Scenario 2: exp sys corrected; theo sys halved

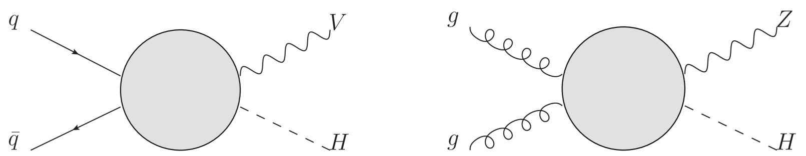
VH Production

$pp \to VH$ is the most sensitive process to $H \to b\bar{b}$ [ATLAS-2007.02873, CMS-1808.08242]

- Two partonic channels in *pp->ZH*:

  $q\bar{q} \to ZH$ - dominant contribution

- $gg \to ZH$ - about 6% of $\sigma(pp \to ZH)$



- Theory prediction in MC codes:

  $q\bar{q} \to ZH$: NNLO accuracy [Han, Willenbrock- '91] [Brein, Djouadi, Harlander- 0307206]

  $gg \to ZH$: LO accuracy → Large scale uncertainties

Production mode	$\Delta_y^{<VH>}$
WH	$\pm 0.7\%$
$q\bar{q} \to ZH$	$\pm 0.6\%$
$gg \to ZH$	$\pm 25\%$

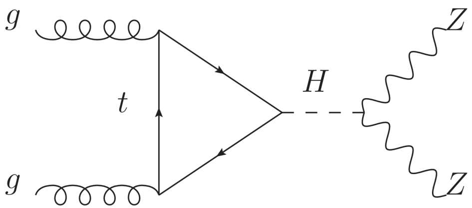
(No *gg*-channel for WH)

[CERN Yellow Report 4]

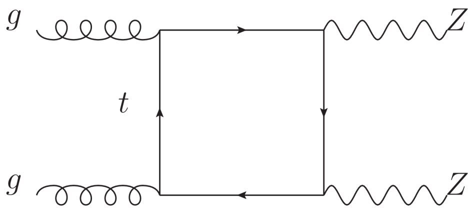
If we really want to improve the theory prediction **we need to go beyond LO in $gg \to ZH$**

ZZ Production

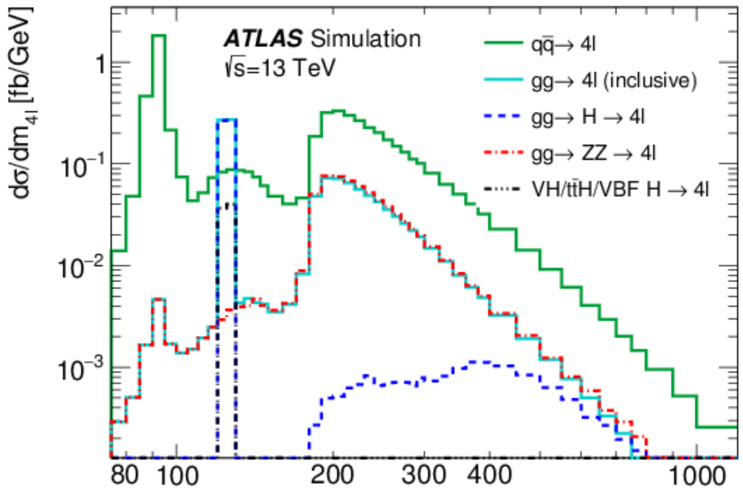
- $pp \to ZZ$ provides access to **single-Higgs** production via gluon fusion
- $q\bar{q} \to ZZ$ gives dominant contribution to hadronic cross section
- $gg \to ZZ$ is about 10% of $\sigma(pp \to ZZ)$



(Higgs-mediated)



(continuum)
Irreducible background



[ATLAS – 1902.05892]

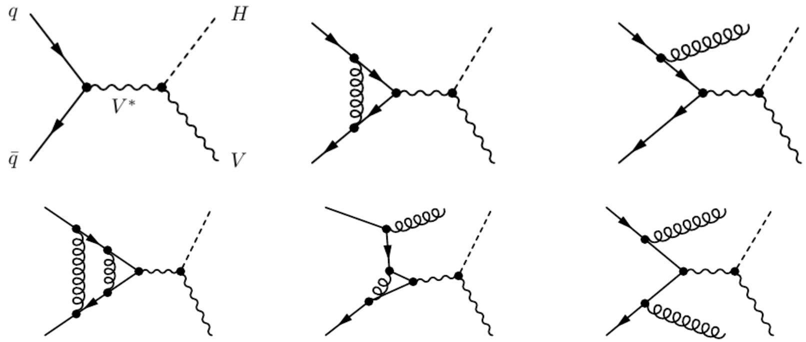
- Knowledge of the background is important for **Higgs width** determination via off-shell measurements

Theoretical predictions for $pp \to ZH$

LO: quark-initiated tree-level contribution

QCD Effects: mainly due to Drell-Yan (DY) production followed by $Z^* \to ZH$ decay

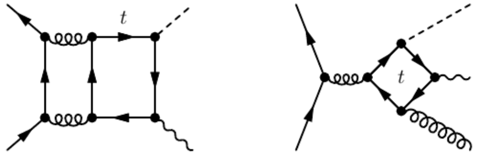
- **Drell-Yan:**



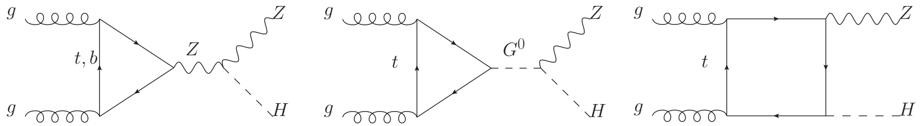
Known through NNLO (O(α_s^2))
(+30% wrt LO) [Han, Willenbrock ('91) ; Hamberg, van Neerven, Matsuura ('92) ; Brein, Djouadi, Harlander - 0307206]

- **Non Drell-Yan:**

Quark-initiated O(1%) wrt LO
[Brein, Harlander, Wiesemann, Zirke - 1111.0761]

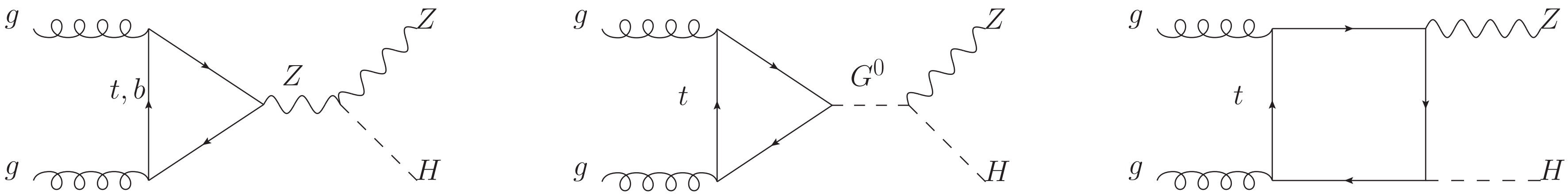


Gluon-initiated



- **EW corrections:** known through NLO (-(5-10%) wrt LO) [Dittmaier et al. - 1211.5015]

$gg \to ZH$ @ LO



- Third generation gives dominant contribution [Kniehl ('90) - Dicus, Kao ('88)]
- $\mathcal{O}(\alpha_s^2)$ correction to $pp \to ZH$ cross section
- NNLO suppression wrt to $q\bar{q} \to ZH$ but gluon luminosity higher at LHC
- Contributes to about 6% of $\sigma(pp \to ZH)$ for $\sqrt{s} = 14$ TeV
- Only LO included in MC $\to$ scale variation leads to **25%** relative uncertainties
- NLO corrections expected to be large in *gg* processes (e.g. *H, HH*)

[Cepeda et al. - 1902.00134]

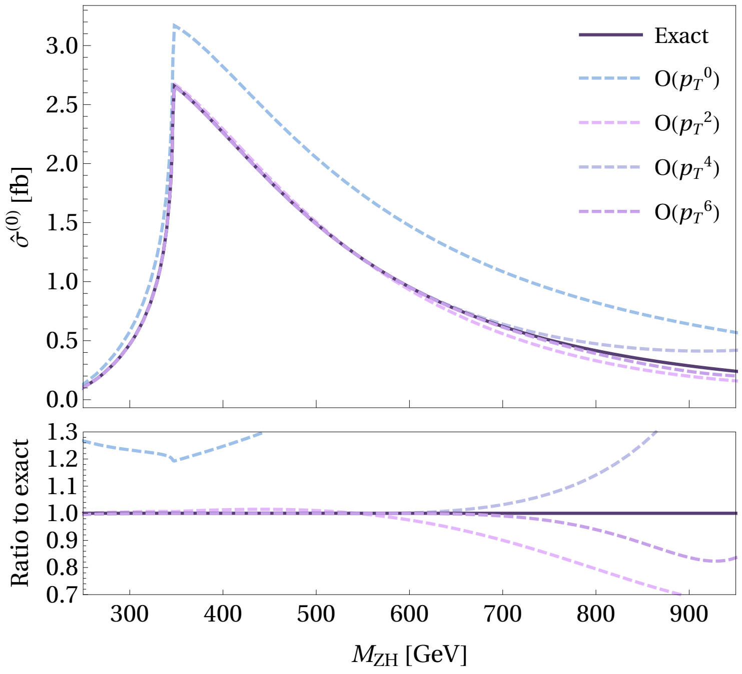
$\sqrt{s}$ [TeV]	$\sigma_{\text{NNLO QCD}\otimes\text{NLO EW}}$ [pb]	Δ_{scale} [%]	$\Delta_{\text{PDF}\oplus\alpha_s}$ [%]
13	0.123	$^{+24.9}_{-18.8}$	4.37
14	0.145	$^{+24.3}_{-19.6}$	7.47
27	0.526	$^{+25.3}_{-18.5}$	5.85

LO Validation

- Three orders sufficient for very good accuracy
- Reliable results for $M_{ZH} \lesssim 700$ GeV
- For $M_{ZH} \gtrsim 700$ GeV the assumption

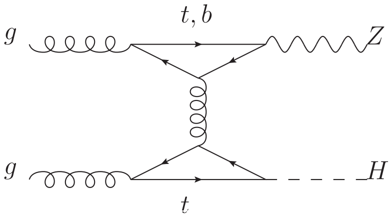
$$p_T^2 \ll 4m_t^2$$

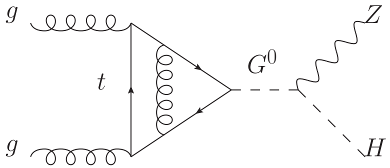
can be violated $\Rightarrow$ the p_T expansion **diverges** (but wait a few slides...)

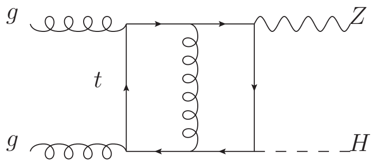


$gg \to ZH$ @ NLO in QCD: all ingredients

Virtual corrections ($2 \to 2$, two loops): merging pt+HE expansions





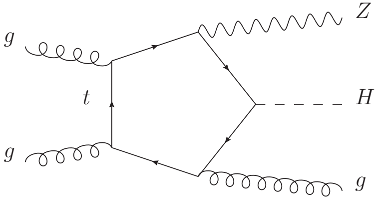


Real emission ($2 \to 3$, one loop): automated evaluation (RECOLA2, MadGraph5)

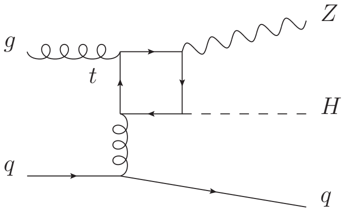
[Denner, Lang, Uccirati - 1711.07388]
[Alwall et al. - 1405.0301]

We included all diagrams that:

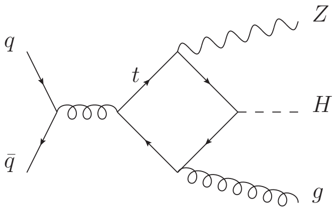
- give O(α_S^3) contribution to the cross section *pp->ZH*
- feature a closed fermion loop



$gg \to ZHg$



$qg \to ZHq$



$q\bar{q} \to ZHg$

Full NLO QCD Results

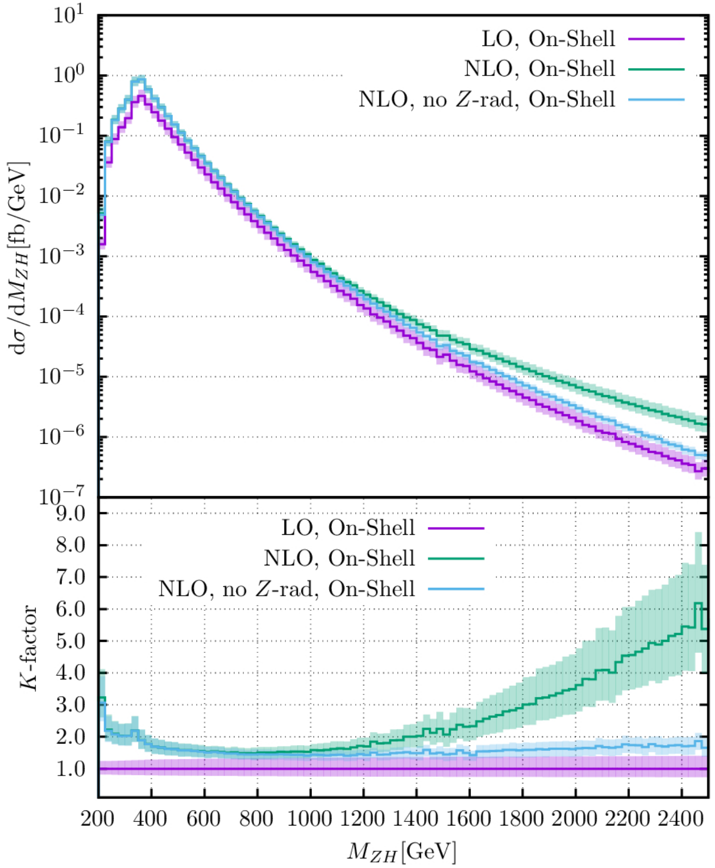
Inclusive cross section

Top-mass scheme	LO [fb]	$\sigma_{LO}/\sigma_{LO}^{OS}$	NLO [fb]	$\sigma_{NLO}/\sigma_{NLO}^{OS}$	$K = \sigma_{NLO}/\sigma_{LO}$
On-Shell	$64.01^{+27.2\%}_{-20.3\%}$	—	$118.6^{+16.7\%}_{-14.1\%}$	—	1.85
$\overline{\text{MS}}, \mu_t = M_{ZH}/4$	$59.40^{+27.1\%}_{-20.2\%}$	0.928	$113.3^{+17.4\%}_{-14.5\%}$	0.955	1.91
$\overline{\text{MS}}, \mu_t = m_t^{\overline{\text{MS}}}(m_t^{\overline{\text{MS}}})$	$57.95^{+26.9\%}_{-20.1\%}$	0.905	$111.7^{+17.7\%}_{-14.6\%}$	0.942	1.93
$\overline{\text{MS}}, \mu_t = M_{ZH}/2$	$54.22^{+26.8\%}_{-20.0\%}$	0.847	$107.9^{+18.4\%}_{-15.0\%}$	0.910	1.99
$\overline{\text{MS}}, \mu_t = M_{ZH}$	$49.23^{+26.6\%}_{-19.9\%}$	0.769	$103.3^{+19.6\%}_{-15.6\%}$	0.871	2.10

- Top mass renormalized both in OS and $\overline{\text{MS}}$ scheme
- NLO corrections are the same size as LO (K~2)
- Scale uncertainties reduced by 2/3 wrt LO
- Agreement with independent calculations
  [Wang et al. - 2107.08206] [Chen et al. - 2204.05225]

M_{ZH} distribution

- K-factor is not flat over M_{ZH} range
- Large NLO enhancement in the high-energy tail (M_{ZH}>1 TeV)



[Degrassi, Gröber, MV, Zhao - 2205.02769]

Full NLO QCD Results

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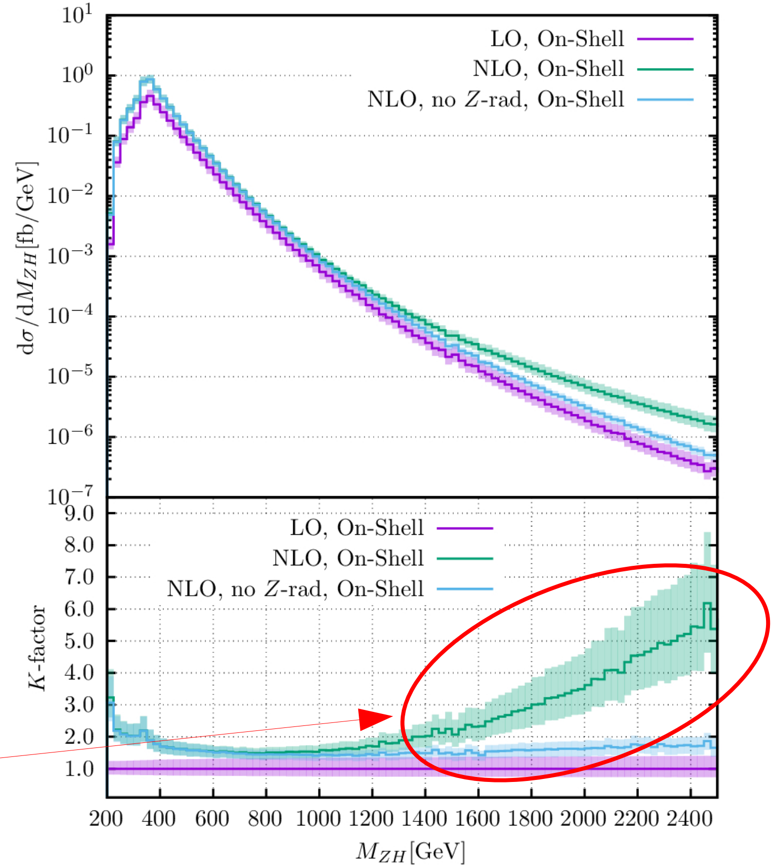
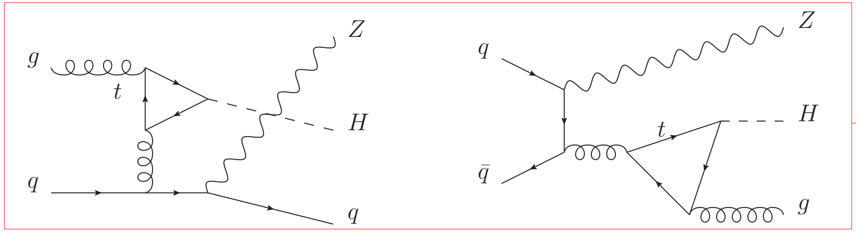


[Degrassi, Gröber, MV, Zhao - 2205.02769]

Z-radiated diagrams

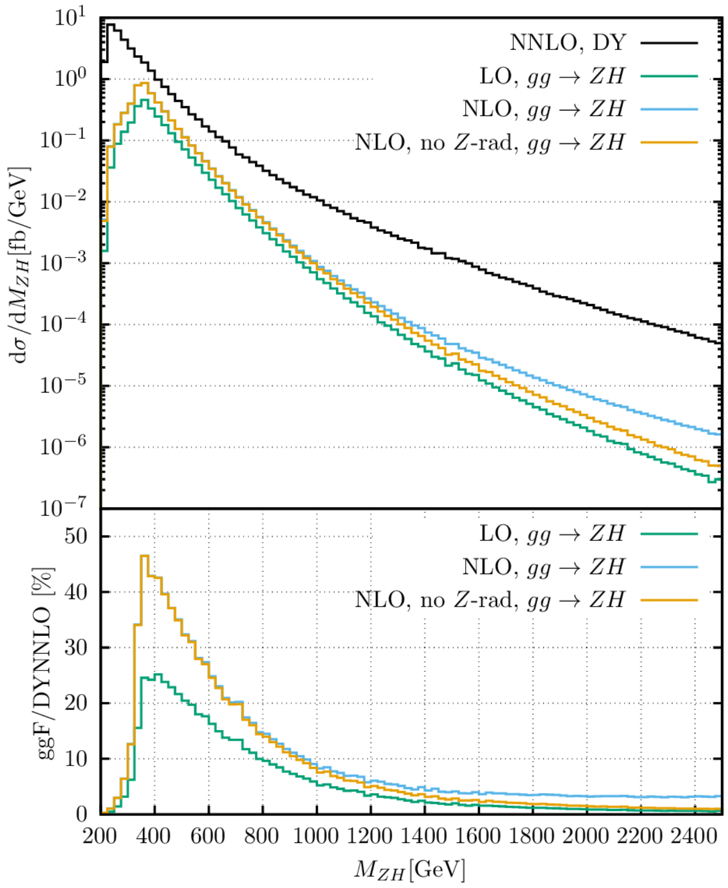
- Large EW Logs?

$$\log^2\left(\frac{m_Z^2}{M_{ZH}^2}\right)$$



$gg \to ZH$ @ NLO: comparing with Drell-Yan contribution

- ***gg→ZH*** is almost 50% of DY near $M_{ZH} \sim 2\, m_t$
- Because of Z -radiated diagrams the gg contribution falls off as rapidly as the DY one (ratio constant at ~ 2%)
- DY obtained using **vh@nnlo**
  [Harlander et al - 1802.04817]



[Degrassi, Gröber, MV, Zhao - 2205.02769]

$gg \to ZH$ @NLO QCD - Top mass schemes

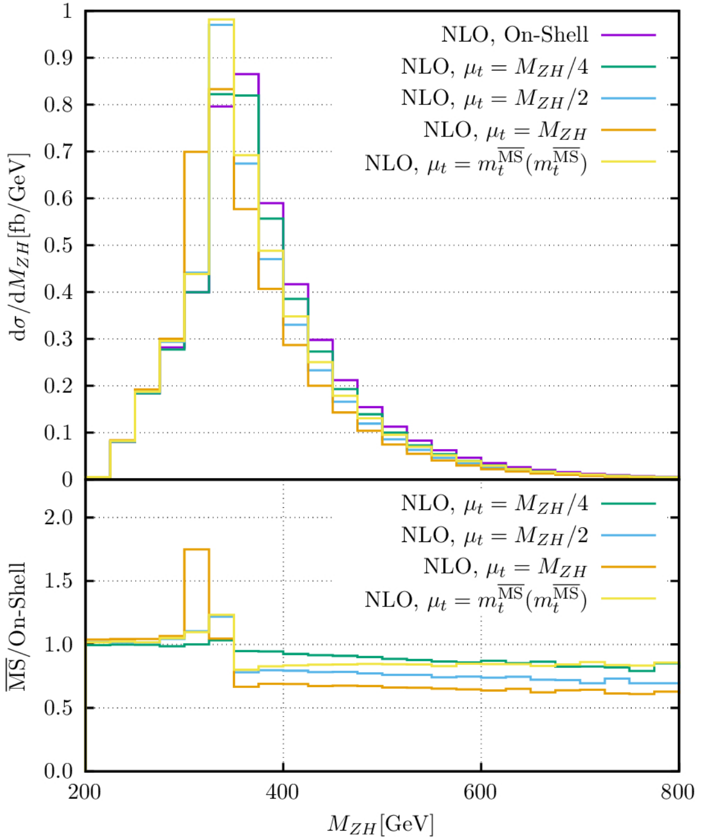
- Take deviations of $\overline{\text{MS}}$ scheme wrt OS result as top mass scheme uncertainty
(used for HH production in [Baglio et al. - 1811.05692, 2003.03227])

- Analytic results → change of top mass scheme is straightforward

$$F_i^{NLO,\overline{\text{MS}}} = F_i^{NLO,\text{OS}} - \frac{1}{4}\frac{\partial F_i^{LO}}{\partial m_t^2}\Delta_{m_t^2} \qquad \Delta_{m_t^2} = 2m_t^2 C_F\left[-4 + 3\log\left(\frac{m_t^2}{\mu^2}\right)\right]$$

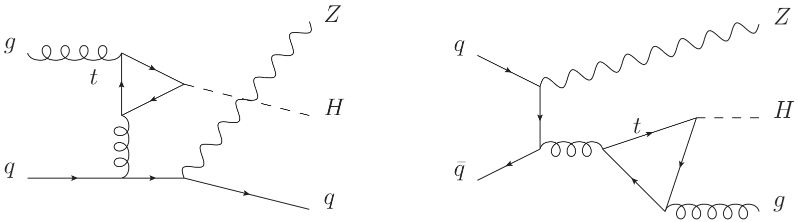
Bin Width [GeV]	LO	NLO
1	$64.01^{+15.6\%}_{-35.9\%}$	$118.6^{+17.2\%}_{-27.0\%}$
5	$64.01^{+15.3\%}_{-35.6\%}$	$118.6^{+14.7\%}_{-24.9\%}$
25	$64.01^{+14.0\%}_{-33.1\%}$	$118.6^{+10.9\%}_{-20.8\%}$
100	$64.01^{+2.0\%}_{-25.3\%}$	$118.6^{+0.6\%}_{-13.7\%}$
∞	$64.01^{+0\%}_{-23.1\%}$	$118.6^{+0\%}_{-12.9\%}$

Avoid overestimate of mt uncertainty



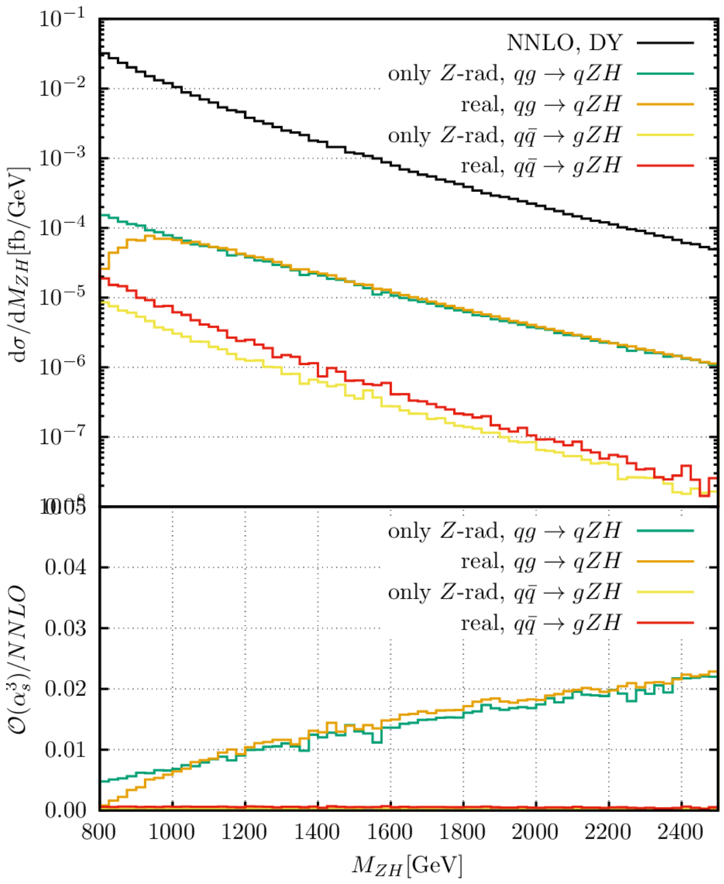
[Degrassi, Gröber, MV, Zhao - 2205.02769]

The effect of Z-radiated diagrams



In the high-energy tail (M_{ZH}> 1 TeV)

- **qg → ZHq channel**
  - Z-radiated diagrams dominate
  - Non-negligible contribution (up to 2% wrt DY)
- ***q$\bar{q}$*** **→ ZHg channel**
  - Z-radiated diagrams dominate
  - Negligible (PDF suppression)



[Degrassi, Gröber, MV, Zhao - 2205.02769]

Merging pT and HE expansions at NLO

Improve the convergence of a series expansion by matching the coefficients of the **Pade approximant** [m/n] [e.g. Fleisher, Tarasov ('94)]

$$f(x) \overset{x\to 0}{\simeq} c_0 + c_1 x + \cdots + c_q x^q \qquad f(x) \simeq [m/n](x) = \frac{a_0 + a_1 x + \cdots + a_m x^m}{1 + b_1 x + \cdots + b_n x^n} \quad (q = m+n)$$

[Bellafronte, Degrassi, Giardino, Gröber, MV -2103.06225]

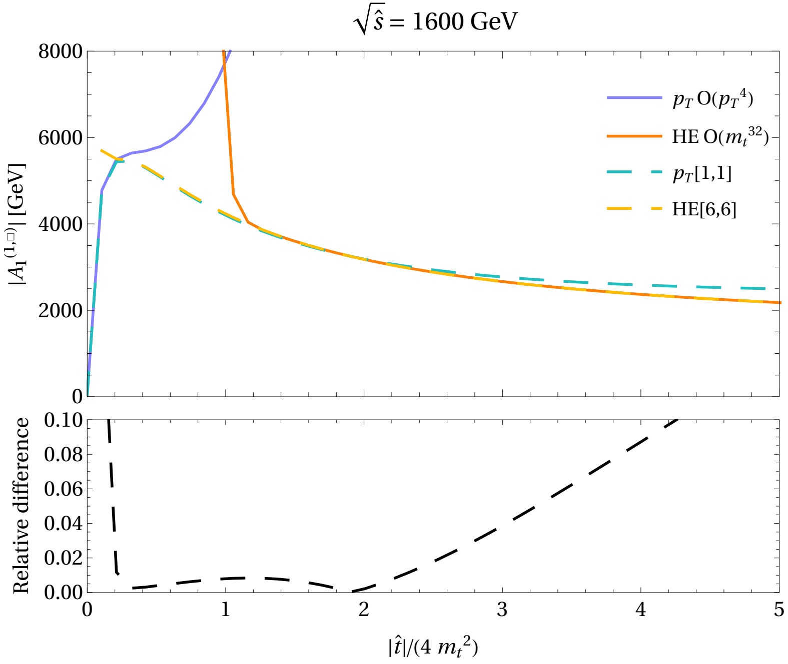
- For each FF we merged the following results

  - pT exp improved by [1/1] Padé

  - HE exp improved by [6/6] Padé

- Padé results are stable and comparable in the region $|\hat{t}| \sim 4 m_t^2 \rightarrow$ can switch without loss of accuracy (% level or below)

- Evaluation time for a phase-space point below 0.1 $s \Rightarrow$ suitable for Monte Carlo



Integration-by-Parts Reduction

Express a scalar integral as a function of denominator exponents

$$I(n_1,\dots,n_N) = \int d^Dk_1 \cdots d^Dk_L \frac{1}{D_1^{n_1}\cdots D_N^{n_N}} \qquad (n_i \in \mathbb{Z})$$

Recurrence relations connecting scalar integrals with different n_i from differentiation

$$\int d^Dk_1 \cdots d^Dk_L \frac{\partial}{\partial k_i^\mu} \frac{q_j^\mu}{D_1^{n_1}\cdots D_N^{n_N}} = 0$$

The process can be **iterated** $\Rightarrow$ each scalar integral in the amplitude can be decomposed along a basis of master integrals

$$I(n_1,\dots,n_N) = \sum_j C^{(j)} MI^{(j)}(\mathbf{z_1},\dots,\mathbf{z_N}) \qquad z_i \in \{0,1,2\}$$

- For $gg \to ZH$ @ NLO: from ~200.000 scalar integrals to 52 MIs
- First simplification with pT expansion $\Rightarrow$ simpler IBP $\Rightarrow$ simpler MIs

Master Integrals

52 MIs already known in the literature

SAME MIs FOR $gg \to HH$, $gg \to ZH$, $gg \to ZZ$

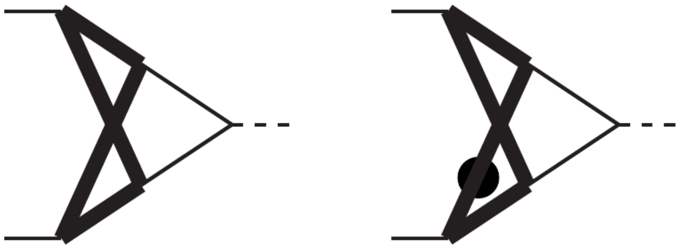
- 50 MIs expressed in terms of Generalized Polylogarithms (GPLs)

  [Bonciani, Mastrolia, Remiddi ('03) - Aglietti et al. ('06) - Anastasiou et al. ('06) - Caron-Huot, Henn ('14) - Becchetti, Bonciani ('17) - Bonciani, Degrassi, Vicini ('10)]

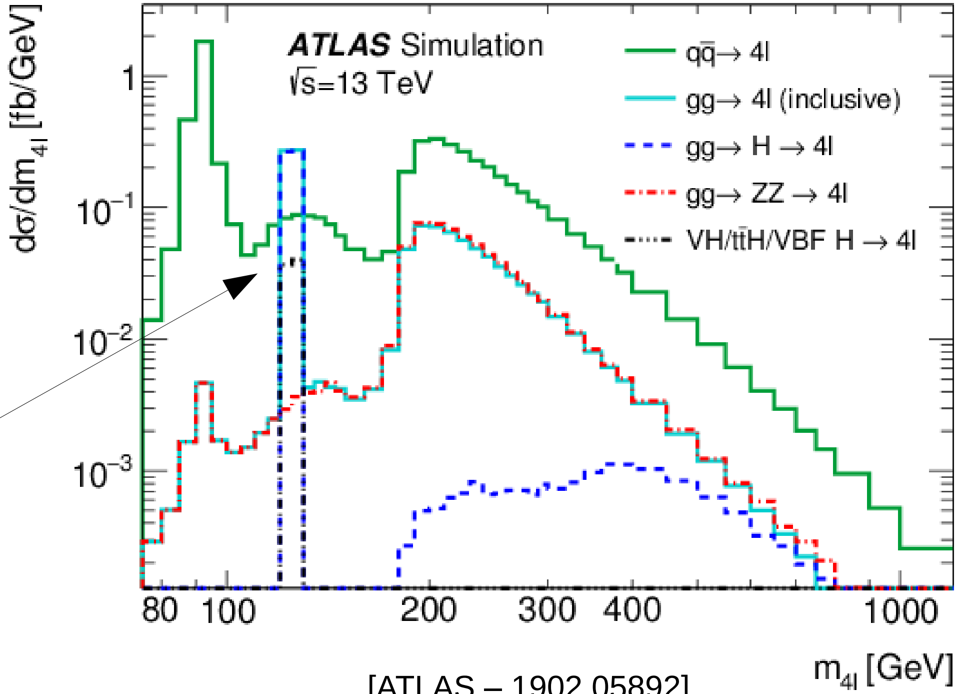
- Two elliptic integrals [von Manteuffel, Tancredi ('17)]

  Semi-analytical evaluation implemented in FORTRAN routine

  [Bonciani, Degrassi, Giardino, Gröber ('18)]



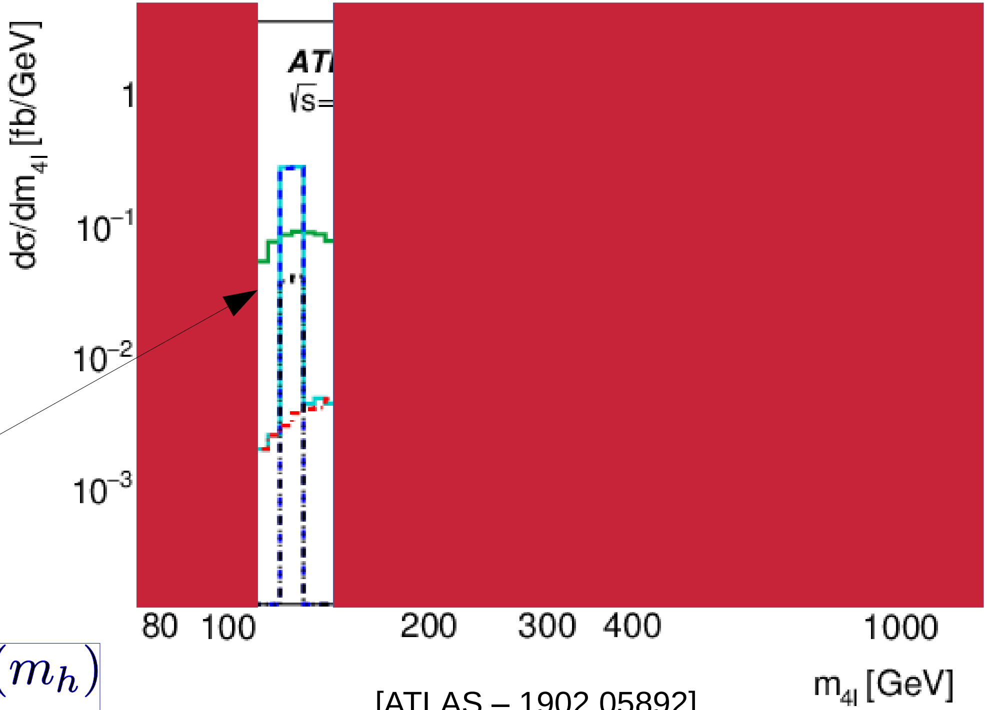
$pp \to ZZ$



On-shell H production

[ATLAS – 1902.05892]

Higgs width from off-shell measurements

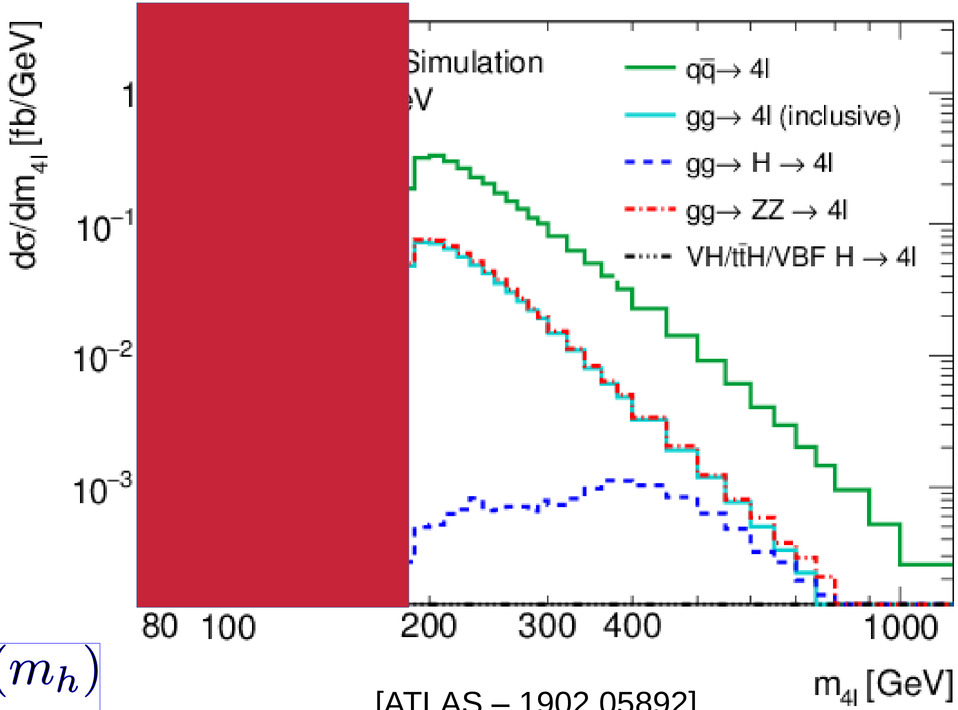


On-shell H production

$$\mu_{\rm on} = \frac{\kappa^2_{ggh}(m_h)\kappa^2_{hZZ}(m_h)}{\Gamma_h/\Gamma_h^{\rm SM}}$$

[ATLAS – 1902.05892]

Higgs width from off-shell measurements



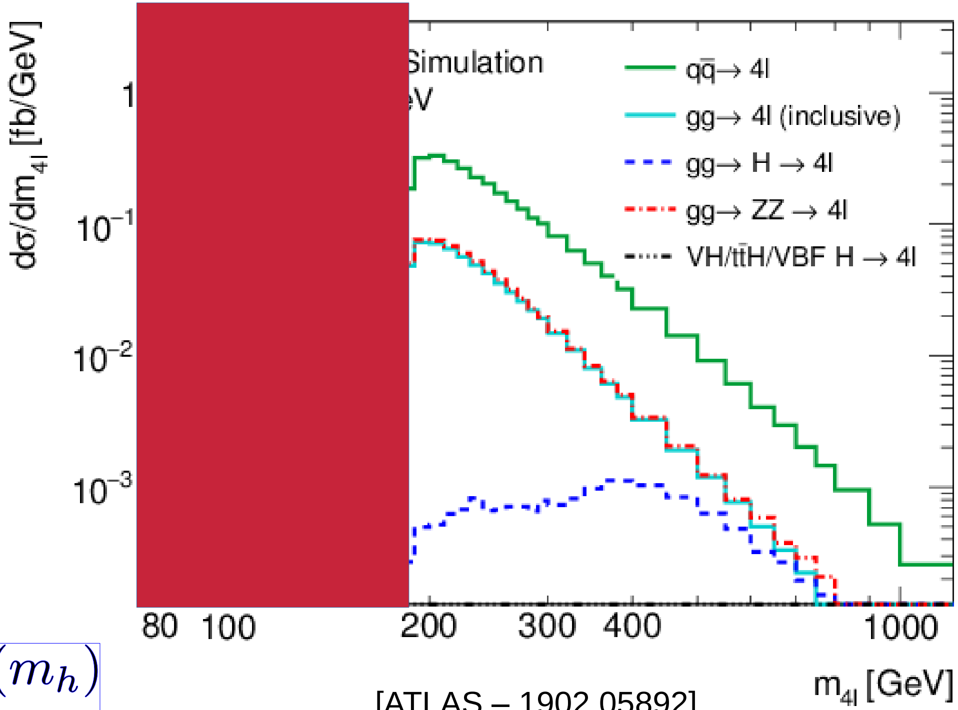
[ATLAS – 1902.05892]

Off-shell region

$$\mu_{\rm on} = \frac{\kappa^2_{ggh}(m_h)\kappa^2_{hZZ}(m_h)}{\Gamma_h/\Gamma_h^{\rm SM}}$$

$$\mu_{\rm off} = \kappa^2_{ggh}(m_{ZZ})\kappa^2_{hZZ}(m_{ZZ})$$

Higgs width from off-shell measurements



[ATLAS – 1902.05892]

Off-shell region

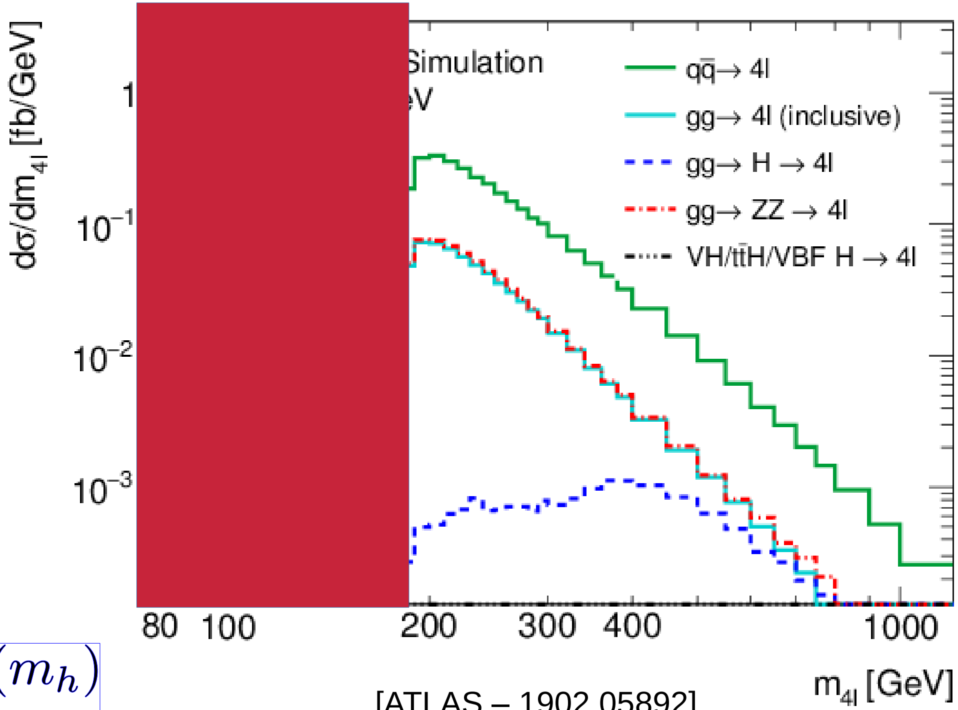
$$\mu_{\rm on} = \frac{\kappa^2_{ggh}(m_h)\kappa^2_{hZZ}(m_h)}{\Gamma_h/\Gamma_h^{\rm SM}}$$

$$\mu_{\rm off} = \kappa^2_{ggh}(m_{ZZ})\kappa^2_{hZZ}(m_{ZZ})$$

$$\frac{\mu_{\rm on}}{\mu_{\rm off}} \propto \frac{\kappa^2_{ggh}(m_h)\kappa^2_{hZZ}(m_h)}{\Gamma_h/\Gamma_h^{\rm SM}} \frac{1}{\kappa^2_{ggh}(m_{ZZ})\kappa^2_{hZZ}(m_{ZZ})},$$

[Kauer, Passarino – 1206.4803]
[Caola, Melnikov – 1307.4935]
[Campbell. Ellis, Williams - 1311.3589]

Higgs width from off-shell measurements



[ATLAS – 1902.05892]

$$\mu_{\rm on} = \frac{\kappa^2_{ggh}(m_h)\kappa^2_{hZZ}(m_h)}{\Gamma_h/\Gamma_h^{\rm SM}}$$

Off-shell region

$$\mu_{\rm off} = \kappa^2_{ggh}(m_{ZZ})\kappa^2_{hZZ}(m_{ZZ})$$

$$\Gamma_H = 3.2^{+2.4}_{-1.7}\,{\rm MeV}$$

[CMS - 2202.06923]

$$\Gamma_H = 4.5^{+3.3}_{-2.5}\,{\rm MeV}$$

[ATLAS - 2304.01532]

Interference @ NLO: massless vs massive

Two-loop boxes are a problem (again)

- Light-quark (~massless) known fully analytically
  [Caola et al. - 1509.06734]

- Heavy quarks

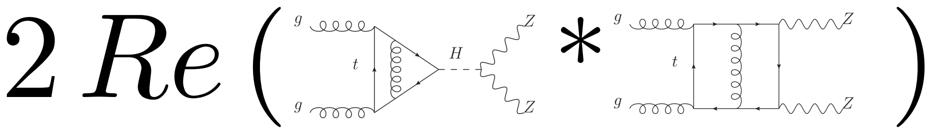
➔ Exact numerical results available
[Agarwal, Jones, von Manteuffel - 2011.15113 ; Brønnum-Hansen, Wang - 2101.12095]

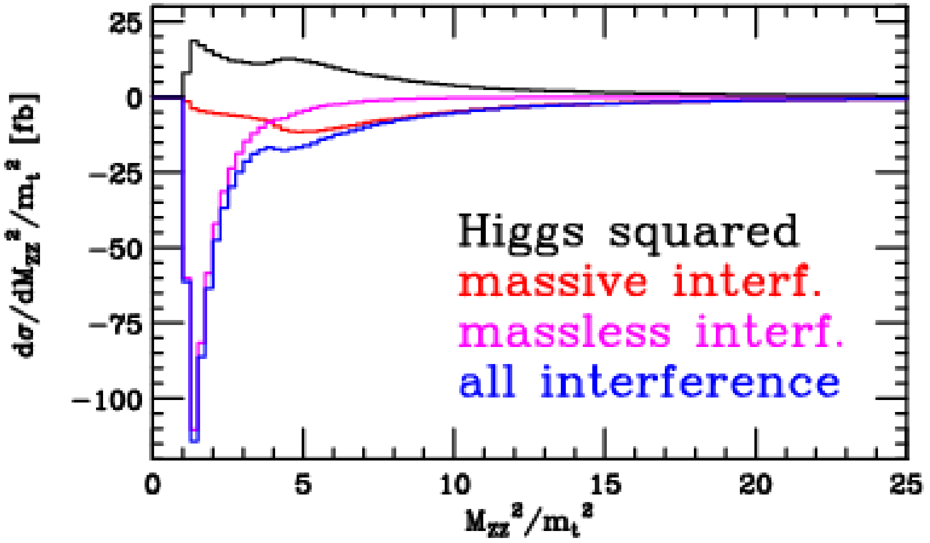
➔ Analytic approximations:

-LME [Melnikov, Dowling - 1503.01274 ; Gröber, Maier, Rauh - 1605.04610]

-High-energy exp [Davies et al. - 2002.05558]

$$m_Z^2 \ll m_t^2 \ll \hat{s}, \hat{t}$$





[Campbell et al. - 1605.01380]

p_T expansion for $gg \to ZZ$

- More involved Lorentz structure → 16 form factors
- More involved intermediate expressions
- ~ 750.000 scalar integrals per form factor
- IBP leads to same 52 MIs as HH and ZH
- Permille accuracy at LO with three orders