Inclusive rare Λ_b decays to photon

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Based on:
P. Colangelo, F. De Fazio, FL, *On the decay mode* $\Lambda_b \to X_s \gamma$, [arXiv:2306.02748]

Outline

Overview

$b \to s\,\gamma$ effective Hamiltonian

Inclusive decays of hadrons with one heavy quark.
The case of Λ_b

Spin effects

Treatment of the singular terms

Conclusions

Outline

Inclusive $H_b \to X_s\,\gamma$ decay

Powerful testground of the Standard Model (SM)

1. Occurring at loop-level in SM, sensitive to heavy particle exchanges
2. $H_{\mathbf{eff}} \sim \sum_i C_i\, O_i$ may be modified by New Physics (NP):
   modified coefficients and/or new operators
3. Observables can constrain operators and coefficients
4. Exploit well defined theoretical framework based on controlled expansions:
   $1/m_b$ (Heavy Quark Expansion (HQE)) and $\alpha_s(m_b)$
5. Intensively analyzed in theory, several measurements available ($B \to X_s\,\gamma$)

What has been studied

1. $\Lambda_b \to X_s\,\gamma$: dependence on the b-baryon spin
2. A way to resum the singular terms of the photon energy spectrum

Outline

Low-energy effective Hamiltonian $\Delta B = -1,\ \Delta S = +1$

$b \to s\gamma$ transition

$$H_{\rm eff}^{b\to s\gamma} = -4\,\frac{G_F}{\sqrt{2}}\,V_{tb}\,V_{ts}^*\sum_i\left[C_i(\mu)\,O_i + C_i'(\mu)\,O_i'\right]$$

Doubly Cabibbo-suppressed terms proportional to $V_{ub}\,V_{us}^*$ neglected

SM operators

- Current-current

$$O_1 = (\bar s_\alpha\,\gamma^\mu\,P_L\,c_\beta)\,(\bar c_\beta\,\gamma_\mu\,P_L\,b_\alpha)$$
$$O_2 = (\bar s\,\gamma^\mu\,P_L\,c)\,(\bar c\,\gamma_\mu\,P_L\,b)$$

- QCD penguins

$$O_3 = (\bar s\,\gamma^\mu\,P_L\,b)\sum_q(\bar q\,\gamma_\mu\,P_L\,q)$$
$$O_4 = (\bar s_\alpha\,\gamma^\mu\,P_L\,b_\beta)\sum_q(\bar q_\beta\,\gamma_\mu\,P_L\,q_\alpha)$$
$$O_5 = (\bar s\,\gamma^\mu\,P_L\,b)\sum_q(\bar q\,\gamma_\mu\,P_R\,q)$$
$$O_6 = (\bar s_\alpha\,\gamma^\mu\,P_L\,b_\beta)\sum_q(\bar q_\beta\,\gamma_\mu\,P_R\,q_\alpha)$$

- Magnetic penguins

$$O_7 = \frac{e}{16\,\pi^2}\left[\bar s\,\sigma^{\mu\nu}\,(m_s\,P_L + m_b\,P_R)\,b\right]F_{\mu\nu}$$
$$O_8 = \frac{g_s}{16\,\pi^2}\left[\bar s_\alpha\,\sigma^{\mu\nu}\left(\frac{\lambda^a}{2}\right)_{\alpha\beta}(m_s\,P_L + m_b\,P_R)\,b_\beta\right]G^a_{\mu\nu}$$

NP operators

- Scalar/tensor structures

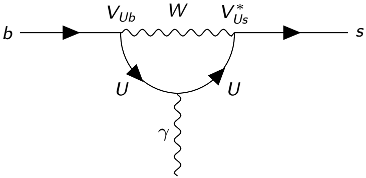
$$O_{15}^q = (\bar s\,P_R\,b)\sum_q(\bar q\,P_R\,q)$$
$$O_{16}^q = (\bar s_\alpha\,P_R\,b_\beta)\sum_q(\bar q_\beta\,P_R\,q_\alpha)$$
$$O_{17}^q = (\bar s\,P_R\,b)\sum_q(\bar q\,P_L\,q)$$
$$O_{18}^q = (\bar s_\alpha\,P_R\,b_\beta)\sum_q(\bar q_\beta\,P_L\,q_\alpha)$$
$$O_{19}^q = (\bar s\,\sigma^{\mu\nu}\,P_R\,b)\sum_q(\bar q\,\sigma_{\mu\nu}\,P_R\,q)$$
$$O_{20}^q = (\bar s_\alpha\,\sigma^{\mu\nu}\,P_R\,b_\beta)\sum_q(\bar q_\beta\,\sigma_{\mu\nu}\,P_R\,q_\alpha)$$

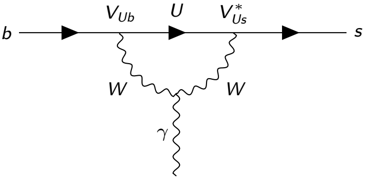
$$P_{R,L} = \frac{1\pm\gamma_5}{2}$$

Primed operators (all NP) have opposite chirality

$b \to s\,\gamma$ in SM

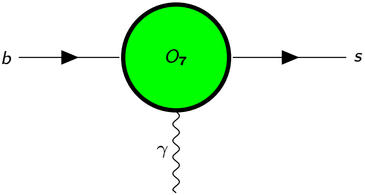
Photon penguin diagrams ($U = \{u, c, t\}$)





Magnetic operator O_7

$$O_7 = \frac{e}{16\,\pi^2}\left[\bar{s}\,\sigma^{\mu\nu}\left(m_s\,P_L + m_b\,P_R\right)b\right]F_{\mu\nu}$$



The only operator contributing to lowest order in QCD

Outline

$$H_b(p,s) \to X_s(p_X)\,\gamma(q,\epsilon)$$

Effective Hamiltonian (SM + NP)

$$H_{\text{eff}}^{b\to s\,\gamma} = -4\,\frac{G_F}{\sqrt{2}}\,V_{tb}\,V_{ts}^*\,\left[C_7^{\text{eff}}\,O_7 + C_7^{\prime\text{eff}}\,O_7'\right] = -4\,\frac{G_F}{\sqrt{2}}\,\lambda_t\,\frac{e}{16\,\pi^2}\sum_{i=7,7'} C_i^{\text{eff}}\,J_{\mu\nu}^i\,F^{\mu\nu}$$

$$\lambda_t = V_{tb}\,V_{ts}^*$$

$$J_{\mu\nu}^i = \bar{s}\,\sigma^{\mu\nu}\left[m_s\,(1-P_i) + m_b\,P_i\right]b \qquad \text{where} \qquad \begin{cases} P_i = P_R & \text{if} \quad i=7 \\ P_i = P_L & \text{if} \quad i=7' \end{cases}$$

Fully differential distribution

$$d\Gamma = [dq]\,\frac{G_F^2\,|\lambda_t|^2}{8\,M_H}\,\frac{\alpha}{\pi^2}\sum_{i,j=7,7'} C_i^{\text{eff}*}\,C_j^{\text{eff}}\,(W^{ij})_{MN}\,\mathcal{F}^{MN}$$

$$\underbrace{[dq] = \frac{d^3q}{(2\,\pi)^3\,2\,q^0}}_{\textbf{phase space}}$$

$(W^{ij})_{MN}$ hadronic tensor $\qquad \mathcal{F}^{MN}$ electromagnetic tensor

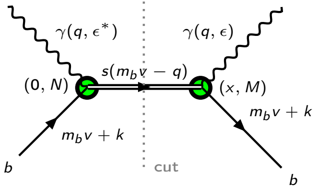
Heavy Quark Expansion (HQE)

$(W^{ij})_{MN}$ from the discontinuity of the forward amplitude

$$(T^{ij})_{MN} = i \int d^4x\, e^{-i\,q\cdot x}\, \langle H_b(p,s)|\mathbb{T}\left\{ J^{i\dagger}_M(x)\, J^j_N(0)\right\}|H_b(p,s)\rangle$$

across the cut

$$(W^{ij})_{MN} = \frac{1}{\pi}\,\mathrm{Im}\left[(T^{ij})_{MN}\right] \qquad \text{(optical theorem)}$$



Hadron momentum expressed in terms of the heavy quark mass and of a residual momentum

$$p = m_H\, v \quad \to \quad p = m_b\, v + k\,, \qquad \frac{k}{m_b} \ll 1$$

Redefinition of the QCD field

$$b(x) = e^{-i\,m_b\,v\cdot x}\, b_v(x) = e^{-i\,(m_b\,v + k)\cdot x}\, b_v(0)$$

$$(T^{ij})_{MN} = \langle H_b(v,s)|\bar b_v(0)\,\bar\Gamma^i_M\, \frac{1}{m_b\,\not v + \not k - \not q - m_s}\,\Gamma^j_N\, b_v(0)|H_b(v,s)\rangle$$

$$\bar\Gamma^i_M = \gamma^0\,\Gamma^{i\dagger}_M\,\gamma^0 \qquad \Gamma^7_M = \sigma_{\mu\nu}\left[m_s\,P_L + m_b\,P_R\right] \qquad \Gamma^{7\prime}_M = \sigma_{\mu\nu}\left[m_s\,P_R + m_b\,P_L\right]$$

Series expansion wrt $k \sim \Lambda_{QCD} \ll m_b$

$$(T^{ij})_{MN} = \sum_{n=0}^{+\infty} \langle H_b(v,s)|\bar b_v(0)\,\bar\Gamma^i_M\,(\not p_s + m_s)\left[i\,\not D\,(\not p_s + m_s)\right]^n \Gamma^j_N\, b_v(0)|H_b(v,s)\rangle\, \frac{(-1)^n}{\Delta_0^{n+1}} \qquad \Delta_0 = p_s^2 - m_s^2$$

I. I. Bigi, M. A. Shifman, N. Uraltsev, A. I. Vainshtein, *Phys. Rev. Lett.* 71 (1993) 496–499, [hep-ph/9304225].

Operator Product Expansion (OPE)

Trace formalism

$$\langle H_b(v,s)|\bar{b}_v(0)\,\overline{\Gamma}^i_M\,(\not{p}_s+m_s)\,\underbrace{i\,\not{D}\,(\not{p}_s+m_s)\dots i\,\not{D}\,(\not{p}_s+m_s)}_{\textbf{n times}}\,\Gamma^j_N\,b_v(0)|H_b(v,s)\rangle =$$

$$= \left[\overline{\Gamma}^i_M\,(\not{p}_s+m_s)\prod_{k=1}^{n}\left[\gamma_{\mu_k}\,(\not{p}_s+m_s)\right]\,\Gamma^j_N\right]_{ab}\underbrace{\langle H_b(v,s)|\bar{b}_v(0)\,iD^{\mu_1}\,\dots\,iD^{\mu_n}\,b_v(0)|H_b(v,s)\rangle_{ba}}_{(\mathcal{M}^{\mu_1\cdots\mu_n})_{ba}}$$

The higher the order of the expansion, the greater the number of the parameters

$$\mathcal{O}(1/m_b^n)\dots\begin{cases}\mathcal{O}(1/m_b^3)\begin{cases}\mathcal{O}(1/m_b^2)\begin{cases}-2\,M_H\,\hat{\mu}_\pi^2 = \langle H_b|\bar{b}_v\,iD^\mu\,iD_\mu\,b_v|H_b\rangle\\ 2\,M_H\,\hat{\mu}_G^2 = \langle H_b|\bar{b}_v\,(-i\sigma_{\mu\nu})\,iD^\mu\,iD^\nu\,b_v|H_b\rangle\end{cases}\\ 2\,M_H\,\hat{\rho}_D^3 = \langle H_b|\bar{b}_v\,iD^\mu\,(iv\cdot D)\,iD_\mu\,b_v|H_b\rangle\\ 2\,M_H\,\hat{\rho}_{LS}^3 = \langle H_b|\bar{b}_v\,(-i\sigma_{\mu\nu})\,iD^\mu\,(iv\cdot D)\,iD^\nu\,b_v|H_b\rangle\end{cases}\\ \dots\end{cases}$$

Dependence on the spin four-vector s_μ must be kept for baryons

Hadronic matrix elements

$$\mathcal{M}^{\rho\sigma\lambda} = M_H \left[\left(\frac{\hat\rho_D^3}{3} \Pi^{\rho\lambda} v^\sigma \mathbf{P}_+ + \frac{\hat\rho_{LS}^3}{6} v^\sigma i\,\epsilon^{\rho\lambda\alpha\beta} v_\alpha \mathbf{s}_\beta \right) - \left(\frac{\hat\rho_D^3}{3} \Pi^{\rho\lambda} v^\sigma s^\mu \mathbf{s}_\mu - \frac{\hat\rho_{LS}^3}{2} v^\sigma i\,\epsilon^{\rho\lambda\alpha\beta} v_\alpha s_\beta \mathbf{P}_+ \right) \right]$$

$$\mathcal{M}^{\rho\sigma} = M_H \left[\left(\frac{\hat\mu_\pi^2}{3} \Pi^{\rho\sigma} \mathbf{P}_+ + \frac{\hat\mu_G^2}{6} i\,\epsilon^{\rho\sigma\alpha\beta} v_\alpha \mathbf{s}_\beta + \frac{\hat\rho_D^3 + \hat\rho_{LS}^3}{24\, m_b} [4\,(i\,\epsilon^{\rho\sigma\alpha\beta} v_\alpha \mathbf{s}_\beta - v^\rho v^\sigma \not{v}) + \right.\right.$$

$$\left. + v^\rho (2\gamma^\sigma + \not{v}\gamma^\sigma - \gamma^\sigma \not{v}) + v^\sigma (2\gamma^\rho + \not{v}\gamma^\rho - \gamma^\rho \not{v})] \right) + \left(-\frac{\hat\mu_\pi^2}{3} \Pi^{\rho\sigma} \mathbf{P}_+ \not{s}\gamma_5 + \right.$$

$$+ \frac{\hat\mu_G^2}{2} i\,\epsilon^{\rho\sigma\alpha\beta} v_\alpha s_\beta \mathbf{P}_+ + \frac{\hat\rho_D^3}{12\, m_b} [6\, i\,\epsilon^{\rho\sigma\alpha\beta} v_\alpha s_\beta + i\,(v^\rho \epsilon^{\sigma\mu\alpha\beta} - v^\sigma \epsilon^{\rho\mu\alpha\beta}) v_\alpha s_\beta \gamma_\mu +$$

$$+ s^\rho v^\sigma \not{v}\gamma_5 + v^\rho s^\sigma (2\gamma_5 + \not{v}\gamma_5) + (2 v^\rho v^\sigma \not{v} - v^\rho\gamma^\sigma - v^\sigma\gamma^\rho) \not{s}\gamma_5] +$$

$$+ \frac{\hat\rho_{LS}^3}{8\, m_b} [4\, i\,\epsilon^{\rho\sigma\alpha\beta} v_\alpha s_\beta + i\,(v^\rho \epsilon^{\sigma\mu\alpha\beta} - v^\sigma \epsilon^{\rho\mu\alpha\beta}) v_\alpha s_\beta \gamma_\mu +$$

$$\left.\left. + (s^\rho v^\sigma + v^\rho s^\sigma)\gamma_5 + (2 v^\rho v^\sigma \not{v} - v^\rho \gamma^\sigma - v^\sigma \gamma^\rho) \not{s}\gamma_5] \right) \right]$$

$$\mathcal{M}^{\rho} = M_H \left[\left(\frac{\hat\mu_\pi^2 - \hat\mu_G^2}{12\, m_b} (v^\rho (3 + 5\not{v}) - 2\gamma^\rho) - \frac{\hat\rho_D^3 + \hat\rho_{LS}^3}{12\, m_b^2} (4 v^\rho \not{v} - \gamma^\rho) \right) + \right.$$

$$+ \left(-\frac{\hat\mu_\pi^2}{12\, m_b} [(v^\rho (3 + 5\not{v}) - 2\gamma^\rho) \not{s}\gamma_5 + 4 s^\rho \mathbf{P}_+ \gamma_5] + \frac{\hat\mu_G^2}{4\, m_b} [(v^\rho (1 + 2\not{v}) - \gamma^\rho) \not{s}\gamma_5 + s^\rho \gamma_5] + \right.$$

$$\left.\left. + \frac{\hat\rho_D^3}{12\, m_b^2} [(v^\rho (1 + 4\not{v}) - 2\gamma^\rho) \not{s}\gamma_5 + s^\rho (2 - \not{v})\gamma_5] + \frac{\hat\rho_{LS}^3}{8\, m_b^2} [(3 v^\rho \not{v} - \gamma^\rho) \not{s}\gamma_5 + s^\rho \gamma_5] \right) \right]$$

$$\mathcal{M} = M_H \left[\left(\mathbf{P}_+ - \frac{\hat\mu_\pi^2 - \hat\mu_G^2}{4\, m_b^2} \right) + \left(\mathbf{P}_+ + \frac{\hat\mu_\pi^2}{24\, m_b^2} (7 + 5\not{v}) - \frac{\hat\mu_G^2}{8 m_b^2} (3 + \not{v}) - \frac{\hat\rho_D^3}{6\, m_b^3} \mathbf{P}_- \right) \not{s}\gamma_5 \right]$$

P. Colangelo, F. De Fazio, FL, *JHEP* 11 (2020) 032, [arXiv:2006.13759]. [Erratum: *JHEP* 12, 098 (2022)].

$H_b(p,s) \to X_s(p_X)\,\gamma(q,\epsilon)$

Double differential decay distribution

$$\frac{d^2\Gamma}{dy\, d\cos\theta_P} = \tilde{\Gamma}_1 + \tilde{\Gamma}_2\,\cos\theta_P$$

- $y \equiv 2\,E_\gamma/m_b$: photon energy
- θ_P: angle between hadron spin s and photon momentum q: $\cos\theta_P \equiv \frac{\mathbf{s}\cdot\mathbf{q}}{|\mathbf{s}||\mathbf{q}|}$

Photon energy spectrum

$$\begin{aligned}\frac{1}{\Gamma_0}\frac{d\Gamma}{dy} = &\left[1 - \frac{\hat\mu_\pi^2}{2\,m_b^2} - \frac{\hat\mu_G^2}{2\,m_b^2}\,\frac{3+5\,z}{1-z} - \frac{10\,\hat\rho_D^3}{3\,m_b^3}\,\frac{1+z}{1-z}\right]\delta(1-z-y)+\\ &+\left[\frac{\hat\mu_\pi^2}{2\,m_b^2}\,(1-z) - \frac{\hat\mu_G^2}{6\,m_b^2}\,(3+5\,z) - \frac{4\,\hat\rho_D^3}{3\,m_b^3}\,(1+2\,z) + \frac{2\,\hat\rho_{LS}^3}{3\,m_b^3}\,(1+z)\right]\delta'(1-z-y)+\\ &+\left[\frac{\hat\mu_\pi^2}{6\,m_b^2}\,(1-z)^2 - \frac{\hat\rho_D^3}{3\,m_b^3}\,(1-z)\,(1+2\,z) + \frac{\hat\rho_{LS}^3}{6\,m_b^3}\,(1-z^2)\right]\delta''(1-z-y)+\\ &-\frac{\hat\rho_D^3}{18\,m_b^3}\,(1-z)^2\,(1+z)\,\delta'''(1-z-y)\end{aligned}$$

$$\Gamma_0 \equiv \frac{\alpha\,G_F^2\,|\lambda_t|^2}{32\,\pi^4}\,m_b^5\,(1-z)^3\,\left[|C_+^{\rm eff}|^2 + |C_+^{\prime\,\rm eff}|^2\right]$$

$$z \equiv \frac{m_s^2}{m_b^2} \qquad C_+^{\rm eff} \equiv C_7^{\rm eff} + \sqrt{z}\,C_7^{\prime\,\rm eff} \qquad C_+^{\prime\,\rm eff} \equiv \sqrt{z}\,C_7^{\rm eff} + C_7^{\prime\,\rm eff}$$

Angular differential distribution

$$\frac{d\Gamma(H_b \to X_s\,\gamma)}{d\cos\theta_P} = A + B\,\cos\theta_P$$

$$A = \frac{1}{2}\,\Gamma(H_b \to X_s\,\gamma) \qquad B = -\frac{\Gamma_0}{2}\,\frac{|C_+^{\rm eff}|^2 - |C_+^{\prime\,\rm eff}|^2}{|C_+^{\rm eff}|^2 + |C_+^{\prime\,\rm eff}|^2}\left[1 - \frac{13\,\hat\mu_\pi^2}{12\,m_b^2} - \frac{3\,\hat\mu_G^2}{4\,m_b^2}\,\frac{5+3\,z}{1-z} - \frac{\hat\rho_D^3}{6\,m_b^3}\,\frac{31+9\,z}{1-z}\right]$$

Decay width

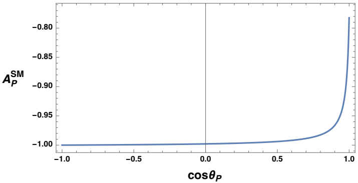
$$\Gamma(H_b \to X_s\,\gamma) = \Gamma_0\left[1 - \frac{\hat\mu_\pi^2}{2\,m_b^2} - \frac{\hat\mu_G^2}{2\,m_b^2}\,\frac{3+5\,z}{1-z} - \frac{10\,\hat\rho_D^3}{3\,m_b^3}\,\frac{1+z}{1-z}\right]$$

Outline

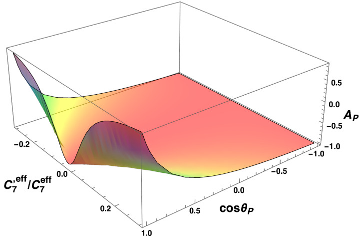
Photon polarization asymmetry: A_P

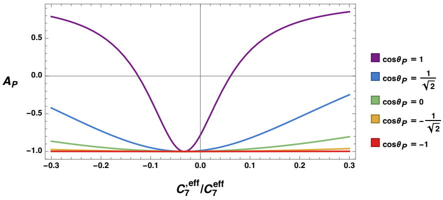
Relative abundance of the LH photons wrt the RH ones ($+$ VS $-$ polarization)

$$A_P(\cos\theta_P) = \frac{\frac{d\Gamma_+}{d\cos\theta_P} - \frac{d\Gamma_-}{d\cos\theta_P}}{\frac{d\Gamma_+}{d\cos\theta_P} + \frac{d\Gamma_-}{d\cos\theta_P}}$$



In SM: $A_P(\cos\theta_P) \simeq -1$





Deviations of A_P from SM can be obtained

Largest effects for $\cos\theta_P \simeq 1$

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Photon energy spectrum

$$\frac{1}{\Gamma}\,\frac{d\Gamma}{dy} = \sum_n \frac{M_n}{n!}\,\delta^{(n)}(1-z-y) \qquad M_n = \sum_{k=n} \frac{M_{n,k}}{m_b^k} = \frac{M_{n,n}}{m_b^n} + \frac{M_{n,n+1}}{m_b^{n+1}} + \dots$$

Moments M_n (computed up to $1/m_b^3$ in our case)

$$M_0 = 1 \qquad M_1 = \frac{\hat\mu_\pi^2}{2\,m_b^2}\,(1-z) - \frac{\hat\mu_G^2}{6\,m_b^2}\,(3+5\,z) - \frac{4\,\hat\rho_D^3}{3\,m_b^3}\,(1+2\,z) + \frac{2\,\hat\rho_{LS}^3}{3\,m_b^3}\,(1+z)$$

$$M_2 = \frac{\hat\mu_\pi^2}{2\,m_b^2}\,(1-z)^2 - \frac{2\,\hat\rho_D^3}{3\,m_b^3}\,(1-z)\,(1+2\,z) + \frac{\hat\rho_{LS}^3}{3\,m_b^3}\,(1-z^2) \qquad M_3 = -\frac{\hat\rho_D^3}{3\,m_b^3}\,(1-z)^2\,(1+z)$$

Photon energy moments VS M_n

$$\langle y^k\rangle \equiv \frac{1}{\Gamma}\int_0^{y_{\max}} dy\, y^k\,\frac{d\Gamma}{dy} = \sum_{j=0}^{k} \binom{k}{j}\,(1-z)^{k-j}\,M_j$$

$$\langle y\rangle = (1-z) + M_1$$
$$\langle y^2\rangle = (1-z)^2 + 2\,(1-z)\,M_1 + M_2$$
$$\sigma_y^2 = \langle y^2\rangle - \langle y\rangle^2 = M_2 - M_1^2$$

$$\text{LO} \;\to\; \frac{1}{\Gamma}\,\frac{d\Gamma}{dy} = \delta(1-z-y) \;\Rightarrow\; \langle y\rangle|_{\text{LO}} = 1-z \qquad \sigma_y^2|_{\text{LO}} = 0$$

$$\mathcal{O}(1/m_b^N) \;\to\; \frac{1}{\Gamma}\,\frac{d\Gamma}{dy} = \sum_{n=0}^{N} \frac{M_n}{n!}\,\delta^{(n)}(1-z-y) \;\Rightarrow\; \langle y\rangle|_{\text{N}} = 1-z+M_1(N) \qquad \sigma_y^2|_{\text{N}} > 0$$

For any order in the $1/m_b$ expansion: photon energy spectrum $\leftrightarrow$ monochromatic line

Fermi motion

The spectrum obtained by the short distance OPE does not account for the Fermi motion of the b quark due to soft interactions with the light degrees of freedom in the hadron.

↓

Important close to the end point region of the photon energy spectrum

Fermi motion has the effect of smearing the spectrum

Taken into account introducing a shape function:

- is a non perturbative quantity
- produces the smearing through convolution
- provides an interpretation of the singular terms in the photon energy spectrum

$$S_s(y) = \sum_{n=0}^{\infty} \frac{M_n}{n!}\,\delta^{(n)}(1 - z - y)$$

← Resum all the singular terms

Shape function

Spectral function VS Shape function

$$S_s(y) = \int dk_+\, \delta\!\left(1 - y - z + \frac{k_+}{m_b}\right) \left[f(k_+) + \mathcal{O}(m_b^{-1})\right]$$

$\Uparrow$ Shape function

Convolution with $f(k_+)$ has the effect of smearing the spectrum

$$\frac{d\Gamma}{dy} = \int dk_+\, f(k_+)\, \frac{d\Gamma}{dy}^{*} \qquad m_b \to m_b^* = m_b + k_+$$

$$k_+ \in [-m_b, M_H - m_b] \qquad \Rightarrow \qquad y \to y = \frac{2\,E_\gamma}{m_b + k_+} \qquad \Rightarrow \qquad E_\gamma \in [0, \underbrace{M_H/2}]$$

physical endpoint ($m_s = 0$) $\uparrow$

Shape function: our ansatz

Notice that

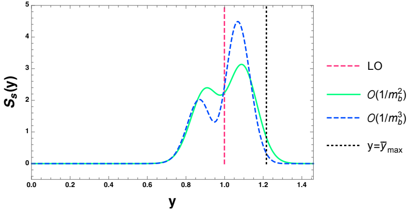
$$\lim_{m_b \to \infty} \langle y \rangle|_{\mathbf{N}} = \langle y \rangle|_{\mathbf{LO}} = 1 - z \qquad \lim_{m_b \to \infty} \sigma_y^2|_{\mathbf{N}} = \sigma_y^2|_{\mathbf{LO}} = 0$$

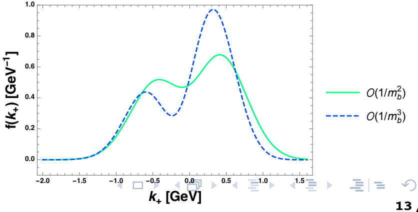
Exploit

$$\delta(b-y) = \lim_{\sigma_y \to 0} \frac{1}{\sqrt{2\,\pi}\,\sigma_y}\, e^{-\frac{(b-y)^2}{2\,\sigma_y^2}} \qquad b = 1 - z = \langle y \rangle|_{\mathbf{LO}}$$

Ansatz

$$S_s(y) \longrightarrow \frac{1}{\sqrt{2\,\pi}\,\sigma_y} \sum_{n=0}^{\infty} \frac{M_n}{n!} \left(\frac{-1}{\sqrt{2}\,\sigma_y}\right)^n e^{-\frac{(b-y)^2}{2\,\sigma_y^2}} \underbrace{H_n\left(\frac{b-y}{\sqrt{2}\,\sigma_y}\right)}_{\text{Hermite polynomials}}$$





Comparison with other approaches

Previous approaches

- Shape function modelled to reproduce the experimental photon spectrum (SIMBA Collaboration for $B \to X_s\,\gamma$)
- Parameters set by the first computed moments M_n (not guaranteed that higher moments are reproduced)
- Moments M_n generally increase with the order n

Our approach

- All moments M_n can be included
- Any shape function moment is obtained by M_n which starts at $\mathcal{O}(1/m_b^n)$
- Expansion in Hermite polynomials not arbitrary
- No parameters

SIMBA Collaboration, F. U. Bernlochner, H. Lacker, Z. Ligeti, I. W. Stewart, F. J. Tackmann, and K. Tackmann, *Phys. Rev. Lett.* 127 (2021) 102001, [arXiv:2007.04320].

Outline

Conclusions

Results

- HQE to compute the inclusive decay width induced by $b \to s\,\gamma$ transition for beauty baryons (e.g. Λ_b)
- $\mathcal{O}(1/m_b^3)$ for non-vanishing s-quark mass, using the baryon matrix elements $\mathcal{M}^{\mu_1 \cdots \mu_n}$
- NP operator O_7' affects the photon polarization asymmetry
- Treatment of the singularities (from $1/m_b$ expansion) systematically improved: $\delta^{(n)}$ distributions in the spectrum replaced with smooth distributions

Back-up

Outline

$b \to s\,\gamma$ effective Hamiltonian

Inclusive decays of hadrons with one heavy quark.
The case of Λ_b

Treatment of the singular terms

Wilson coefficient C_7

Renormalization Group Evolution

At $\mu_b \simeq \mathcal{O}(m_b)$ involves O_8 and $O_{1,\dots,6}$

- the mixing generates large logarithms (strong enhancement of the rate)
- anomalous dimension matrix turns out to be regularization scheme dependent
- $C_7(\mu_b) \to C_7^{\text{eff}}(\mu_b)$: effective coefficient (contributions of $O_{1,\dots,6}$)

O_7 dominant contribution to $b \to s\,\gamma$ [in SM $C_i(m_b)$ known at $\mathcal{O}(\alpha_s^2)$]*

Physics beyond SM can produce a sizable effect

For a quantitative insight on the possible deviation from SM, we consider ranges for

- Assumption: both coefficients are real
- Exploiting the results of a global fit of the $b \to s$ transitions†

$$\frac{C_7'^{\text{eff}}}{C_7^{\text{eff}}} \in [-0.3, 0.3]$$

* **M. Misiak, *Acta Phys. Polon. B* 49 (2018) 1291 - 1300.**
† **A. Paul, D. M. Straub, *JHEP* 04 (2017) 027, [arXiv:1608.02556].**

Scheme dependence for m_b

$\Gamma(B \to X_s\,\gamma)$ computed using m_b in $\begin{cases} \text{1S scheme } (^*) \\ \text{kinetic scheme } (^\star) \end{cases}$

For $E_\gamma > 1.6$ GeV

$$\mathcal{B}(\overline{B} \to X_s\,\gamma) = \begin{cases} (3.15 \pm 0.23)\,10^{-4} & (^*) \\ (3.26 \pm 0.24)\,10^{-4} & (^\star) \end{cases} \qquad \mathcal{B}(\overline{B} \to X_s\,\gamma) = \begin{cases} (3.55 \pm 0.24 \pm 0.09)\,10^{-4} & (^\dagger) \\ (3.50 \pm 0.17)\,10^{-4} & (^\ddagger) \end{cases}$$

Agreement with SM at $1.2\,\sigma$

Measurements (rates and moments) aimed at constraining C_7

Numerics

$$m_b = 4.62\text{ GeV} \qquad m_s = 0.150\text{ GeV}$$

$$\hat{\mu}_\pi^2 = 0.5\text{ GeV}^2 \qquad \hat{\mu}_G^2 = 0\text{ GeV}^2 \qquad \hat{\rho}_D^3 = 0.17\text{ GeV}^3 \qquad \hat{\rho}_{LS}^2 = 0\text{ GeV}^3$$

[*] **M. Misiak, H. M. Asatrin, K. Bieri, M. Czakov, A. Czarnecki, *Phys. Rev. Lett.* 98 (2007) 022002, [**arXiv:0609232**].**
[⋆] **P.Gambino, P. Giordano, *Phys. Lett. B* 669 (2008) 69-73, [**arXiv:0805.0271**].**
[†] **HFLAV Collaboration, Y. S. Amhis et al, *Phys. Rev. D* 107 (2023) 052008, [**arXiv:2206.07501**].**
[‡] **M. Artuso, E. Barberio, S. Stone, *PMC Phys. A* 3 (2009) 3, [**arXiv:0902.3743**].**

Outline

Outline of the calculation

Electromagnetic tensor

- Summing and averaging on the photon polarization

$$\mathcal{F}^{MN} \equiv \mathcal{F}^{\mu\nu\mu'\nu'} = \sum_{\epsilon} 4\,q^{\nu}\,q^{\nu'}\,\epsilon^{\mu}\,\epsilon^{*\mu'} = -4\,q^{\nu}\,q^{\nu'}\,g^{\mu\mu'}$$

- Specifying the photon polarization

$$\mathcal{F}^{MN}_{\pm} = 4\,q^{\nu}\,q^{\nu'}\,\epsilon^{\mu}_{\pm}\,\epsilon^{*\mu'}_{\pm} \qquad \text{with} \qquad \epsilon_{\pm} = \mp\frac{1}{\sqrt{2}}(0,1,\pm i,0)$$

$(T^{ij})_{MN} \sim \langle H_b(v,s)|\bar{b}_v(0)\,iD^{\mu_1}\,\ldots\,iD^{\mu_n}\,b_v(0)|H_b(v,s)\rangle$

$$k = n \;\to\; h_v \text{ is taken into account} \;\to\; \text{EoM: } (i\,v\cdot D)\,h_v = 0 \;\to\; \text{(HQET)}$$

$$0 \leq k < n \;\to\; b_v \text{ is taken into account} \;\to\; \text{EoM: } (i\,v\cdot D)\,b_v = -\frac{i\,\not{D}\,i\,\not{D}}{2\,m_b}\,b_v \;\to\; \text{(QCD)}$$

Outline of the calculation

Full decomposition

$$\sum_{i,j=7,7'} C_i^{\text{eff}*}\, C_j^{\text{eff}}\, (T^{ij})_{MN}\, \mathcal{F}^{MN} = \left[(m_b^2 + m_s^2)\,(|C_7^{\text{eff}}|^2 + |C_7'^{\text{eff}}|^2) + 4\, m_b\, m_s\, \text{Re}[C_7^{\text{eff}}\, C_7'^{*\text{eff}}]\right]\, \tilde{T} +$$

$$+ (m_b^2 - m_s^2)\,(|C_7^{\text{eff}}|^2 - |C_7'^{\text{eff}}|^2)\, \tilde{S}$$

$$\tilde{T} = 16\, M_H\, (v\cdot q)^2 \sum_{i=1}^{4} \left(\frac{m_b}{\Delta_0}\right)^n \tilde{T}_n \qquad \text{and} \qquad \tilde{S} = 16\, M_H\, (v\cdot q)\,(q\cdot s) \sum_{i=1}^{4} \left(\frac{m_b}{\Delta_0}\right)^n \tilde{S}_n$$

$$\tilde{T}_1 = 1 + \frac{5}{6}\left[\frac{\hat{\mu}_\pi^2}{m_b^2} - \frac{\hat{\mu}_G^2}{m_b^2}\right] - \frac{2}{3}\left[\frac{\hat{\rho}_D^3}{m_b^3} + \frac{\hat{\rho}_{LS}^3}{m_b^3}\right]$$

$$\tilde{T}_2 = \frac{7\, v\cdot q}{3}\,\frac{\hat{\mu}_\pi^2}{m_b^2} + \frac{4\, m_b - 5\, v\cdot q}{3}\,\frac{\hat{\mu}_G^2}{m_b^2} + \frac{2}{3}\left[(4\, m_b - 3\, v\cdot q)\,\frac{\hat{\rho}_D^3}{m_b^3} + (2\, m_b - 3\, v\cdot q)\,\frac{\hat{\rho}_{LS}^3}{m_b^3}\right]$$

$$\tilde{T}_3 = \frac{4\,(v\cdot q)^2}{3}\,\frac{\hat{\mu}_\pi^2}{m_b^2} + \frac{4\,(m_b - v\cdot q)\,(v\cdot q)}{3}\left[2\,\frac{\hat{\rho}_D^3}{m_b^3} + \frac{\hat{\rho}_{LS}^3}{m_b^3}\right] \qquad \tilde{T}_4 = \frac{8\,(m_b - v\cdot q)\,(v\cdot q)^2}{3}\,\frac{\hat{\rho}_D^3}{m_b^3}$$

$$\tilde{S}_1 = 1 + \frac{1}{4}\left[\frac{\hat{\mu}_\pi^2}{m_b^2} + \frac{\hat{\mu}_G^2}{m_b^2}\right] + \frac{1}{6}\,\frac{\hat{\rho}_D^3}{m_b^3}$$

$$\tilde{S}_2 = \frac{7\, v\cdot q}{3}\,\frac{\hat{\mu}_\pi^2}{m_b^2} + (2\, m_b - v\cdot q)\,\frac{\hat{\mu}_G^2}{m_b^2} + \frac{2\,(4\, m_b - 3\, v\cdot q)}{3}\,\frac{\hat{\rho}_D^3}{m_b^3}$$

$$\tilde{S}_3 = \frac{4\,(v\cdot q)^2}{3}\,\frac{\hat{\mu}_\pi^2}{m_b^2} + \frac{8\,(m_b - v\cdot q)\,(v\cdot q)}{3}\,\frac{\hat{\rho}_D^3}{m_b^3} \qquad \tilde{S}_4 = \frac{8\,(m_b - v\cdot q)\,(v\cdot q)^2}{3}\,\frac{\hat{\rho}_D^3}{m_b^3}$$

Outline

$b \to s\gamma$ effective Hamiltonian

Inclusive decays of hadrons with one heavy quark.
The case of Λ_b

Treatment of the singular terms

Photon energy moments

δ distribution

$$S_s(y) = \sum_{n=0}^{\infty} \frac{M_n}{n!}\, \delta^{(n)}(1-z-y) \quad \longrightarrow \quad \langle y^k \rangle_\delta \equiv \int_0^{y_{\max}} dy\, y^k\, S_s(y) = \sum_{j=0}^{k} \binom{k}{j} (1-z)^{k-j} M_j$$

First $N+1$ moments: $\{M_0, M_1, \ldots, M_N\}$

$$M_n = \sum_{k=n}^{N} \frac{M_{n,k}}{m_b^k} = \frac{M_{n,n}}{m_b^n} + \frac{M_{n,n+1}}{m_b^{n+1}} + \cdots + \frac{M_{n,N}}{m_b^N} \equiv M_n(N)$$

Up to $\mathcal{O}(1/m_b^3)$

$$\langle y \rangle = (1-z)\left[1 + \frac{\hat\mu_\pi^2}{2\, m_b^2} - \frac{\hat\mu_G^2}{6\, m_b^2}\, \frac{3+5\,z}{1-z} - \frac{4\,\hat\rho_D^3}{3\, m_b^3}\, \frac{1+2\,z}{1-z} + \frac{2\,\hat\rho_{LS}^3}{3\, m_b^3}\, \frac{1+z}{1-z}\right]$$

$$\langle y^2 \rangle = (1-z)^2\left[1 + \frac{4\,\hat\mu_\pi^2}{3\, m_b^2} - \frac{\hat\mu_G^2}{3\, m_b^2}\, \frac{3+5\,z}{1-z} - \frac{10\,\hat\rho_D^3}{3\, m_b^3}\, \frac{1+2\,z}{1-z} + \frac{5\,\hat\rho_{LS}^3}{3\, m_b^3}\, \frac{1+z}{1-z}\right]$$

$$\sigma_y^2 = (1-z)^2\left[\frac{\hat\mu_\pi^2}{3\, m_b^2} - \frac{2\,\hat\rho_D^3}{3\, m_b^3}\, \frac{1+2\,z}{1-z} + \frac{\hat\rho_{LS}^3}{3\, m_b^3}\, \frac{1+z}{1-z}\right]$$

	LO	$O(1/m_b^2)$	$O(1/m_b^3)$
$\langle y \rangle$	0.999	1.011	1.008
$\langle y^2 \rangle$	0.998	1.029	1.023
σ_y^2	0	0.008	0.007

$m_b \to \infty$ (fixed order of the expansion)

$$\lim_{m_b\to\infty} \langle y^k \rangle|_{\mathbf{N}} = \lim_{m_b\to\infty} \sum_{j=0}^{k} \binom{k}{j} (1-z)^{k-j} M_j(N) = (1-z)^k = \langle y^k \rangle|_{\mathbf{LO}}$$

Photon energy moments

$\mathcal{N}$ distribution

$$S_s(y) = \frac{1}{\sqrt{2\,\pi}\,\sigma_y} \sum_{n=0}^{\infty} \frac{M_n}{n!} \left(\frac{-1}{\sqrt{2}\,\sigma_y}\right)^n e^{-\frac{(b-y)^2}{2\,\sigma_y^2}} H_n\left(\frac{b-y}{\sqrt{2}\,\sigma_y}\right) \longrightarrow$$

$$\longrightarrow \qquad \langle y^k \rangle_{\mathcal{N}} = \int_0^{y_{\max}} dy\, y^k\, S_s(y) = \sum_{j=0}^{k} \binom{k}{j} b^{k-j} \sum_{n=0}^{\infty} M_n\, (-\sqrt{2}\,\sigma_y)^{j-n}\, \Phi_{j,n}$$

$$\Phi_{j,n} \equiv \frac{1}{\sqrt{\pi}} \sum_{m=0}^{\lfloor \frac{n}{2} \rfloor} \frac{(-1)^m}{m!\,(n-2\,m)!} 2^{n-2\,m} \frac{1}{2} \left[\gamma\left(\frac{j+n-2\,m+1}{2}, x_{\max}^2\right) - (-1)^{j+n-2\,m+1}\,\gamma\left(\frac{j+n-2\,m+1}{2}, x_{\min}^2\right)\right]$$

if $\mathrm{Re}(j+n-2\,m) > -1$ $\qquad \gamma(s,x) = \Gamma(s) - \Gamma(s,x) \qquad x_{\max} = \frac{b-y_{\min}}{\sqrt{2}\,\sigma_y} > 0 > x_{\min} = \frac{b-y_{\max}}{\sqrt{2}\,\sigma_y}$

From $\mathcal{N}$ to δ

$\Phi_{j,n}$ depends on σ_y only through $x_{\max(\min)}$ ⟶ $\lim_{\sigma_y \to 0} x_{\max(\min)} = +(-)\infty$ ⟶ $\lim_{\sigma_y \to 0} \Phi_{j,n} = \begin{cases} 0 & \text{for } n \neq j \\ 1 & \text{for } n = j \end{cases}$

$$\langle y^k \rangle_{\mathcal{N}} = \sum_{j=0}^{k} \binom{k}{j} b^{k-j} M_j + \mathcal{O}\left(\frac{1}{m_b^{N+1}}\right) \longrightarrow \lim_{\sigma_y \to 0} \langle y^k \rangle_{\mathcal{N}} = \langle y^k \rangle_{\delta}$$

Leading order in HQE

Shape function in HQET

Considering the process $H_b \to X_s\,\gamma$, one defines the spectral function $S_s(y)$

$$S_s(y) = \left\langle \delta\left[1 - y - z + \frac{2}{m_b}\,(v - \hat{q}) \cdot i\,D\right] \right\rangle \qquad \hat{q} = \frac{q}{m_b}\ , \qquad \langle \mathcal{O} \rangle = \frac{\langle H_b(v)|\overline{h}_v\,\mathcal{O}\,\overline{h}_v|H_b(v)\rangle}{\langle H_b(v)|\overline{h}_v\,\overline{h}_v|H_b(v)\rangle}$$

Introducing the vector $n_\mu + \delta\,n_\mu$ (in the shape function region)

$$n_\mu + \delta\,n_\mu = 2\,(v - \hat{q})|_{y=\mathbf{1}-z} \qquad n^{\mathbf{2}} = 0 \qquad v \cdot n = 1 \qquad n \cdot \delta n \sim \mathcal{O}(\Lambda_{\mathbf{QCD}}/m_b)$$

$$S_s(y) = \left\langle \delta\left[1 - y - z + \frac{i\,D_+}{m_b}\right] \right\rangle = \int dk_+\,\delta\left(1 - y - z + \frac{k_+}{m_b}\right)\left[f(k_+) + \mathcal{O}(m_b^{-\mathbf{1}})\right] \qquad k_+ \equiv n \cdot k$$

Broadening of the spectrum

Perturbative gluon bremsstrahlung AND Motion of the b quark in the hadron

Difficulties

1. Singular terms appear at higher orders in the HQE

$$\frac{d\Gamma}{dE_\gamma} \sim \sum_n c_n\, \delta^{(n)}(E_\gamma - E_\gamma^{\max})$$

Endpoint: partonic (~~hadronic~~) kinematics!

Gap governed by nonperturbative physics responsible of bound state effects

- related to the Fermi motion of the heavy quark in the decaying hadron
- accounted for introducing a shape function which encodes information on the distribution of the b quark residual momentum in the hadron

2. Resolved photon contribution (RPC) related to the photon couplings different from the effective weak interaction vertex

   2.1 most important contribution: operators O_2 and O_8

   2.2 effects at $\mathcal{O}(1/m_b)$ not explainable by HQE (but with subleading shape functions)

Other models of the shape function

Example[†]

1. Only the most singular terms in the M_n expansion are considered
2. A single Gaussian distribution is obtained
3. Symmetric shape function is established

Example[‡]

1. Based on a choice of a functional representation able to reproduce the photon spectrum, with parameters set by the first computed moments M_n
2. Such representations generally do not guarantee that higher moments are reproduced
3. In such models the moments M_n generally increase with the order n

Example[⋆]

1. Shape function assumed positive
2. Arbitrary complete set of orthonormal function is used (Legendre polynomials)
3. Parameters required to reproduce the shape function

† M. Neubert, *Analysis of the photon spectrum in inclusive $B \to X_s\,\gamma$ decays*, Phys. Rev. D 49 (1994) 4623–4633, [hep-ph/9312311].

‡ T. Mannel, *Inclusive Semi-Leptonic B Decays, in Pushing the Limits of the Theoretical Physics, Mainz, 08-12 May 2023, indico.mitp.uni-mainz.de/event/341.*

⋆ Z. Ligeti, I. W. Stewart, and F. J. Tackmann, *Treating the b quark distribution function with reliable uncertainties*, *Phys. Rev. D* 78 (2008) 114014, [arXiv:0807.1926].