

New Frontiers in Theoretical Physics
XXXVII Convegno Nazionale di Fisica Teorica

Nucleon spin structure in Holographic QCD

Federico Castellani

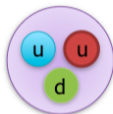
F. Bigazzi and F. Castellani arXiv:2308.16833



UNIVERSITÀ
DEGLI STUDI
FIRENZE

Nucleon internal structure

Nucleons are composite systems of three valence quarks



Protone



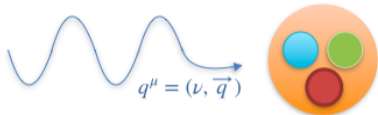
Neutrone

Scattering processes probe \rightarrow $\left\{ \begin{array}{l} \text{at High Energy the physics of the single constituents;} \\ \text{at Low Energy collective response of internal structure;} \end{array} \right.$

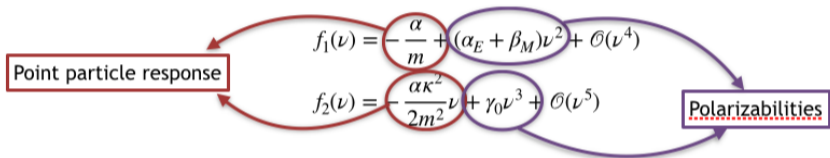
Collective properties: mass, spin, charge, polarizabilities...

Polarizabilities quantify the internal rearrangement of a body under an e.m. field.

Polarizabilities



- Real photon: $Q^2 = q^2 = 0$;
 - Response described by two Compton scattering amplitudes



- Virtual photon: $Q^2 = q^2 > 0$;
 - Polarizabilities acquire a Q^2 dependence \rightarrow **generalized polarizabilities**
 - Further polarizability appears: $\delta_{LT}(Q^2)$

Generalized spin polarizabilities $\delta_{LT}(Q^2)$ and $\gamma_0(Q^2)$ describe the spin precession of the nucleon under a probing external electromagnetic field.

Generalized spin polarizabilities

- Hadronic tensor:

- $W^{\mu\nu} = \frac{1}{4\pi} \sum_X (2\pi)^4 \delta^4(p_X - p - q) \langle N | J^\mu | X \rangle \langle X | J^\nu | N \rangle;$

- $W_{pol}^{\mu\nu} = i\epsilon^{\mu\nu\rho\sigma} q_\rho \left[\frac{s_\sigma}{p \cdot q} (\mathbf{g}_1 + \mathbf{g}_2) - \frac{q \cdot S}{(p \cdot q)^2} p_\sigma \mathbf{g}_2 \right];$

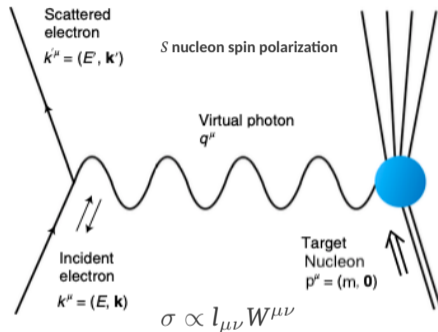
- Generalized spin polarizabilities:

- $\gamma_0(Q^2) = \frac{16\alpha m^2}{Q^6} \int_0^{x_0} dx x^2 \left[\mathbf{g}_1(x, Q^2) - \frac{4m^2 x^2}{Q^2} \mathbf{g}_2(x, Q^2) \right];$

- $\delta_{LT}(Q^2) = \frac{16\alpha m^2}{Q^6} \int_0^{x_0} dx x^2 \left[\mathbf{g}_1(x, Q^2) + \mathbf{g}_2(x, Q^2) \right];$

- Dynamical parameters:

- Bjorken parameter $x = -\frac{q^2}{2p \cdot q}$, Pion-production threshold $x_0 = \frac{Q^2}{Q^2 + (m_\pi + m)^2 - m^2};$



Recent experimental results

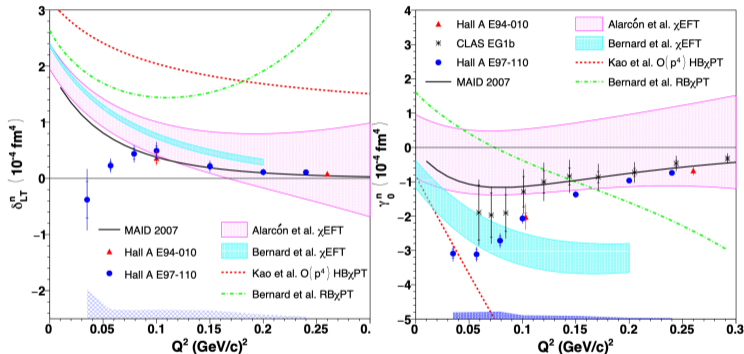
- Novel experimental results on the low energy behaviour of $\delta_{LT}(Q^2)$ and $\gamma_0(Q^2)$:
 - 2021 - Jefferson Lab E97-110 - neutron;
 - 2022 - Jefferson Lab g_2p - proton;
- Inelastic scattering processes with **polarized electrons** and **polarized nucleon targets**;
- Low energy-momentum transfer squared $Q^2 \lesssim 0.2 \text{ GeV}^2$

Low energy regime

- Low Q^2 regime \rightarrow pQCD can not be applied:
 - QCD displays **asymptotic freedom**;
 - strong coupling constant $g \gg 1$ at low energy;
 - **Perturbative** approaches can not be used;
- Necessity of non-perturbative theoretical approaches:
 - **Lattice QCD**;
 - Chiral Effective Field Theories χ **EFT**;
 - **Phenomenological models**;

Experimental results: Neutron

V. Sulkosky et al. Nature Phys. 17, no.6, 687-692 (2021) - Jefferson Lab E97-110

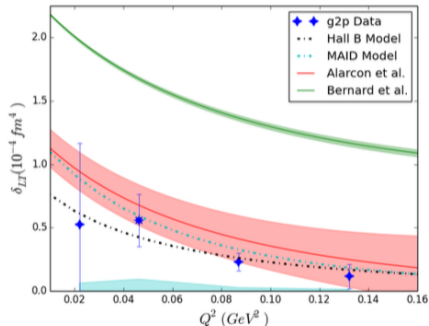


- Blue points: data from Jefferson Lab;

- Colored bands: theoretical predictions from χ EFTs;

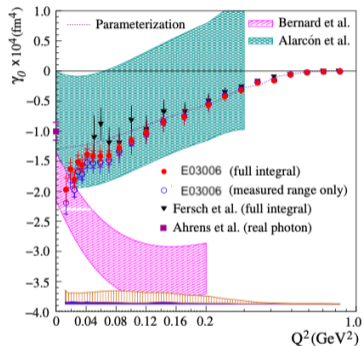
Experimental results: Proton

D. Ruth et al. Nature Phys. 18, no.12, 1441-1446 (2022) - Jefferson Lab Hall A g_2p



- Blue points: data from Jefferson Lab;

- Colored bands: theoretical predictions from $\chi EFTs$;



- Blue and Red points: data from Jefferson Lab;

- Colored bands: theoretical predictions for $\chi EFTs$;

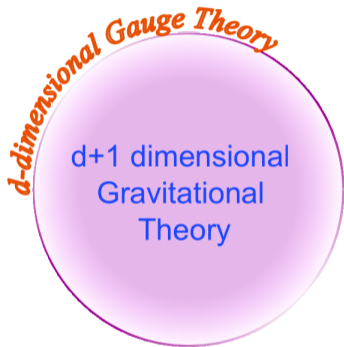
Goals

- Analyze generalized nucleon spin polarizabilities at low energies using the **holographic correspondence**;
- Account for low-lying resonance contributions to nucleon spin polarizabilities;
- Compare our theoretical predictions with JLab experimental data through numerical analysis;

Holographic Correspondence

J. M. Maldacena, Adv. Theor. Math. Phys.2 (1998), 231

- Conjecture based on the duality between **gauge theories** and **gravity theories** in higher dimensions;
- “Weak/strong duality”;
- Countless checks of its validity;
- Applied in various physics research areas → QCD-like models;



Witten Sakai Sugimoto model

E. Witten, Adv. Theor. Math. Phys. 2, 505-532 (1998),

T. Sakai and S. Sugimoto, Prog. Theor. Phys. 113, 843-882 (2005)

Gauge theory

- QCD-like $SU(N_c)$ + Adjoint $SU(N_c)$ massive modes ($N \sim \Lambda_{QCD}$);
- $N_c \gg 1$;
- Strongly coupled $\lambda = g^2 N_c \gg 1$;
- Confinement;
- $N_f \ll N_c$ massless quarks ($N_f = 2$);
- $\chi SB : U(N_f)_L \times U(N_f)_R \rightarrow U(N_f)_V$;

Gravity theory

- Classical gravity limit of a type IIA string theory;
- Specific curved background;
- N_f extended sources ($D8$ -branes);
- Sources' minimal energy configuration;

Hadrons in the WSS model

T. Sakai and S. Sugimoto, Prog. Theor. Phys. 113, 843-882 (2005),

H. Hata, T. Sakai, S. Sugimoto and S. Yamato, Prog. Theor. Phys. 117, 1157 (2007)

Low energy theory of the hadron sector



$U(N_f)$ Yang-Mills Chern-Simons theory in 5-dim curved background

$$S = -\frac{\lambda N_c}{216\pi^3} \int d^4x dz \operatorname{Tr} \left[\frac{1}{2} (1+z^2)^{-1/3} \mathcal{F}_{\mu\nu}^2 + (1+z^2) \mathcal{F}_{\mu z}^2 \right] + \frac{N_c}{24\pi^2} \int_{\mathbb{M}^4 \times \mathbb{R}} \omega_5(\mathcal{A})$$

- Pions, vector and axial mesons: modes of the gauge field $\mathcal{A}(x, z)$;
- **Barions**: solitonic solutions of the 5-dim gauge theory;

Nucleon spin structure in the WSS model

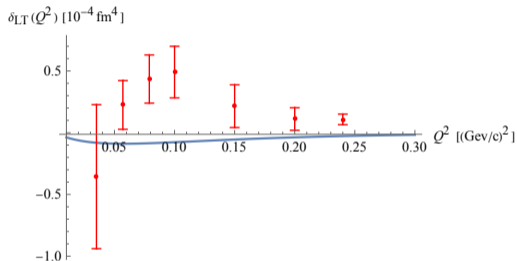
F. Bigazzi and F. Castellani arXiv:2308.16833

- Evaluate e.m. current one-point functions $\langle X|J^\mu|N\rangle$:
 - e.m. current holographically evaluated in the WSS model;
 - one point functions between quantized baryons solutions;
 - Identify $\langle N|J^\mu|X\rangle = \langle N|J^\mu|X\rangle_{WSS}$;
- Build up the polarized hadronic tensor $W_{\text{pol}}^{\mu\nu}$;
- Extract polarized structure functions $g_1(x, Q^2)$ and $g_2(x, Q^2)$;
- Obtain expressions for the generalized spin polarizabilities;
- Extrapolating WSS model parameters to real QCD...

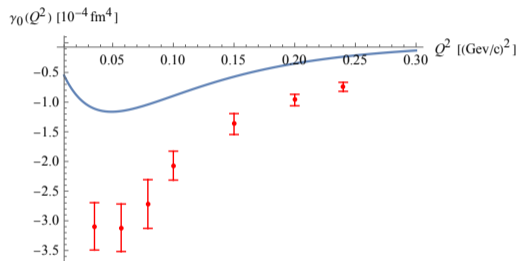
Our results: Neutron

F. Bigazzi and F. Castellani arXiv:2308.16833

$$\delta_{LT}(Q^2)$$



$$\gamma_0(Q^2)$$



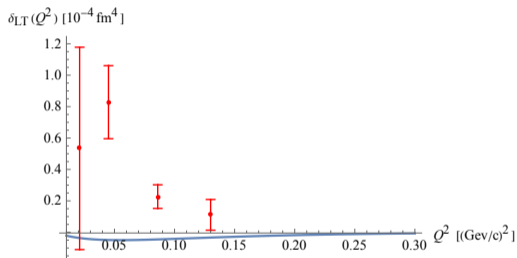
- Quantitative gap: our results are multiplied by a factor 20;
- Main contribution from the $N(1535)$ resonance;

- Better quantitative and qualitative agreement;
- Main contribution from the $\Delta(1232)$ resonance;

Our results: Proton

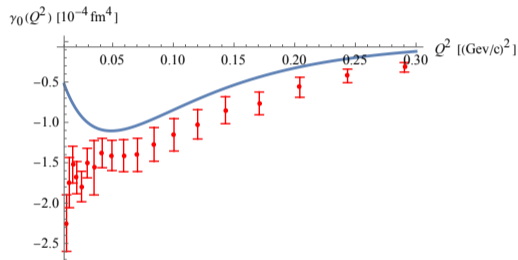
F. Bigazzi and F. Castellani arXiv:2308.16833

$$\delta_{LT}(Q^2)$$



- Quantitative gap: our results are multiplied by a factor 10;
- Qualitative disagreement with the data;

$$\gamma_0(Q^2)$$

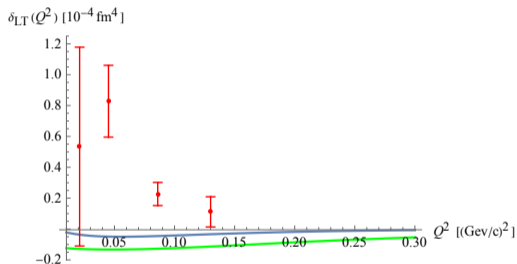


- Better quantitative and qualitative agreement;
- Main contribution from the $\Delta(1232)$ resonance;

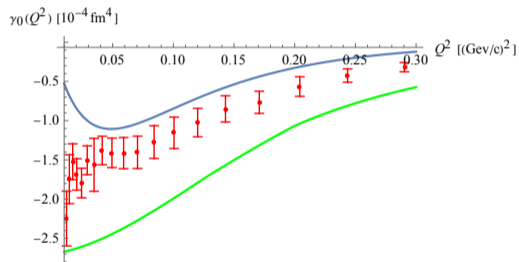
Resonance contributions: data interpolation

F. Bigazzi and F. Castellani arXiv:2308.16833

$$\delta_{LT}(Q^2)$$



$$\gamma_0(Q^2)$$



- **Green line:** data interpolation from only resonance contributions
- Qualitative agreement between our results and data interpolation curves

Conclusions and future perspectives

- Holographic results match qualitatively with experimental data extrapolations of resonance contributions only, suggesting that the WSS model is qualitatively reliable also on this set of observables;
- Both nucleon analyses show that resonances alone are not enough to explain experimental results;
- This suggests us to explore other contributions that could be captured by an analysis of the holographic **two-point functions**:

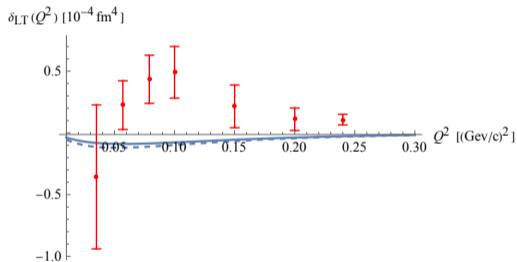
$$W^{\mu\nu} = \frac{1}{4\pi} \int d^4x e^{iq \cdot x} \langle N | [J^\mu(\mathbf{x}), J^\nu(0)] | N \rangle$$

Thank you for listening!

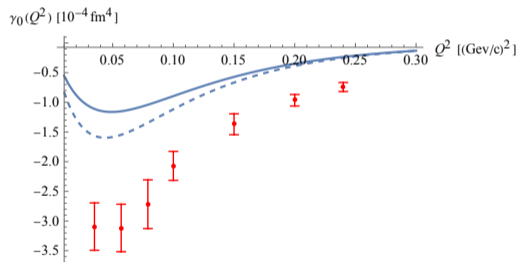
Not-sharp resonances: Neutron

F. Bigazzi and F. Castellani arXiv:2308.16833

$\delta_{LT}(Q^2)$



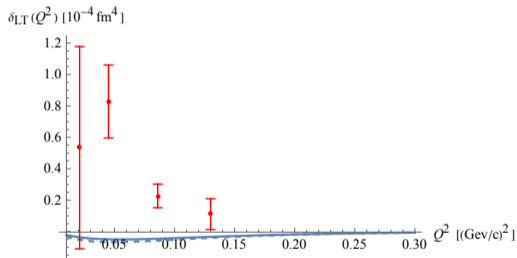
$\gamma_0(Q^2)$



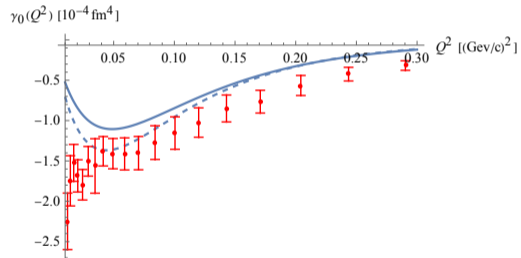
Not-sharp resonances: Proton

F. Bigazzi and F. Castellani arXiv:2308.16833

$\delta_{LT}(Q^2)$



$\gamma_0(Q^2)$



Comments on WSS model

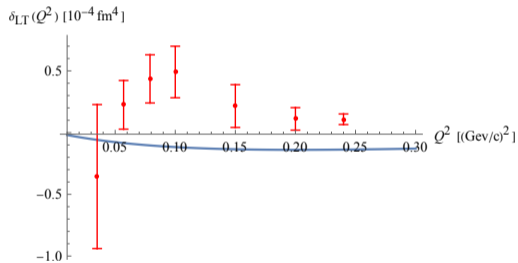
T. Sakai and S. Sugimoto, Prog. Theor. Phys. 113, 843-882 (2005)

- At high energy WSS model is very different from QCD: there is **no asymptotic freedom**;
- $\lambda \sim 1/(l_s^2 M_{KK}^2)$ parameter is not present in QCD;
- Probe approximation $N_f \ll N_c$: we are neglecting quark dynamics;
- Infinite tower of adjoint massive modes which are not decoupled in classical gravity limit;
- Extrapolation to $N_c = 3, N_f = 2, M_{KK} = 488$ MeV and $\lambda \sim 54$;

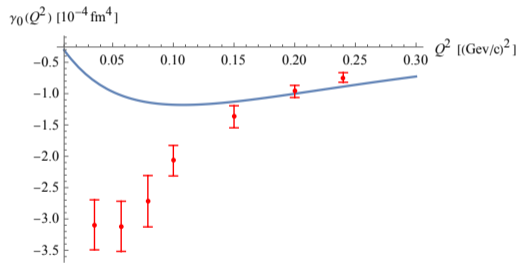
Our Results: parameters dependence

F. Bigazzi and F. Castellani arXiv:2308.16833

- “Resonance choice”: $M_{KK} = 0.488$ MeV and $\lambda \sim 54 \rightarrow m_{n(p)} = 940$ MeV and $m_{\Delta(1232)} = 1232$ MeV;
- “Meson choice”: $M_{KK} = 0.949$ MeV and $\lambda \sim 16 \rightarrow m_{\rho} \simeq 776$ MeV and $f_{\pi} \simeq 92.4$ MeV;



Results multiplied by a factor of 2×10^2 ;



Results multiplied by a factor of 10;

Some details on the WSS model

E. Witten, Adv. Theor. Math. Phys. 2, 505-532 (1998),

T. Sakai and S. Sugimoto, Prog. Theor. Phys. 113, 843-882 (2005)

- Type IIA string theory on a curved background, sourced by N_c D4-Branes wrapped along a S^1 circle of radius $1/M_{KK}$;
- Susy breaking b.c. conditions on the compactification S^1 ;
- Background metric solution:

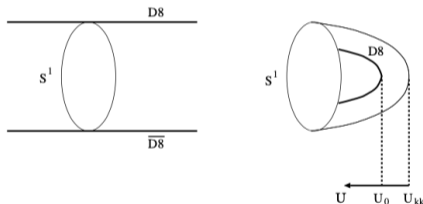
$$ds^2 = \left(\frac{U}{R}\right)^{3/2} (\eta_{\mu\nu} dx^\mu dx^\nu + f(U) dx_4^2) + \left(\frac{R}{U}\right)^{3/2} \left(\frac{dU^2}{f(U)} + U^2 d\Omega_4^2\right),$$

$$f(U) = 1 - \frac{U_{KK}^3}{U^3}, \quad e^\phi = g_s \left(\frac{U}{R}\right)^{3/4}, \quad F_4 = dC_3 = \frac{2\pi N_c}{V_4} \epsilon_4, \quad R^3 = \pi g_s N_c l_s^3, \quad V_4 = 8\pi^2/3;$$

Some details on the WSS model

T. Sakai and S. Sugimoto, Prog. Theor. Phys. 113, 843-882 (2005)

- $N_f \ll N_c$ $D8/\overline{D8}$ -Branes orthogonally posed along S^1 ;
- $D8/\overline{D8}$ -Branes join in the transverse direction respect with S^1 , geometrically realizing χSB ;



- Classic gravity limit: $\lambda \sim \frac{1}{l_s^2 M_{KK}^2} \gg 1$;

Some details on JLab experiments

V. Sulkosky et al. Nature Phys. 17, no.6, 687-692 (2021) - Jefferson Lab E97-110

D. Ruth et al. Nature Phys. 18, no.12, 1441-1446 (2022) - Jefferson Lab Hall A g_2p

Neutron - Jefferson Lab E97-110

- Longitudinal polarized electron beam;
- Polarized neutrons in ^3He nuclei target;
- Beam energies ranged from 1.1 GeV^2 to 4.4 GeV^2 ;
- Scattered electrons detected by a High Resolution Spectrometer;

Proton - Jefferson Lab Hall A g_2p

- Longitudinal polarized electron beam;
- Polarized protons in solid NH_3 target;
- Beam energies ranged from 1.2 GeV^2 to 3.4 GeV^2 ;
- Scattered electrons detected by a High Resolution Spectrometer;