The anomalous magnetic moment of the muon & the new g-2 puzzle

Francesco Sanfilippo, INFN Roma Tre

XXXVII Convegno Nazionale di Fisica Teorica

27–29 Sept 2023

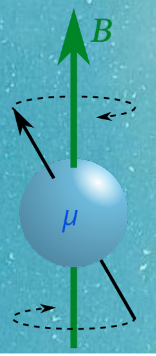
Extended Twisted Mass Collaboration

25th 1998-2023 INFN ROMA TRE

ICSC Centro Nazionale di Ricerca in HPC, Big Data and Quantum Computing

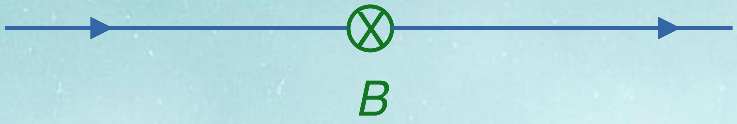
New Frontiers in Theoretical Physics 2023 CONVEGNO NAZIONALE DI FISICA TEORICA

The magnetic moment



Parametrization of the interaction of a Muon with an external magnetic field

$$\mu = g\frac{e}{2m}S$$



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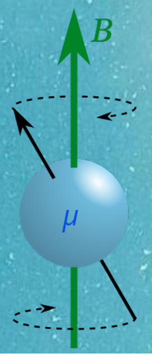

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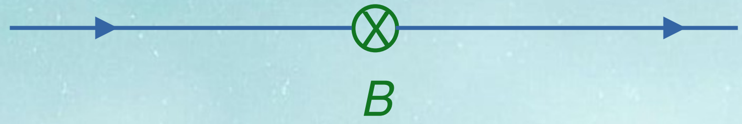

Giromagnetic factor g: relation to the particle spin S

$$\begin{aligned} \text{Proton} \quad g_p &= \quad 5.5856946893 \\ \text{Neutron} \quad g_n &= \quad -3.82608545 \\ \text{Electron} \quad g_e &= \quad -2.00231930436256 \\ \text{Muon} \quad g_\mu &= \quad -2.0023318418 \end{aligned}$$

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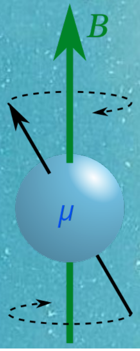

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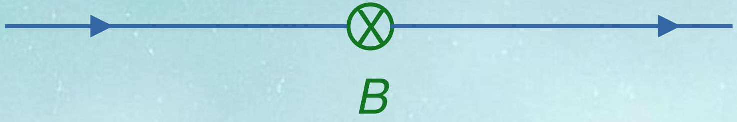


Giromagnetic factor g: relation to the particle spin S

Proton	$g_p =$	5.5856946893
Neutron	$g_n =$	-3.82608545
Electron	$g_e =$	-2.00231930436256
Muon	$g_\mu =$	-2.0023318418

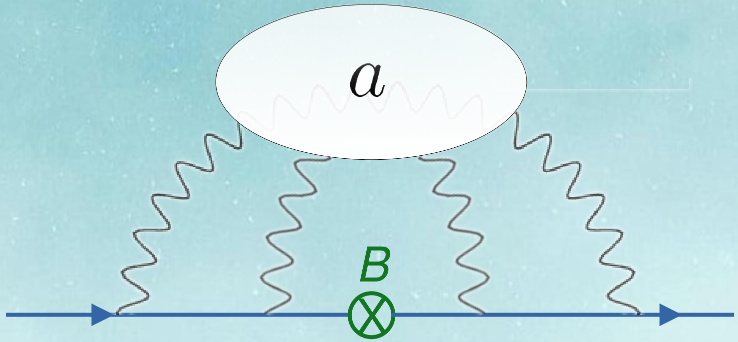
Pointlike particles: $g = 2$ Dirac equation 1928

Loop correction to *g*=2

Vacuum polarization renormalizes *g*

$$a = \frac{g-2}{2}$$

Anomalous magnetic moment



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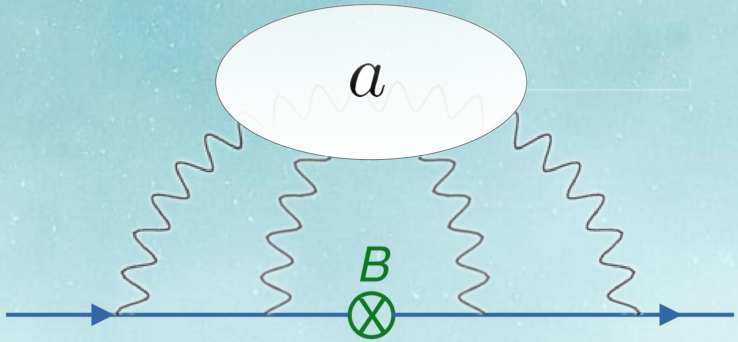

$$a = a^{QED} + a^{weak} + a^{HVP} + a^{LBL}$$

… and possibly, any sort of unknown particle

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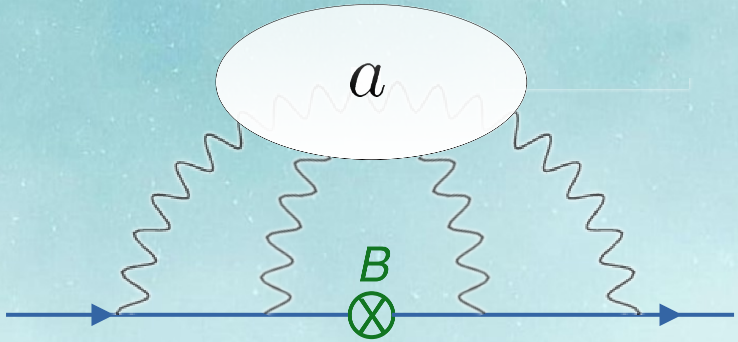

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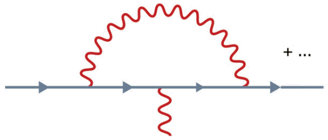

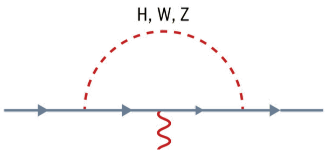

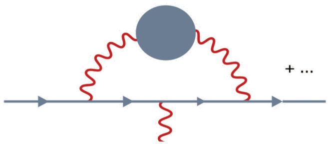

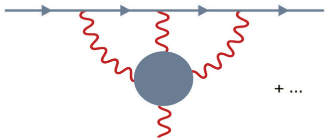

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Measure precisely $a \longrightarrow$ probe completeness of the Standard Model

The anomalous magnetic moment of leptons

$$\text{electron}: \quad a_e = 0.00115965218073$$

$$\text{muon}: \quad a_\mu = 0.00116592089$$

$$\text{tau}: \quad a_\tau = 0$$

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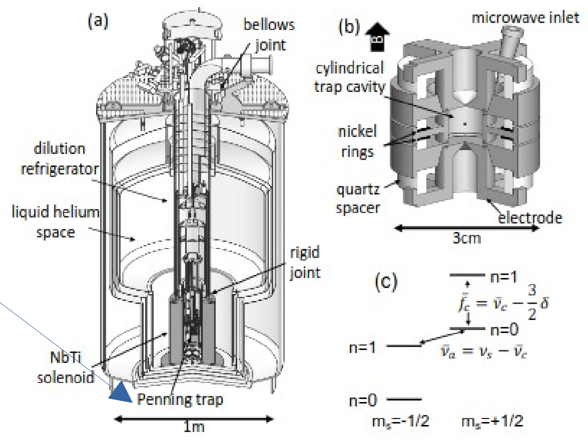

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Tau would be even better, but decays too fast to measure (but there are ideas)

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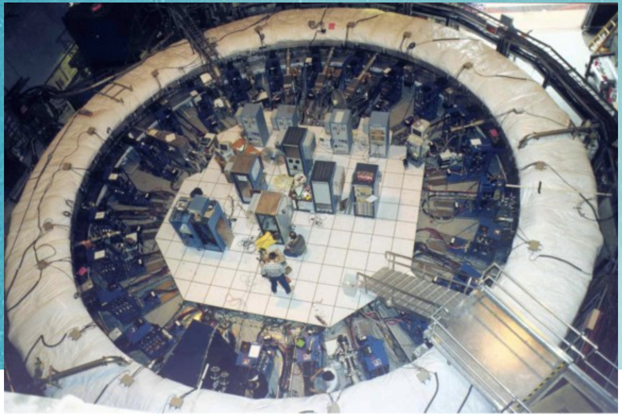

BNL E821 exp. up to 2006

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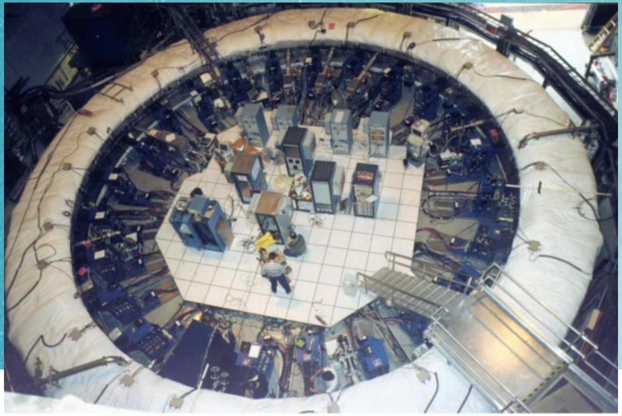

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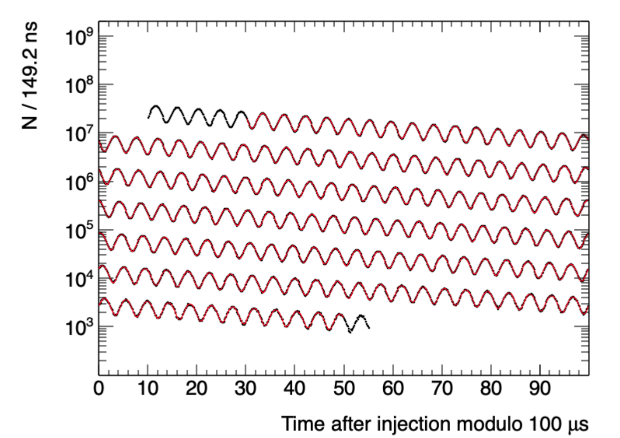

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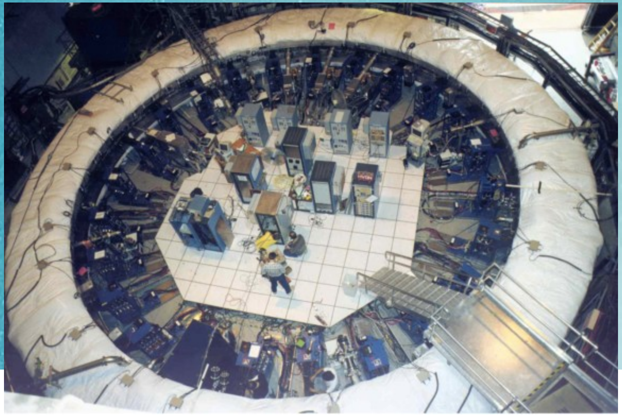

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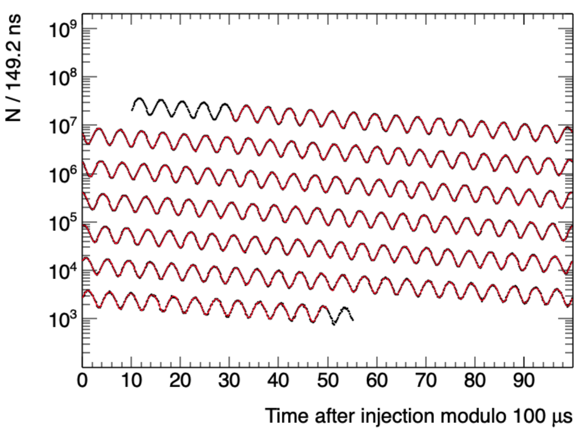

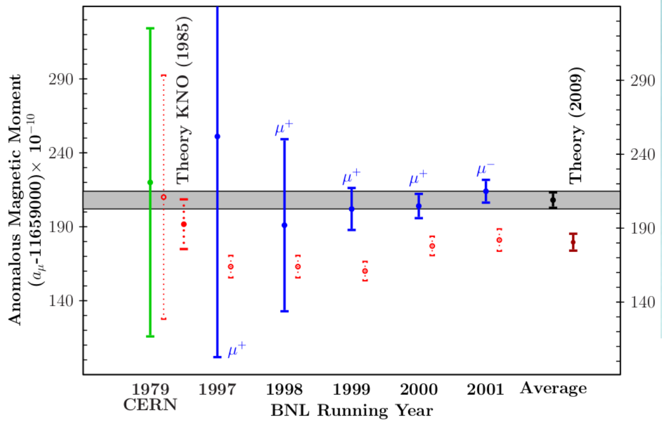

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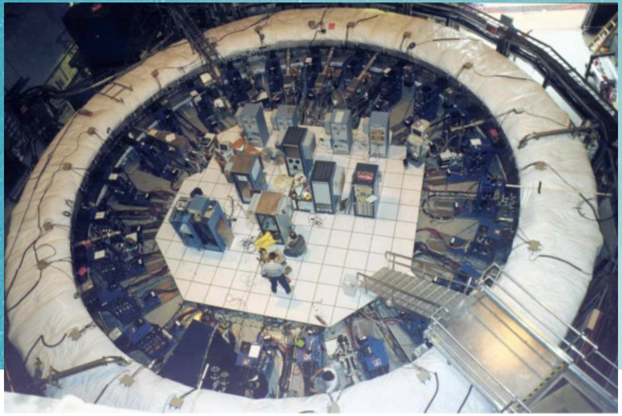

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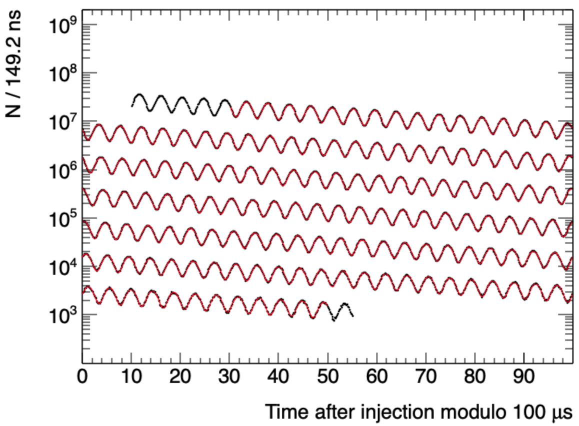

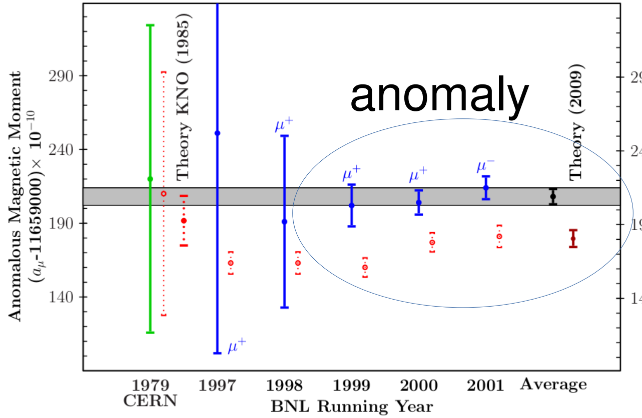

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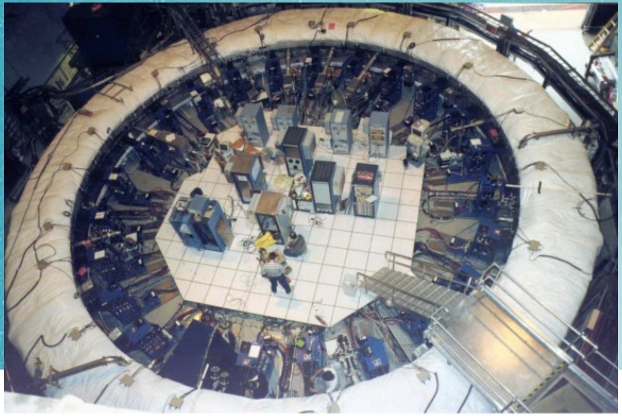

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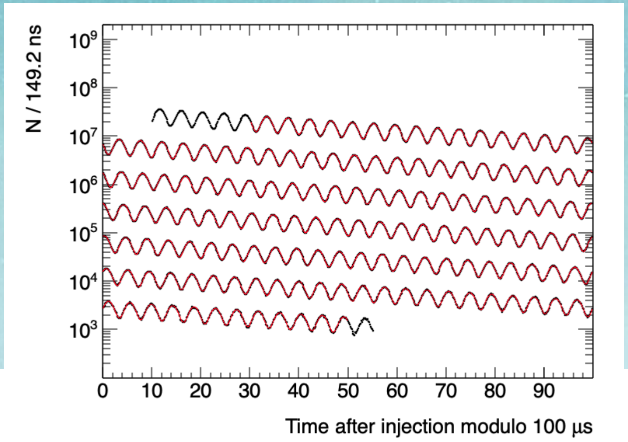

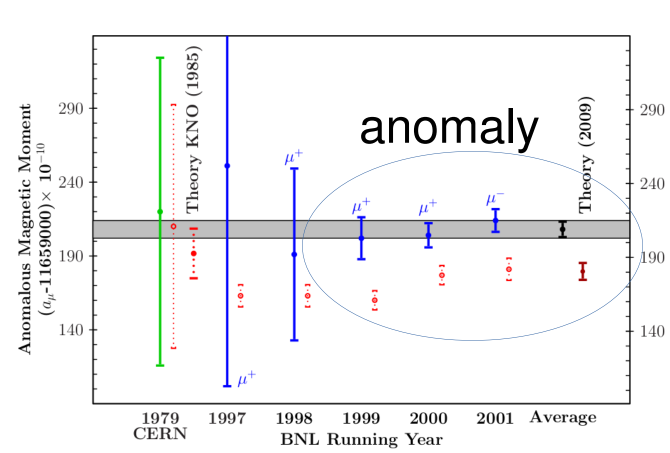

Transfer of the ring to Fermilab

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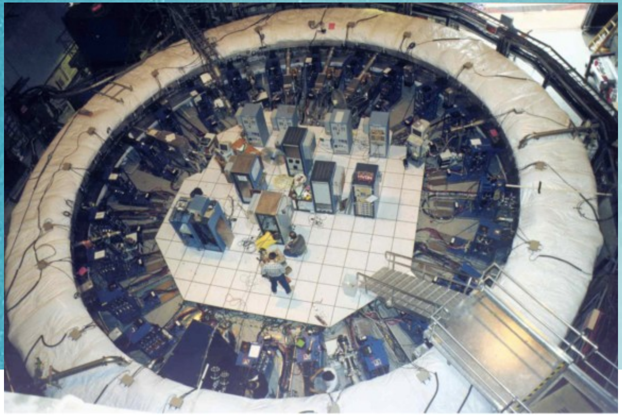

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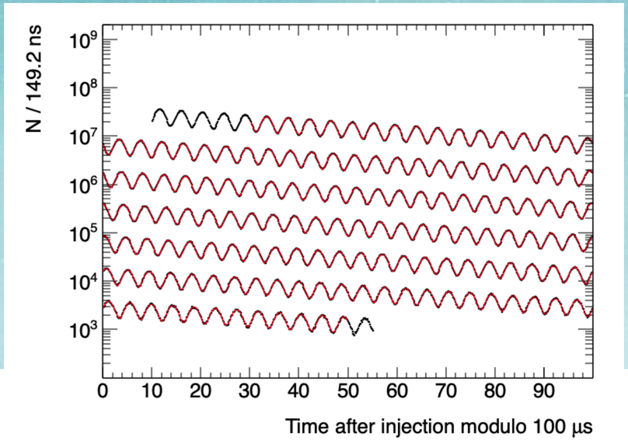

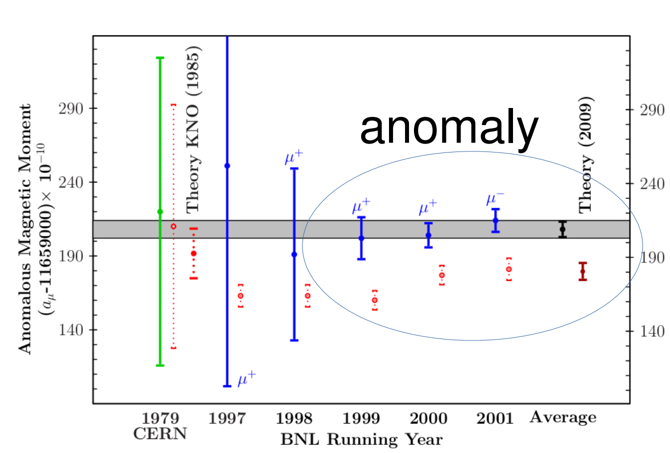

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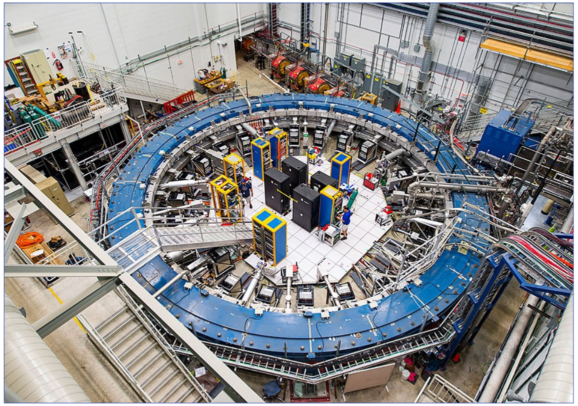

g_μ-2 experiment @ Fermilab

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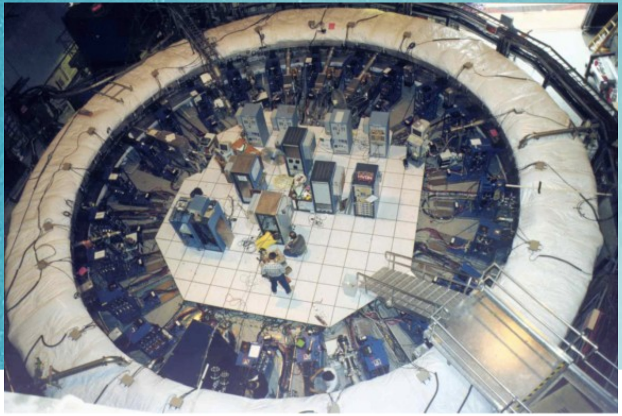

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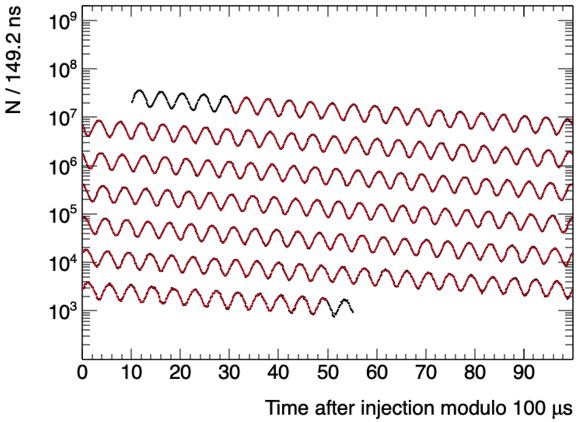

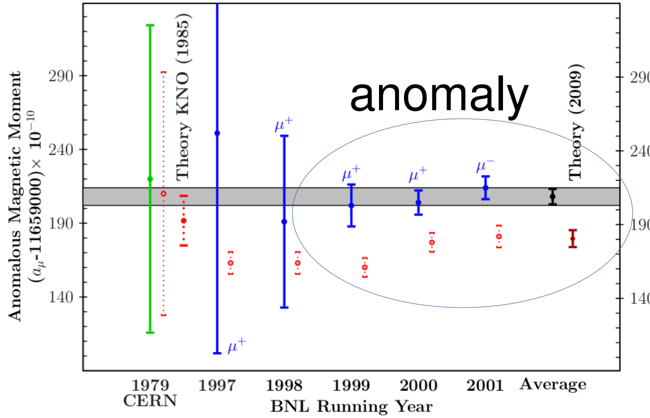

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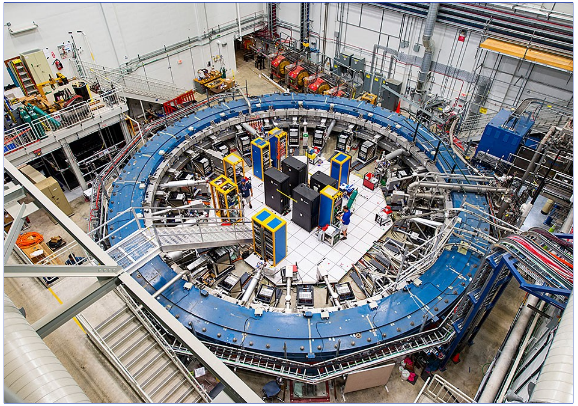

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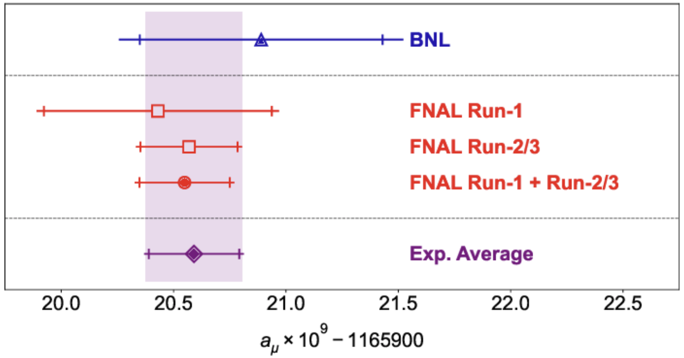

2023: measurement confirmed

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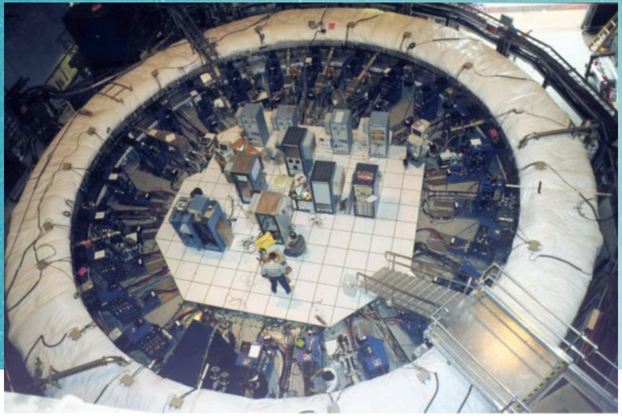

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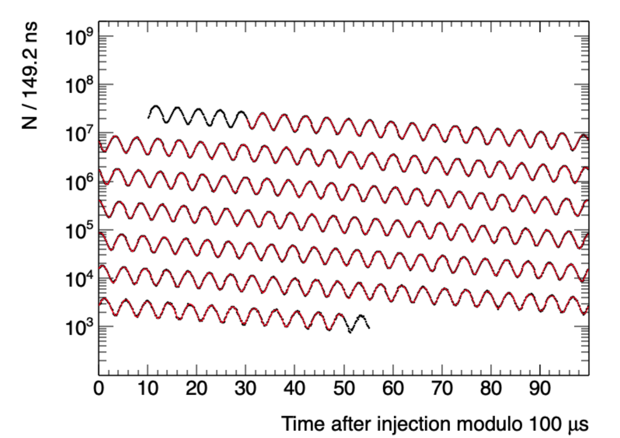

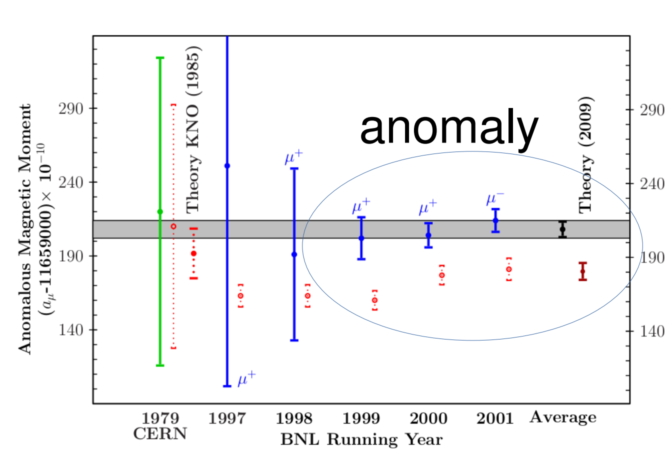

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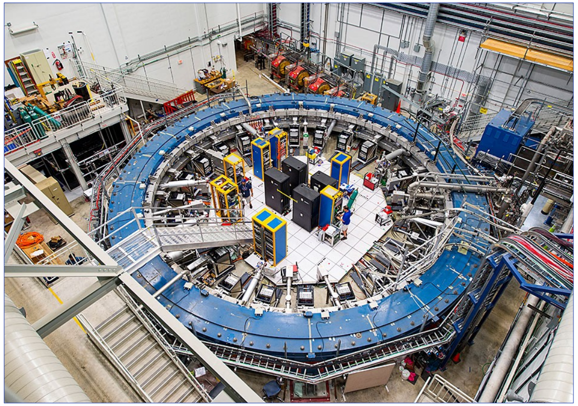

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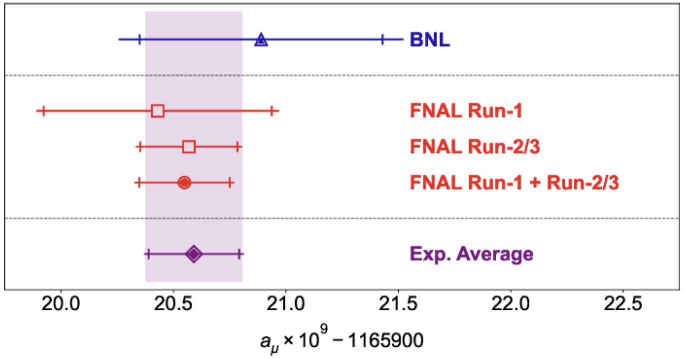


2023: measurement confirmed
What about the theory?!?

Muon g-2 Theory Initiative – from 2017

https://muon-gm2-theory.illinois.edu/

Target: match the theory precision & accuracy with the upcoming g-2 experiment

White paper: Physics Reports 887 (2020) 1-166 [arXiv:2006.04822]

Regular meetings: latest in Bern, 4-8 September 2023

The anomalous magnetic moment of the muon in the Standard Model

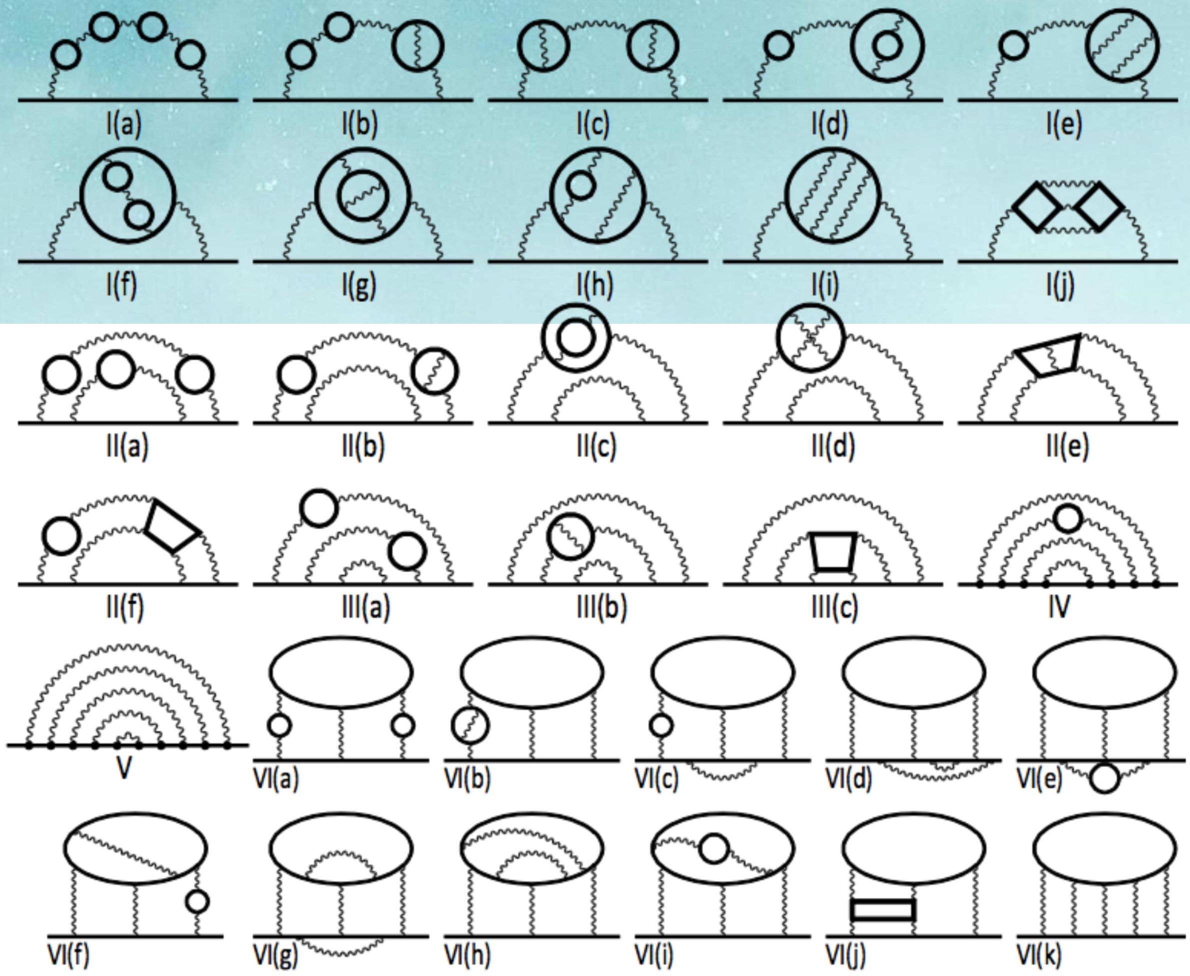
T. Aoyama[1,2,3], N. Asmussen[4], M. Benayoun[5], J. Bijnens[6], T. Blum[7,8], M. Bruno[9], I. Caprini[10],
C. M. Carloni Calame[11], M. Cè[9,12,13], G. Colangelo[†14], F. Curciarello[15,16], H. Czyż[17], I. Danilkin[12], M. Davier[†18],
C. T. H. Davies[19], M. Della Morte[20], S. I. Eidelman[†21,22], A. X. El-Khadra[†23,24], A. Gérardin[25], D. Giusti[26,27],
M. Golterman[28], Steven Gottlieb[29], V. Gülpers[30], F. Hagelstein[14], M. Hayakawa[31,2], G. Herdoíza[32], D. W. Hertzog[33],
A. Hoecker[34], M. Hoferichter[†14,35], B.-L. Hoid[36], R. J. Hudspith[12,13], F. Ignatov[21], T. Izubuchi[37,8], F. Jegerlehner[38],
L. Jin[7,8], A. Keshavarzi[39], T. Kinoshita[40,41], B. Kubis[36], A. Kupich[21], A. Kupść[42,43], L. Laub[14], C. Lehner[†26,37],
L. Lellouch[25], I. Logashenko[21], B. Malaescu[5], K. Maltman[44,45], M. K. Marinković[46,47], P. Masjuan[48,49],
A. S. Meyer[37], H. B. Meyer[12,13], T. Mibe[†1], K. Miura[12,13,3], S. E. Müller[50], M. Nio[2,51], D. Nomura[52,53],
A. Nyffeler[†12], V. Pascalutsa[12], M. Passera[54], E. Perez del Rio[55], S. Peris[48,49], A. Portelli[30], M. Procura[56],
C. F. Redmer[12], B. L. Roberts[†57], P. Sánchez-Puertas[49], S. Serednyakov[21], B. Shwartz[21], S. Simula[27],
D. Stöckinger[58], H. Stöckinger-Kim[58], P. Stoffer[59], T. Teubner[†60], R. Van de Water[24], M. Vanderhaeghen[12,13],
G. Venanzoni[61], G. von Hippel[12], H. Wittig[12,13], Z. Zhang[18],

M. N. Achasov[21], A. Bashir[62], N. Cardoso[47], B. Chakraborty[63], E.-H. Chao[12], J. Charles[25], A. Crivellin[64,65],
O. Deineka[12], A. Denig[12,13], C. DeTar[66], C. A. Dominguez[67], A. E. Dorokhov[68], V. P. Druzhinin[21], G. Eichmann[69,47],
M. Fael[70], C. S. Fischer[71], E. Gámiz[72], Z. Gelzer[23], J. R. Green[9], S. Guellati-Khelifa[73], D. Hatton[19],
N. Hermansson-Truedsson[14], S. Holz[36], B. Hörz[74], M. Knecht[25], J. Koponen[1], A. S. Kronfeld[24], J. Laiho[75],
S. Leupold[42], P. B. Mackenzie[24], W. J. Marciano[37], C. McNeile[76], D. Mohler[12,13], J. Monnard[14], E. T. Neil[77],
A. V. Nesterenko[68], K. Ottnad[12], V. Pauk[12], A. E. Radzhabov[78], E. de Rafael[25], K. Raya[79], A. Risch[12],
A. Rodríguez-Sánchez[6], P. Roig[80], T. San José[12,13], E. P. Solodov[21], R. Sugar[81], K. Yu. Todyshev[21], A. Vainshtein[82],
A. Vaquero Avilés-Casco[66], E. Weil[71], J. Wilhelm[12], R. Williams[71], A. S. Zhevlakov[78]

Electroweak contributions

T. Aoyama, M. Hayakawa,
T. Kinoshita, M. Nio

[PRLs, 2012]



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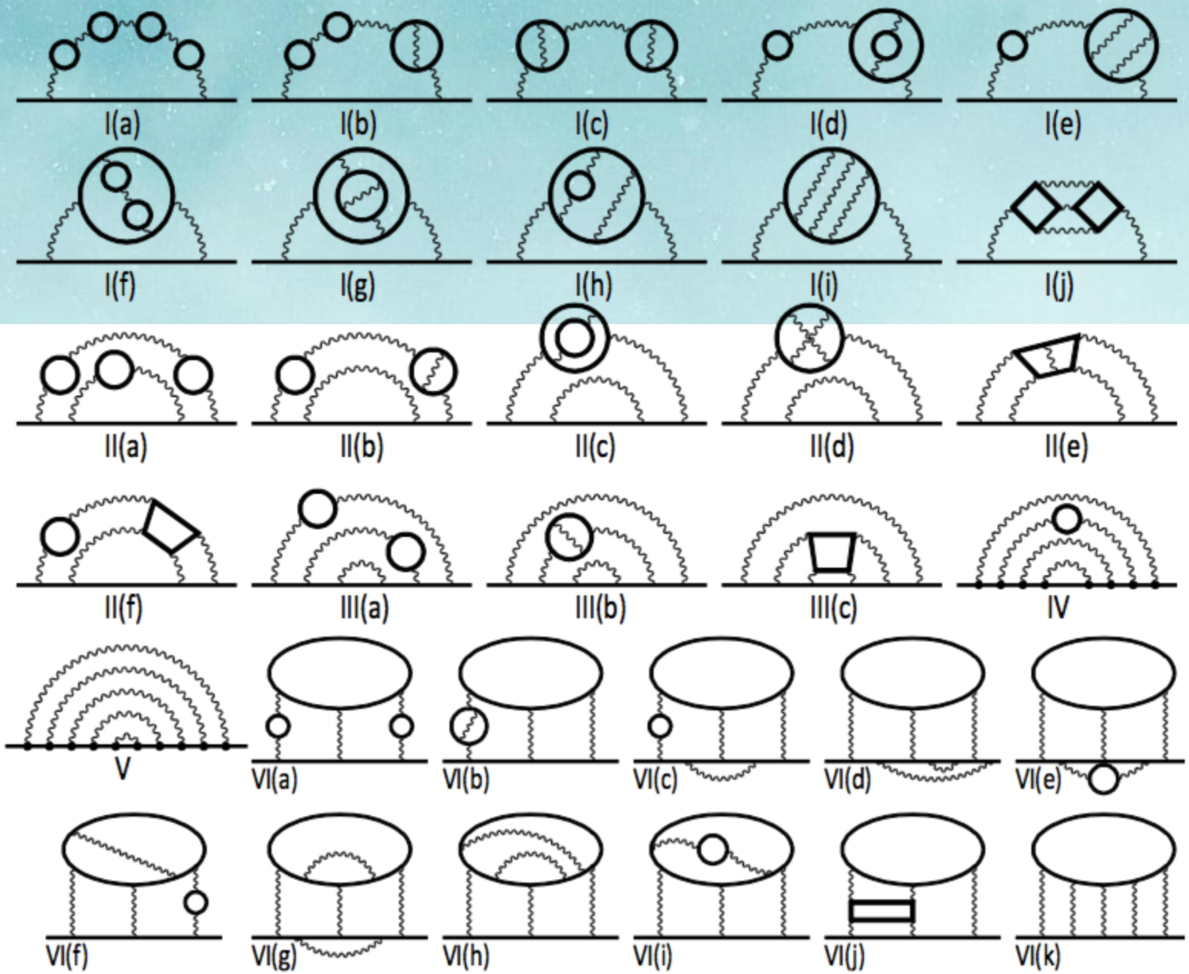
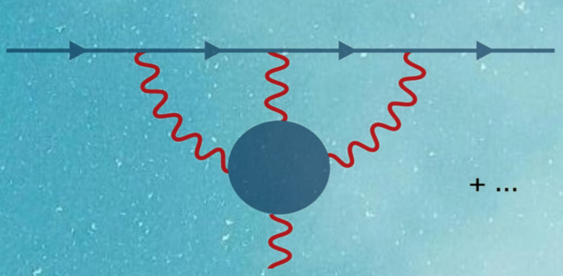


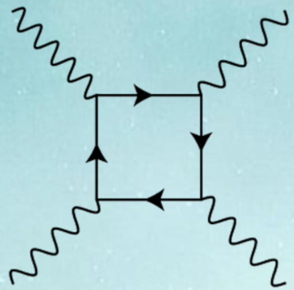
Computed up to 5th order!!!

~10000 diagrams

Light-by-light contribution



- Related to **two-photons scattering**
- Nasty hadronic contribution
- Long distance effects hard to compute
- Nonperturbative contribution



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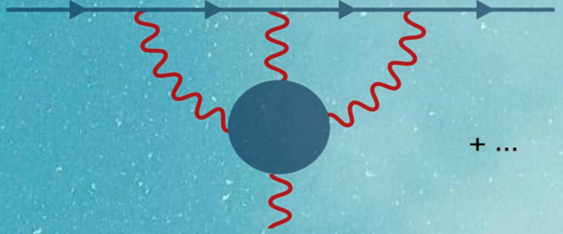

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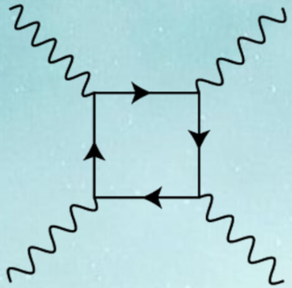
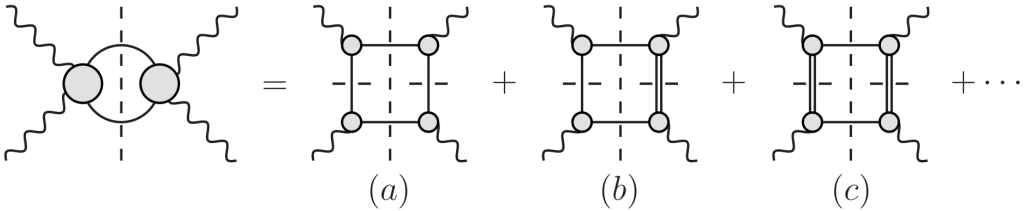


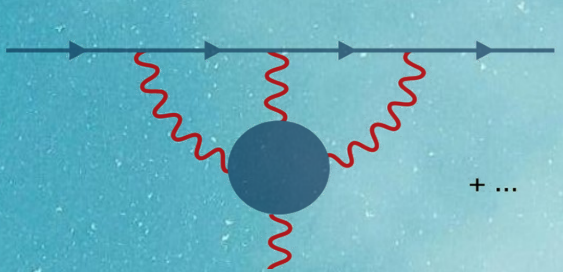
Dispersive approach



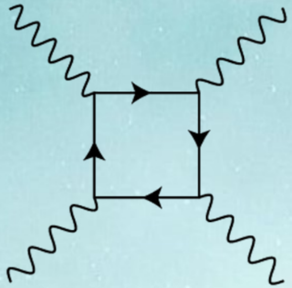
$$a_\mu^{lbl,had} = 92(19) \cdot 10^{-11}$$

[several contributions put together]

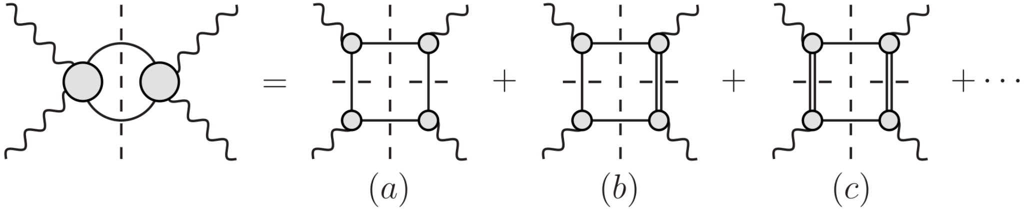
Light-by-light contribution



- Related to **two-photons scattering**
- Nasty hadronic contribution
- Long distance effects hard to compute
- Nonperturbative contribution



Dispersive approach



$$a_\mu^{lbl,had} = 92(19) \cdot 10^{-11}$$

[several contributions put together]

- Tiny, but model-dependent

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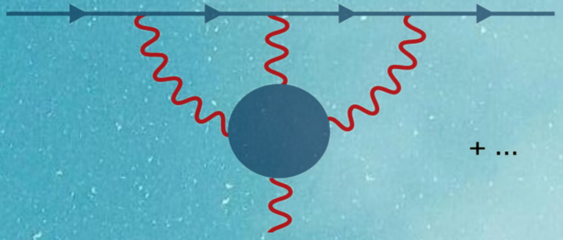

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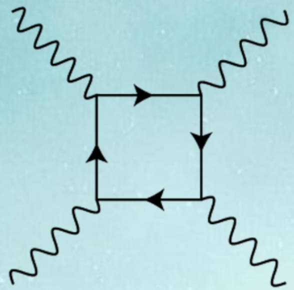

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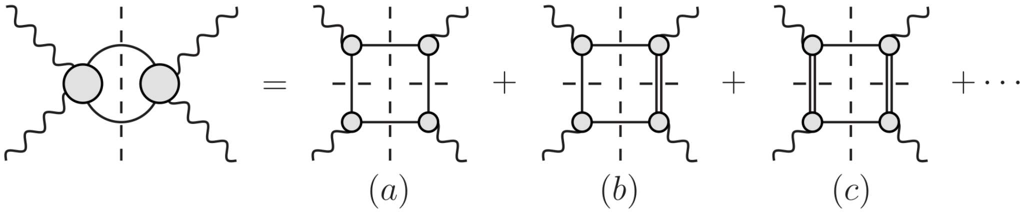

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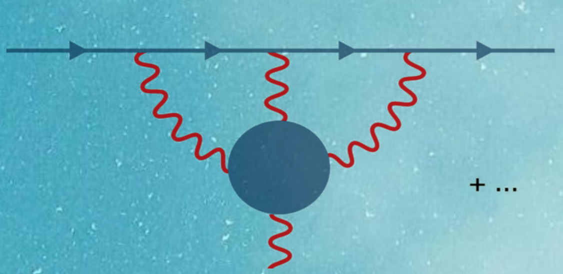

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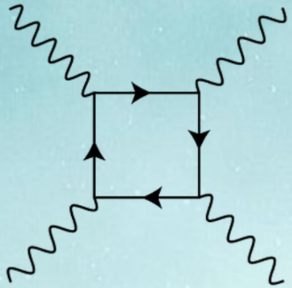

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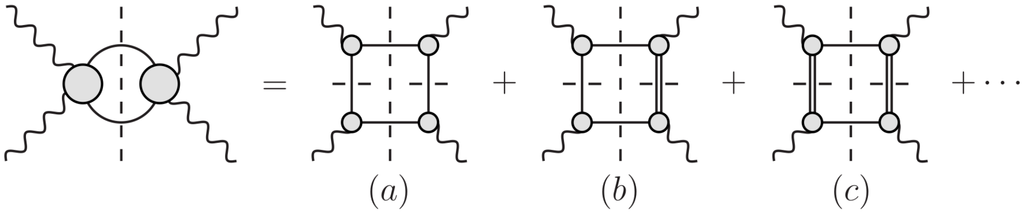

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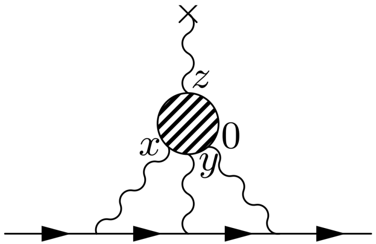
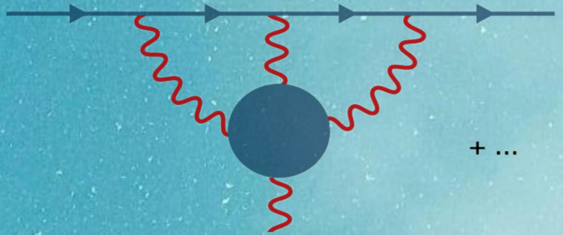
Lattice calculations



$$a_\mu^{lbl,had} = 78(35) \cdot 10^{-11}$$

[RBC/UKQCD coll, PRL 124, 2020]

Light-by-light contribution



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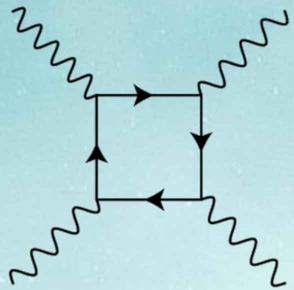

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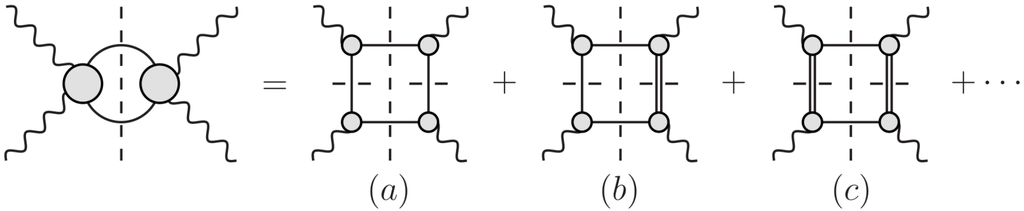

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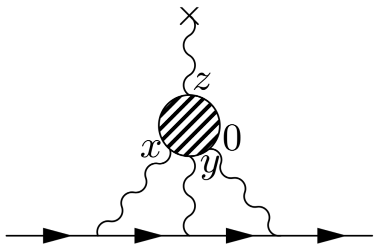

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- First principle calculation

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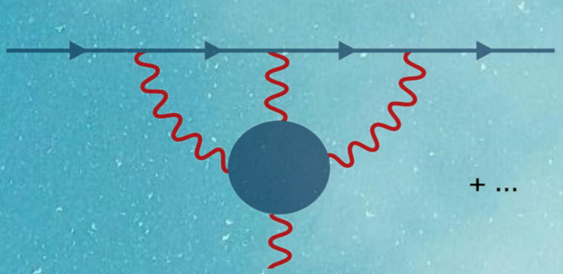

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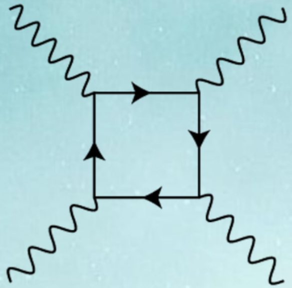

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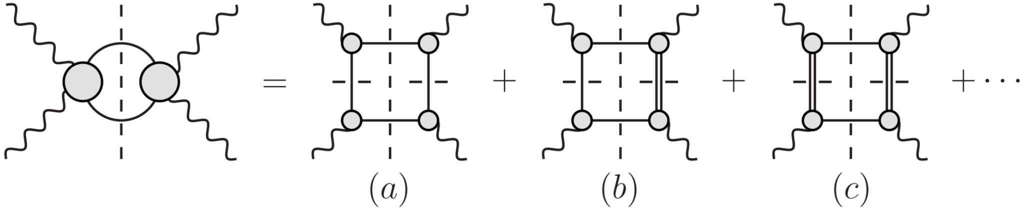

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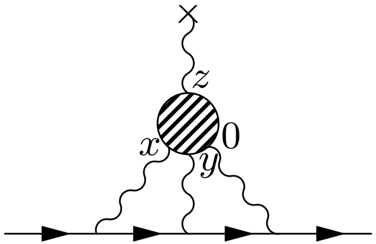

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[RBC/UKQCD coll, PRL 124, 2020]

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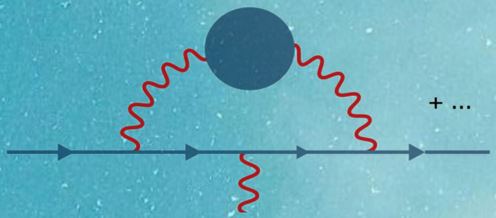
☑ Larger error, but validates model

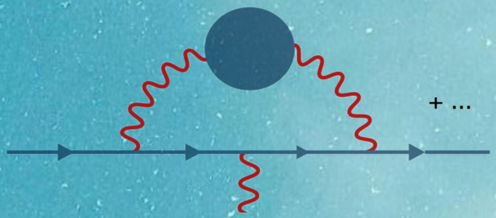
Hadronic vacuum polarization



$$a_\mu^{HVP} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dQ^2 f\left(Q^2\right) \left[\Pi\left(Q^2\right) - \Pi\left(0\right)\right]$$

analytic kernel vectorial polarization

Hadronic vacuum polarization



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Vector Correlation function

$$C_{\mu\nu}(x) = \langle j_\mu(x) j_\nu(0) \rangle$$

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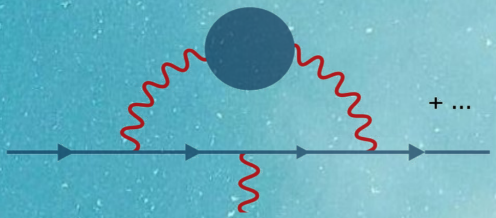

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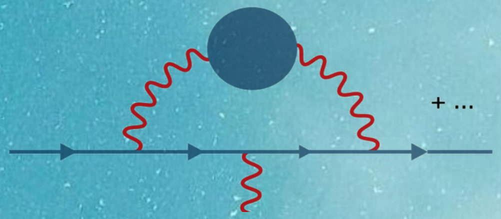

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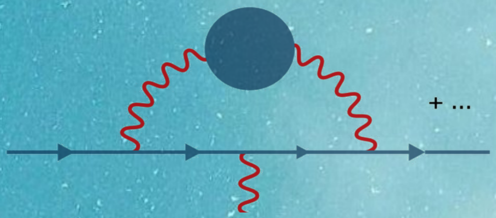

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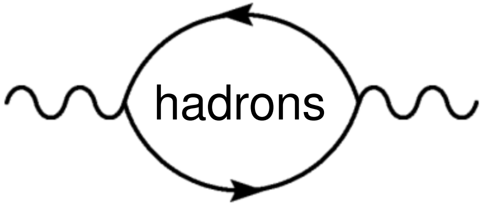

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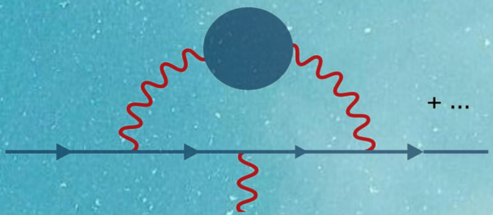

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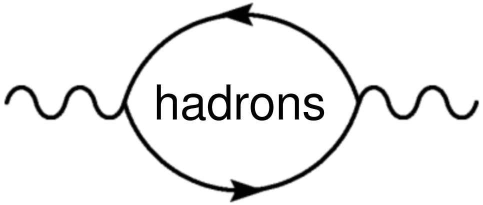

- Long distance contributions

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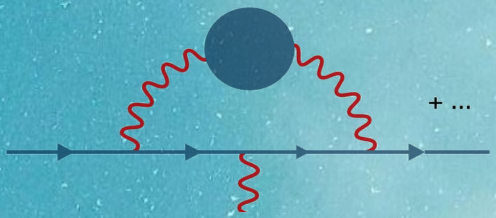

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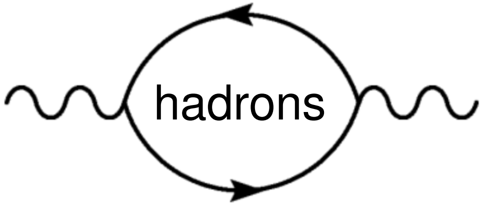

- Long distance contributions
- Nonperturbative QCD effects

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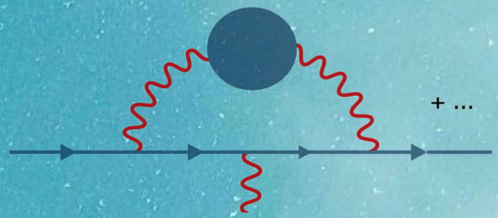

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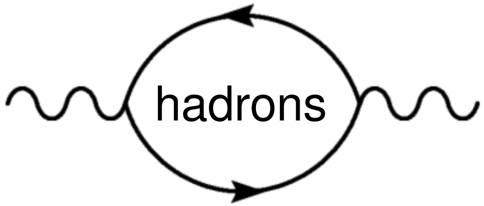

- Long distance contributions
- Nonperturbative QCD effects
- How to evaluate?!!?

How to evaluate the HVP?

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...replace it with another, **unrelated** experimental measurement!

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Optical theorem

Elastic scattering amplitude = Total $e^+ e^-$ cross section

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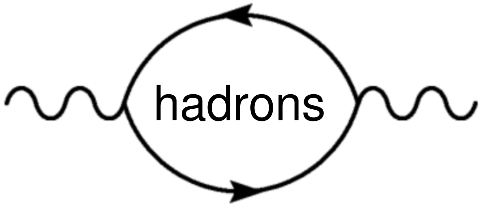

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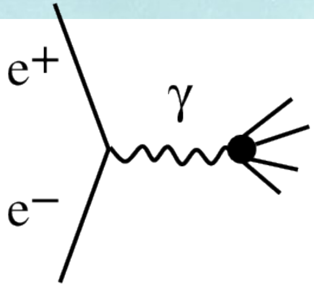

hadrons

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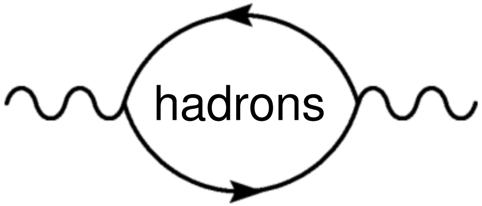

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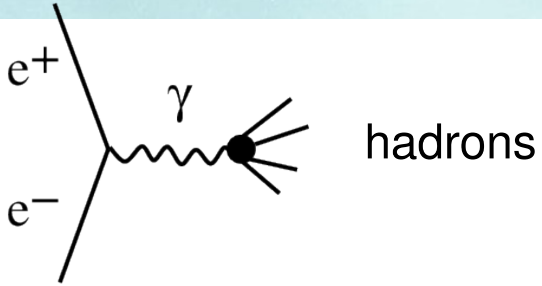

$$a_\mu^{HVP} = \int_0^\infty dQ^2 K\left(Q^2\right)\hat{\Pi}\left(Q^2\right) = \int_0^\infty R(E)K(E)dE$$

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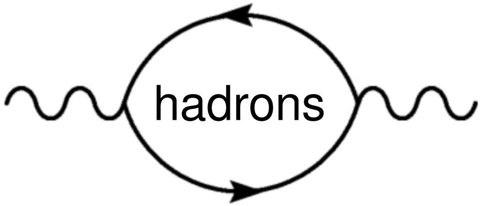

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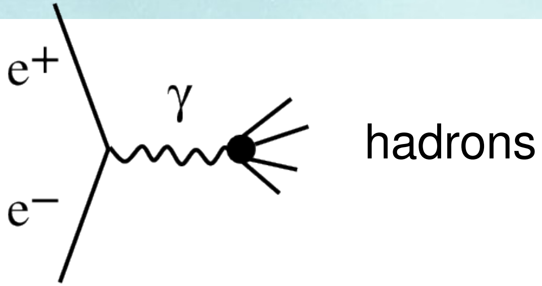

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Can we call this “theoretical prediction”…?

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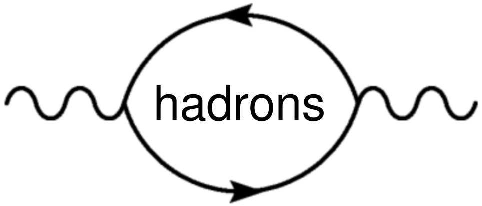

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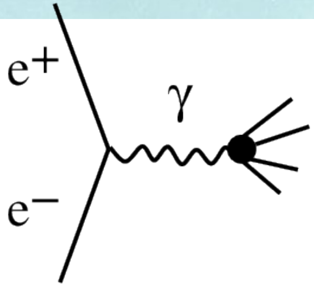

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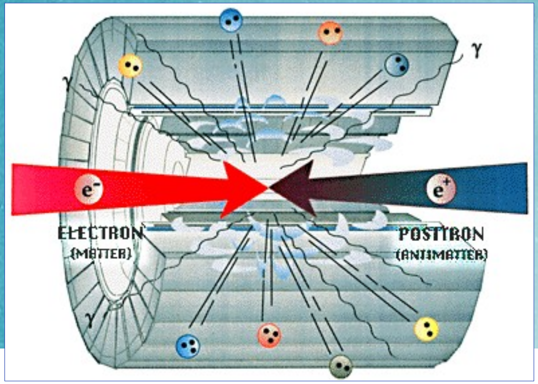
Can we call this "theoretical prediction"…?

NO! We are plugging a **substantial** experimental input

Electron-positron cross section σ



Probability of electron-positrons to annihilate into hadrons

$$R(E) = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

normalizing each energy *E* with the annihilation into muons

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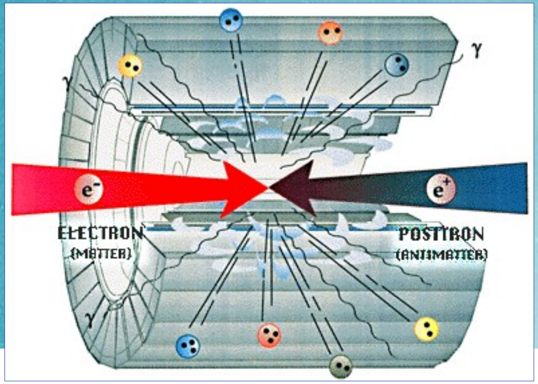

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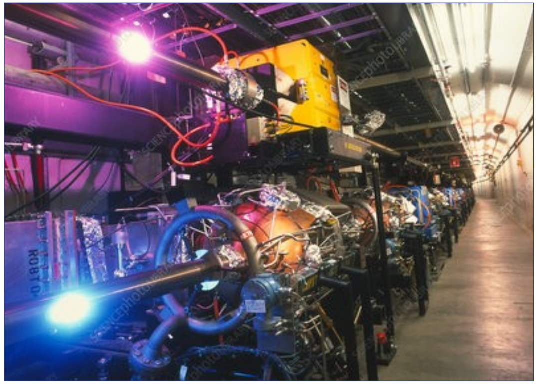
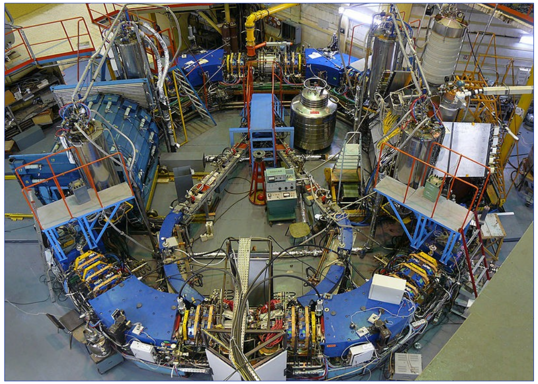
A number of worldwide experiments since the early ‘60

KLOE @ DAΦNE
FRASCATI

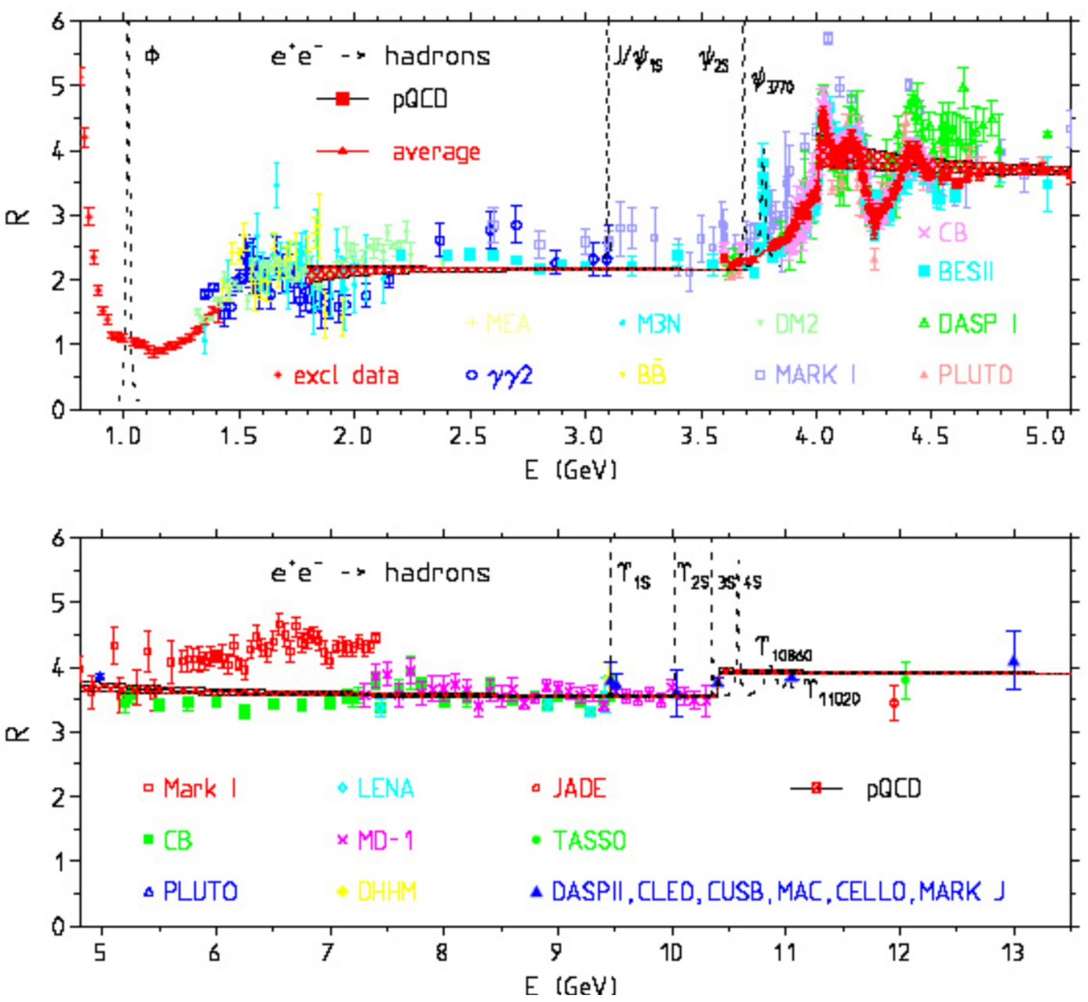


BABAR @ SLAC
STANFORD

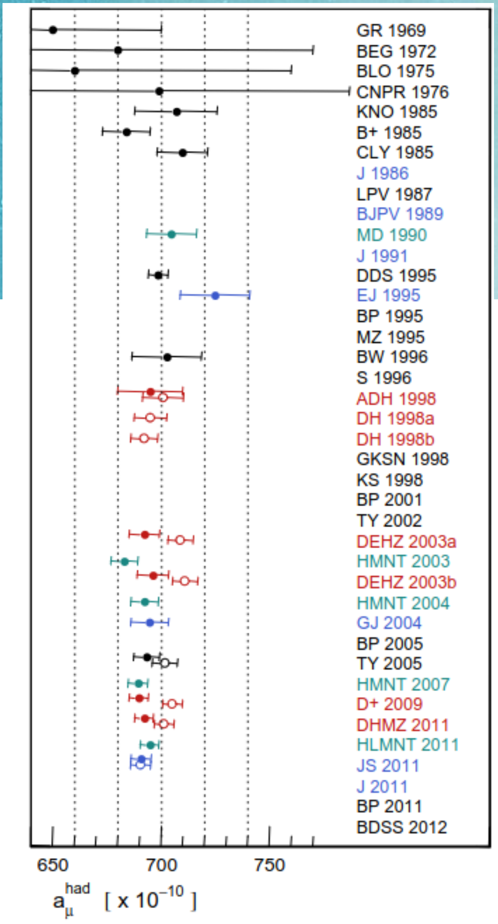


CMD3 @ VEPP-2000
NOVOSIBIRSK

R-ratio: combination of many experiments



HVP from phenomenological R-ratio



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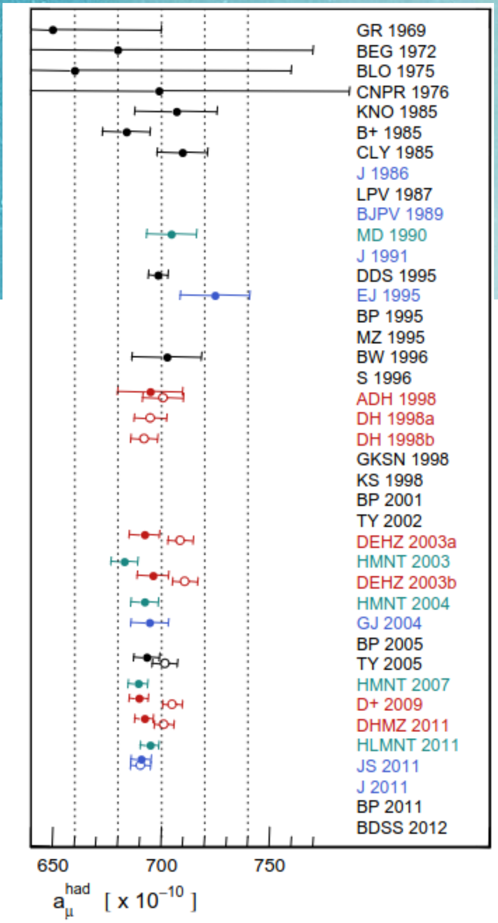
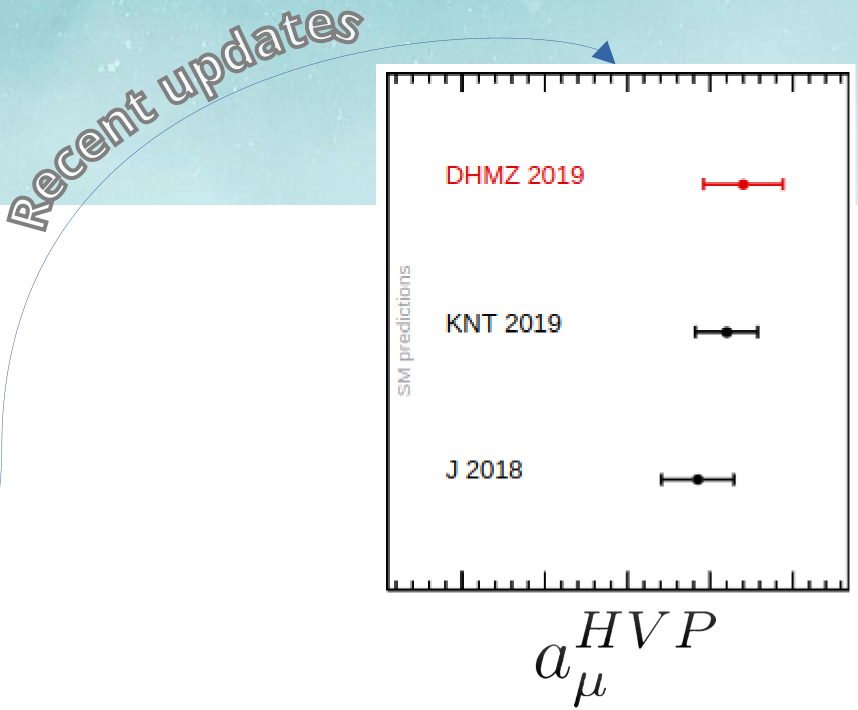
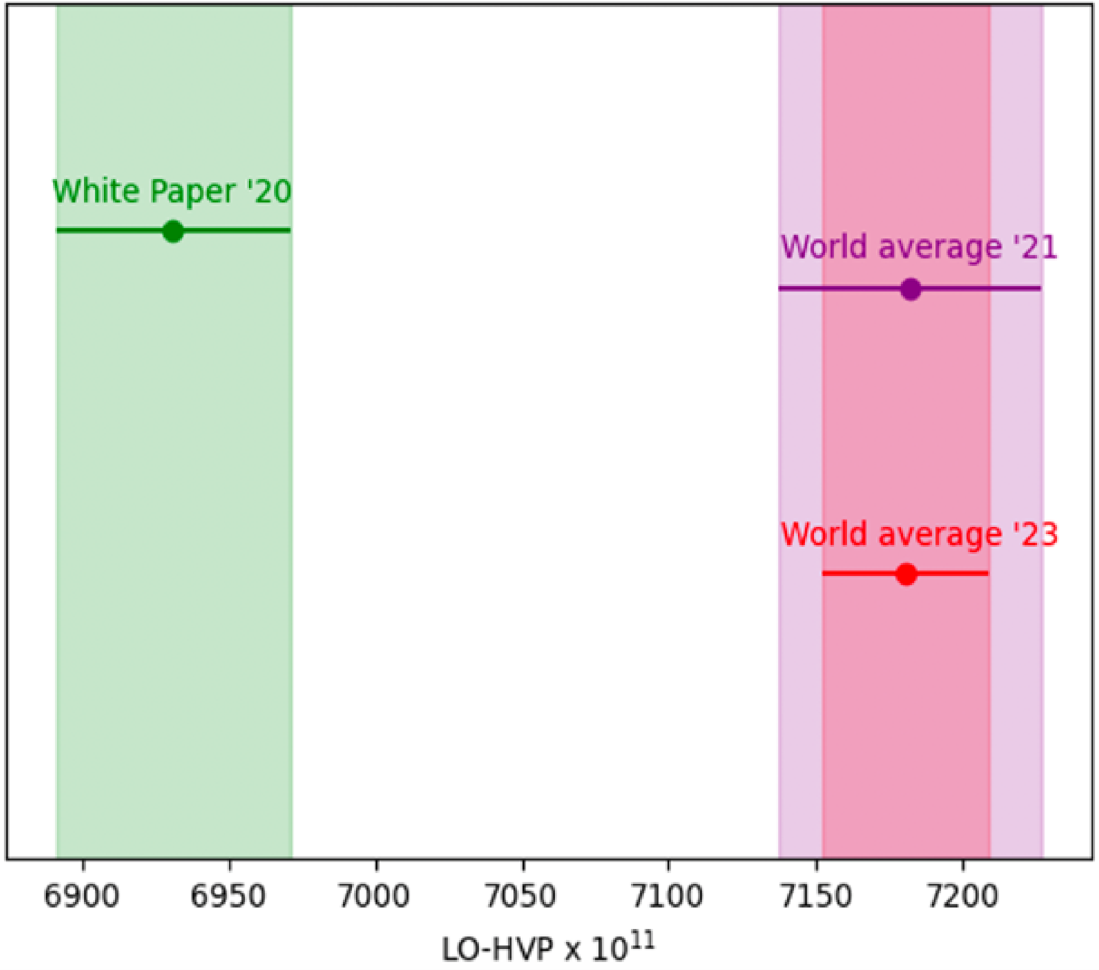




M. Davier, A. Hoecker, B. Malaescu, Z. Zhang,

[Eur. Phys. J. C 80 (2020) 241]

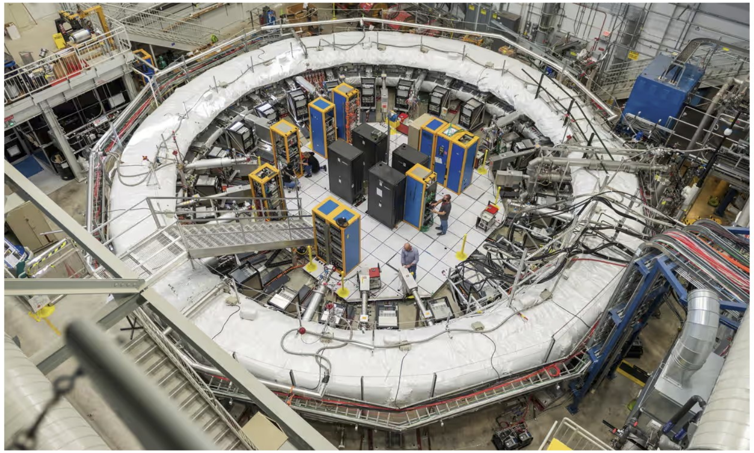
The renowned g-2 puzzle



August 2023: release of Run III results by Fermilab g-2 experiment

Scientists may be on brink of discovering fifth force of nature

Experts closing in on potentially identifying new force after surprise wobble of subatomic particle



The muon g-2 ring sits in its detector hall at the Fermilab in Illinois. Photograph: Ryan Postel/Fermi national accelerator laboratory/Reuters

The tantalising theory that a fifth force of nature could exist has been given a boost thanks to unexpected wobbling by a subatomic particle, physicists have revealed.

Hold on, fifth force!

Do we really control the theory uncertainties?

After all, we are replacing HVP with a combination of other experiments

Let us look back at the R-ratio...

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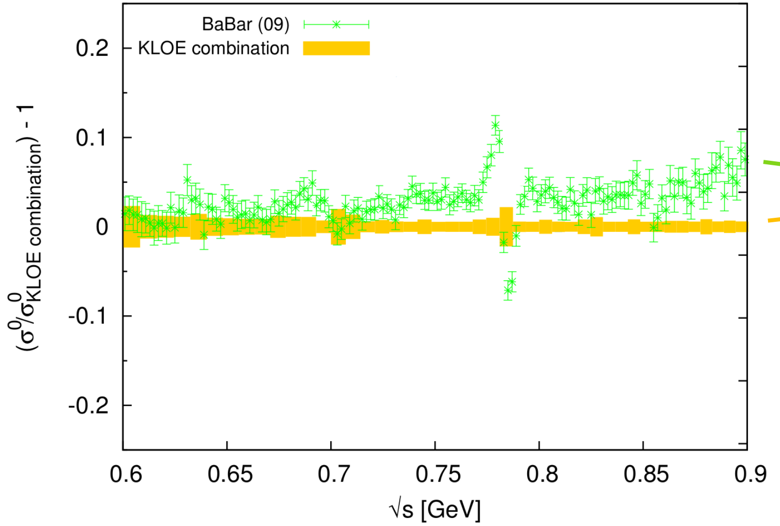
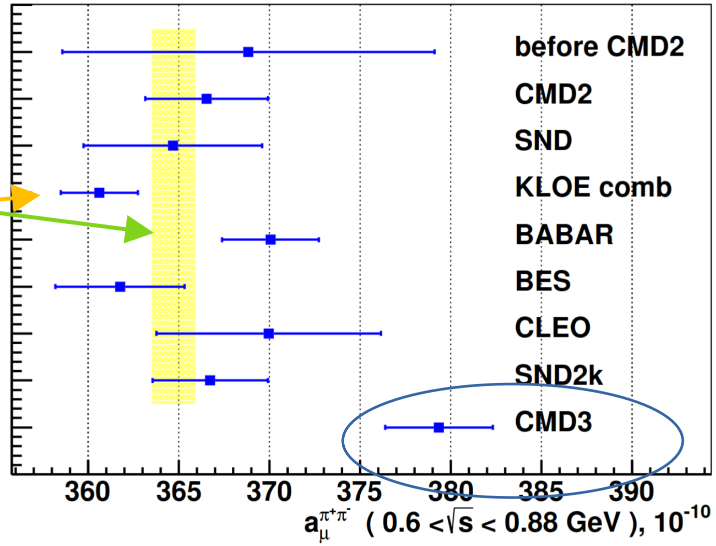
Let us look back at the R-ratio...

Hints of tension in the two-pions final state



Disagreement of 2023 CMD3 measurement



Let’s put the questions back on track

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- "Computing" HVP via dispersive method is the <u>weakest part</u> of the story

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    - Can we compute for real HVP from the first principle of the theory?

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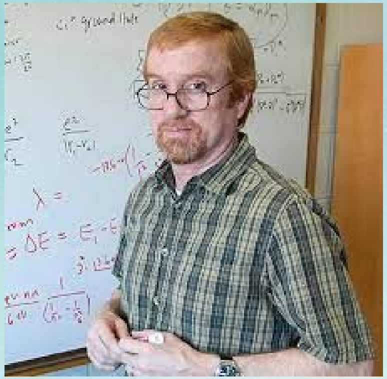
  - Can we compute for real HVP from the first principle of the theory?

    → Lattice QCD comes to the rescue!

Computing HVP from the first principles



Original proposal

"*Lattice Calculation of the Lowest-Order Hadronic Contribution to the Muon Anomalous Magnetic Moment*"

[T. Blum, PRL 91 (2003)]

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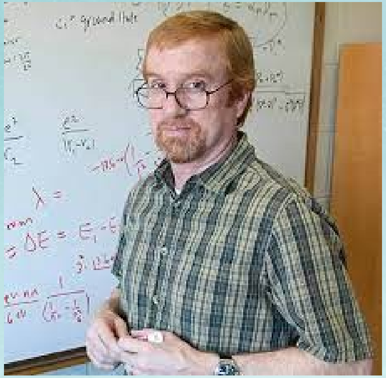
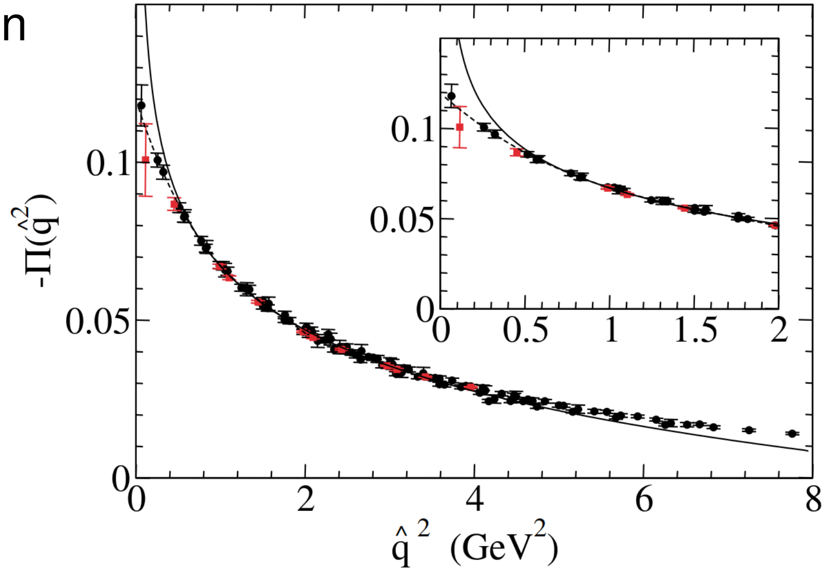


Fourier transform of lattice-computed correlation function

$$\Pi_{\mu\nu}\left(Q^2\right) = \int d^4x e^{iQx} C_{\mu\nu}(x)$$

"simple" two points correlation function



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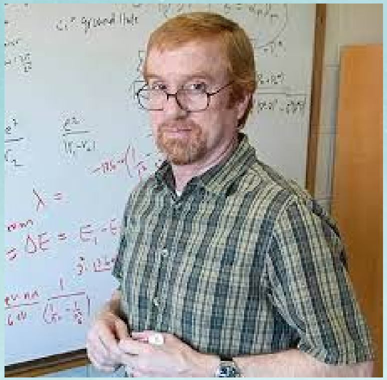

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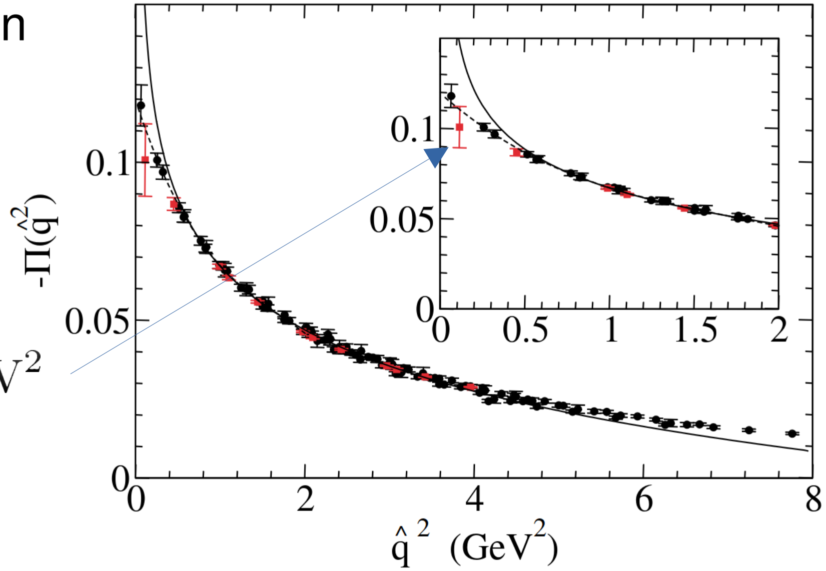
Issue: Convolution kernel enhances $Q^2 \sim m_\mu^2 \sim 0.01\mathrm{GeV}^2$

- Momenta on the lattice are quantized
- Lowest momenta are very noisy

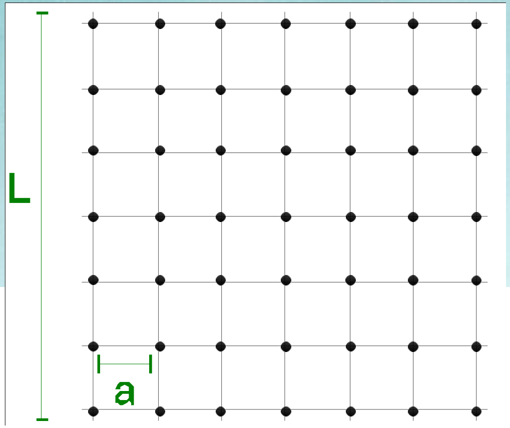


Lattice QCD simulation

First principle simulation of strong interactions

Quantum Chromodynamics on a Lattice

Euclidean spacetime with $O(10^{10})$ degrees of freedom



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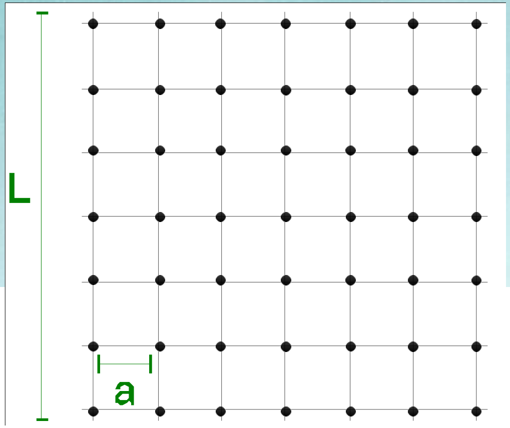

Hybrid Monte Carlo + Molecular Dynamics simulations

Numerical solution of the discrete Dirac Equation
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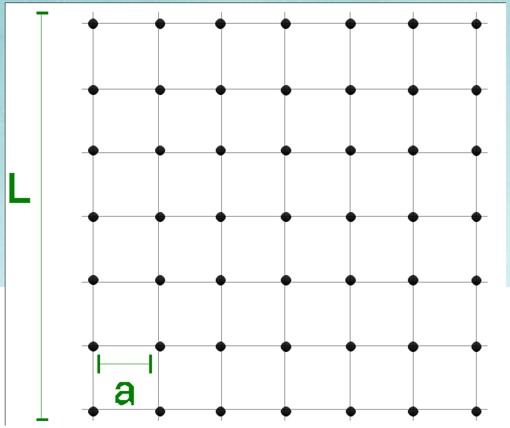

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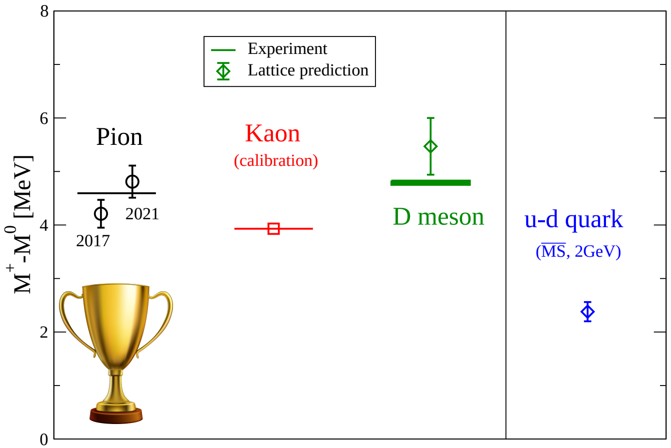
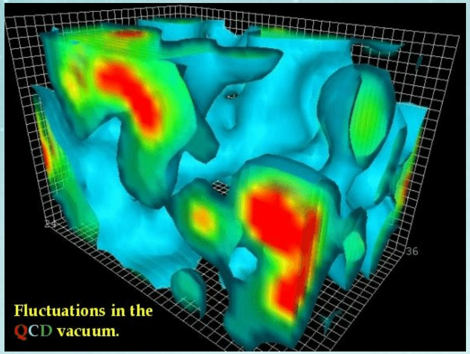
A long list of scientific achievements:

- ‰ reconstruction of the hadron spectrum,
- thermodynamics of strong interactions,
- ...



Correlation function in lattice QCD

Task #1: Producing O(100-1000) “**configurations**” of gluonic fields.



- 1 configuration: O(1-50 GB data) ~ 1 day of simulation on **O(5000) cores**.
- 100s MCoreHours in national, European & worldwide supercomputers
- Similar in spirit to storing collision events at particle accelerators
  → a handful of collaboration worldwide

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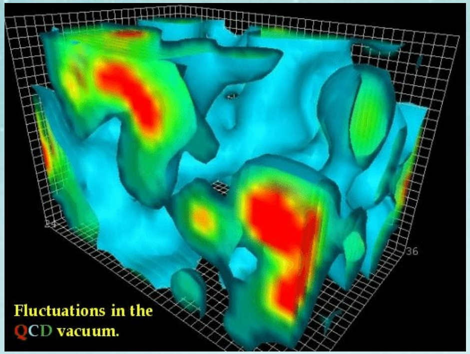

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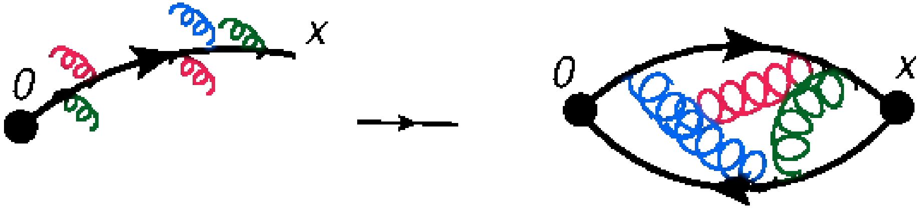

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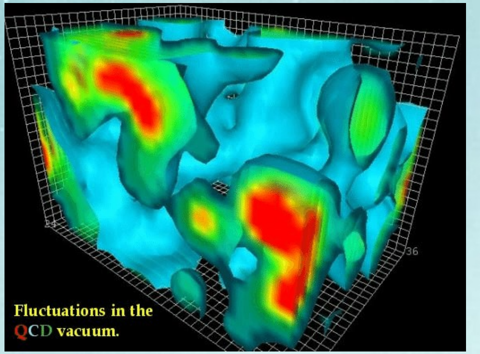

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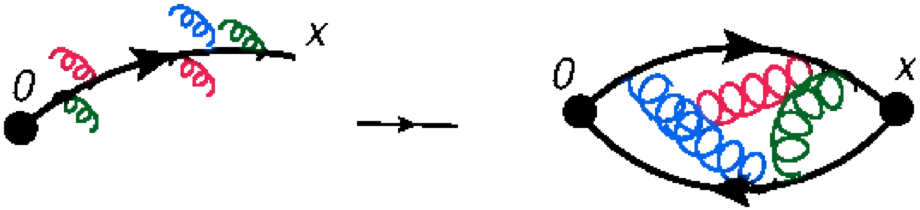

Key point: Lattice is a real (Euclidean) space method
So let's stay in real space!

HVP from real space

By simply taking **Laplace** transform:

$$a_\mu^{HVP} = \int_0^\infty dQ^2 K\left(Q^2\right)\hat{\Pi}\left(Q^2\right) \quad \rightarrow \quad \int_0^\infty dt \tilde{K}(t) C(t)$$

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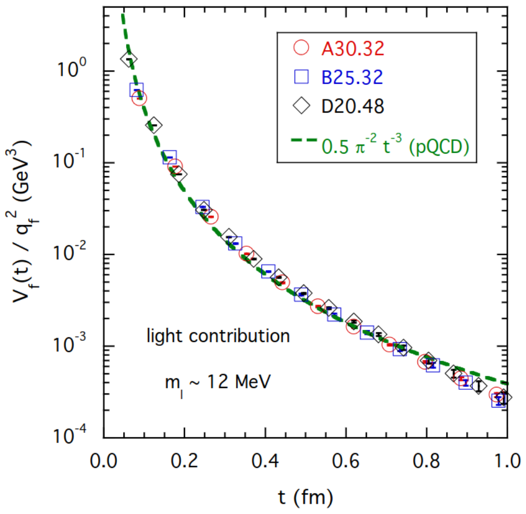

Integral converges

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$$C(t) \to e^{-At}$$

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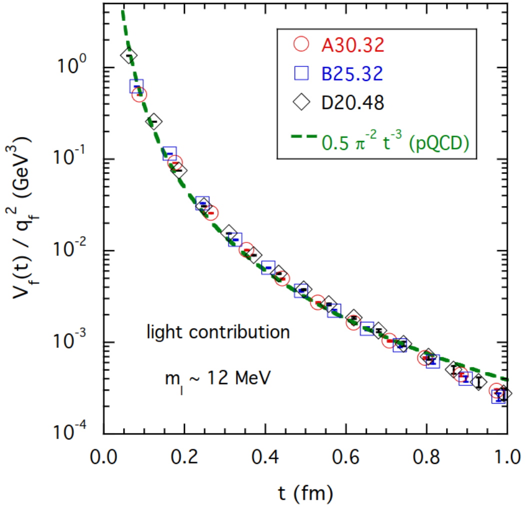

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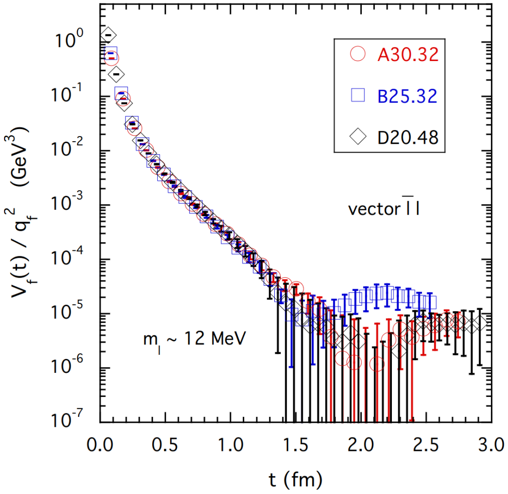


Issue

Exponentially large noise at large time

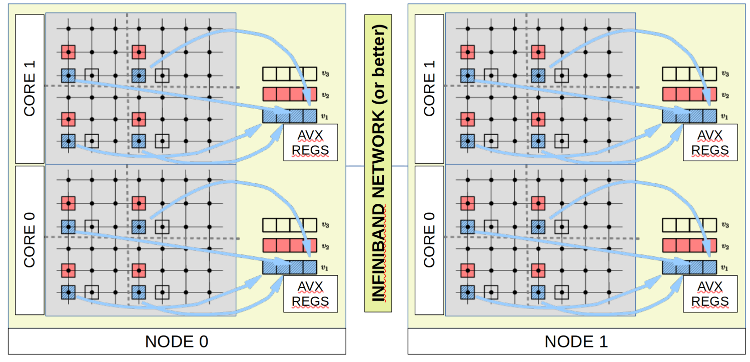
$$\frac{S}{N}(t) \rightarrow e^{-Bt}$$

[G.Parisi, 1984]

Difficult task: hard work, special tools

POWERFUL
SUPERCOMPUTERS
&
GOOD USAGE

"La potenza è nulla senza il controllo"



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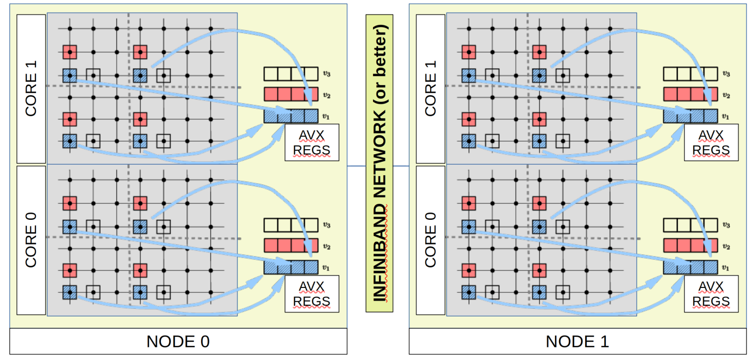
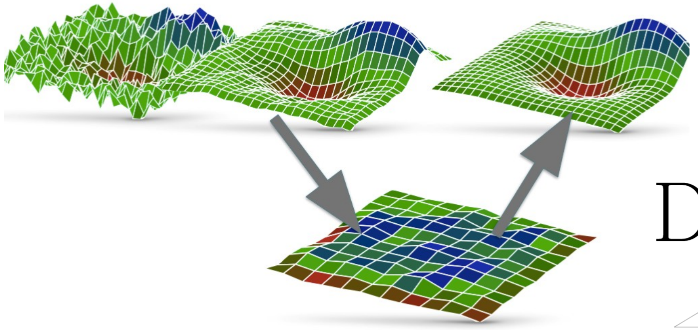
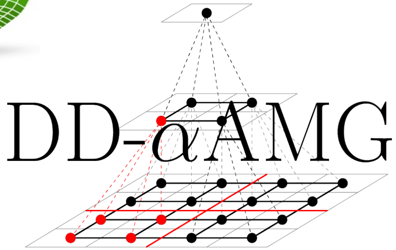


MODERN
ALGORITHMS
&
NEW METHODS



Adaptative solvers



Multigrid

Eigendeflation

Difficult task: hard work, special tools

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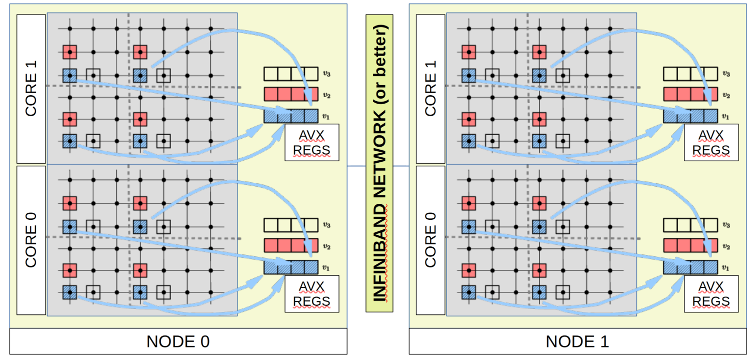

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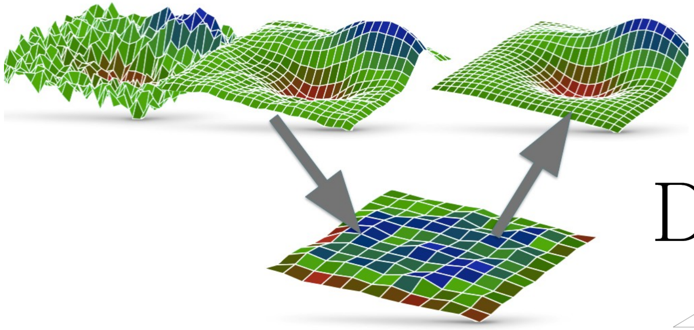

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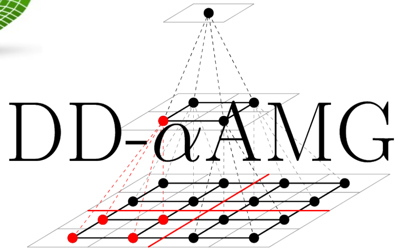


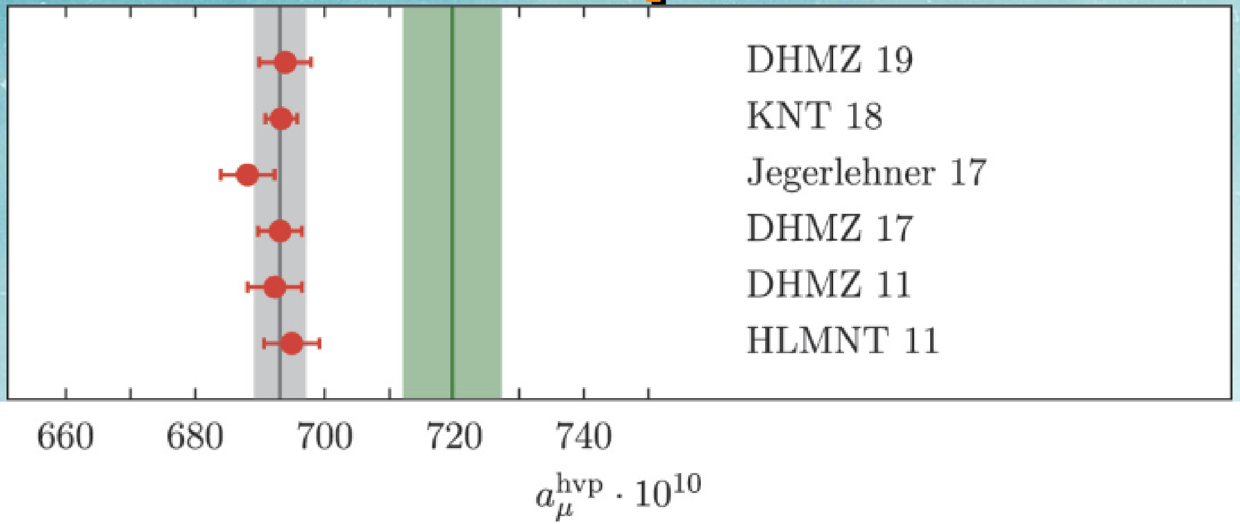
Multigrid

Eigendeflation

… and a lot of attention from the community/CPU hours funding

Lattice results vs. dispersive results

Dispersive results
incompatible
with the
experiments



Lattice results vs. dispersive results

Dispersive results
incompatible
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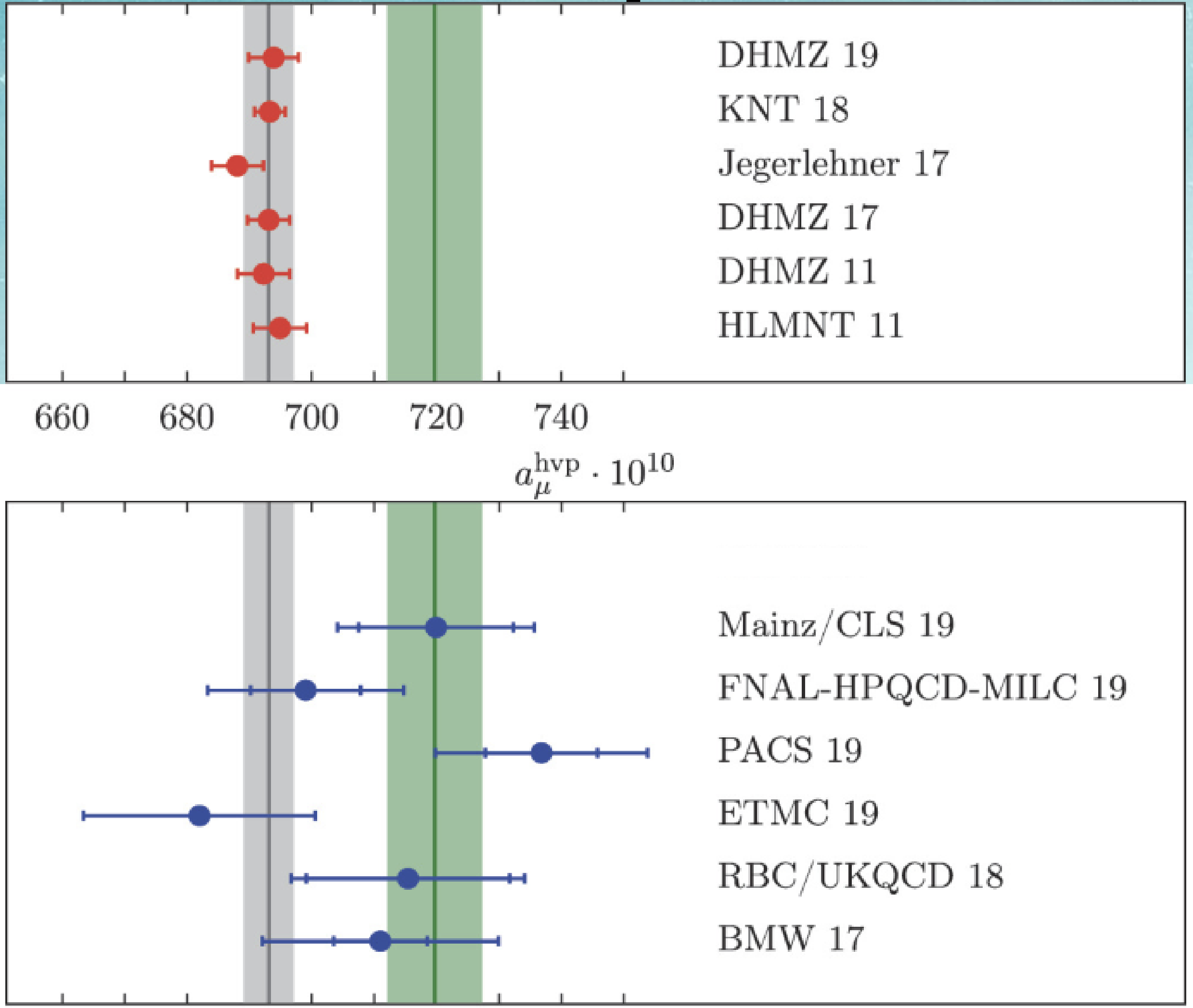
Until 2020:
Lattice results
were **compatible**
with...
everything



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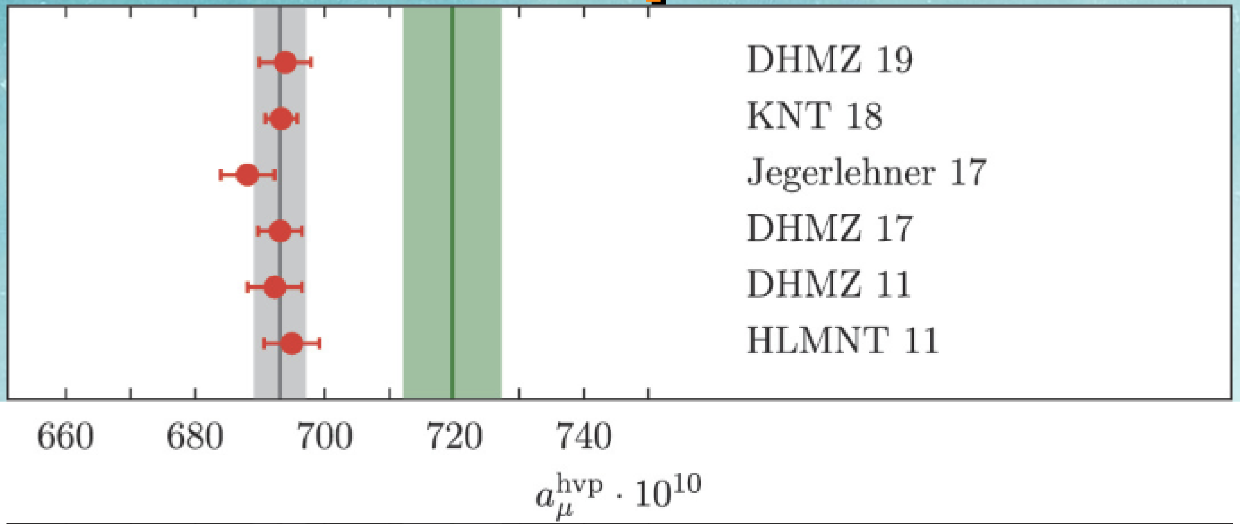

From 2020:
Lattice results
are **compatible**
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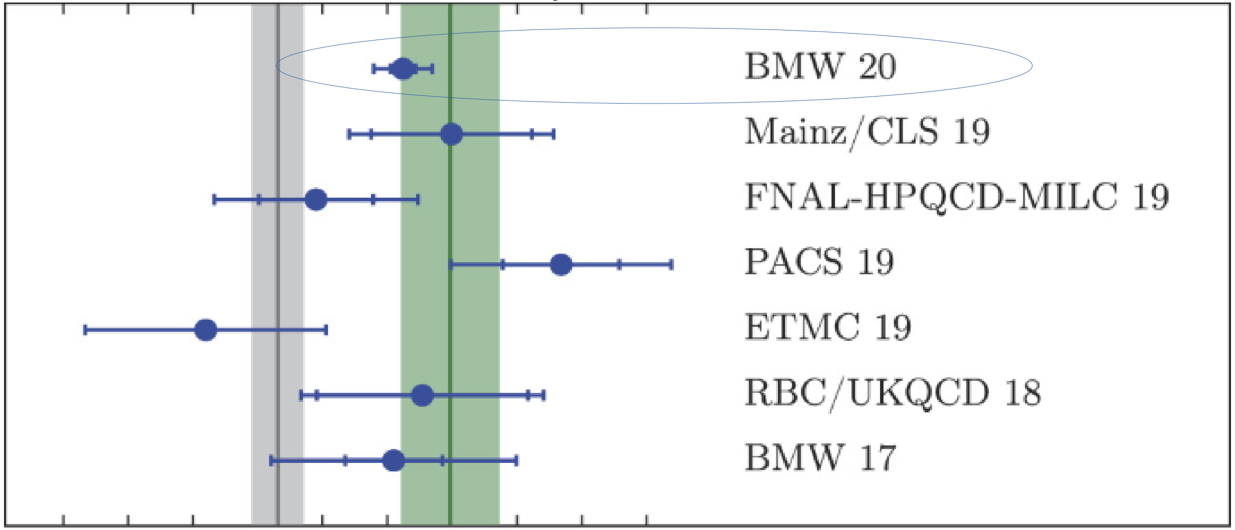
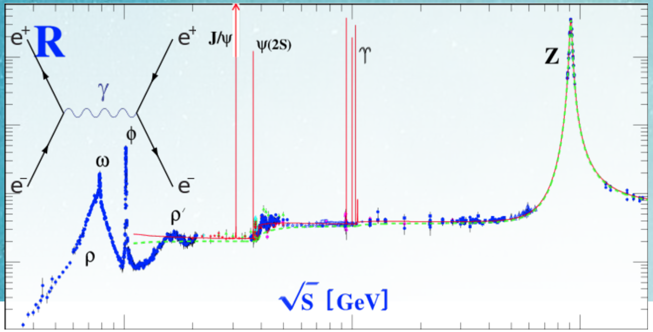
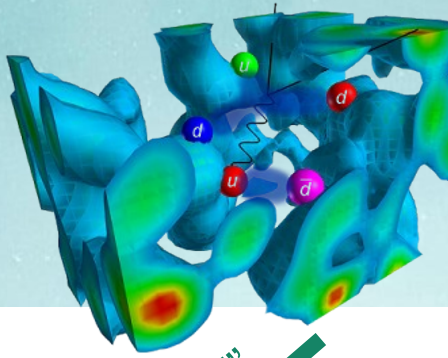


The new g-2 puzzle (L.Darmé, G.G di Cortona, E.Nardi, 2022)

DISPERSIVE PREDICTION



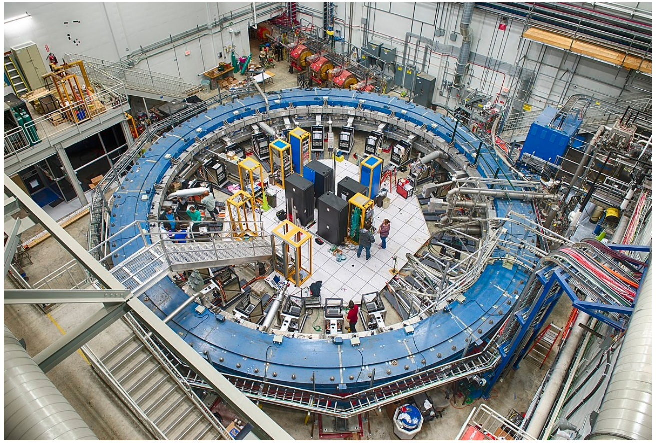
LATTICE PREDICTION



5.1σ DISAGREEMENT

1.7σ “AGREEMENT”

g-2 EXPERIMENT



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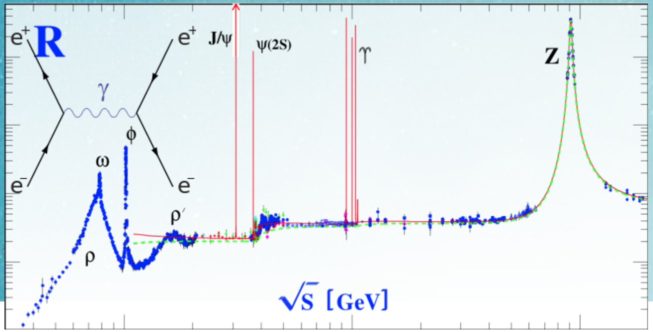

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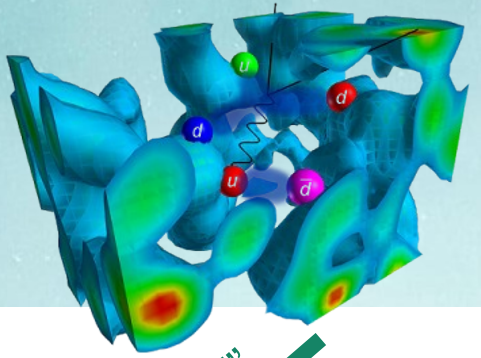

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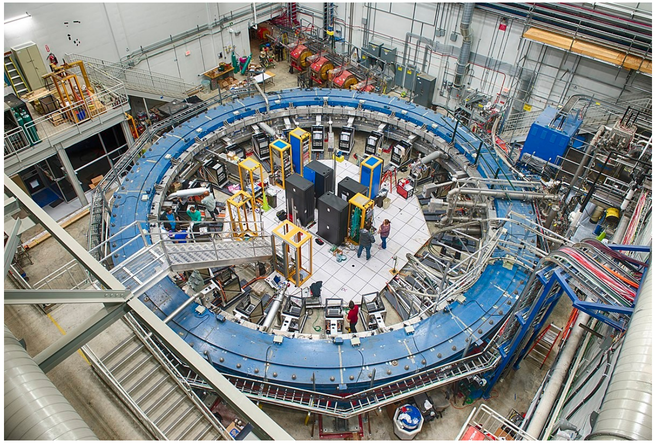

PROBLEMS IN R-RATIO?

- NEW PHYSICS?
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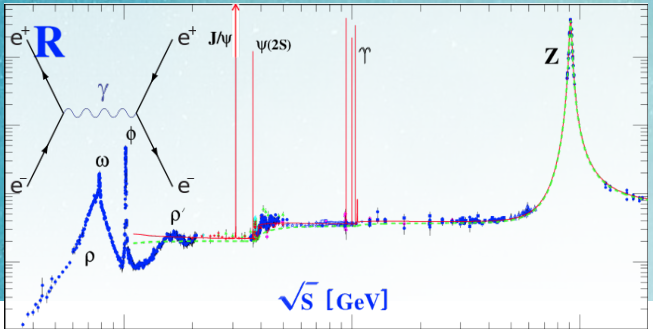

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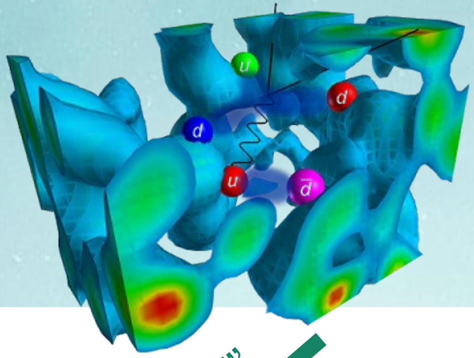

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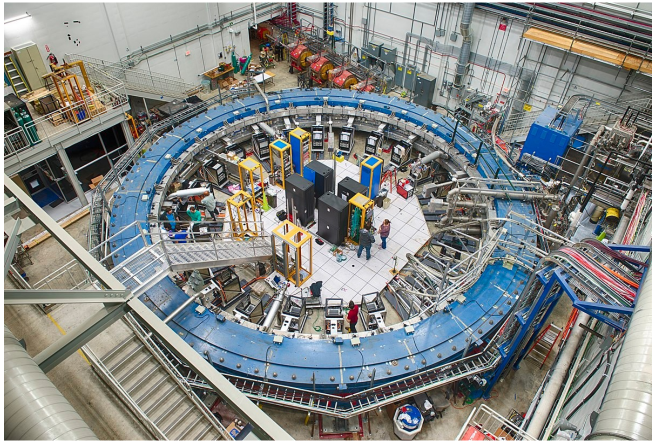

PROBLEMS IN R-RATIO?

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PROBLEMS IN LATTICE?

- BMW IS ALONE
- OTHERS ARE ARRIVING

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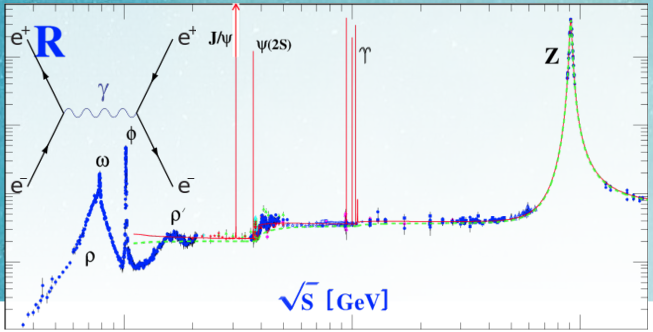

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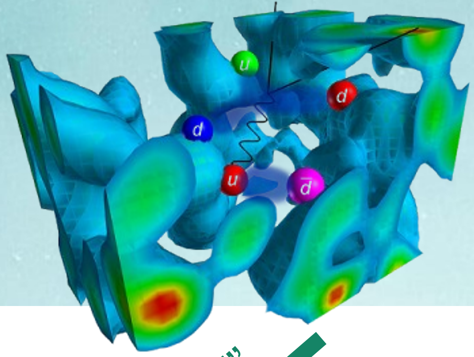

DIRECT COMPARISON?

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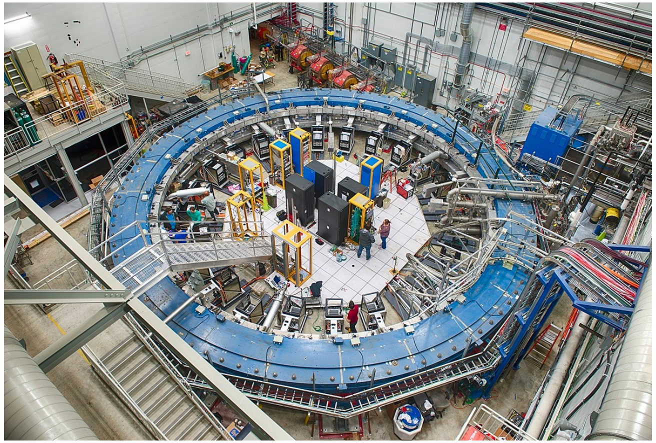

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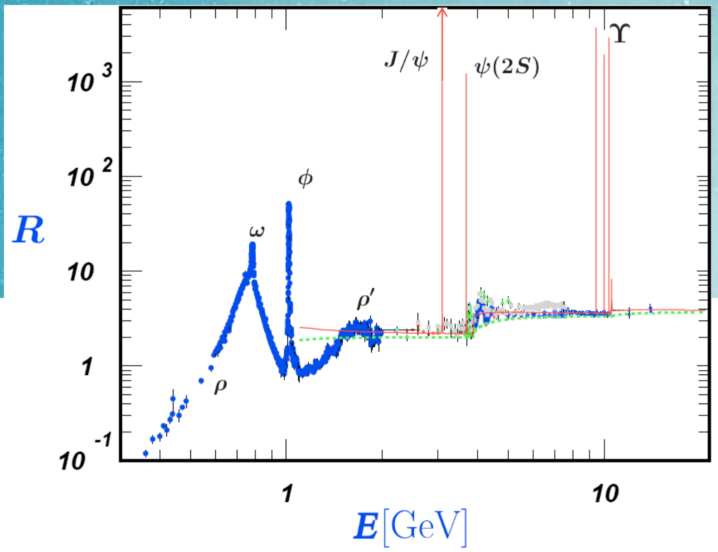
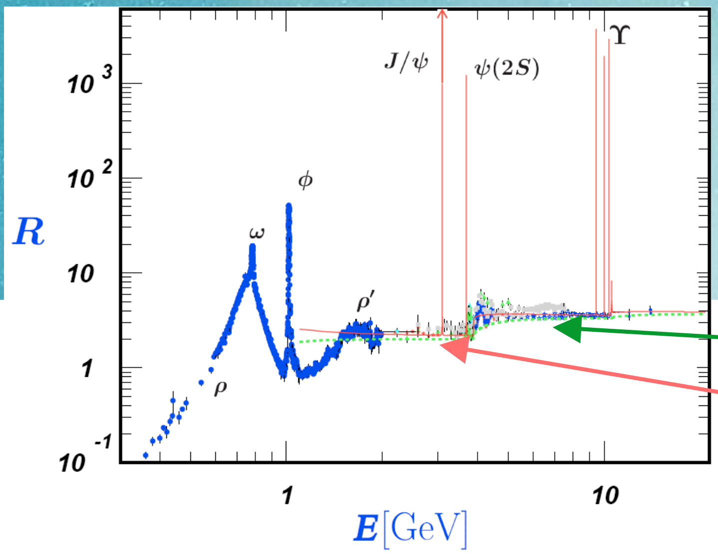
Theoretical understanding of *R(E)*



$$R(E) = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

A number of important features are qualitatively understood

Theoretical understanding of *R(E)*



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R(E)~3 at large Energies: color gauge theory

J/Ψ narrow peak: charm quark (GIM)

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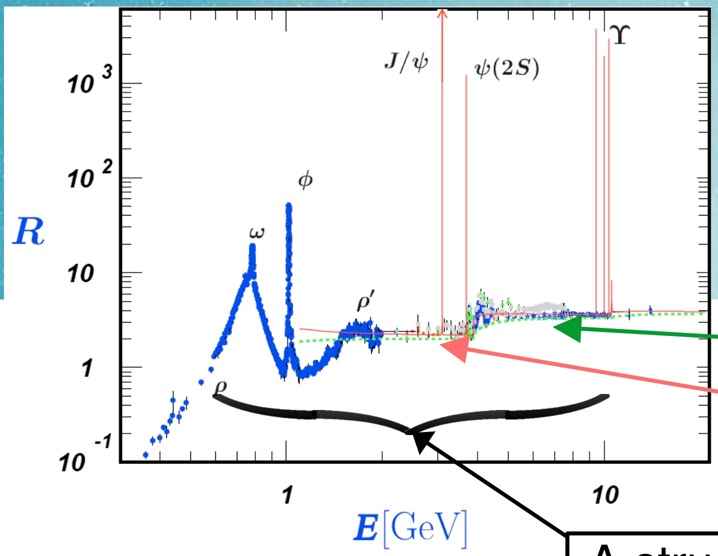

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A structure surprisingly reach of details

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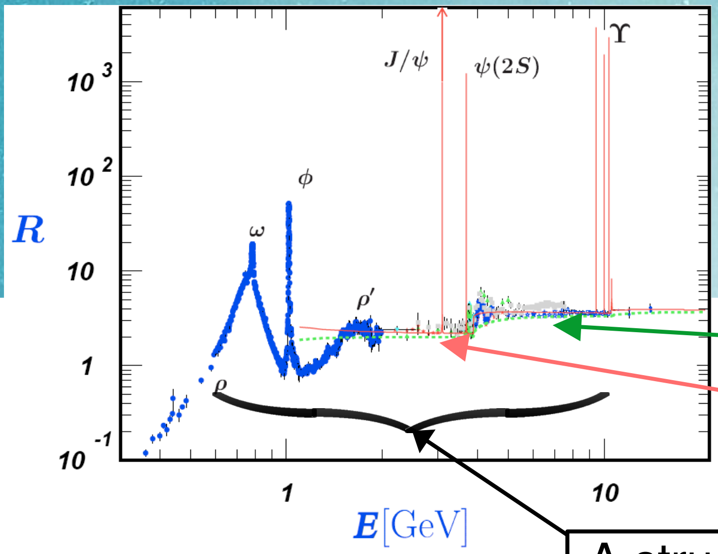

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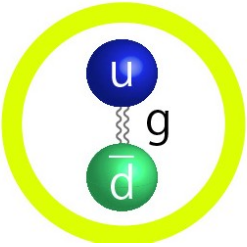
R(E)~3 at large Energies: color gauge theory

J/Ψ narrow peak: charm quark (GIM)

A structure surprisingly reach of details

Hadrons are confined states of quarks interacting <u>nonperturbatively</u>

R(E) Semi-quantitatively described only by <u>models/effective theories</u>



Direct determination of *R(E)* from *V(t)*

$$\underbrace{\frac{\alpha_{em}^2}{3\pi^2}\int_0^\infty \frac{dE^2}{E^2}\tilde{K}(E)\,\boldsymbol{R}(\boldsymbol{E})}_{disperisve,\,experimental} = a_\mu^{HVP} = \underbrace{2\alpha_{em}^2\int_0^\infty dt\; t^2 K(m_\mu t)\,\mathbf{V}(\mathbf{t})}_{lattice,\,SM}$$

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R(E) is the <u>inverse Laplace transform</u> of *V(t)*

- To be computed in presence of noise & finite sampling
- Notoriously, an ill-posed problem (needs regularization)

Direct determination of *R(E)* from *V(t)*

$$\underbrace{\frac{\alpha_{em}^2}{3\pi^2}\int_0^\infty \frac{dE^2}{E^2}\tilde{K}(E)\,\boldsymbol{R}(\boldsymbol{E})}_{disperisve,\,experimental} = a_\mu^{HVP} = \underbrace{2\alpha_{em}^2\int_0^\infty dt\; t^2 K(m_\mu t)\,\boldsymbol{V}(\boldsymbol{t})}_{lattice,\,SM}$$

R(E) is the <u>inverse Laplace transform</u> of *V(t)*

- To be computed in presence of noise & finite sampling
- Notoriously, an ill-posed problem (needs regularization)

...and we are working hard on this!!!

"*Probing the Energy-Smeared R-Ratio Using Lattice QCD*" [PRL 130 (2023)]

see A.De Santis firetalk, today @10:45 am

Complementary observables: windows

$$\underbrace{\frac{\alpha_{em}^2}{3\pi^2}\int_0^\infty \frac{dE^2}{E^2}\tilde{K}(E)\,\boldsymbol{R(E)}}_{disperisve,\,experimental}\,\underline{\tilde{\Theta}(E)} = a_\mu^\Theta = \underbrace{2\alpha_{em}^2\int_0^\infty dt\; t^2 K(m_\mu t)\,\boldsymbol{V(t)}}_{lattice,\,SM}\,\underline{\Theta(t)}$$

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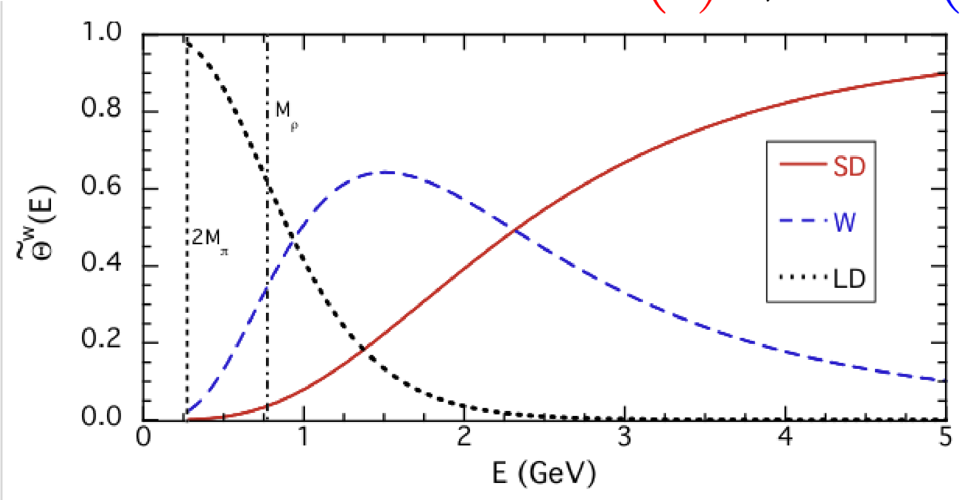
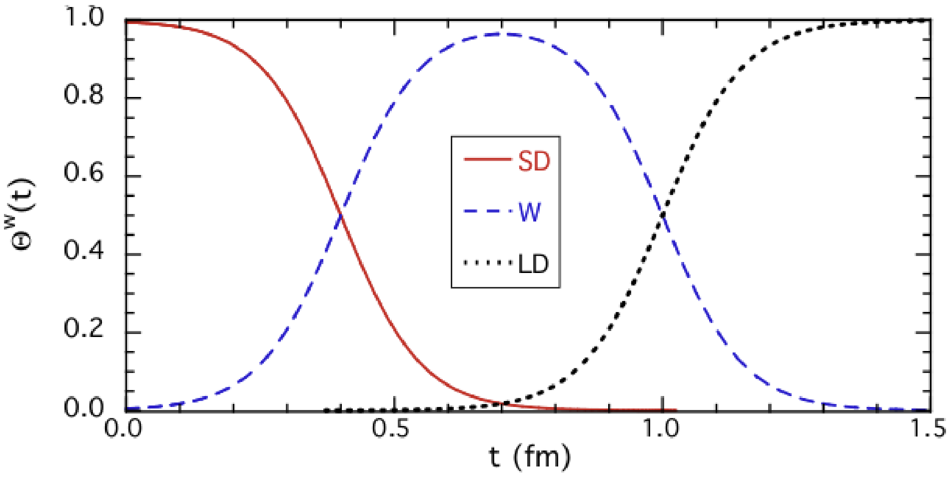
Modified version of HVP, more localized in energy

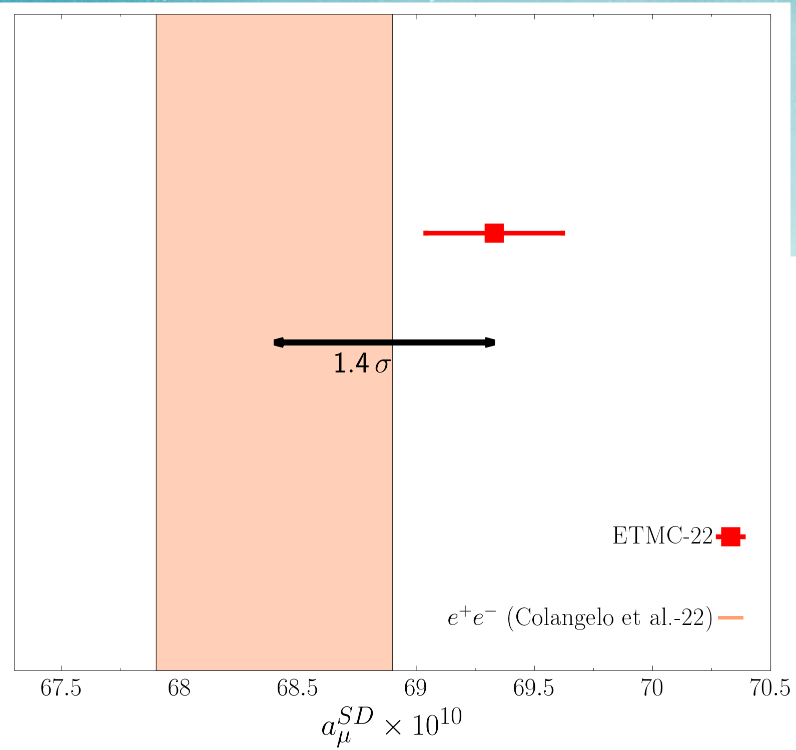
$$\Theta^{SD}(t) + \Theta^{W}(t) + \Theta^{LD}(t) = 1$$





Short & Intermediate windows

Short Distance = Large Energies
(mostly perturbative)



Lattice compatible with Dispersive

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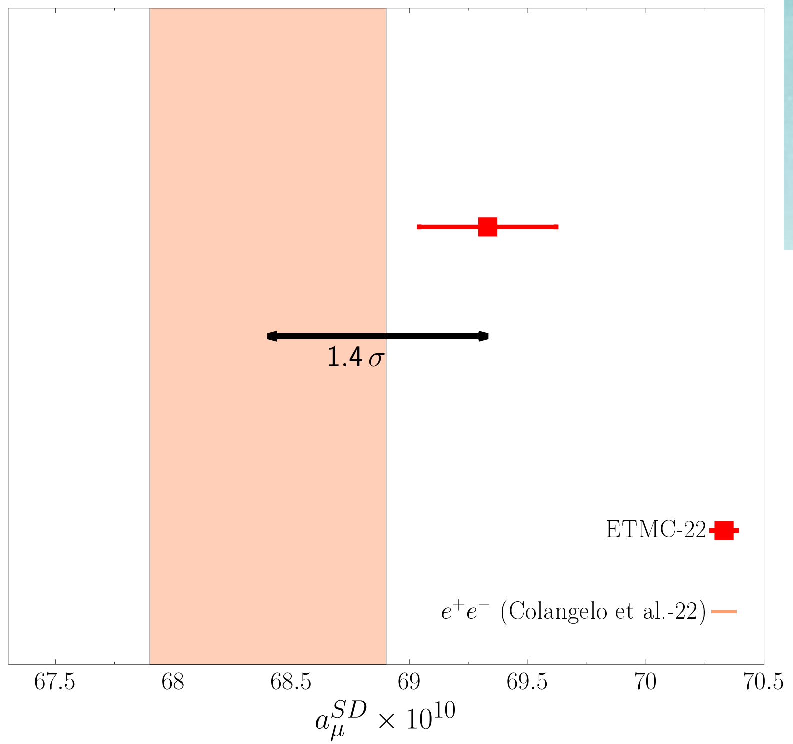
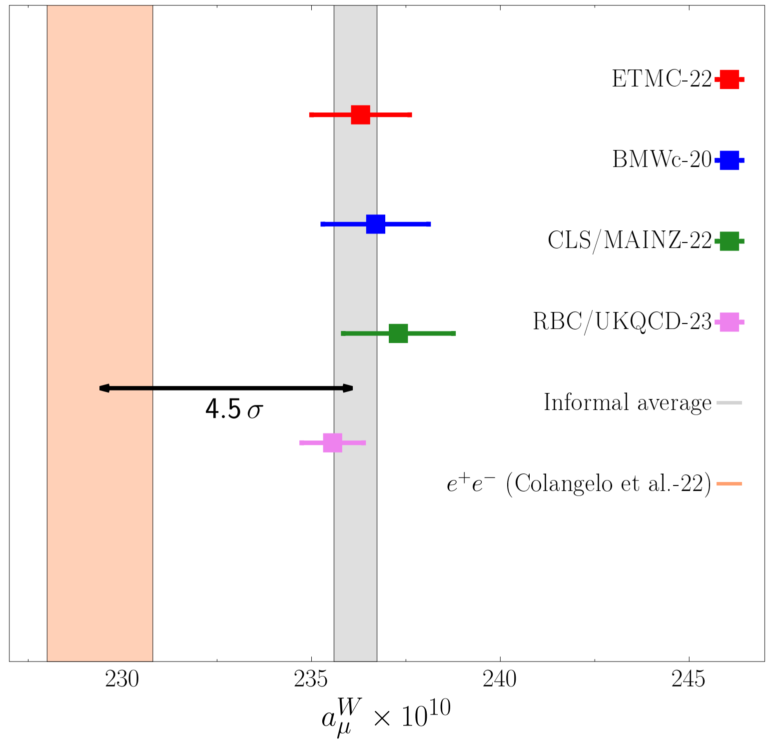


Lattice compatible with Dispersive

Intermediate Distances ~ 1-2 GeV
(two pions final state)



Lattice **incompatible** with Dispersive!

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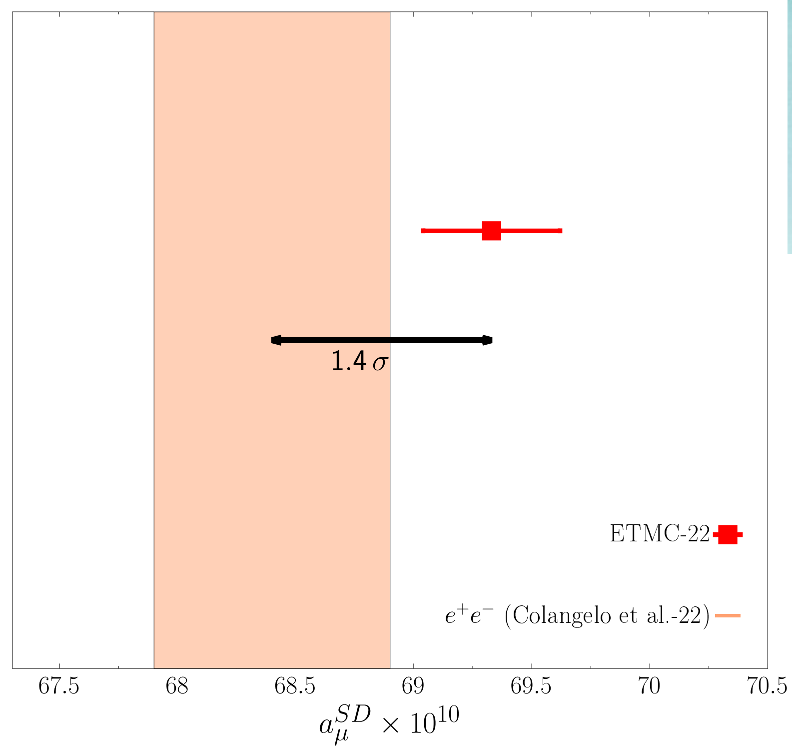

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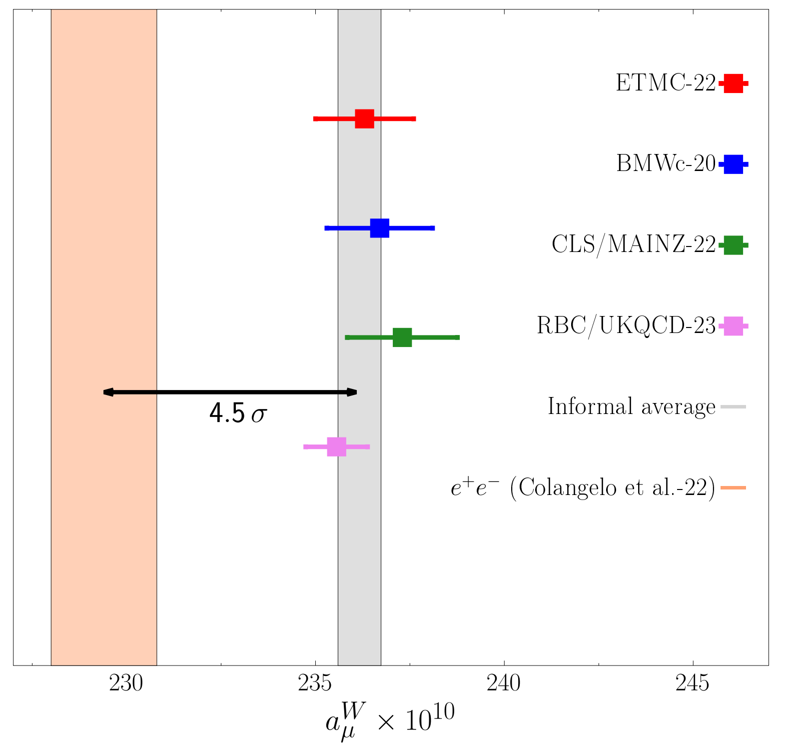


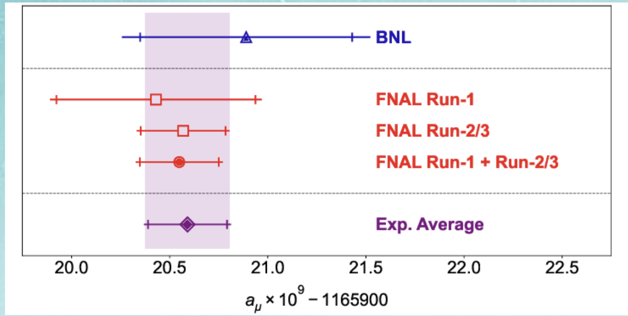
Lattice **incompatible** with Dispersive!

Intermediate window accounts for most of the difference in HVP

Bottom line

Experimental status

- Muon anomalous magnetic is measured since 40 years
- Striking agreement within recent measurements
- Precision will improve in 2024 (Run IV at Fermilab g-2)



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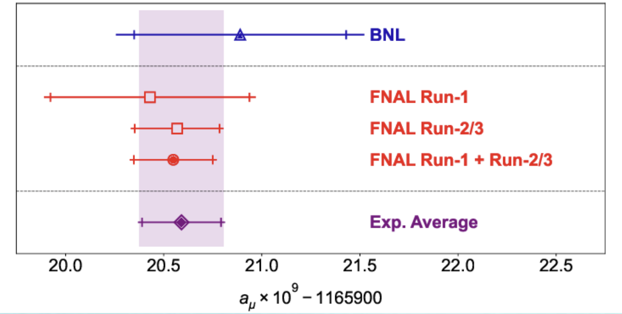
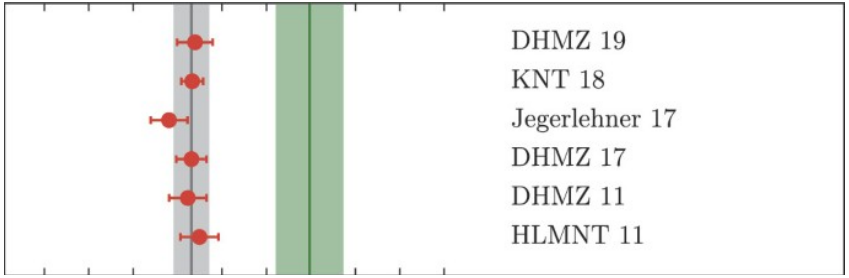
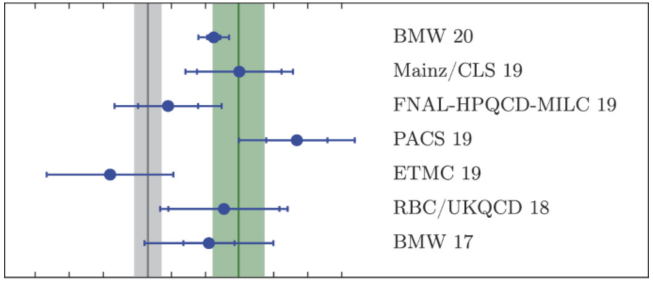


Theoretical status

- All contributions but HVP are well understood
- Old g-2 puzzle: disagreement with experiments using R-ratio to “compute” HVP
- New g-2 puzzle: ~agreement when using HVP from recent lattice calculation





Possible solutions to the puzzles

Assuming NO NEW PHYSICS

(the so called "*everybody go home*" scenario)

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Who is wrong?	Solves Puzzle 1?	Solves Puzzle 2?
Mistake in lattice prediction of HVP	NO	YES
Mistake in g-2 experiments	YES	NO
Mistake in inclusive e^+e^- cross section	YES	YES

Possible solutions to the puzzles

Allowing for NEW PHYSICS and no experimental/lattice mistake

Possible solutions to the puzzles

Allowing for NEW PHYSICS and no experimental/lattice mistake

→ Difficult to explain at the same time both puzzles

New physics behind the new muon g-2 puzzle?

Luca Di Luzio,[1,2] Antonio Masiero,[1,2] Paride Paradisi,[1,2] and Massimo Passera[2]
[1] *Dipartimento di Fisica e Astronomia 'G. Galilei', Università di Padova, Italy*
[2] *Istituto Nazionale di Fisica Nucleare, Sezione di Padova, Padova, Italy*

The recent measurement of the muon g-2 at Fermilab confirms the previous Brookhaven result. The leading hadronic vacuum polarization (HVP) contribution to the muon g-2 represents a crucial ingredient to establish if the Standard Model prediction differs from the experimental value. A recent lattice QCD result by the BMW collaboration shows a tension with the low-energy $e^+e^- \to$ hadrons data which are currently used to determine the HVP contribution. We refer to this tension as the new muon g-2 puzzle. In this Letter we consider the possibility that new physics contributes to the $e^+e^- \to$ hadrons cross-section. This scenario could, in principle, solve the new muon g-2 puzzle. However, we show that this solution is excluded by a number of experimental constraints.

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...but more complicated scenarios are not ruled out

In the future

Experimental side

- More precise data from g-2 experiment
- Reanalysis of old e^+e^- experiment KLEO in progress
- Additional measurements from ongoing e^+e^- experiments
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STAY TUNED!

Thank you!