New results for the Schwinger model from the lattice

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Based on [Dempsey, Klebanov, Pufu, BZ '22] and [Dempsey, Klebanov, Pufu, Søgaard, BZ '23]

The Schwinger model

Schwinger model: $1+1$ dimensional QED.

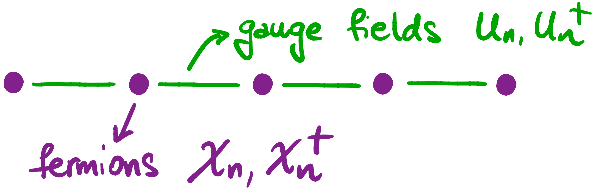
$$\mathcal{L} = -\frac{1}{4g^2} F_{\mu\nu} F^{\mu\nu} - \frac{\theta}{4\pi} \epsilon^{\mu\nu} F_{\mu\nu} + \bar{\Psi}(i\partial\!\!\!/ - A\!\!\!/ - m)\Psi \,.$$

Massless fermion $m = 0$: theory is solvable! It has a mass gap
[Schwinger '62]

Exhibits: **screening** ($m = 0$), **confinement** ($m \neq 0$), chiral symmetry breaking ($\langle \bar{\psi}\psi \rangle \neq 0$), ...

The lattice Schwinger model

Not solvable for $m \neq 0$! Study it by putting it on the lattice in the Hamiltionian formulation [Banks, Kogut, Susskind '76]



Lattice theories in the Hamiltonian formulation are promising (no sign problem, real time dynamics, . . .), but are less developed than those in Lagrangian formulation!

The lattice Schwinger model

In the last decade, a lot of interest in the Schwinger model from several communities

- Effort to simulate gauge theories in the Hamiltonian formulation using **tensor network** methods. [many people, e.g. the Munich group: Cirac, Bañuls, . . .]

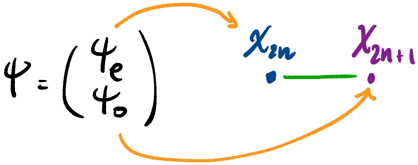
- Quantum simulation of gauge theories: the Schwinger model was **experimentally realized** using trapped ions in the case of 4 sites [Martinez et al. '16] and 6 site [Nguyen et al, '21]

The lattice Schwinger model

The Hamiltonian for the Schwinger model is ($A_0 = 0$ gauge)

$$H = \int dx \left[\frac{g^2}{2} \left(E(x) + \frac{\theta}{2\pi} \right)^2 - i\psi^\dagger \gamma^5 (i\partial_1 - A_1)\psi + m\psi^\dagger \gamma^0 \psi \right]$$

Discretize this by using **staggered fermions** [Kogut, Susskind '75].
Avoids fermion doubling problem!



The lattice Schwinger Hamiltonian

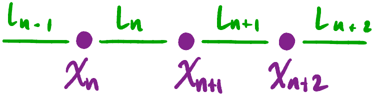
From the continuum

$$H = \int dx \left[\frac{g^2}{2} \underline{\left(E(x) + \frac{\theta}{2\pi} \right)^2} - \underline{i\psi^\dagger \gamma^5 (i\partial_1 - A_1)\psi} + \underline{m\psi^\dagger \gamma^0 \psi} \right]$$

to the lattice

$$H = \frac{g^2 a}{2} \sum_n \underline{\left(L_n + \frac{\theta}{2\pi} \right)^2} - \underline{\frac{i}{2a} \sum_n \left(\chi_n^\dagger U_n \chi_{n+1} - \mathrm{h.c.} \right)} + \underline{m_{\mathrm{lat}} \sum_n (-1)^n \chi_n^\dagger \chi_n}$$

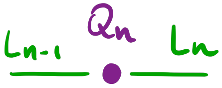
with the commutation relations $\{\chi_n^\dagger, \chi_m\} = \delta_{nm}$ and $[L_n, U_m] = \delta_{nm} U_m$.



Gauge invariance

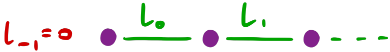
We only care about gauge invariant states: **Gauss law**

$$L_n - L_{n-1} = Q_n \equiv \chi_n^\dagger \chi_n - \frac{1-(-1)^n}{2}$$

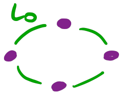


Arbitrary choice, but adopted by almost all of the literature (exception: [Berruto, Grignani, Semenoff, Sodano '98]).

Gauss law allows us to integrate out the electric field



Open boundary conditions



Closed boundary conditions

The Schwinger anomaly

Continuum mass m and lattice mass m_{lat} are not the same.

Continuum massless theory $m = 0$: chiral symmetry $\psi \to \exp(i\alpha\gamma_5)\psi$ is **anomalous**!

Take V_α, the operator that implements a chiral transformation, $V_\alpha \psi V_\alpha^{-1} = e^{i\alpha\gamma_5}\psi$. On the Hamiltonian it acts

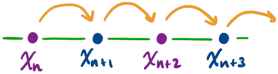
$$V_\alpha H_\theta V_\alpha^{-1} = H_{\theta - 2\alpha}$$

Chiral transformation changes the theta angle.

What happens on the lattice?

The Schwinger anomaly on the lattice

Discrete $\pi/2$ chiral transformation on the lattice: **one site lattice translation**!



Given the operator that implements this $V\chi_n V^{-1} = \chi_{n+1}$ we find that

$$VH_\theta V^{-1} = H_{\theta+\pi} \qquad \Leftrightarrow \qquad m_{\mathrm{lat}} = -\frac{g^2 a}{8}$$

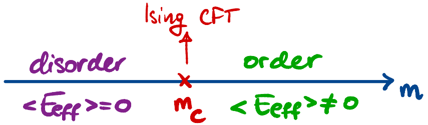
Therefore lattice and continuum mass related by [Dempsey, Klebanov, Pufu, BZ '22]

$$\boxed{m_{\mathrm{lat}} = m - \frac{g^2 a}{8}}$$

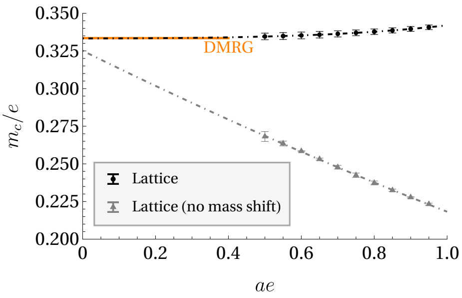
Subleading in the continuum limit $a \to 0$, but greatly improves convergence.

Improved convergence: $N_f = 1$

$\theta = \pi$: $\mathbb{Z}_2$ symmetry, order parameter is $\langle E_{\text{eff}} \rangle \equiv \frac{1}{N} \sum_n \langle L_n + \frac{\theta}{2\pi} \rangle$.



Exact diagonalization computation of m_c.



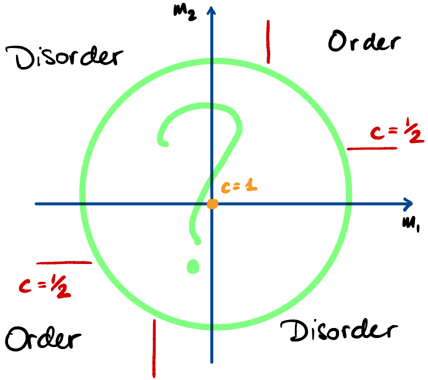
Tensor networks (DMRG) (more precise, $O(500)$ sites)

- No mass shift: 0.3335 ± 0.0002 [Byrnes et al. 2002]
- With mass shift: 0.33352 ± 0.00003

Not just a quantitative improvement: $N_f = 2$

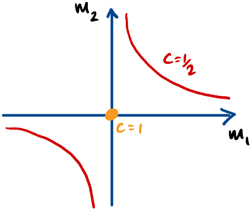
Two flavor Schwinger model at $\theta = \pi$: fermion masses m_1 and m_2.

- For $m_1 = m_2 = 0$: gapless, $SU(2)_1$ WZW model ($c = 1$ CFT)
- One large mass $\rightarrow$ single flavor Schwinger model

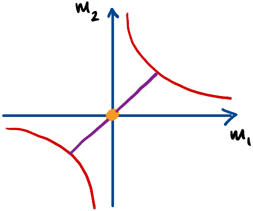


$N_f = 2$

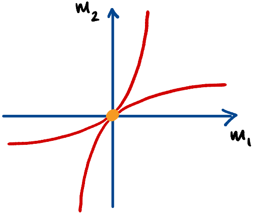
Possible scenarios:



Scenario 1 [Coleman '76]

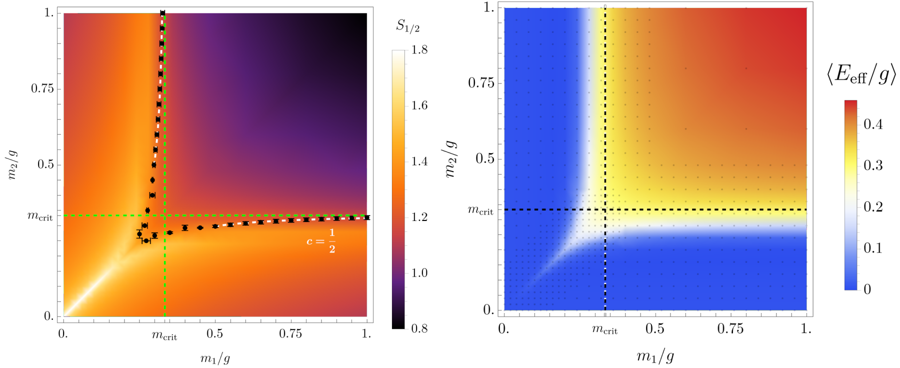


Scenario 2 [Georgi '22]



Scenario 3

Numerical results

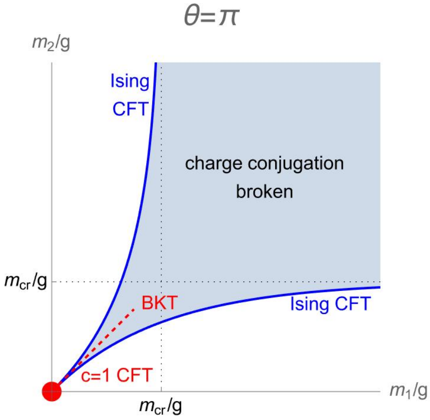


Scenario 3 is the correct one!

The physics

- Lines of Ising CFTs all the way to the origin
- Around the origin, nonperturbative small wedge where $\mathbb{Z}_2$ is broken
- For small $m_1 = m_2 = m$, marginally relevant perturbation.

Similar to CP breaking in QCD_4 at $\theta = \pi$.



Summary and future directions

Today:

1. Careful analysis of continuum theory
2. Improvement of lattice discretization of the model
3. New physics for two dimensional gauge theories.

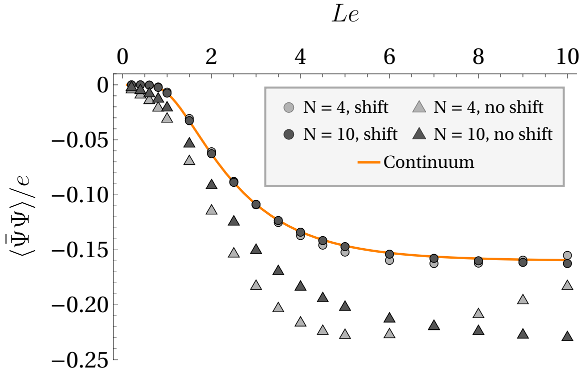
Path forward:

- **Non-abelian gauge theories in** $2d$: e.g. adjoint QCD_2 on the lattice. recent renewed interest.
- **Abelian theories in** $3d$: go to one dimension higher, consider Abelian theories such as QED_3, . A similar setup has been studied [Felser, Silvi, Collura, Montangero '20], but mass shift will probably play a role here.

Backup slides

Improved convergence: $N_f = 1$

'Exact' diagonalization with periodic boundary conditions Keep $L = Na$ fixed, send $N \to \infty$.



Improved convergence: $N_f = 2$