Bootstrapping Effective Field Theories of Photons

Francesco Russo
francesco.russo@phd.unipi.it

Università di Pisa

27/09/2023

Based on 2107.13009 and 2203.08164
with J. Henriksson, B. McPeak and A. Vichi

Introduction

Effective field theory (EFT)

What is it?

When different energy scales of a system are well-separated, we can integrate out high-energy excitations and focus only on the low-energy ones, which will be described by a new set of interactions

A bit more rigorous:

A quantum field theory (QFT), valid only up to a certain energy scale, in which the low-energy content of the theory is specified in a controlled series of higher dimensional operators. The coefficients of this expansion are called Wilson (or EFT) coefficients.

$$\mathcal{L} = -\frac{1}{2}(\partial_\mu\phi)^2 - \frac{g}{3!}\phi^3 - \frac{\lambda}{4!}\phi^4 + g_2((\partial_\mu\phi)^2)^2 + \\ + g_3(\partial_\mu\partial_\nu\phi)^2(\partial_\sigma\phi)^2 + \dots$$

Bootstrap

The idea:

Systematic application of general principles to constrain observable quantities. The usual assumptions are causality, unitarity and crossing symmetry.

In practice:

In Conformal field theories, we can use unitarity and crossing symmetry to constrain n-point function. Using this method we are able to compute the dimension of some operators and some OPE coefficients.

In a general QFT we "bootstrap" scattering amplitudes.

$$\mathcal{A}_{low} = -g^2\left(\frac{1}{s} + \frac{1}{t} + \frac{1}{u}\right) - \lambda + g_2\left(s^2 + t^2 + u^2\right) + g_3(stu) + \ldots$$

Bootstrapping scattering amplitudes

What is it?

We use S-matrix axioms (plus some assumptions) to constrain $2 \to 2$ scattering amplitudes.

Axioms and Assumptions:

- Unitarity
- Causality
- Crossing symmetry
- Partial wave decomposition in the ultraviolet (UV)
- Infrared (IR) of the theory parametrised by a tree level EFT

Positivity bounds

The idea:

Using only a small set of constraints – where unitarity is explicit – to get one sided bounds on Wilson coefficients

Results:

People have used this method to get information on QCD, theories with gravity, SMEFT... [Pham, Truong '85][Ananthanarayan, et al. '95][Adams, et al. '06]

An example:

For scattering of identical scalars $g_2 \geq 0$.

Using bootstrap methods

What is it?

Follow the same strategy and same goals as the Positivity bounds, but the method uses of potentially all constraints coming from the assumptions.

Results:

We are able to get double-sided bounds for most of the coefficients.

Example:

Bounds on higher order coefficients, $-10.3 < \frac{g_3 M^2}{g_2} < 3$

Bootstrapping photons

Idea:

Bootstrap of $2 \to 2$ scattering of photons. This was the first generalisation to scattering of massless spinning particles of the method.

Aim:

The goal is to investigate the allowed space for EFTs of photons, e.g. QED or the Standard Model

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \alpha_1 (F_{\mu\nu} F^{\mu\nu})^2 + \alpha_2 (F_{\mu\nu} \tilde{F}^{\mu\nu})^2 + \dots .$$

… with gravity

What is it?

Bootstrap of $2 \to 2$ scattering photons with exchange of gravitons.

Challenges:

Graviton exchange create extra terms which are difficult to deal with. In this framework we will get weaker bounds.

$$\mathcal{L} = \sqrt{-g}\Big(\frac{M_{\mathrm{P}}^2}{2}R - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \alpha_1(F_{\mu\nu}F^{\mu\nu})^2 + \\ + \alpha_2(F_{\mu\nu}\tilde{F}^{\mu\nu})^2 + \beta W_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma} + \dots\Big).$$

Method

Assumptions

Let us focus on $2 \to 2$ scattering of identical scalars to explain the method.

Infrared:

The IR is parameterised by a tree-level EFT. The amplitude is

$$\mathcal{A}_{low} = -g^2(\frac{1}{s} + \frac{1}{t} + \frac{1}{u}) - \lambda + g_2(s^2 + t^2 + u^2) + g_3(stu) + \ldots$$

Ultraviolet:

In the UV we use the partial wave decomposition

$$\mathcal{A}_{high} = \sum_J f_J(s)\mathcal{P}_J(1 + \frac{2t}{s}),$$

where $\mathcal{P}_J(x)$ are the partial waves, known for scalar scattering.

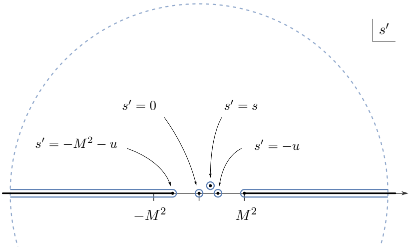
Relation between IR and UV

Dispersion relations:

Solving

$$\int_\gamma \frac{ds'}{2\pi i} \frac{\mathcal{A}(s',u)}{s'((s'-s)(s'+u))^{k/2}} \,,$$

where γ is the following path



Sum rules:

We will get an infinite number of sum rules relating the IR with the UV,

$$
\begin{aligned}
&2g_2 - g_3 t + 8 g_4 t^2 + \ldots = \langle \ldots \mathcal{P}_J \rangle\,, \\
&4g_4 + \ldots = \langle \ldots \mathcal{P}'_J \rangle\,, \\
&\ldots\,.
\end{aligned}
$$

Notice that $\langle . \rangle$ is a positive functional. Using the forward limit $t \to 0$ we are able to isolate EFT coefficients,

$$
\begin{aligned}
g_2 &= \langle \frac{1}{s^2} \rangle\,, \\
g_3 &= \langle \frac{3 - 2\mathcal{J}}{s^3} \rangle\,, \\
&\ldots\,.
\end{aligned}
$$

How to get bounds

Optimisation method:

The strategy to get bounds from sum rules is the following,

$$\begin{cases} \text{Max/Min} & A \\ \text{s.t.} & 0 \leqslant (\pm A, 1) \cdot (g_2(s, J), g_3(s, J)), \end{cases}$$

valid for each value of J and $s > M^2$.

Example of result:

Using for instance `Mathematica` we can solve it and obtain the result $-10.3 \leq \frac{g_3 M^2}{g_2} \leq 3$. In the same way we can get bounds for other coefficients.

Generalisations

Spinning particles:

We have to deal with multiple amplitudes, and positivity translates into semidefinite positivity.

With gravity:

We cannot use the forward limit, due to the graviton pole,

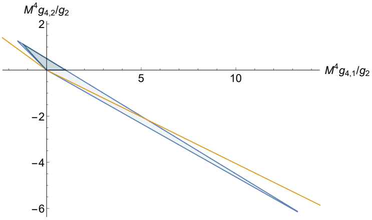
$$\mathcal{A}_{low} = 8\pi G\left(\frac{st}{u} + \frac{su}{t} + \frac{tu}{s}\right) + \ldots . \tag{1}$$

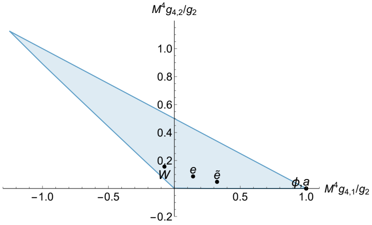
We generalise from a derivative functional to an integral one.

Conclusions

Results without gravity

An example of bounds of EFT coefficients.
In the left figure we show various upgrades of the method with different colours. In the other figure we zoom in the allowed region, adding some partial UV completions.

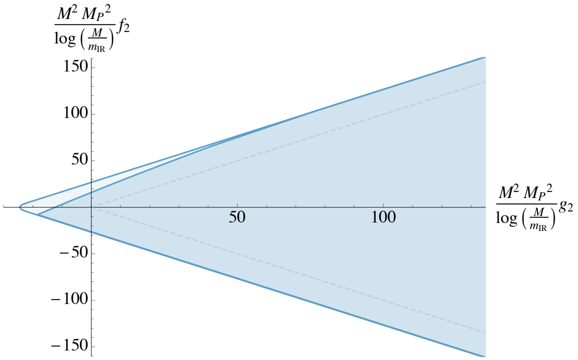




Results with gravity

An example of bound with gravity.

- Addition of extra IR regulator to have well define bounds.
- Bounds not strong enough to prove the Weak Gravity Conjecture.



Sum up

Our aim is to try to understand how much we can constrain the IR physics from very general UV assumptions.

Results:

- generalise the bootstrap to scattering of spinning particles
- ... with gravity
- starting the exploration of allowed space for tree-level EFTs of photons

Future plans

Explore theories of gravity in higher dimensions using spinning partial waves [Burić, F.R., Vichi '23]

More information in the bootstrap $\rightarrow$ stronger bounds.

- Adding scattering of other particles
- Multiparticle scattering

Thank you!