Callan-Rubakov effect in a chiral gauge theory

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Based on a work in progress with: Stefano Bolognesi and Bruno Bucciotti

Fermion-monopole scattering

Take a $U(1)$ theory, a Dirac monopole (very heavy $M_{monopole} \to \infty$)

$$\vec{B} = \frac{m\hat{r}}{2r^2}\ , \quad m \in \mathbb{Z}$$

and scatter a charged ($q \in \mathbb{Z}$) left-handed fermion on it.

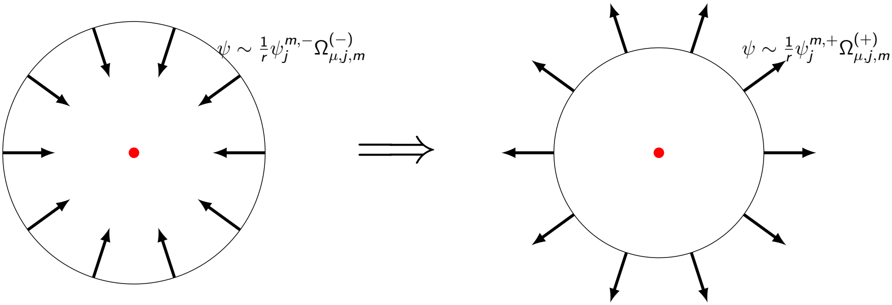
The $SO(3)_{rot}$ symmetry is unbroken, but the angular momentum is deformed

$$\boxed{j = \mu - \frac{1}{2}\ ,\ \ \mu + \frac{1}{2}\ ,\ \ \mu + \frac{3}{2}\ , \ldots \quad \text{where} \ \ \mu = \frac{1}{2}|qm|}$$

$$j = 0, 1, 2, \ldots \quad \text{for} \ \ q = m = 1$$

$$j = \frac{1}{2}, \frac{3}{2}, \ldots \quad \text{for} \ \ q = 2 \ \ m = 1$$

The scattering of the $j = \mu + \frac{1}{2}\,,\ \ \mu + \frac{3}{2}\,, \ldots$ modes is simple. At 0th order



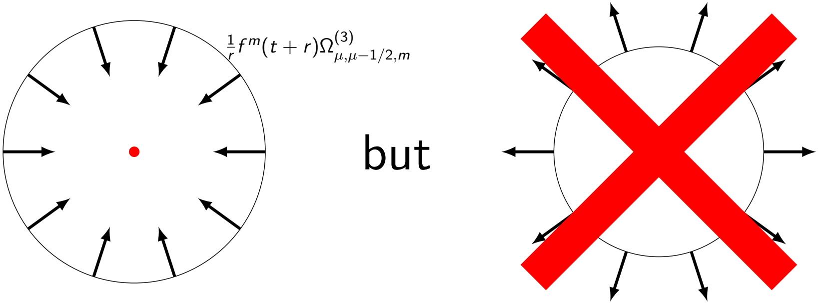
...they bounce on a centrifugal barrier and never reach the core!

Two modes for each j, m, described by

$$\left(i\gamma^{\mu}_{2d}\partial_{\mu} - \frac{\tilde{\ell}}{r}\right)\psi^{2d}_{\mu,j,m} = 0 \qquad \tilde{\ell} \geq 1/2$$

automatically a well-defined Hamiltonian (Calogero problem).

The scattering of the $j = \mu - \frac{1}{2}$ is a bit of a puzzle!



The mode is purely in-coming (out-going) depending on the chirality (charges):

$$\psi = \frac{1}{r} f^m(t \pm r)\Omega^{(3)}_{\mu,\mu-1/2,m}$$

But a low-energy fermion cannot be simply absorbed:

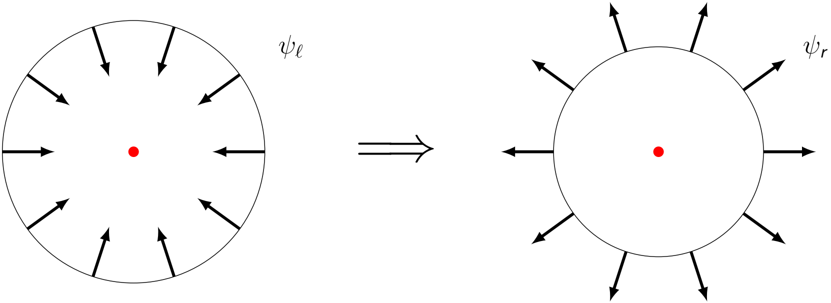
$$M_{dyon} - M_{monopole} \gg E$$

So, what happens?

In QED we know what happens thanks to Callan and Rubakov:

- Around the monopole there is a $< \psi_\ell \psi_r > \propto r^{-3}$ condensate.
- This allows the non-conservation of $U(1)_A$:

$$\psi_\ell + M \to \psi_r + M$$



(For the multi-flavor ($N_f \geq 4$) case the out state has fractional fermion numbers... the interpretation is still debated...)

But what happens in a chiral gauge theory?

Two questions:

- Are some symmetries broken by the monopole?
- What is the outcome of a scattering event?

There are two main approaches:

I) Stick with the IR theory, look at symmetries and self-consistency

☺ It can be applied always.

☹ It does not tell you what actually happens! E.g. some symmetries might be broken dynamically.

Marike van Beest et al [2306.07318 [hep-th]], Philip Boyle Smith et al [2106.06402 [hep-th]]...

II) Embed the theory in a UV-completion and perform a concrete calculation à lá Rubakov.

☺ It tells you what happens.

☹ You need a UV completion.

We will follow the second route!

The UV model

Our UV theory is the $\psi\chi\eta$ model

$$SU(N)\ YM \ + \ \psi^{(i,j)} \in \square\!\square \ + \ \chi_{[i,j]} \in \overline{\begin{smallmatrix}\square\\\square\end{smallmatrix}} \ + \ 8 \cdot \eta_i \in \overline{\square}$$

plus a (real) adjoint scalar Φ and a suitable Yukawa coupling and potential

$$\mathcal{L} \subset \Phi^i_j \psi^{(j,k)} \chi_{[ki]} + h.c. \,, \qquad V(\Phi) \,.$$

The $\psi\chi\eta$ symmetry is

$$U(1)_{\psi\eta} \times \tilde{U}(1) \times SU(8)_\eta \to \tilde{U}(1) \times SU(8)_\eta$$

~~$$U(1)_{\psi\eta} : \begin{cases} \psi \to e^{i(N-2)\alpha}\psi \\ \chi \to e^{-i(N+2)\alpha}\chi \\ \eta \to \eta \end{cases}$$~~

$$\tilde{U}(1) : \begin{cases} \psi \to e^{2i\alpha}\psi \\ \chi \to e^{-2i\alpha}\chi \\ \eta \to e^{-i\alpha}\eta \end{cases}$$

As

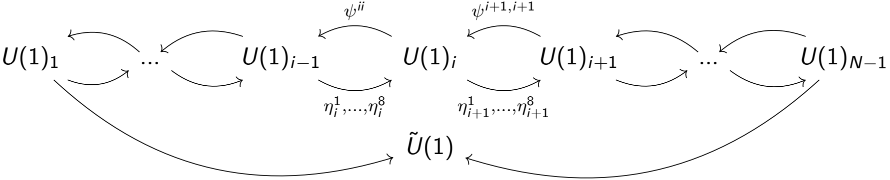
$$< \Phi >= \begin{pmatrix} a_1 & & & \\ & a_2 & & \\ & & \ddots & \\ & & & a_N \end{pmatrix} \ , \quad a_i \neq a_j \gg \Lambda_{\psi\chi\eta}$$

$SU(N) \to U(1)^{N-1}$ and ψ^{ij} gap with χ_{ij} $(i \neq j)$.

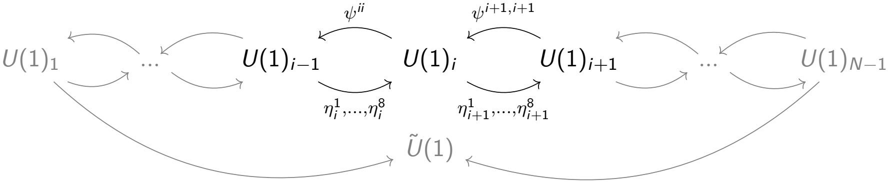
At energies $E << a_1, ..., a_N$ remains:

$$N-1 \text{ photons} \qquad \eta_i^A \qquad \psi^{ii} \ , \quad A = 1, ..., 8, \ i = 1, ..., N$$

forming a *mildly* chiral gauge theory



Let's take a Dirac monopole for $U(1)_i$:



The relevant degrees of freedom are

	$U(1)_i$	$U(1)'$	$SU(8)$	$SU(2)_{rot}$	IN/OUT
ψ^{ii}	2	2	$(\cdot)$	**2**	IN
$\psi^{i+1,i+1}$	-2	2	$(\cdot)$	**2**	OUT
η_i^A	-1	-1	$\square$	**1**	OUT
η_{i+1}^A	1	-1	$\square$	**1**	IN

Here $U(1)'$ is generated by

$$Q' = diag(\underbrace{0, ..., 0}_{i-2}, -1, 1, 1, -1, \underbrace{0, ..., 0}_{N-i-2}) = Q_{i+1} - Q_{i-1}$$

Something funny happens. If you look at the far IR (free 2d fermions), bosonize

$$\begin{pmatrix} \psi^i_M \\ \psi^{i+1}_M \end{pmatrix}, \begin{pmatrix} \eta^{i+1}_A \\ \eta^i_A \end{pmatrix} \xleftrightarrow{\textit{bosonization}} \Phi^M_\psi, \ \Phi^A_\eta$$

and impose $U(1)_i \times U(1)'$ preserving boundary conditions, you obtain simultaneously:

$$\text{Neumann:} \ \partial_r \left(2 \sum_M \Phi^M_\psi - \sum_A \Phi^A_\eta \right) \Big|_{r=0} = 0$$

$$\text{Dirichlet:} \ \left(2 \sum_M \Phi^M_\psi - \sum_A \Phi^A_\eta \right) \Big|_{r=0} = 0$$

Something is odd!

Perhaps either $U(1)_i$ or $U(1)'$ is broken!

Following the Rubakov insight...

...we regularize the Dirac monopole as a 't Hooft Polyakov one.

$$\Phi_{\text{background}} = h(r) \begin{pmatrix} 1 & & \\ & \hat{r}\cdot\vec{\tau} & \\ & & 1 \end{pmatrix}$$

$$\vec{A}_{backgroud} = \frac{1-K(r)}{er} \begin{pmatrix} 0 & & \\ & \hat{r}\times\vec{\tau} & \\ & & 0 \end{pmatrix}$$

and consider the $U(1)_i$ and $U(1)'$ electric fluctuations around the background

$$\vec{A} = \vec{A}_{\text{background}} + a_1(t,r)\,\hat{r}\,(\hat{r}\cdot\tau) + v_1(r,t)\,\hat{r}\,Q'$$

$$A_0 = a_0(r,t)\;(\hat{r}\cdot\tau) + v_0(r,t)\;Q'$$

Regularity at $r \to 0$ implies

$$\boxed{a_0|_{r=0} = 0 \quad \partial_r a_1|_{r=0} = 0} \quad \text{and} \quad \boxed{\partial_r v_0|_{r=0} = 0 \quad v_1|_{r=0} = 0}$$

We can pack together the $j = \mu - 1/2$ components of

$$\psi^{i,i},\ \psi^{i+1,i+1} \to \psi^m_{2d},\ m = 1, 2$$

$$\eta^A_{i+1},\ \eta^A_i \to \eta^A_{2d},\ A = 1, ..., 8$$

where ψ^m_{2d} and η^A_{2d} are 2d **Dirac** fermions.

The 't Hooft Polyakov regularization fixes (see [Rubakov 82'])

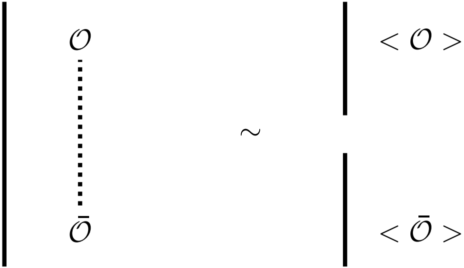
$$\boxed{\psi^m_{2d}|_{r=0} = \eta^A_{2d}|_{r=0} = 0}$$

The fermions couples axially with a_i and vectorially with v_i.

$$S^{eff}_{2d} = \int drdt\ \ \frac{4\pi}{g_a^2} r^2 f_{ij}(a) f^{ij}(a) + \frac{4\pi}{g_v^2} r^2 f_{ij}(v) f^{ij}(v) +$$
$$\bar{\psi}^m \gamma^i \left(\partial_i - 2i\gamma^5 a_i - 2iv_i\right) \psi^m + \bar{\eta}^A \gamma^i \left(\partial_i + i\gamma^5 a_i + iv_i\right) \eta^A$$

We can compute $< \mathcal{O} >$ by using:

Cluster Decomposition



$$\lim_{\Delta t \to \infty} < \bar{\mathcal{O}}(r, \Delta t)\mathcal{O}(r, 0) > = < \bar{\mathcal{O}} >< \mathcal{O} >$$

Surprisingly this correlator is exactly calculable:

- Trade a_i and v_i with ρ^a and ρ^v:

$$a_i = \epsilon_{ij}\partial_j \rho^a + \partial_i \lambda^a \ , \quad (a \leftrightarrow v)$$

- Then decouple fermions and scalars

$$\psi = e^{2\rho^a + 2\rho^v \gamma^5}\psi_{free} \quad \eta = e^{\rho^a + \rho^v \gamma^5}\eta_{free} \quad \mathcal{O} = e^{\tilde{Q}_A \rho^A + \tilde{Q}_V \rho^v}\mathcal{O}_{free}$$

The correlation function of $< \bar{\mathcal{O}}\mathcal{O} >$ scales as

$$\underbrace{(\Delta t)^{\frac{\tilde{Q}_A^2}{16}}}_{\rho^a \ \text{propagators}} \ \overbrace{(\Delta t)^0}^{\rho^v \ \text{propagators}} \ \underbrace{(\Delta t)^{-f}}_{\text{free fermions contribution}} \sim (\Delta t)^{\nu \leq 0}$$

- The effect of ρ^a and ρ^v is different because of different boundary conditions!

$$\underbrace{e^{log(\Delta t^2) + log(\Delta t^2 + 2\Delta R^2)} \sim \Delta t^2}_{\text{Neumann}} \quad \underbrace{e^{log(\Delta t^2) - log(\Delta t^2 + 2\Delta R)} \sim \Delta t^0}_{\text{Dirichlet}}$$

thus we have a master equation

$$\nu = \frac{\tilde{Q}_A^2}{16} - f = 0 \iff < \mathcal{O} >= 0$$

There is a single solution

$$\mathcal{O} = \bar{\psi}_\ell^1 \bar{\psi}_r^1 \bar{\psi}_\ell^2 \bar{\psi}_r^2 \eta_\ell^{A_1} \ldots \eta_\ell^{A_4} \eta_r^{A_5} \ldots \eta_r^{A_8} \epsilon_{A_1 \ldots A_8} \qquad < O >= \frac{\#}{r^6}$$

That leaves unbroken $U(1)_i \times SU(2)_{rot} \times SU(8)$ but breaks

$$U(1)' \to \mathbb{Z}_{16}$$

Uplifting to 4d we have different condensates:

$$< (\psi^i \epsilon \psi^i)(\psi^{i+1} \epsilon \psi^{i+1})(\eta_i^{A_1} \epsilon \eta_{i+1}^{A_2}) \ldots (\eta_i^{A_7} \epsilon \eta_{i+1}^{A_8}) > = \# / r^9 \qquad \text{scalar}$$

$$< (\psi^i \epsilon \psi^i)(\psi^{i+1} \epsilon \psi^{i+1})(\eta_i^{A_1} \bar{\sigma}^{[\mu} \sigma^{\nu]} \eta_{i+1}^{A_2}) \ldots (\eta_i^{A_7} \epsilon \eta_{i+1}^{A_8}) > = \frac{\#}{r^9} \begin{pmatrix} 0 & i\hat{x} \\ -i\hat{x} & \epsilon_{ijk} \hat{x}^k \end{pmatrix} \qquad \text{self-dual tensor}$$

As $U(1)'$ is broken scattering can happen. In the bosonized language:

$$\text{Neumann: } \partial_r \left(2 \sum_M \Phi_\psi^M - \sum_A \Phi_\eta^A \right) \Big|_{r=0} = 0$$

$$\text{Dirichlet: } \left(2 \sum_M \Phi_\psi^M - \sum_A \Phi_\eta^A \right) \Big|_{r=0} = 0$$

The condensate $< \bar{\psi}^4 \eta^8 >$ suggest a scattering like

$$\eta_{i+1} \xrightarrow{<\bar{\psi}^4 \eta^8>} 2\psi_i + 2\psi_{i+1} + 4\bar{\eta}_{i+1} + 3\bar{\eta}_i$$

But in "s-wave" ψ_i and $\bar{\eta}_{i+1}$ have only in-going modes.

There are different possible solutions:

- Some results of the scattering lie outside the "s-wave"
- The asymptotic states of the "s-wave" theory ought to be rethought (the theory is 2d and interacting, problem is subtle)

Work in progress here!

Take home messages:

- The $\psi\chi\eta$ model is a gym to study the Callan-Rubakov effect in a (*mildly*) chiral gauge theory.

- The UV completion is fundamental to pin down the UV boundary conditions, which determine which symmetries are broken.

TODOs:

- Given a certain abelian theory with monopoles, can we construct an ($SU(N)$) UV completion?

- We can compute all the correlators of "s-wave" fields ($j = \mu - 1/2$). Can we identify also the asymptotic states?

Thanks for listening!