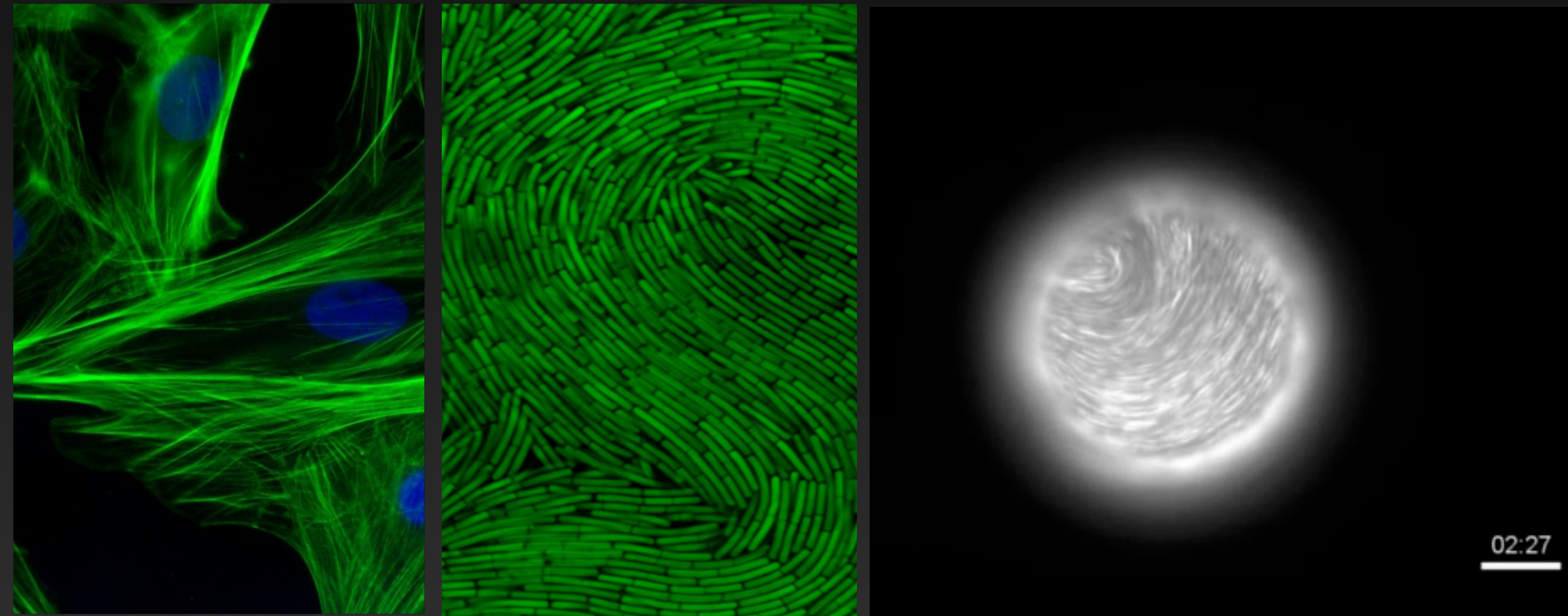


# Dynamical phase transitions in complex fluids and non-equilibrium systems

## WP1 Spoke 2 b5 Physics of Complex Systems (UniBa)

- Ordering properties in complex fluids and non-equilibrium systems  
Active fluids (dense suspensions of bacteria, cytoskeletal extracts, Microtubules bundles)
- Hydrodynamic and Statistical mechanics modelling
- High Performance Computing



- Prof. Giuseppe Gonnella
- Antonio Suma (RTB)
- Antonio Lamura

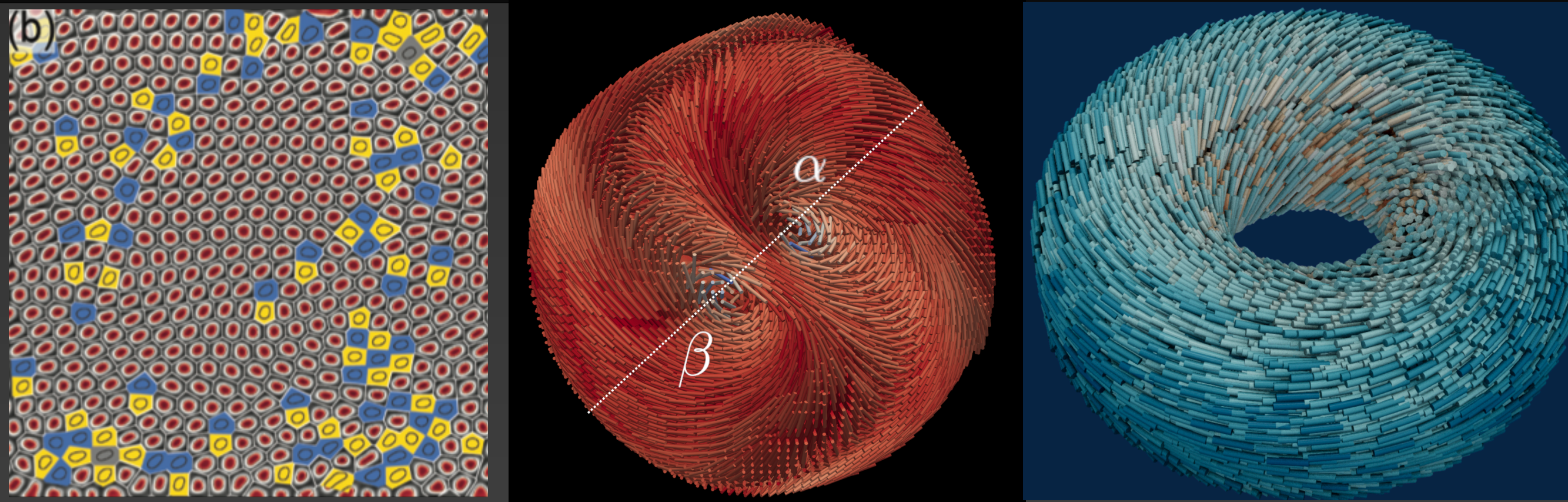
**Dr. Giuseppe Negro**

6/03/2023

[gnegro2@ed.ac.uk](mailto:gnegro2@ed.ac.uk)

# USE CASES

Phase Field Theories of complex and active fluids



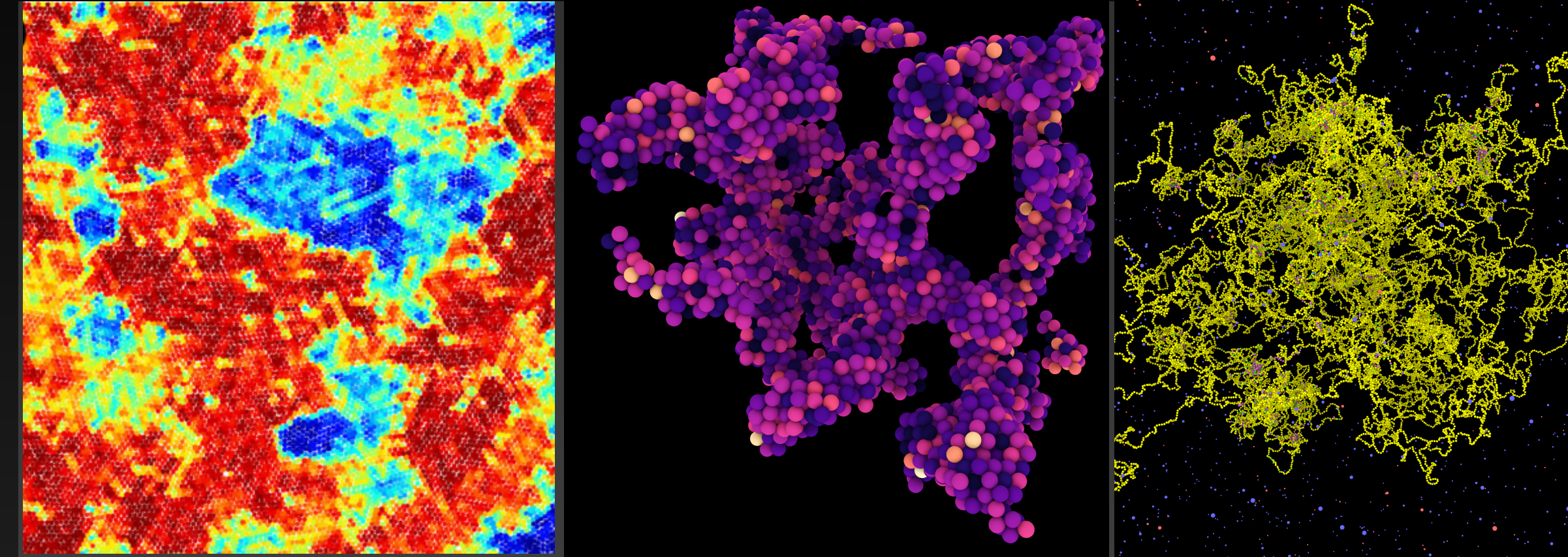
Topological Phase transitions of Cholesteric Shells Confined in Shells

Self-Propulsions of active droplets in 3D

Active Turbulence

**Lattice Boltzmann Methods**  
**MPI**

Non-equilibrium statistical models for self propelled particles and DNA transcription



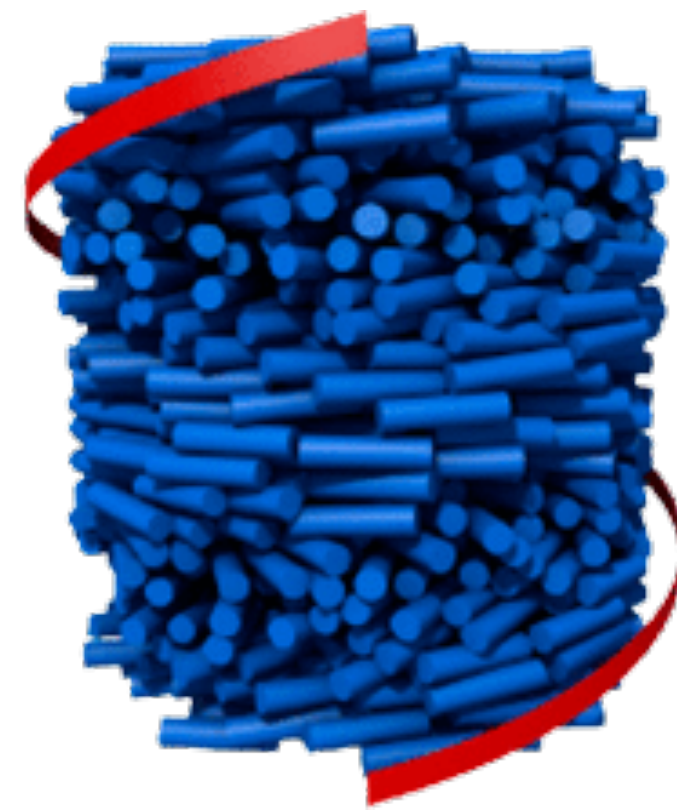
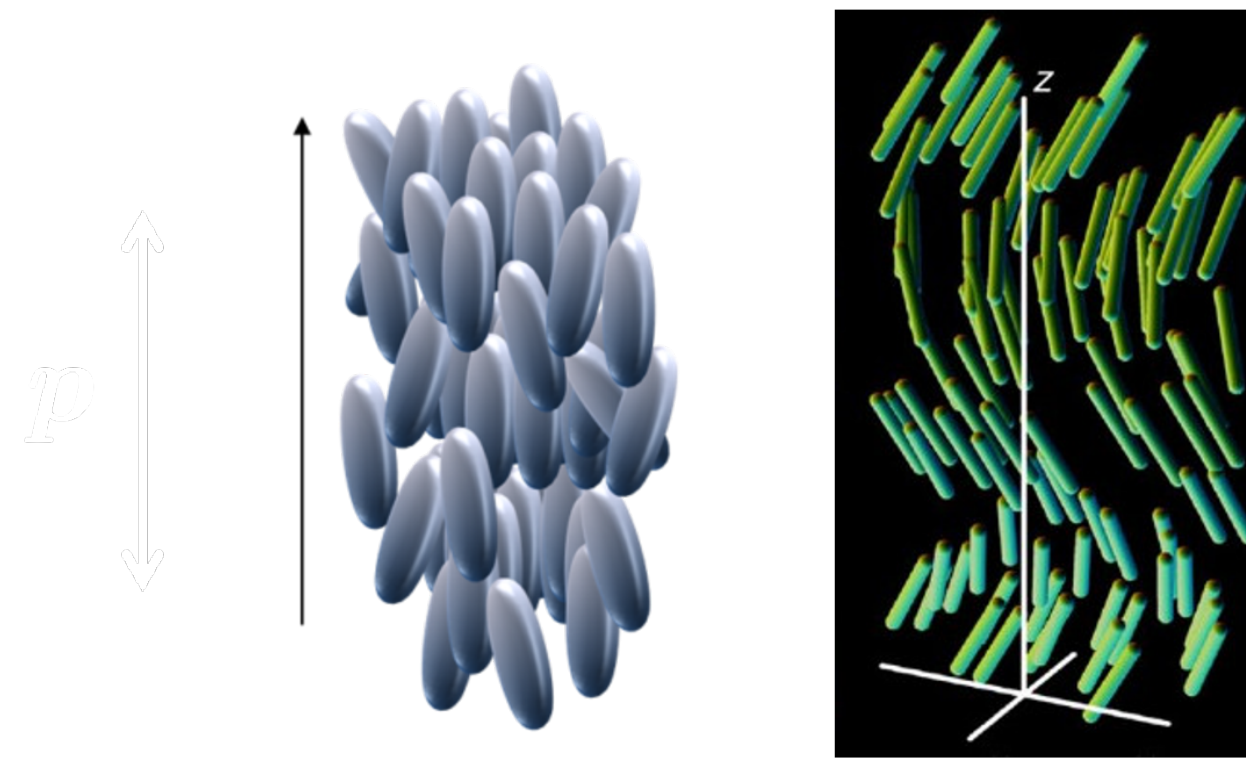
Non equilibrium Phase transitions in Active Brownian Particles

Polymer models to study transcription and dynamics in Human Chromosomes

**Molecular dynamics simulations**  
**with implicit and explicit solvents**  
**(LAMMPS+ CustomMulti Particle**  
**collision Dynamics Package)**

# PHASE FIELD THEORIES

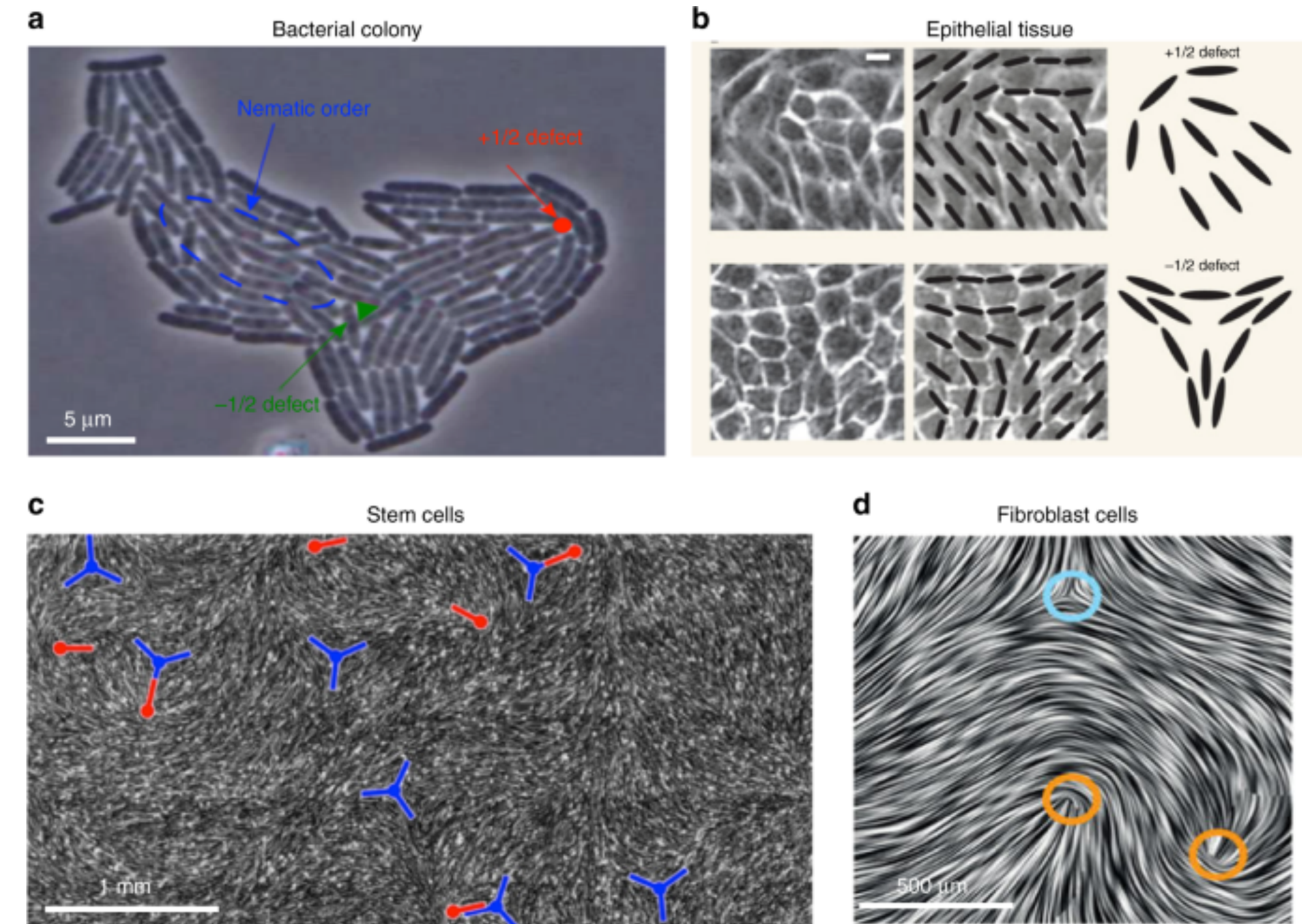
# liquid crystals



Oriental Order  $\longrightarrow$  Smooth Deformations

**Described by a second rank  
traceless symmetric Tensor  
Order Parameter**

$$Q_{\alpha\beta} = S \left\langle \mathbf{n}_\alpha \mathbf{n}_\beta - \frac{1}{d} \delta_{\alpha\beta} \right\rangle$$



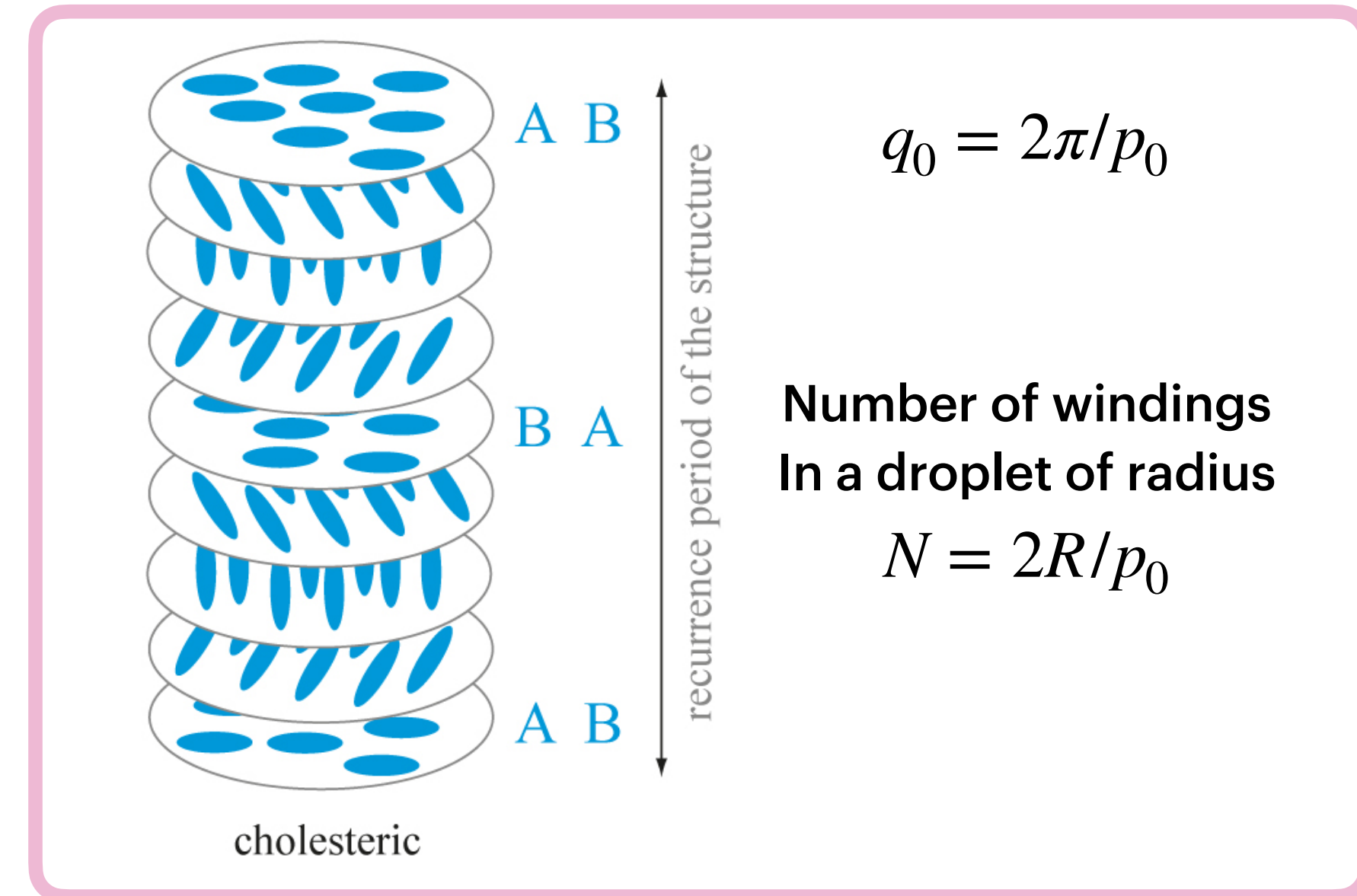
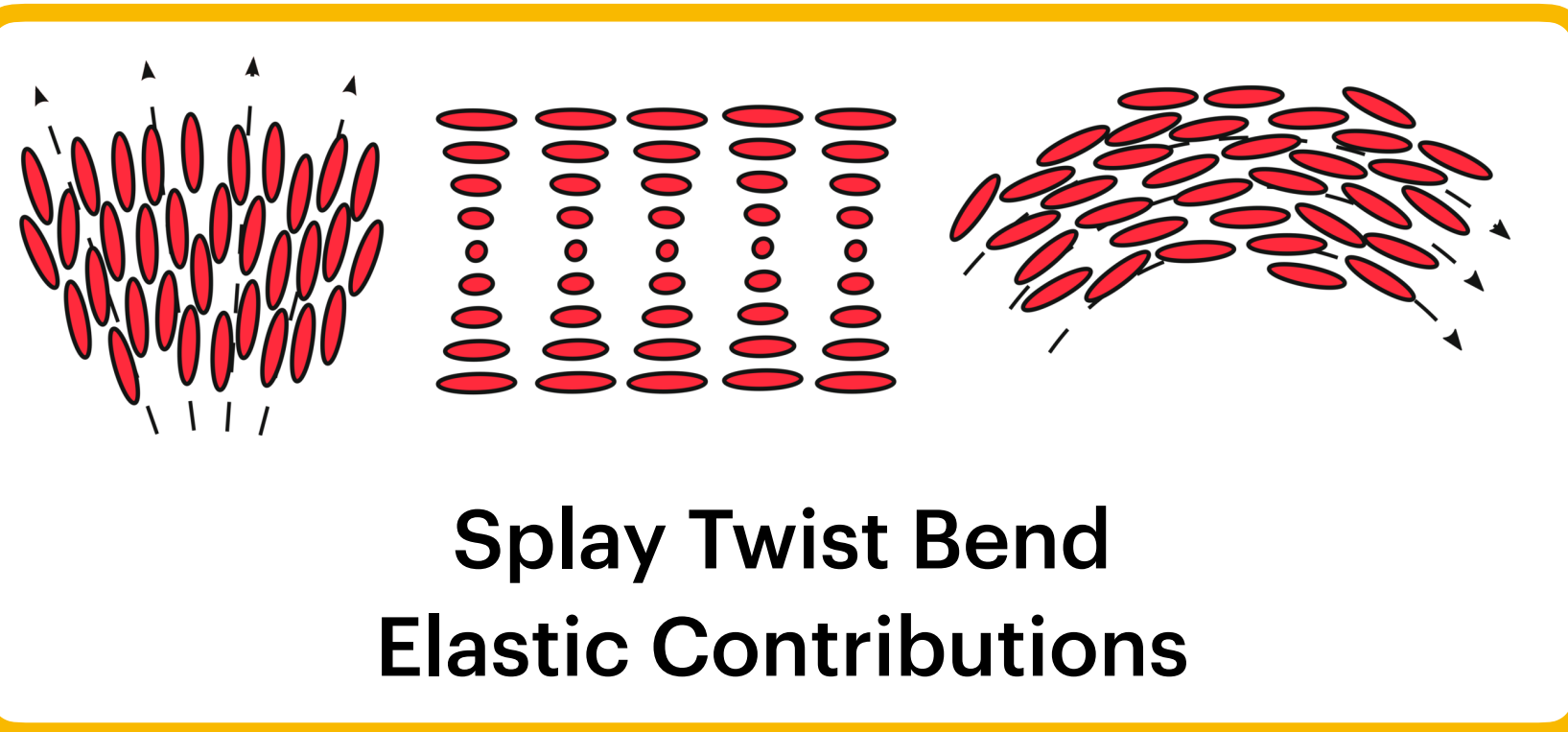
# Landau-De Gennes Theory

Tensor order parameter  
First order transition for orientational order

$$\mathcal{F}[\phi, Q_{\alpha\beta}] = \int dV \left[ A_0 \left[ \frac{1}{2} \left( 1 - \frac{\chi(\phi)}{3} \right) \mathbf{Q}^2 - \frac{\chi(\phi)}{3} \mathbf{Q}^3 + \frac{\chi(\phi)}{4} \mathbf{Q}^4 \right] + \frac{L}{2} \left[ (\nabla \cdot \mathbf{Q})^2 + (\nabla \times \mathbf{Q} + 2q_0 \mathbf{Q})^2 \right] \right]$$

First order I-N phase transition

$$\chi(\phi) = \chi_0 + \chi_s (\nabla \phi)^2 > 2.7$$



3 Control Parameters

Radius of Confinement

Chirality

Elastic energy

Activity

# DYNAMICAL EQUATIONS

## Dynamical fields

- Nematic tensor  $Q_{\alpha\beta}$
- Velocity field  $\mathbf{v}$
- Concentration field  $\phi$

### Advection relaxation for the tensor order parameter

$$(\partial_t + \mathbf{v} \cdot \nabla) \mathbf{Q} - \mathbf{S}(\mathbf{W}, \mathbf{Q}) = -\frac{1}{\Gamma} \mathbf{H}$$

$$\mathbf{H} = -\frac{\delta \mathcal{F}}{\delta \mathbf{Q}} + \frac{\mathbf{I}}{3} \text{Tr} \left( \frac{\delta \mathcal{F}}{\delta \mathbf{Q}} \right)$$

### Navier Stokes equation for the velocity field

$$(\partial_t + \mathbf{v} \cdot \nabla) \mathbf{v} = \nabla \cdot \boldsymbol{\sigma}$$

### Advection diffusion for concentration field

$$\partial_t \phi + \nabla \cdot (\phi \mathbf{v}) = \nabla \cdot \left( M \nabla \frac{\delta \mathcal{F}}{\delta \phi} \right)$$

$$\sigma_{\alpha\beta}^{el} = -\xi H_{\alpha\gamma} \left( Q_{\gamma\beta} + \frac{1}{3} \delta_{\gamma\beta} \right) - \xi \left( Q_{\alpha\gamma} + \frac{1}{3} \delta_{\alpha\gamma} \right) H_{\gamma\beta}$$

$$+ 2\xi \left( Q_{\alpha\beta} - \frac{1}{3} \delta_{\alpha\beta} \right) Q_{\gamma\mu} H_{\gamma\mu} + Q_{\alpha\gamma} H_{\gamma\beta} - H_{\alpha\gamma} Q_{\gamma\beta}$$

$$\sigma^{bm} = \left( f - \frac{\delta \mathcal{F}}{\delta \phi} \right) \delta_{\alpha\beta} - \frac{\delta \mathcal{F}}{\delta (\partial_\beta \phi)} \partial_\alpha \phi$$

**Numerically solved via a hybrid Lattice Boltzmann Scheme**

# LATTICE BOLTZMANN MODELS

Based on phase-space discretisation form of the Boltzmann equation

$$\partial_t f + \xi_\alpha \partial_\alpha f + \frac{F_\alpha}{\rho} \partial_{\xi_\alpha} f = \Omega(f)$$

Discretised both in real and velocity space

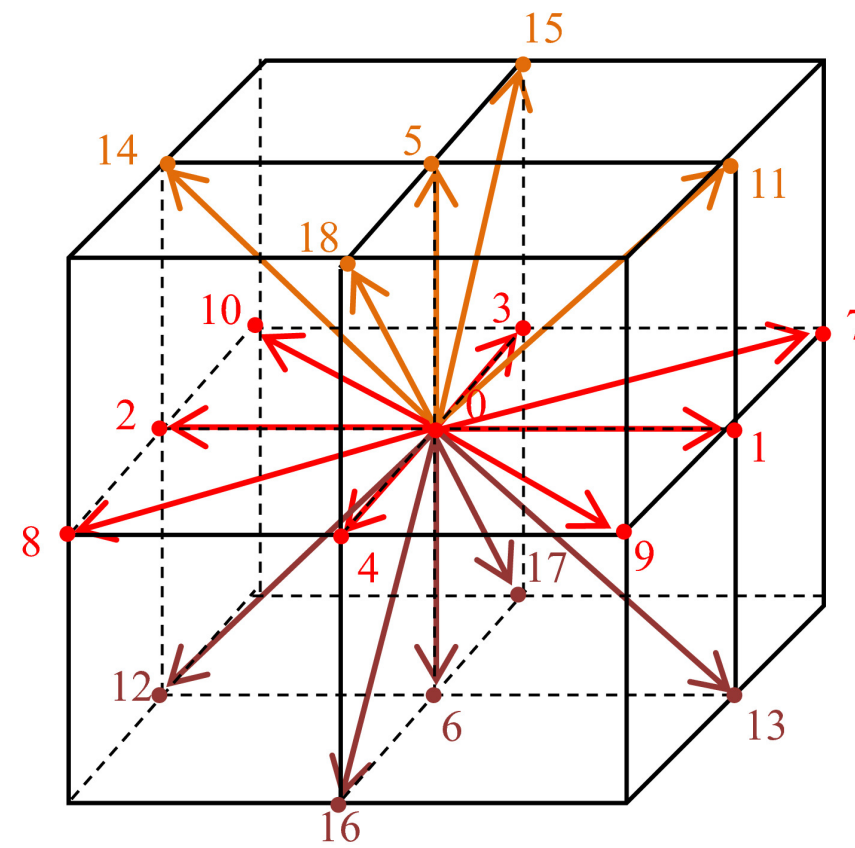
$$f_i(\vec{r} + \vec{\xi}_i \Delta t, t + \Delta t) - f_i(\vec{r}, t) = \mathcal{C}(f_i, t)$$

$$\mathcal{C}(f_i, t) = -\frac{1}{\tau}(f_i - f_i^{eq})$$

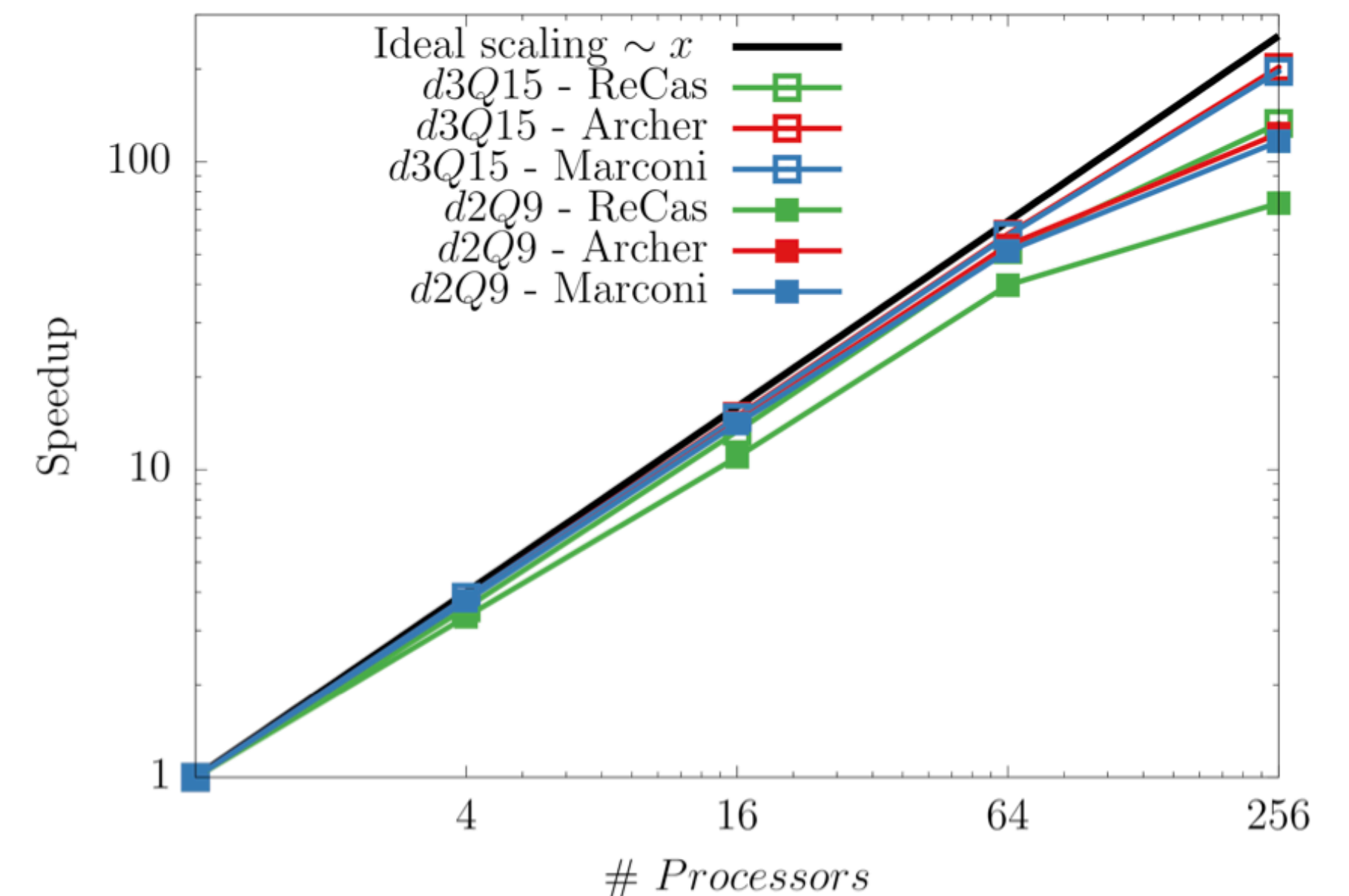
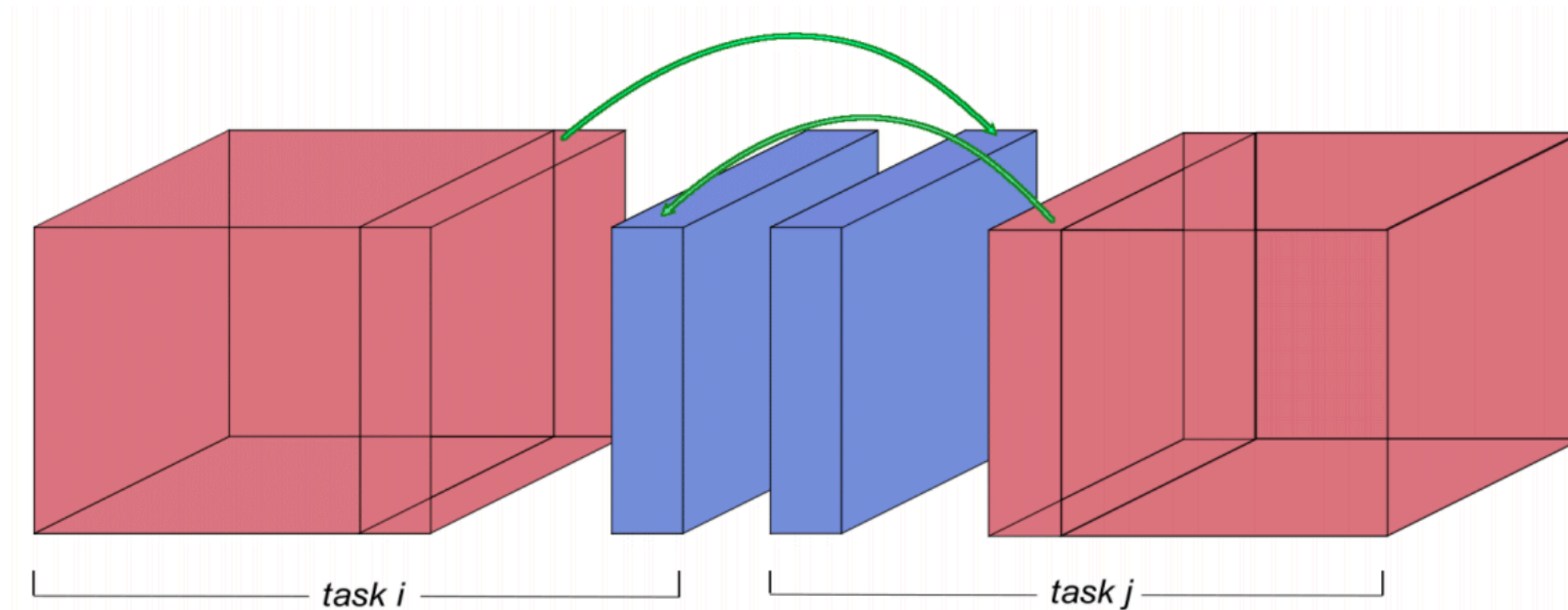
Mass and momentum density are defined as

$$\sum_i f_i^{eq} \xi_{i\alpha} = \rho u_\alpha \quad \sum_i f_i^{eq} = \rho$$

$$\sum_i f_i^{eq} \xi_{i\alpha} \xi_{i\beta} = \rho u_\alpha u_\beta - \sigma_{\alpha\beta}$$



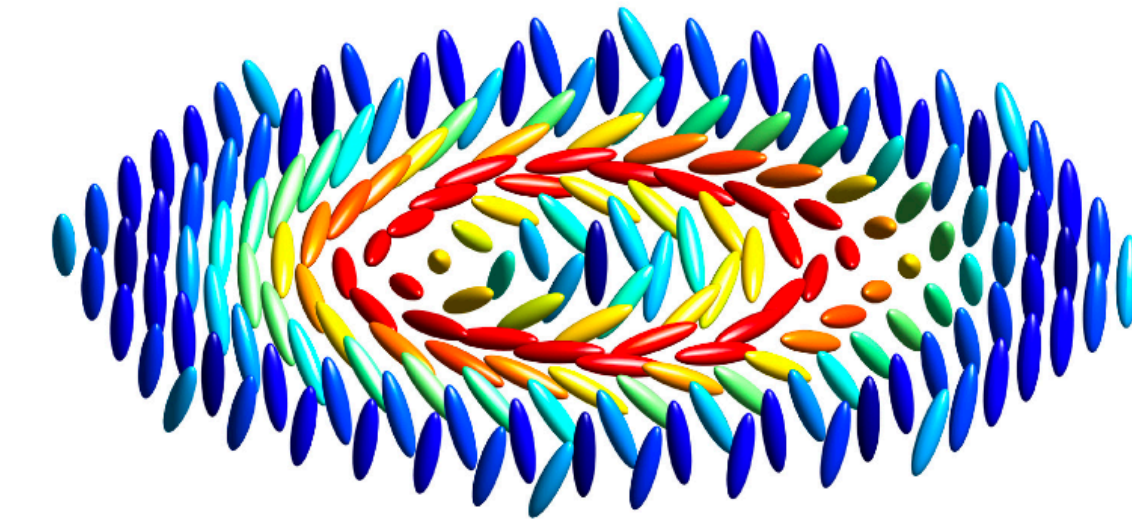
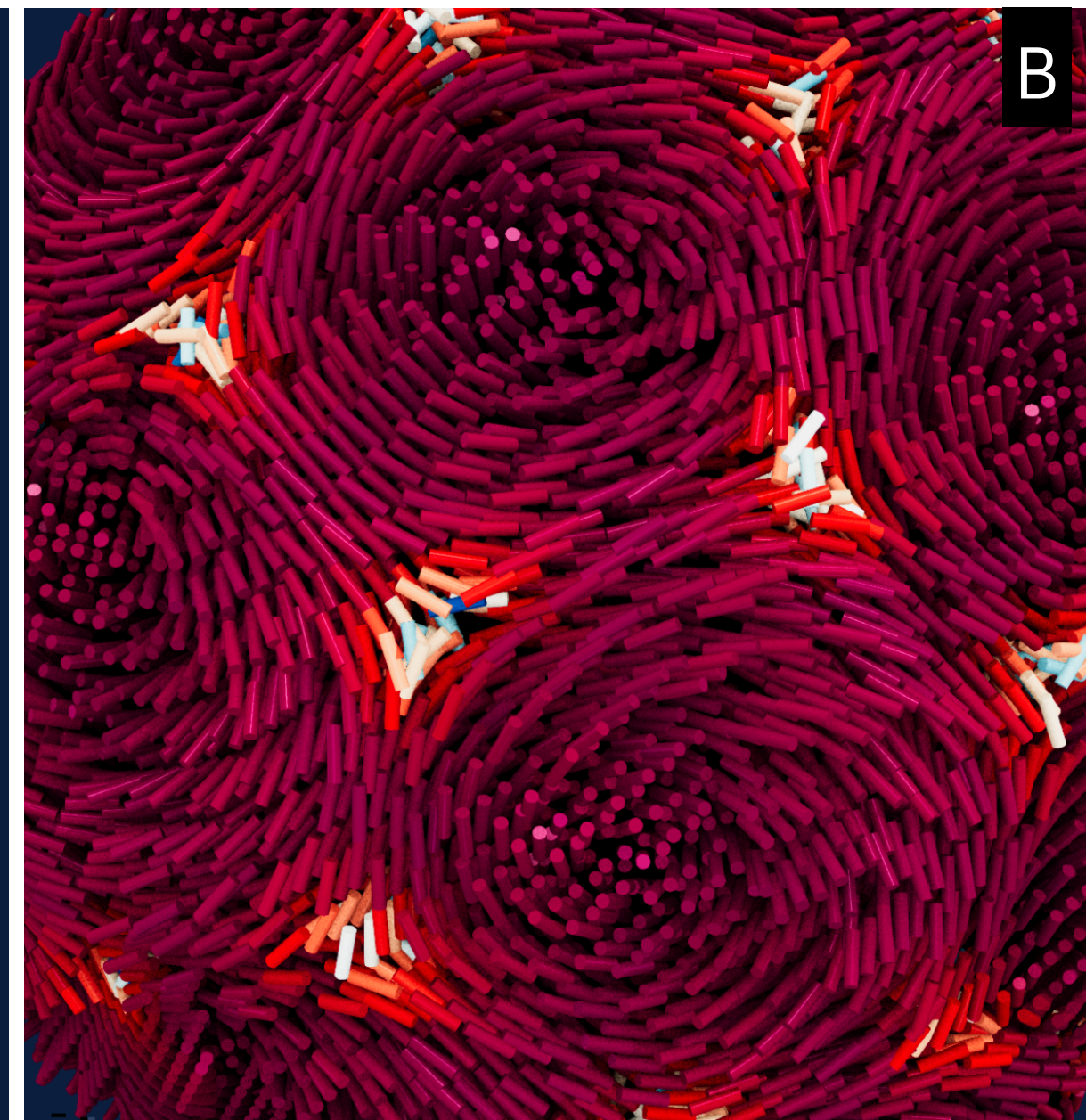
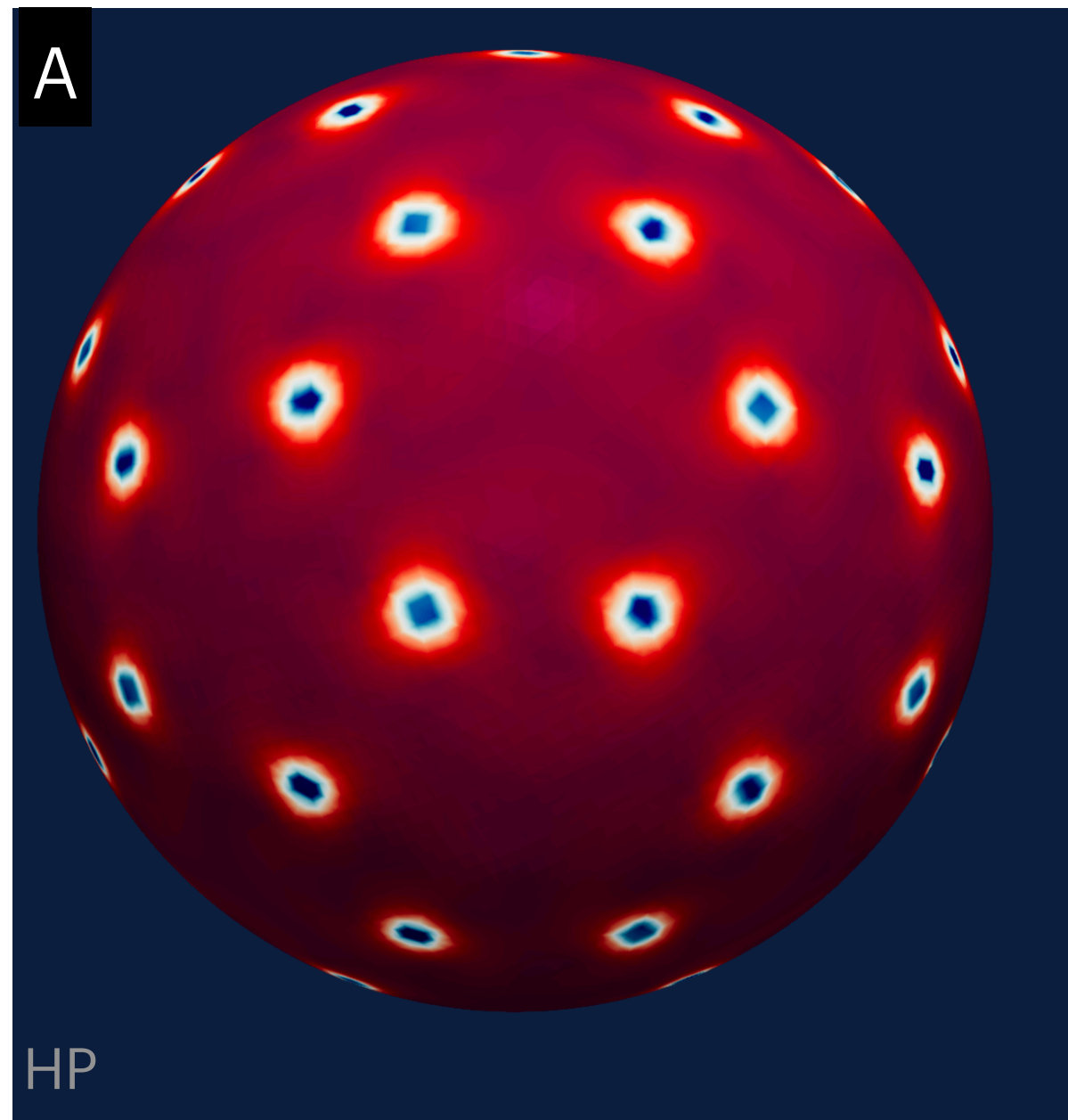
MPI Parallelization



- The Equilibrium distribution functions are expanded up to a given order in the fluid velocity
- The Navier Stokes equations can be formally derived from the Lattice Boltzmann equation in the long wave length limit through Chapman-Enskog expansion

HPC facilities Cineca (Bologna), Recas (Bari), Archer2 (Edinburgh), Snellius (Amsterdam)

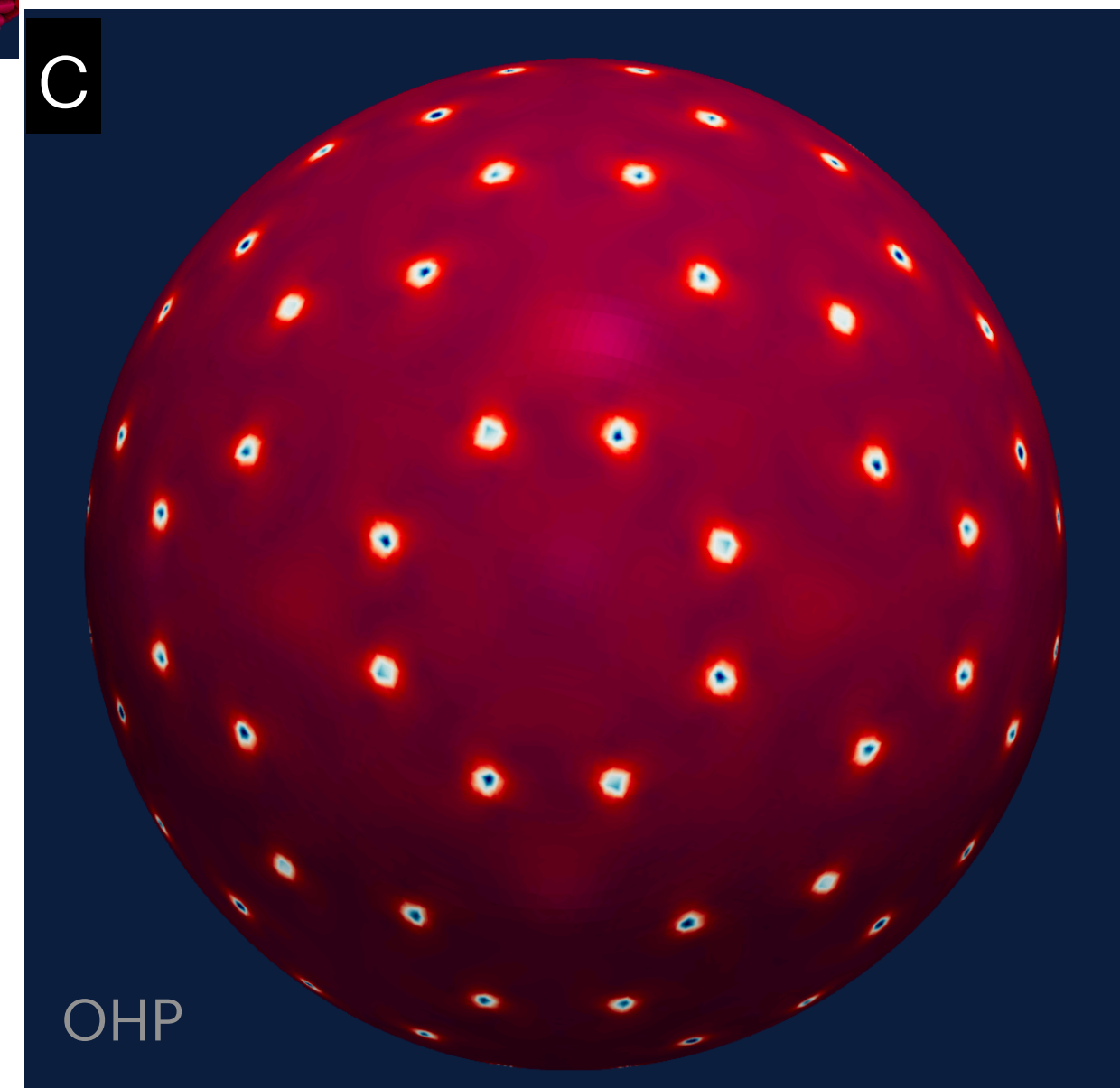
# Chiral liquid crystals confined in shells



BABY SKYRMION

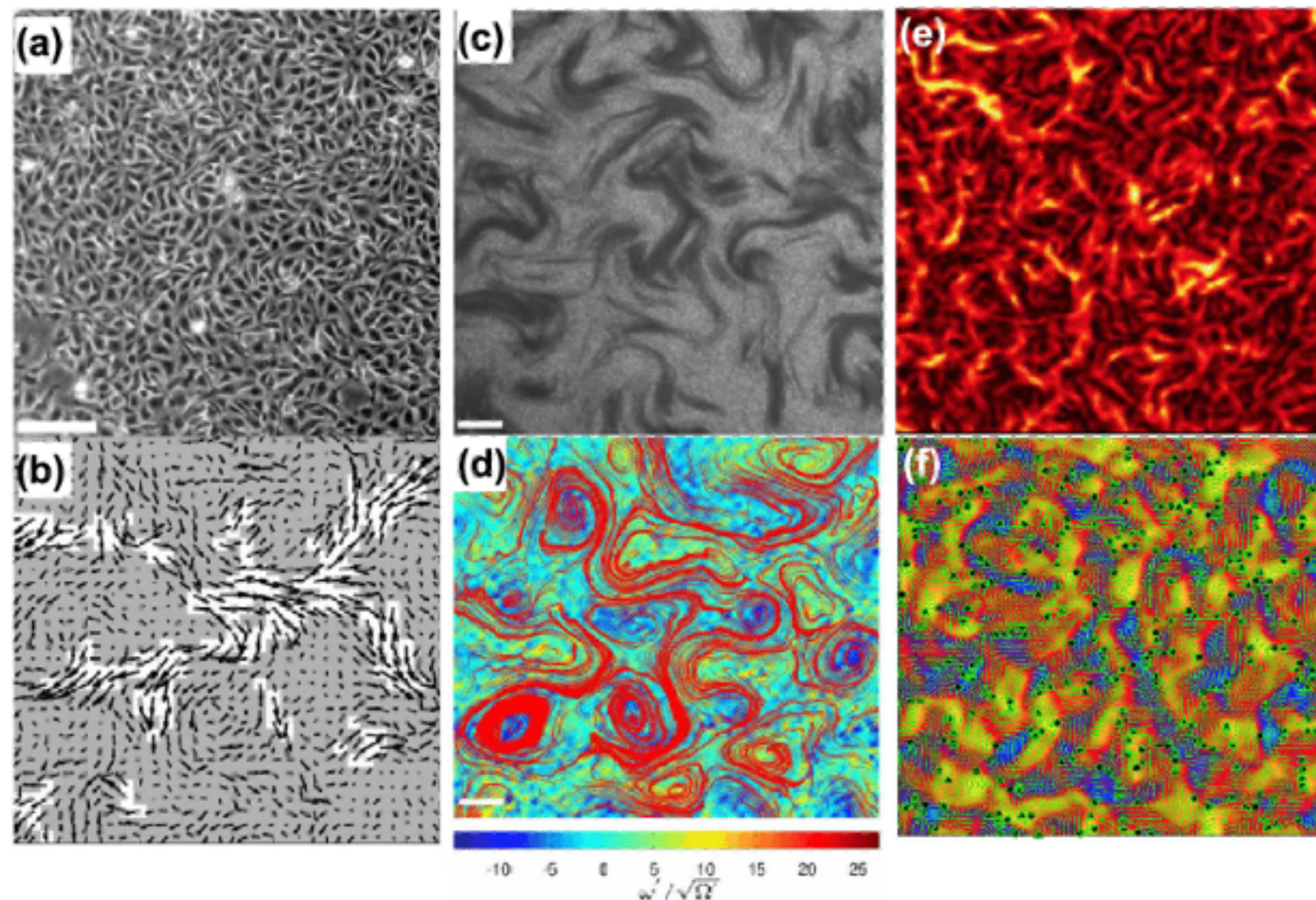
Football configuration

Increasing chirality or increasing R

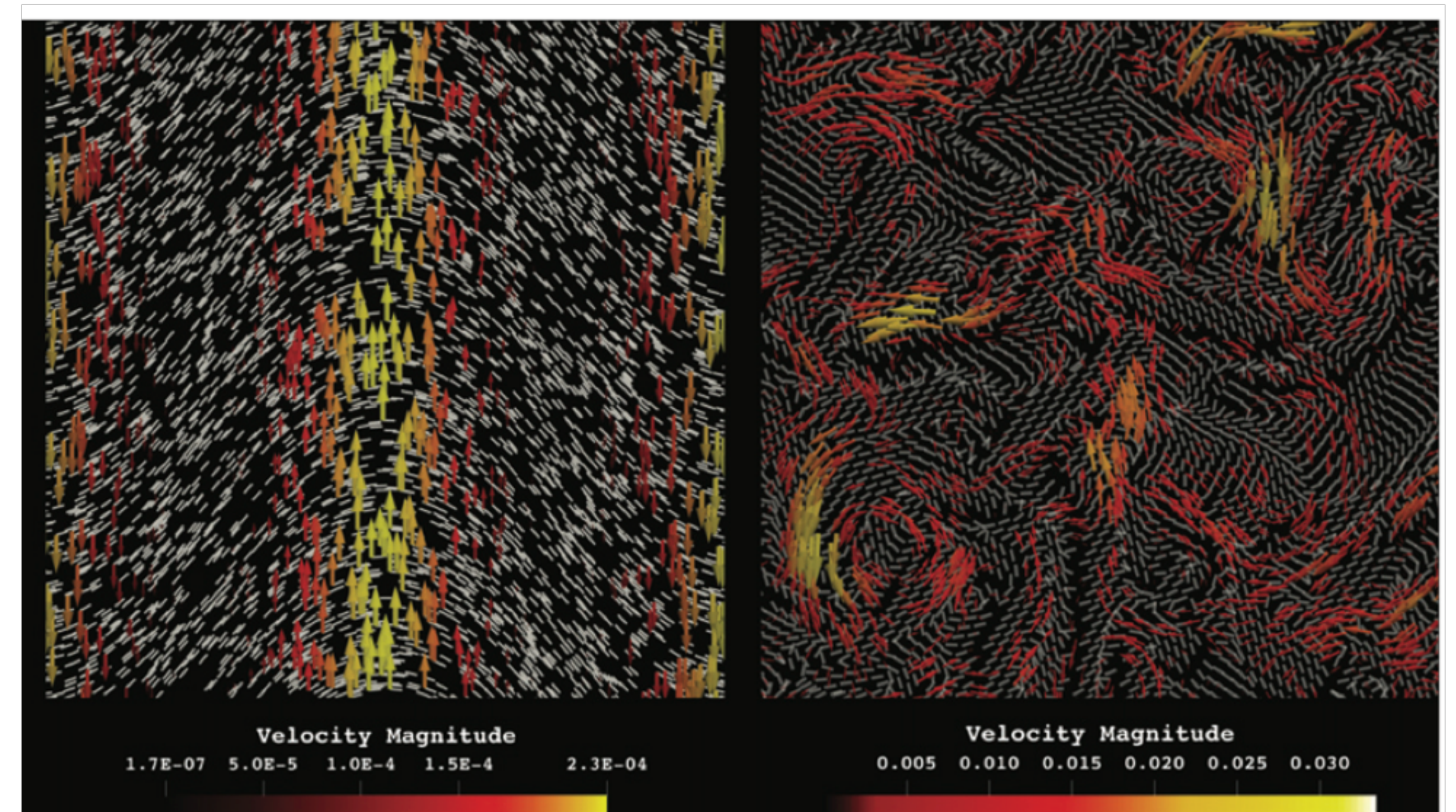




# ACTIVE TURBULENCE



Jörn Dunkel, Sebastian Heidenreich, Knut Drescher, Henricus H. Wensink, Markus Bär, and Raymond E. Goldstein  
Phys. Rev. Lett. **110**, 228102 – Published 28 May 2013



[Cascade or not cascade? Energy transfer and elastic effects in active nematics](#)

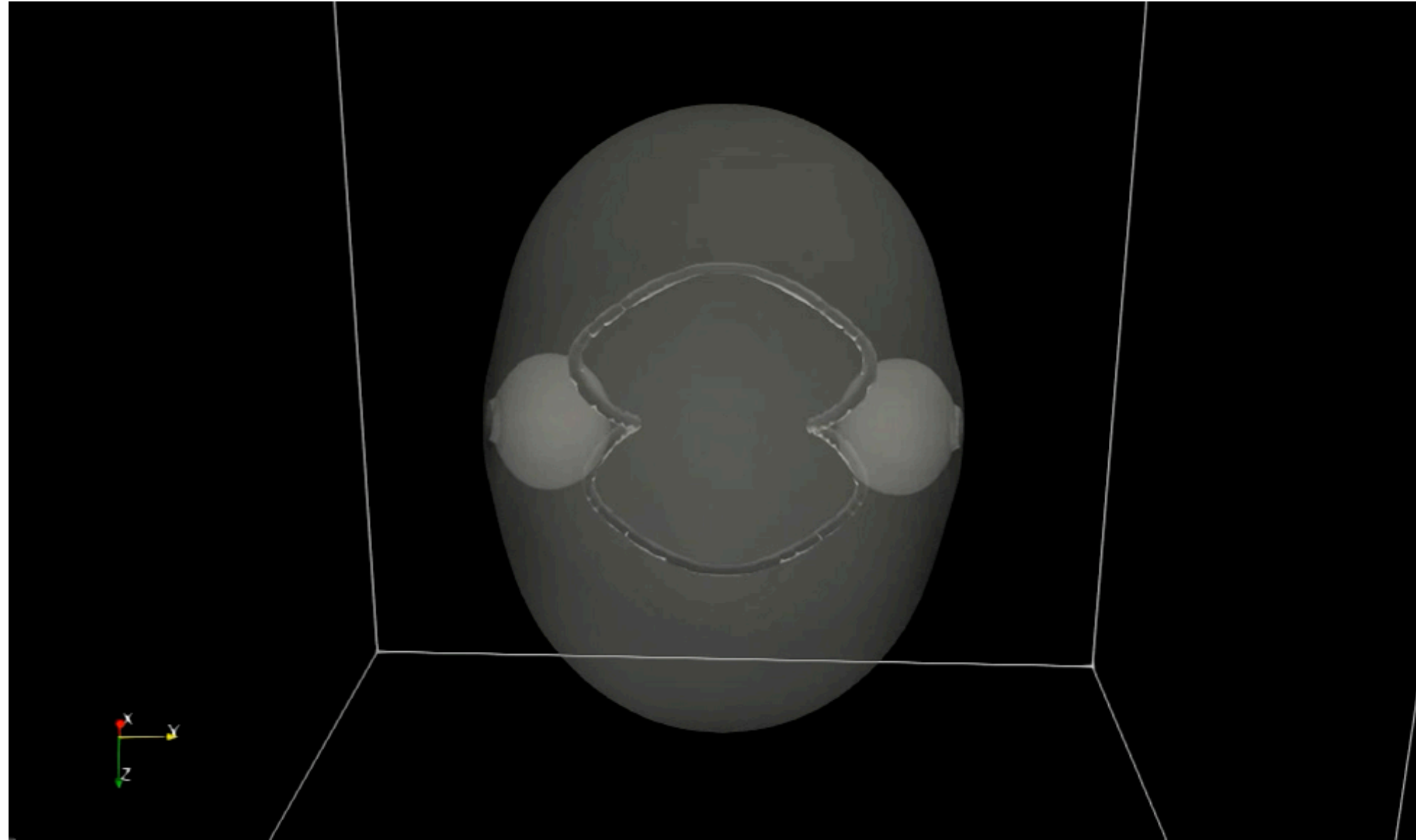
LN Carenza, L Biferale, G Gonnella  
Europhysics Letters 132 (4), 44003 (2021)

[Multiscale control of active emulsion dynamics](#)

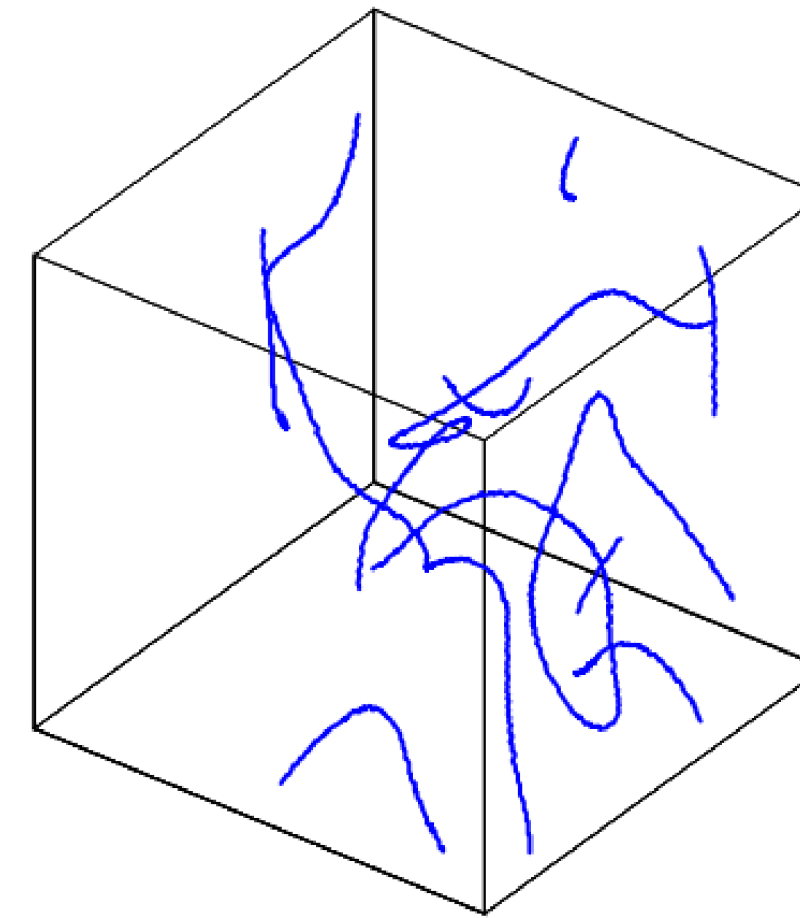
LN Carenza, L Biferale, G Gonnella  
Physical Review Fluids 5 (1), 011302 (2020)

# Planned Activity

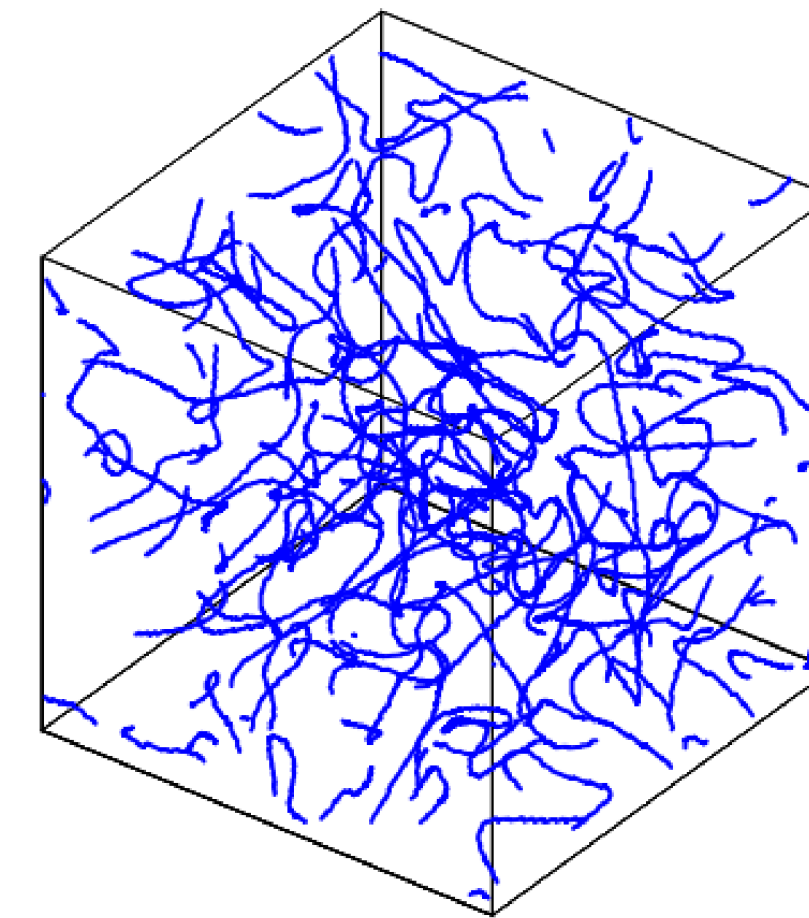
## Multi-Phase models



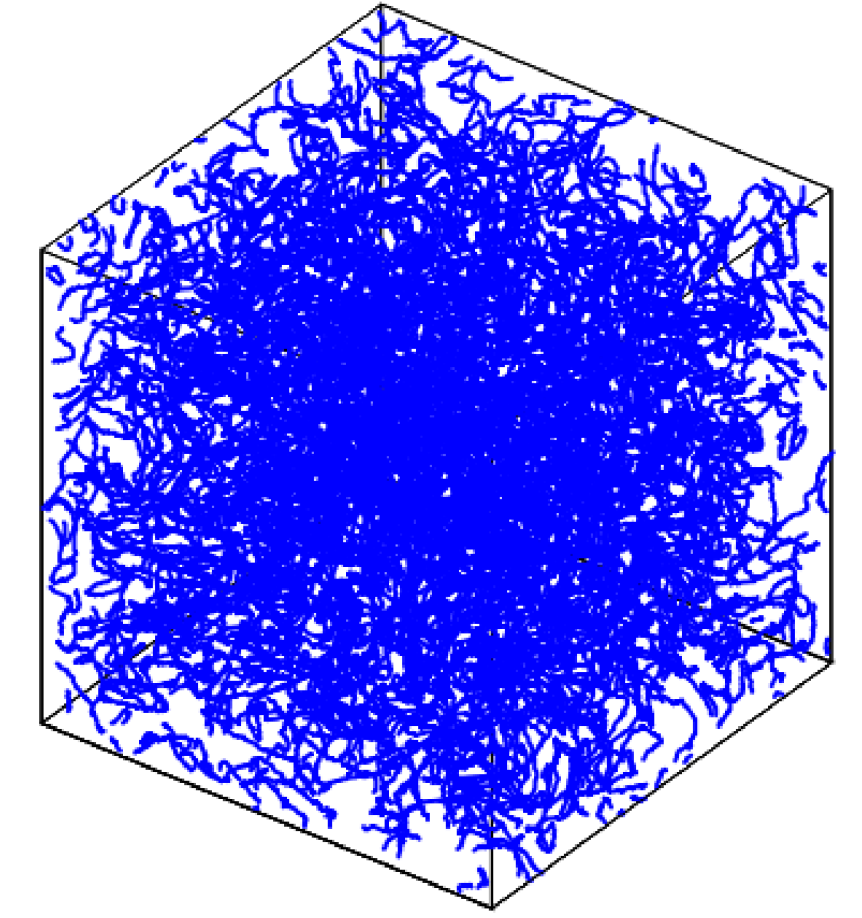
## Active Turbulence in 3D



$\alpha=0.0001$



0.001

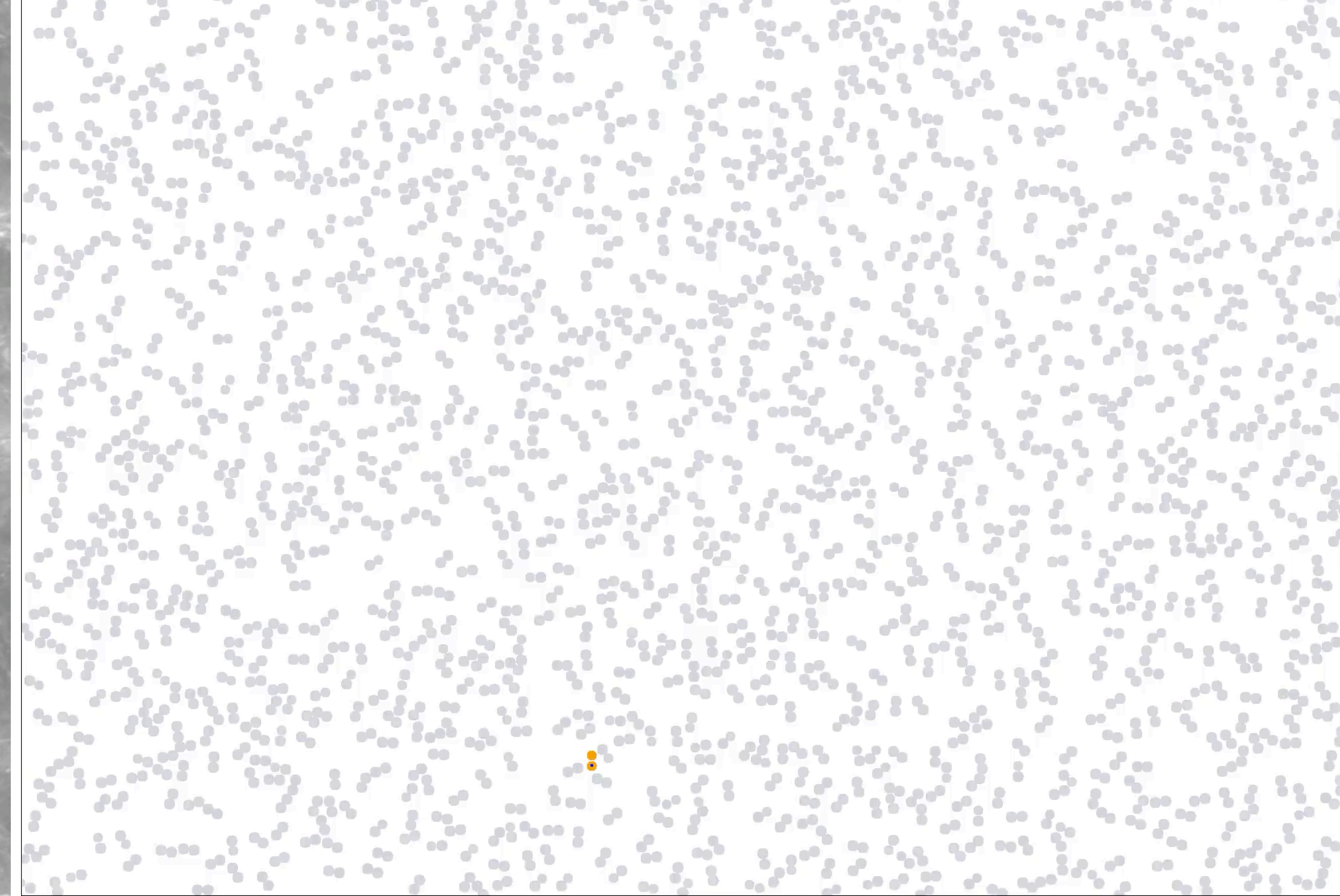
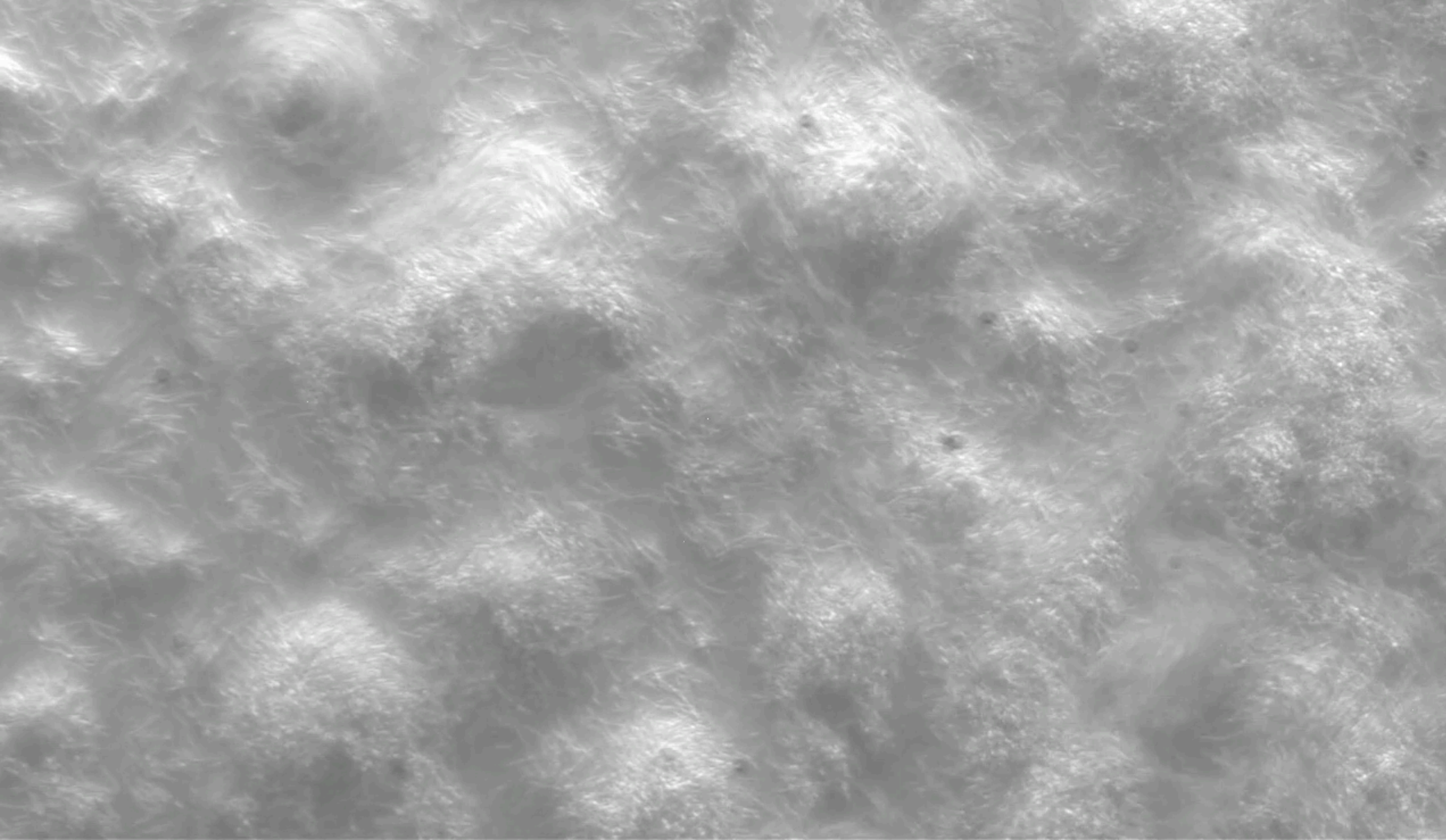


0.01

Defects tracking in 3D Active turbulence

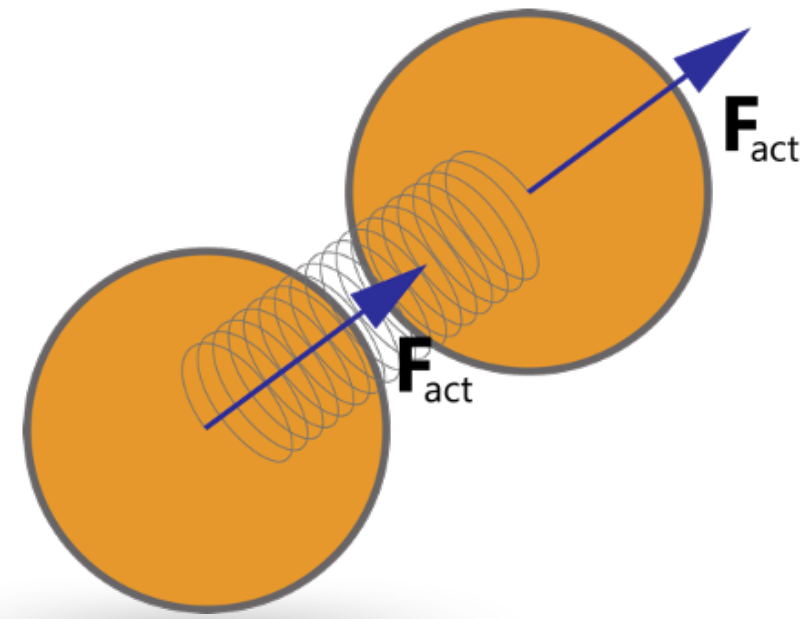
Typical Runs: 128/256 cores for 48 hours  
50 exploratory runs  
307200 core hours

**Non-equilibrium statistical models for self propelled particles and DNA  
transcription**



- $U(r) = \begin{cases} U_{\text{Mie}}(r) - U_{\text{Mie}}(r_{\text{min}}) & \text{if } r < r_{\text{min}} \\ 0 & \text{if } r \geq r_{\text{min}} \end{cases}$

- $\mathbf{F}_{FENE} = -\frac{k(\mathbf{r}_i - \mathbf{r}_j)}{1 - r_{ij}^2/r_0^2}$

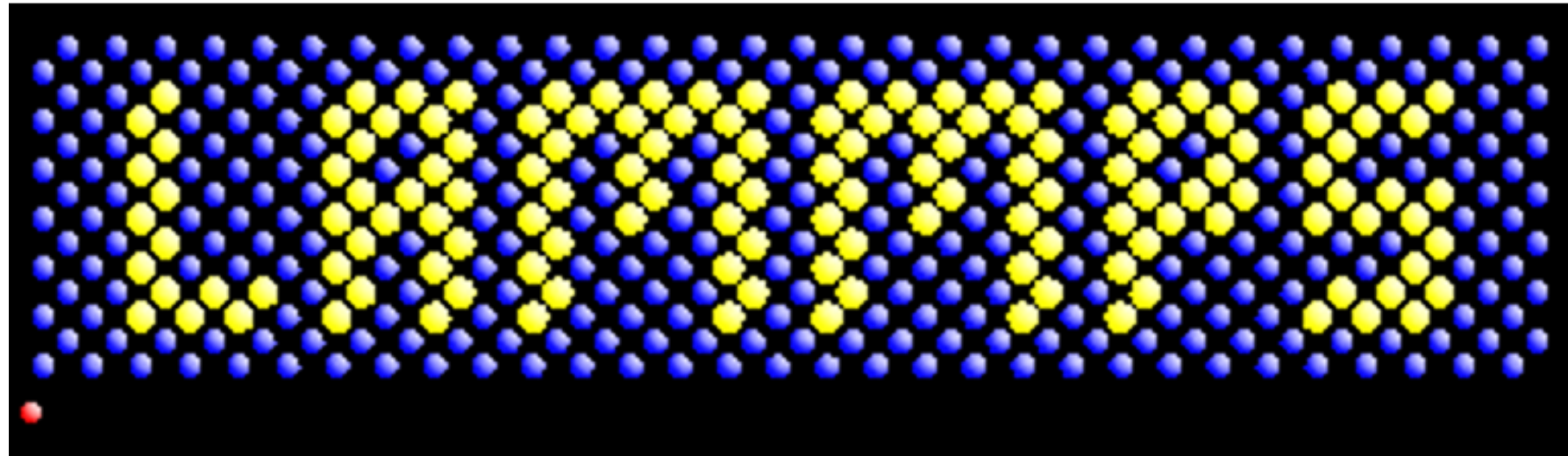


$$m\ddot{\mathbf{r}}_i = -\gamma\dot{\mathbf{r}}_i - \frac{\partial U_{FENE}}{\partial r_{i,i+1}} \hat{\mathbf{r}}_{i,i+1} - \sum_{j \neq i}^{2N} \frac{\partial U}{\partial r_{ij}} \hat{\mathbf{r}}_{ij} + F_{\text{act}} \hat{\mathbf{n}}_i + \sqrt{2D_0} \eta_i ,$$

$$m\ddot{\mathbf{r}}_{i+1} = -\gamma\dot{\mathbf{r}}_{i+1} + \frac{\partial U_{FENE}}{\partial r_{i,i+1}} \hat{\mathbf{r}}_{i,i+1} - \sum_{j \neq i+1}^{2N} \frac{\partial U}{\partial r_{i+1,j}} \hat{\mathbf{r}}_{i+1,j} + F_{\text{act}} \hat{\mathbf{n}}_{i+1} + \sqrt{2D_0} \eta_{i+1}$$

# ACTIVE PARTICLES

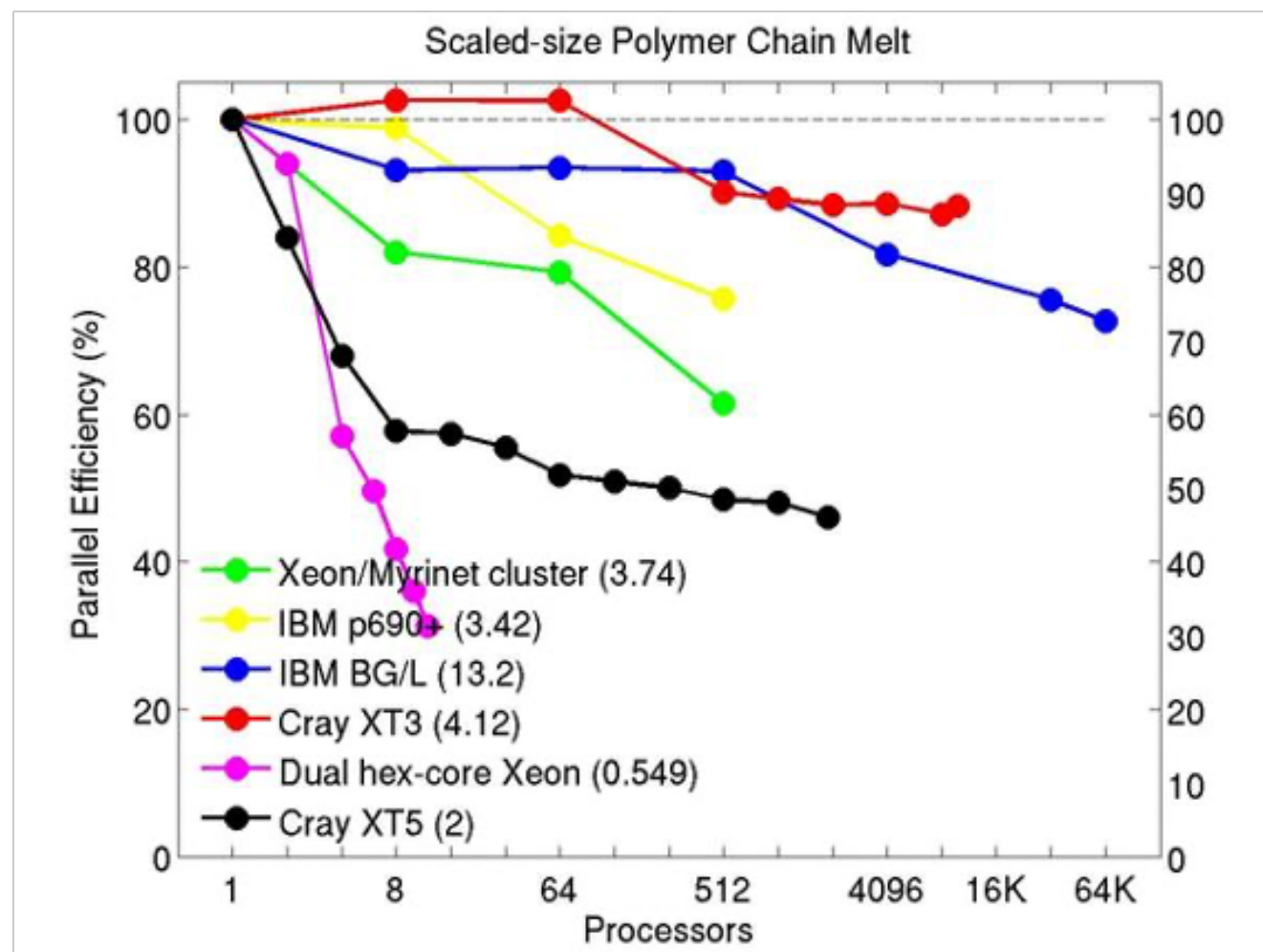
# Molecular Dynamics Methods



LAMMPS is a classical molecular dynamics code with a focus on materials modeling.

It's an acronym for **Large-scale Atomic/Molecular Massively Parallel Simulator**.

It can be used with multiple CPUs and with a GPU

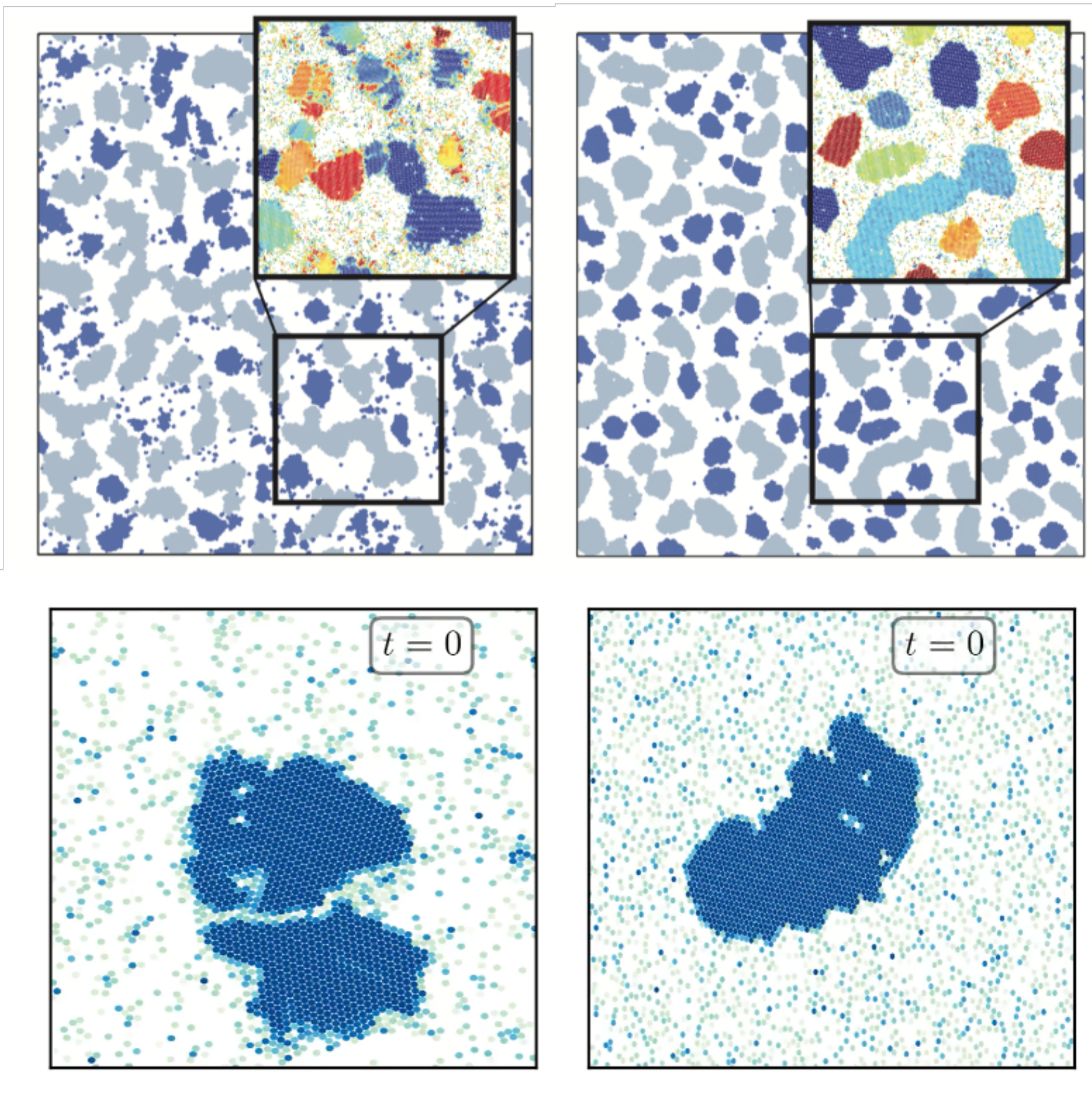


Bead-spring polymer melt with 100-mer chains and FENE bonds:

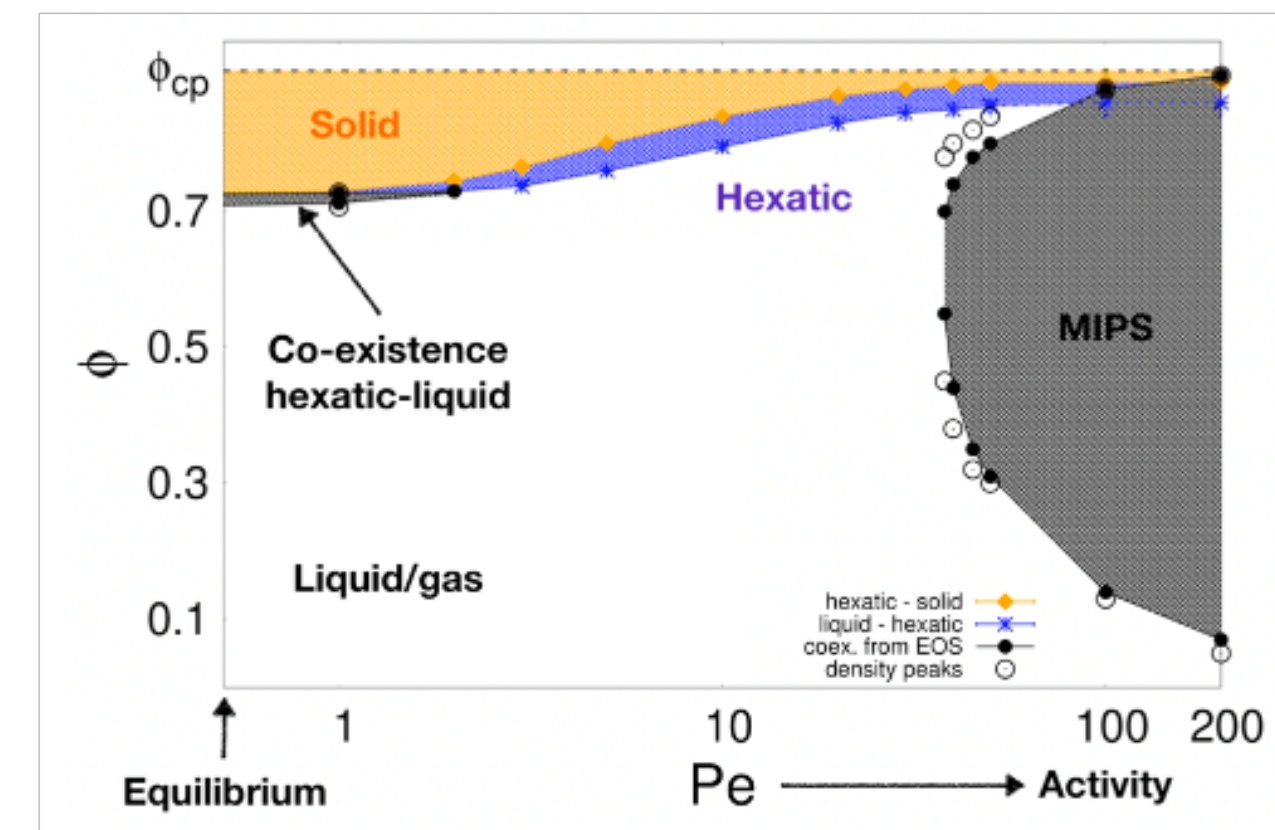
- 32,000 atoms for 100 timesteps
- reduced density 0.8442 (liquid)
- force cutoff of  $2^{(1/6)}$  sigma
- neighbor skin = 0.4 sigma
- neighbors/atom = 5 (within force cutoff)
- NVE time integration

# Active self-propelled particles

Active (with repulsion) Passive (with attraction)



- Micro and macro phase separation of active systems
- Self-organization and dynamics of active clusters
- Phase diagram



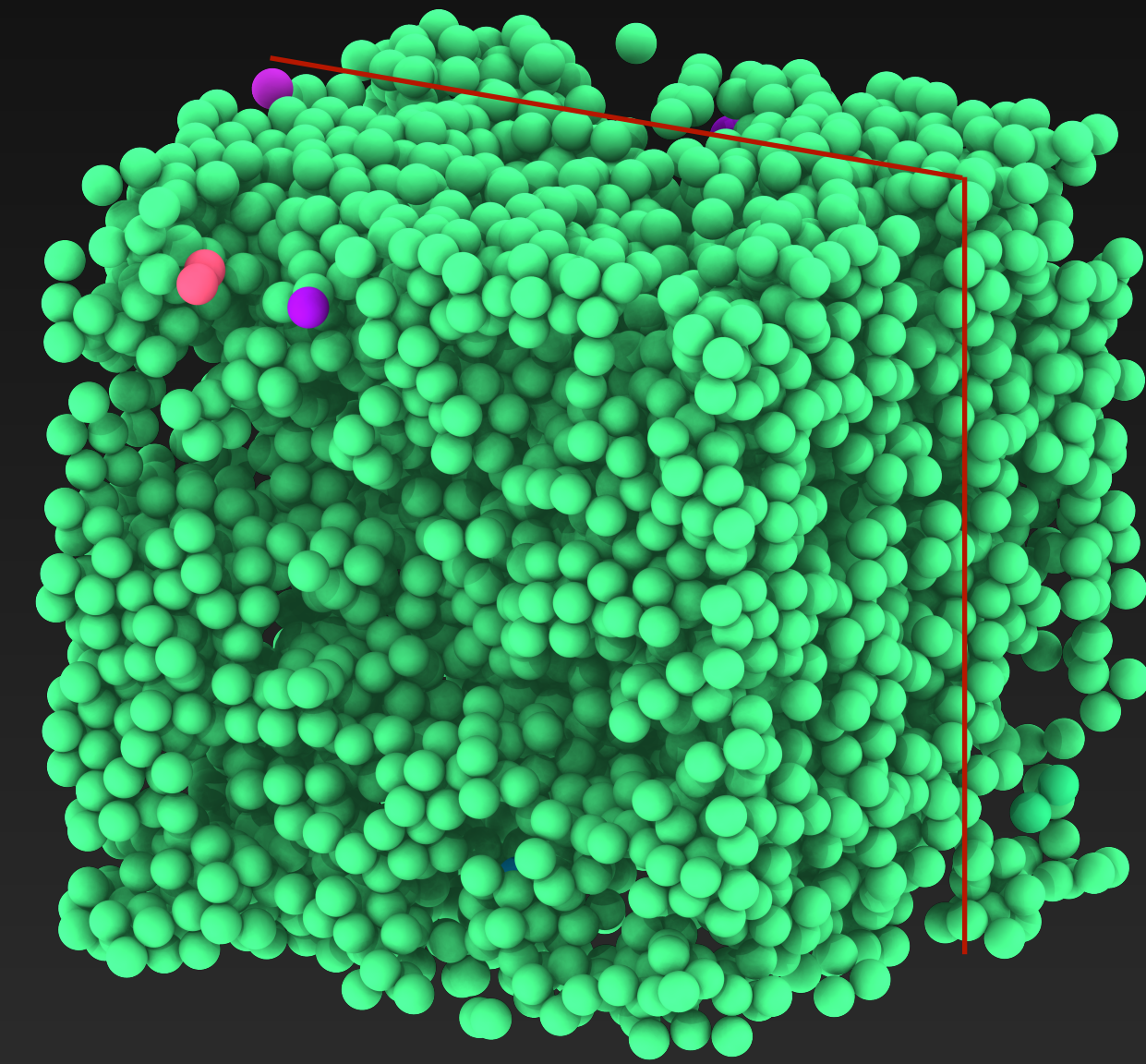
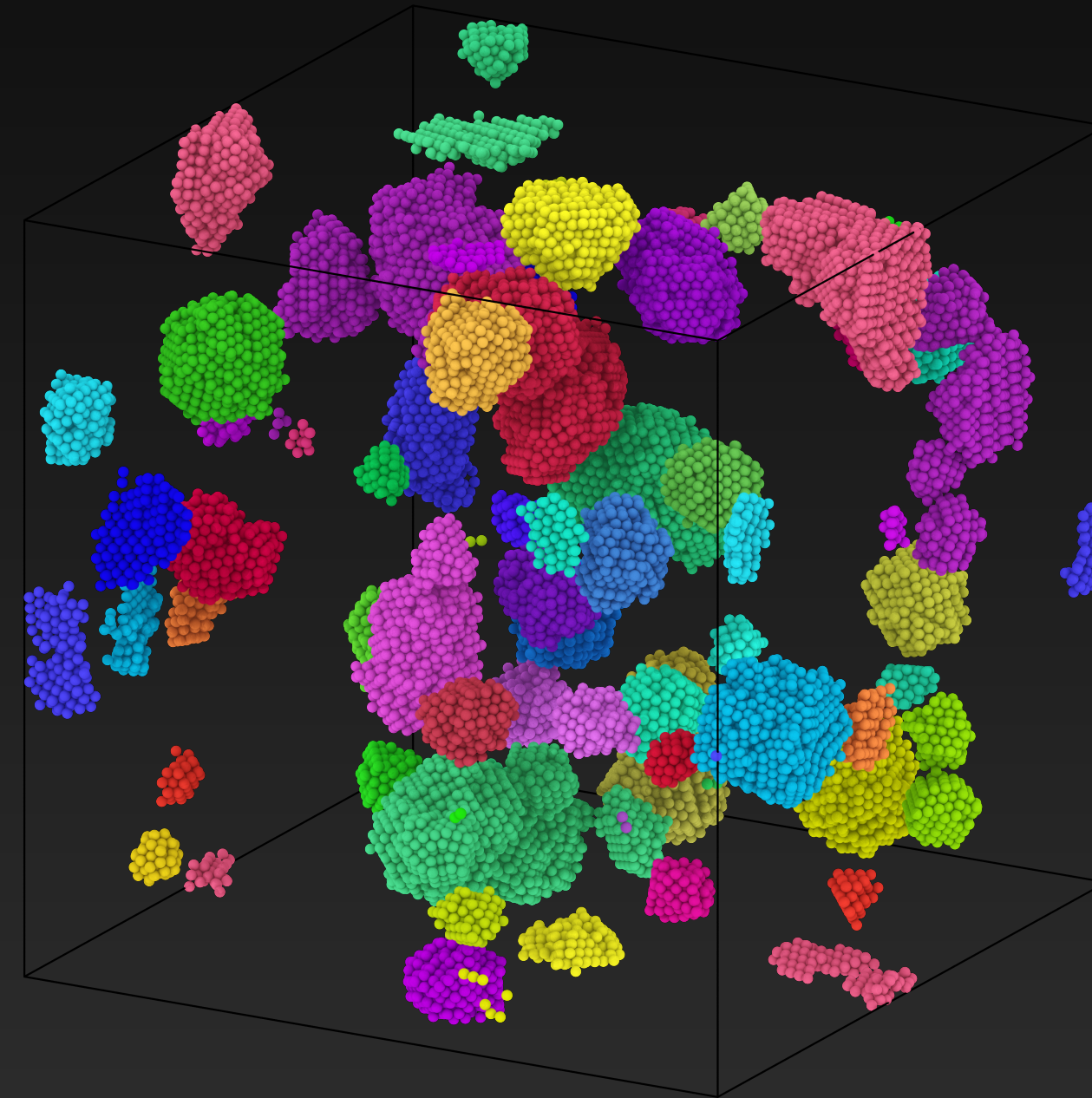
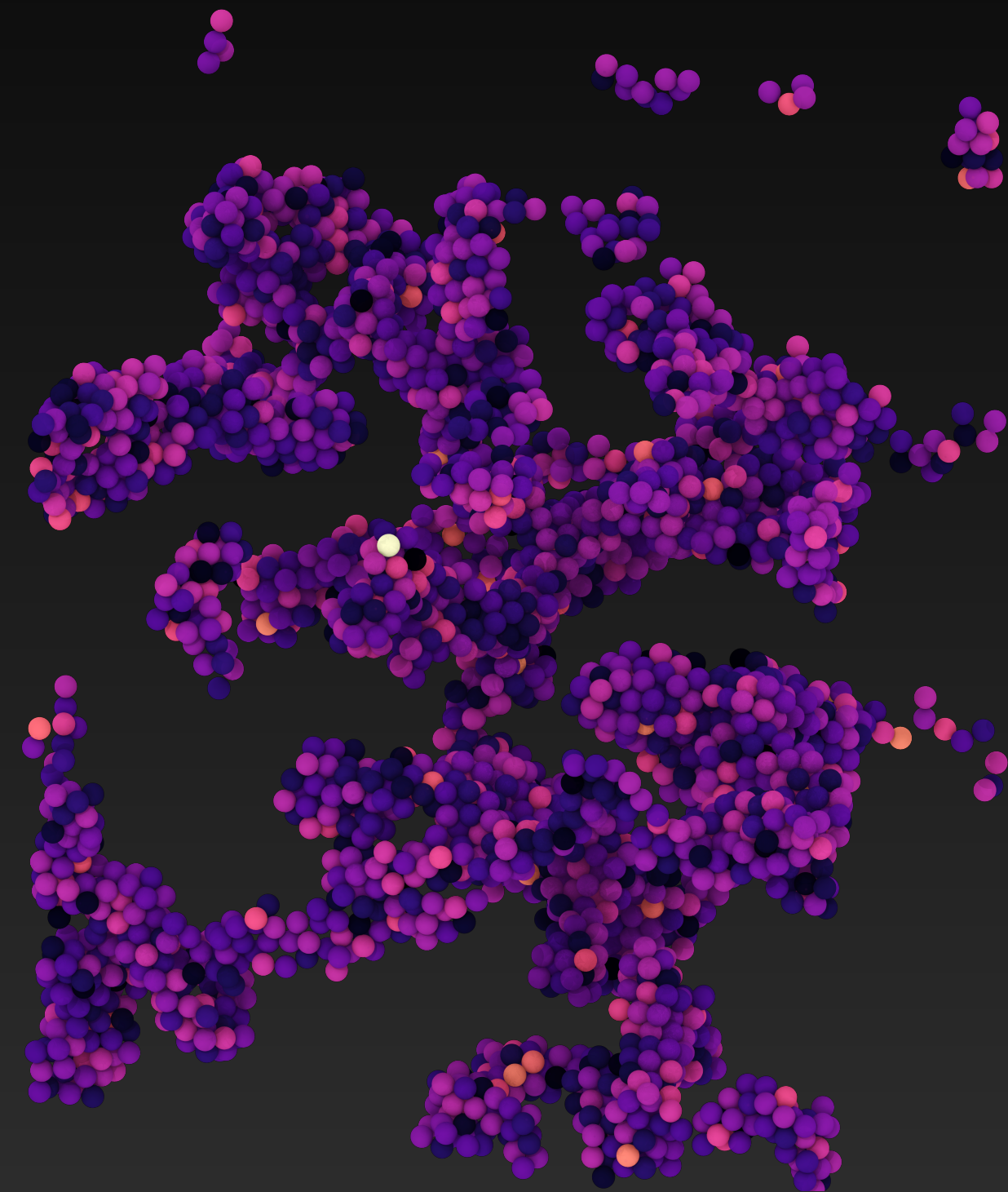
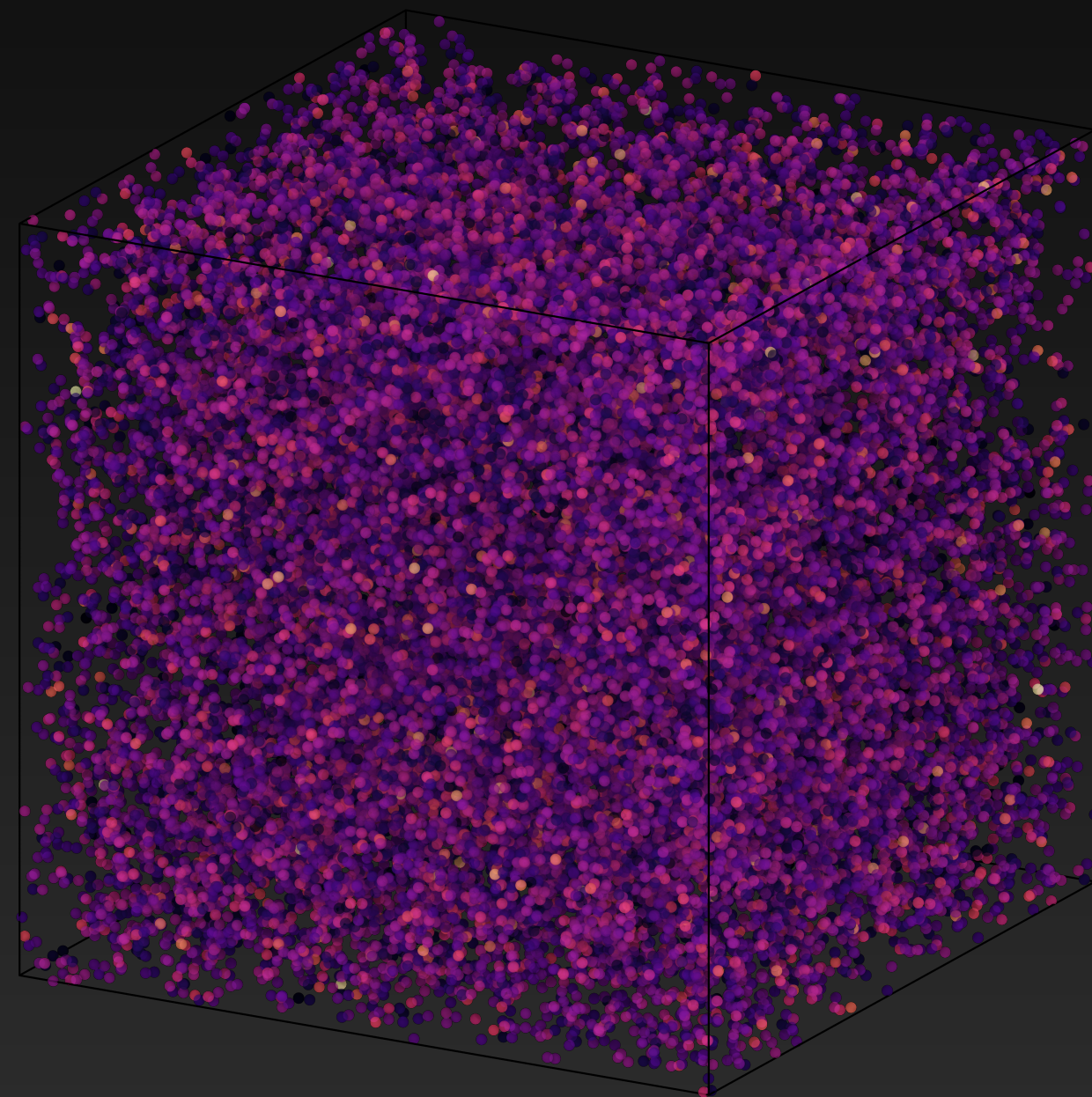
Cugliandolo et al. PRL 119 (26), 268002 2017

Digregorio et al. PRL 121, 098003 2018

Caporusso et al. PRL 125 (17), 178004 2020

# Planned Activity

Three control parameters Attraction Activity Density



No Attraction, Any value of density and activity

Attraction, low activity, low density:  
Gel Phase

Attraction, higher activity, low density:  
Arrested Phase Separation

Attraction, high activity, high density:  
Percolating Network

Typical Runs: 256 cores for 48 hours  
30 exploratory runs  
368640 core hours

**THANK YOU**



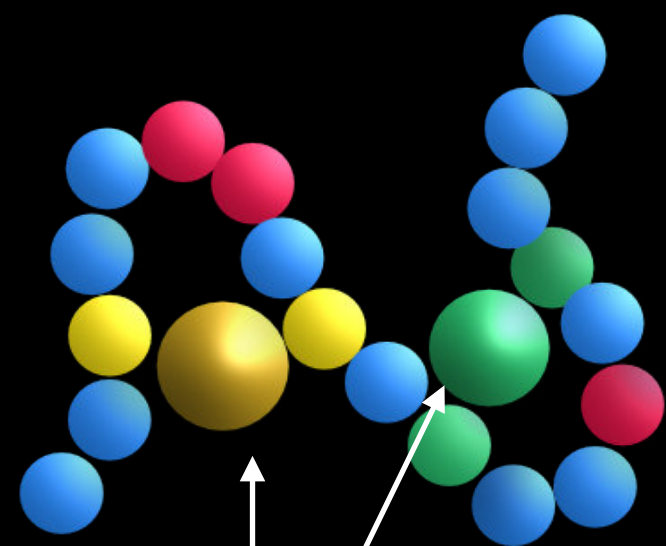
# POLYMER MODEL FOR GENE ORGANIZATION AND TRANSCRIPTION



● Neutral

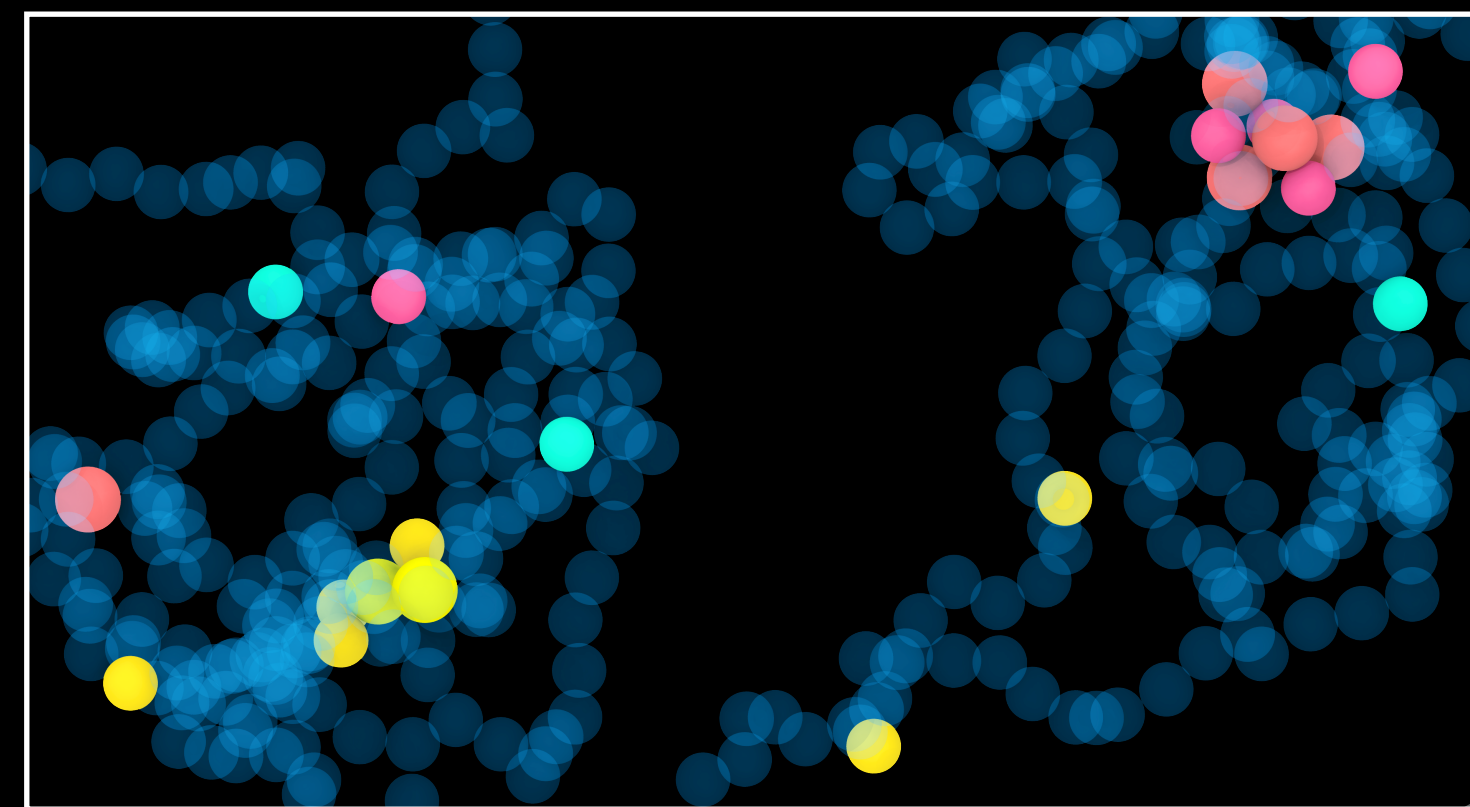
● ● ● Binding sites (TU)

3Mbp → 1000 beads (each 30nm in diameter → 3Kbp)

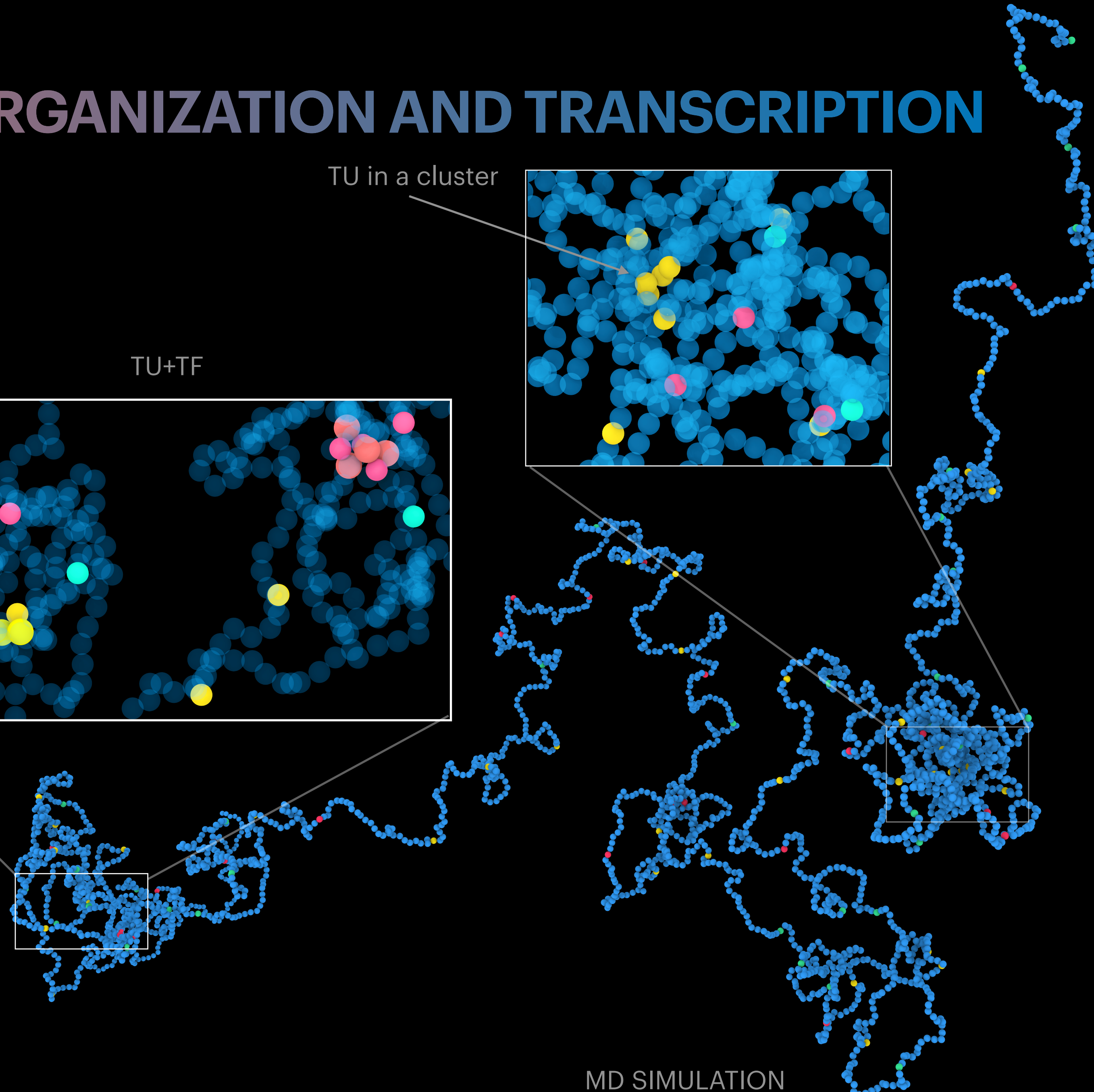
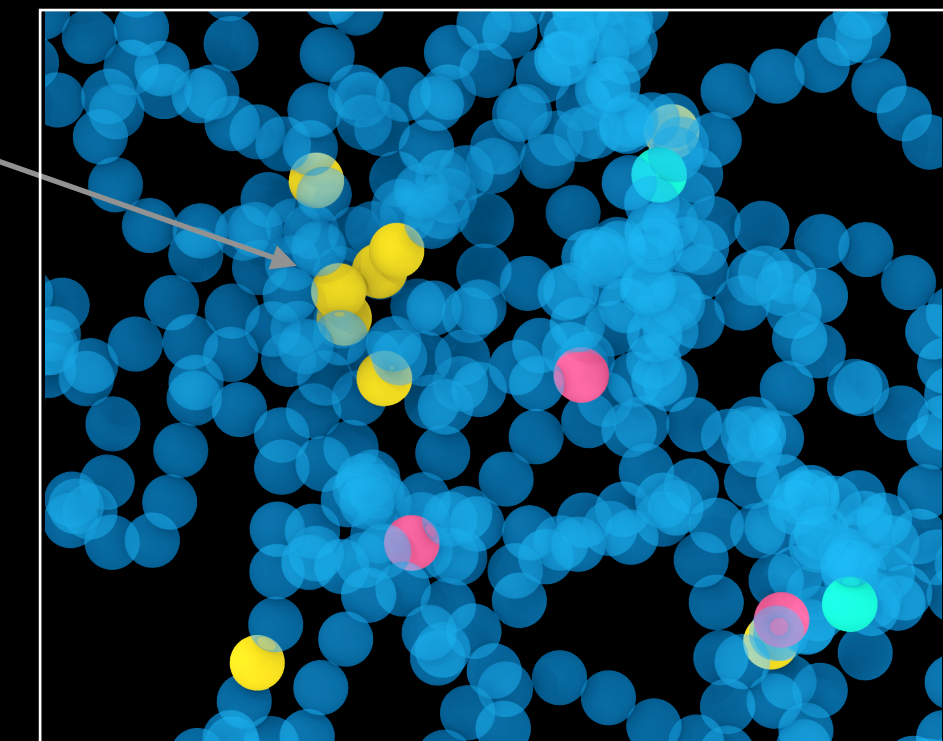


Binders (TF) Weakly with all others

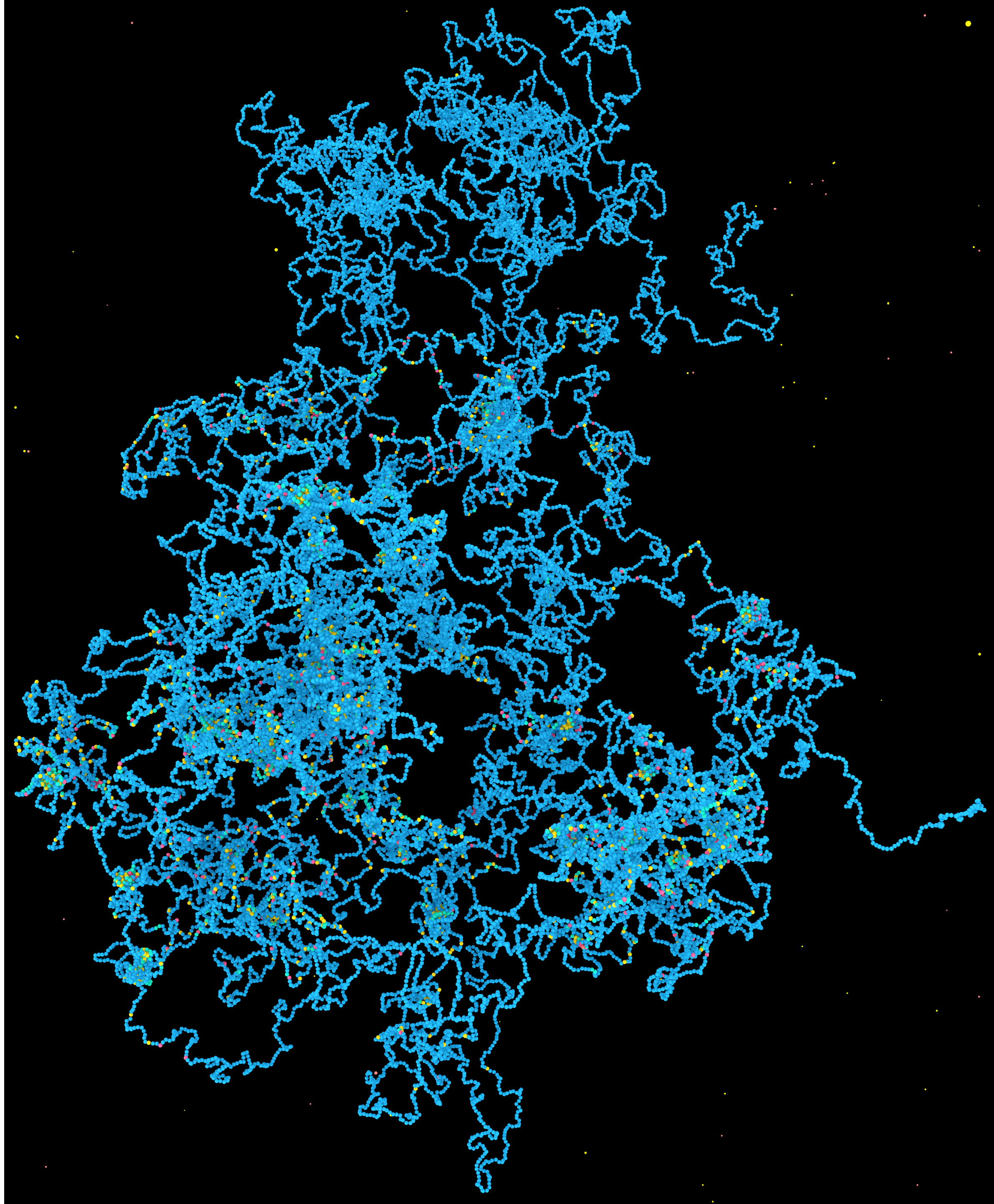
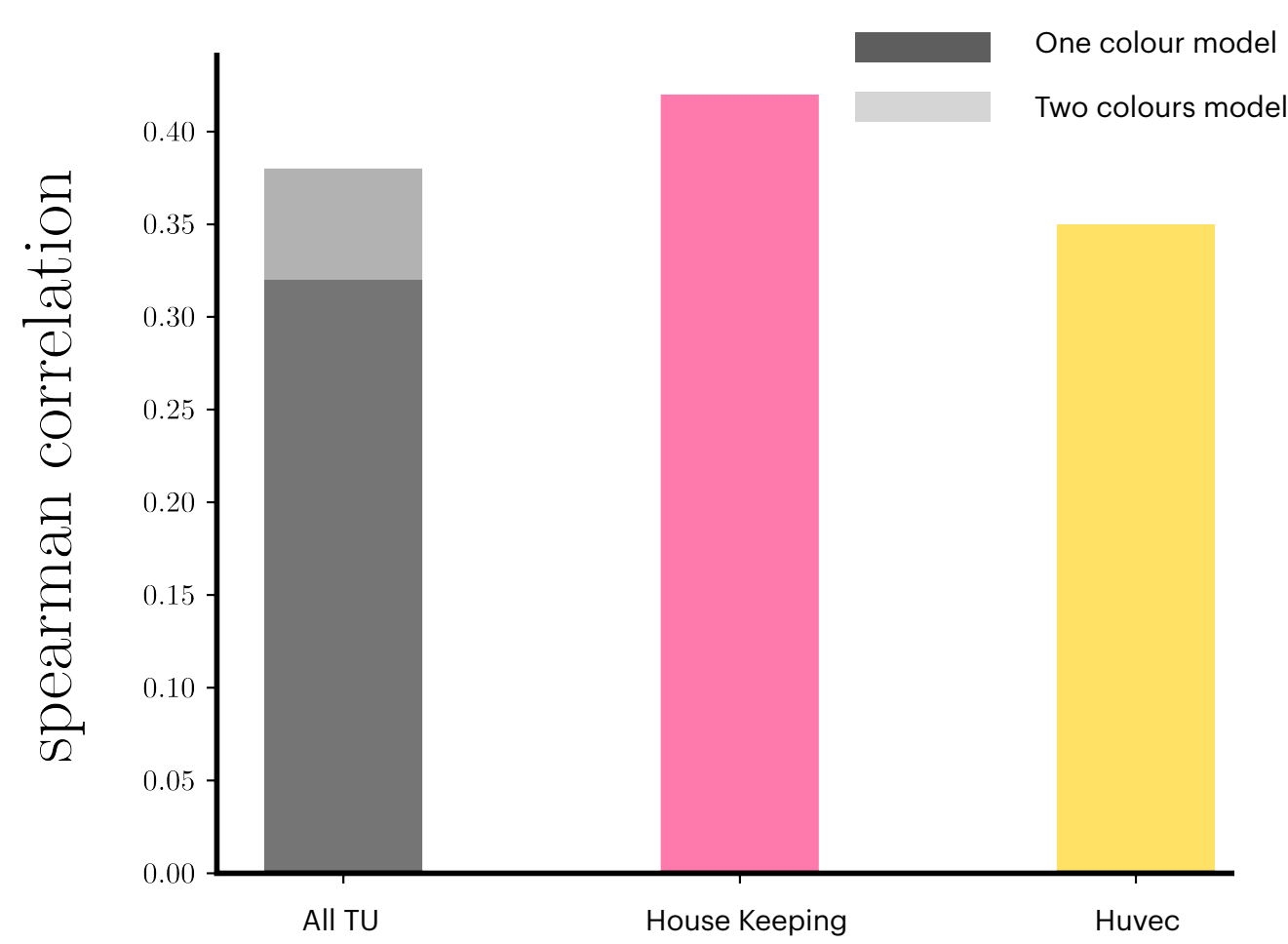
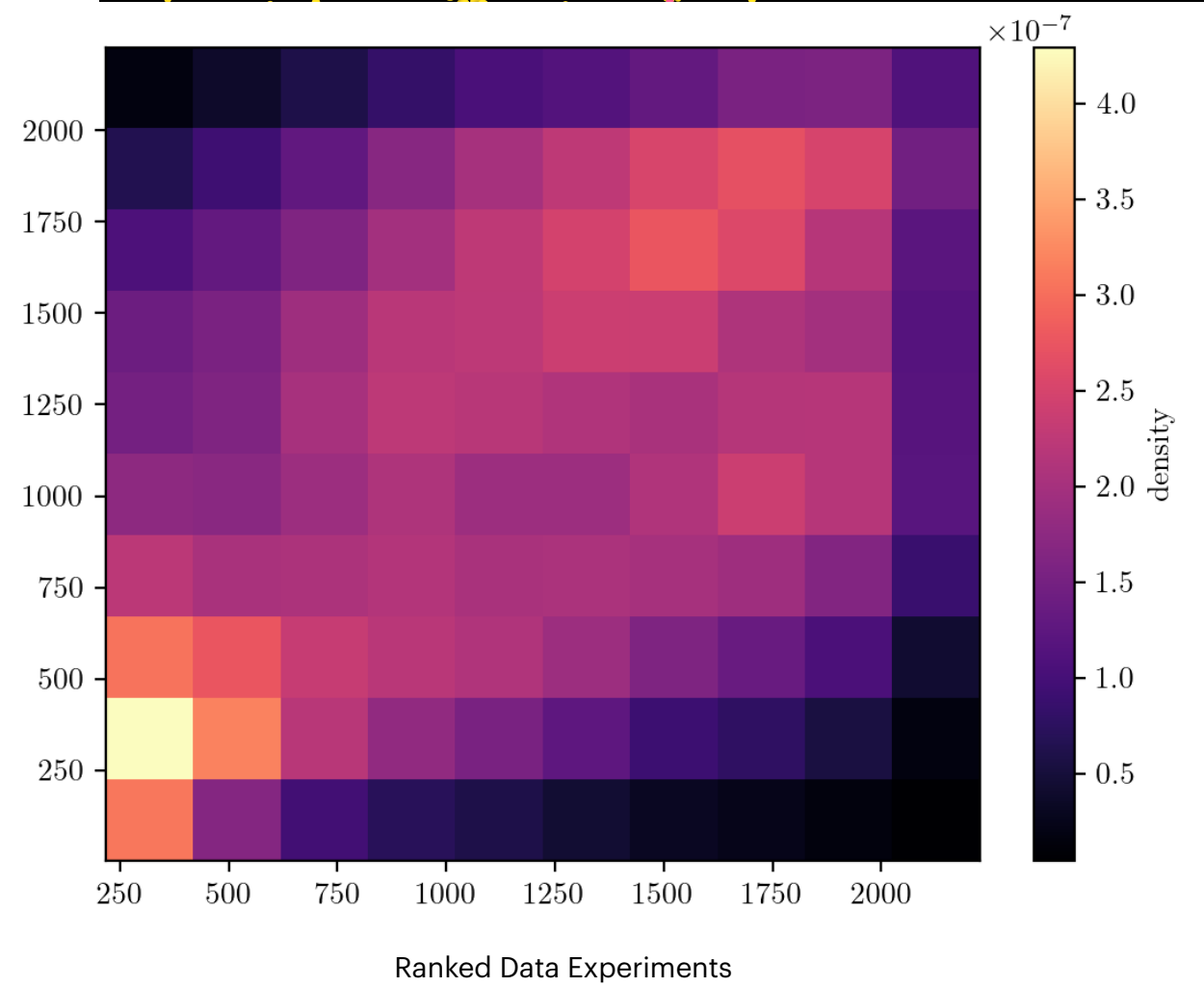
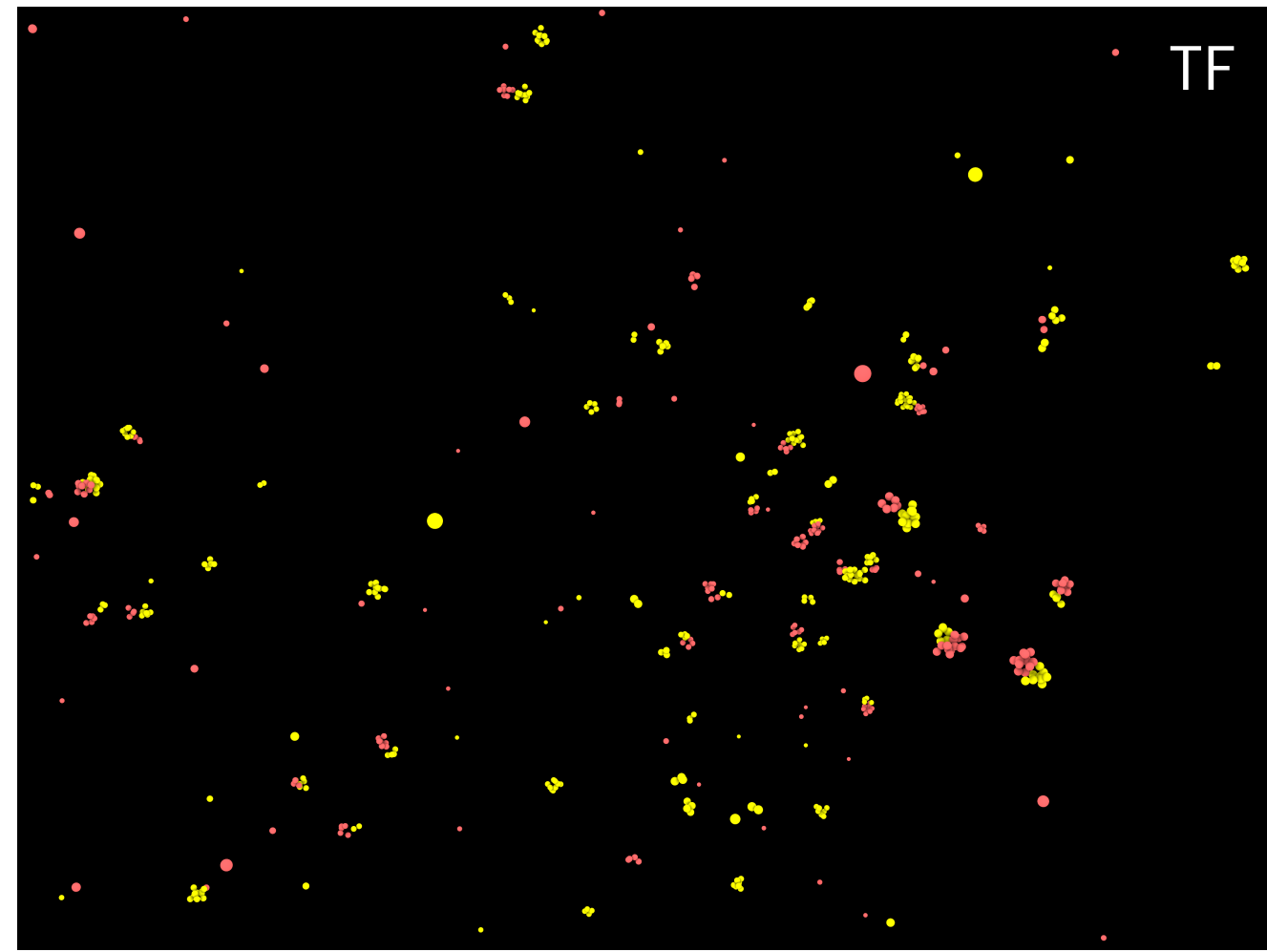
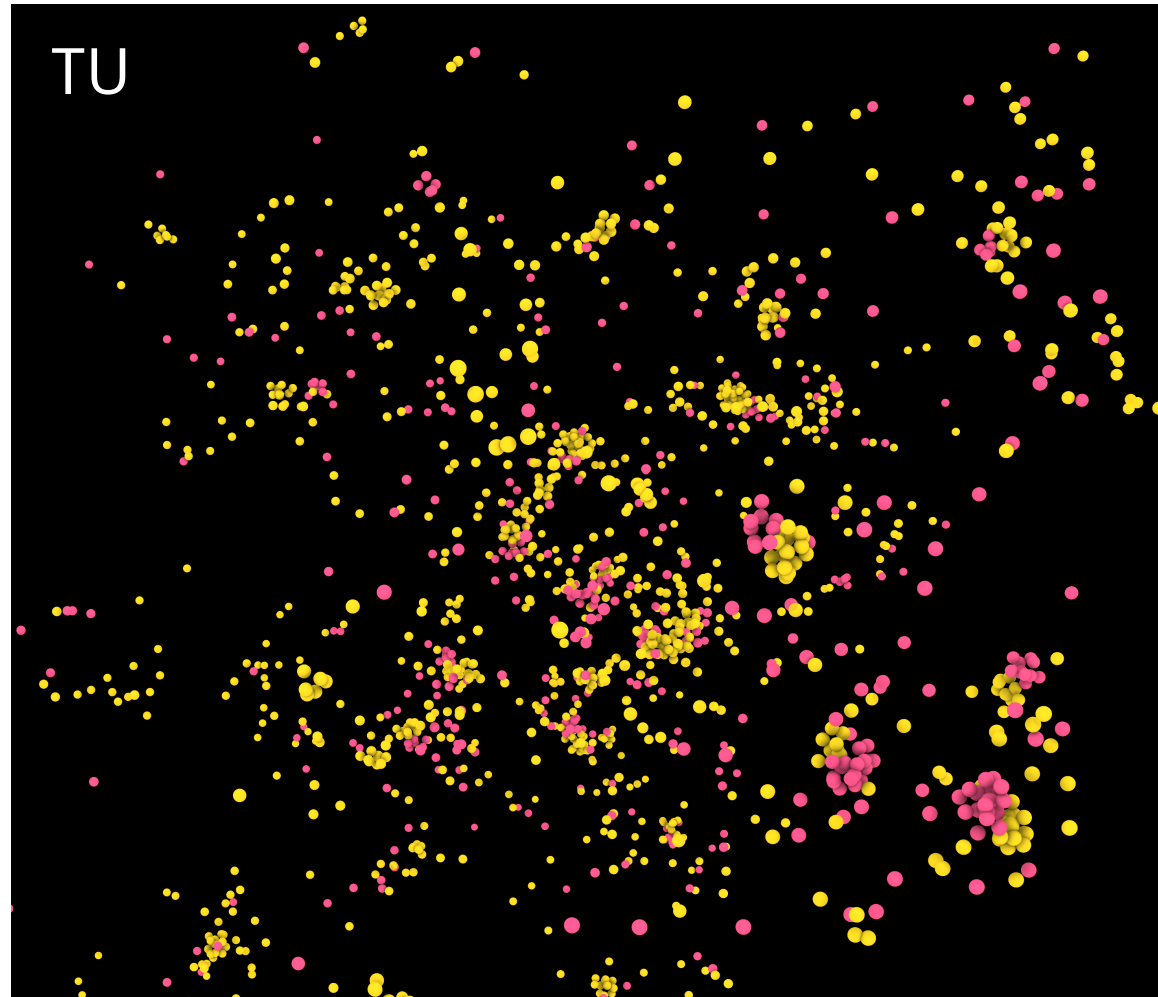
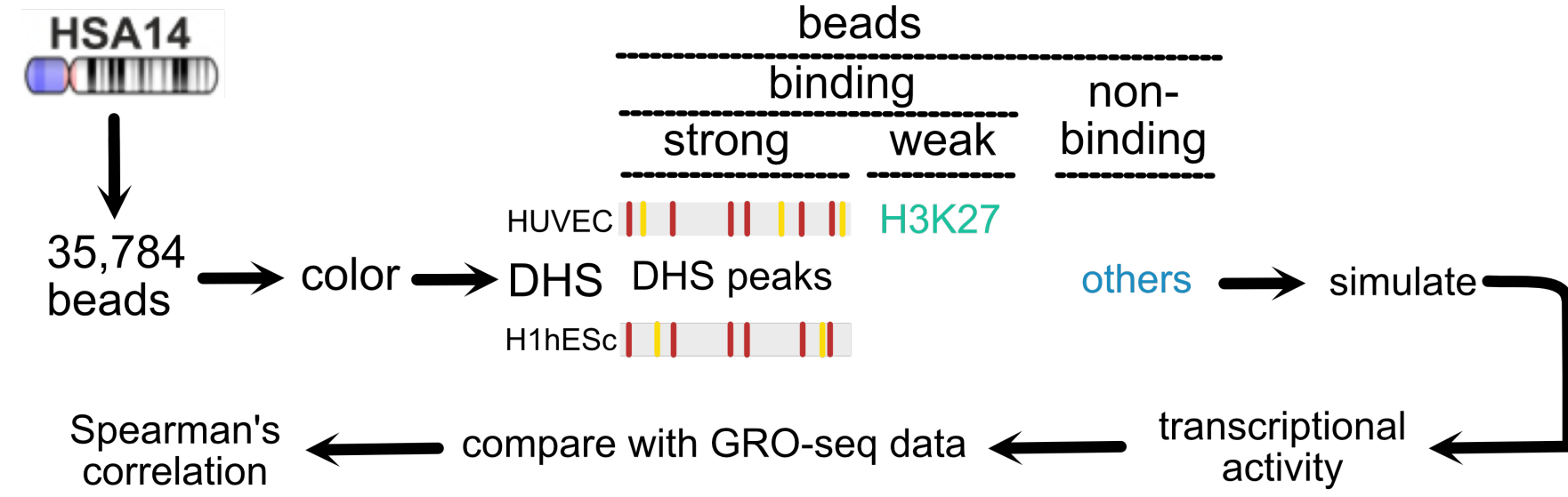
TFs SWITCH BETWEEN ACTIVE AND INACTIVE STATES  
Rapid exchange of factors and polymerases between bound and free states seen in photobleaching experiments



TU in a cluster



# CHR 14



# Active Stress Tensor

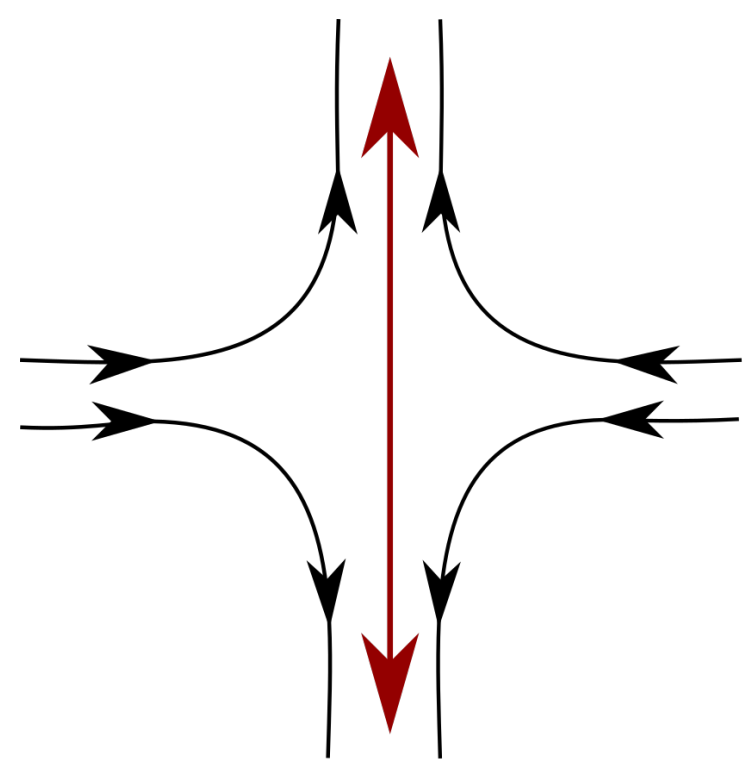
$$(\partial_t + \mathbf{v} \cdot \nabla) \mathbf{v} = \nabla \cdot [\sigma^{pass} + \sigma^{act}]$$

Dissipative/reactive term

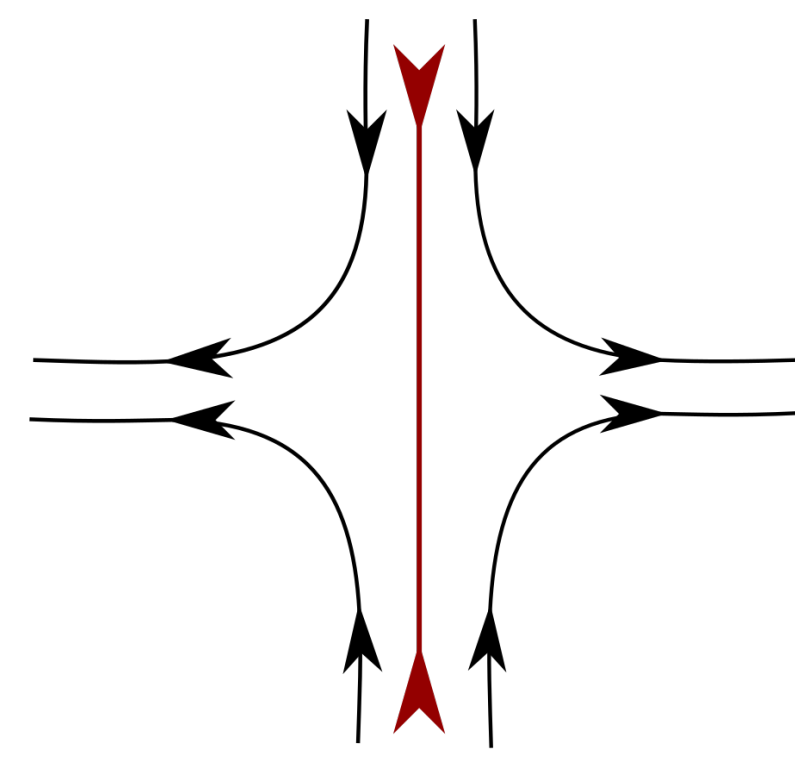
Non-Equilibrium terms

$$\sigma_{\alpha\beta}^{act} = -\zeta\phi Q_{\alpha\beta} - \bar{\zeta}\epsilon_{\alpha\mu\nu}\partial_\mu(\phi Q_{\nu\beta})$$

## Force Dipoles

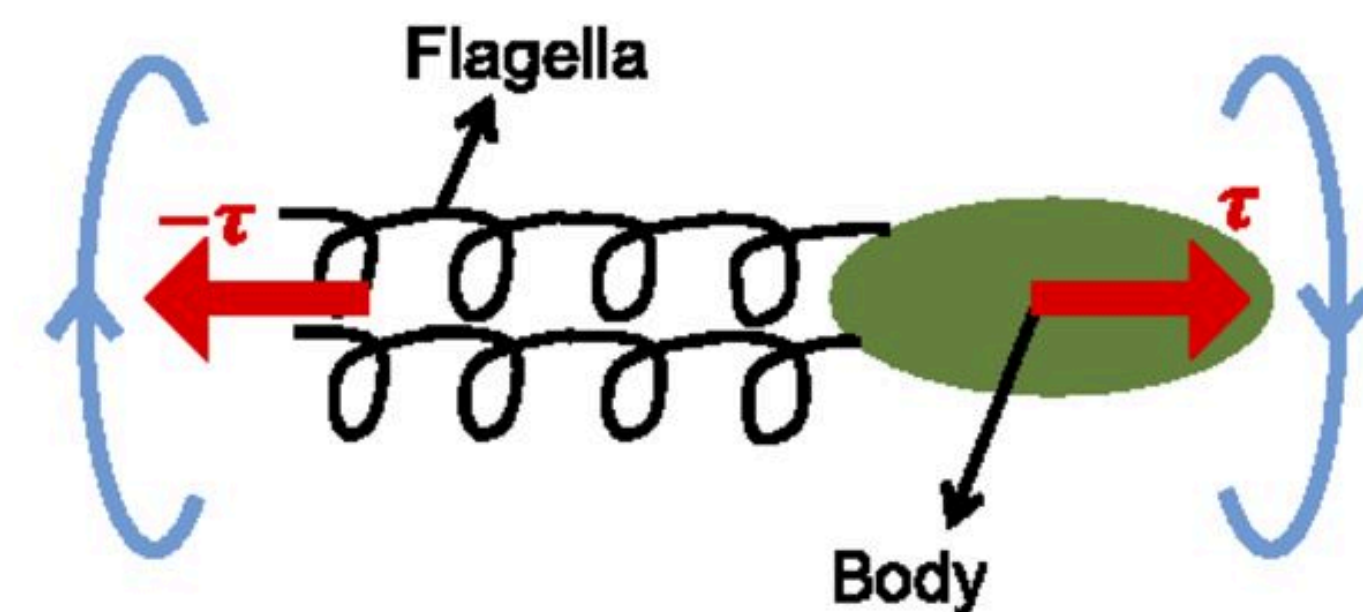


Extensile



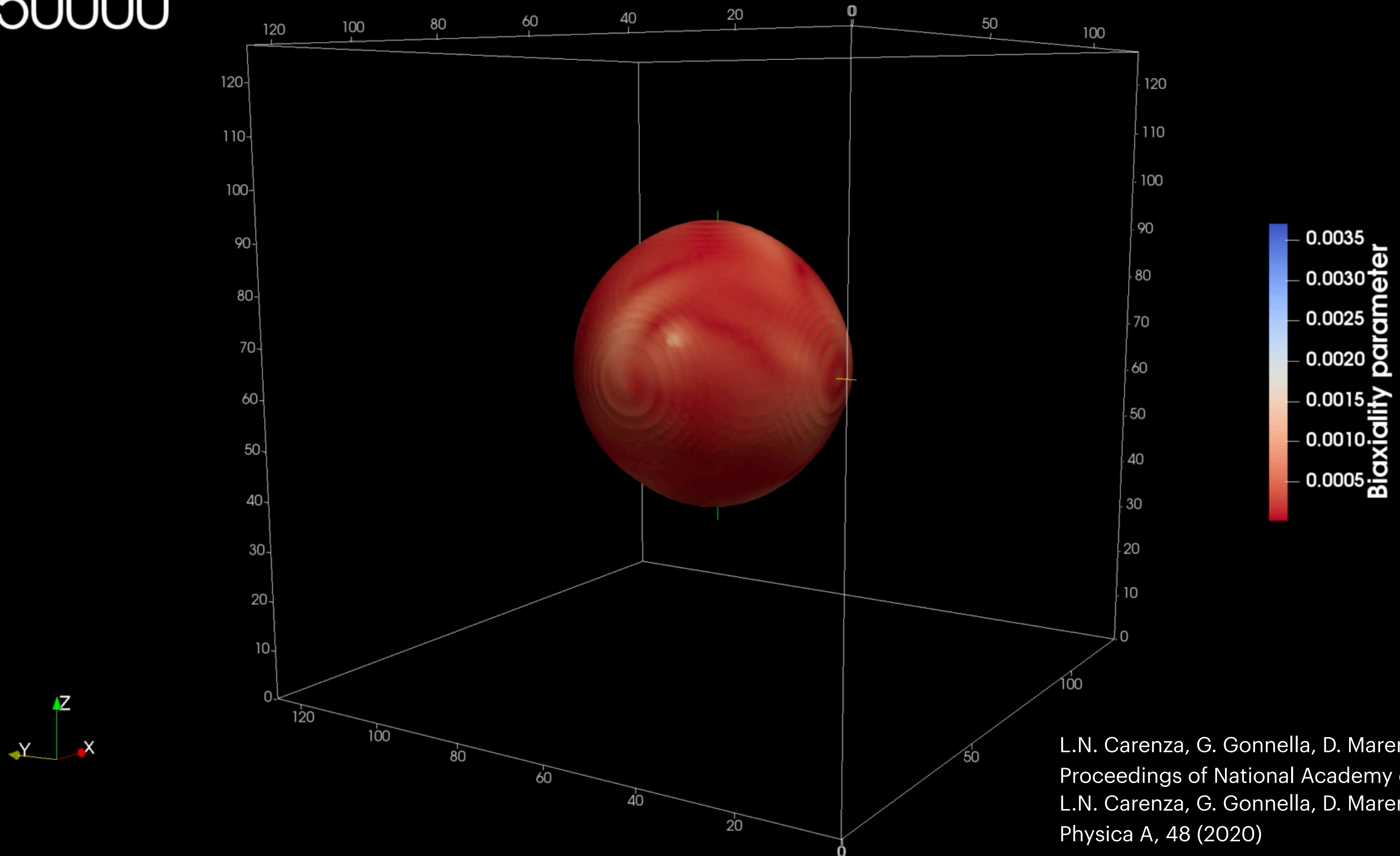
Contractile

## Torque Dipoles



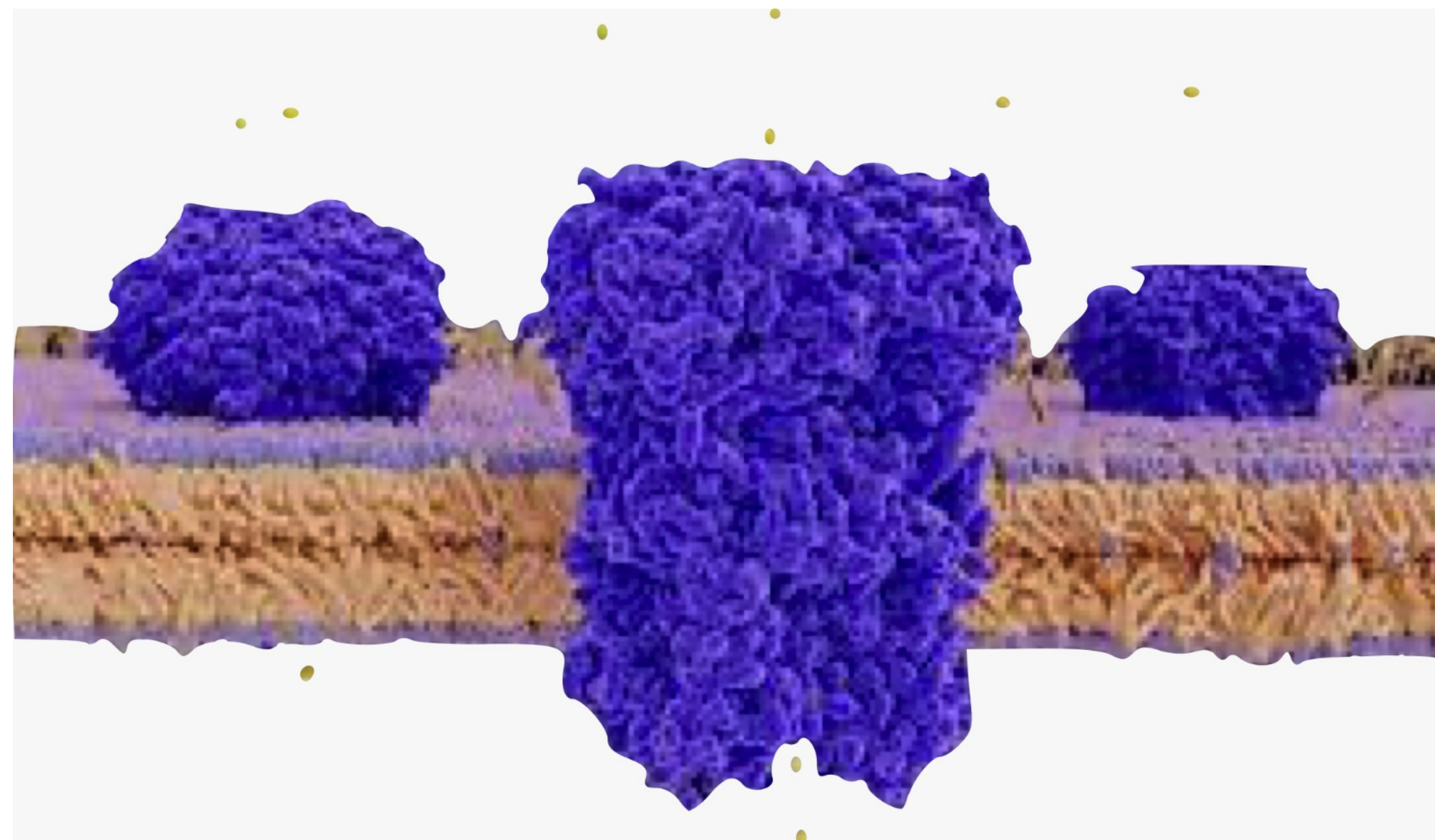
# Active Cholesteric Droplet: A novel Motility mode

150000

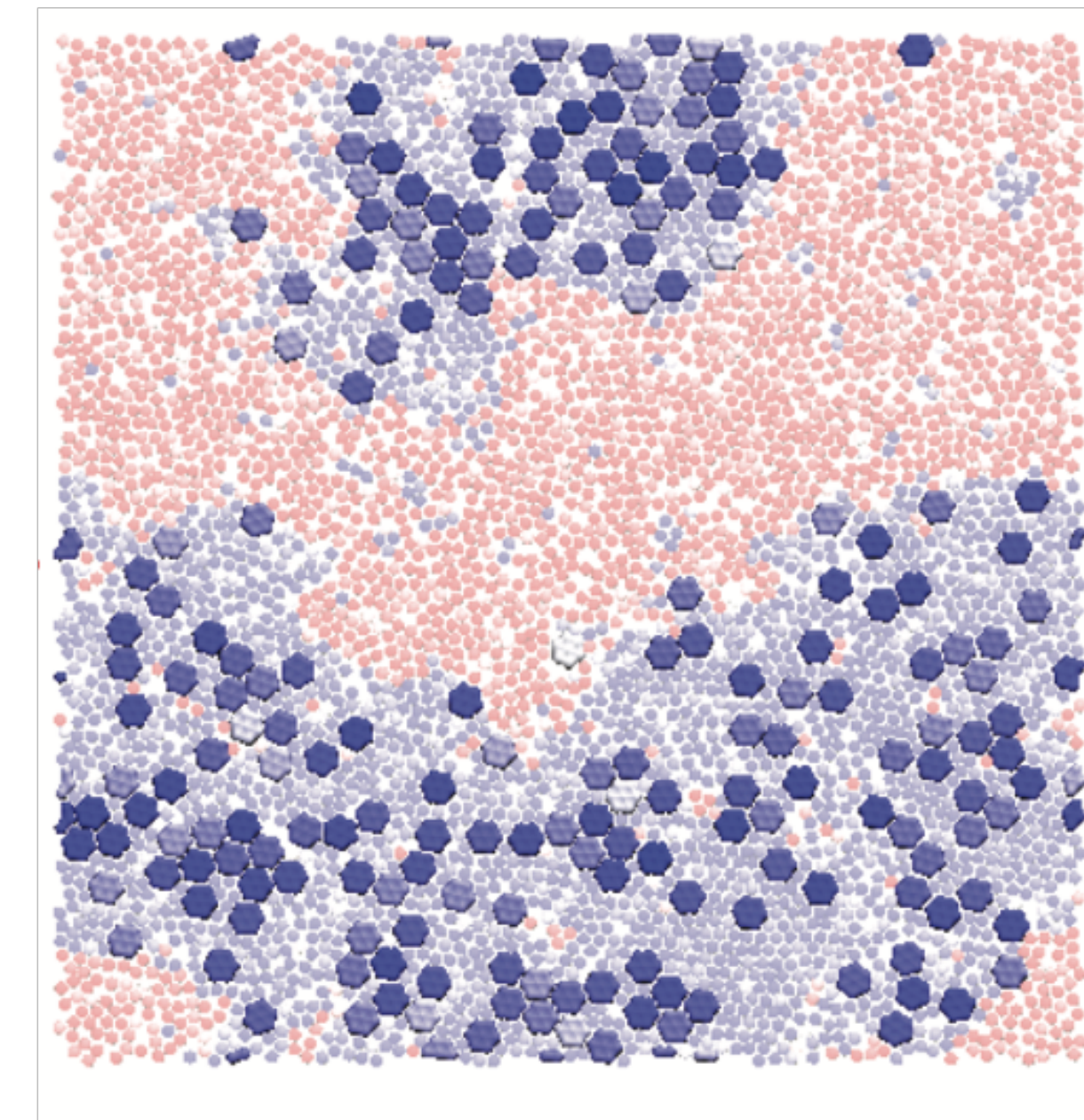
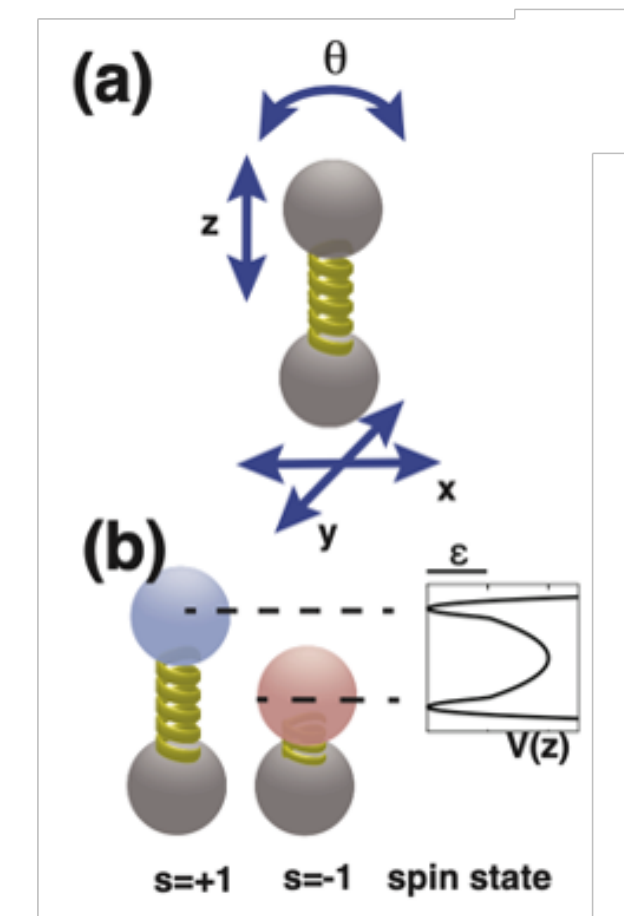


L.N. Carenza, G. Gonnella, D. Marenduzzo, and G. Negro\*  
Proceedings of National Academy of Science, 116 (2019)  
L.N. Carenza, G. Gonnella, D. Marenduzzo, and G. Negro\*  
Physica A, 48 (2020)

# Model for lipid bilayers and ion channels



Coarse-grained



Typical Runs: 128 cores for 48 hours

45 exploratory runs

276480 core hours