Lecture 4

Statistical Hadronization

and its Origin

basic observation in all high energy multihadron production

$\boxed{\text{statistical production pattern}}$

Fermi, Landau, Pomeranchuk, Hagedorn

- species abundances $\sim$ ideal resonance gas at T_H
- universal $T_H \simeq 150 - 200$ MeV for all (large) $\sqrt{s}$
- thermal transverse momentum spectra with same T_H

caveats: baryon density, strangeness, heavy flavors, flow

begin by recalling what is "statistical" and what are the experimental features to be described

1. Statistical Hadron Production

what is "statistical"?

- **equal *a priori* probabilities for all states in accord with a given overall average energy $\Rightarrow$ temperature T;**
- **partition function of ideal resonance gas**

$$\ln Z(T) = V \sum_i \frac{d_i}{(2\pi)^3} \phi(m_i, T)$$

Boltzmann factor $\phi(m_i, T) = 4\pi m_i^2 T K_2(m_i/T)$

- **relative abundances** $$\frac{N_i}{N_j} = \frac{d_i \phi(m_i, T)}{d_j \phi(m_j, T)}$$
- **transverse momenta** $$\frac{dN}{dp_T^2} \sim \exp -\frac{1}{T}\sqrt{m_i^2 + p_T^2}.$$

Hadronization features in elementary & nuclear collisions

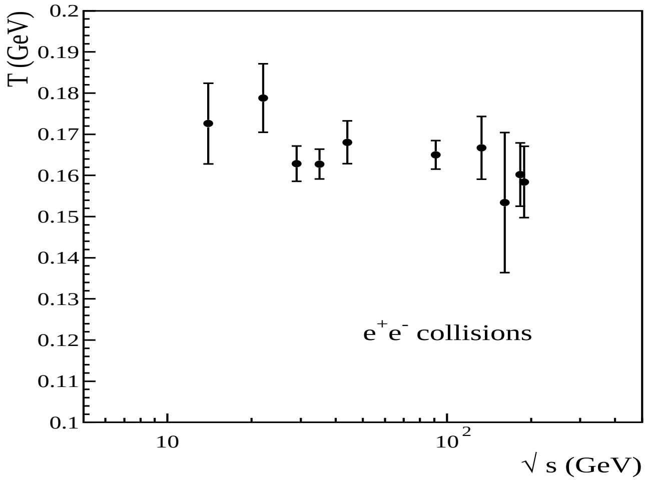
- **jet structure**
- **multiplicity per jet ln $\sqrt{s}$**
  (caveat multi-jets, evolution)
- **universal hadronization temperature**
  – from species abundances
  (caveat: strangeness, heavy flavors)
  – from transverse mass spectra
  (caveat: flow in nuclear collisions)
- **initial state quantum number structure**
  – baryon number, heavy flavor

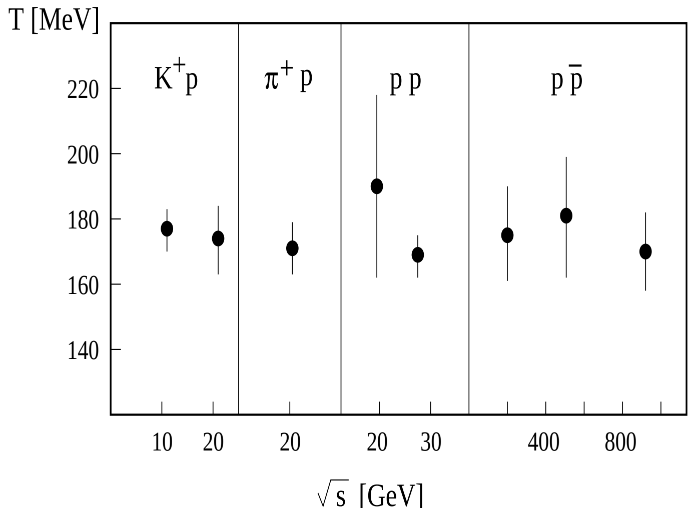
$\exists$ a universal scenario (including the caveats)?

summarize experimental situation on abundances & p_T

Species abundances in elementary collisions

[Becattini et al. 1996 - 2008]





Conclude:

$$\boxed{T_H = 170 \pm (10-20)\ \mathrm{MeV};\ \gamma_s \simeq 0.5 - 0.7}$$

independent of $\sqrt{s}$, incident production configuration

Transverse momentum spectra in elementary collisions

requires resonance decay code; model dependence?

[Becattini & Passaleva 2001]

pp at $\sqrt{s} = 27.4$ GeV:

average $T = 163$ MeV

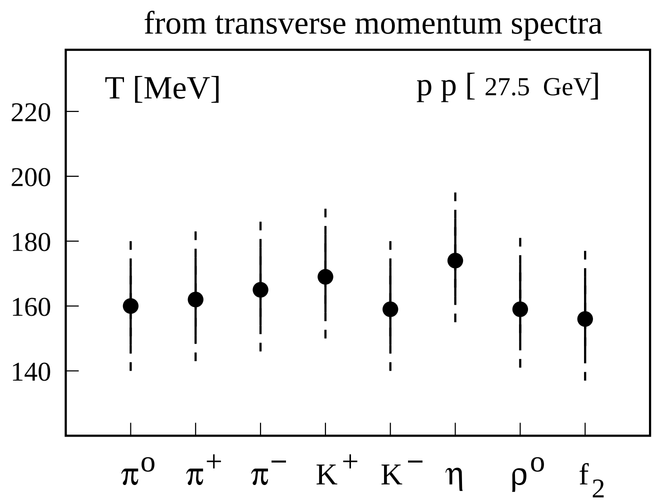
similar analyses for

K^+p at $\sqrt{s} = 21.7$ GeV:

average $T = 165$ MeV

π^+p at $\sqrt{s} = 21.7$ GeV:

average $T = 160$ MeV



Conclude:

$$\boxed{T_H = 163 \pm ? \ \text{MeV}}$$

independent of species

Heavy ion collisions

- temperature T, baryochem. pot. μ_B; $\mu_B \Downarrow$ for $\sqrt{s} \Uparrow$
- elementary high energy collisions low baryon content
- compare to species abundances for RHIC, peak SPS

SPS (Pb-Pb), $\sqrt{s} = 17$ GeV

$$T_H = 157.8 \pm 2.5 \text{ MeV}, \ \mu_B = 248.9 \pm 9.0 \text{ MeV}$$

RHIC (Au-Au), $\sqrt{s} = 130$, $y = 0$ GeV

$$T_H = 163.8 \pm 4.1 \text{ MeV}, \ \mu_B = 36.3 \pm 10.2 \text{ MeV}$$

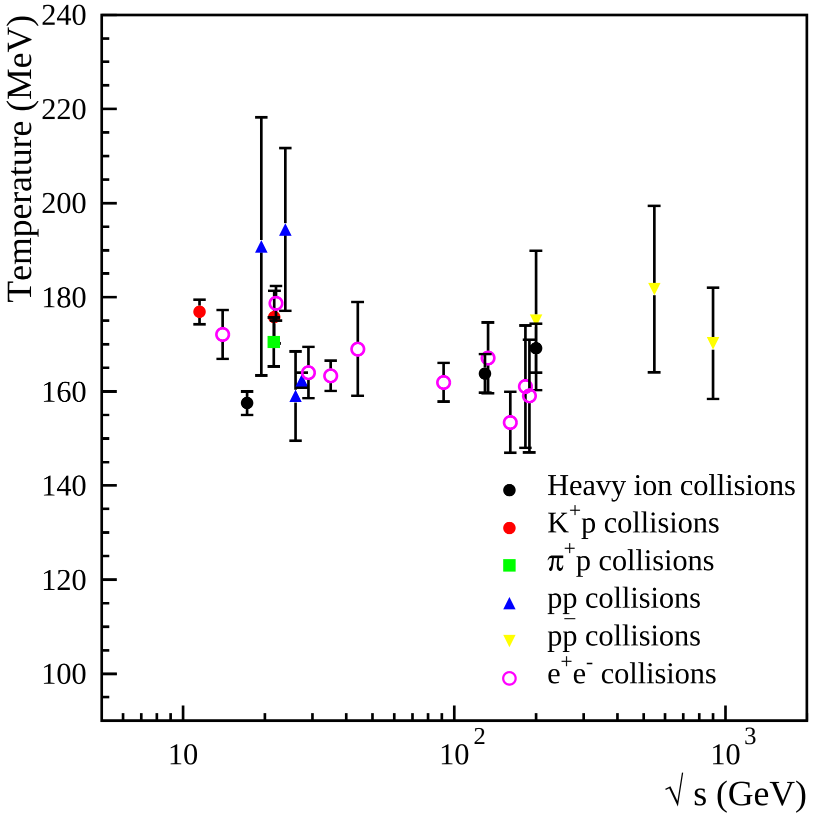
RHIC (Au-Au), $\sqrt{s} = 200$ GeV

$$T_H = 169.2 \pm 5.2 \text{ MeV}, \ \mu_B = 29.5 \pm 11.2 \text{ MeV}$$

in general $\gamma_s \simeq 0.8 - 1.1$

[Andronic, Braun-Munzinger & Stachel 2006, Becattini & Manninen 2008]

Data Summary



Conclude:

The hadron abundances in all high energy collisions (e^+e^- annihilation, hadron-hadron & nuclear collisions) are specified by an ideal resonance gas of a universal temperature

$$T_H \simeq 170 \pm 20 \text{ MeV}.$$

The transverse momentum spectra in elementary collisions are in accord with such "thermal" behavior; $\exists$ broadening (flow) in nuclear collisions.

Strangeness production in elementary collisions is systematically reduced; strangeness suppression is weakened or removed in nuclear collisions.

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WHY?

Why should high energy collisions produce statistical behavior?

Multiple parton interactions $\longrightarrow$ kinetic thermalization?

nucleus-nucleus maybe; e^+e^-, hadron-hadron not

$\exists$ a "non-kinetic" mechanism producing statistical features?

$\exists$ a common origin of statistical hadron production in all high energy collisions?

Passing color charge disturbs vacuum, vacuum recovers by hadron production according to maximum entropy

What does that mean?

Color Confinement $\Rightarrow$ *Event Horizon* $\Rightarrow$ *Hawking-Unruh Radiation*

[Castorina, Kharzeev, HS 2007]

2. Event Horizons & Hawking-Unruh Radiation

• Unruh radiation [Unruh 1976]

event horizon arises for systems in uniform acceleration

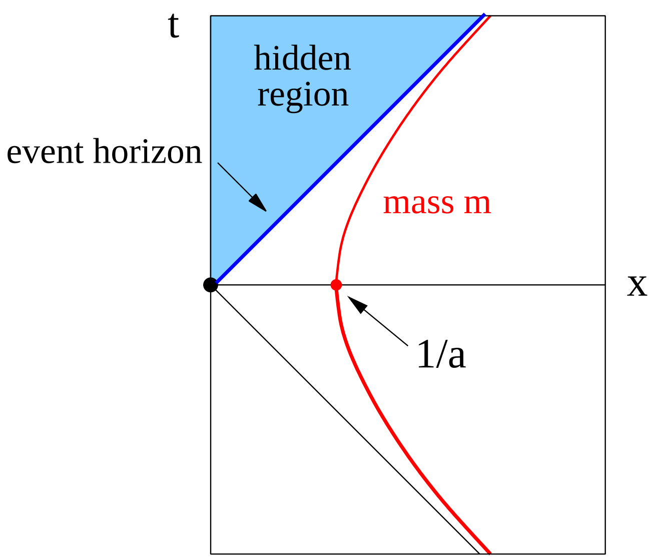
mass m in uniform acceleration a

$$\frac{d}{dt}\frac{mv}{\sqrt{1-v^2}} = F$$

$$v = dx/dt,\ F = ma,\ c = 1$$

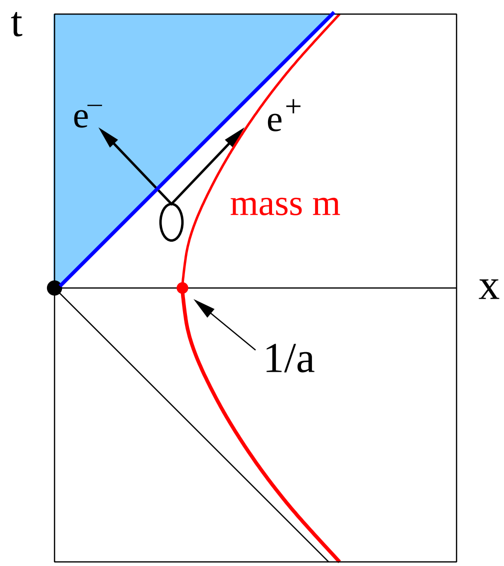
solution: hyperbolic motion

$$x = \frac{1}{a}\cosh a\tau \quad t = \frac{1}{a}\sinh a\tau$$



$\exists$ event horizon: m cannot reach hidden region
observer in hidden region cannot communicate with m

m passes through vacuum, can use part of acceleration energy to excite vacuum fluctuations on-shell



e^+ absorbed in detector on m
e^- disappears beyond event horizon

equivalent:
e^- tunnels through event horizon

"quantum entanglement"
$\sim$ Einstein-Podolsky-Rosen effect

observer on m as well as observer in hidden region have incomplete information: $\Rightarrow$ each sees <u>thermal radiation</u>

observer on m:
physical vacuum $\sim$ thermal medium of temperature T_U

observer in hidden region:
passage of $m \longrightarrow$ thermal radiation of temperature T_U

Unruh temperature $\quad T_U = \frac{\hbar a}{2\pi c}$

relativistic (c) quantum ($\hbar$)effect

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Applications

- Black Holes

event horizon $R = 2GM$ (Schwarzschild radius)

$$F = ma = G\frac{Mm}{R^2} \quad \Rightarrow \quad a = \frac{GM}{R^2} = \frac{1}{4GM}$$

$$\Rightarrow \quad T_U = \frac{a}{2\pi} = \frac{1}{8\pi GM} = T_{BH}$$

obtain temperature T_{BH} of Hawking radiation

[Hawking 1975]

- Schwinger Mechanism

in strong electric field $\mathcal{E}$, vacuum becomes unstable against pair production

$F = e\mathcal{E} = (m/2)a$ leads to production of pair of charges of mass m

$$T_U = \frac{a}{2\pi} = \frac{e\mathcal{E}}{\pi m}$$

$$P(m, \mathcal{E}) \sim \exp\{-m/T_U\} = \exp\{-\pi m^2/e\mathcal{E}\}$$

obtain Schwinger production probability $P(m, \mathcal{E})$

[Schwinger 1951]

In general: [T. D. Lee 1986, Parikh & Wilczek 2000]

event horizon $\sim$ information transfer forbidden

$\Rightarrow$ quantum tunnelling $\sim$ thermal radiation

Caveat

Color event horizon in QCD reasonable, but speculative

Gravitation: space-time metric is given by

$$ds^2 = g_0\, dt^2 - g_0^{-1} dr^2 - d^2\Omega$$

for flat space, $g_0 = 1$; solution of Einstein equations gives

$$g_0 = \left(1 - \frac{2GM}{r}\right)$$

determines Schwarzschild radius $R = 2GM$ as <u>event horizon</u>

non-linear electrodynamics $\longrightarrow$ space compactification,
photon trapping

QCD? $\longrightarrow$ million dollar question Nr. 7,
Clay Mathematics Institute

3. Pair Production and String Breaking

Basic process:

two-jet e^+e^- annihilation, cms energy $\sqrt{s}$:

$$e^+e^- \longrightarrow \gamma* \longrightarrow q\bar{q} \longrightarrow \text{ hadrons}$$

$q\bar{q}$ separate subject to constant confining force $F = \sigma$

initial quark velocity $\quad v_0 = \dfrac{p}{\sqrt{p^2 + m^2}} \ , \quad p \simeq \sqrt{s}/2$

Solve $ma = \sigma$ (hyperbolic motion): [Hosoya 1979, Horibe 1979]

$$\tilde{x} = [1 - \sqrt{1 - v_0\tilde{t} + \tilde{t}^2}] \ , \ \tilde{x} = x/x_0 \ , \ \tilde{t} = t/x_0$$

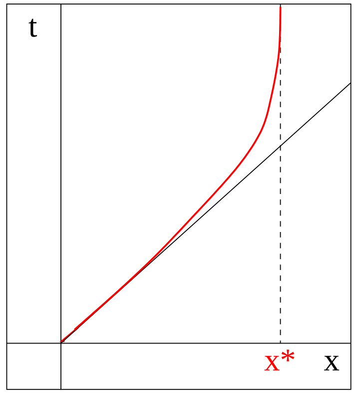
with $\quad x_0 = \dfrac{m}{\sigma}\dfrac{1}{\sqrt{1 - v_0^2}} = \dfrac{m}{\sigma}\,\gamma = \dfrac{1}{a}\,\gamma$

classical turning point $v(t^*) = 0$ at

$$x^* = x(t^*) = \frac{m}{\sigma}\gamma\left[1 - \sqrt{1-(v_0/2)^2}\right] \simeq \frac{\sqrt{s}}{2\sigma}$$

$q\bar{q}$ can separate arbitrarily far
if $\sqrt{s}$ is large enough

What's wrong?



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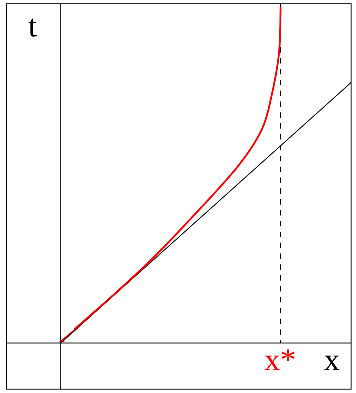
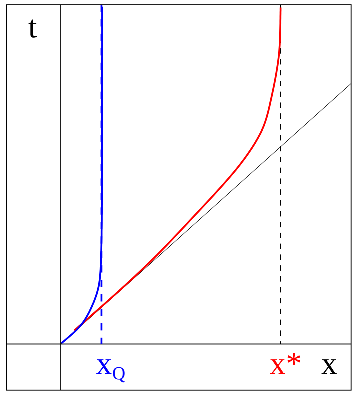


What's wrong? classical event horizon

Strong field $\Rightarrow$ vacuum unstable
against pair production [Schwinger 1951]

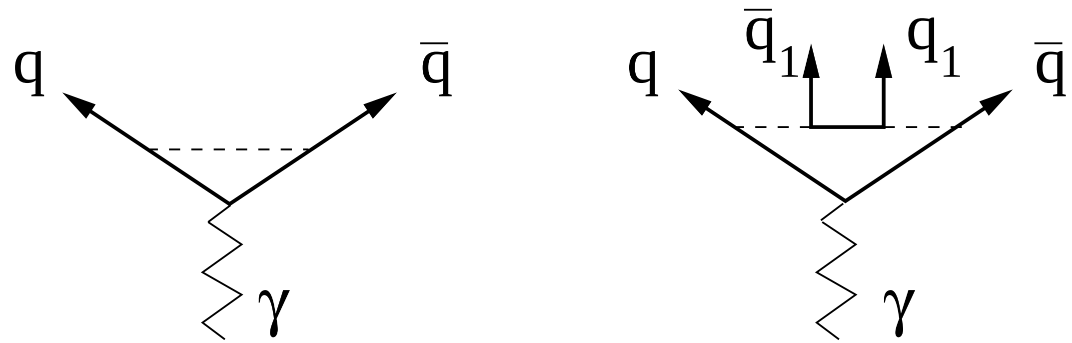
when $\sigma x > \sigma x_Q \equiv 2m$
string connecting $q\bar{q}$ breaks



Result: quantum event horizon

Hadron production in e^+e^- annihilation:

"inside-outside cascade" [Bjorken 1976]



$q\bar{q}$ flux tube has thickness $r_T \simeq \sqrt{\frac{2}{\pi\sigma}}$

$q_1\bar{q}_1$ at rest in cms, but $k_T \simeq \frac{1}{r_T} \simeq \sqrt{\frac{\pi\sigma}{2}}$

$q\bar{q}$ separation at $q_1\bar{q}_1$ production $\sigma x(q\bar{q}) = 2\sqrt{m^2 + k_T^2}$

q_1 screens $\bar{q}$ from q, hence string breaking at

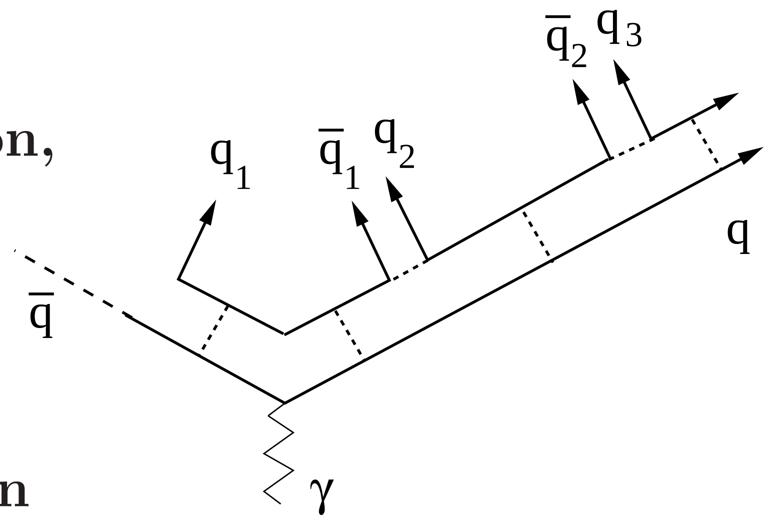
$$x_q \simeq \frac{2}{\sigma}\sqrt{m^2 + (\pi\sigma/2)} \simeq \sqrt{2\pi/\sigma} \simeq 1 \text{ fm}$$

new flux tubes $q\bar{q}_1$ and $\bar{q}q_1$
stretch $q_1\bar{q}_1$
to form new pair $q_2\bar{q}_2$

$$\sigma x(q_1\bar{q}_1) = 2\sqrt{m^2 + k_T^2}$$

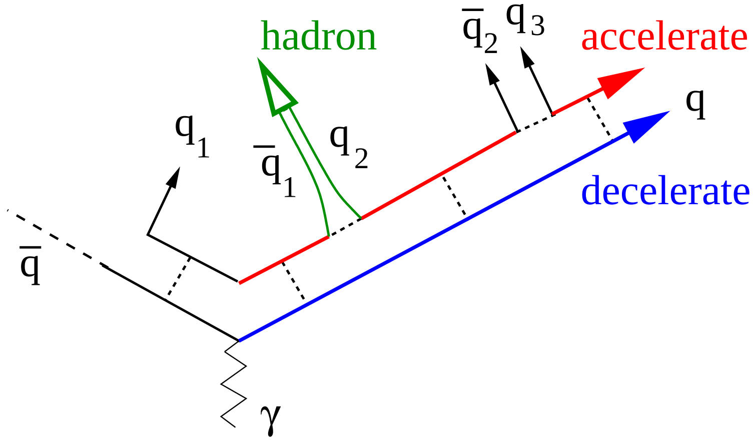
equivalent:
$\bar{q}_1$ reaches $q_1\bar{q}_1$ event horizon,
tunnels to become $\bar{q}_2$

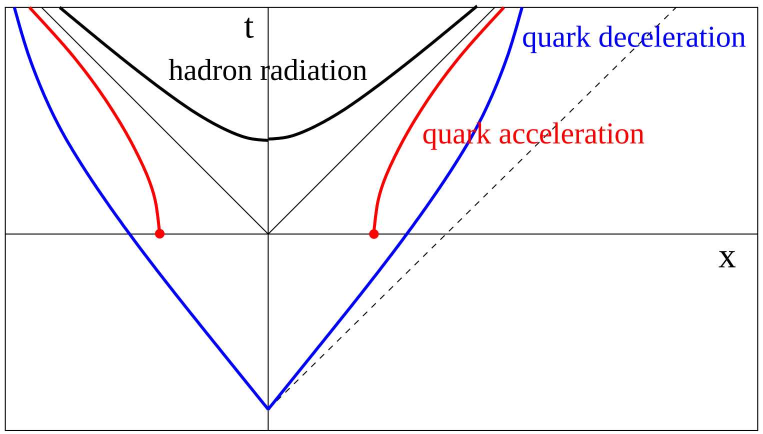
emission of hadron $\bar{q}_1 q_2$
as Hawking-Unruh radiation



self-similar pattern:

screening
string breaking
tunnelling
quark acceleration
/deceleration
Hawking-Unruh radiation





temperature of H-U radiation: what acceleration?

$$(\bar{q}_1 \longrightarrow \bar{q}_2 \longrightarrow \bar{q}_3 \longrightarrow \ldots)$$

$$a = F/m \;\Rightarrow\; a_q = \frac{\sigma}{w_q} = \frac{\sigma}{\sqrt{m_q^2 + k_q^2}}$$

string breaking & thickness determine $k_q \simeq \sqrt{\pi\sigma/2}$

$$\Rightarrow \quad a_q \simeq \frac{\sigma}{\sqrt{m_q^2 + (\sigma/2\pi)}}$$

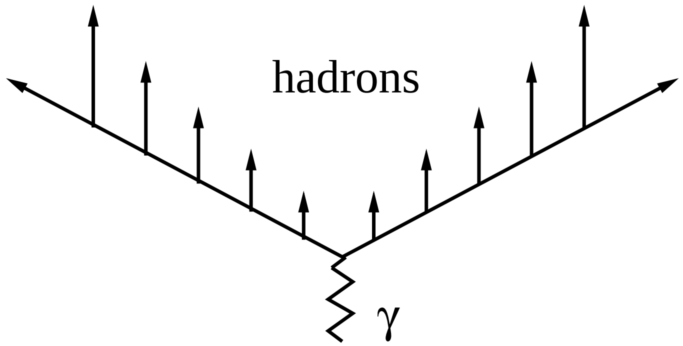
for light quarks, $m_q \ll \sqrt{\sigma} \simeq 420$ **MeV, hence**

$$\boxed{T = \frac{a}{2\pi} \simeq \sqrt{\frac{\sigma}{2\pi}} \simeq 170 \text{ MeV}}$$

temperature of hadronic Hawking-Unruh radiation

hadronization pattern:

hadron multiplicity?



thickness of classical "overstretched" string:

$$R_T^2 = \frac{2}{\pi\sigma} \sum_{k=0}^{K} \frac{1}{2k+1} \simeq \frac{2}{\pi\sigma} \ln 2K \simeq \frac{2}{\pi\sigma} \ln \sqrt{s}$$

quantum breaking at $x_q \sim r_T$, hence hadron multiplicity

$$\boxed{\nu(s) \simeq \frac{R_T^2}{r_T^2} \simeq \ln \sqrt{s}}$$

NB: parton evolution (minijets), multiple jets lead to stronger increase

4. Strangeness Production

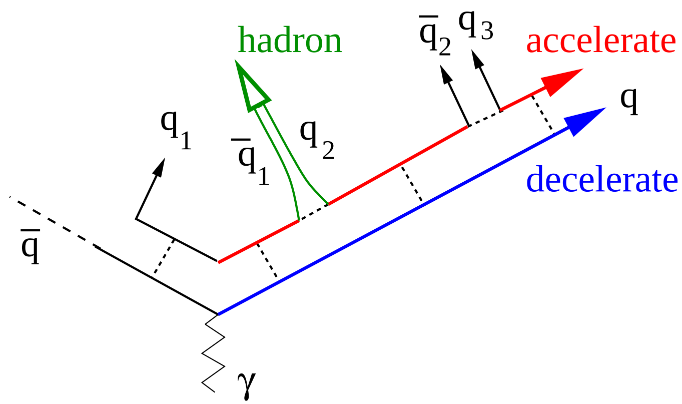
[Becattini, Castorina, Manninen, HS 2008]

Unruh temperature $\sim$ 1 / mass of secondary

we had for finite quark mass m_q

$$a_q \simeq \frac{\sigma}{\sqrt{m_q^2 + (\sigma/2\pi)}} \quad \Rightarrow \quad T_U = \frac{a_q}{2\pi}$$

produced meson consists of quarks $\bar{q}_1$ and q_2



meson containing two different quark masses
will have average acceleration

$$\bar{a}_{12} = \frac{w_1 a_1 + w_2 a_2}{w_1 + w_2} = \frac{2\sigma}{w_1 + w_2}; \quad w_i \simeq \sqrt{m_i^2 + (\sigma/2\pi)}$$

leading to

$$T(12) \simeq \frac{a_{12}}{2\pi}$$

easily extended to baryons; result: five temperatures

$T(00) = T(000);\ T(s0);\ T(ss) = T(sss);\ T(00s);\ T(0ss)$

fully determined by σ and m_s

for $\sigma \simeq 0.17\ \mathrm{GeV}^2$ and $m_s \simeq 0.08$ GeV

obtain temperatures:

T	[GeV]
$T(00)$	0.164
$T(0s)$	0.156
$T(ss)$	0.148
$T(000)$	0.164
$T(00s)$	0.158
$T(0ss)$	0.153
$T(sss)$	0.148

does this work?

analyse all high energy e^+e^- data

hadron production data in e^+e^- annhilation exist at

$$\sqrt{s} = 14,\ 22,\ 29,\ 35,\ 43, 91, 180\ \mathrm{GeV}$$

(PETRA, PEP, LEP)

example:

long-lived hadrons produced at LEP for $\sqrt{s} = 91.25$ GeV

fit data in terms
of σ and m_s

result:

$\sigma = 0.169 \pm 0.002$ GeV^2

$m_s = 0.083$ GeV

χ^2/dof $= 23/12$

standard values:

$\sigma = 0.195 \pm 0.030$ GeV^2

$m_s = 0.095 \pm 0.025$ GeV

$e^+e^-\ \sqrt{s} = 91.2\ GeV$				
species	measured			fit
π^+	8.50	$\pm$	0.10	8.30
π^0	9.61	$\pm$	0.29	9.67
K^+	1.127	$\pm$	0.026	1.089
K^0	1.038	$\pm$	0.001	1.049
η	1.059	$\pm$	0.996	0.910
ω	1.024	$\pm$	0.059	0.971
p	0.519	$\pm$	0.018	0.557
η'	0.166	$\pm$	0.047	0.096
ϕ	0.0977	$\pm$	0.0058	0.1060
Λ	0.1943	$\pm$	0.0038	0.1891
Σ^+	0.0535	$\pm$	0.0052	0.0437
Σ^0	0.0389	$\pm$	0.0041	0.0444
Σ^-	0.0410	$\pm$	0.0037	0.0400
Ξ^-	0.01319	$\pm$	0.0005	0.01269
Ω	0.00062	$\pm$	0.0001	0.00077

illustration:
ϕ production in H-U vs. standard statistical model

ϕ **production density in standard statistical model**

$$\langle n \rangle_\phi = 3 \frac{T m^2}{2\pi^2} \mathrm{K}_2(m/T)\ \gamma_S^2$$

with $T \simeq 165$ **MeV,** $\gamma_S \simeq 0.65$**:** $\langle n \rangle_\phi \simeq 1.85\ \gamma_S^2 \simeq 0.078$

NB: $\gamma_S^2 \simeq 0.42$ **reduces equilibrium rate by more than 2**

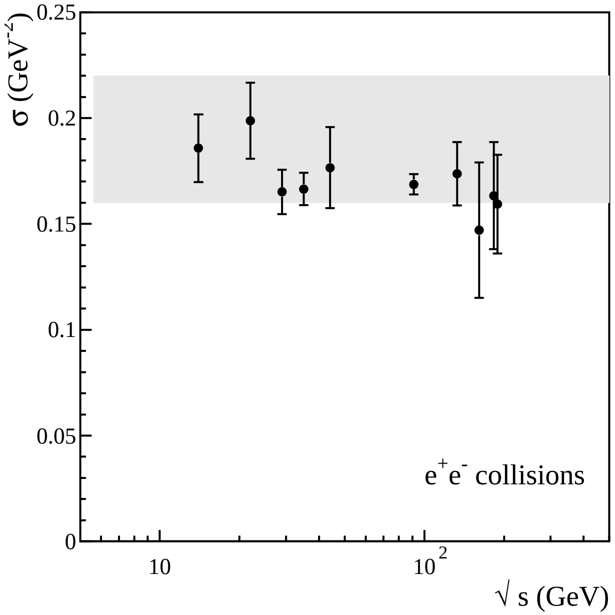
ϕ **production density in H-U statistical model**

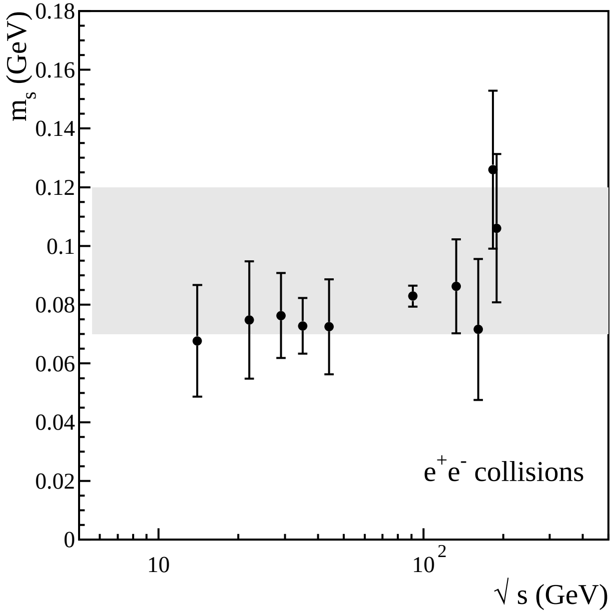
$$\langle n \rangle_\phi = 3 \frac{T(ss) m^2}{2\pi^2} \mathrm{K}_2(m/T(ss))$$

with $T(ss) \simeq 148$ **(*vs.* 164) MeV:** $\langle n \rangle_\phi \simeq 0.077$

[NB: actual production rates $\sim$ **heavy flavor decay]**

results from all data





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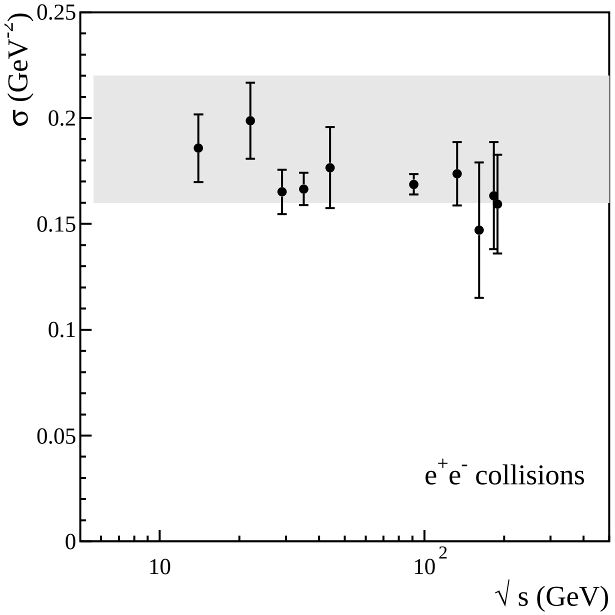
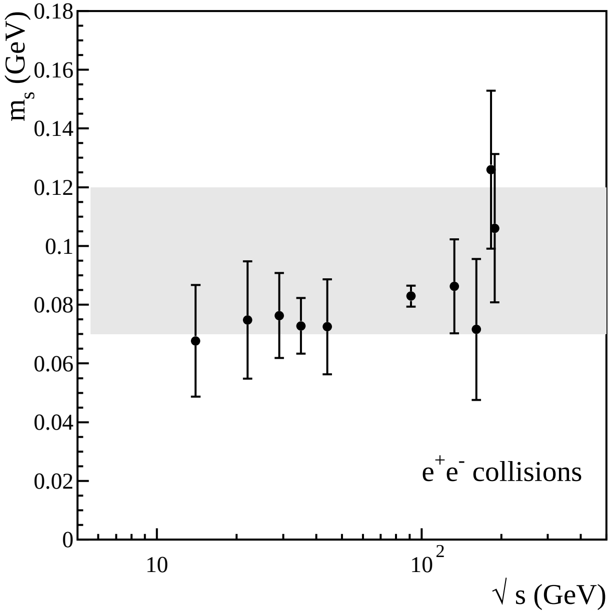


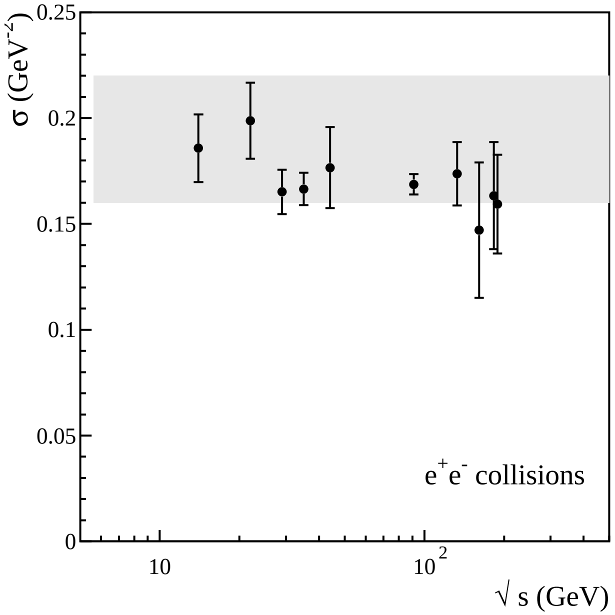


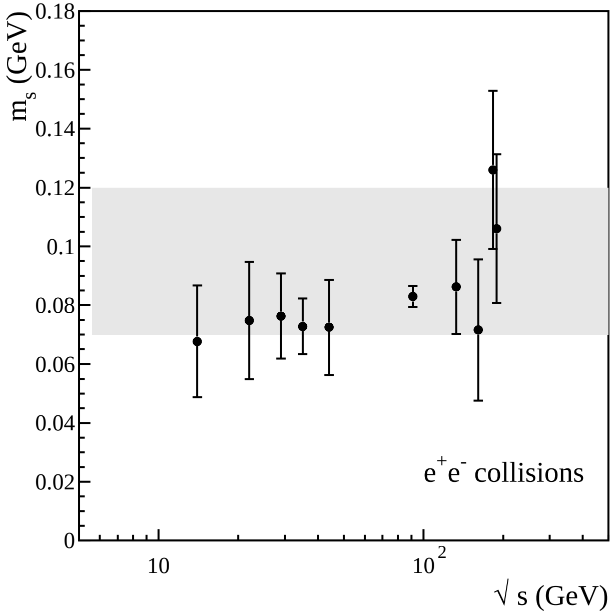
Conclude

thermal hadron production in e^+e^- annihilation, includ'g strangeness suppression, is reproduced parameter-free as

Hawking-Unruh radiation of QCD

results from all data





<u>Conclude</u>

thermal hadron production in e^+e^- annihilation, includ'g strangeness suppression, is reproduced parameter-free as

Hawking-Unruh radiation of QCD

$\Rightarrow$ $pp/p\bar{p}$ (straight-forward); heavy ions (interesting)

Heavy Ions

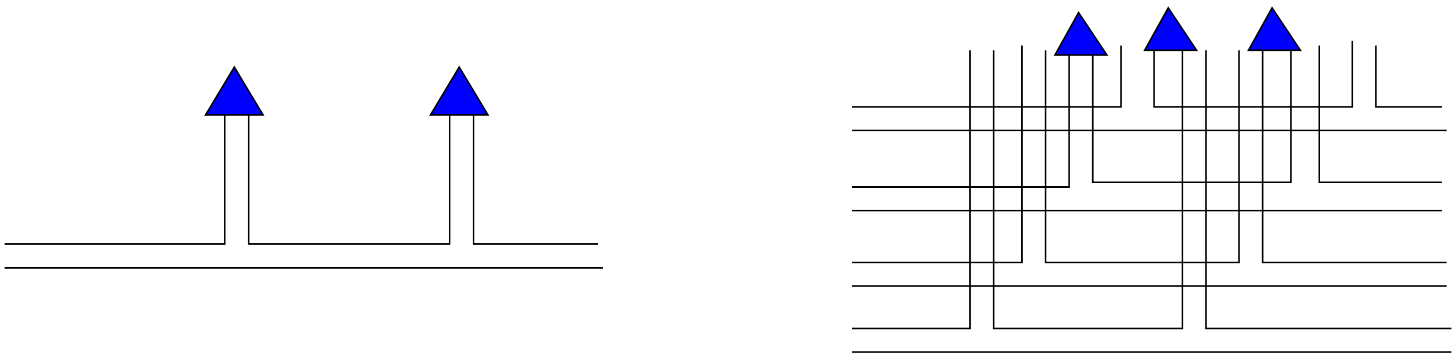
- elementary collisions

  sequential $q\bar{q}$ pair production $\Rightarrow$ independent hadron emission

- nuclear collisions

  superposition of $q\bar{q}$ pair production, interference

  exogamous pairing, not hadronic scattering



elementary nuclear

result: increase in strange hadron temperatures

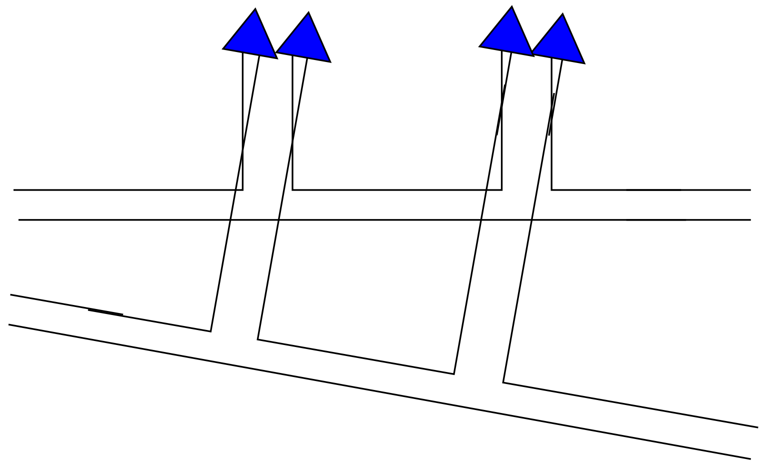
$$T(0s) \;\rightarrow\; [T(00)+T(0s)]/2 \equiv T_r(0s) > T(0s)$$

$$T(ss) \;\rightarrow\; [T(0s)+T(0s)]/2 \equiv T_r(ss) > T(ss)$$

T_r			T
$T_r(00)$	$T(00)$	0.164	0.164
$T_r(0s)$	$[T(00)+T(0,s)]/2$	0.160	0.156
$T_r(ss)$	$T(0s)$	0.156	0.148
$T_r(00s)$	$[2\,T(00)+T(0s)]/3$	0.161	0.158
$T_r(0ss)$	$[T(00)+2\,T(0s)]/3$	0.159	0.153
$T_r(sss)$	$T(0s)$	0.156	0.148

corresponds to $\gamma_s \simeq 0.82$ (vs. 0.65)
strangeness suppression is considerably reduced

Further nuclear effect:
transverse momentum broadening



- **initial state collisions → rotation of emission axes**
- **quarks from different NN collisions not collinear**
- **exogamous pairing broadens p_T distribution**
- **NB: combination of initial & final state effects**

5. Kinetic vs. Stochastic Thermalization

Kinetic thermalization:

time evolution of given non-equilibrium configuration
(two parallel colliding parton beams)
through multiple collisions
to a time-independent equilibrium state
(quark-gluon plasma)
requires

- **many constituents**
- **sufficiently large interaction cross sections**
- **sufficiently long time**

thermal hadron production in e^+e^-, $pp/p\bar{p}$?

Hagedorn: *the emitted hadrons are "born into equilibrium"*

Hawking-Unruh radiation:

- **final state produced at random from the set of all states corresponding to temperature T_H determined by confining field**

- **this set of all final states is same as that produced by kinetic thermalization**

- **measurements cannot tell if the equilibrium was reached by thermal evolution or by throwing dice:**

$\Rightarrow$ <u>Ergodic Equivalence Principle</u> $\Leftarrow$

gravitation $\sim$ acceleration **kinetic $\sim$ stochastic**

6. Summary

- **Physical vacuum: event horizon for colored quarks & gluons; thermal hadrons: Hawking-Unruh radiation from quark tunnelling through event horizon.**

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- Hadronization temperature T_H: quark acceleration and deceleration in color field at (quantum) horizon.

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- Hadronization temperature T_H: quark acceleration and deceleration in color field at (quantum) horizon.
- Hadron multiplicity: $\nu(s) \sim \ln s$.

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- Given string tension σ and strange quark mass m_s, obtain parameter-free description of thermal hadron production in high energy interactions.

God does play dice, but He sometimes throws them where they can't be seen.

Stephen Hawking