Lecture 2

The Phase Diagram of

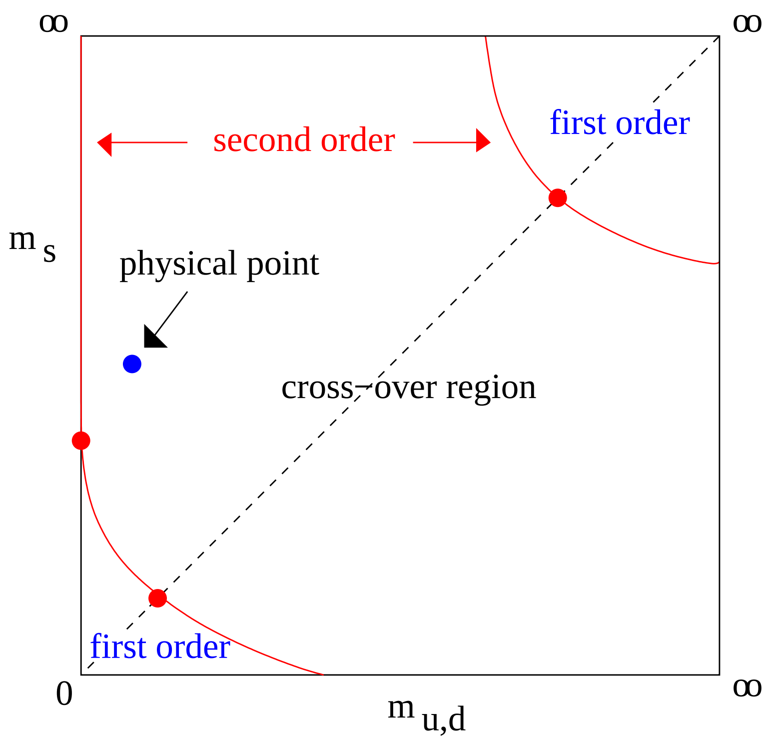
Strongly Interacting Matter

For $m_q \to \infty$ in $SU(N)$ gauge theory, Z_N symmetry breaking:
genuine thermal phase transition

For $m_q \to 0$ in $N_f \geq 2$ flavor QCD, chiral symmetry breaking and restoration: genuine thermal phase transition

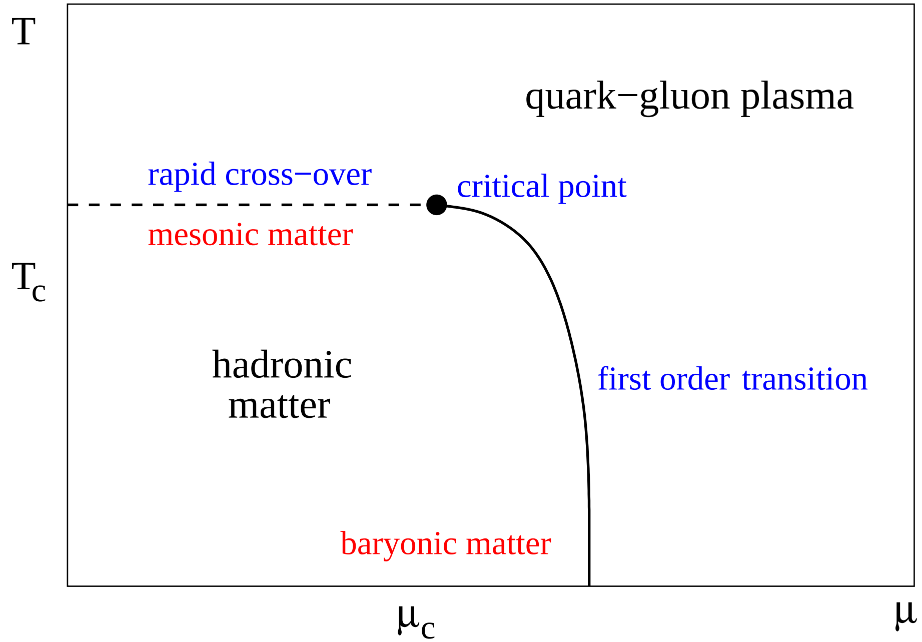
in-between?

what determines this structure?



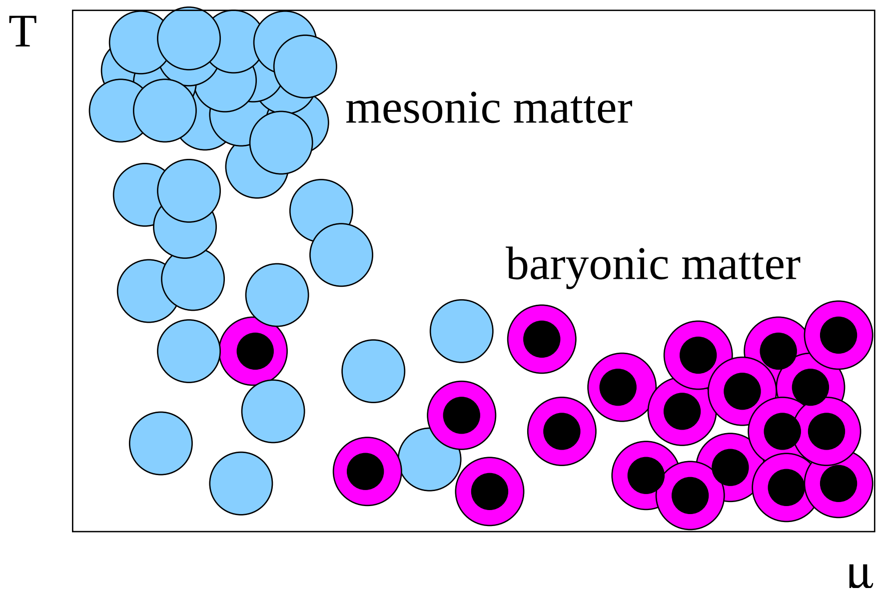
At sufficiently high temperature or large baryon number density:

Limits of Hadronic Matter



- different limit forms in different T, μ regions
- does this arise from different hadronic interactions?
- does this lead to different deconfined states of matter?

Constituent Structure of Hadronic Matter



- low μ: with increasing T, mesonic medium of increasing density
  mesons experience attraction $\longrightarrow$ resonance formation
  mesons are permeable (overlap) $\longrightarrow$ resonances $\sim$ same size

- low T: with increasing μ, baryonic medium of increasing density
  nucleons experience attraction $\longrightarrow$ formation of nuclei
  nucleons repel (hard core) $\longrightarrow$ nuclei grow linearly with A

In both cases, $\exists$ clustering

$\exists$ relation between clustering and critical behavior? Frenkel 1939
Essam & Fisher1963

consider spin systems, e.g., Ising model

- for $H = 0$,
  spontaneous Z_2 symmetry breaking $\longrightarrow$ magnetization transition

- but this can be translated into cluster formation and fusion

  critical behavior via cluster fusion: percolation $\equiv$
  critical behavior via spontaneous symmetry breaking

  Fisher 1967, Fortuin & Kasteleyn 1972, Coniglio & Klein 1980

- for $H \neq 0$, Isakov 1984
  partition function is analytic, no thermal critical behavior
  but clustering & percolation persists Kertész 1989

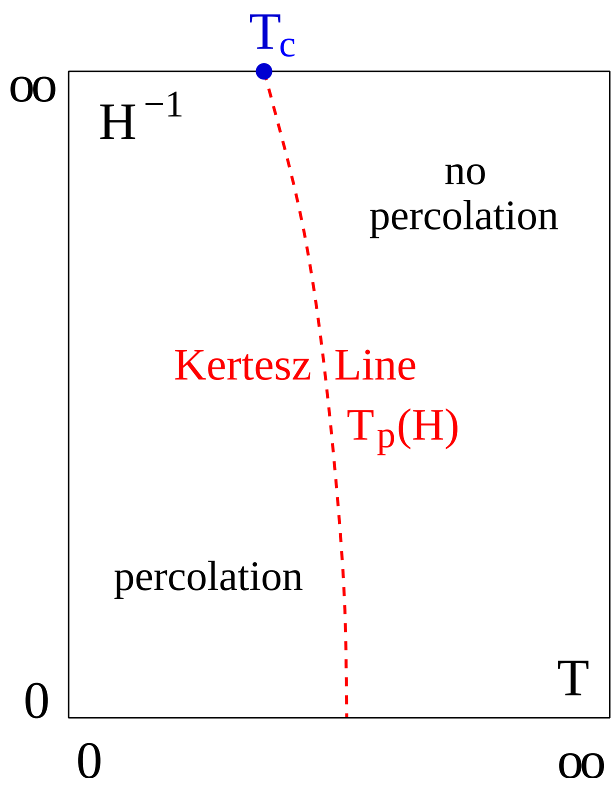
$\exists$ geometic critical behavior

In spin systems,

$\exists$ geometric critical behavior
for all values of H;

for $H = 0$, this becomes identical
to thermal critical behavior, with
non-analytic partition function
& Z_2 exponents

for $H \neq 0$, $\exists$ Kertész line
geometric transition with
singular cluster behavior
& percolation exponents



For spin systems,

thermal critical behavior $\subset$ geometric critical behavior

Also in QCD? Hadrons have intrinsic size, with increasing density
they form clusters & eventually percolate

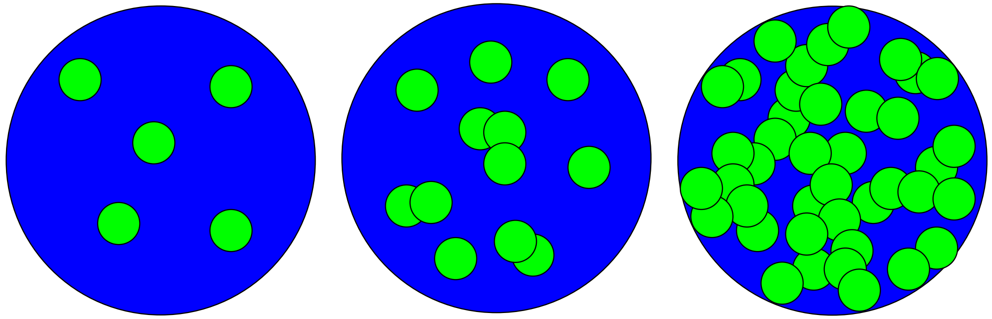
Hadron Percolation $\sim$ Color Deconfinement

Pomeranchuk 1951

Baym 1979, Çelik, Karsch & S. 1980

Recall percolation

- 2-d, with overlap:
  lilies on a pond



- 3-d: N spheres of volume V_h in box of volume V, with overlap

  increase density $n = N/V$ until largest cluster spans volume:

  <u>percolation</u>

  critical percolation density $n_p \simeq 0.34/V_h$

  at $n = n_P$, 30 % of space filled by overlapping spheres,
  70 % still empty

how dense is the percolating cluster? Digal, Fortunato & S. 2004

critical cluster density $n_m \simeq 1.2/V_h$

$$R_h \simeq 0.8 \text{ fm} \;\Rightarrow\; n_m \simeq \frac{0.6}{\text{fm}^3} \quad \text{as deconfinement density}$$

so far, cluster constituents were allowed arbitrary overlap

what if they have a hard core?

percolation for spheres of radius R_0
with a hard core of radius $R_{hc} = R_0/2$ Kratky 1988

hard cores tend to prevent dense clusters;
higher density needed to achieve percolating clusters

$$n_b \simeq \frac{2.0}{V_0} = \frac{0.25}{V_{hc}} \simeq \frac{1.0}{\text{fm}^3} \simeq 6 \text{ n}_0$$

for the deconfinement density of baryonic matter

$\exists$ two percolation thresholds in strongly interacting matter:

- mesonic matter, full overlap: $n_m \simeq 0.6/\mathrm{fm}^3$
- baryonic matter, hard core: $n_b \simeq 1.0/\mathrm{fm}^3$

now apply to determine critical behavior

If interactions are resonance dominated,

interacting medium $\equiv$ ideal resonance gas

Beth & Uhlenbeck 1937; Dashen, Ma & Bernstein 1969

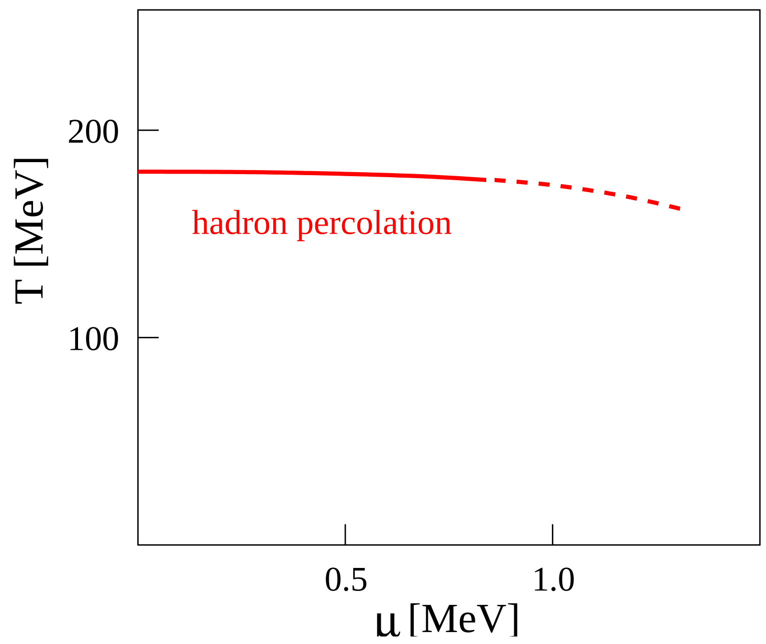
include all PDG states for $M \leq 2.5$ GeV, partition function

$$\ln Z(T,\mu,\mu_S,V) = \sum_{\text{mesons i}} \ln Z_M^i(T,V,\mu_S) + \sum_{\text{baryons i}} \ln Z_B^i(T,\mu,\mu_S,V)$$

for mesonic and baryonic contributions; enforce $S=0$

• low baryon-density limit: percolation of overlapping hadrons

$$n_h(T_h, \mu) = \frac{\ln Z(T, \mu, V)}{V} = 0.6/\mathrm{fm}^3$$



Obtain at $\mu = 0$

$$T_h \simeq 180\ \mathrm{MeV}$$

deconfinement temperature

based on hadron percolation

baryons included, but hard core effects ignored

slow decrease of transition temperature with μ,
due to associated production

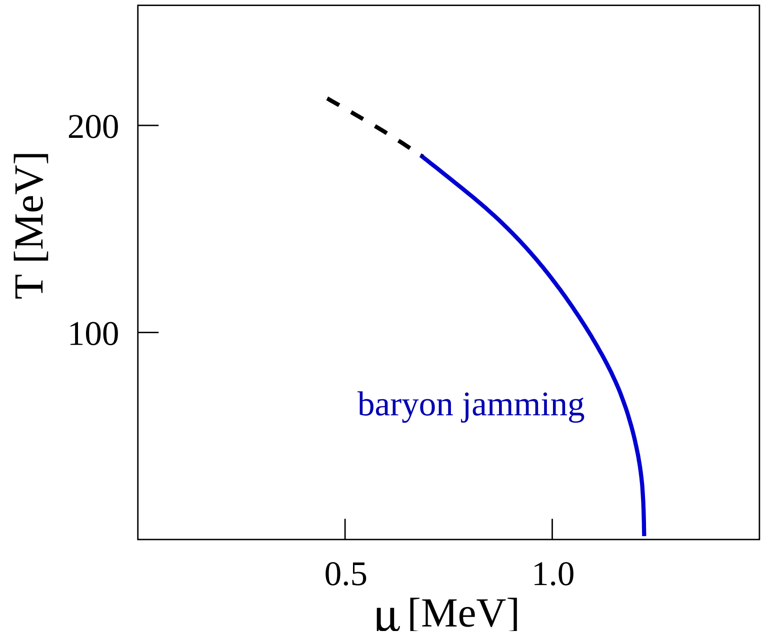
• **high baryon-density limit: percolation of hard-core baryons**

density of pointlike baryons

$$n_b^0 = \frac{1}{V}\left(\frac{\partial\ T \ln Z_B(T,\mu,V)}{\partial\mu}\right)$$

hard core $\Rightarrow$ excluded volume
(Van der Waals)

$$n_b = \frac{n_b^0}{1 + V_{hc} n_b^0}$$



percolation threshold
$\longrightarrow$ transition line

$$n_b^c(T,\mu) = \frac{2.0}{V_0} = \frac{0.9}{\mathrm{fm}^3} \simeq 5\ n_0$$

combine the two mechanisms:

phase diagram of hadronic matter

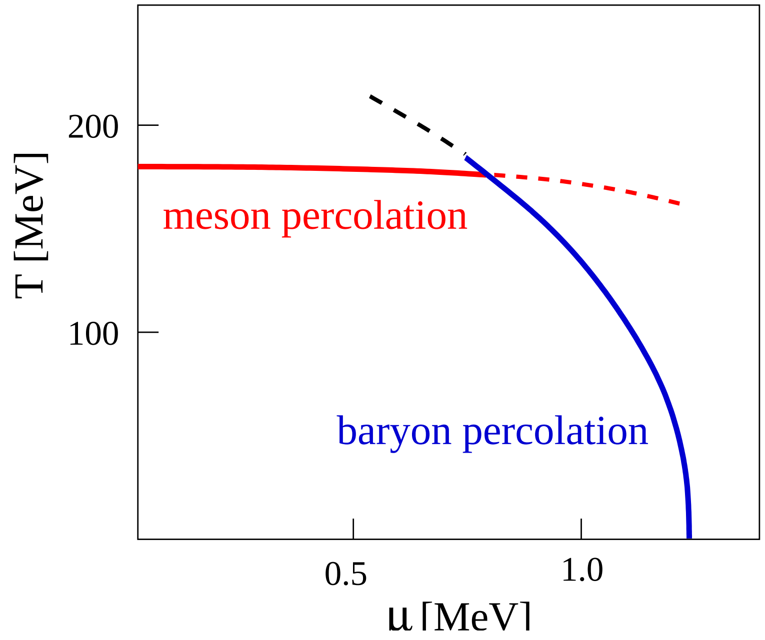
• low baryon density:

percolation of overlapping hadrons

clustering $\sim$ attraction

• high baryon density:

percolation of hard-core baryons



NB:

nuclear <u>attraction</u> plus hard-core <u>repulsion</u> $\longrightarrow$ <u>1^{st} order transition</u>

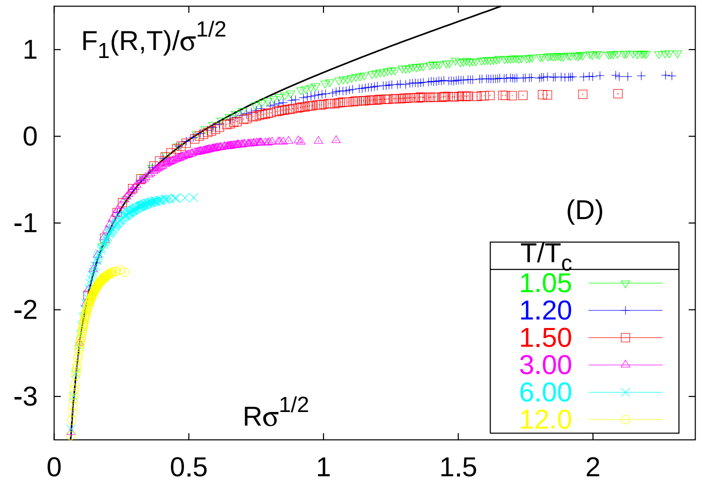
clustering and percolation can provide
a conceptual basis for the limits of hadronic matter
in the QCD phase diagram

What happens beyond the limits?

There are two roads to deconfinement:

- Increase quark density so that several quarks/antiquarks within confinement radius $\longrightarrow$ pairing ambiguous or meaningless.
- Increase temperature so much that gluon screening forbids communication between quarks/antiquarks distance r apart.

Illustration of the second case:
heavy Q correlations, quenched QCD



Quarks separated by about 1 fm
no longer "see" each other for $T \geq T_c$

mesonic matter:
when quark density is high enough,
gluon screening radius is short enough, so both coincide

baryonic matter?

in hadrons & in hadronic matter $\exists$ chiral symmetry breaking

$\Rightarrow$ confined quarks acquire effective mass $M_q \simeq 300$ MeV
effective size $R_q \simeq R_h/3 \simeq 0.3$ fm
through surrounding gluon cloud

what happens at deconfinement? Possible scenarios:

- **plasma of massless quarks and gluons,**
  ground state shift re physical vacuum $\rightarrow$ bag pressure B
- **plasma of massive "constituent" quarks, all gluon effects in M_q**

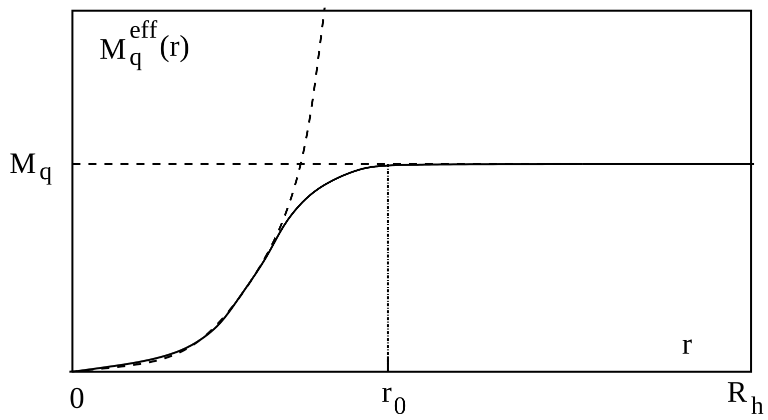
"effective" quark? $\sim$ depends on how you look:

- **hadronic distances, soft probes: massive constituent quark**
  (additive quark model)
- **sub-hadronic distances, hard probes: bare current quark**
  (deep inelastic scattering)

Origin of constituent quark mass?
quark polarizes gluon medium $\longrightarrow$ gluon cloud around quark

$$M_q \sim m_q + \epsilon_g r^3$$

where ϵ_g is the change
in energy density of the gluon field
due to the presence of the quark



QCD:
non-abelian gluon screening
limits “visibility” range to r_g

$\longrightarrow$ energy density of gluon cloud and screening radius
determine “asymptotic” constituent quark mass $\sim$ gluon cloud

relation to chiral symmetry breaking?

estimates from perturbative QCD Politzer 1976 effective quark mass $M_q^{\mathrm{eff}}(r)$ at distance r

$$M_q^{\mathrm{eff}}(r) = 4\ g^2(r)\ r^2 \left[\frac{g^2(r)}{g^2(r_0)}\right]^{-d} \langle\bar{\psi}\psi(r_0)\rangle$$

with reference point r_0 for determination of $\langle\bar{\psi}\psi(r_0)\rangle$; coupling is

$$g^2(r) = \frac{16\pi^2}{9}\frac{1}{\ln[1/(r^2\Lambda_{\mathrm{QCD}}^2)]}$$

for $N_f = 3,\ N_c = 3\ \rightarrow\ d = 4/9$

constituent quark mass is defined as solution of

$$M_q = M_q^{\mathrm{eff}}(r = 1/2M_q)$$

giving M_q in terms of r_0 and $\langle\bar{\psi}\psi(r_0)\rangle$

With $r_0 = 1/2M_q$ (meeting of perturbative and non-perturbative)

$$M_q^3 = \left\{ \frac{16\pi^2}{9} \frac{1}{\ln(4M_q^2/\Lambda_{QCD}^2)} \right\} \langle \bar{\psi}\psi(r_0) \rangle$$

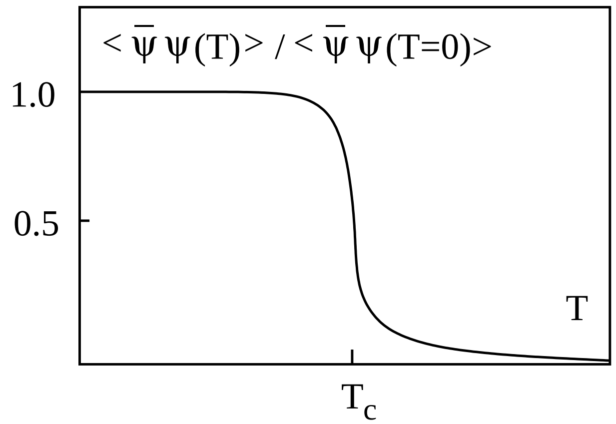
and with $\Lambda_{QCD} = 0.2$ GeV, $\langle \bar{\psi}\psi(r_0) \rangle^{1/3} = 0.2$ GeV

$$M_q = 375 \text{ MeV}; \quad R_q = 0.26 \text{ fm}$$

constituent quark mass determined by chiral condensate

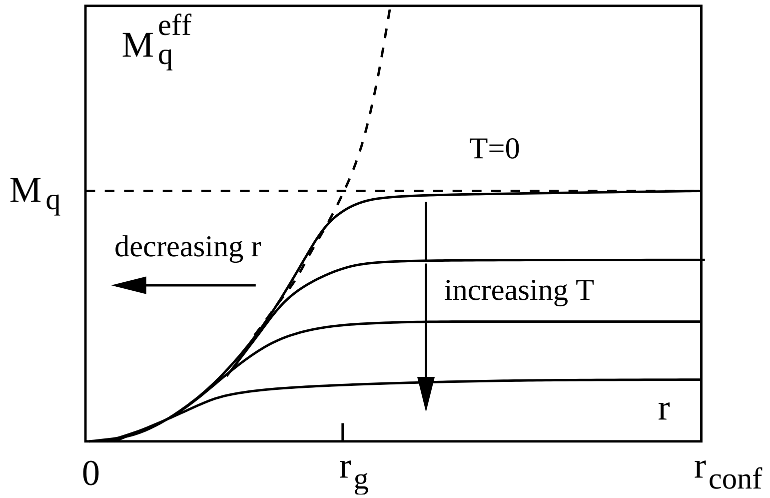
how does $\langle \bar{\psi}\psi(T) \rangle^{1/3}$ change with temperature?

gluon cloud evaporates, constituent quark mass vanishes as $T \rightarrow T_c$



So there are two ways to make the effective quark mass vanish

- decrease interquark distance
- increase temperature

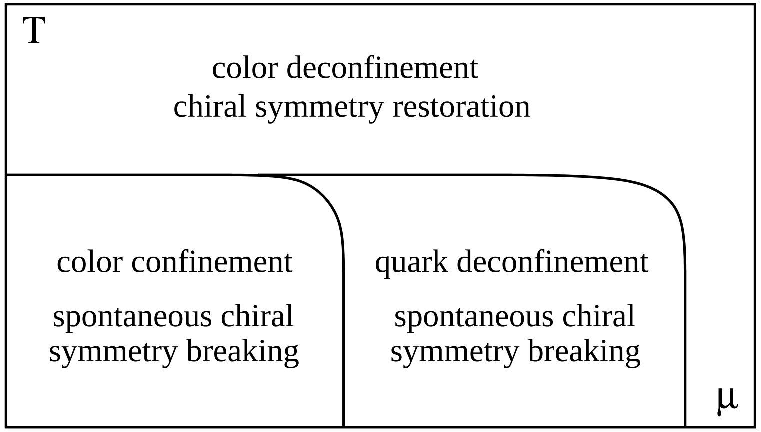


now consider different $T - \mu$ regions:

- $\mu \simeq 0,\ T \simeq T_c$: interquark distance $\sim$ 1 fm but hot medium makes gluon cloud evaporate $\Rightarrow\ M_q^{eff} \simeq 0$
- $T \simeq 0,\ \mu \simeq \mu_c$: interquark distance $\sim$ 1 fm and cold medium, gluon cloud does not evaporate $\Rightarrow\ M_q^{eff} \simeq M_q$

in cold dense matter, $M_q^{eff} \longrightarrow 0$ requires short interquark distance $\sim$ constituent quark percolation

intermediate massive quark plasma for $0.3 < r < 1$ fm and $T \lesssim T_c$

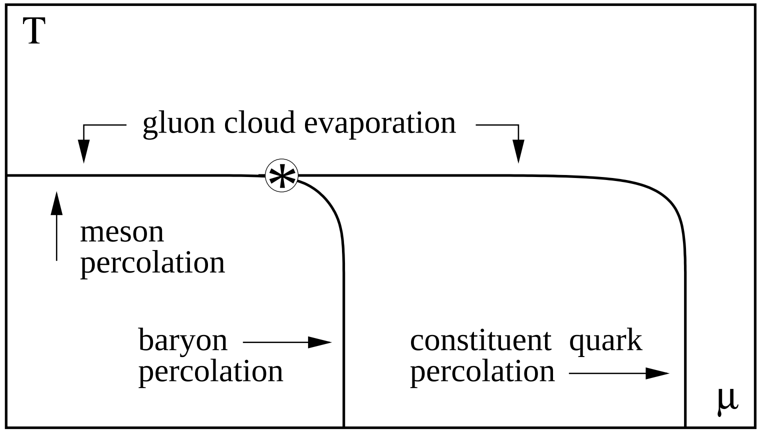


color deconfinement, but chiral symmetry remains broken; constituents: massive colored quarks, gluons only as quark dressing

baryon density limit through quark percolation $n_b^c \simeq 3.5$ fm^{-3}

- **nuclear matter $n_b \leq 0.9$ fm^{-3}**
- **quark plasma 0.9 fm$^{-3} \leq n_b \leq 3.5$ fm^{-3}**
- **quark-gluon plasma $n_b \geq 3.5$ fm^{-3}**

Transitions:



Nature of massive quark plasma

– massive quarks and (at higher T) some massive antiquarks

– no gluons, "chiral pions"?

no color confinement, but colored bound states possible

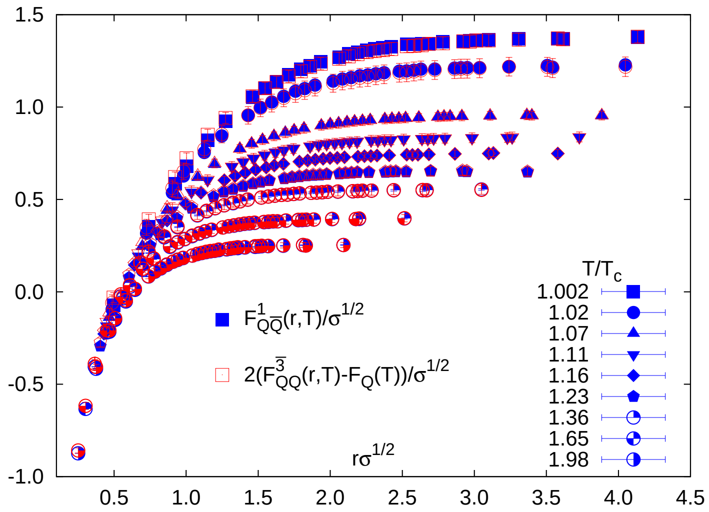
anti-triplet qq bound states = diquarks
(genuine two-body states, not Cooper pairs)

attractive interaction for

$qq \longrightarrow$ color anti-triplet,

$q\bar{q} \longrightarrow$ color singlet,

with same functional form of potential in r, T

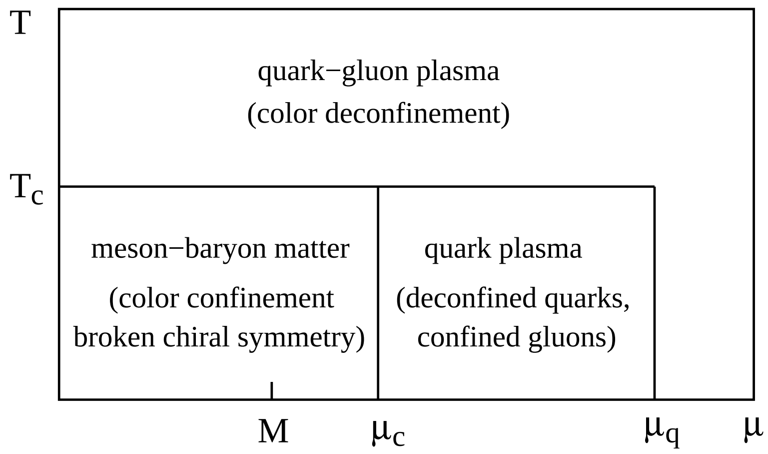


Bielefeld Lattice Group 2002

constituent quark plasma can be structurally similar to hadron gas:

- massive quarks
- (antitriplet) diquark and (singlet) $q\bar{q}$ states
- higher excitations (colored resonance gas)
- also possible: glueballs, chiral pions
- all states have intrinsic finite size, hence $\exists$ percolation limit

Conclusion



$\Rightarrow$ **Three State Phase Diagram** (modulo color superconductor)

- **Hadronic mattter at low T, μ:**
  quarks and gluons confined to hadrons, broken chiral symmetry
- **Quark plasma at low T, large(r) μ:**
  massive deconfined quarks, broken chiral symmetry
- **Quark-gluon plasma at large T, μ:**
  deconfined massless quarks and gluons, restored chiral symmetry

Back-Up

quark plasma has effective color degrees of freedom

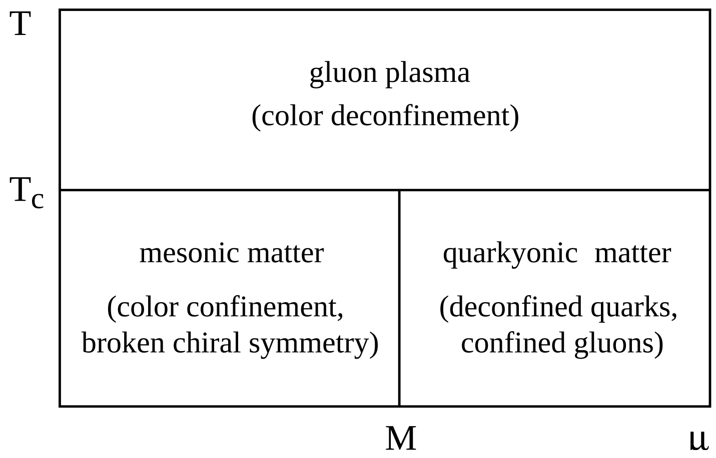
- hadron gas: $d_{\text{eff}} = 1$
- massive quark plasma: $d_{\text{eff}} = N_c$
- quark-gluon plasma: $d_{\text{eff}} = N_c^2$

relation to quarkyonic matter? McLerran & Pisarski 2007

phase structure of QCD for $N_c \to \infty$:

- confined hadronic matter is purely mesonic,
  since $n_b \sim \exp\{(\mu - M)\}$, and $\mu,\ M \sim N_c$.
- quark-gluon plasma becomes gluon plasma,
  since gluon sector $\sim N_c^2$, quark sector $\sim N_c$.
- quarkyonic matter proposed to have
  color degrees of freedom $\sim N_c$, hence no "free" gluons.
- quark plasma, with $n_q \sim N_c(\mu_q^2 - M_q^2)$, contracted to $\mu_q = M_q$.

quarkyonic



quark plasma