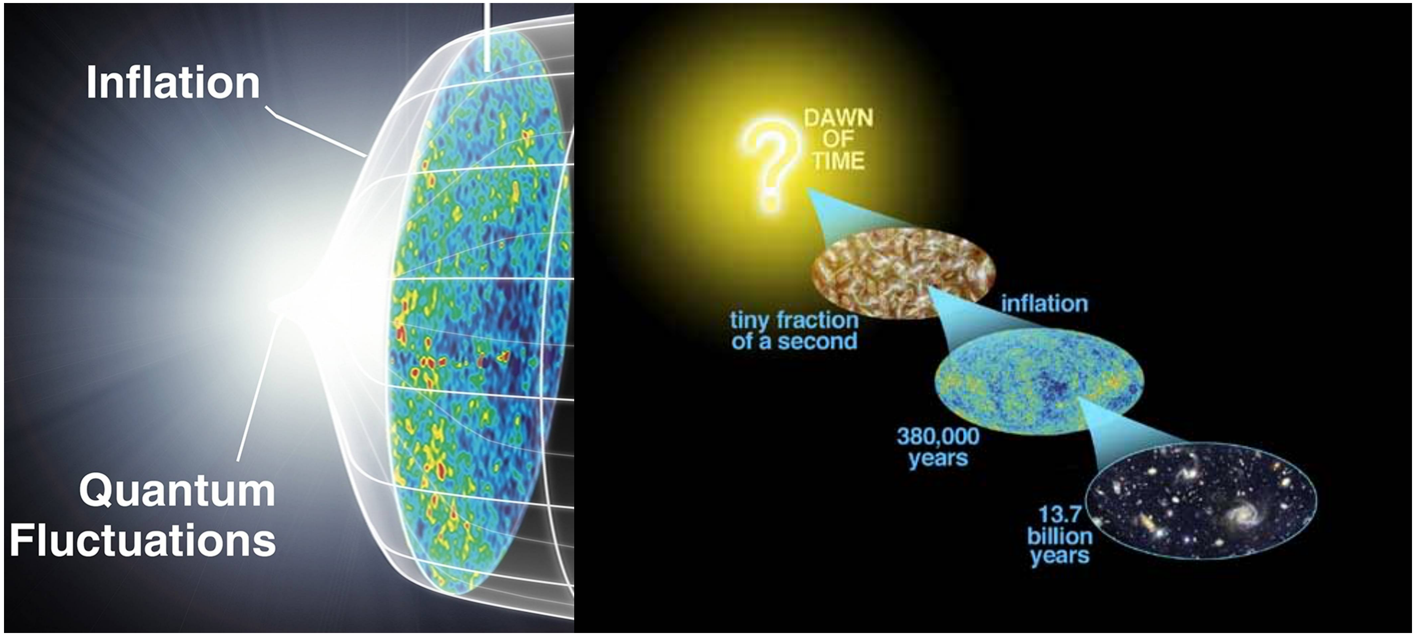

An introduction to some cosmological observables: from Inflation & gravitational waves to non-Gaussianity

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UNIVERSITÀ DEGLI STUDI DI PADOVA

COSMOSTATS, Asiago 12-15 September 2023

Outline

- Some keywords in Cosmology and the overall picture
- Theory meets cosmological observables: basic statistical tools
- Inflation and primordial Gravitational waves
- Physical processes behind distinct statistical signatures,

  e.g.: - non-Gaussianity as a precision test of inflation

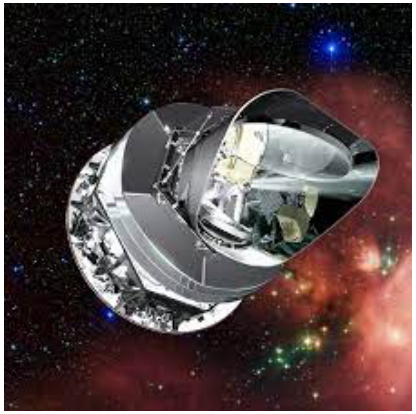
  - statistical anomalies in the Cosmic Microwave Bakcground
  - rare events from inflation??

Some key words in Cosmology

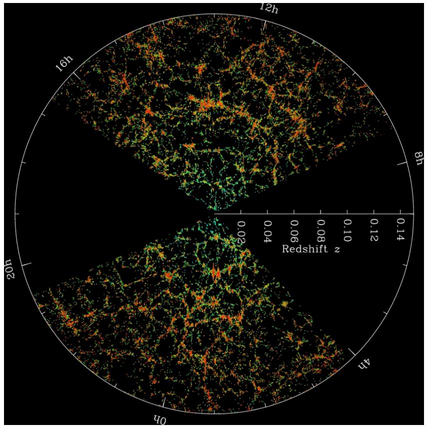
&

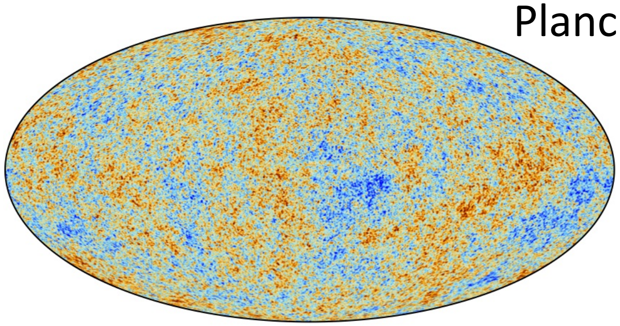
the overall picture

A long journey: from raw data to models



SDSS survey

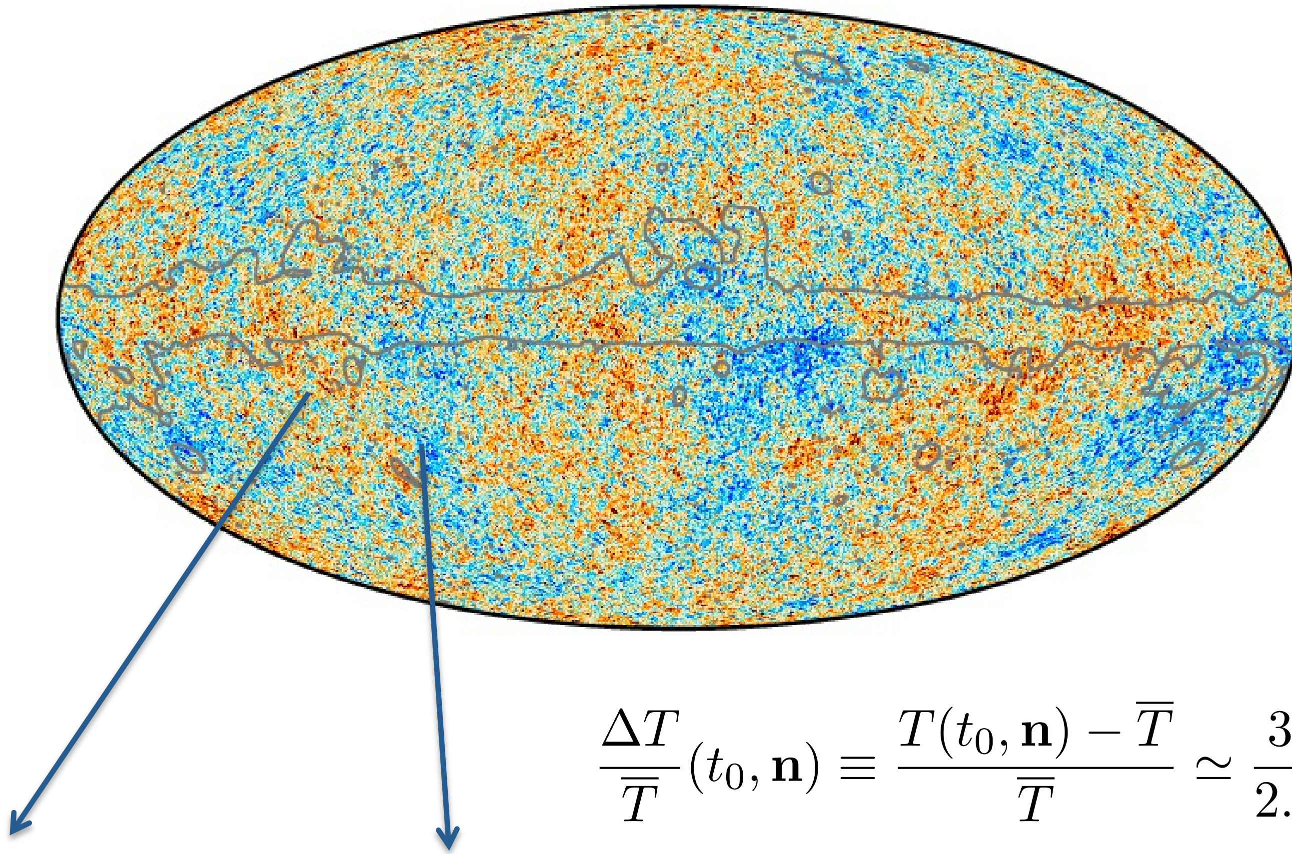




Planck CMB map

The (almost) ``smooth'' isotropic universe

Full-sky map of CMB temperature anisotropies from *Planck* satellite (2018)



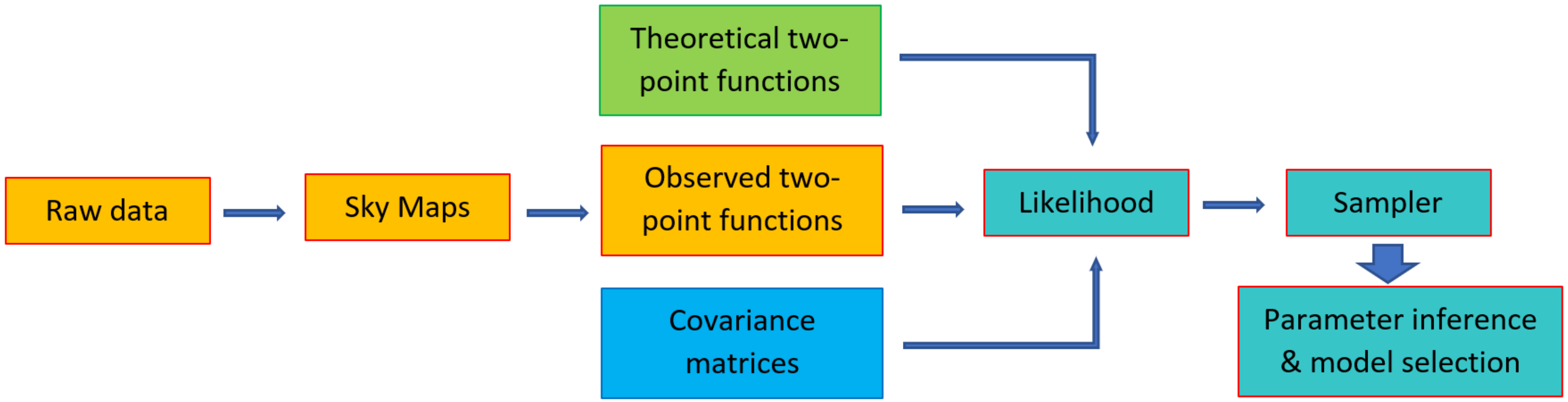
$$\frac{\Delta T}{\overline{T}}(t_0, \mathbf{n}) \equiv \frac{T(t_0, \mathbf{n}) - \overline{T}}{\overline{T}} \simeq \frac{35\,\mu K}{2.725K} \simeq 10^{-5}$$

T=2.7251 K

T=2.7252 K

Angular resolution ~ 5 arcminutes

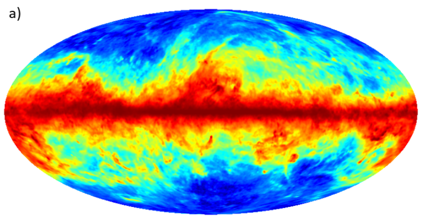
A long journey: from raw data to models



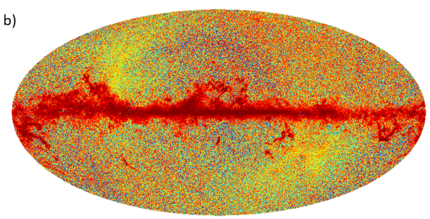
From PhD Thesis of Thomas Colas, adapted from "Modern Cosmology" by S. Dodelson

A long journey: from raw data to models

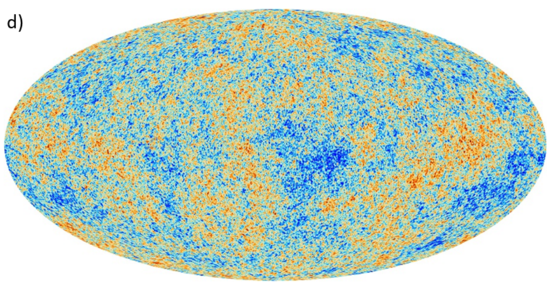
a)



b)



d)



Data must be processed
(Foregrounds subtraction, masks etc.)

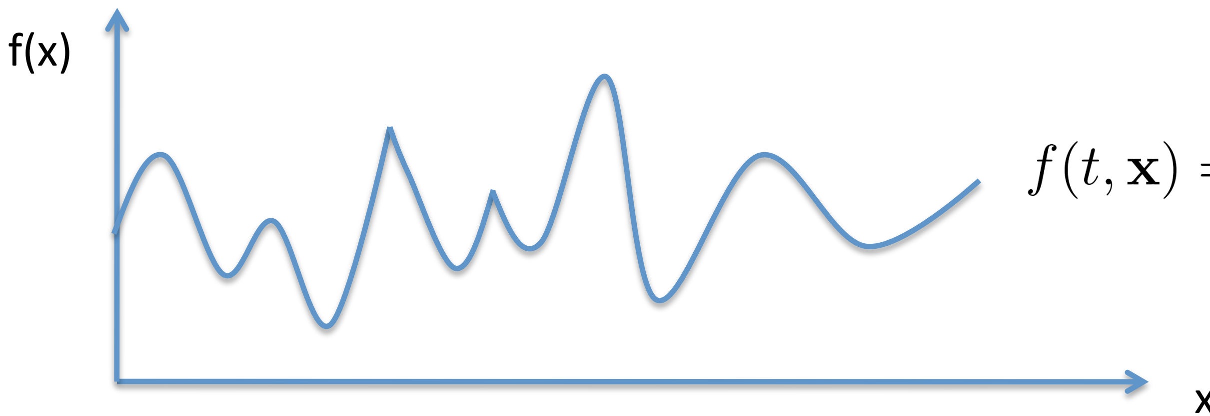
Cosmological signal

See talks by C. Baccigalupi & M. Liguori on CMB data analysis; E. Sarpa & M. Raveri about LSS data analysis.

Theory meets cosmological observables: basic statistical tools

Power spectrum of cosmological perturbations

Consider a random field f(t,**x**):



$$f(t,\mathbf{x}) = \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}}\, e^{i\mathbf{k}\cdot\mathbf{x}}\, f_{\mathbf{k}}(t)$$

$$\langle f_{\mathbf{k}_1} f^*_{\mathbf{k}_2} \rangle = (2\pi)^3 P_f(k_1)\delta^{(3)}(\mathbf{k}_1 - \mathbf{k}_2)$$

→ Dirac Delta because of homogenity

depends only on the modulus of because of isotropy

f(t,**x**) can be the fractional energy density perturbation δρ/ρ, or the (quantum) scalar field fluctuations (if quantum the brackets denote the expectation value on the vacuum state and it can be computed using creation and annihiliaton operators)

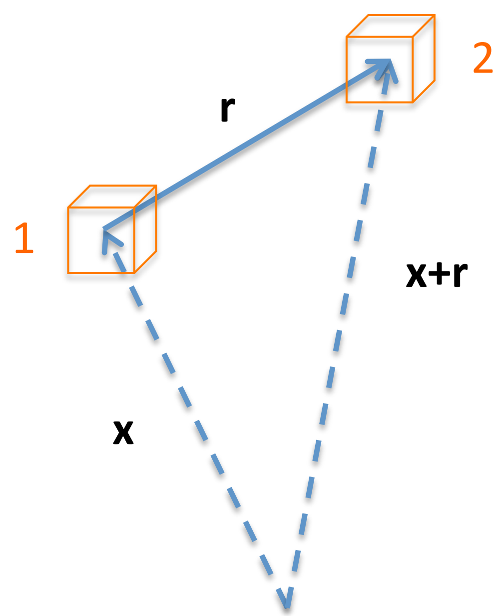
N.B.: $f^*_{\mathbf{k}2} = f_{-\mathbf{k}2}$ if f is real

Power spectrum of cosmological perturbations

You can show that the power-spectrum is the Fourier trasnform of the spatial two point-correlation function

N.B.: of course to statistically characterize the level of perturbations one cannot take simply $\langle f(\mathrm{x},t)\rangle$ given that $\langle f(\mathrm{x},t)\rangle = 0$

$$\xi(r) = \langle f(\mathbf{x}+\mathbf{r},t) f(\mathbf{x},t)\rangle$$



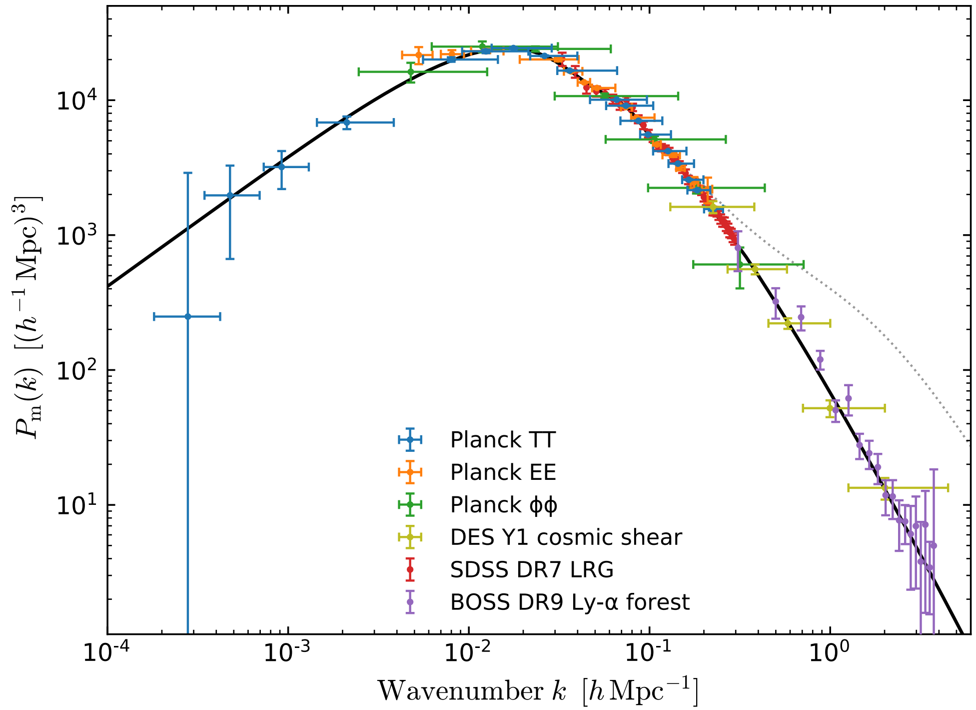
$$\delta P_{1,2} = \bar{n}^2 \, \delta V_1 \delta V_2 (1 + \xi(r))$$

ξ(r) describes the excess probability (w.r.t to a Poisson distribution) of finding two galaxies separated by r
→ e.g. statistical characterization of galaxy clustering

Power spectrum of cosmological perturbations

Essentially

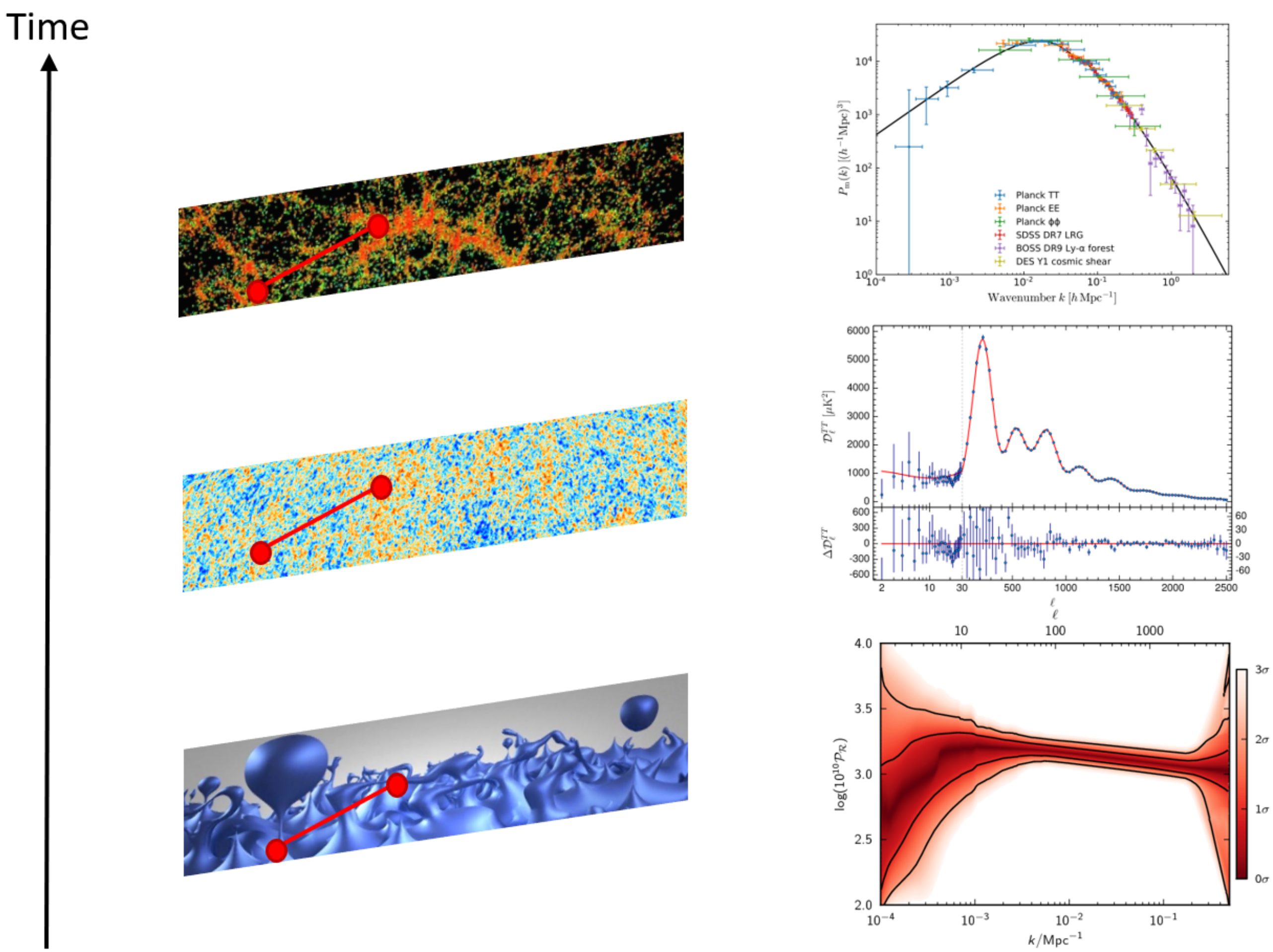
$$P_f(k) = |f_{\mathbf{k}}|^2$$



- You can also easily show that the variance of the fluctuation is given by

$$\sigma^2 = \langle f(\mathbf{x},t) f(\mathbf{x},t)\rangle = \frac{1}{2\pi^2}\int_0^\infty dk k^2 P(k) = \int_0^\infty \frac{dk}{k}\mathcal{P}(k)$$

Such a statistics can be "declined" in different ways in cosmology, according to the kind of data (e.g., CMB, large-scale galaxy surveys, etc.)



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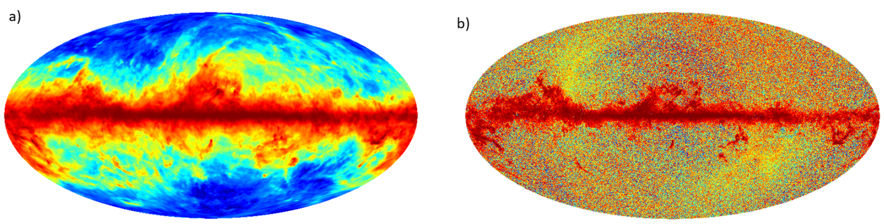
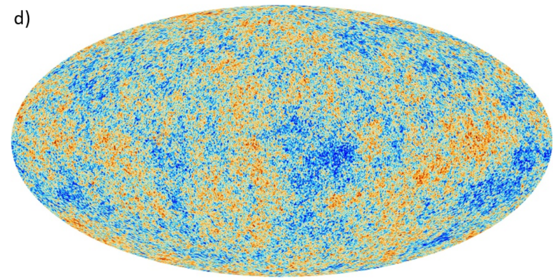
A long journey: from raw data to models

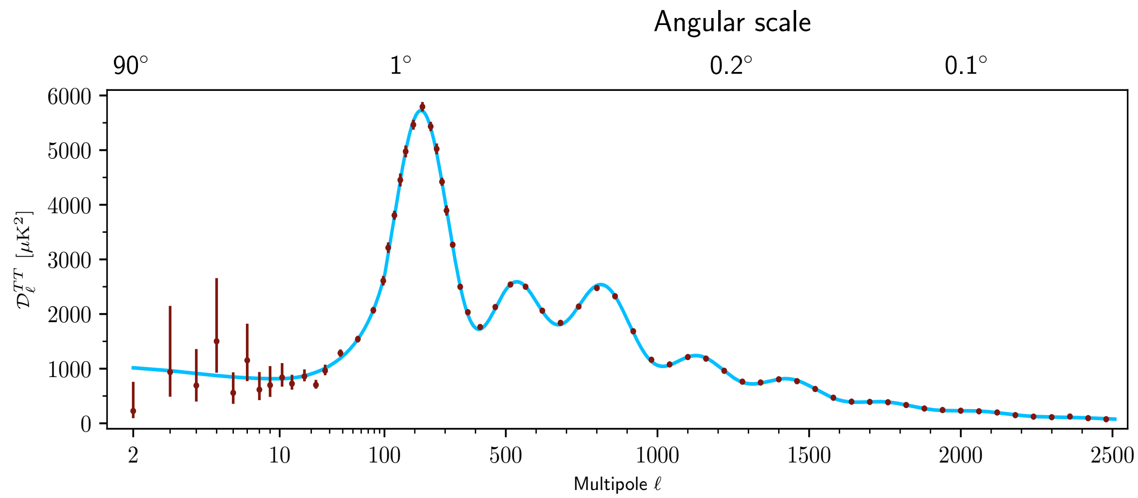




Data must be processed
(Foregrounds subtraction, masks etc.)

Cosmological signal

Statistics of objects
(here angular power-spectrum
of CMB temperature anisotropies)

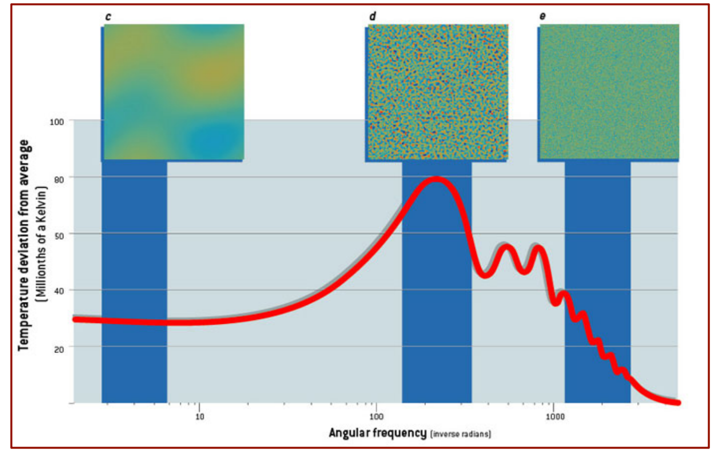


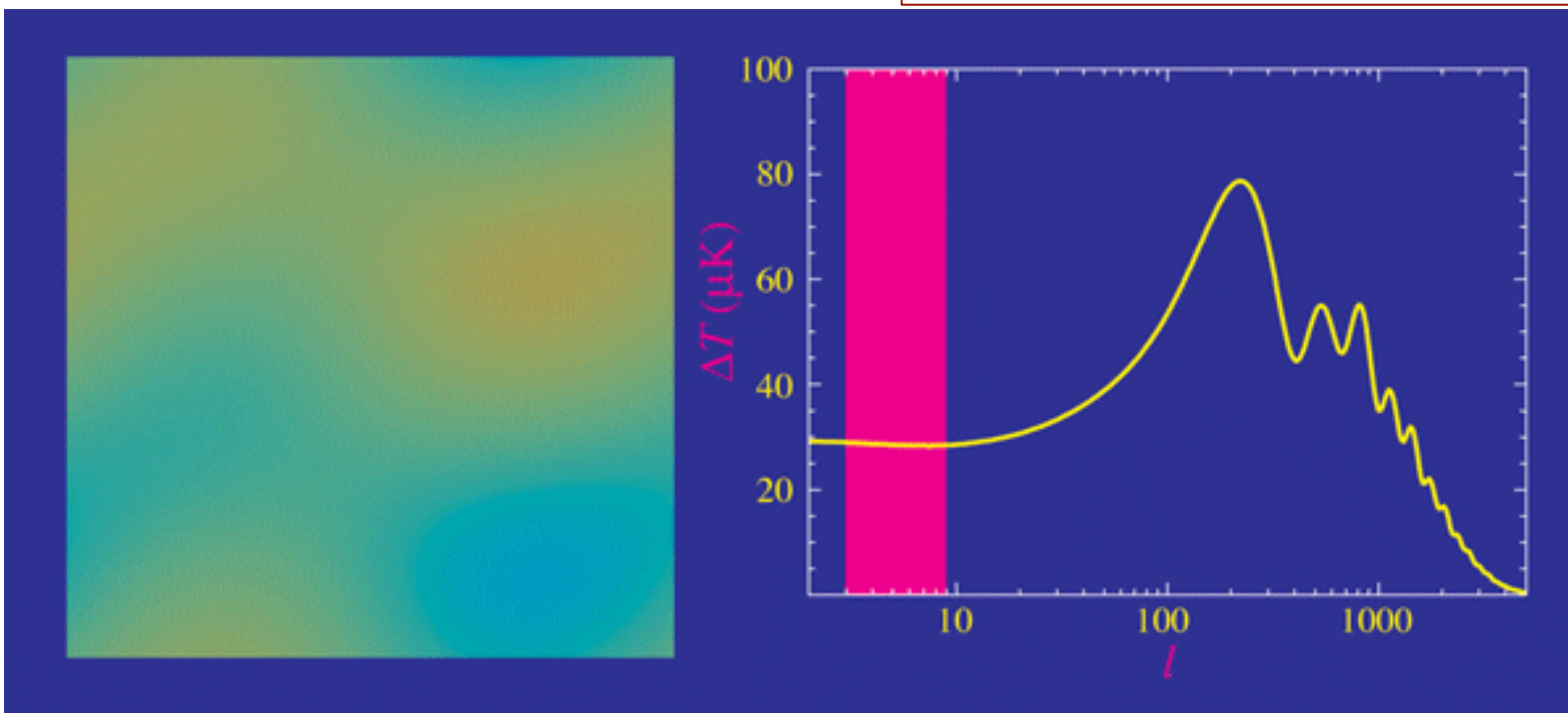
See talks by C. Baccigalupi & M. Liguori on CMB data analysis; E. Sarpa & M. Raveri about LSS data analysis.

The CMB power spectrum

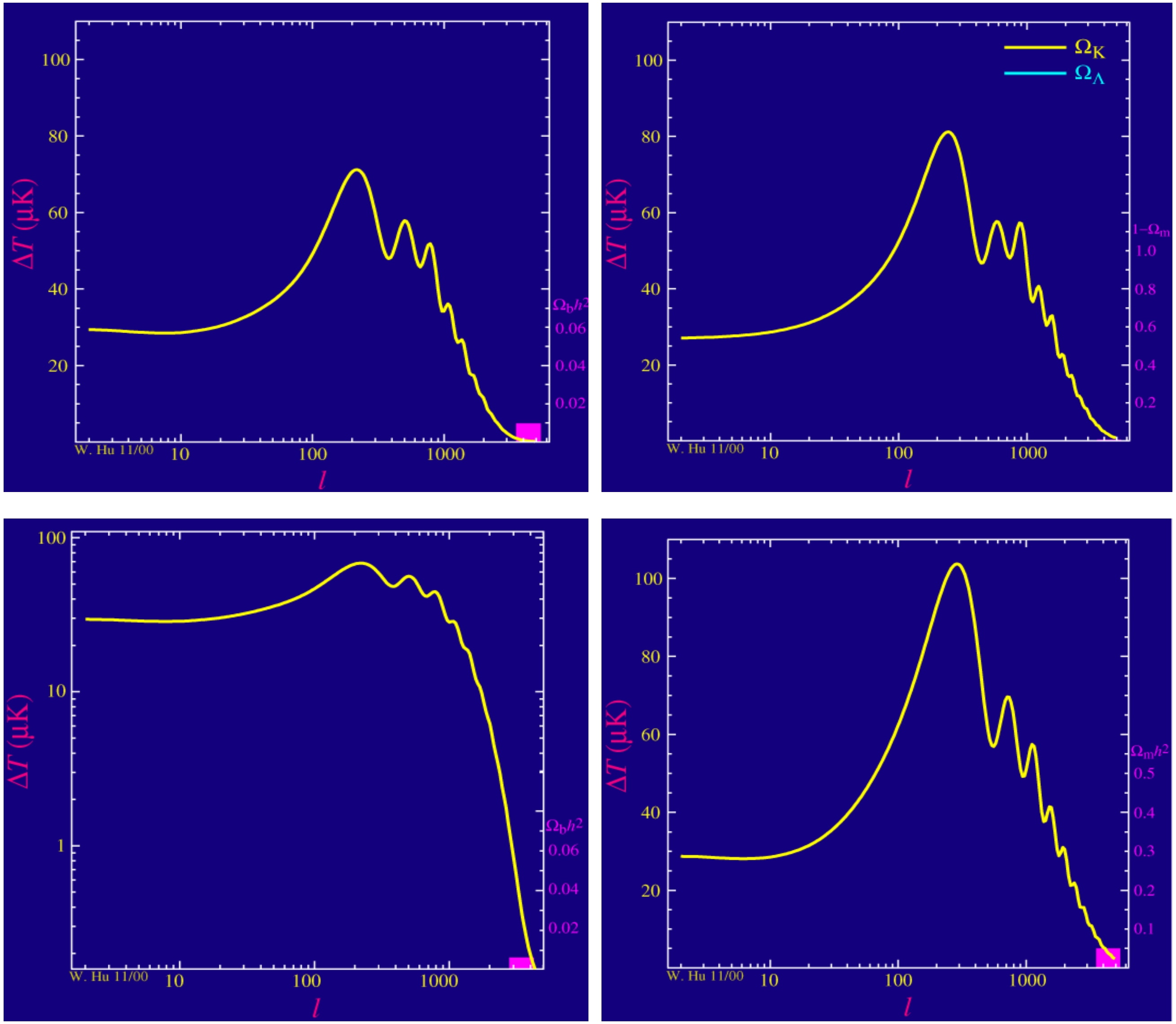
$$\frac{\Delta T}{T}(\vartheta,\varphi)=\sum_{\ell=2}^{\infty}\ \sum_{m=-\ell}^{\ell}a_{\ell m}Y_{\ell m}(\vartheta,\varphi)$$

$$\left\langle a^{*}_{\ell_1 m_1}a_{\ell_2 m_2}\right\rangle = C_{\ell_1}\delta_{\ell_1}^{\ell_2}\delta_{m_1}^{m_2}$$





Effect of changing parameters on C_l



A long journey: from raw data to models

The last step: confrontation with theory, statistical inference of model parameters

E.g.: in a Baeysian framework

$$\ln \mathcal{L}(\mathcal{D}|\mathcal{M}_i,\theta_{ij}) = -\frac{1}{2}\sum_{\alpha\beta}\left[\widehat{P}_g(k_\alpha) - \widehat{P}_{\rm th.}(k_\alpha|\mathcal{M}_i,\theta_{ij})\right]\mathrm{Cov}^{-1}_{\alpha\beta}\left[\widehat{P}_g(k_\beta) - \widehat{P}_{\rm th.}(k_\beta|\mathcal{M}_i,\theta_{ij})\right]$$

Theoretical power spectrum (at scale k_α) within model M for the set of parameters θ_{ij}

Statistical estimator from the data

Likelihood: probability of getting the data given a model M and a set of parameters θ_{ij}

- Bayes theorem

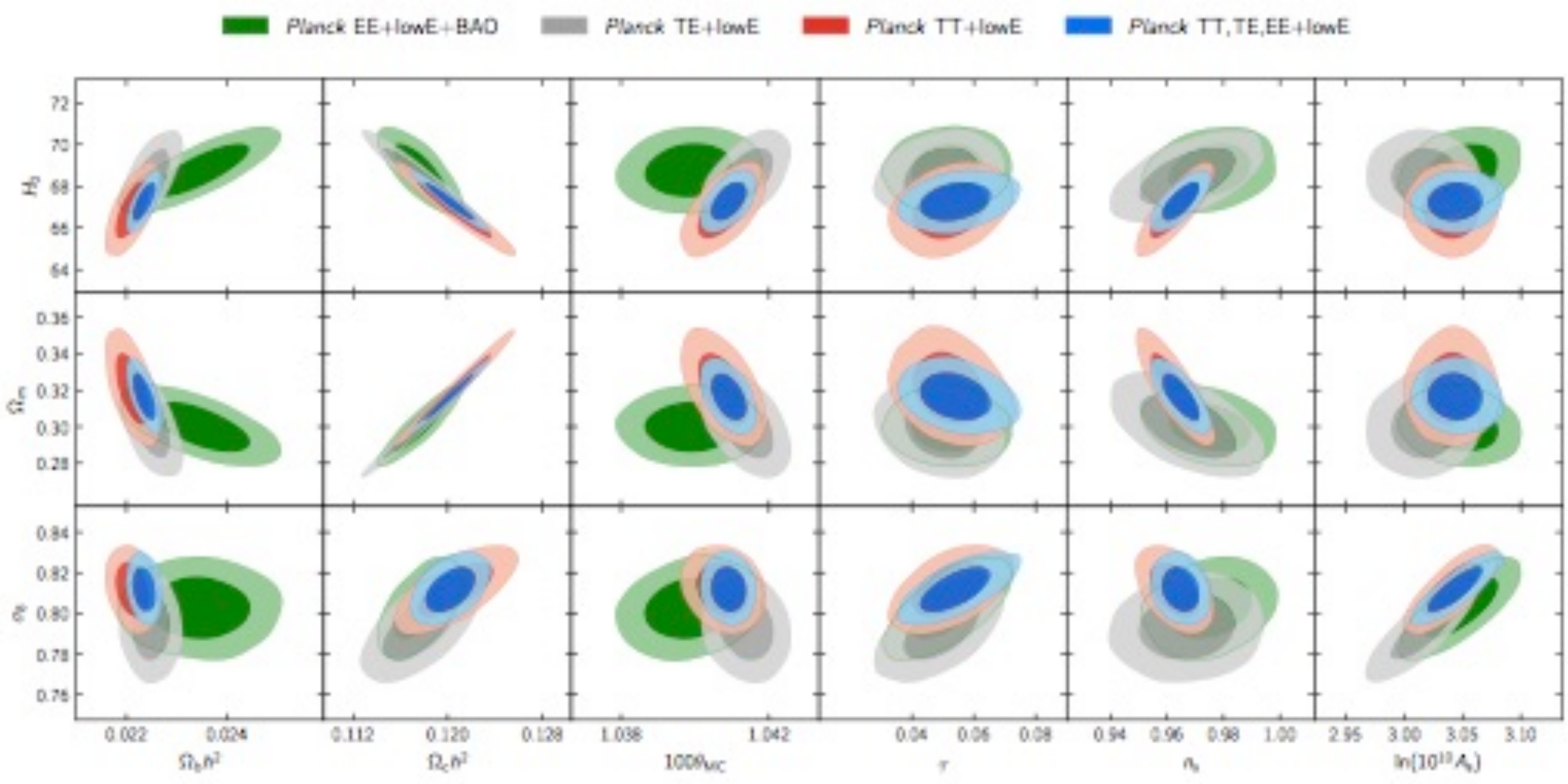
Prior distribution

$$p(\theta_{ij}|\mathcal{D},\mathcal{M}_i) = \frac{\mathcal{L}(\mathcal{D}|\mathcal{M}_i,\theta_{ij})\pi(\theta_{ij}|\mathcal{M}_i)}{\mathcal{E}(\mathcal{D}|\mathcal{M}_i)}$$

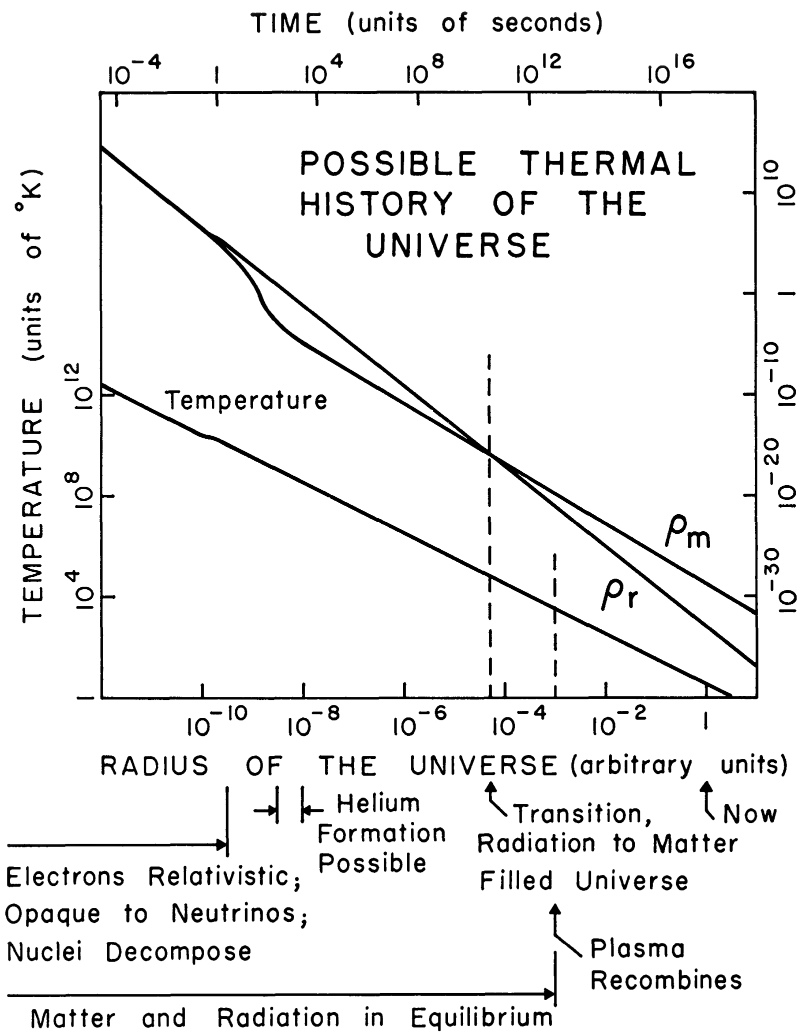
Posterior probability for the parameters

A long journey: from raw data to models

Cosmological parameters



The standard cosmological model



James Peebles, Nobel Prize 2019

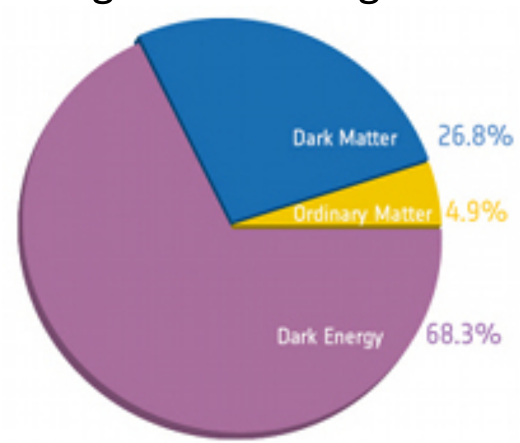
The big picture: precision (accuracy??) cosmology

ΛCDM: The standard cosmological model

Credit: L. Verde

just 6 numbers…….
describe the Universe composition and evolution

Homogeneous background

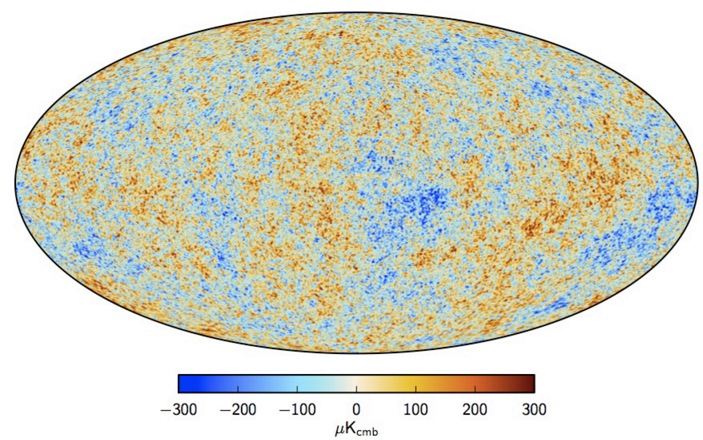


$\Omega_{b,}\ \Omega_{c,}\ \Omega_{\Lambda,} H_{0,}\ \tau$

- atoms 4%
- cold dark matter 23%
- dark energy 73%

Λ?? CDM??

Perturbations



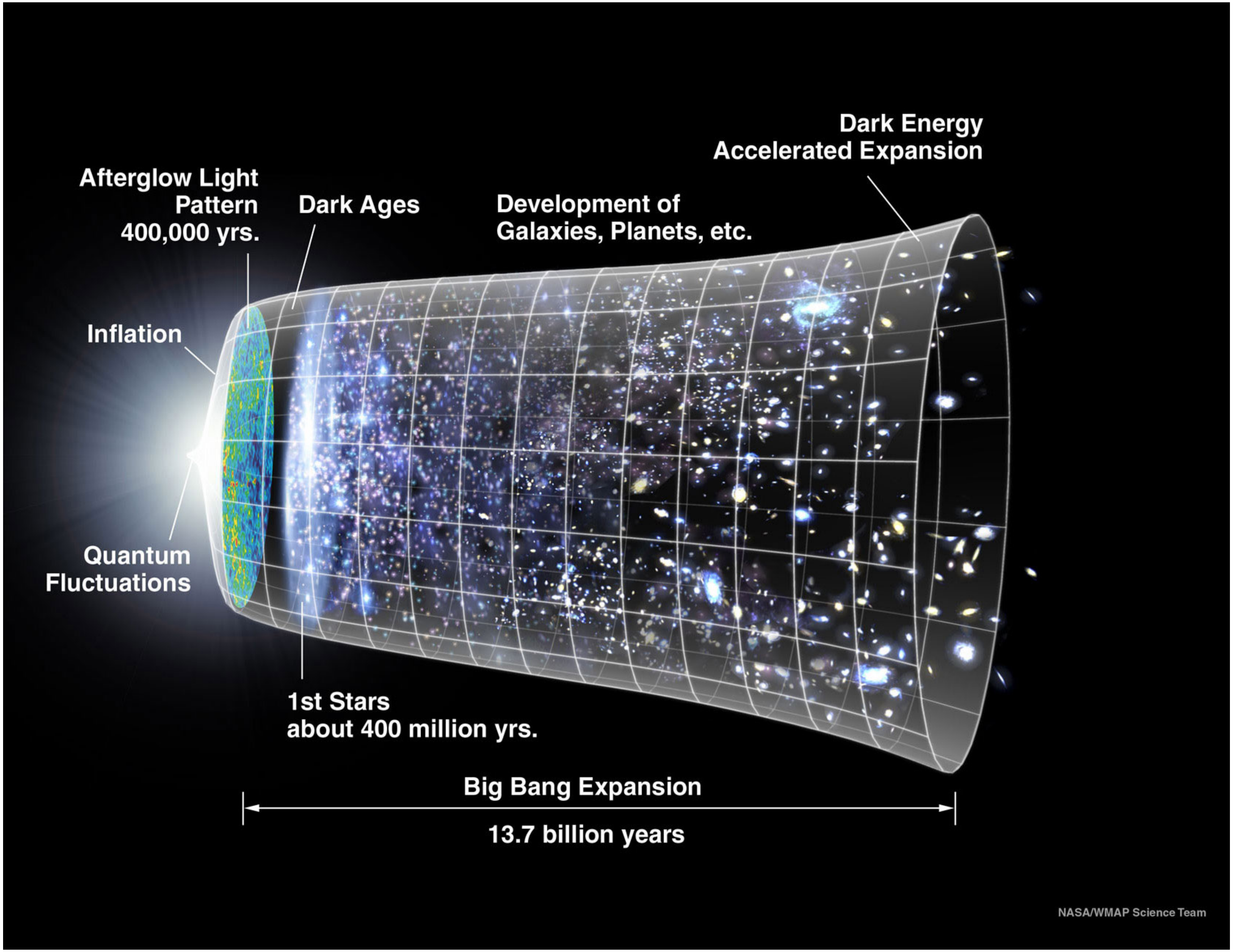
$A_{s,}\ n_{s,}\ r$

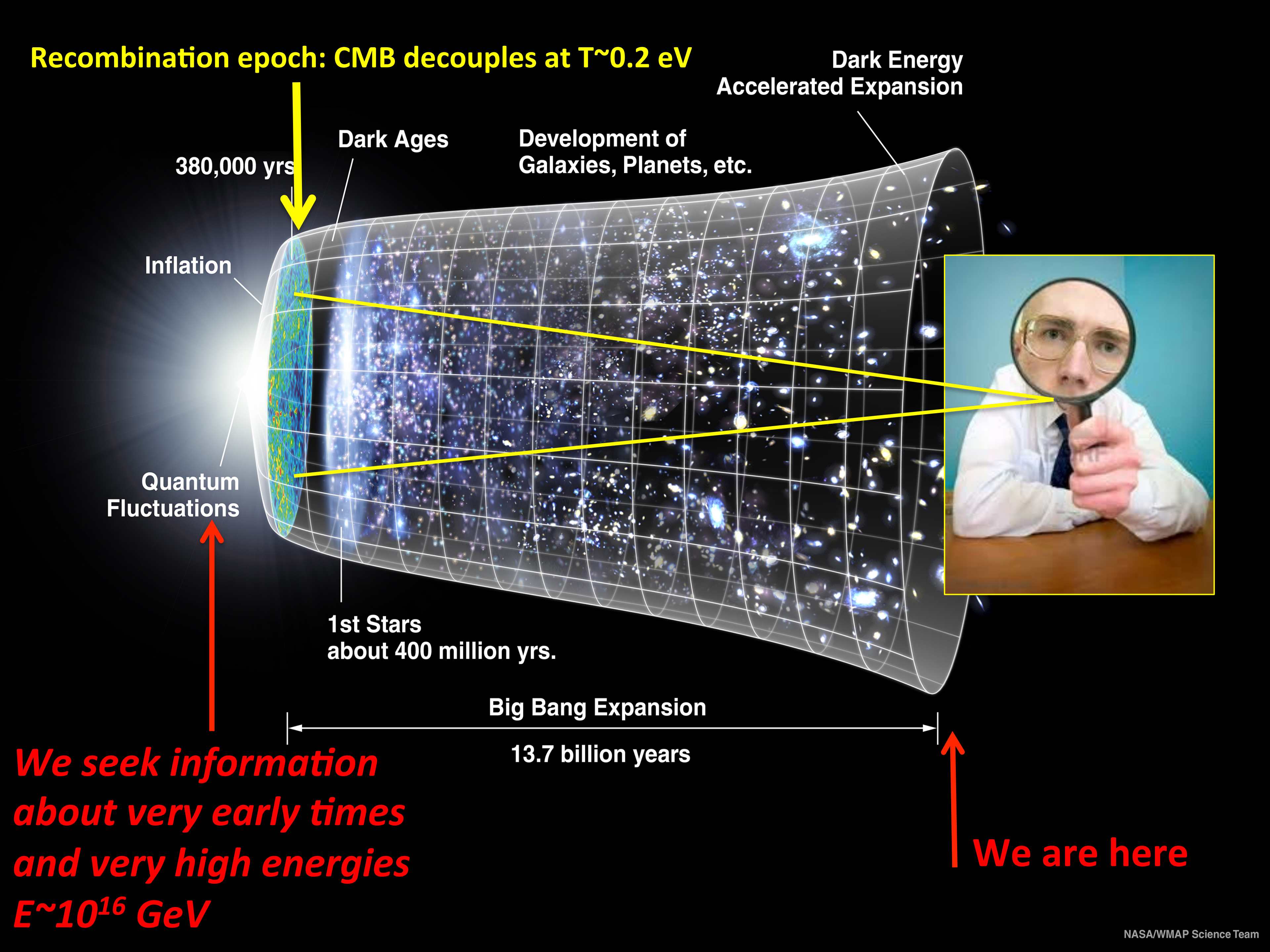
- nearly scale-invariant
- adiabatic
- (almost) Gaussian

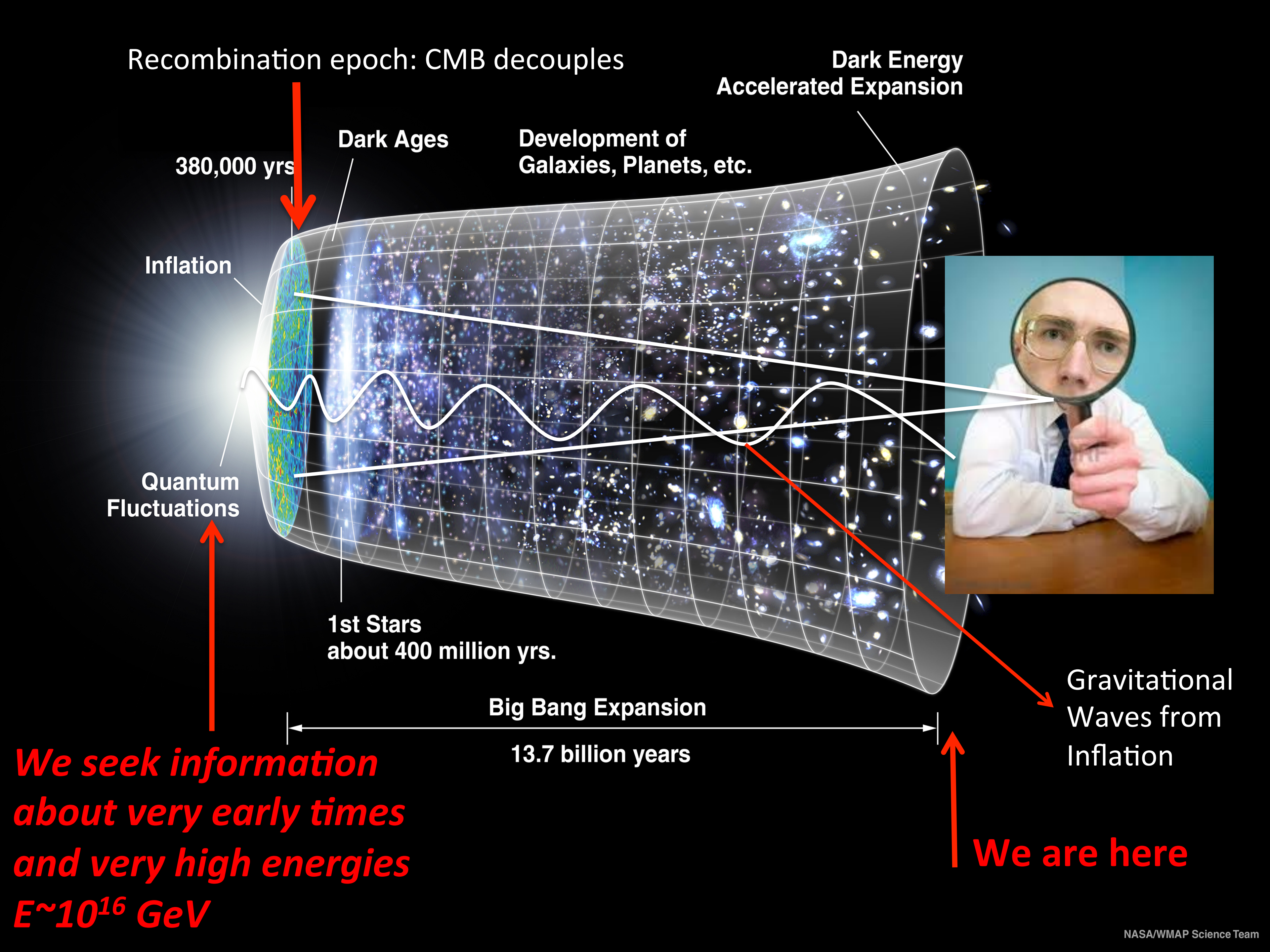
ORIGIN???→ Inflation(?)

Inflation &
primordial gravitational waves

Fitting into the Big Picture




Recombination epoch: CMB decouples at T~0.2 eV
Dark Energy
Accelerated Expansion
Dark Ages
Development of
Galaxies, Planets, etc.
380,000 yrs
Inflation
Quantum
Fluctuations
1st Stars
about 400 million yrs.
Big Bang Expansion
13.7 billion years
We seek information
about very early times
and very high energies
E~10^16 GeV
We are here
NASA/WMAP Science Team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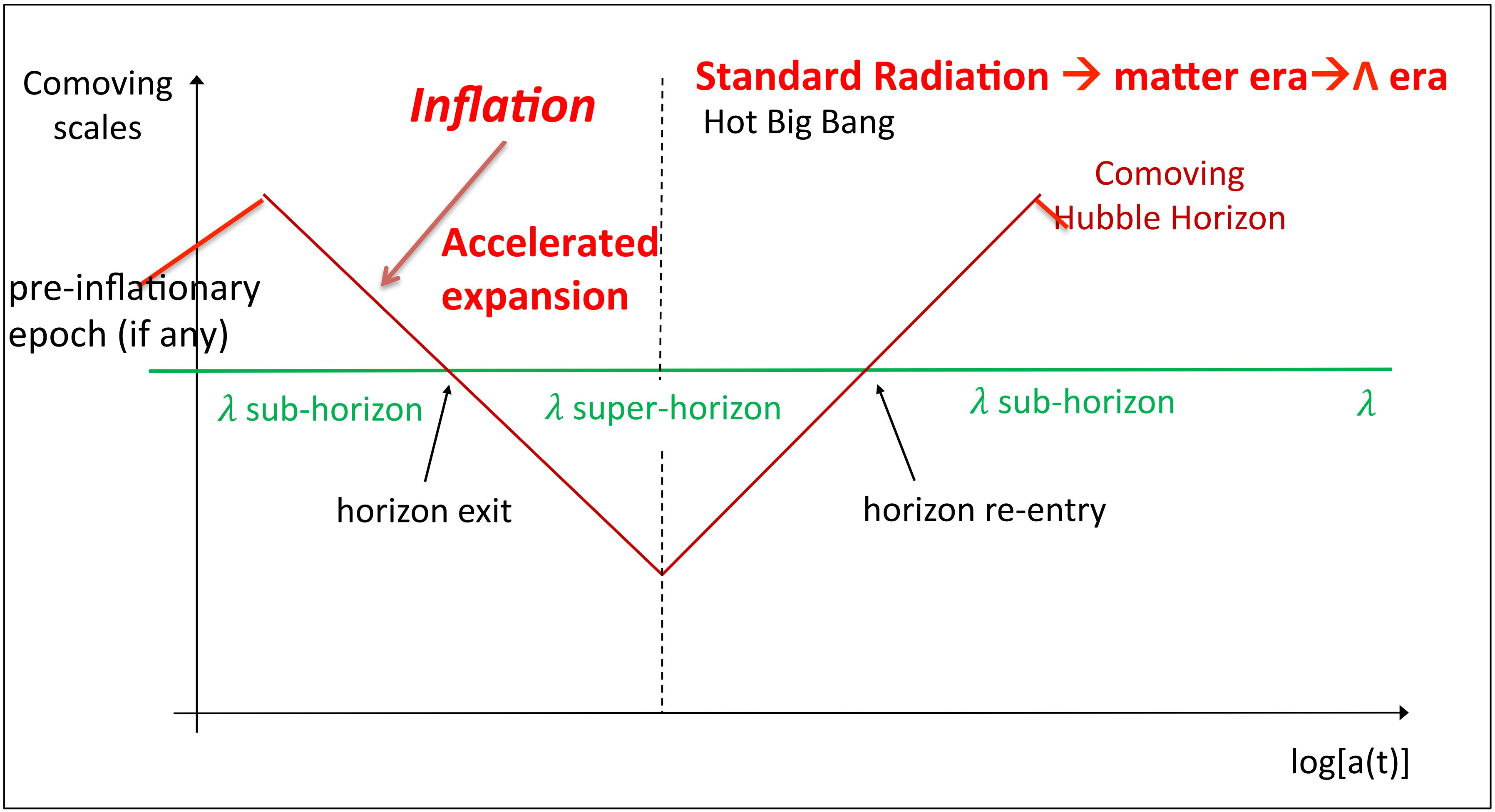
The rise and fall … of the comoving Hubble horizon



4 FACTS INFLATION CAN EXPLAIN

- The Universe is old
- The Universe is homogeneous and isotropic (for a comoving observers on large scales)
- The Universe today is very close to be spatially flat
- **Most importantly**: Structures grew out of tiny, *nearly* scale invariant (*almost* Gaussian) perturbations

SOME BASICS 1

A homogeneous and isotropic universe: $ds^2 = c^2dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right]$

$a(t)$ cosmological scale factor (physical length $\lambda_{\rm phys} \propto a(t)$) (Robertson-Walker metric)

From Einstein+continuity equations

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}$$

$$\dot{\rho} + 3H(\rho + p) = 0$$

and

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)$$

with the total energy density of the universe and pressure $\rho = \rho(t), \;\; p = p(t)$

Equation of state w: $p = w\rho$ **; for collisionless matter** $w = 0$

for radiation $w = 1/3$

SOME BASICS 1

Einstein equations $G^{\mu}{}_{\nu} = 8\pi G\, T^{\mu}{}_{\nu}$ $\qquad T^{\mu}{}_{\nu} = \mathrm{diag}(\rho, -p, -p, -p)$

$$\dot{\rho} + 3H(\rho + p) = 0 \rightarrow \rho(t) \propto a^{-3(1+w)} \begin{cases} \rho_m \propto a^{-3} & \text{For a matter dominated universe} \\ \rho_r \propto a^{-4} & \text{For a radiation dominated universe} \end{cases}$$

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2} \rightarrow a(t) \propto t^{\frac{2}{3(1+w)}} \begin{cases} a(t) \propto t^{1/2} & \text{matter} \\ a(t) \propto t^{2/3} & \text{radiation} \end{cases}$$

(for k=0 or at early times)

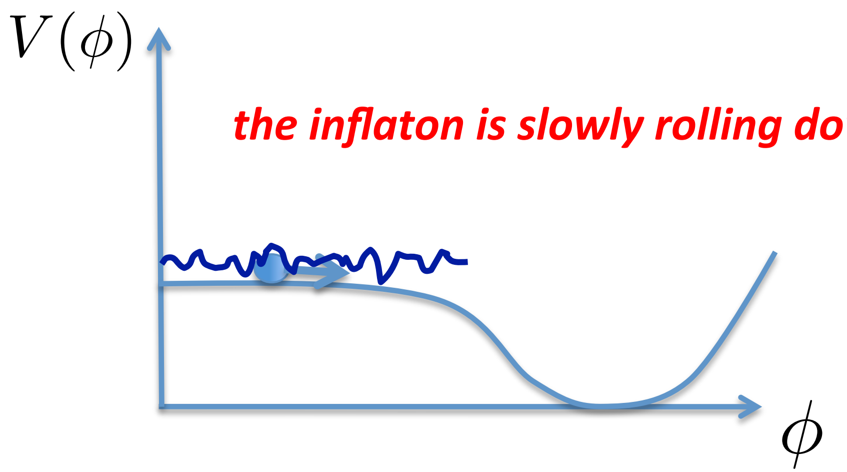
N.B.: $\dfrac{\ddot{a}}{a} < 0$ for a matter or dominated universe

Inflation

A single real quantum scalar field with a canonical kinetic term on top of a rather ***flat potential***

$$\mathcal{L} = \frac{1}{2} M_{\rm Pl}^2 R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi)$$

(and minimally coupled to gravity; GR; Bunch-Davies vacuum)

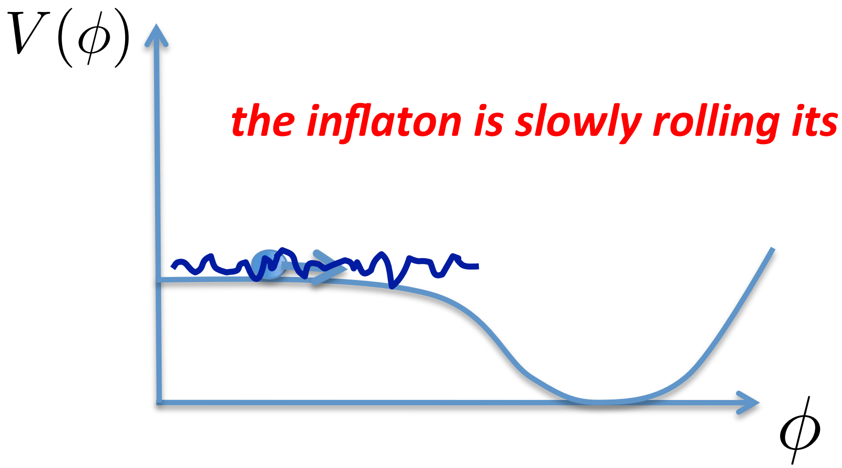


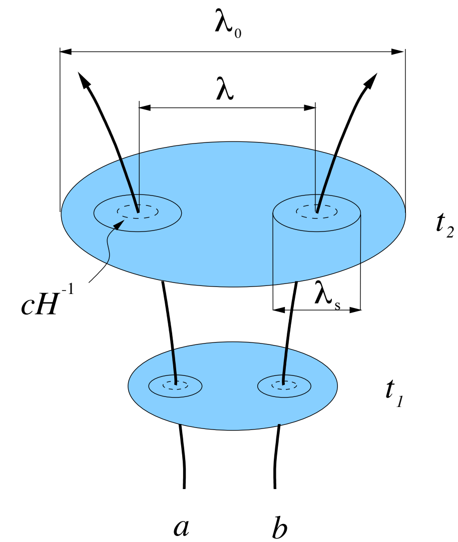
✓ $V(\phi) >> \frac{1}{2}\dot{\phi}^2 \longrightarrow H^2 = \frac{8\pi G}{3} V(\phi) \simeq const. \longrightarrow a(t) \simeq e^{Ht}$ ***Accelerated expansion***

$$\varepsilon = \frac{M_{Pl}^2}{2}\left(\frac{V_\phi}{V}\right)^2 << 1$$

$$\eta = M_{Pl}^2 \frac{V_{\phi\phi}}{V} << 1$$

Inflation





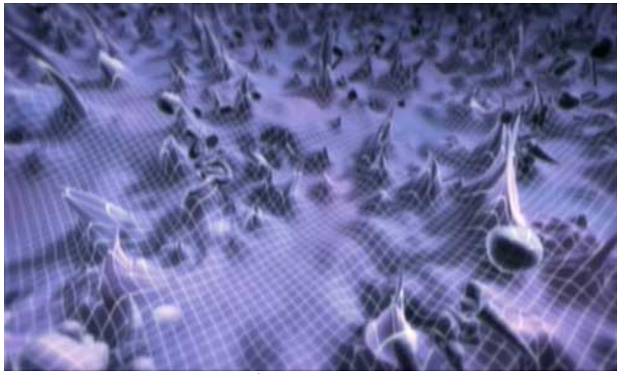
- On large (super-horizon scales) each region in the universe goes through the same expansion history but at slightly different times:

$$\phi(\mathbf{x}, t) = \phi_0(t - \delta t(\mathbf{x})) \longrightarrow \delta\phi(\mathbf{x}, t) = -\delta t(\mathbf{x})\dot{\phi}_0(t)$$

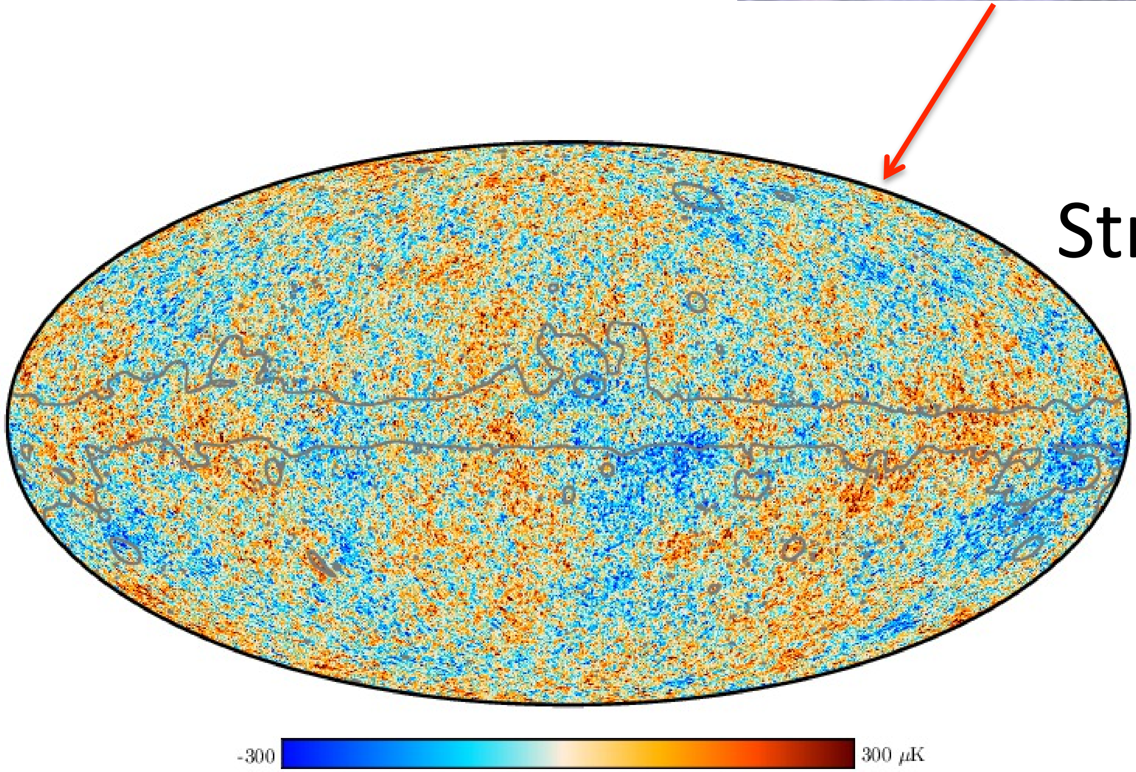
- Fluctuations in the inflaton produce fluctuations in the universe expansion from place to place $\longrightarrow$

$$H^2 \simeq \frac{8}{3}\pi G \rho(\phi)$$

Primordial seeds for stuctures in the Universe

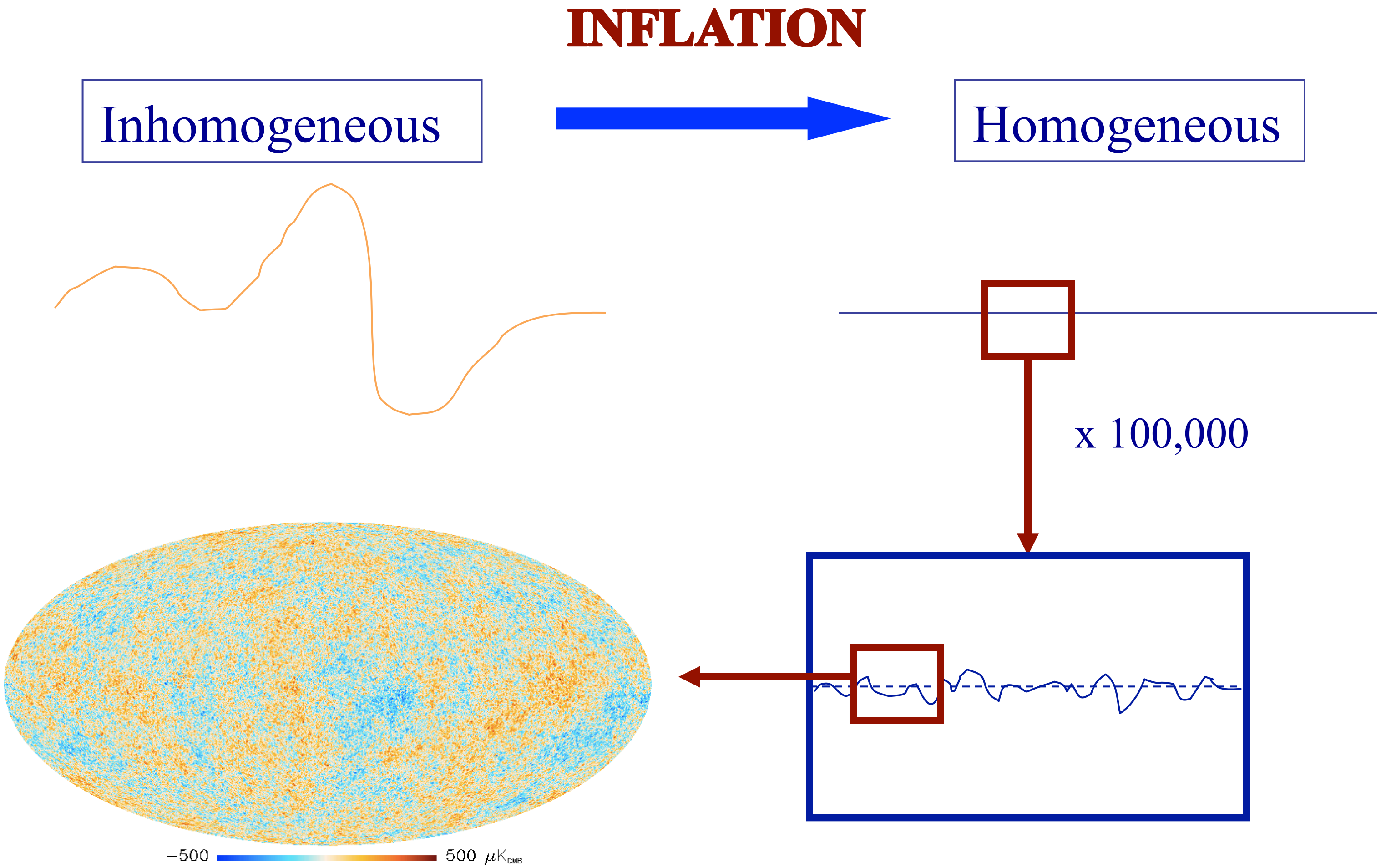


Initial quantum fluctuations



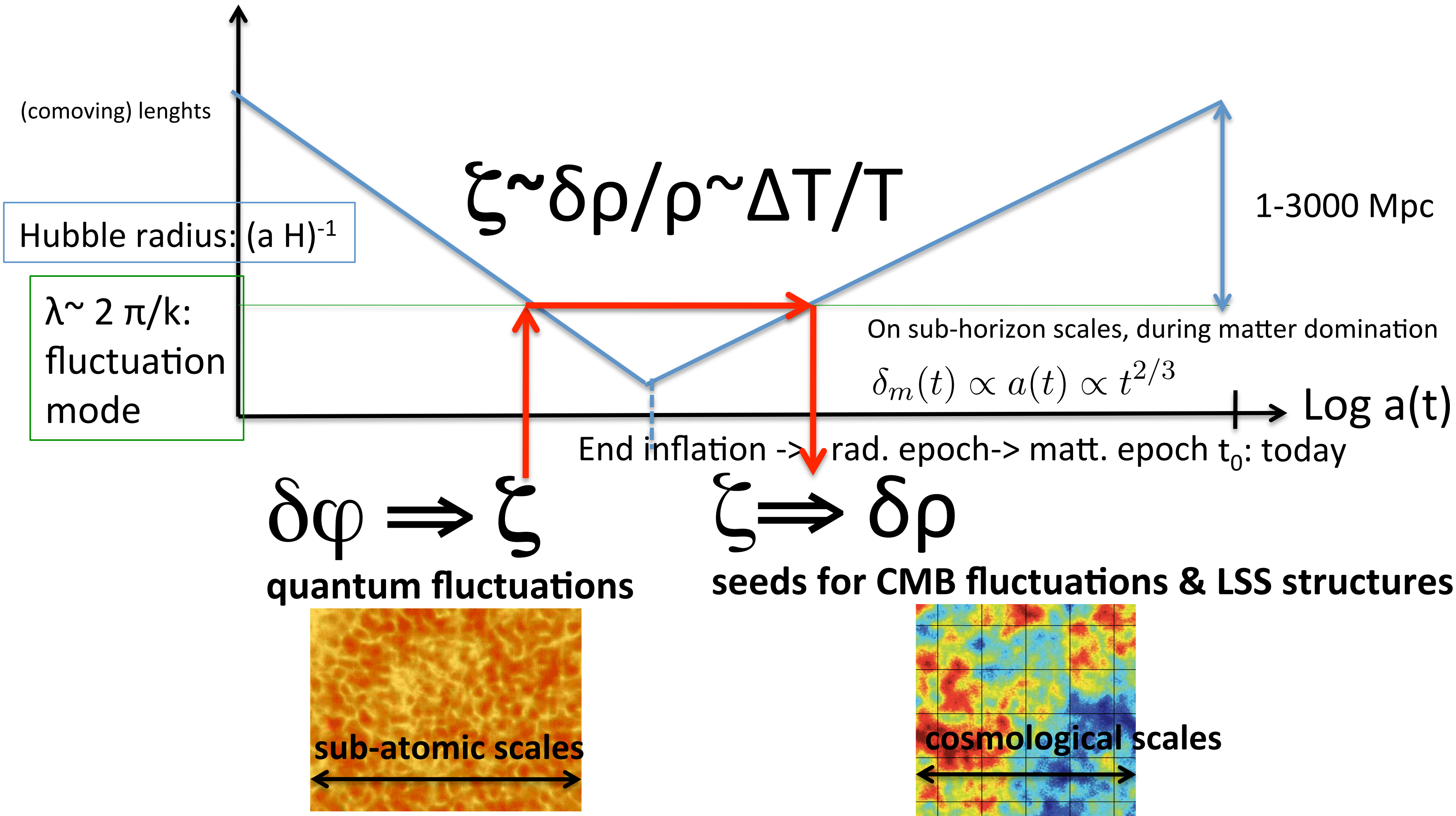
Structures we see today

Initial conditions



Structure formation within the inflationary scenario

Quantum fluctuations are streched from microscopic to cosmological scales



Primordial gravitational waves

GWs are tensor perturbations of the metric. Restricting ourselves to a flat FRW background (and disregarding scalar and vector modes)

$$ds^2=a^2(\tau)[-d\tau^2+(\delta_{ij}+h_{ij}(\underline{x},\tau))\,dx^i\,dx^j]$$

where h_{ij} are tensor modes which have the following properties

$h_{ij} = h_{ji}$ (symmetric)

$h^i_i = 0$ (traceless)

$h^i_{j|i}= 0$ (transverse, i.e. divergence free)

and satisfy the equation of motion

$$h''_{ij}+2\frac{a'}{a}h'_{ij}-\nabla^2 h_{ij}=0$$

$' = d/d\tau$

i,j=1,2,3

Primordial gravitational waves

GWs have only (9→6-1-3=) 2 independent degrees of freedom, corresponding to the 2 polarization states of the graviton

$$h_{ij}(\vec{x},\tau) = \int \sum_{\lambda=+,\times} \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\bullet\vec{x}} h_\lambda(\vec{k},\tau)\varepsilon_{ij}^\lambda(\vec{k})$$

$$h_\lambda'' + 2\frac{a'}{a}h_\lambda' + k^2 h_\lambda = 0$$

polarization tensor

free massless, minimally coupled scalar field

behaviour:

k « aH (outside the horizon) **h ≈ const + decaying mode**

k » aH (inside the horizon**)** **h ≈ $e^{\pm ik\tau}/a$** gravitational wave; it freely streams, experiencing redshift and dilution, like a free photon)

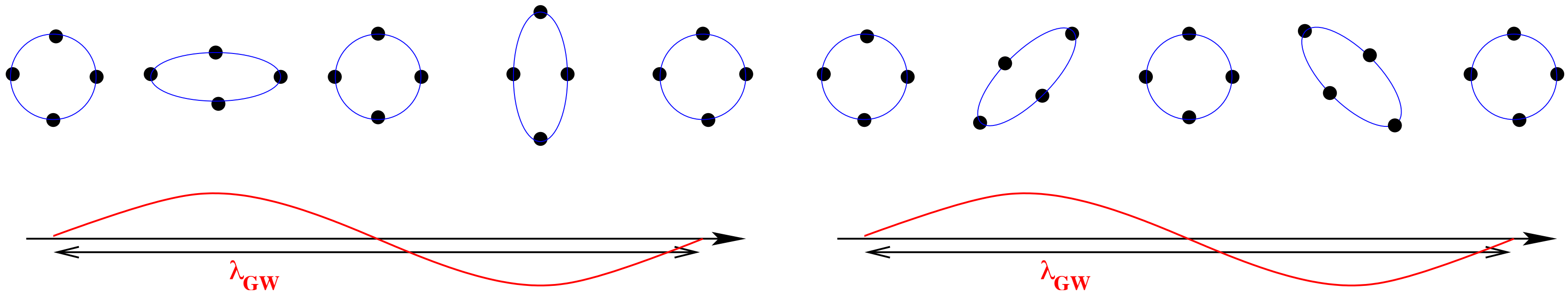
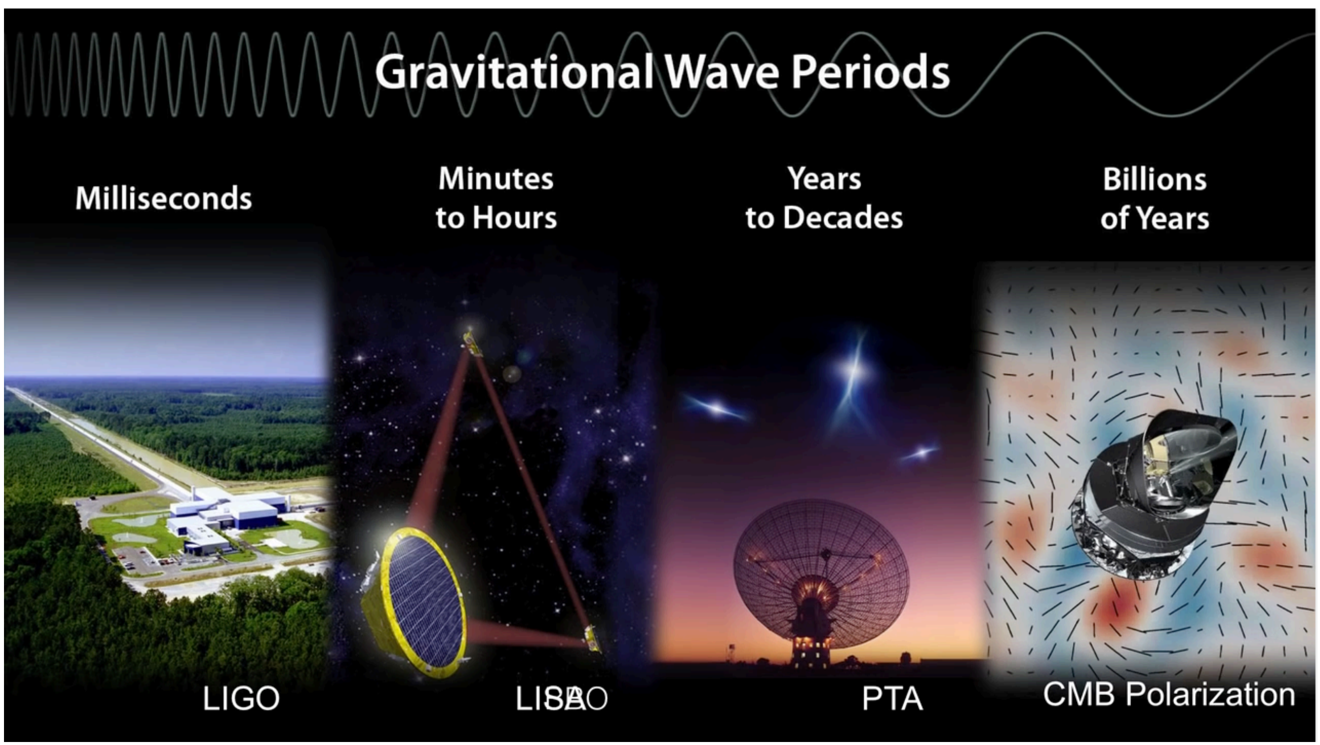
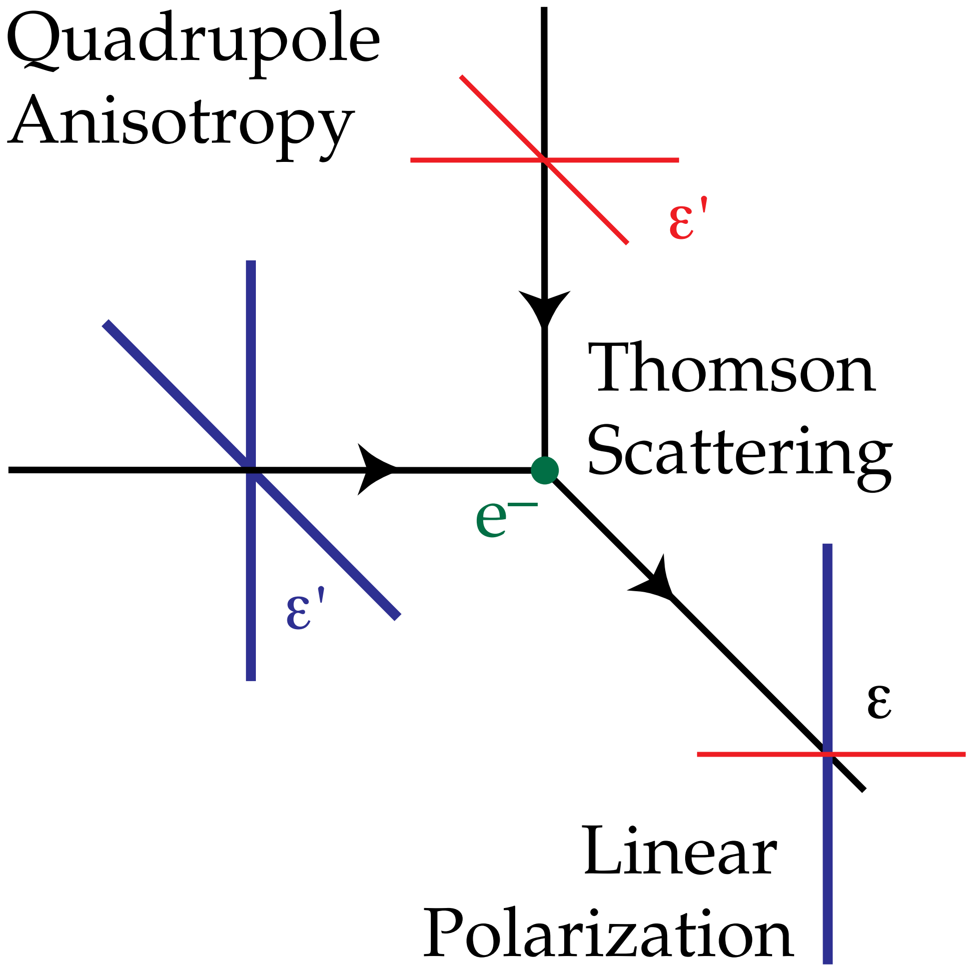


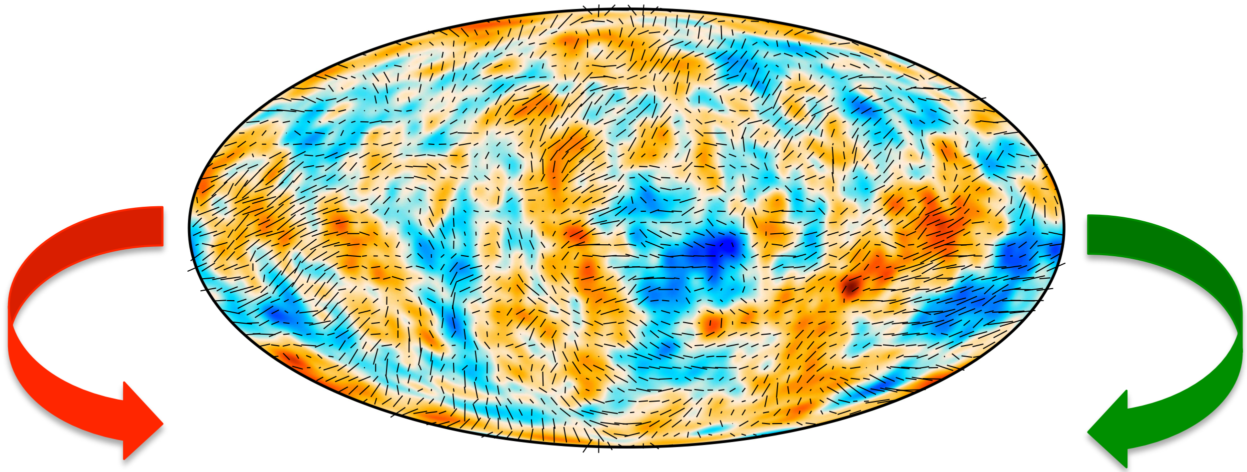
Fig. 1. We show how point particles along a ring move as a result of the interaction with a GW propagating in the direction perpendicular to the plane of the ring. The left panel refers to a wave with $+$ polarization, the right panel with $\times$ polarization.



Looking for gravitational waves via CMB polarization

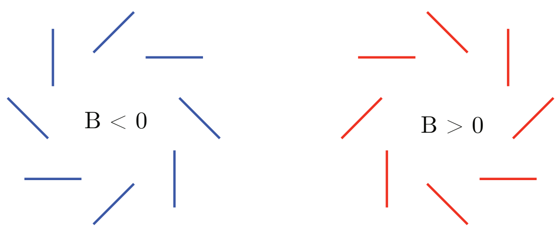


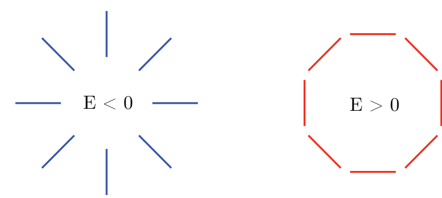
Looking for gravitational waves via CMB polarization



B-modes

E-modes





Sourced by tensor (and vector) perturbations

$$P_{\rm T} \sim \left(\frac{V}{M_{\rm Pl}}\right)^4$$

Sourced by scalar and tensor (and vector) perturbations

Primary goal for future CMB experiments

Observational predictions

- **Primordial density (scalar) perturbations**

$$\mathcal{P}_\zeta(k) = \frac{16}{9}\frac{V^2}{M_{\rm Pl}^4\dot\phi^2}\left(\frac{k}{k_0}\right)^{n-1}$$

amplitude

spectral index: $n-1 = 2\eta - 6\epsilon$
(or ``tilt'')

$$\epsilon = \frac{M_{\rm Pl}^2}{16\pi}\left(\frac{V'}{V}\right)^2 \ll 1;\ \eta = \frac{M_{\rm Pl}^2}{8\pi}\left(\frac{V''}{V}\right) \ll 1$$

- **Primordial (tensor) gravitational waves:** ***a smoking gun for inflation***

$$\mathcal{P}_{\rm T}(k) = \frac{128}{3}\frac{V}{M_{\rm Pl}^4}\left(\frac{k}{k_0}\right)^{n_{\rm T}}$$

Tensor spectral index: $n_{\rm T} = -2\epsilon$

Energy scale of inflation

- **Tensor-to-scalar perturbation ratio**

$$r = \frac{\mathcal{P}_{\rm T}}{\mathcal{P}_\zeta} = 16\epsilon$$

- **Consistency relation** (valid for *all* single field models of slow-roll inflation):

$$r = -8n_T$$

Current observational status

Constraints from CMB: *Planck*

- Primordial density perturbations: Amplitude $\ln(10^{10}A_s) = 3.044 \pm 0.014 \quad (68\%\,\mathrm{CL})$

$$n_s = 0.9649 \pm 0.0042 \quad (68\%\,\mathrm{CL})$$

n_s=1 (Harrison Zeld' ovich spectrum) excluded at 8.4 sigmas!!

Two fundamental observational constants of cosmology in addition to three very well known (Ω_b,Ω_{cdm},Ω_Λ)

Latest constraints

- **Primordial gravitational waves:**

Cosmological model ΛCDM+r	Parameter	*Planck* TT,TE,EE +lowEB+lensing	*Planck* TT,TE,EE +lowE+lensing+BK15	*Planck* TT,TE,EE +lowE+lensing+BK15+BAO
	r	< 0.11	< 0.061	< 0.063
	$r_{0.002}$	< 0.10	< 0.056	< 0.058
	n_s	0.9659 ± 0.0041	0.9651 ± 0.0041	0.9668 ± 0.0037

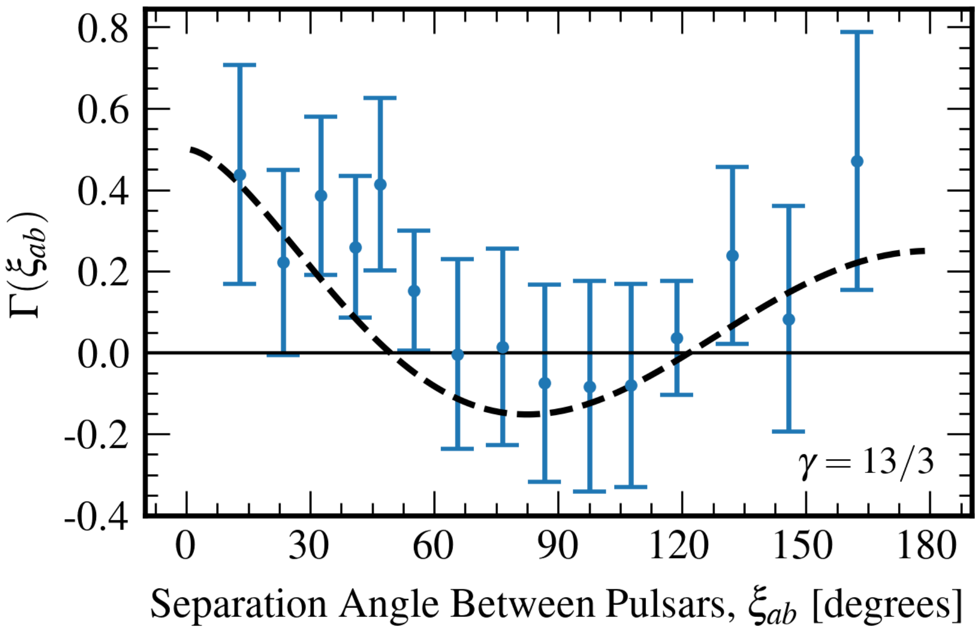
Latest constraints: from *Planck TT, EE,* EB, BB PR3/PR4 + BICEP/Keck15/18 data+Ligo/Virgo/Kagra

$$r_{0.01} < 0.028 \quad @95\%\mathrm{CL}$$

Energy scale of inflation $V^{1/4} < 1.6 \times 10^{16}\,\mathrm{GeV}$

A new era (the CMB B-mode era) has started!
Target of future CMB experiments: $r < 10^{-3}$

Nanograv 15 year data

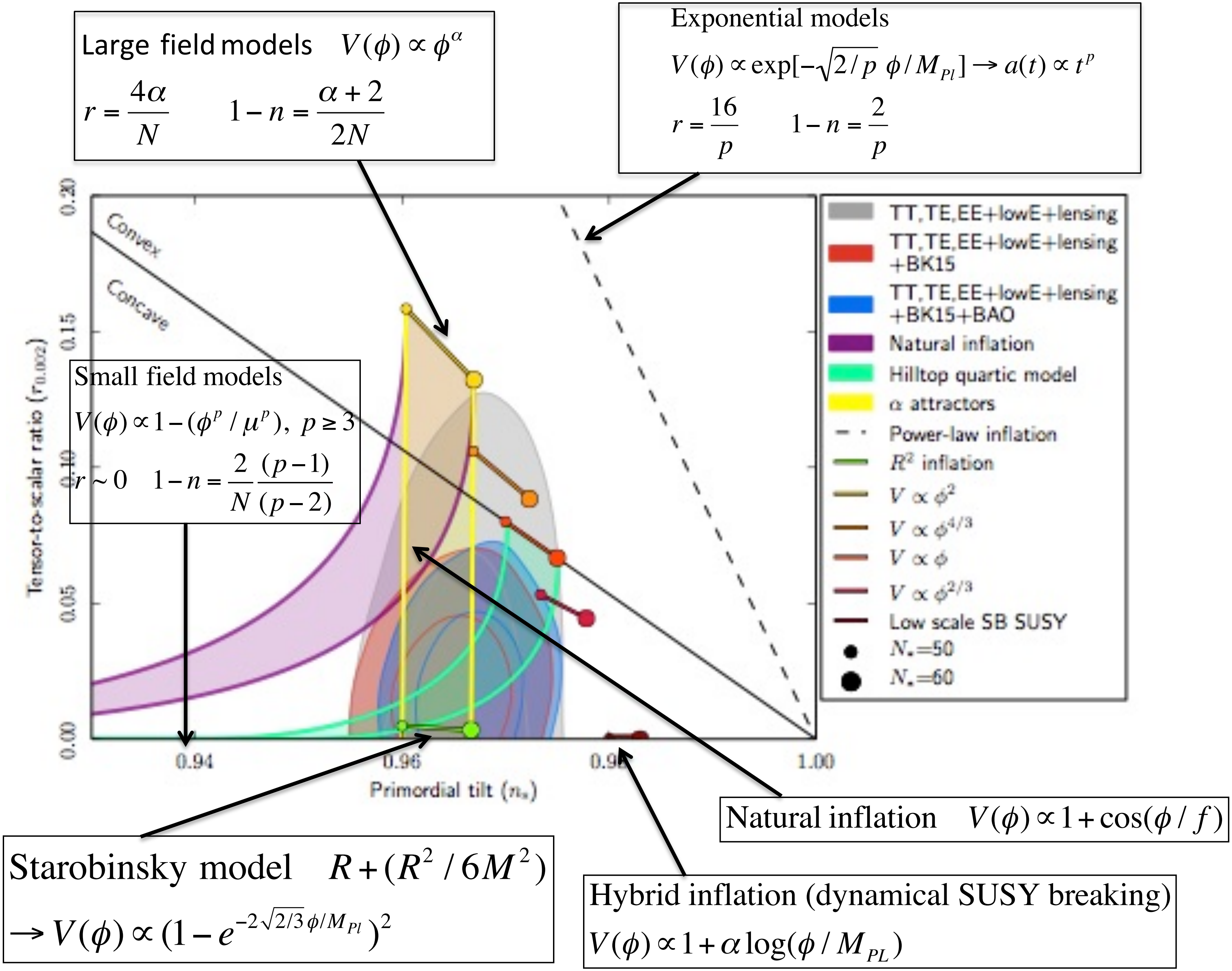


Pulsar timing array (PTA) collaborations, NANOGrav, EPTA, PPTA, and CPTA, have presented evidence for an isotropic stochastic gravitational wave background (GWB).

A Cosmological origin is a possibility (if from inflation for sure models beyond the standard ones).

What are the implications for inflationary models ?**

** I am talking here about single-field slow roll models of inflation


Large field models $V(\phi) \propto \phi^{\alpha}$
$r = \frac{4\alpha}{N}$ $1-n = \frac{\alpha+2}{2N}$
Exponential models
$V(\phi) \propto \exp[-\sqrt{2/p}\,\phi/M_{Pl}] \rightarrow a(t) \propto t^{p}$
$r = \frac{16}{p}$ $1-n = \frac{2}{p}$
Small field models
$V(\phi) \propto 1-(\phi^{p}/\mu^{p}),\ p \geq 3$
$r \sim 0$ $1-n = \frac{2}{N}\frac{(p-1)}{(p-2)}$
Convex
Concave
Tensor-to-scalar ratio ($r_{0.002}$)
Primordial tilt (n_s)
0.00
0.05
0.10
0.15
0.20
0.94
0.96
0.98
1.00
TT,TE,EE+lowE+lensing
TT,TE,EE+lowE+lensing +BK15
TT,TE,EE+lowE+lensing +BK15+BAO
Natural inflation
Hilltop quartic model
α attractors
Power-law inflation
R^2 inflation
$V \propto \phi^2$
$V \propto \phi^{4/3}$
$V \propto \phi$
$V \propto \phi^{2/3}$
Low scale SB SUSY
$N_*=50$
$N_*=60$
Natural inflation $V(\phi) \propto 1+\cos(\phi/f)$
Starobinsky model $R+(R^2/6M^2)$
$\rightarrow V(\phi) \propto (1-e^{-2\sqrt{2/3}\,\phi/M_{Pl}})^2$
Hybrid inflation (dynamical SUSY breaking)
$V(\phi) \propto 1+\alpha\log(\phi/M_{PL})$

So far so good.....but.....
what is the precise mechanism behind inflation?

$$S = \int d^4x \sqrt{-g} \left[???? \right]$$

Standard single-field slow-roll
simple power-law spectra

Multiple-fields

Features in the potential

Modified gravitational sector

Higher-order derivative interactions or
non-canonical kinetic term

$\mathcal{L}(\phi, X)$ with $X = \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$

At least two (main) avenues:

- ***gravitational waves***
- ***primordial non-Gaussianity***

We have seen that in cosmology (spatial) correlators are among the most important statistical estimators

So what about higher-order correlators?

↕

(Primordial) non-Gaussianity

(aka: going beyond the (r-n_s) plane)

Primordial NG

ζ(**x**): primordial perturbations

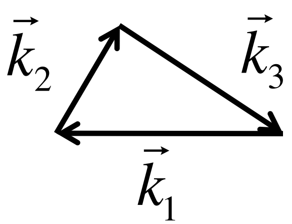
If the fluctuations are Gaussian distributed then their statistical properties are completely characterized by the two-point correlation function, $<\zeta(\mathbf{x}_1)\zeta(\mathbf{x}_2)>$ or its Fourier transform, the power-spectrum.

Thus a non-vanishing ***three point function***, or its Fourier transform, the ***bispectrum is an indicator of non-Gaussianity***

$$\left\langle \zeta(\vec{k}_1)\zeta(\vec{k}_2)\zeta(\vec{k}_3)\right\rangle = (2\pi)^3\delta^{(3)}(\vec{k}_1+\vec{k}_2+\vec{k}_3)f_{NL}F(k_1,k_2,k_3)$$

Amplitude

Shape



$$\rightarrow \left\langle \frac{\Delta T}{T}(n_1)\frac{\Delta T}{T}(n_2)\frac{\Delta T}{T}(n_3)\right\rangle$$

Primordial NG

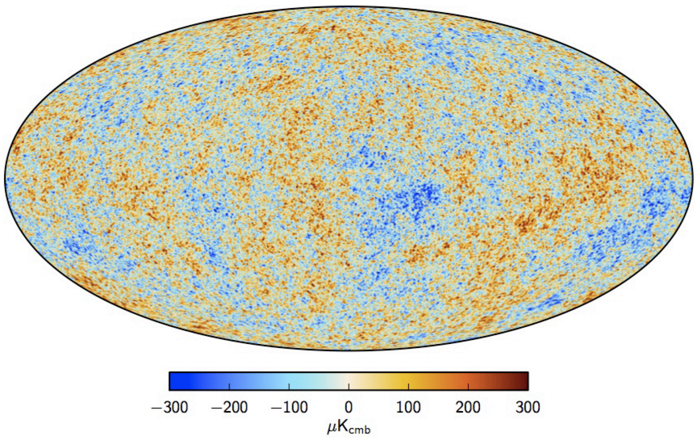
Gaussian ⟺ free (i.e. non-interacting) field, linear theory

Collection of independent harmonic oscillators
(no mode-mode coupling)

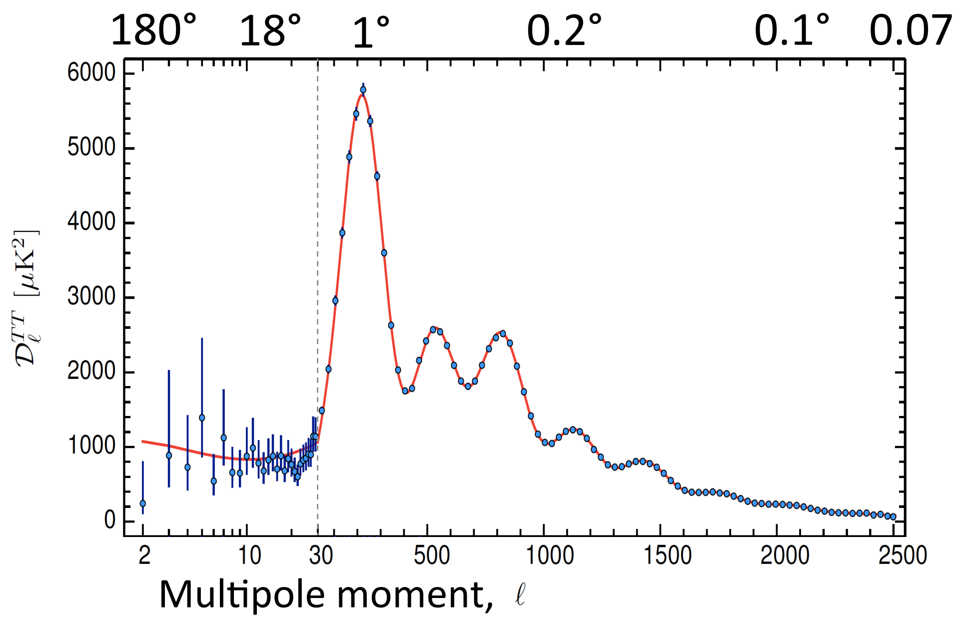
Physical origin of primordial NG:
self-interactions of the inflaton field, e.g. $\lambda\,\phi^3$,
interactions between different fields,
non-linear evolution of the fields during inflation,
gravity itself is non linear…..

Why primordial NG is important?

Bispectrum vs power spectrum information



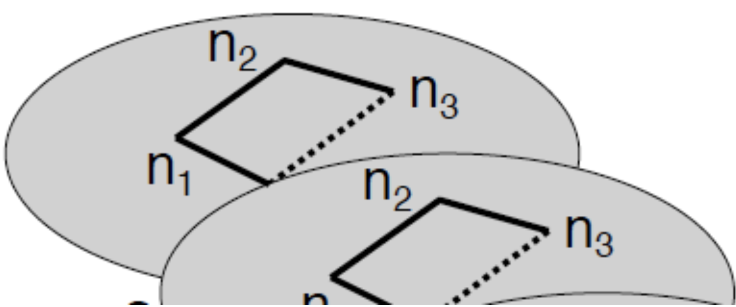
Planck 2015 Results. I. Overview of products and scientific results



$5{\times}10^6$ pixels compressed
into ~2500 numbers:
O.K. only if gaussian

If not we could miss
precious information

Measure 3 point-function
and higher-order



One (among many) good reason:

f_{NL} and shape are model dependent:
e.g.: standard single-field models of slow-roll inflation predict

$$f_{NL} \sim O(\varepsilon,\eta) << 1$$

(Acquaviva, Bartolo, Matarrese, Riotto 2002; Maldacena 2002)

A detection of a primordial $|f_{NL}| \sim 1$ would rule out *all* standard single-field models of slow-roll inflation

One (among many) very good reason:

f_{NL} and shape are model dependent:

e.g.: floor set by standard single-field models of slow-roll inflation is

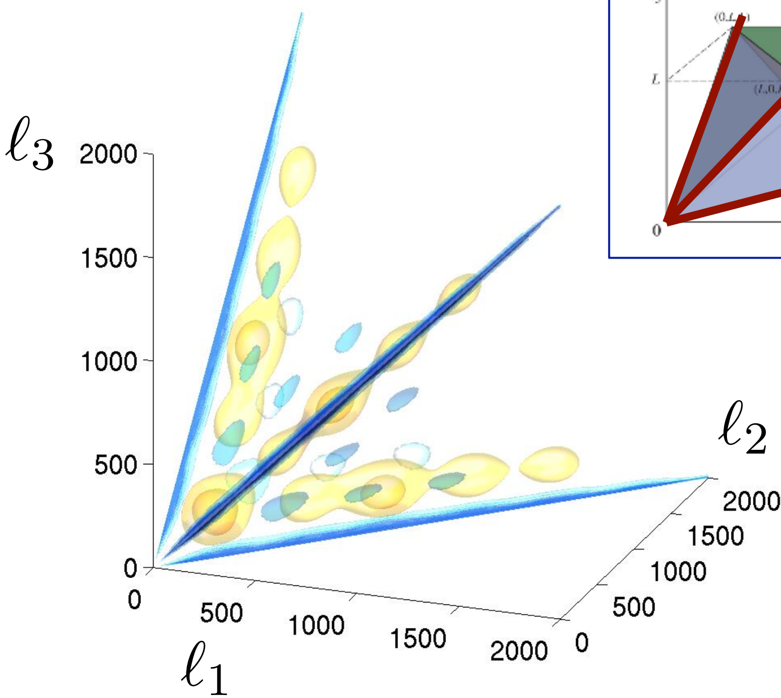
$$f_{NL} \sim O(\varepsilon,\eta) \ll 1$$

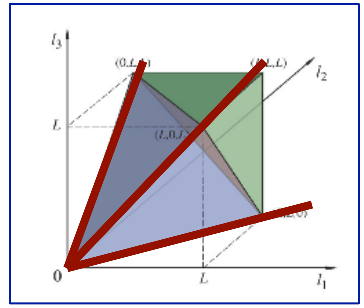
(Acquaviva, Bartolo, Riotto, Matarrese Nucl. Phys. B 2003; Maldacena JHEP 2003)

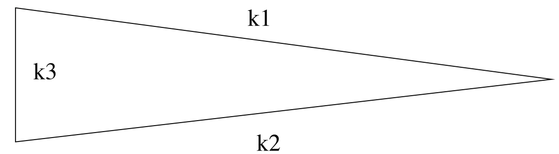
A detection of a primordial $|f_{NL}| \sim 1$ would rule out *all standard* single-field models of slow-roll inflation

Shapes of NG: local NG

Bispectrum peaks for squeezed triangles $k_3 << k_1 \sim k_2$







$$\frac{\Delta T}{T}(\vartheta,\phi) = \sum_{\ell=2}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m} Y_{\ell m}(\vartheta,\phi)$$

$$B_{\ell_1 \ell_2 \ell_3} \sim \left\langle a_{\ell_1}^{m_1} a_{\ell_2}^{m_2} a_{\ell_3}^{m_3} \right\rangle$$

Babich et al. 04

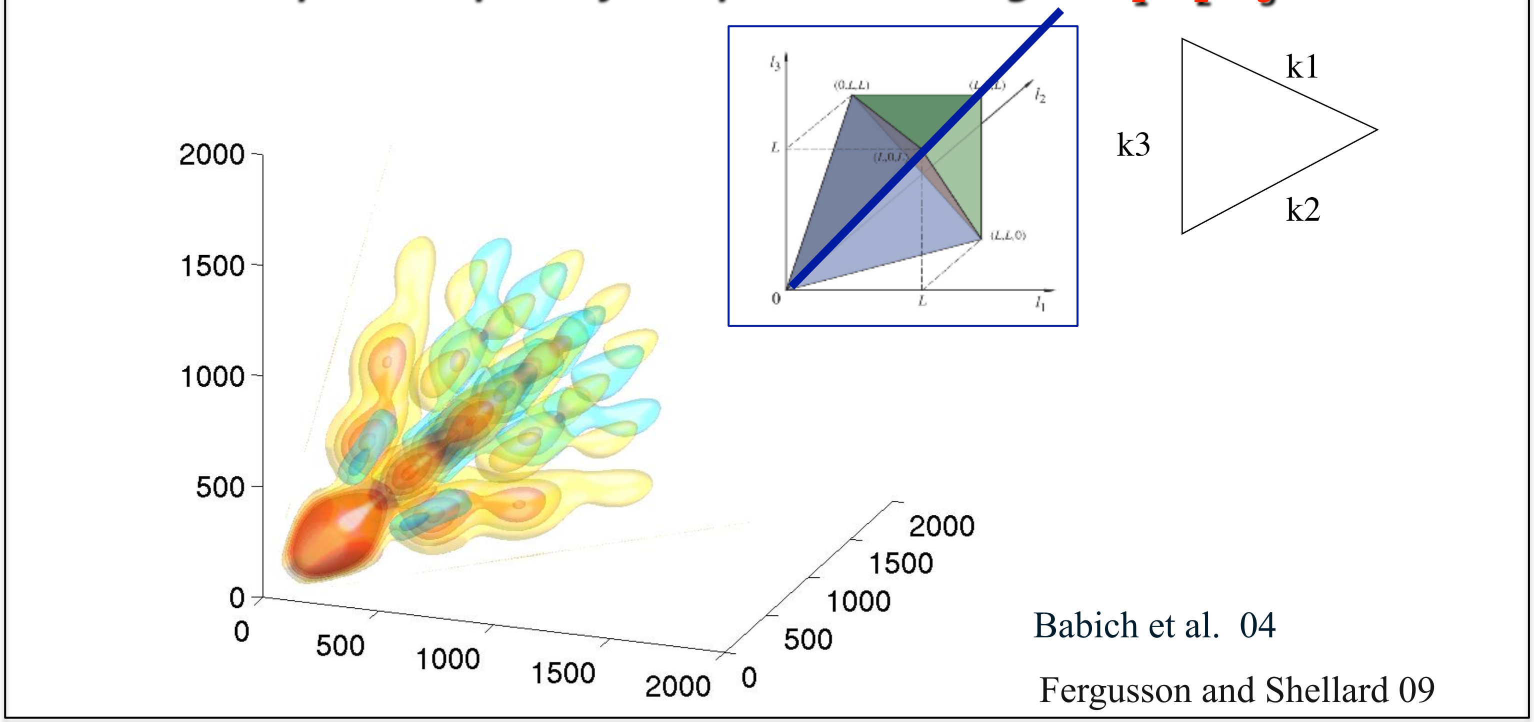
Fergusson and Shellard 09

$$\zeta(\mathbf{x}) = \zeta_g(\mathbf{x}) + \frac{3}{5} f_{\rm NL} \zeta_g^2(\mathbf{x})$$

Non-linearities develop outside the horizon during or immediately after inflation (e.g. ***multifield models of inflation***)

Equilateral NG

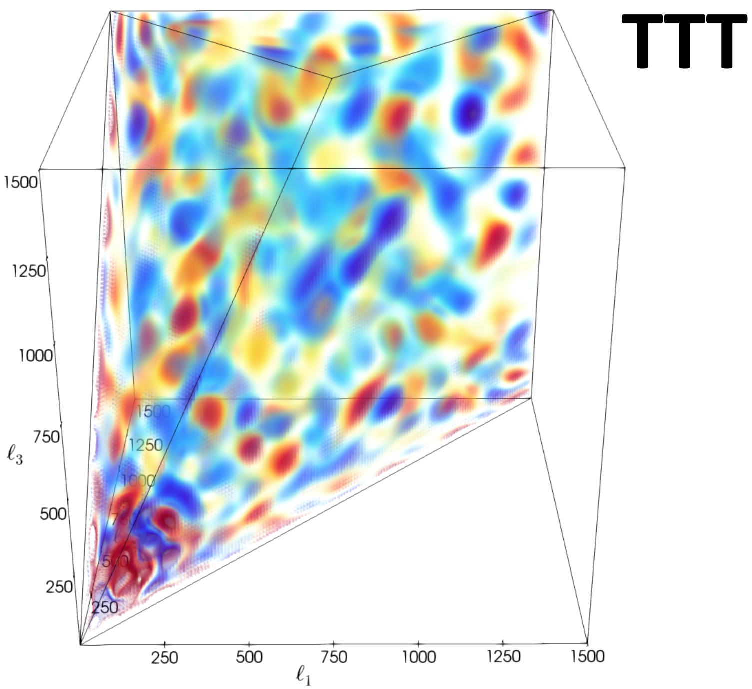
Bispectrum peaks for equilateral triangles: $k_1=k_2=k_3$



Single field models of inflation with non-canonical kinetic term L=P(φ, X) where X=(∂ φ)^2 (DBI or K-inflation) where NG comes from higher derivative interactions of the inflaton field

Example: $\delta\dot{\phi}(\nabla\delta\phi)^2$

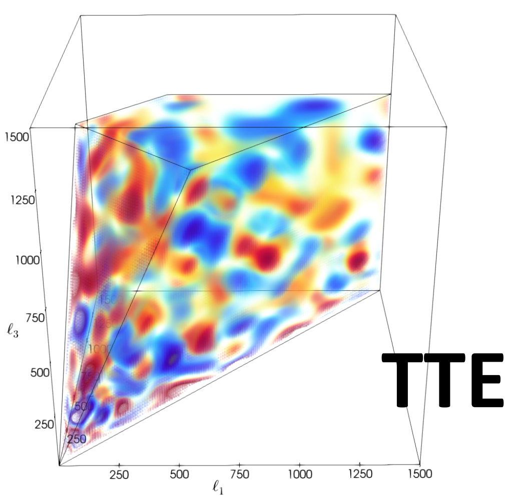
The CMB bispectrum as seen by Planck

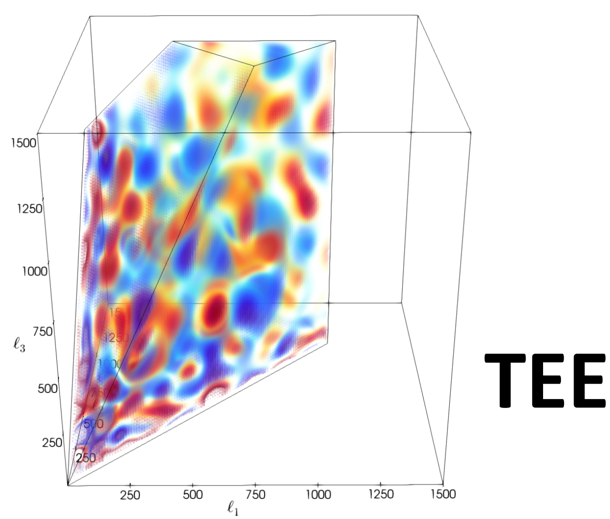


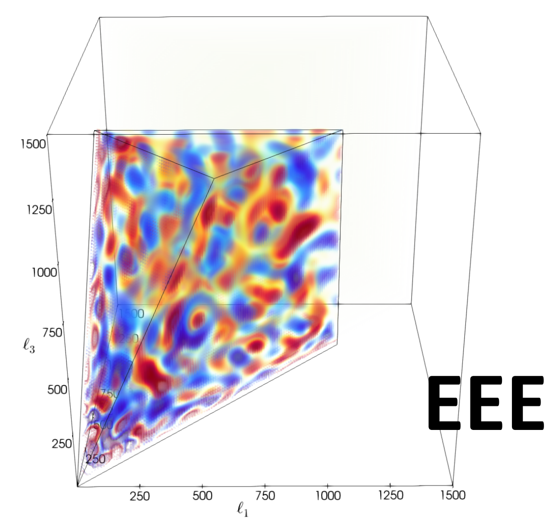
$$\frac{\Delta T}{T}(\vartheta,\phi) = \sum_{\ell=2}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m} Y_{\ell m}(\vartheta,\phi)$$

$$B_{\ell_1\ell_2\ell_3} = \sum_m \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ m_1 & m_2 & m_3 \end{pmatrix} \left\langle a_{\ell_1}^{m_1} a_{\ell_2}^{m_2} a_{\ell_3}^{m_3} \right\rangle;$$

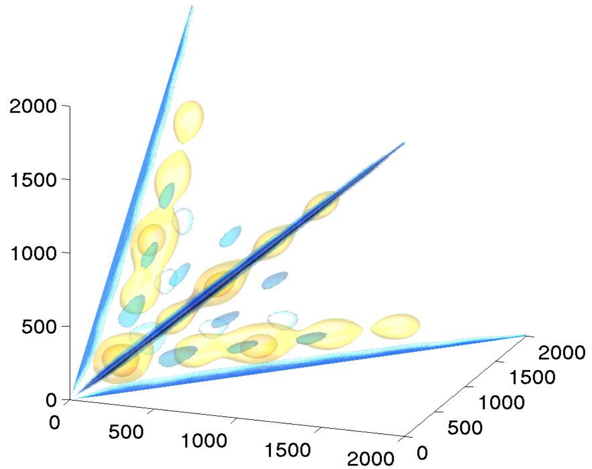
$$B_{\ell_1\ell_2\ell_3} = h_{\ell_1\ell_2\ell_3} b_{\ell_1\ell_2\ell_3}$$







LESSON: NG…IT'S NOT JUST A NUMBER



Not only an amplitude but also shape of non-Gaussianities, with a huge amount of information associated to triangular configurations

Constraints on f_{NL} translates into constraints of the coefficients of the interactions of the inflaton Lagrangian

$$\mathcal{L}[\phi] = \mathcal{L}_2 + \mathcal{L}_3 + \ldots$$

quadratic Lagrangian=
linear evolution

Cubic Lagrangian=interactions -> non-gaussianity

Limits set by Planck

See Planck 2018 results. IX. Constraints on primordial non-Gaussianity

Observational limits set by Planck

	f_{NL}(KSW)	
Shape	Independent	Lensing subtracted
	SMICA T	
Local	6.7 ± 5.6	-0.5 ± 5.6
Equilateral	4 ± 67	5 ± 67
Orthogonal	-38 ± 37	-15 ± 37
	SMICA $T+E$	
Local	4.1 ± 5.1	-0.9 ± 5.1
Equilateral	-25 ± 47	-26 ± 47
Orthogonal	-47 ± 24	-38 ± 24

e.g. models with non-standard kinetic terms

e.g. multi-field models of inflation

Planck 2018 results. IX. Constraints on primordial non-Gaussianity.

Implications for inflation models

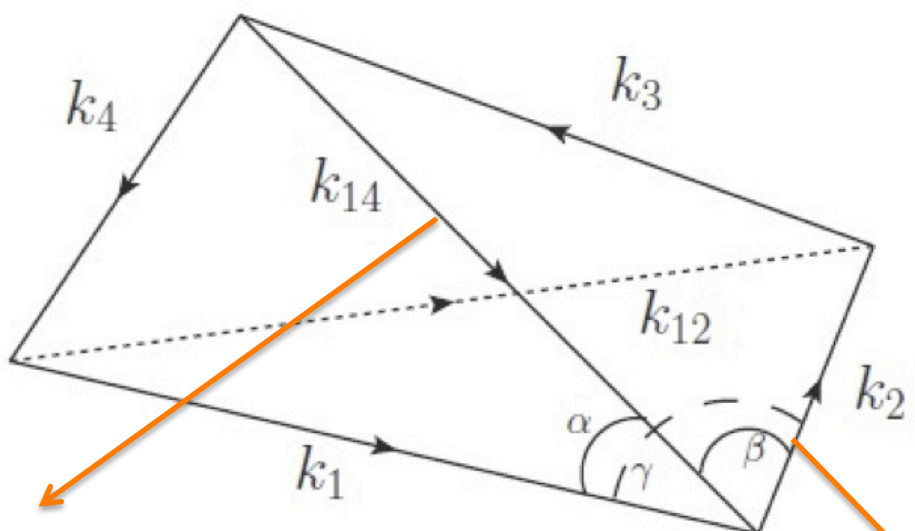
- The standard models of single-field slow-roll inflation has survived the most stringent tests of Gaussianity to-date: ***deviations from primordial Gaussianity are less than 0.01% level. This is a fantastic achievement, one of the most precise measurements in cosmology!***

$$\Phi(\mathbf{x}) = \Phi^{(1)}(\mathbf{x}) + f_{\mathrm{NL}} \left(\Phi^{(1)}(\mathbf{x})\right)^2 + \ldots\ldots$$

~10^{-5} ~few ~10^{-10}

- ***The NG constraints*** on different primordial bispectrum shapes ***severly limit/rule out specific key (inflationary) mechanisms alternative to the standard models of inflation***

Looking at the inflationary trispectra



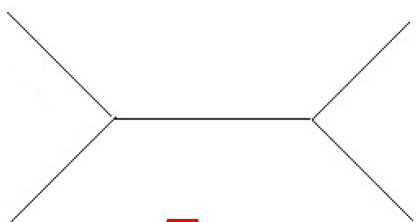
e.g. k_14 -> 0
corresponds to τ_NL:
a modulation of the two power spectra

e.g. k_2 -> 0
corresponds to g_NL:
a modulation of the bispectrum

Planck limits on trispectra

$$\langle \hat{\zeta}_{\vec{k}_1} \hat{\zeta}_{\vec{k}_2} \hat{\zeta}_{\vec{k}_3} \hat{\zeta}_{\vec{k}_4} \rangle = (2\pi)^3 \delta^{(3)}(\vec{k}_1 + \vec{k}_2 + \vec{k}_3 + \vec{k}_4) T_\zeta(\vec{k}_1, \vec{k}_2, \vec{k}_3, \vec{k}_4)$$

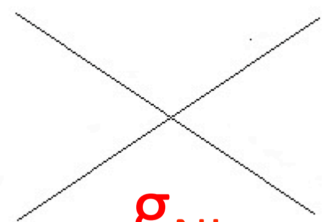
Scalar exchange:
comes from terms in the 3-oder action, e.g. $(\delta\phi)^3$



τ_{NL}

$$\tau_{\rm NL}^{\rm loc} < 2800 \quad (95\%\,{\rm CL})$$

Contact interaction: e.g. $\lambda\,(\delta\phi)^4$ (intrinsic contributions from the 4-th order action)



g_{NL}

$$g_{\rm NL}^{\rm local} = (-5.8 \pm 6.5) \times 10^4$$
$$g_{\rm NL}^{\dot\sigma^4} = (-0.8 \pm 1.9) \times 10^6$$
$$g_{\rm NL}^{(\partial\sigma)^4} = (-3.9 \pm 3.9) \times 10^5$$

Also From LSS

$-4.5 \times 10^5 < g_{\rm NL} < 1.6 \times 10^5$ 95%CL *(Giannantonio et al. 2013)*

Primordial non-Gaussianity allows to answer to some very simple, but fundamental questions you might have about inflation:

- ***What is the sound speed the inflaton fluctuations propagate with?***
- ***Are there other particles other than the inflaton?***
- ***What are their masses and spins?***

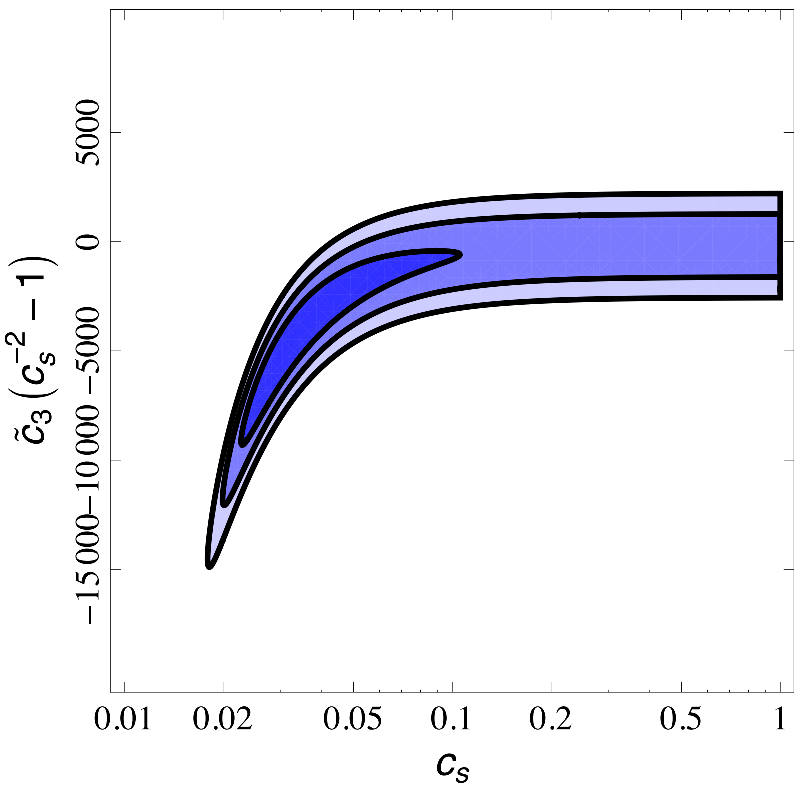
Measuring the of sound speed of the inflation

- General single-field models of inflation: Implications for Effective Field Theory of Inflation

$$S = \int d^4x \sqrt{-g}\left[-\frac{M_{\rm Pl}^2 \dot{H}}{c_s^2}\left(\dot{\pi}^2 - c_s^2 \frac{(\partial_i \pi)^2}{a^2}\right) - M_{\rm Pl}^2 \dot{H}(1 - c_s^{-2})\dot{\pi}\frac{(\partial_i\pi)^2}{a^2} + \left(M_{\rm Pl}^2 \dot{H}(1-c_s^{-2}) - \frac{4}{3}M_3^4\right)\dot{\pi}^3 \right]$$

$$f_{\rm NL} \propto \frac{1}{c_s^2}$$

(Cheung et al. 08; Weinberg 08)
for extensions see also N.B., Fasiello, Matarrese, Riotto 10)



Constraints obtained from

$$f_{\rm NL}^{\rm equil} = -26 \pm 47 \quad (68\%\ {\rm CL})$$

$$f_{\rm NL}^{\rm ortho} = -38 \pm 24 \quad (68\%\ {\rm CL})$$

$$c_s \geq 0.021 \quad \text{at } 95\%\ {\rm CL}$$

The nature of inflation: new avenues

2. Tensor non-Gaussianities

B-mode non-Gaussianity can open up an unexplored window into the early Universe

Motivations: the nature of gravitational waves (I)

- ✓ A detection of GW would not by itself determine the precise mechanism generating the the tensor modes: ***alternative and new observational probes***

- ✓ ***<u>Go beyond the power spectrum and look for the statistical properties of GW:</u>***

$$\langle \gamma_{\mathbf{k}_1} \gamma_{\mathbf{k}_2} \gamma_{\mathbf{k}_3} \rangle \qquad \langle \gamma\zeta\zeta \rangle \qquad \langle \zeta\gamma\gamma \rangle$$

Full-sky	$\sum_n \ell_n = \text{even}$	$\sum_n \ell_n = \text{odd}$
Flat-sky	left-handed = right-handed	left-handed = (−) right-handed
Non-vanishing in parity-conserving universe	$\langle TTT \rangle$, $\langle TEE \rangle$, $\langle TTE \rangle$, $\langle EEE \rangle$, $\langle BBE \rangle$, $\langle BBT \rangle$	$\langle BTT \rangle$, $\langle BEE \rangle$, $\langle BET \rangle$, $\langle BBB \rangle$

$$a^T_{\ell m} \to (-1)^\ell a^T_{\ell m}\,,$$
$$a^E_{\ell m} \to (-1)^\ell a^E_{\ell m}\,,$$
$$a^B_{\ell m} \to (-1)^{\ell+1} a^B_{\ell m}$$

N.B.: single-field models do predcit these signals

M. Shiraishi, D. Nitta, and S. Yokoyama, `11; J.Maldacena, G. Pimenthel `11; X. Gao, T. Kobayashi, M. Shiraishi, M. Yamaguchi, J. Yokoyama, and S. Yokoyama, '13; M. Shiraishi, M.Liguori, J. Fergusson `15; Meerburg et al. `16; L. Dai, D. Jeong, M. Kamionkowski `13 and many more Refs.

The nature of gravitational waves (I)

✓ ***These observables can be signatures of <u>new physics.</u>***

* ***anisotropic evolution during inflation***
(e.g., N.B., Matarrese, Peloso, Ricciardone, '13; Akhshik, Emami,Firouzjahi, Wang '14; Endlich, Horn, Nicolis, Wang, '14; Bordin, Creminelli et al. '16)

* ***extra light spin-2 or higher spin particles*** (Harkani-Hamed, Maldacena '16)

* ***symmetry breaking patterns different w.r.t single-field models ("solid-like" models of inflation)*** (Endlich, Nicolis,Wang'13; N.B, Cannone, Ricciardone, Tasinato '16)

**** parity breaking signatures in the gravity sector***
(e.g., Madacena & Pimentel '11; Soda, Kodama, Nozawa '11; Shiraishi, Nitta, Yokoyama '11; N.B., Orlando Shiraishi, '17 & '19)

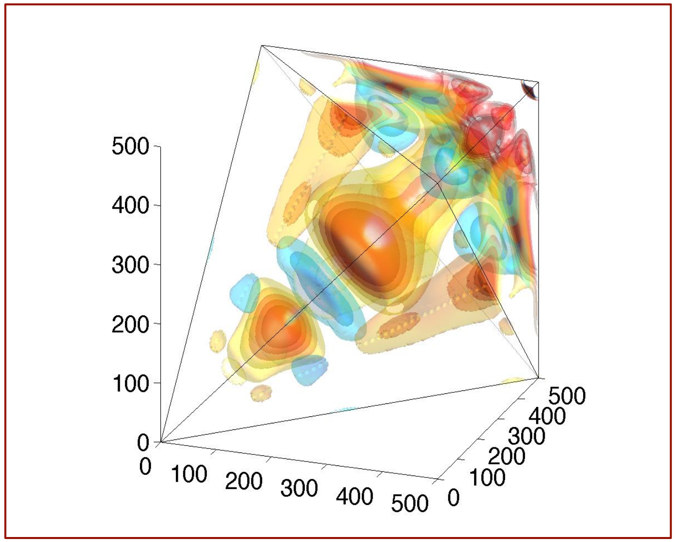
✓ Analyses already carried out within *Planck: different groups are developing the tools to build a full pipeline to fully characterize tensor non-Gaussianities.*

Some examples

Present constraints on tensor NG $\langle h_{\mathbf{k}_1} h_{\mathbf{k}_2} h_{\mathbf{k}_3} \rangle$

Planck

Parity-odd



	Even	Odd	All
SMICA			
T	4 ± 17	100 ± 100	6 ± 16
E	33 ± 67	-570 ± 720	29 ± 67
T+E	11 ± 14	1 ± 18	8 ± 11
SEVEM			
T	4 ± 17	90 ± 100	6 ± 16
E	75 ± 75	-790 ± 830	70 ± 75
T+E	16 ± 14	2 ± 20	13 ± 12
NILC			
T	4 ± 17	90 ± 100	6 ± 16
E	-16 ± 81	-540 ± 820	-19 ± 80
T+E	6 ± 14	3 ± 21	5 ± 11
Commander			
T	5 ± 17	90 ± 100	6 ± 16
E	21 ± 69	-1200 ± 700	13 ± 69
T+E	10 ± 14	-2 ± 19	7 ± 11

We fit parity-odd and parity-even bispectra to *Planck* T+E data

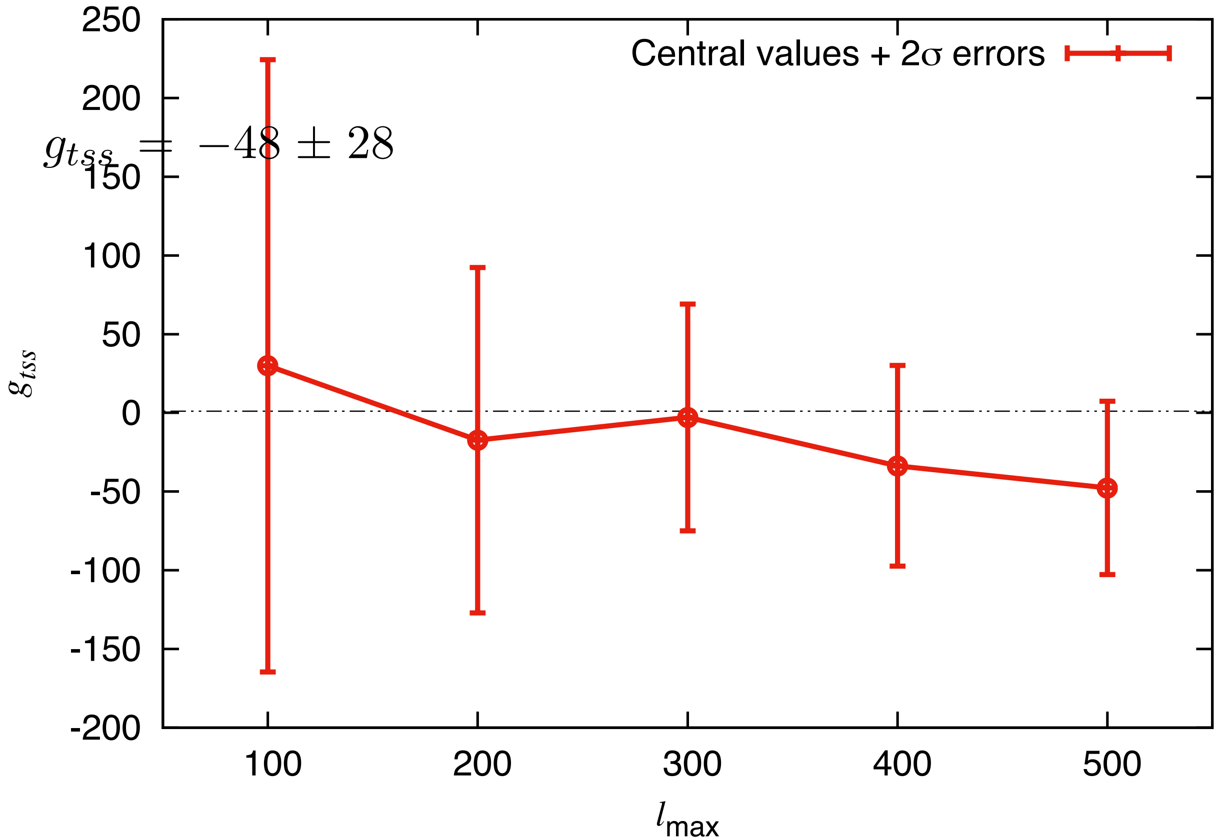
- ***Go beyond power spectrum: look for statistical properties of GWs ←→ graviton interactions***
- These correlators (and mixed ones $\langle h\,\zeta\zeta \rangle$ & $\langle h\,h\,\zeta \rangle$) signal **new physics** (e.g. anisotropic inflation, higher spin particles, parity breaking effects, solid-like models of inflation)
- e.g.: parity axion models of inflation with U(1)-gauge field production

$$f_{\rm NL}^{\rm tens} = 6.4 \times 10^{11}\, \epsilon\, \mathcal{P}_\zeta^3 \,\frac{e^{6\pi\xi}}{\xi^9} \longrightarrow \xi < 3.3 \; (95\,\%\ {\rm CL})$$

Present constraints on tensor NG

$\langle \gamma \zeta \zeta \rangle$ from <TTT>

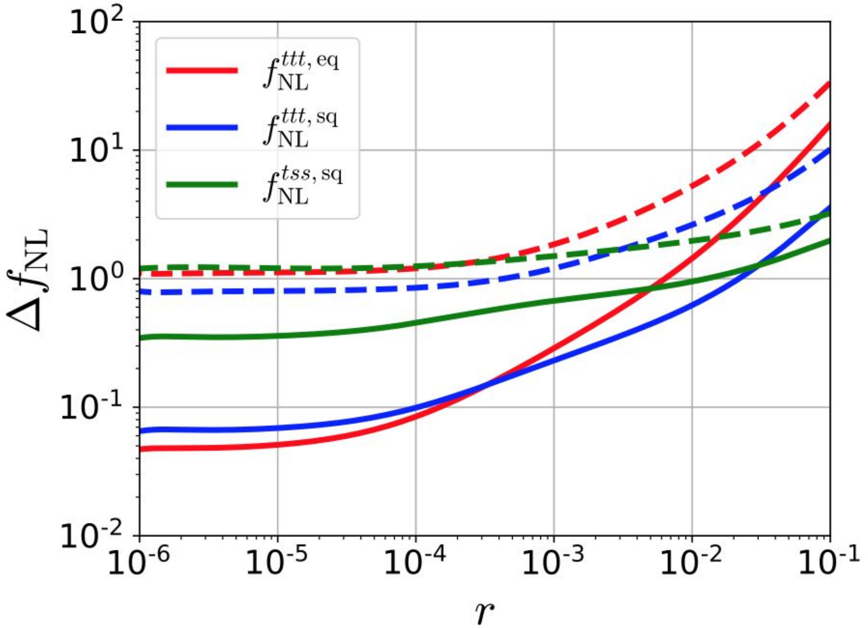
$g_{tss} = -48 \pm 28$



Shiraishi, Liguori, Fergusson, arXiv:1710.06778

Present constraints on tensor NG and forecasts

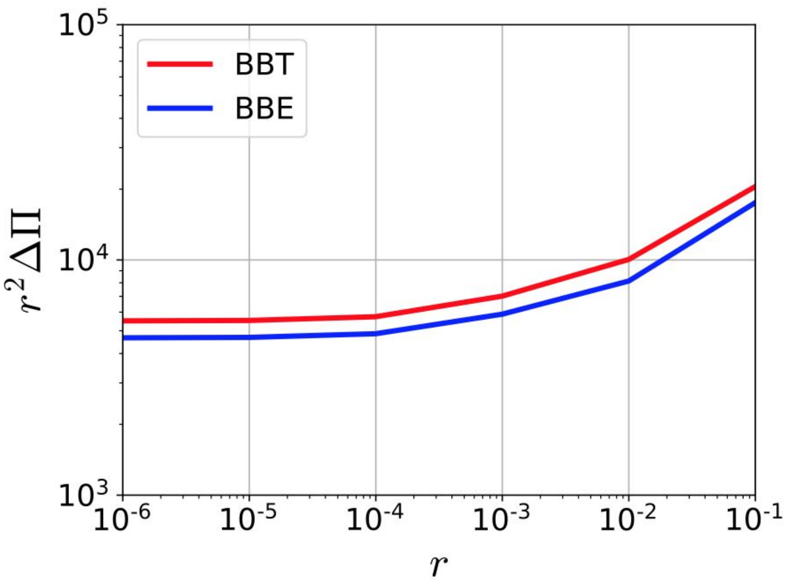
Forecasts for LiteBIRD



Chern-Simons Gravity $S = \int d^4x f(\phi) \tilde{W} W$

(Bartolo, Orlando & Shiraishi '17,'19)

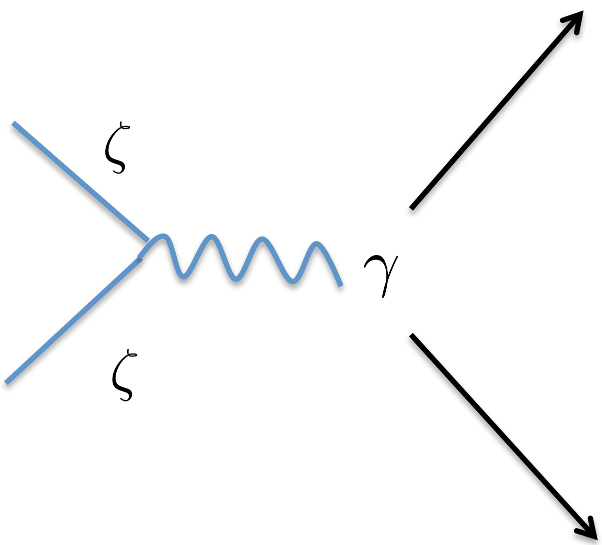
$r^2 \Pi \propto < h^2 \zeta >$

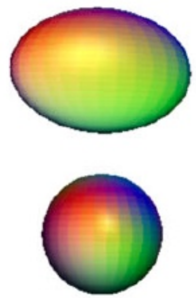


error bars on f_NL amplitudes 2-3 orders of magnitude better than present limits;

Determining the nature of Gravitational Waves (II)

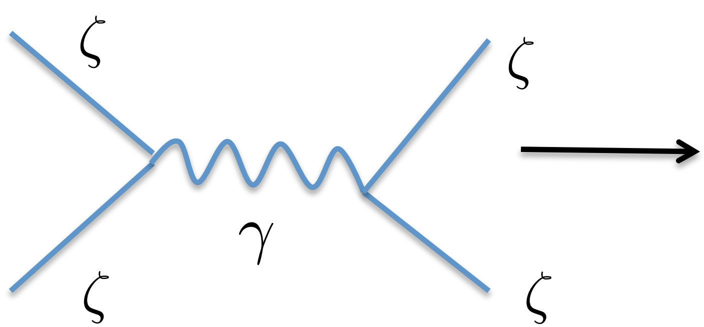
✓ ***Also, very interestingly, we could be missing some specific signatures from the primordial GW sector that might already be in the data!!***





Tensor fossils: quadrupole distortion of the curvature (matter) power spectrum induced by a long-wavelength tensor mode
(e.g., Dai, Jeong and Kamionkowski '12,'13; Dimastrogiovanni,Fasiello,Kamionkowski '14,'15)

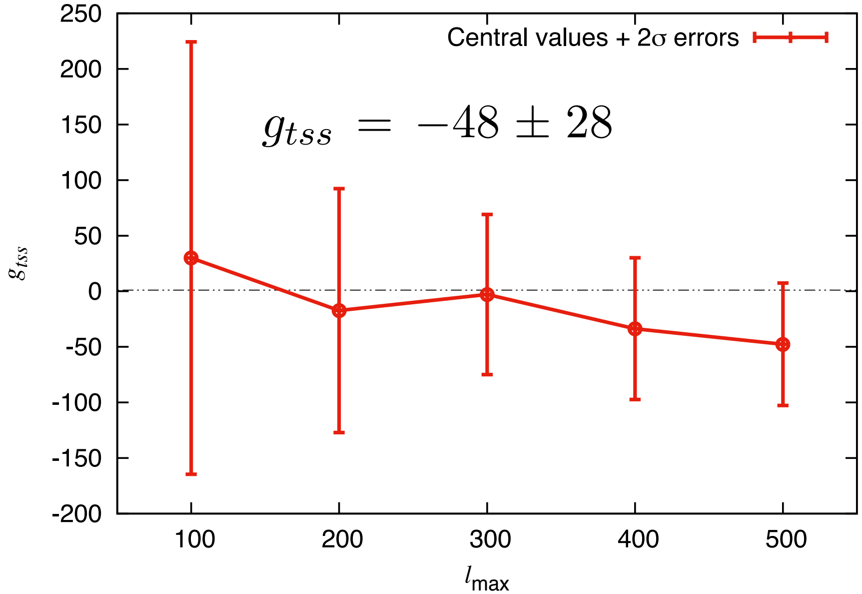
CMB (or LSS) trispectrum similar to τ_{NL}- trispectrum but with a different shape (Bordin et al arXiv:1605.08424; Bellomo, N.B., Jimenez, Matarrese, Verde '18).



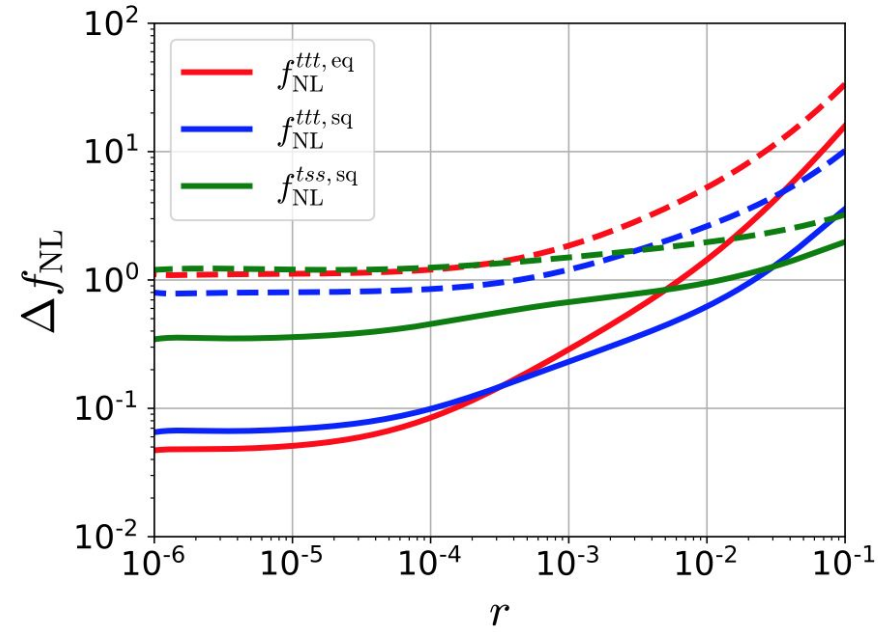
$$\langle TTTT \rangle$$

Present constraints on tensor NG and forecasts

$\langle \gamma \zeta \zeta \rangle$ From <TTT>



Forecasts for LiteBIRD

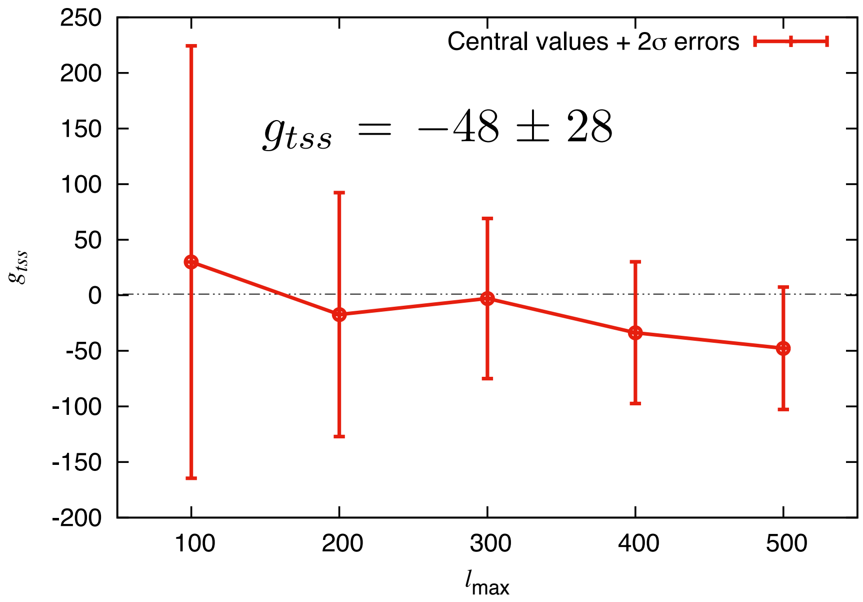


Shiraishi, Liguori, Fergusson, arXiv:1710.06778

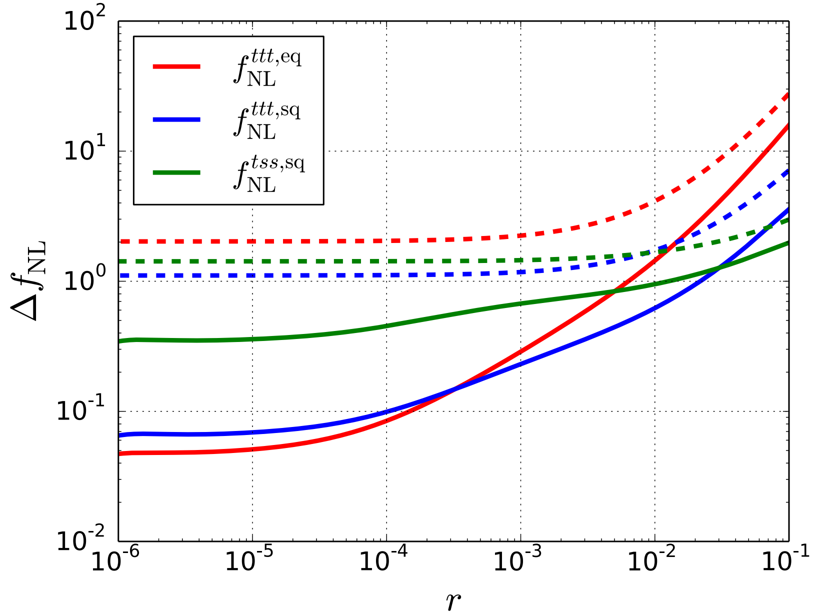
error bars on f_NL amplitudes 2-3 orders of magnitude better than present limits;

Present constraints on tensor NG and forecasts

$\langle \gamma \zeta \zeta \rangle$ From <TTT>



Forecasts for LiteBIRD-like

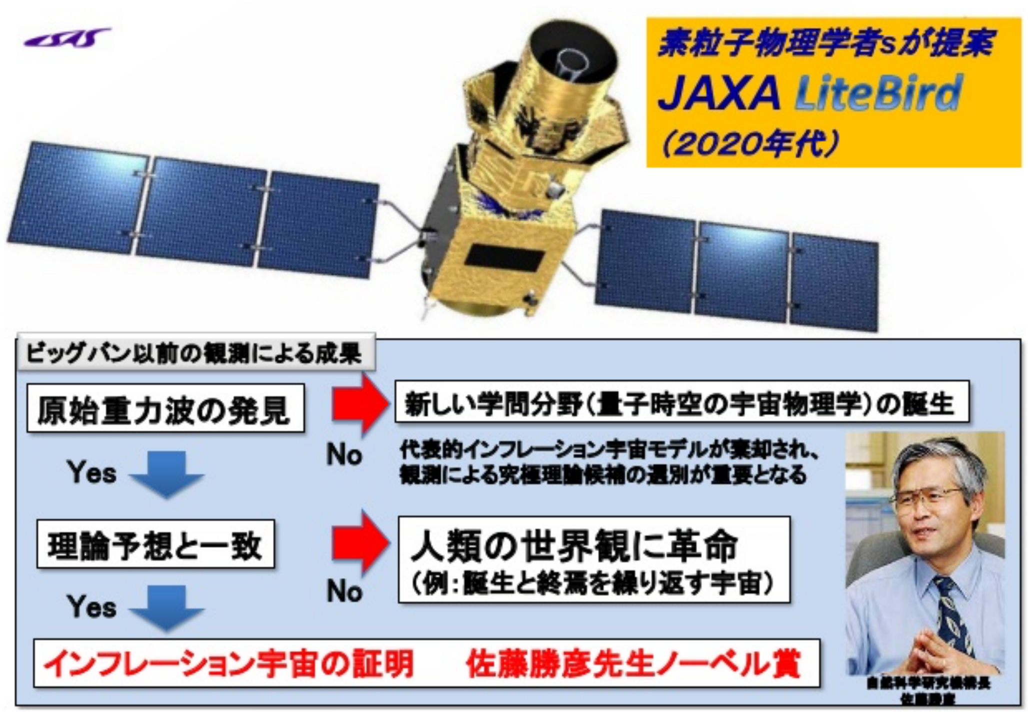


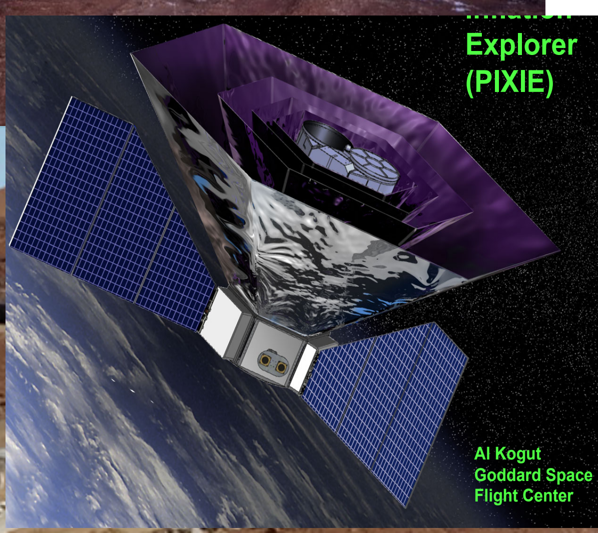
Shiraishi, Liguori, Fergusson, arXiv:1710.06778

error bars on f_NL amplitudes 2-3 orders of magnitude better than present limits;

Looking ahead:
What can we expect in the future?

CMB B-modes a smoking gun of inflation (see Baccigalupi's talk)

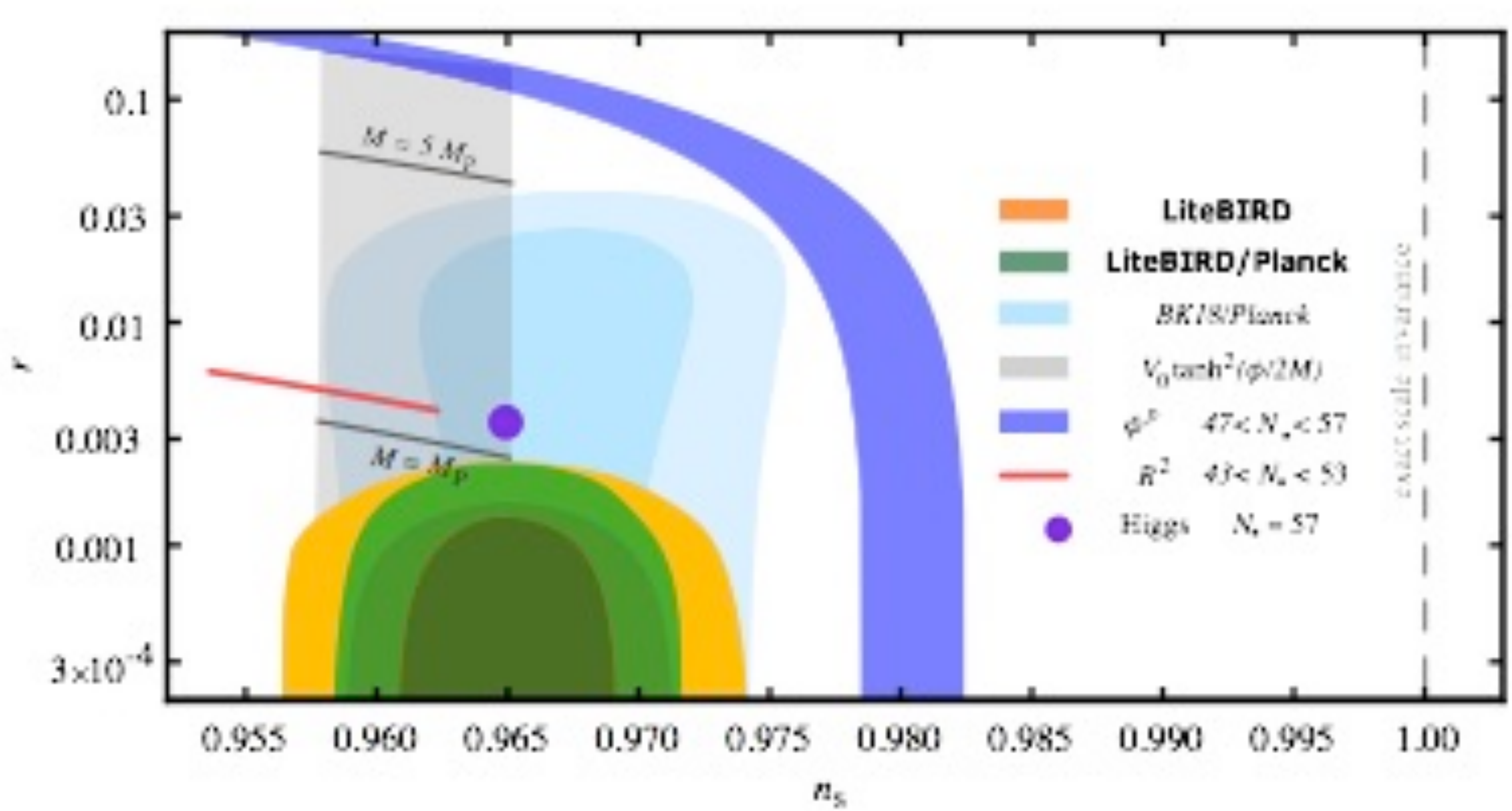




Forecasts for tensor-to-scalar ratio r

➢ **For future space CMB missions (LiteBIRD satellie)**

From "Probing Cosmic Inflation with the LiteBIRD CMB polarization survey" The LiteBIRD coll. PTEP 4, 2023



Main well-motivated theoretical thershold to reach: r~4 ×10^{-3} , corresponding to Starobinsky model of inflation (at present the model that is most compatible with data).

Future observational probes
for non-Gaussianity
(see Michele Liguori's talk)
Here the threshold to reach is f_{NL}*~1*

CMB Bispectrum forecasts

$$\left\langle \zeta(\vec{k}_1)\zeta(\vec{k}_2)\zeta(\vec{k}_3)\right\rangle = (2\pi)^3 \delta^{(3)}(\vec{k}_1+\vec{k}_2+\vec{k}_3) f_{NL} F(k_1,k_2,k_3)$$

	LiteCORE 80	LiteCORE 120	**CORE M5**	COrE+	Planck 2015	LiteBIRD	ideal 3000
T local	4.5	3.7	**3.6**	3.4	(5.7)	9.4	2.7
T equilat	65	59	**58**	56	(70)	92	46
T orthog	31	27	**26**	25	(33)	58	20
T lens-isw	0.15	0.11	**0.10**	0.09	(0.28)	0.44	0.07
E local	5.4	4.5	**4.2**	3.9	(32)	11	2.4
E equilat	51	46	**45**	43	(141)	76	31
E orthog	24	21	**20**	19	(72)	42	13
E lens-isw	0.37	0.29	**0.27**	0.24		1.1	0.14
T+E local	2.7	2.2	**2.1**	1.9	(5.0)	5.6	1.4
T+E equilat	25	22	**21**	20	(43)	40	15
T+E orthog	12	10.0	**9.6**	9.1	(21)	23	6.7
T+E lens-isw	0.062	0.048	**0.045**	0.041		0.18	0.027

New observational strategies

CMB is a privileged laboratory for cosmic inflation. However different observables can be competitive, and in the future, have a better sensitivity to, e.g., primordial non-Gaussianity

- Optical LSS Galaxy Surveys:
  Bispectrum+PS (scale-dependent halo bias)
- Future high-redshift large radio surveys
  Bispectrum+PS (scale-dependent halo bias)

} Talk by Michele Liguori

- CMB spectral distortions
- High-redshift 21cm fluctuations
- Intensity mapping

There is a huge potential improvement!

Are there (other) specific topics that might be of interest to this mixed audience, in particular from the point of view of statistical tools, both theoretical and for data analysis…?

e.g.

- How to extract non-Gussian information from data (see talk by Michele Liguori)

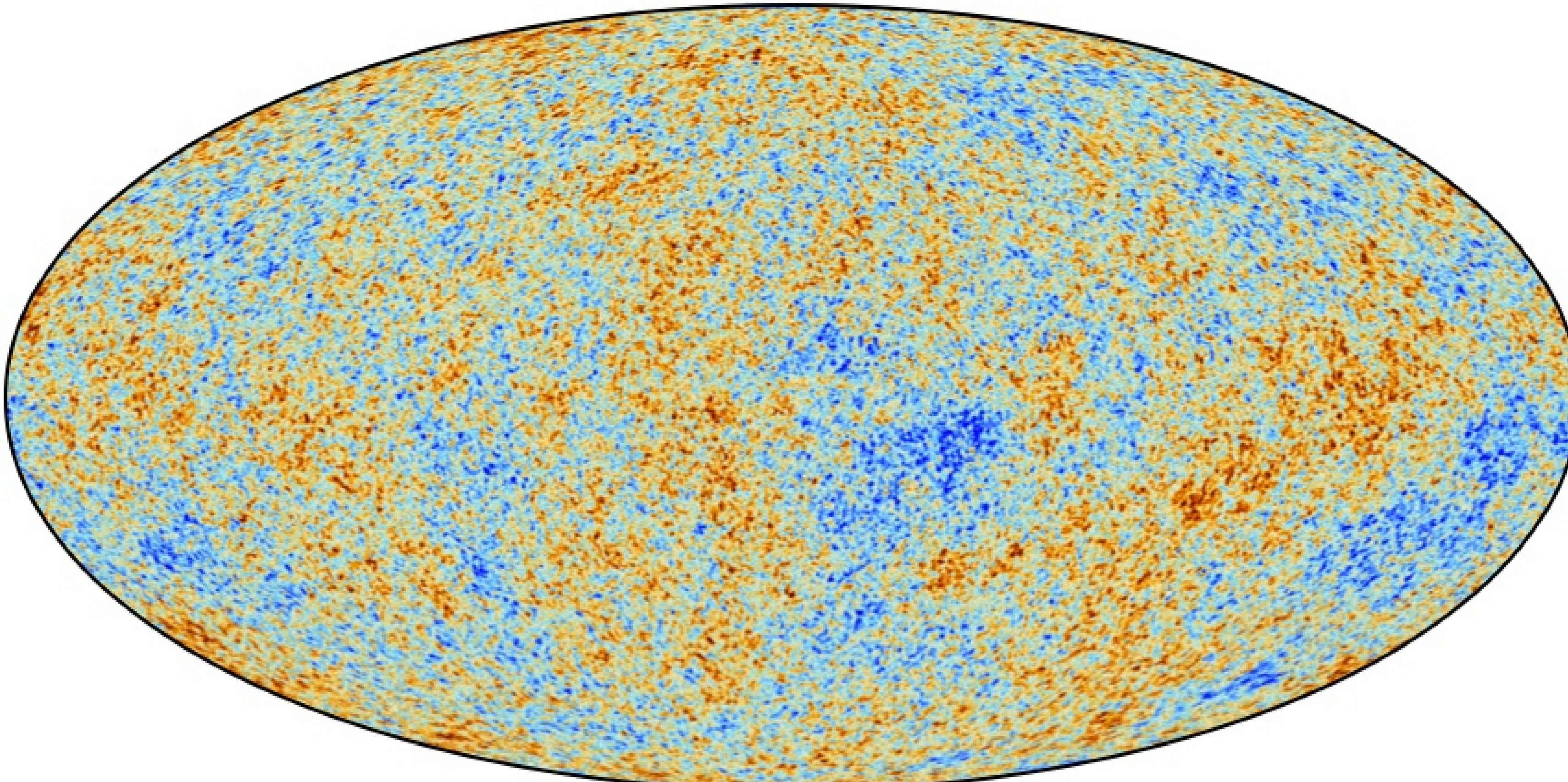
- How to properly deal with tensions from different datasest (see the example of the Hubble tension issue and the talk by Marco Raveri)

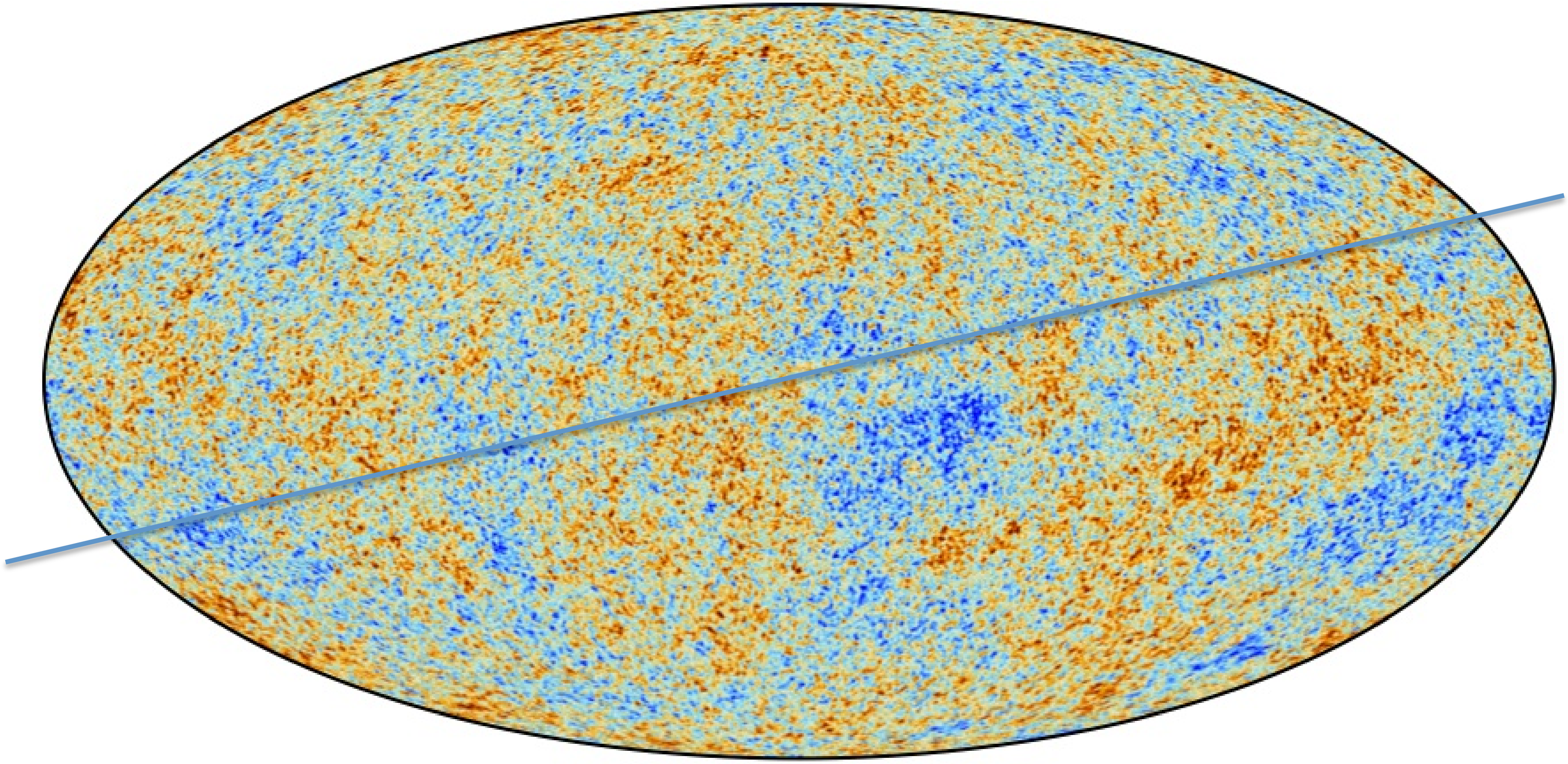
- So called Cosmic Microwve Background anomalies

- Rare events and, e.g., formation of Primordial Black Holes

Cosmic Microwave Background “anomalies”

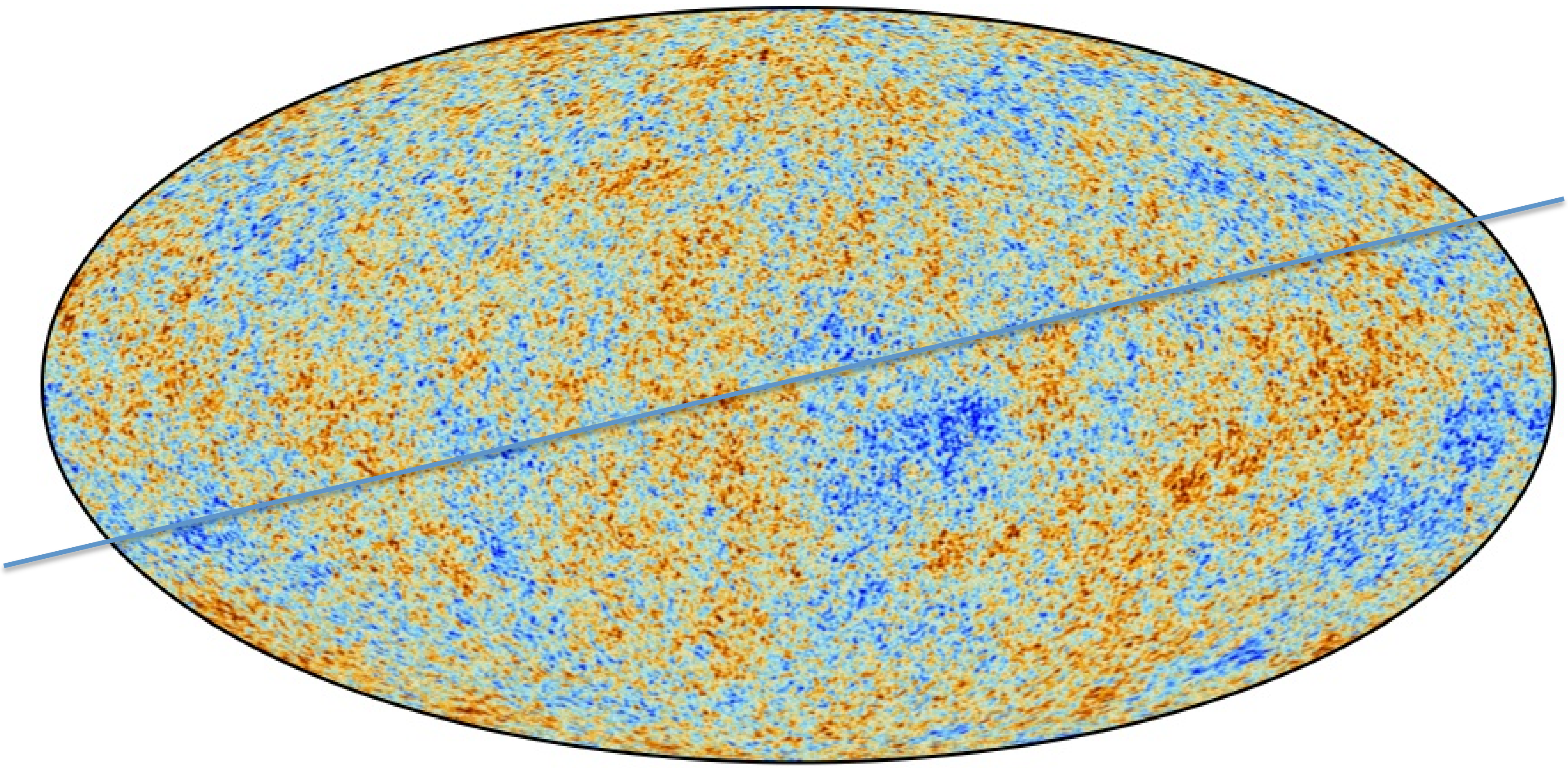
CMB Anomalies



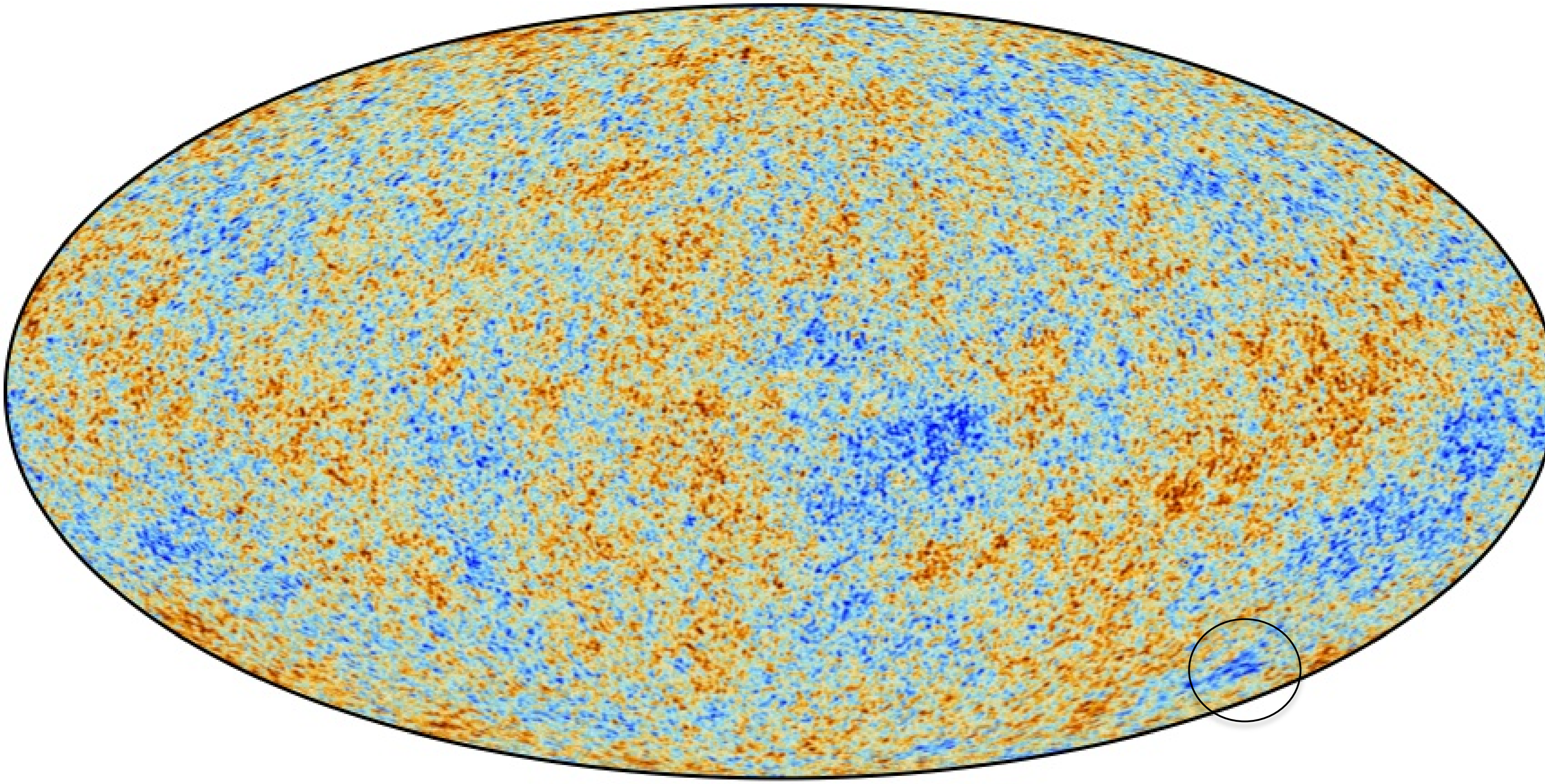


1.Large scale hemispherical asymmetry (dipolar modulation)

2.Small scale hemispherical asymmetry (alignment asymmetry)



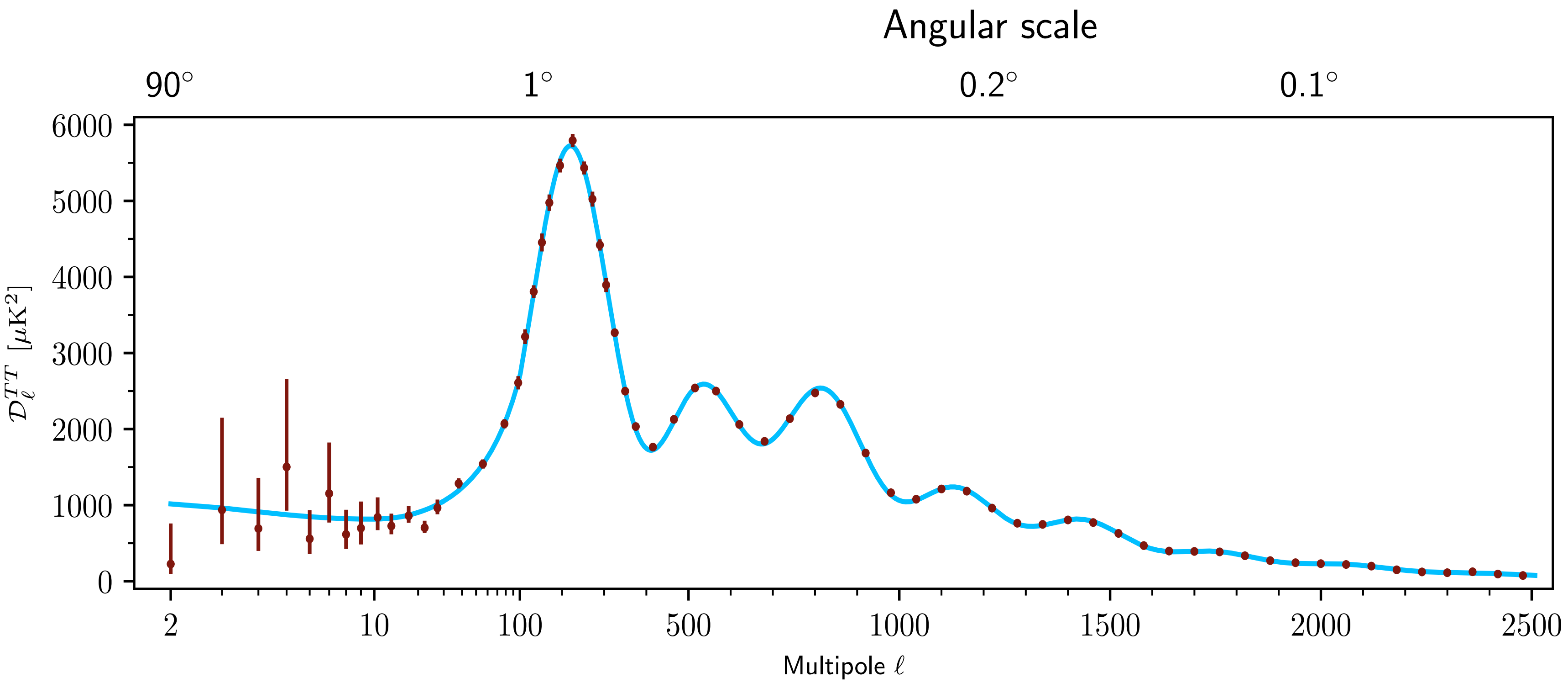
1.Large scale hemispherical asymmetry (dipolar modulation)



3. non-Gaussian cold spot

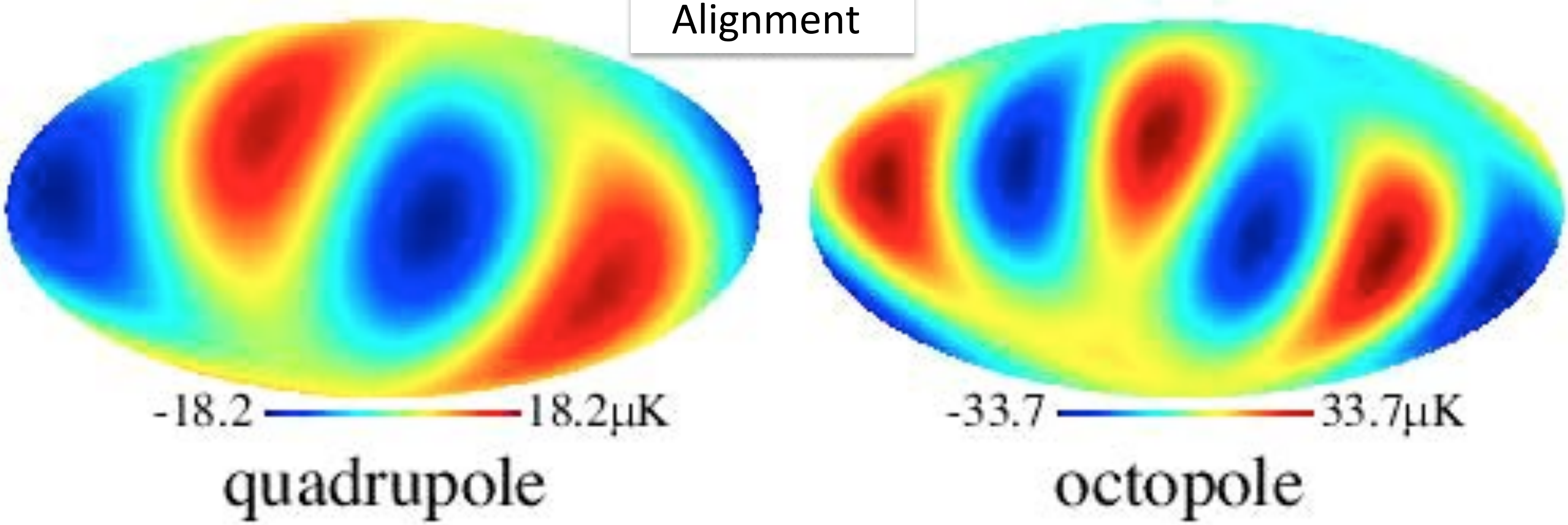
4.Low power spectrum and quadrupole

6.Parity asymmetry



5.Quadru-/octopole alignment

Alignment



CMB Anomalies

Statistical significance at the 2-3σ level (and issues related to a posteriori choiche of statistics & to look-elsewhere effects):
Still:

- It might indicate a (local, apparent) breaking of statistical isotropy on the laregest scales

- It might be indicate new physics, relevant for understanding the underlying inflationary mechanism (and is there a common orgin?)

- New physcis or statistical fluke/foregrounds/systematics?

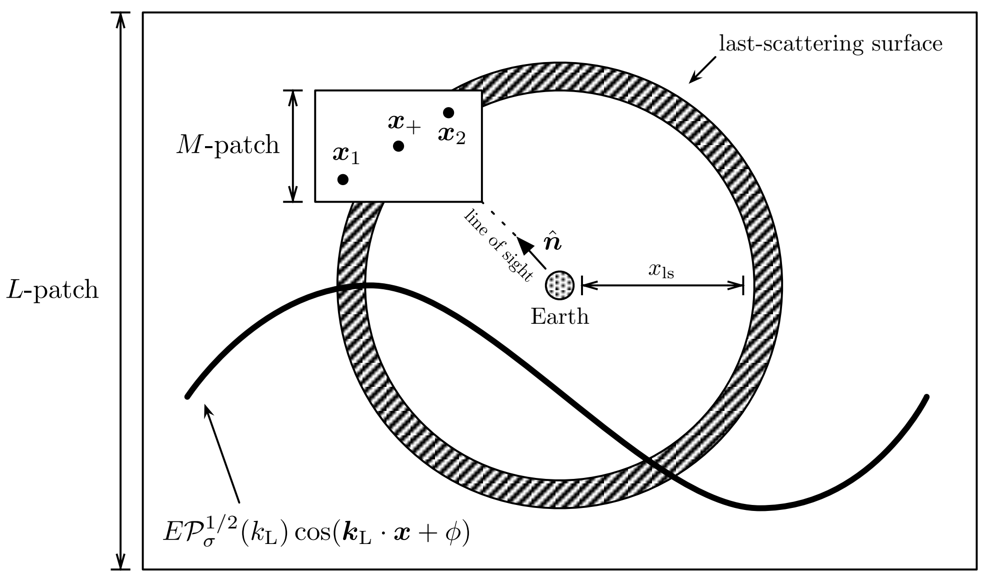
Models for CMB anomalies

- A (maybe too simplified) model can be a dipolar modulation

$$\zeta(\vec{x}) = g(\vec{x})[1 + h(\vec{x})]$$

Sub-horizon scale fluctuations

Super-horizon scale modulating field



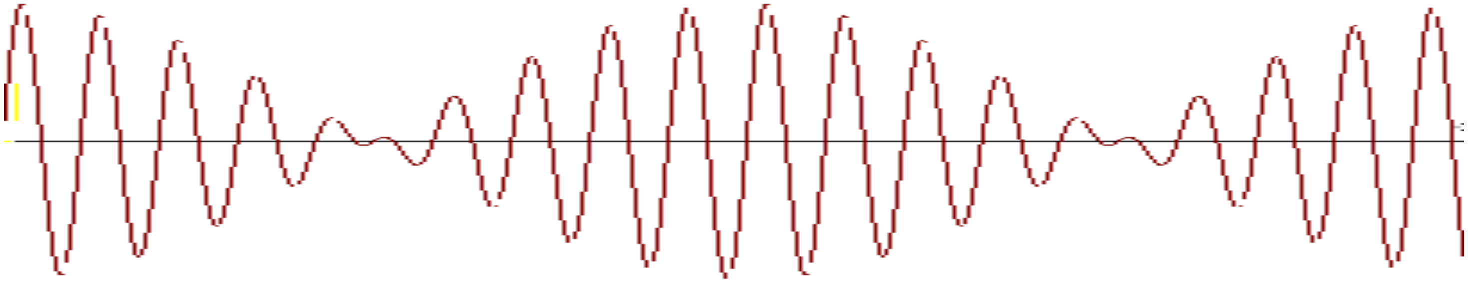
spontaneous (apparent) breaking of statistical isotropy: large-scale (super-horizon) fluctuations that modulate small scale power (so called "local" non-Gaussianity)

Models for CMB anomalies

- Invoke local-like non-Gaussianity

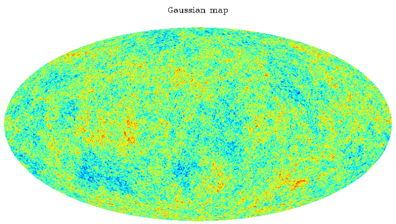
$$\zeta(\mathbf{x}) = \zeta_g(\mathbf{x}) + \frac{3}{5} f_{\rm NL} \, (\zeta_g(\mathbf{x}))^2$$

$$\zeta_g(\mathbf{x}) = \zeta_{g,\rm short}(\mathbf{x}) + \zeta_{g,\rm long}(\mathbf{x})$$



Dipolar modulation

$$T(\theta,\phi) = T_0(\theta,\phi) \text{ x } (1 + \beta\, T^{MOD}(\theta,\phi))$$



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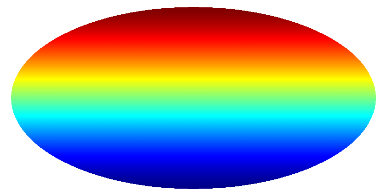
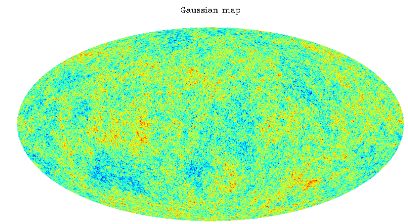


X



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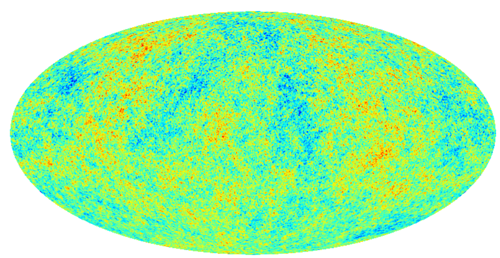


The toy-model

$$T(\theta,\phi) = T_{GAUSS}(\theta,\phi) \times (1 + \beta T^{MOD}(\theta,\phi))$$

$$T^{MOD}(\theta,\phi) = (T^{FILT}(\theta,\phi))^2$$

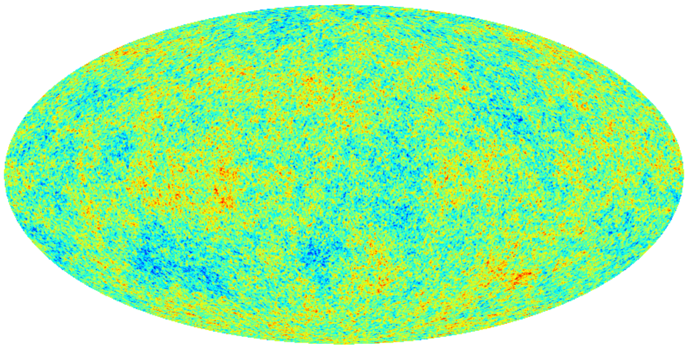
$$T(\theta,\phi) = T_{GAUSS}(\theta,\phi) + \beta\, T_{GAUSS}(\theta,\phi) \times (T^{FILT}(\theta,\phi))^2$$

See Hansen, Trombetti, Bartolo, Natale , Liguori, Banday and Gorski, A&A 2019

The motivations

- ***All the scales will be correlated with the largest scales*** → random dipolar distribution of power on the largest scales will be imprinted on the smaller scales→ ***A2***
- A modulation field which is the ***square*** of the filtered original map $T^{MOD}(\theta,\phi) = (T^{FILT}(\theta,\phi))^2$ will amplify both positive and negative fluctuations → enhance the dipole → ***A1***
- ***enhance hottes and coldest spots***, via a non-Gaussian cubic term→ cold spot with excess kurtosis ***A3***

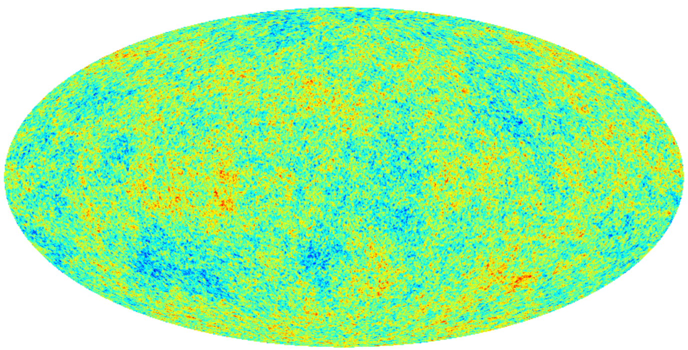
Gaussian map



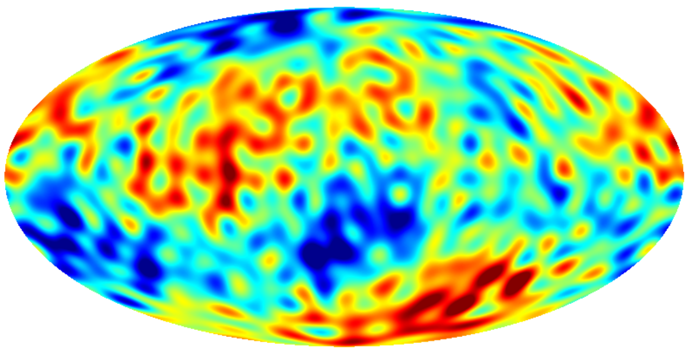
Modulation

$$T(\theta,\phi) = T_0(\theta,\phi) \times (1 + \beta\, T^{MOD}(\theta,\phi))$$

Gaussian map

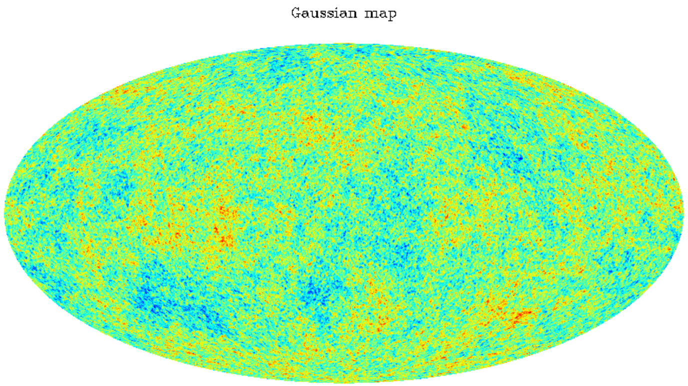


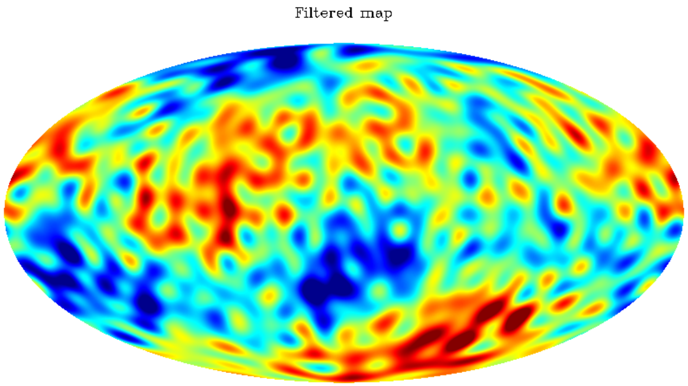
Filtered map

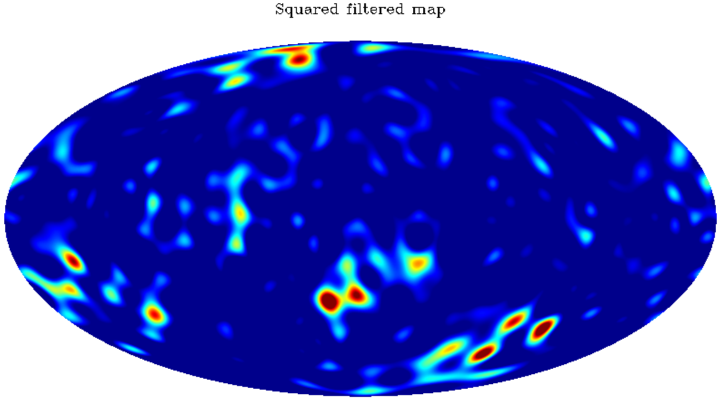


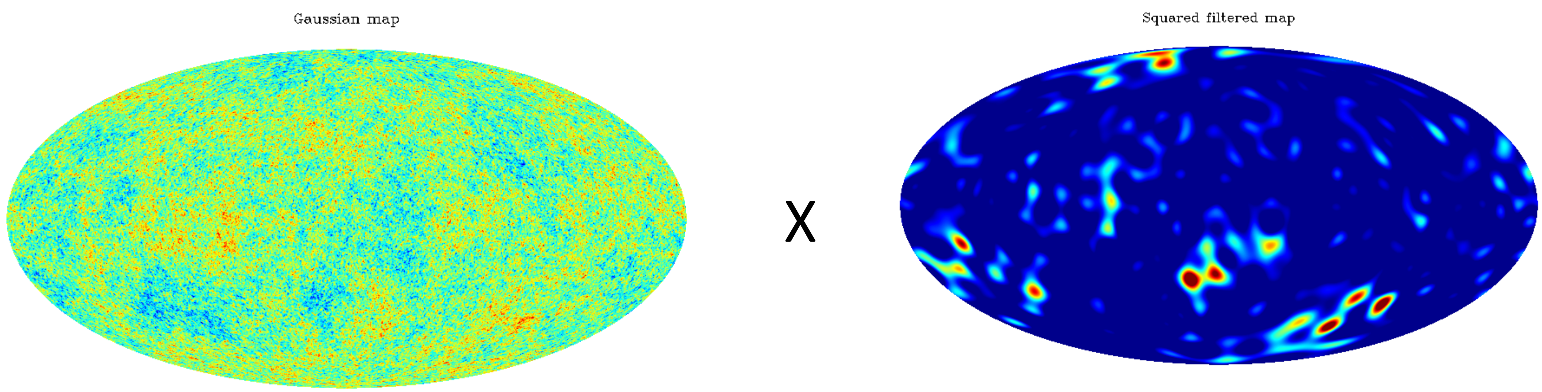
Modulation

$$T(\theta,\phi) = T_0(\theta,\phi) \times (1 + \beta\, T^{MOD}(\theta,\phi))$$

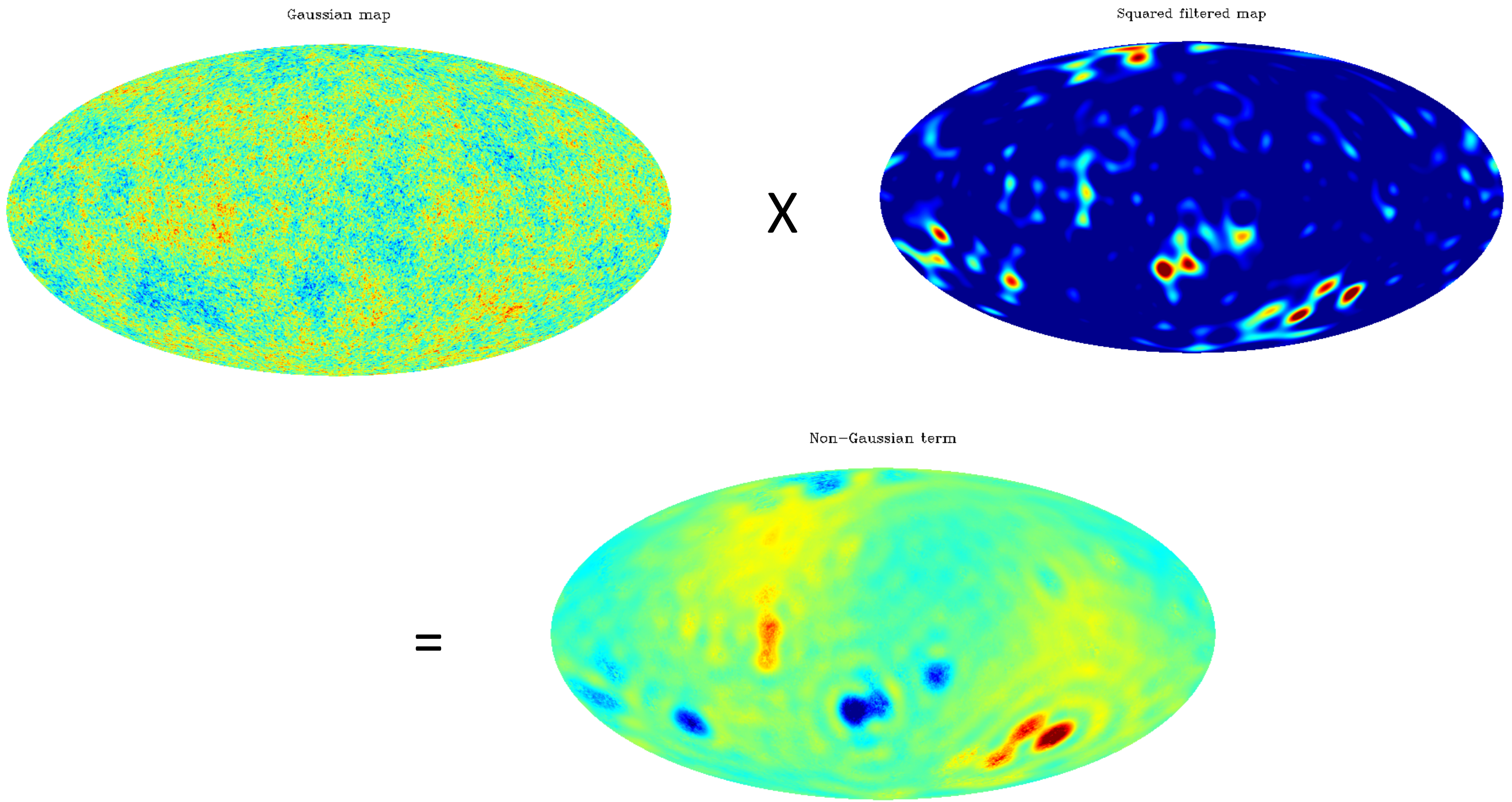


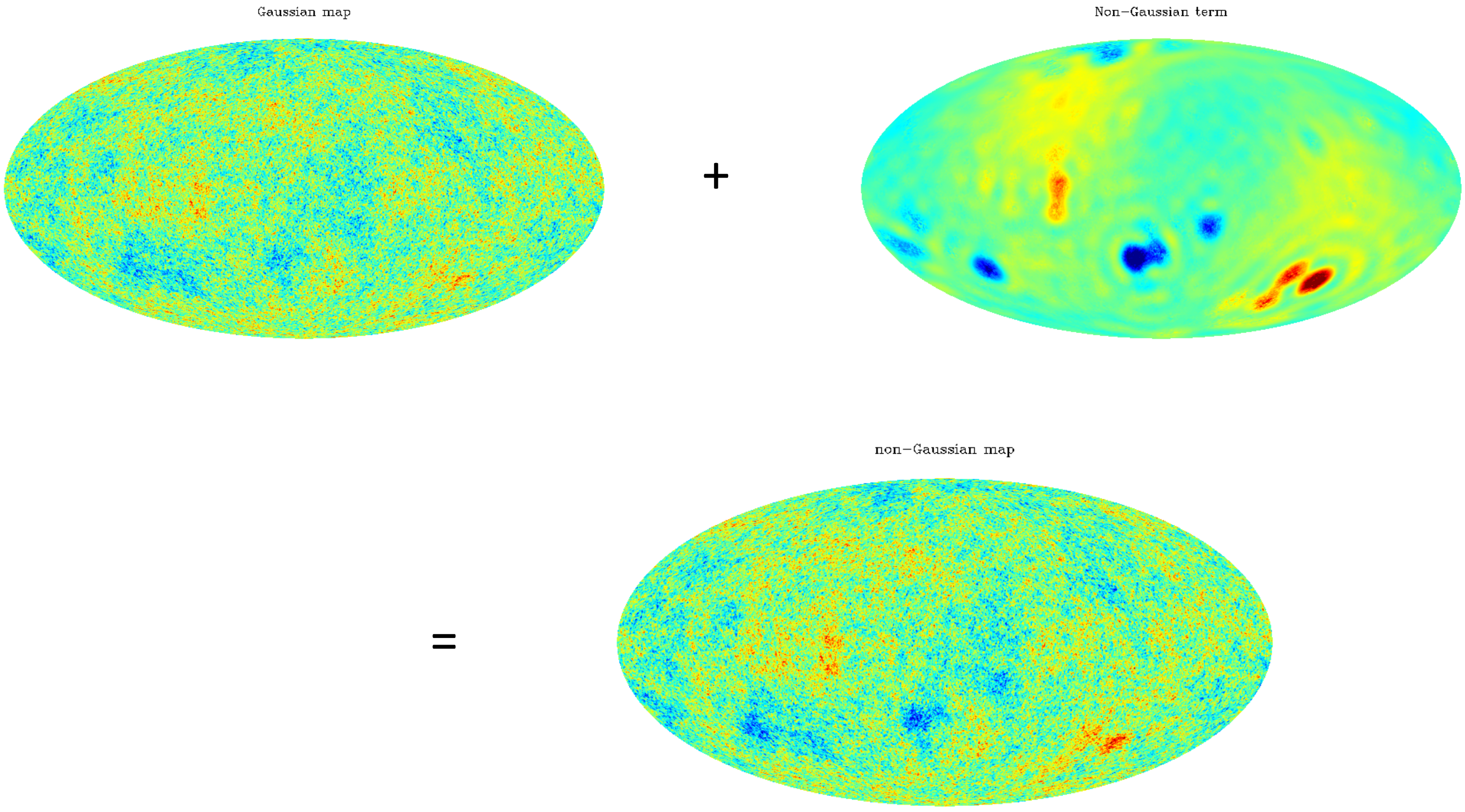


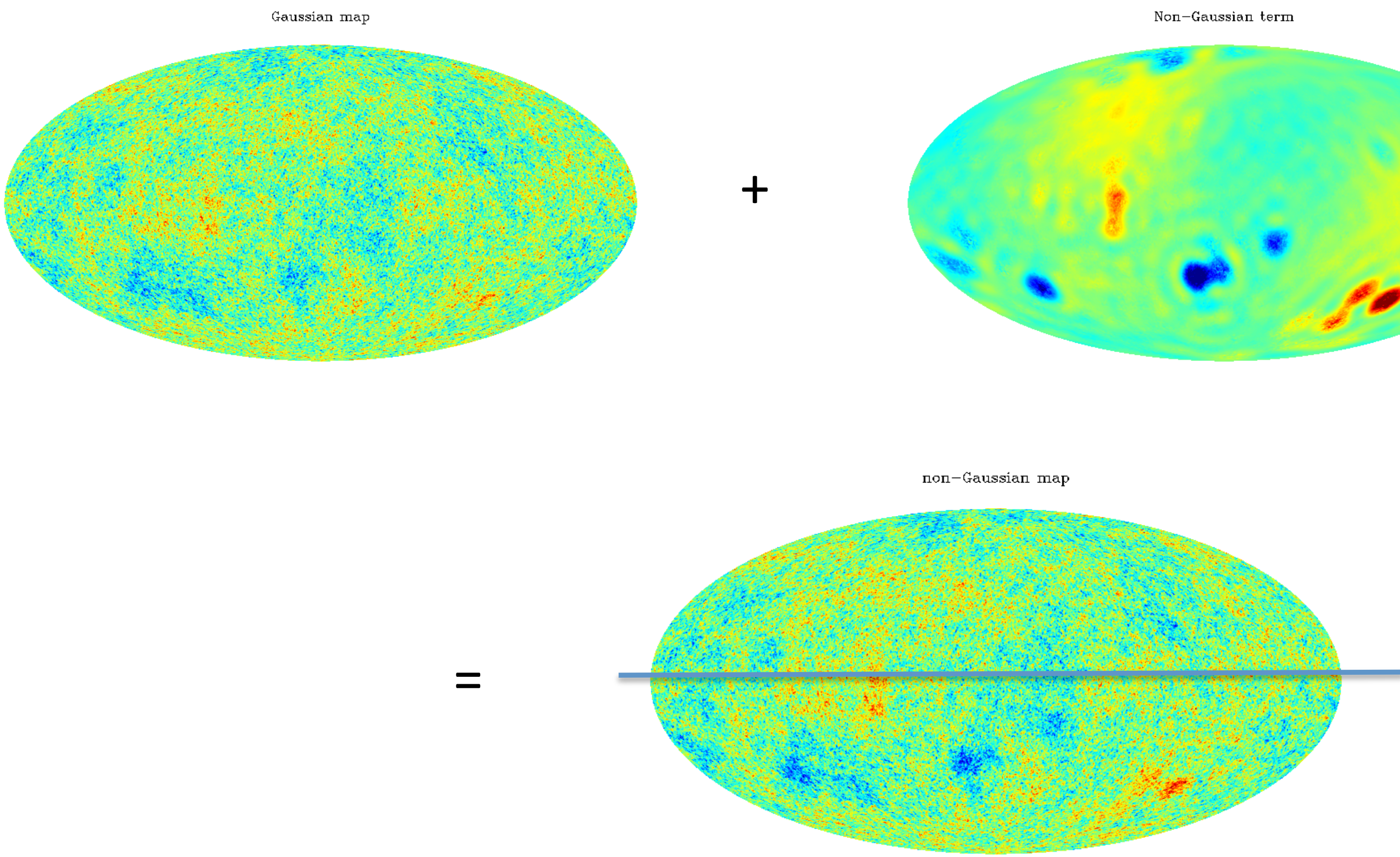


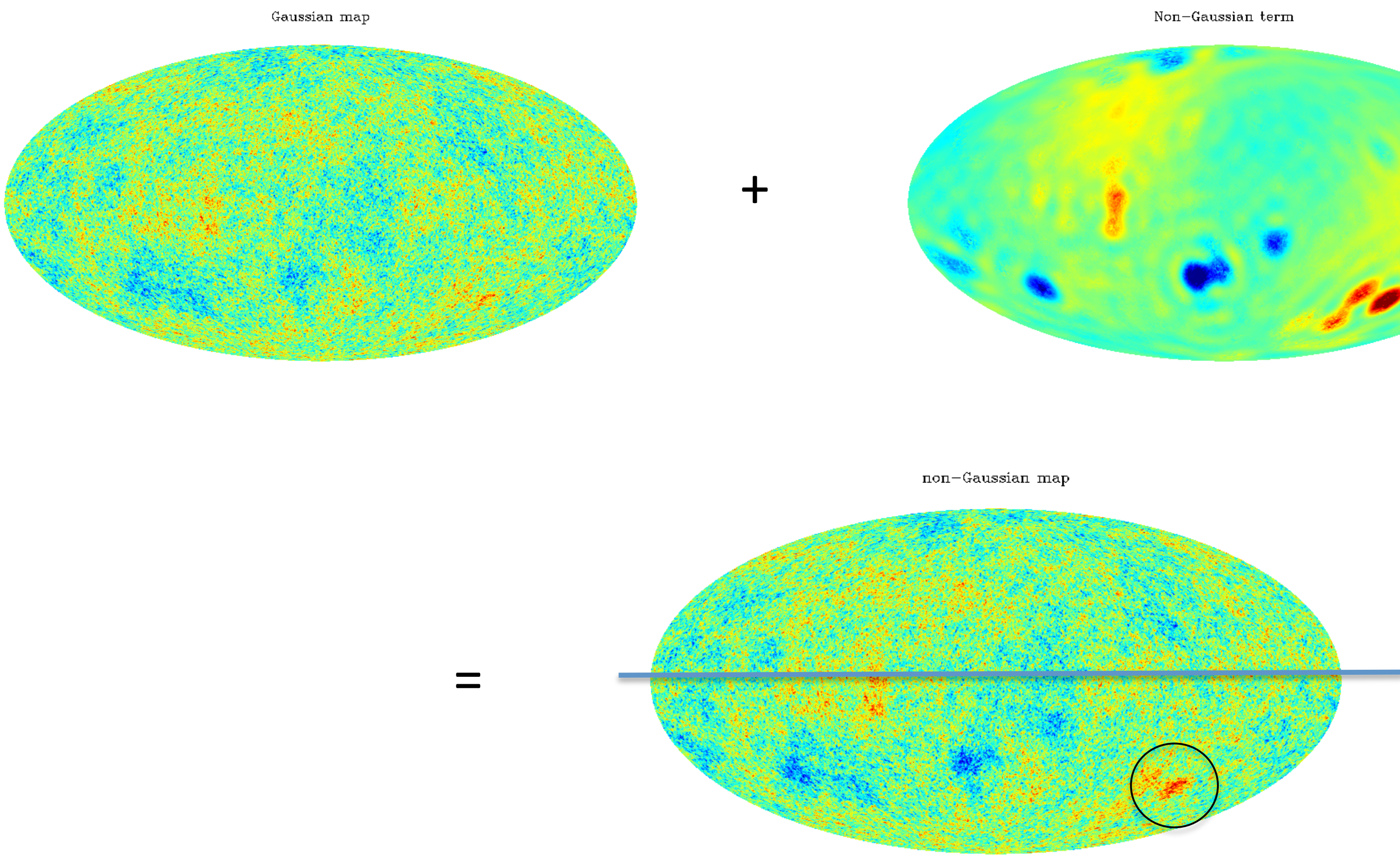


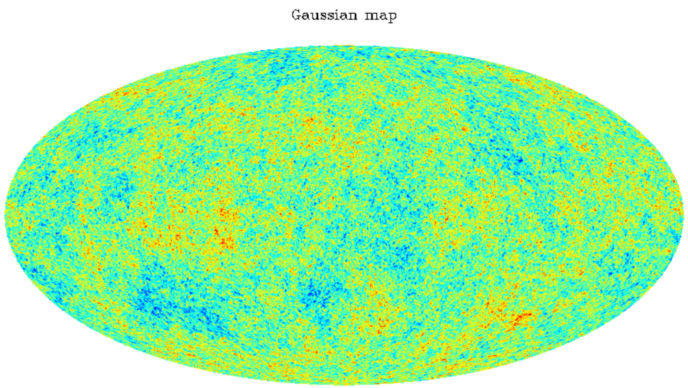
Gaussian map
X
Squared filtered map


Gaussian map
X
Squared filtered map
=
Non−Gaussian term

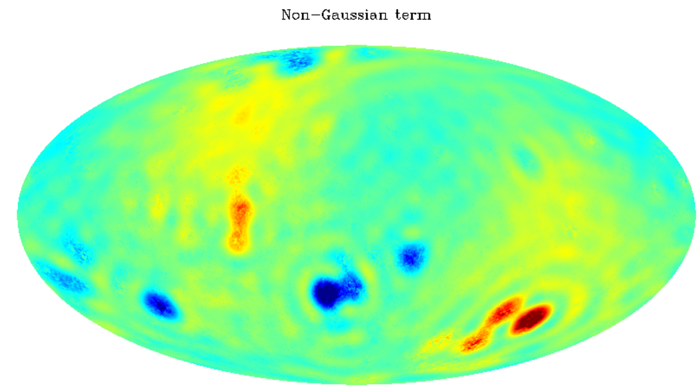

Gaussian map
+
Non−Gaussian term
=
non−Gaussian map


Gaussian map
+
Non−Gaussian term
=
non−Gaussian map


Gaussian map
+
Non−Gaussian term
=
non−Gaussian map



+



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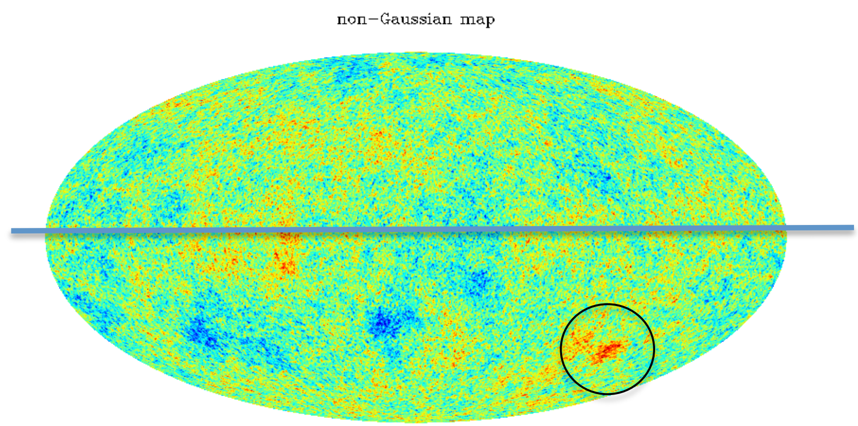
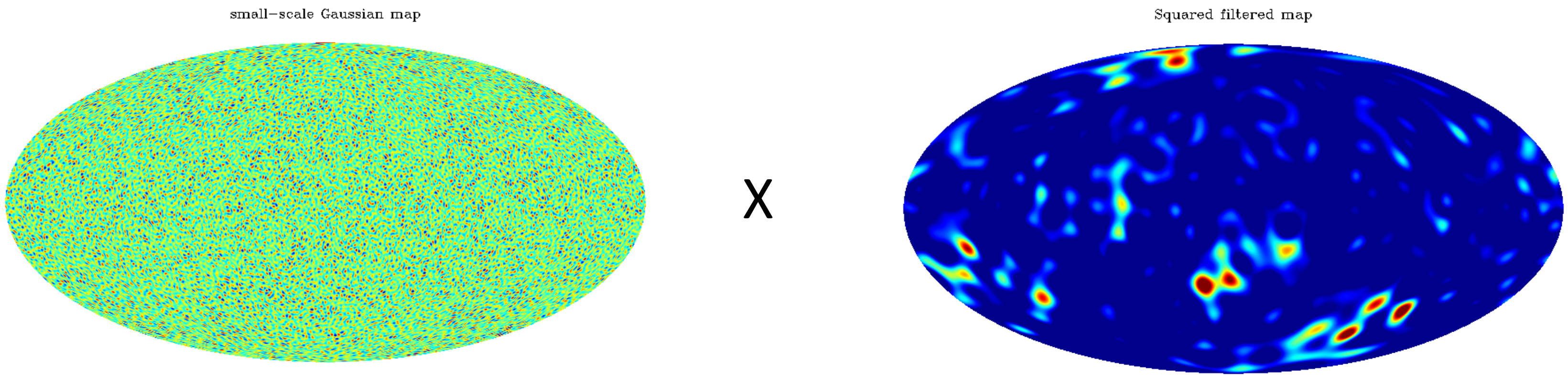
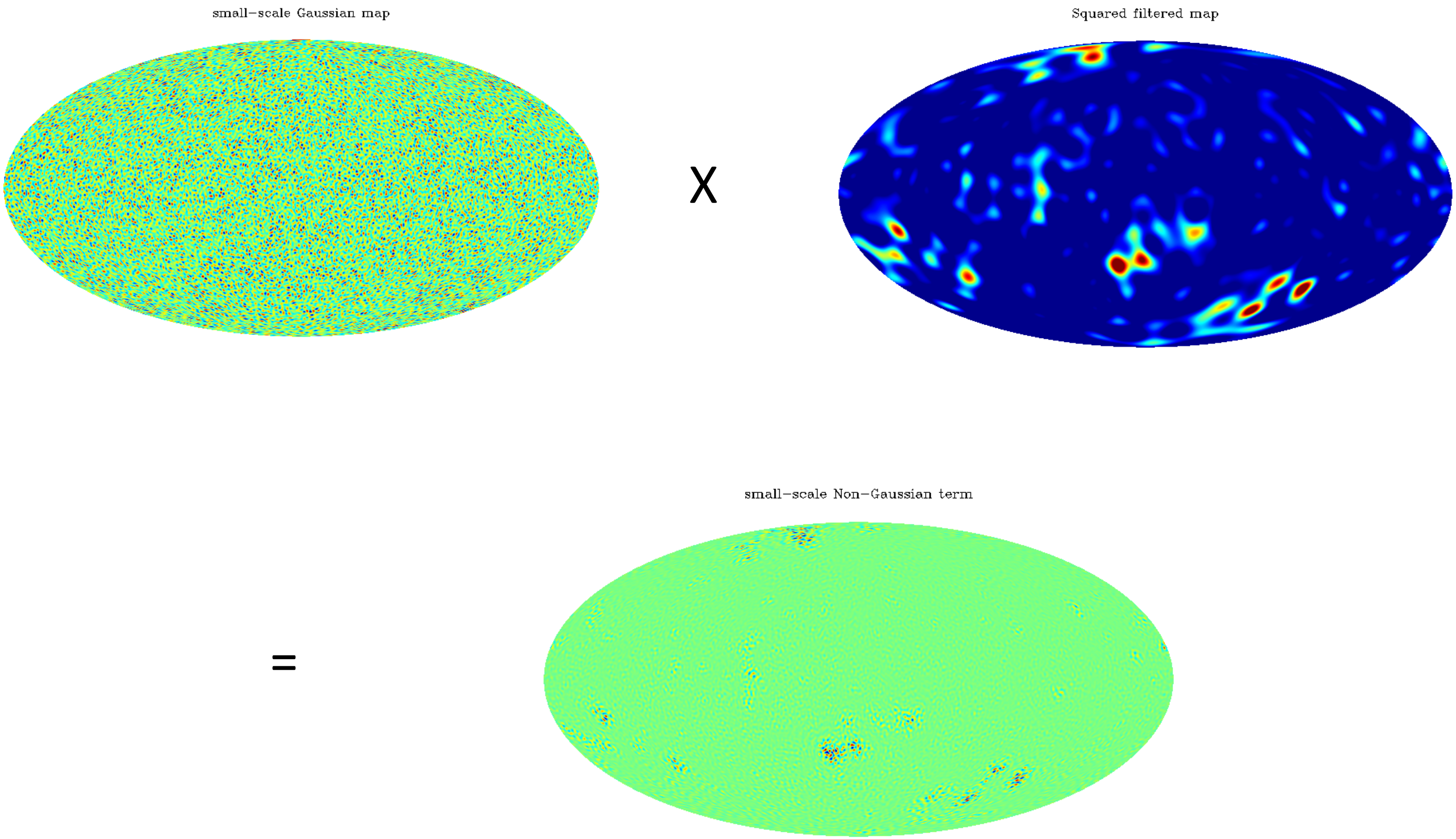
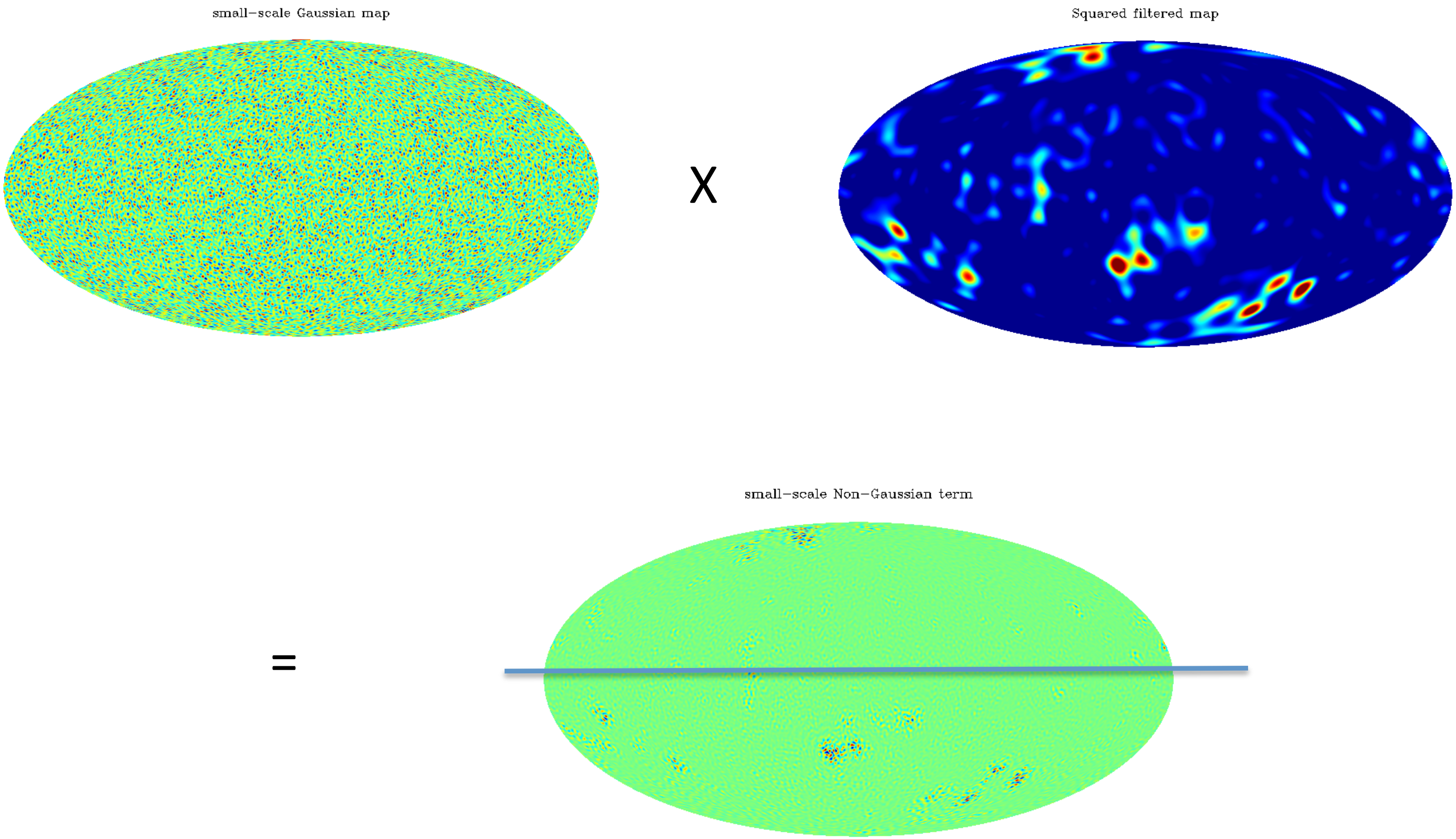


3. Non-Gaussian cold/hot spot

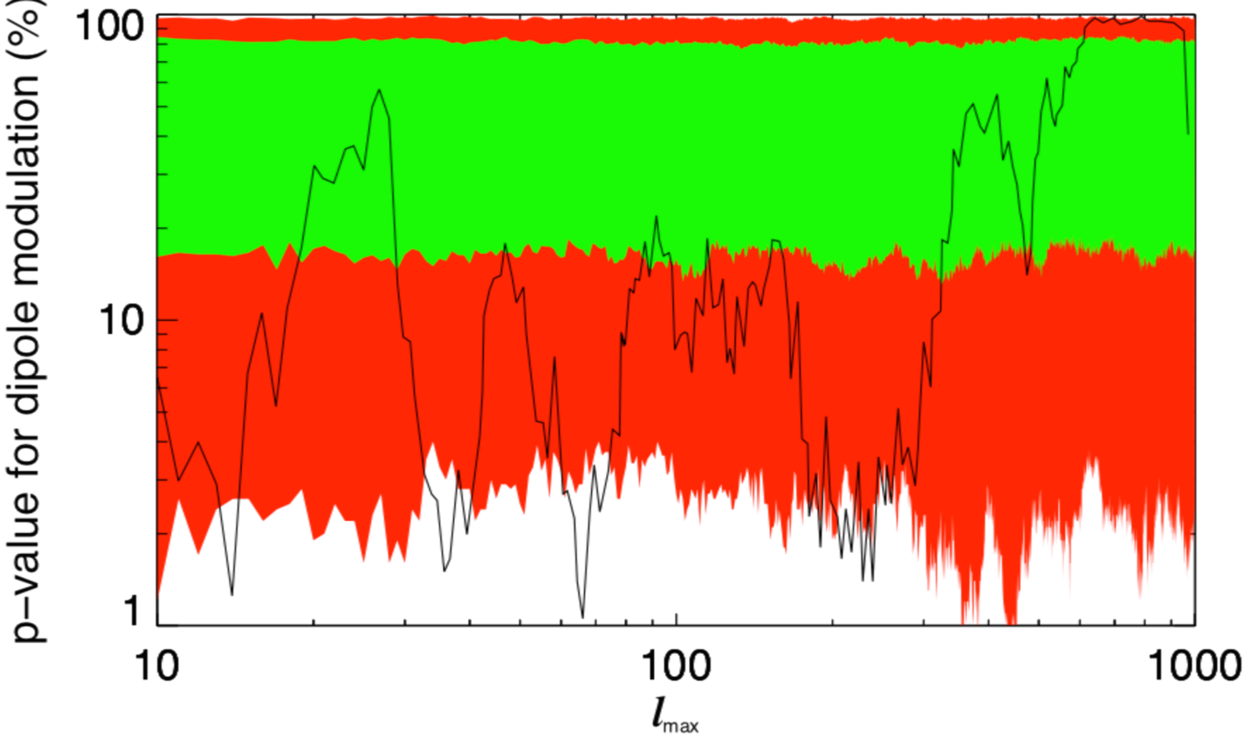
1.Large scale hemispherical asymmetry (dipolar modulation)


small-scale Gaussian map
X
Squared filtered map

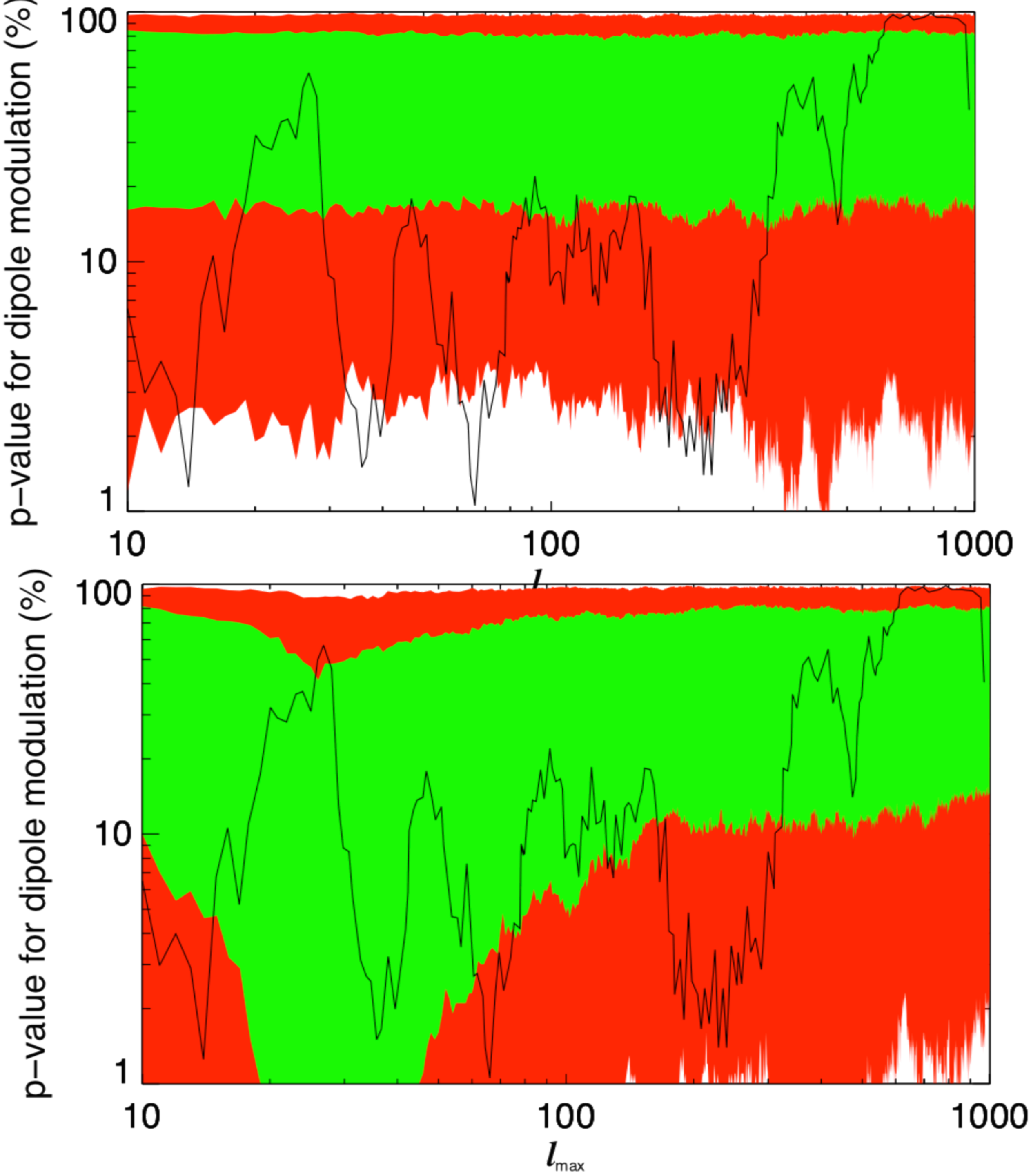

small-scale Gaussian map
X
Squared filtered map
=
small-scale Non-Gaussian term

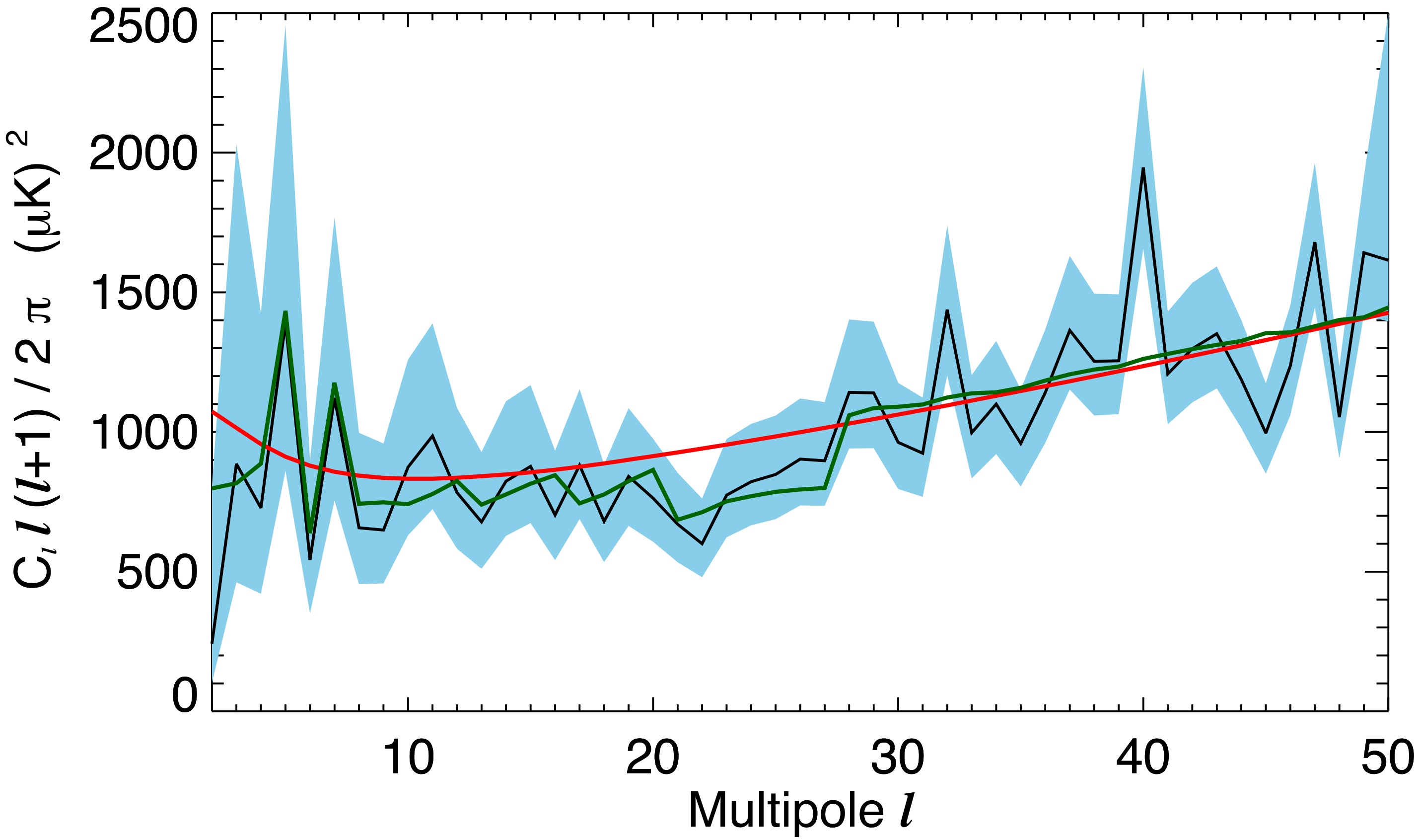


2.Small scale hemispherical asymmetry (alignment asymmetry)



p–value for dipole modulation (%)
100
10
1
10
100
1000
l_{max}


p–value for dipole modulation (%)
100
10
1
10
100
1000
l
p–value for dipole modulation (%)
100
10
1
10
100
1000
l_{max}



2500
2000
1500
1000
500
0
10
20
30
40
50
C_l l (l+1) / 2 π (μK)²
Multipole l

- Not sure I will show the following slides. Forse solo una quella dl trispetto.
- ricorda di citare che hi contribuito negli anni......compresa Daniela Montes Doria.

In search for an inflationary model

Work in progress

In collaboration with Daniela Montes Doria

In search for an inflationary model

1 Find a correspondence between the phenomenological model and primordial curvature perturbation ζ

2 Find a concrete inflation model realization

$$T(\theta,\phi) = T_{GAUSS}(\theta,\phi) + \beta\,[T_{GAUSS}(\theta,\phi) \times (T^{FILT}(\theta,\phi))^2]^{FILT2}$$

↕

$$\Phi(\boldsymbol{x}) = \Phi_G(\boldsymbol{x}) + f_{\rm NL}(\Phi_G^2(\boldsymbol{x}) - \langle\Phi_G^2(\boldsymbol{x})\rangle) + g_{\rm NL}\Phi_G^3(\boldsymbol{x})$$

???

$$T(\theta,\phi) = T_{GAUSS}(\theta,\phi) + \beta \, [T_{GAUSS}(\theta,\phi) \times (T^{FILT}(\theta,\phi))^2]^{FILT2}$$

↕

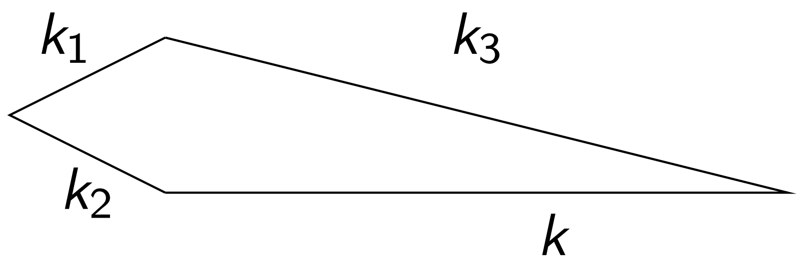
$$T = D^{TRANSFER}[\phi_{GAUSS} + g_{NL} \, [\phi_{GAUSS} \times (\phi^{FILT})^2]^{FILT2}$$

???

Any physical model which can be written in this way can give rise to all CMB anomalies!

In search for an inflationary model

The kind of non-Gaussianity (trispectrum) which leads to the phenomenological model we are interested in, might be realized following the approach of Shandera et al. in a two field inflationary scenario (e.g., inflaton+curvaton). It is just a possibility.
Strongly scale dependent trispectrum, not tested yet in the data.



Shape of trispectrum:

$k_1, k_2 << k, k_3$

N.B.: even if we allow for f_NL term, this would not change our main conclusions

Primordial Black Holes from rare events

Primordial Black Holes (PBHs)

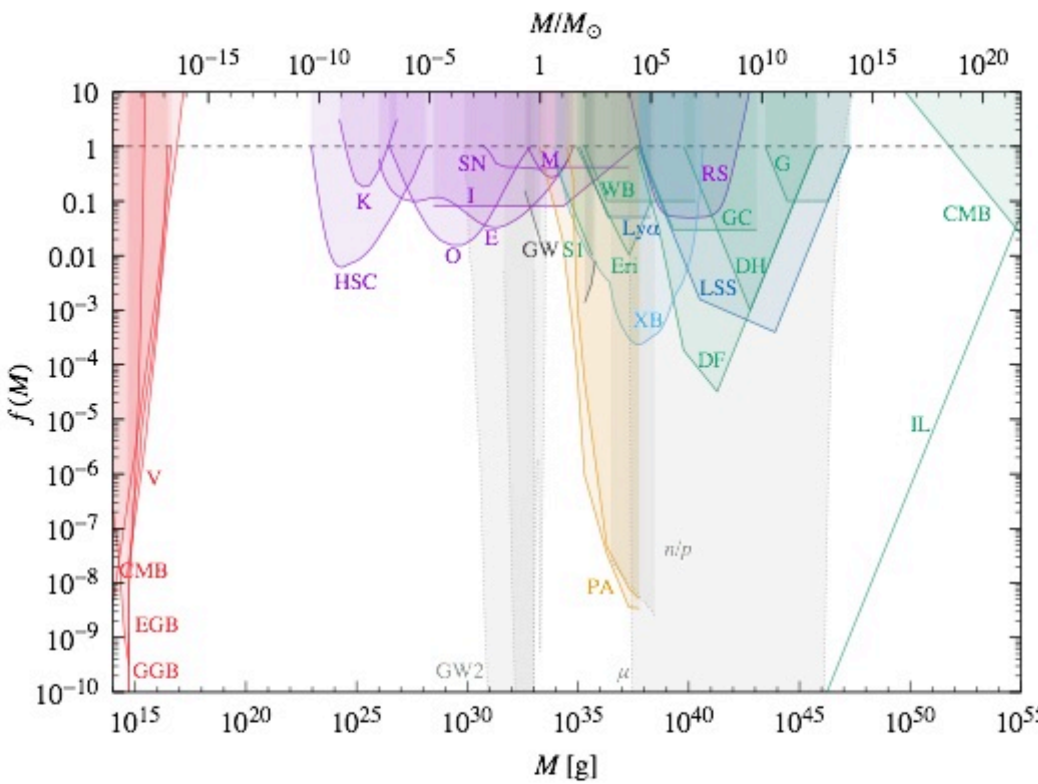
- Black Holes forming at very early early times, deep in the radiation dominated epoch, much before any galaxy has formed, on small scales (much samller than CMB scales)

- How can they form?
  For sufficiently large density fluctuations matter can collapse to a BH
  (Hawking,1971; Carr and Hawking, 1974)

- PBHs can contribute to dark matter

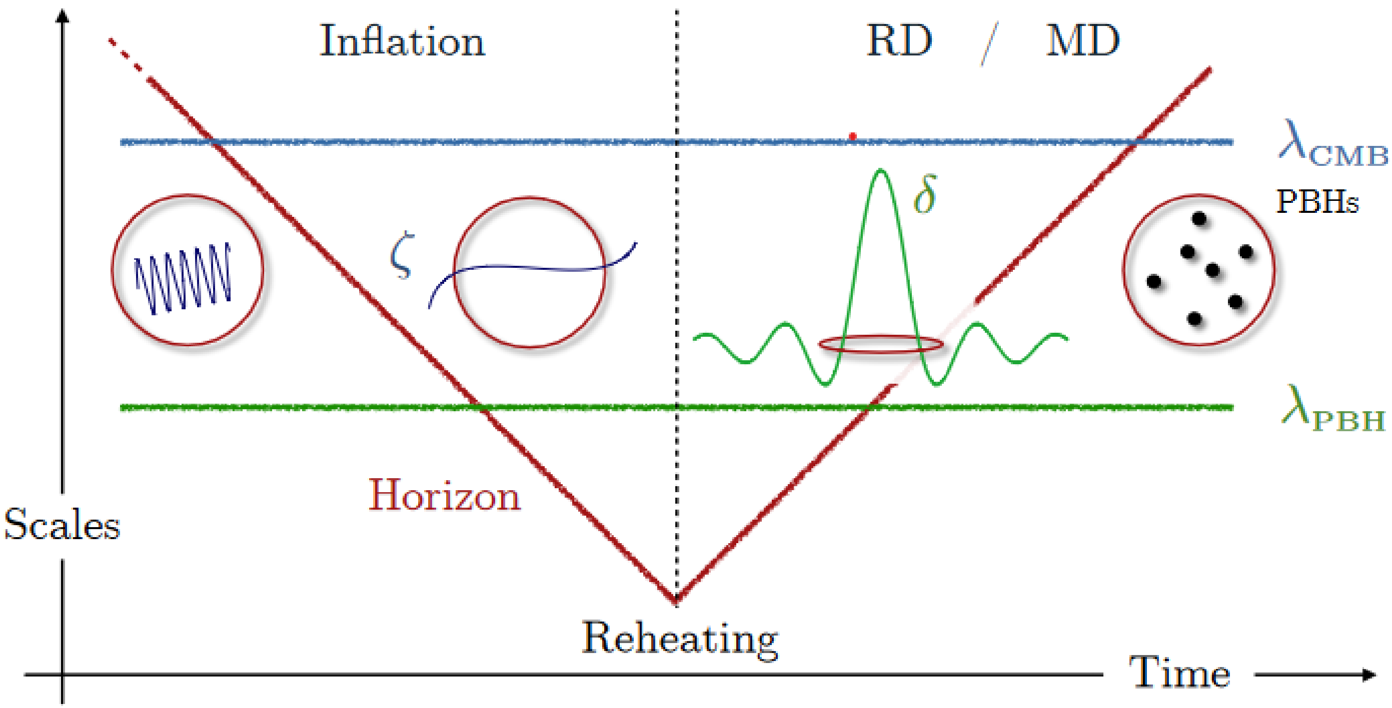
- PBHs can be sources of Gravitational Waves

- They can be produced by specific models of inflation



PBH and the tail of PDF

- For sufficiently large density fluctuations matter can collapse to a BH

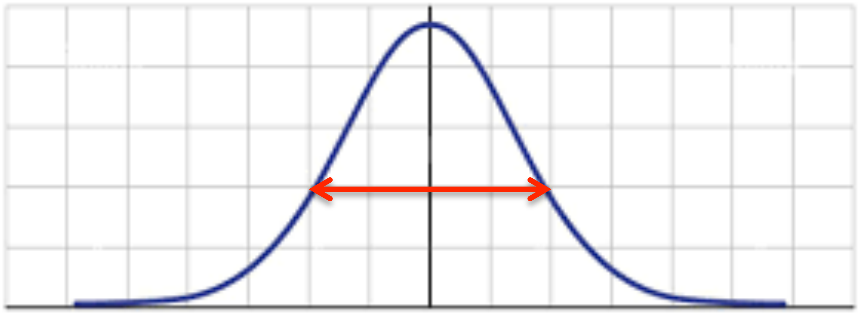


Power spectrum and PDF

- Underlying physical processes determine the statistical properties of the cosmological perturbations

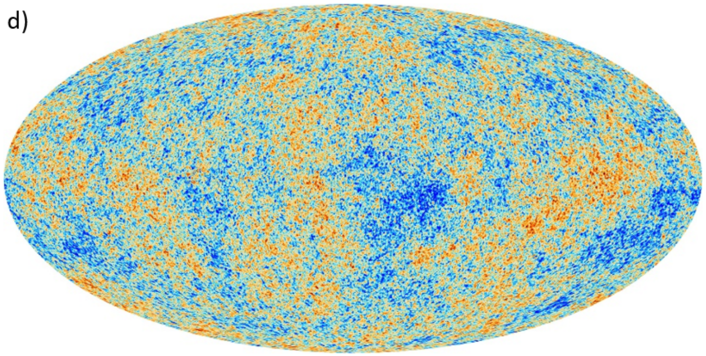
- Given the CMB constraints on primordial non-Gaussianity we can say that perturbations at CMB scales are compatible with Gaussianity

PDF



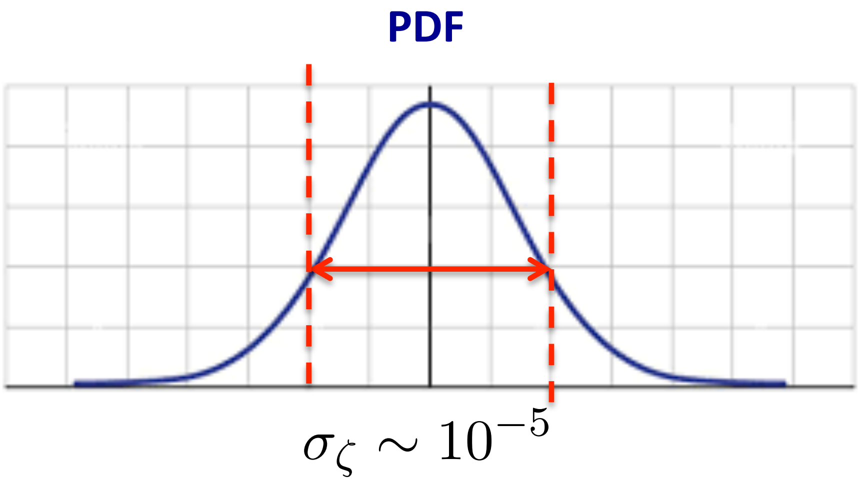
$$\sigma_\zeta \sim 10^{-5}$$

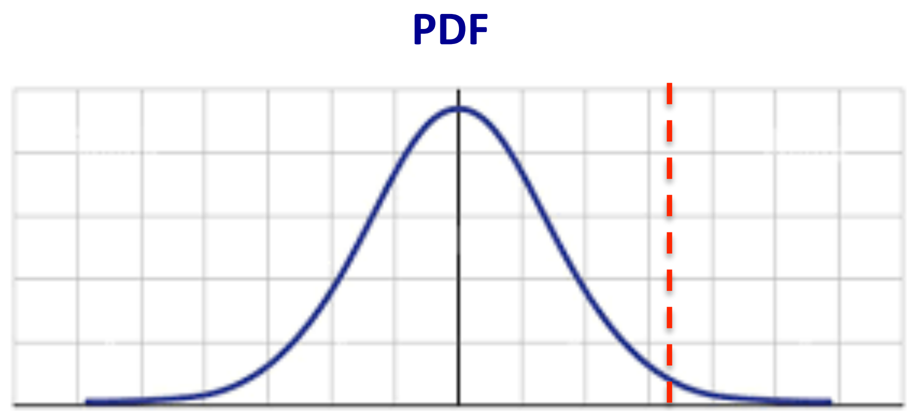
d)



PBH and the tail of PDF

- For sufficiently large density fluctuations matter can collapse to a BH





ζ_c

$$\beta = \int_{\zeta_c}^{\infty} \mathrm{PDF}[\zeta] d\zeta$$

Mass fraction of the Universe that ends up In PBH

N.B.: Actually this quanity must be calculated for the matter density perturbations

$$\Delta = \frac{\rho - \bar{\rho}}{\bar{\rho}} = \frac{2}{3} \frac{k^2}{a^2 H^2} \zeta$$ (formula which ignores non-linear effects).

Modelling of PDF tail is important

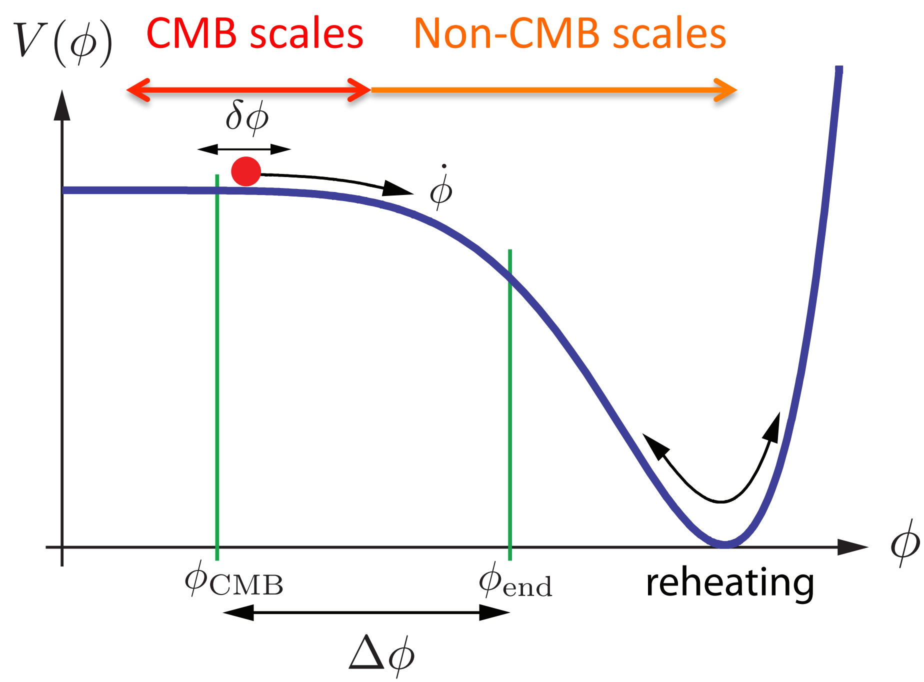
- Rare events can lead to important effects
  (here e,.g., PBH)

- They can shed light on new phenomena
  (e.g. physics governing the Early Univesre, Inflation models)

PDF tail, PBH and Inflation

- They can shed light on new phenomena
  (e.g. physics governing the Early Univeree, Inflation models)

If the the dynamics of the inflaton field
Is smooth (driven by a smooth potential)
then the fluctuations are almost Gaussian
and the tail is extremely suppressed
(at all scales).
CMB scales are compatible
with such a condition.

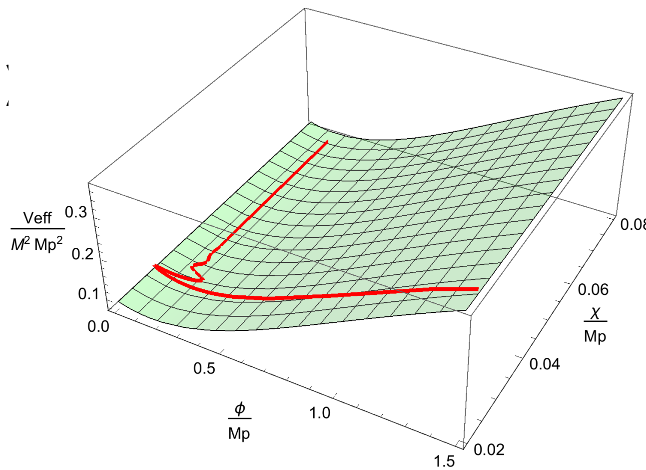


PDF tail, PBH and Inflation

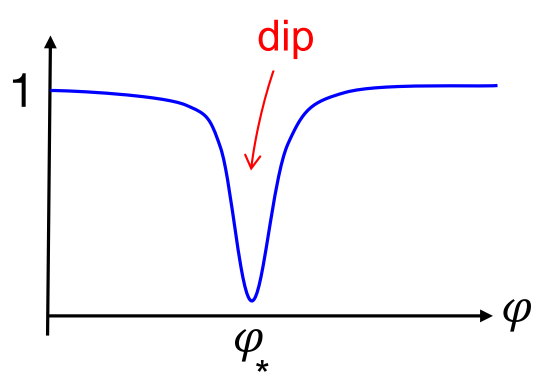
- They can shed light on new phenomena
  (e.g. physics governing the Early Univeree, Inflation models

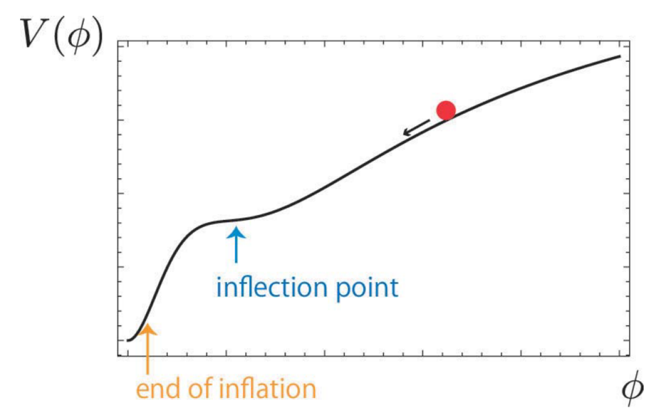
If the the dynamics of the inflaton field
Is characetrized by some features
(beyond the standard models of slow-roll)
then the power-spectrum of primordial
fluctuations can be enhanced and
moreover inflationary fluctuations
can become non-Gaussian.

Inflation models leading to PBH



Multi-field models

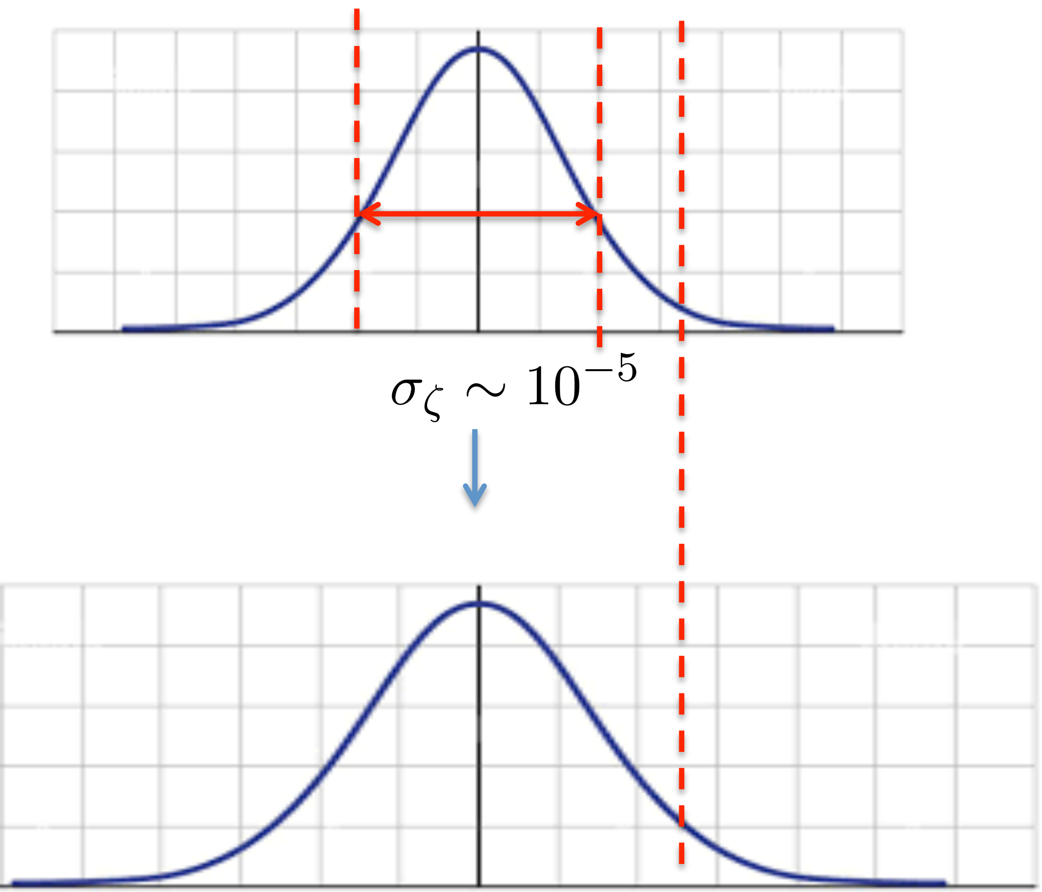




PDF tail, PBH and Inflation

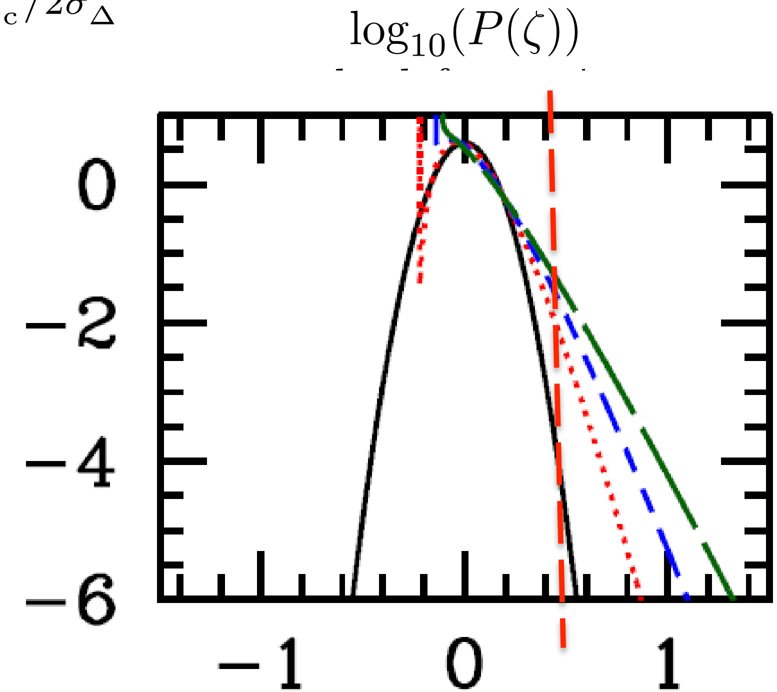
If Gaussian PDF

$$\beta_M = \int_{\Delta_c}^{\infty} \frac{\mathrm{d}\Delta}{\sqrt{2\pi}\,\sigma_\Delta} e^{-\Delta^2/2\sigma_\Delta^2} \simeq \frac{\sigma_\Delta}{\Delta_c\sqrt{2\pi}} e^{-\Delta_c^2/2\sigma_\Delta^2}$$



$\sigma_\zeta \sim 10^{-5}$

$\sigma_\zeta \sim 10^{-3} - 10^{-2}$



Effects of non-Gaussianity on the tail for f_NL=2,3,5

(see e.g., Byrnes et al 2012; Franciolini, Kehagias, Matarrese & Riotto,

Modelling of PDF tail is important

In a nutshell: since PBH are formed on the tails of the probability distribution of the curvature perturbation (aka they are rare events), they are very sensitive to changes in those tails and therefore in non-Gaussianities of the probability distribution.

Modelling of PDF tail is important

Various methods studied

➢

$$P(\zeta_R > \zeta_c) = \left(\sqrt{2\pi\left(\frac{\zeta_c}{\sigma_R}\right)^2}\right)^{-1} \exp\left\{ -\left(\frac{\zeta_c}{\sqrt{2}\sigma_R}\right)^2 + \sum_{n=3}^{\infty}\frac{(-1)^n}{n!}\xi_R^{(n)}(0)\left(\frac{\zeta_c}{\sigma_R^2}\right)^n\right\}$$

Matarrese, Lucchin, Bonometto ApJ 310 (1986) and related works. Path-integral approach that allows to compute exactly the mass fraction of PBHS at formation in presence of non-Gaussianity.

➢ Stochastic effects, Fokker-Planck equation

➢ Resummation techinques within an Effective Fiel Theory methods

➢ Other non-linear approaches to inflaton flactuations

Conclusions

- Standard cosmological model well-defined
  (but eventually see tensions: accuracy cosmology).
- Inflation as the generator of all the structures we see in the Universe: consitent with all data; its basic principles well-understood. Still the precise mechanism is not known.
- Proper Statistical tools essential to dig into these issues.
- For example: can statistics of extreme values be used to either stretength the statisical signficance of some CMB anomalies or to better investigate them? Which further informationit can bring on non-Gaussianity?

Can it help in PBH investigations?

Back-up slides

The power spectrum of cosmological perturbations: a quick definition

- For example:

$$\begin{aligned}\xi(r) &= \langle f(\mathbf{x}+\mathbf{r},t) f(\mathbf{x},t)\rangle = \frac{1}{(2\pi)^3}\int d^3\mathbf{k}\int d^3\mathbf{k}' e^{i\mathbf{k}\cdot(\mathbf{x}+\mathbf{r})} e^{i\mathbf{k}'\cdot\mathbf{x}}\langle f_{\mathbf{k}} f_{\mathbf{k}'}\rangle \\ &= \frac{1}{(2\pi)^3}\int d^3\mathbf{k}\int d^3\mathbf{k}' e^{i\mathbf{k}\cdot(\mathbf{x}+\mathbf{r})} e^{i\mathbf{k}'\cdot\mathbf{x}} P(k)\delta^{(3)}(\mathbf{k}+\mathbf{k}') \\ &= \frac{1}{(2\pi)^{3/2}}\int d^3\mathbf{k} e^{i\mathbf{k}\cdot\mathbf{r}}\frac{P(k)}{(2\pi)^{3/2}}\end{aligned}$$

So that the variance turns out to be

$$\sigma^2 = \xi(o) = \frac{1}{(2\pi)^{3/2}}\int d^3\mathbf{k}\frac{P(k)}{(2\pi)^{3/2}} = \frac{1}{2\pi^2}\int_0^\infty dk k^2 P(k) = \int_0^\infty \frac{dk}{k}\mathcal{P}(k)$$

Spectral index of the power spectrum: definition

$$n_s - 1 = \frac{d \ln \mathcal{P}(k)}{d \ln k}$$

So, if n_s is a constant

$$\mathcal{P}(k) = \mathcal{P}(k_0) \left(\frac{k}{k_0} \right)^{n_s - 1}$$

- So the spectral index describes the shape of the power spectrum (i.e. its dependence with k~(2 π)/λ, or equivalently with the cosmological scales).

- If n_s=1 we have an exact scale-invariant power spectrum which is also called Harrison-Zel’ dovich power-spectrum: the amplitude of the initial fluctuations is the same on all cosmological scales.

- In case n_s=n_s(k), i.e. it depends on the scale, one could also define a ***running of the spectral index*** as

$$\frac{dn_s}{d \ln k} \quad \rightarrow \quad \mathcal{P}(k) = \mathcal{P}(k_0) \left(\frac{k}{k_0} \right)^{n_s - 1 + \frac{1}{2} dn_s / d \ln k \ln(k/k_0) + \frac{1}{6} d^2 n_s / d \ln k^2 (\ln(k/k_0))^2 + ...}$$

CMB basics

- (Afer inflation) the Universe is initially in a hot and dense state
- Free electrons and nuclei interact with photons via Compton scattering
- As the Universe cools down, electrons combine with photons to form Hydrogen atoms (recombination) ➔ matter-radiation decoupling
- Time of decoupling ~ 300000 yrs. Temperature at decoupling ~ 3000 K.
- Due to Universe expansion the CMB has today a blackbody spectrum with color temperature T ~ 2.7 K
- The Early Universe is nearly, but not perfectly homogeneous and isotropic. Matter and radiation accrete onto overdense regions ➔ anisotropies in the CMB spatial temperature distribution

$$\frac{\Delta T}{\bar{T}} \sim 10^{-5} \qquad \bar{T} = 2.755 K$$

Generation of temperature anisotropies

Actors:

✓ Photons-baryons glued together in a single fluid by Compton scattering until last-scattering epoch z~1100.

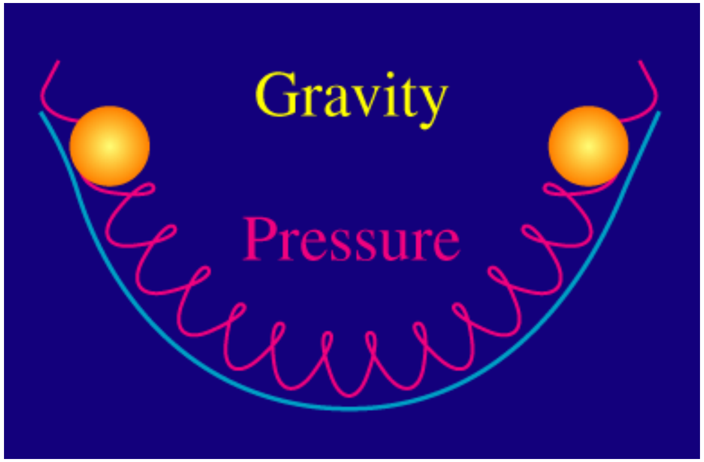
✓ dark matter+ neutrinos+cosmological constant

- *On large scales:*

density fluctuations at last scattering + gravitational redshift (SW effect)

- *On intermediate scales:*

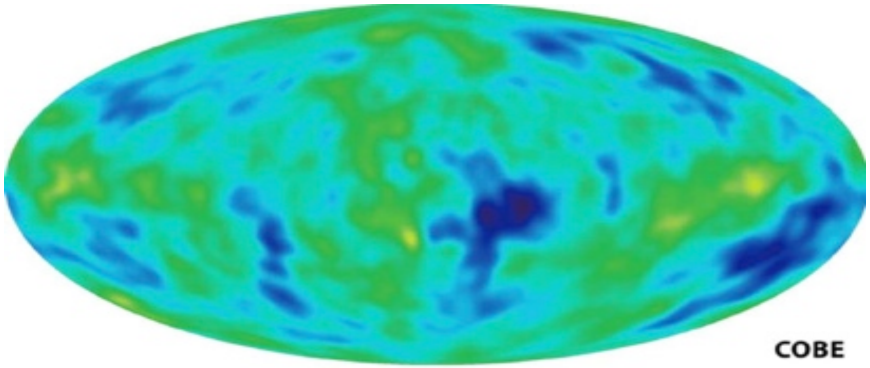
gravity (mainly due to Dark Matter)+pressure
== acoustic oscillations



- *On small scales:*

Damping due to photon free streaming (Silk damping)

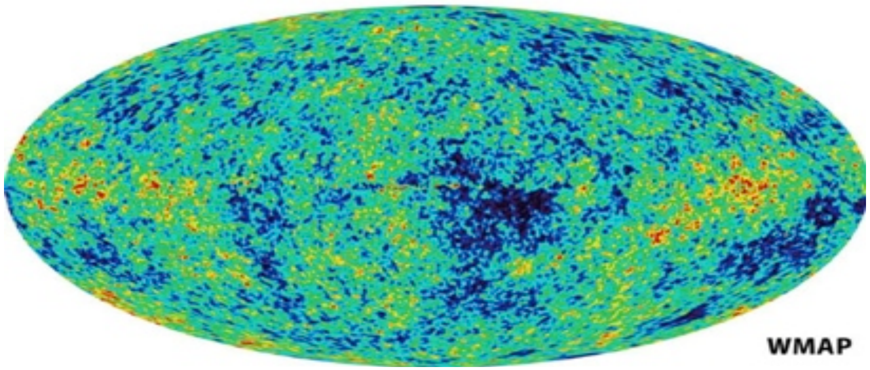
COBE, WMAP, Planck



N_{pix} ~ 10^4

FWHM ~ 7^o

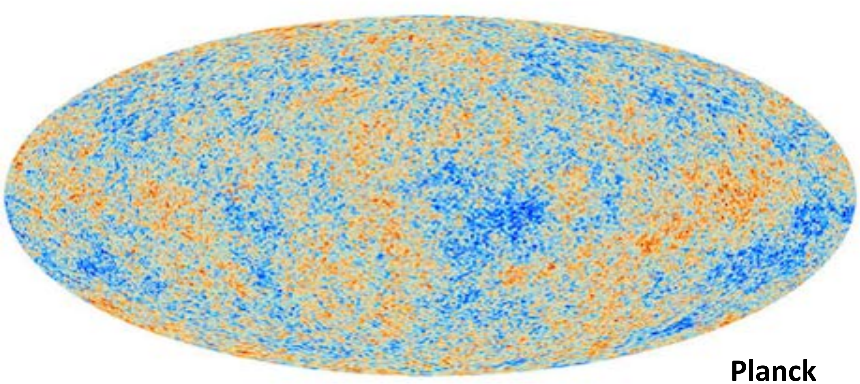
l_{max} ~ 30



N_{pix} ~ 3×10^6

FWHM ~ 12 arcmin

l_{max} ~ 1000

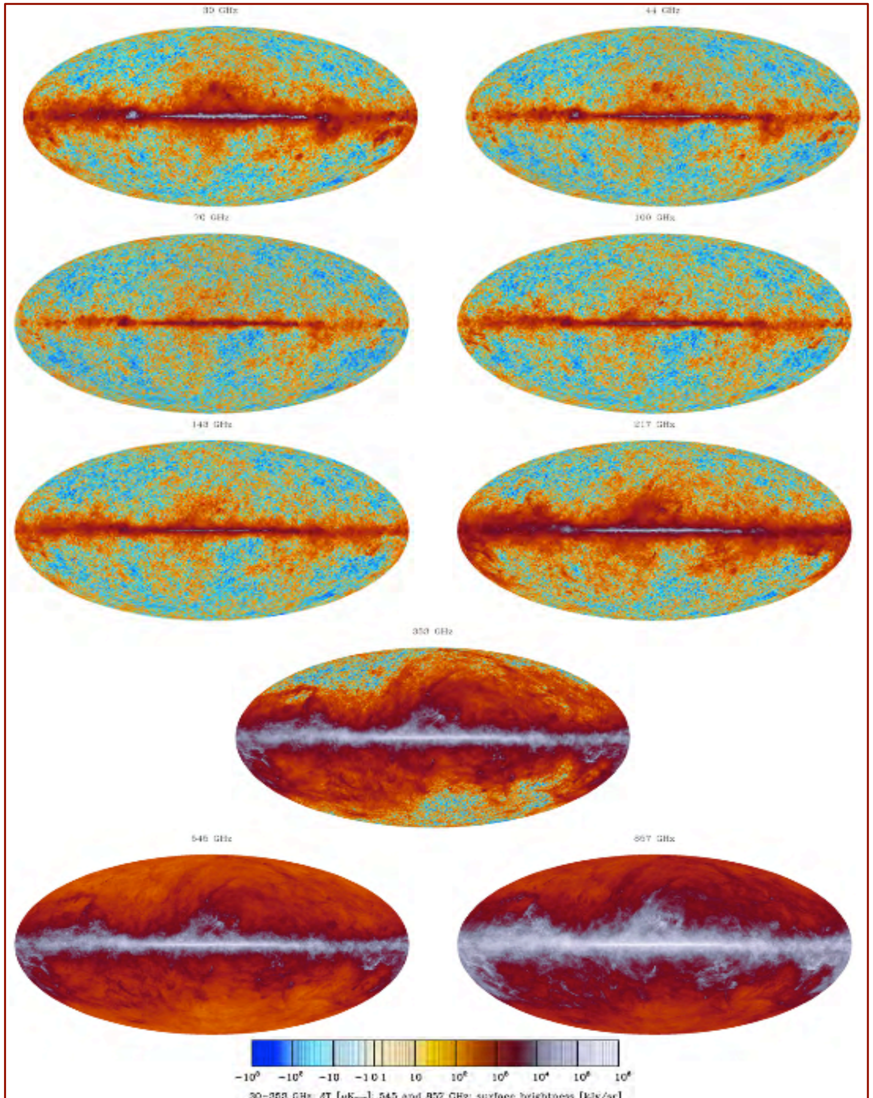


N_{pix} ~ 5×10^7

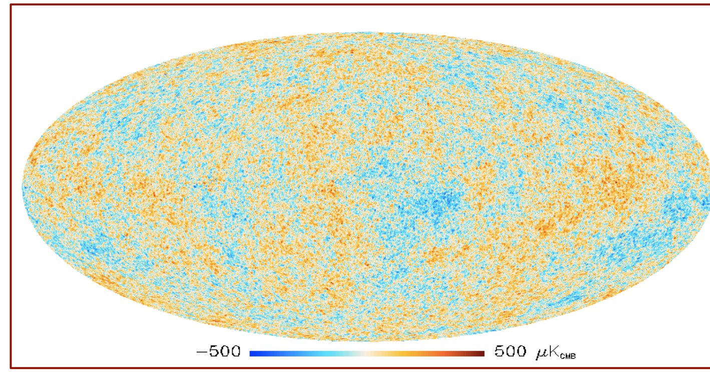
FWHM ~ 5 arcmin

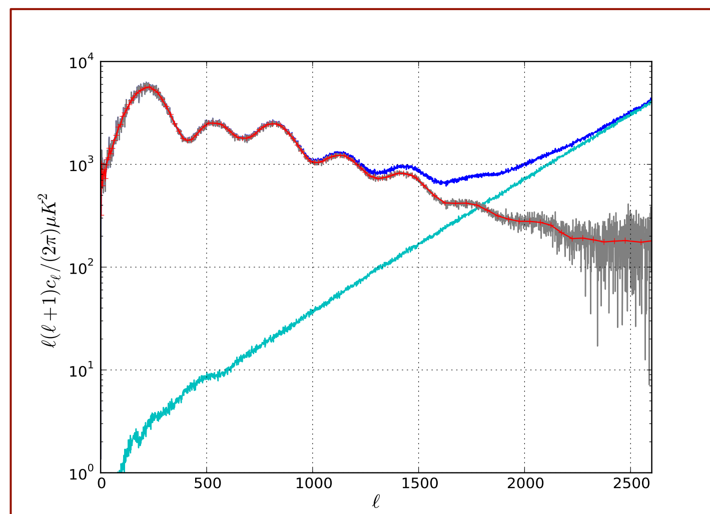
l_{max} ~ 3000

Frequency maps



"cleaning + coadding"





Sensitivity

$$\sigma_{\hat{C}_\ell} = \sqrt{\frac{2}{(2\ell+1) f_{sky}}} \left(b_\ell^2 C_\ell + N_\ell\right)$$

instrumental noise

sky coverage

beam

signal

- Even for an ideal noiseless experiment error bars are not 0 due to *cosmic variance*

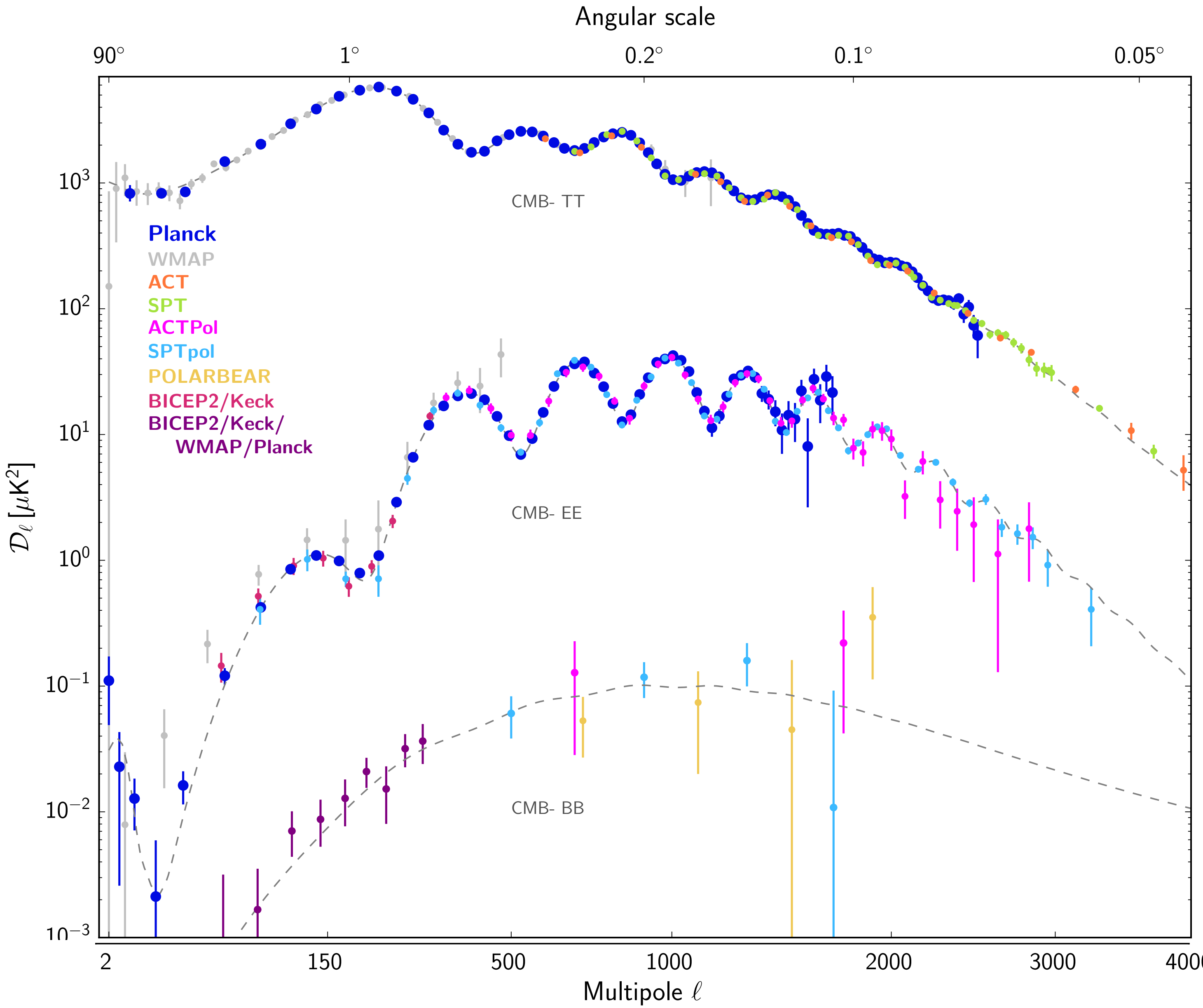
- An experiment is:

  ✓ *Cosmic variance dominated* where the error budget is dominated by the cosmic variance term (instrumental noise is negligible, low l)

  ✓ *Signal dominated* where $C_l > N_l$ (low l)

  ✓ *Noise dominated* when $N_l > C_l$ (high l)

WMAP+*Planck*+ACT+SPT+Bicep+……



Cosmological parameters

The Universe observed by Planck is well-fit by a 6 parameter ΛCDM model (& strong constraints provided on deviations from this model).

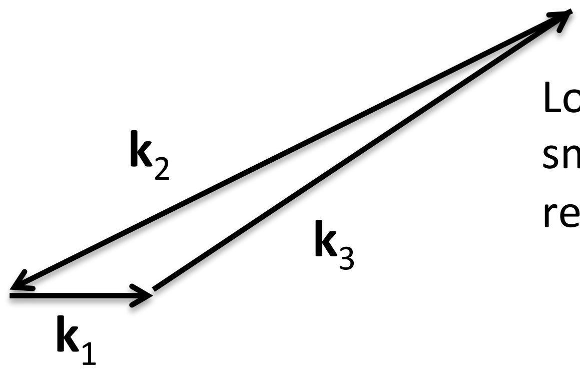
- Baryon density: Ω_b
- Matter density: Ω_m
- Acoustic scale (angular size): θ_{MC}
- Optical depth to reionization: τ
- Amplitude of primordial scalar fluctuations: A_s
- Scalar Spectral index: n_s

Another good reason: Maldacena consistency relations

For all single-field models of inflation, independently of the specific model, the ***bispectrum in the squeezed limit ($k_1 << k_2 \sim k_3$)*** is

$$\langle \zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\rangle|_{\mathbf{k}_1\to 0} = -(2\pi)^3\,\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)\;(n_s-1)\,P_\zeta(k_1)P_\zeta(k_2)$$

(Maldacena 2002)



Long mode is already frozen when the smaller modes freeze and it acts as a rescaling of the smaller modes

A convincing detection of primordial NG of the squeezed configuration can rule out ***all*** single-field models of inflation

N.B.: similar consistency relations do hold also for tensors and higher-order correlators mixing tensor & scalar fluctuations

What are some well motivated thresholds of f_{NL} for future (futuristic) experiments?

- f_{NL} ~1 is the next threshold to reach

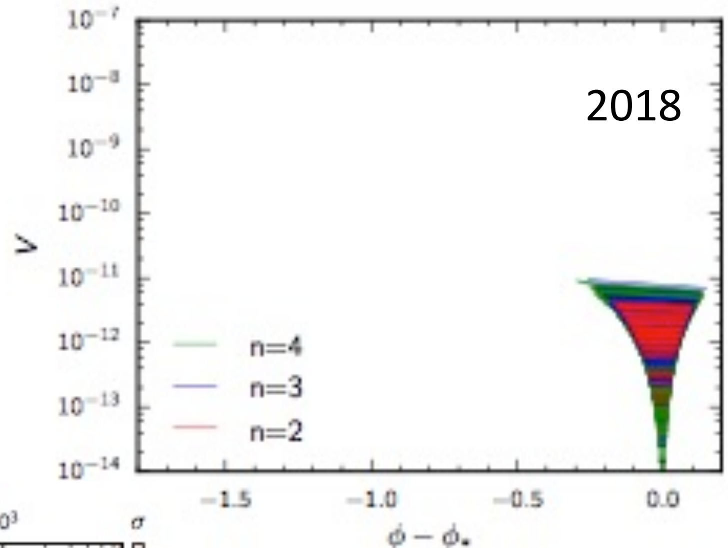
	$f_{\rm NL}^{\rm loc} \lesssim 1$	$f_{\rm NL}^{\rm loc} \gtrsim 1$
$f_{\rm NL}^{\rm eq,\, orth} \lesssim 1$	Single-field slow-roll	Multi-field
$f_{\rm NL}^{\rm eq,\, orth} \gtrsim 1$	Single-field non-slow-roll	Multi-field

↓

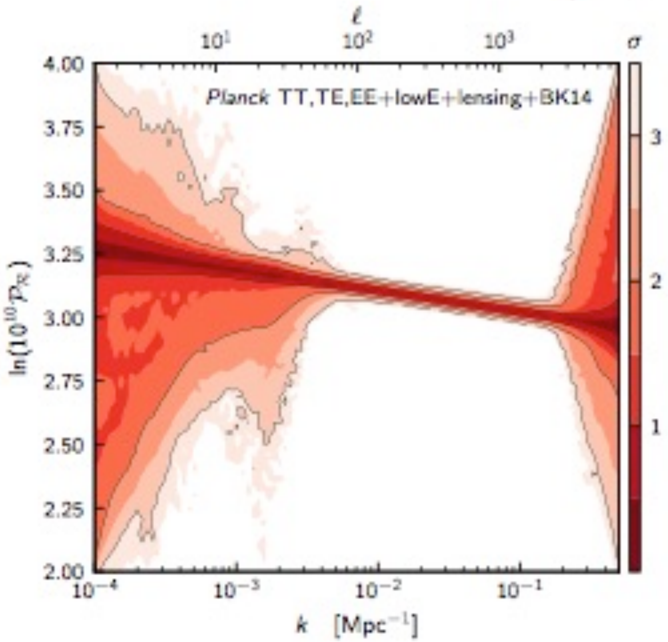
- f_{NL} ~10^{-2}-10^{-3} is *the* threshold one would really reach: ***Another fundamental test of inflation!***

It is the prediction of standard single-field slow-roll models: f_{NL}~O(ε,η) (Acquaviva, Bartolo, Matarrese, Riotto 2002; Maldacena 2002).

Beyond-slow roll: Reconstructing the inflationary potential and the primordial power spectrum



$$\zeta \simeq \frac{H\delta\phi}{\dot{\phi}}$$



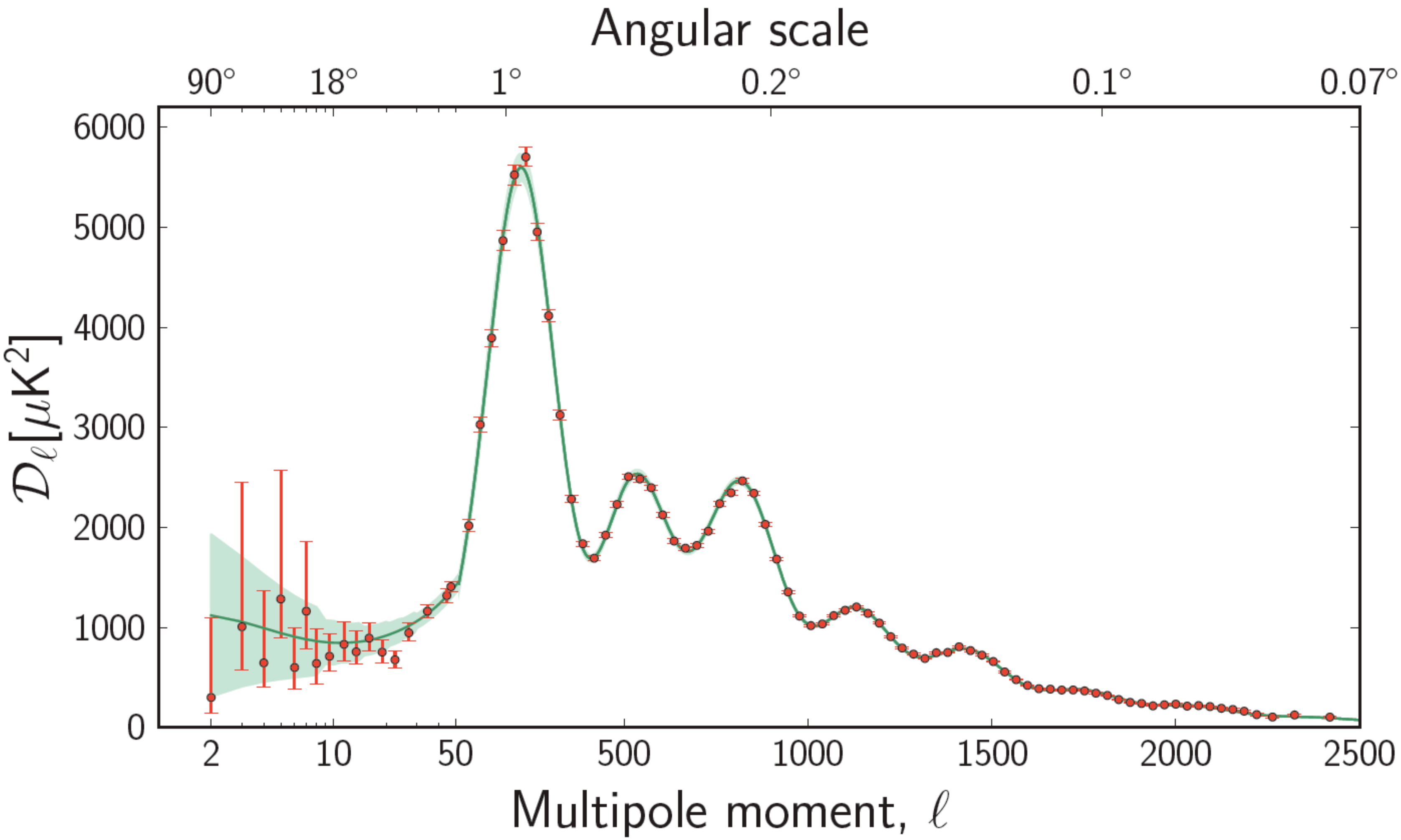
No evidence of deviations from a featureless power-spectrum

In agreement with

$$dn_s/d\ln k = 0.013 \pm 0.012 \ (0.002 \pm 0.010)$$

$$d^2n_s/d\ln k^2 = 0.022 \pm 0.012 \ (0.010 \pm 0.013)$$

Planck CMB power spectrum



Primordial gravitational waves

In a similar way one can compute the power spectrum of the gravitational waves

$$\ddot{h}_\lambda + 3H\dot{h}_\lambda + \frac{k^2}{a^2}h_\lambda = 0$$

We see that the 2 polarization states corresponds to 2 massless minimally coupled scalar fields. Then we have (a "*" here indicates evaluation at horizon crossing during inflation)

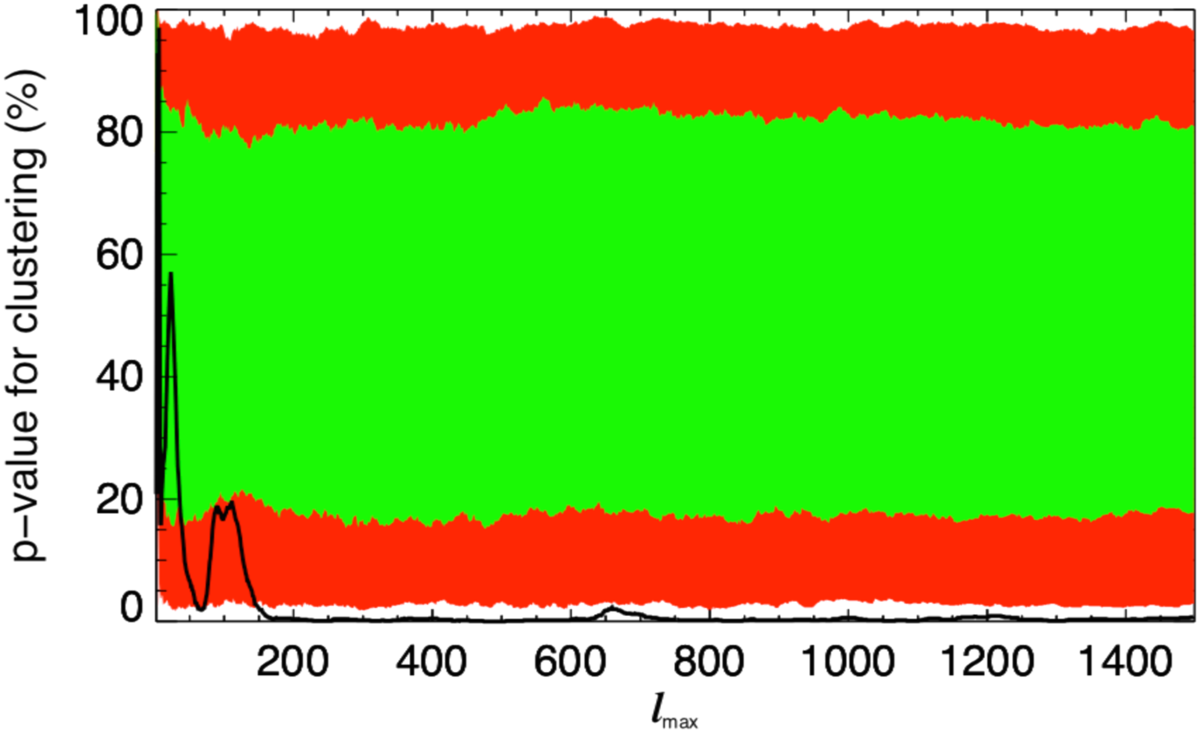
> This equality holds because, on super-horizon scales, tensor fluctuations remain constant in time (see results for a massless scalar field) and so its value on those scales is fixed at horizon-crossing during inflation (similarly to what we did for the curvature perturbations)

$$\mathcal{P}_{h_{+,\times}} = 32\pi G\,\mathcal{P}_{\phi_{+,\times}} = \frac{4}{M_{\rm Pl}^2}\left(\frac{H_*}{2\pi}\right)^2 = \frac{4}{M_{\rm Pl}^2}\left(\frac{H}{2\pi}\right)^2\left(\frac{k}{aH}\right)^{-2\epsilon}$$

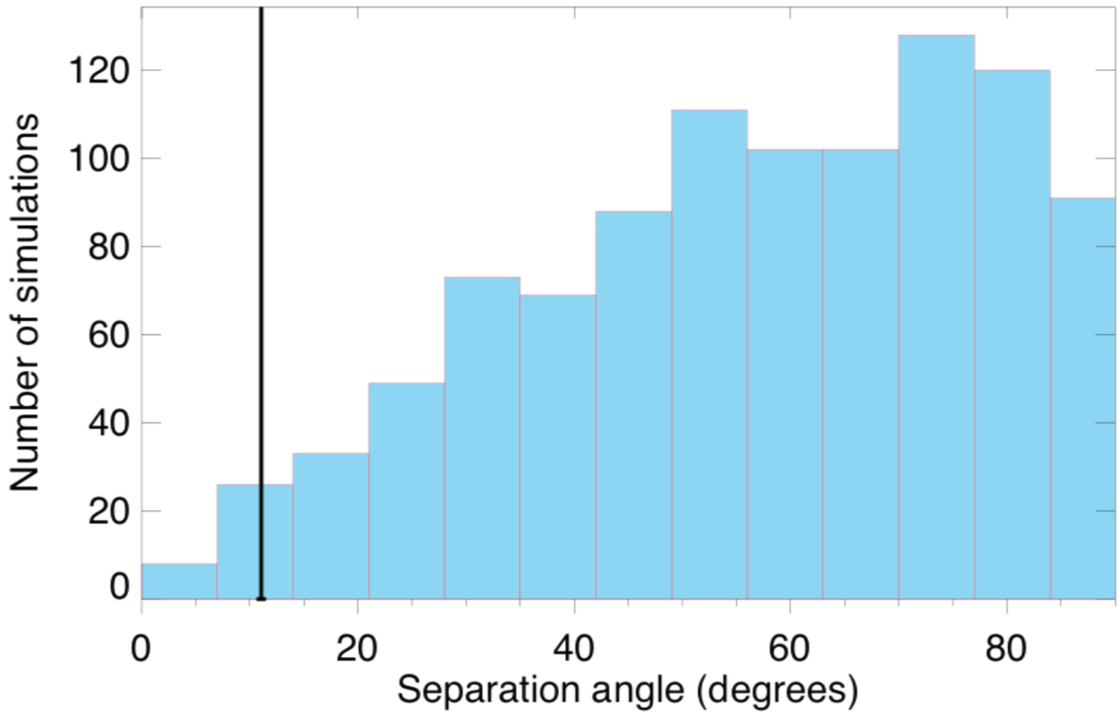
And hence, summing over the 2 polarization states:

$$\mathcal{P}_T = \frac{8}{M_{\rm Pl}^2}\left(\frac{H}{2\pi}\right)^2\left(\frac{k}{aH}\right)^{-2\epsilon}$$

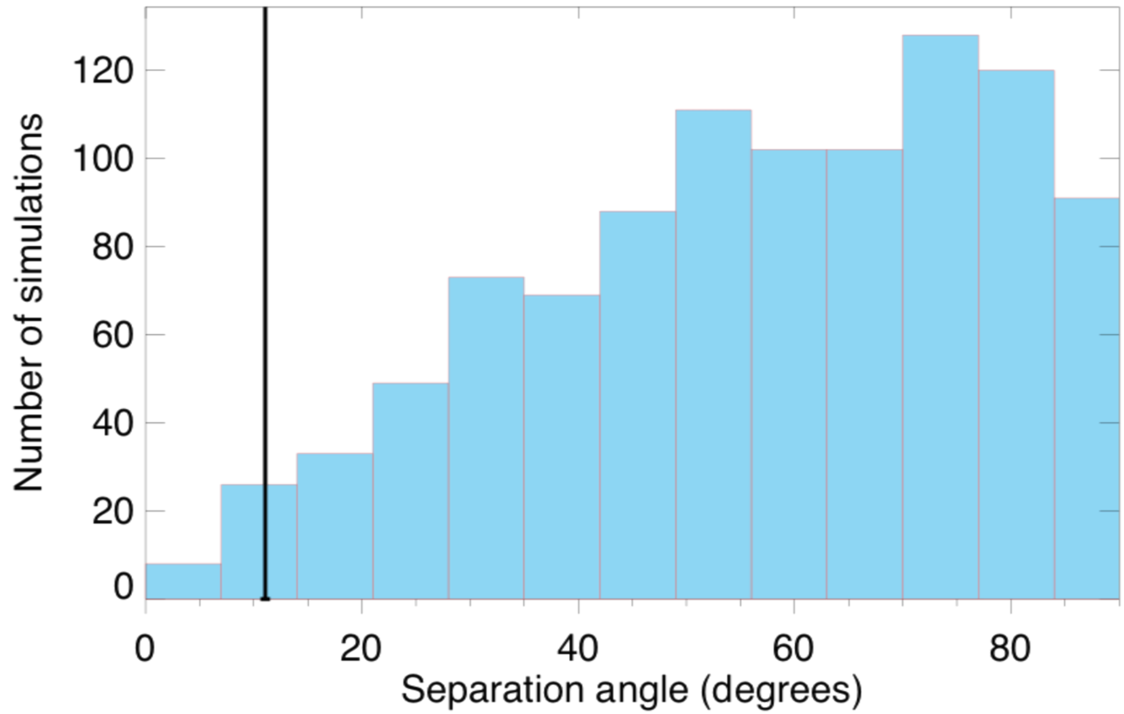
with ***tensor spectral index*** $n_T = \frac{d\ln\mathcal{P}_T}{d\ln k} = -2\epsilon$



p–value for clustering (%)
0
20
40
60
80
100
200
400
600
800
1000
1200
1400
l_{max}



Number of simulations
0
20
40
60
80
100
120
0
20
40
60
80
Separation angle (degrees)



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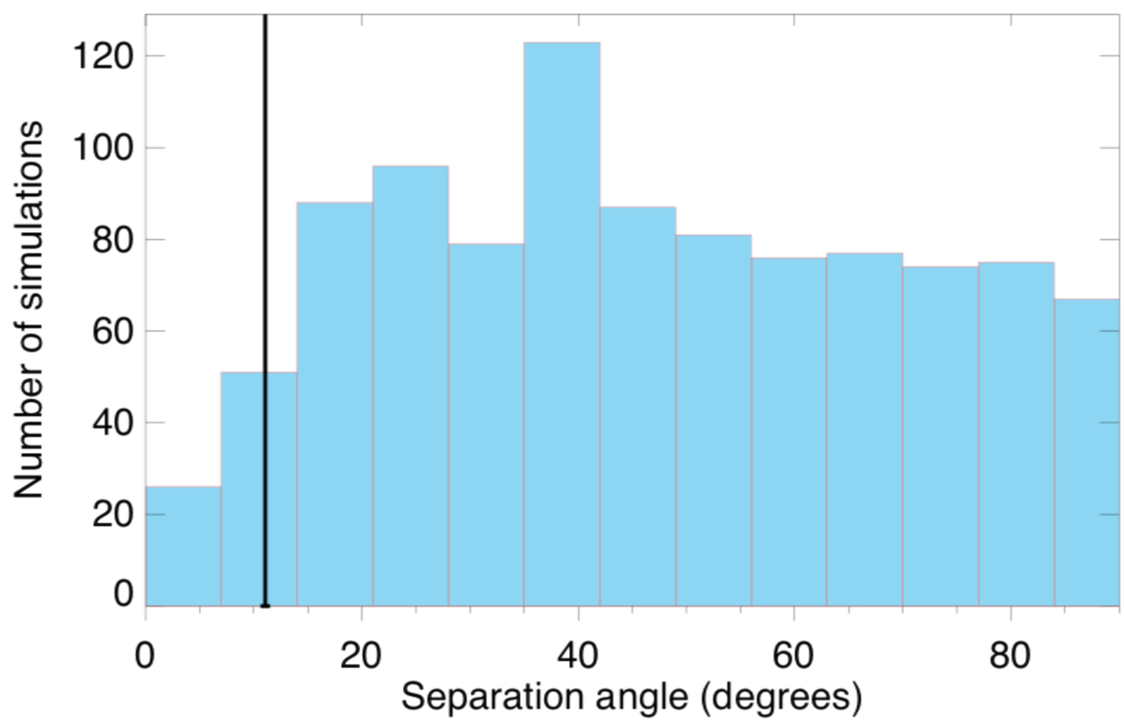

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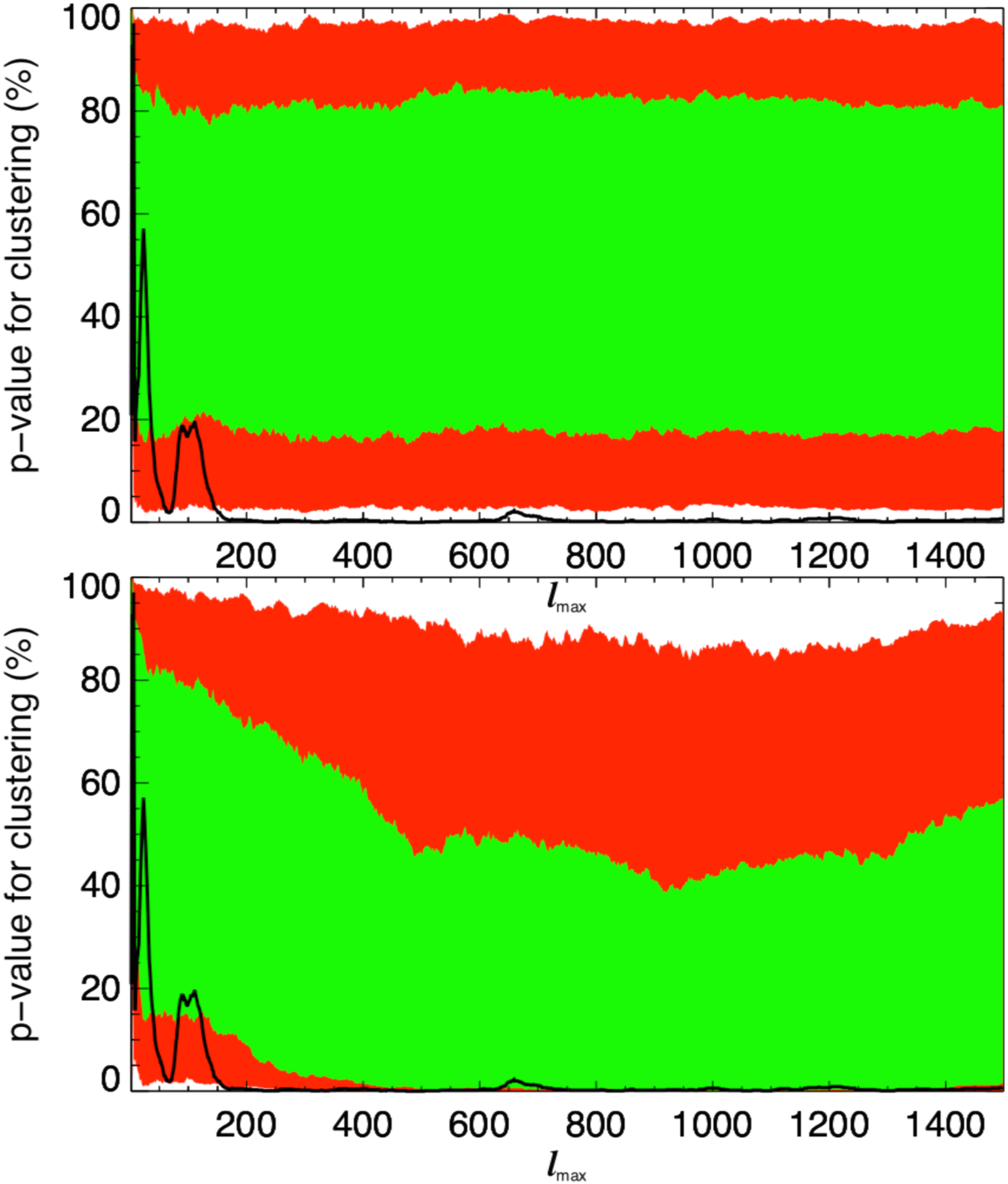
120


p−value for clustering (%)
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l_{max}
p−value for clustering (%)
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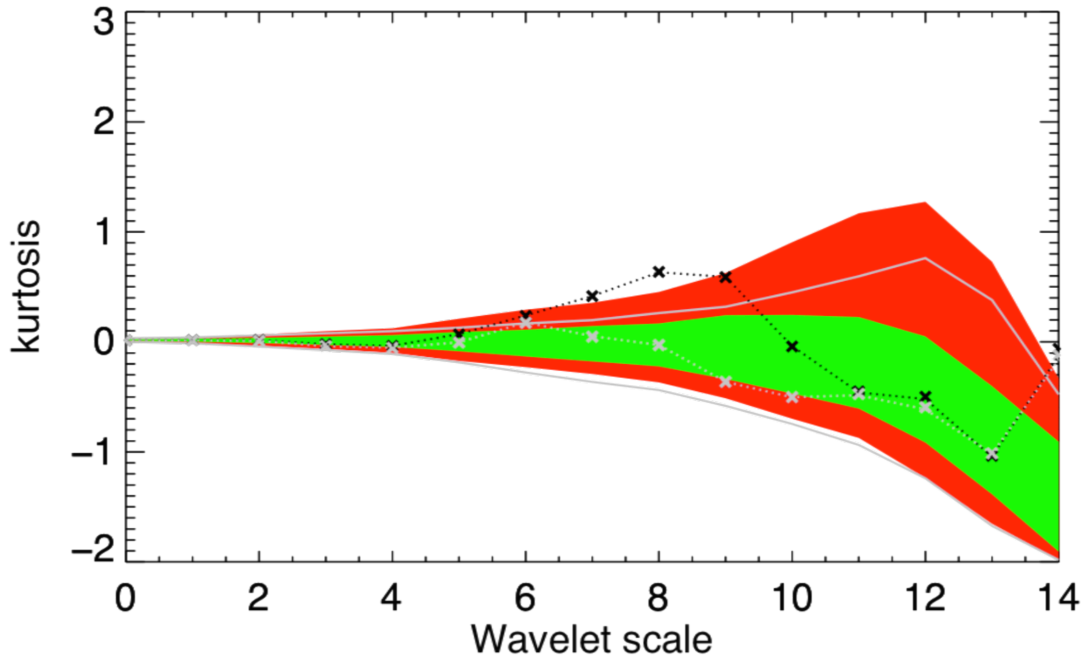
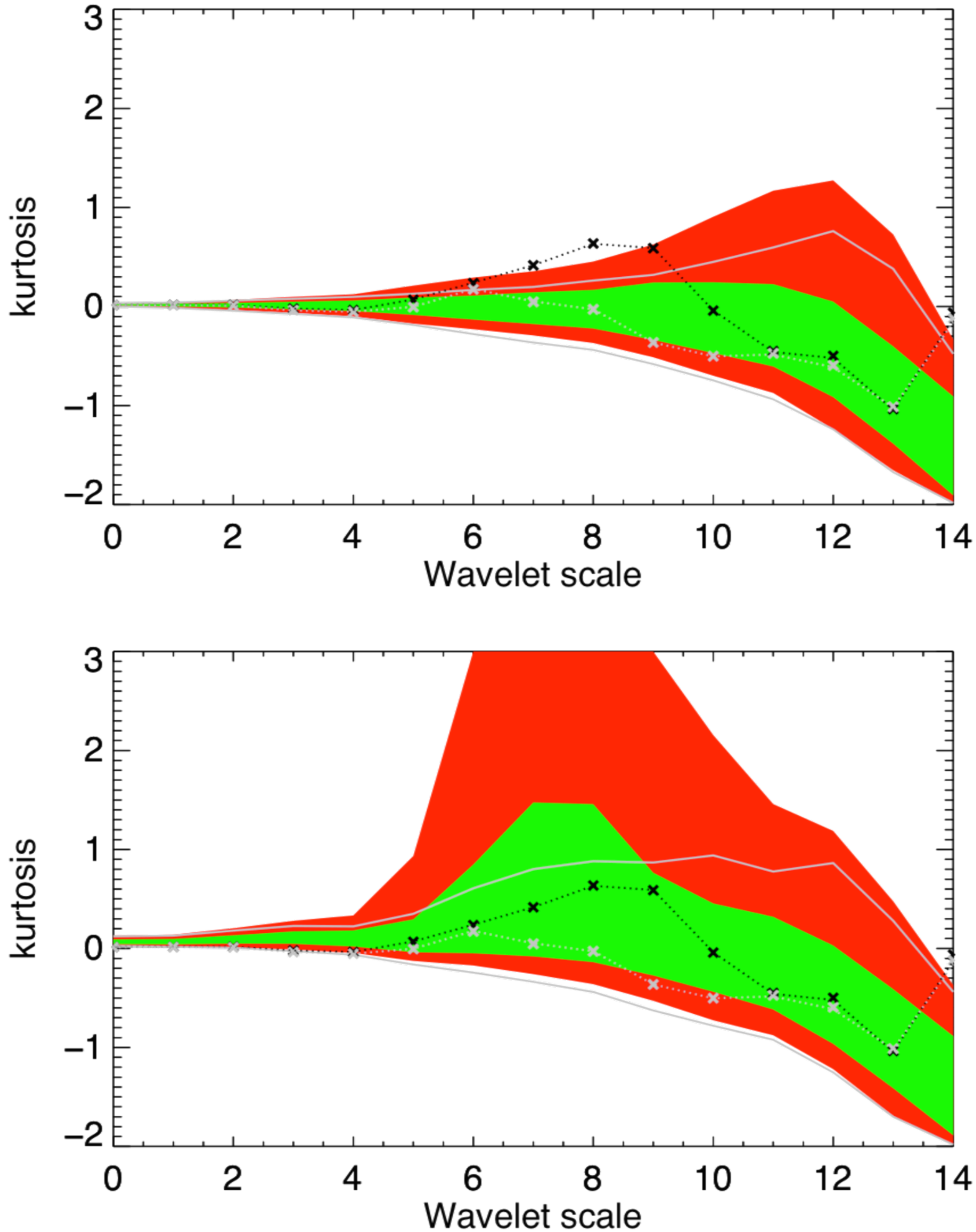
1200
1400
l_{max}



3
2
1
0
−1
−2
kurtosis
0
2
4
6
8
10
12
14
Wavelet scale



kurtosis
Wavelet scale
3
2
1
0
−1
−2
0
2
4
6
8
10
12
14
kurtosis
Wavelet scale

The toy-model

$$T(\theta,\phi) = T_{GAUSS}(\theta,\phi) + \beta\, T_{GAUSS}(\theta,\phi) \times (T^{FILT}(\theta,\phi))^2$$

The toy-model

$$T(\theta,\phi) = T_{GAUSS}(\theta,\phi) + \beta\,[T_{GAUSS}(\theta,\phi) \times (T^{FILT}(\theta,\phi))^2]^{FILT2}$$

1000 simulated toy-model maps

O 3000???

What about primordial non-Gaussianity?

We can consider two filters, W(k) and G(k)

- W(k) filters $k>k_M$
- G(k) filters $k>>k_M$

$$\zeta(\mathbf{k}) = \zeta_G(\mathbf{k}) + g_{NL} \prod_{j=1}^{3} \int \frac{d^3\mathbf{k}_j}{(2\pi)^3} W(k_1) W(k_2) G(k) \delta^{(3)}\left(\sum_{i=1}^{3} \mathbf{k}_i - \mathbf{k}\right) \zeta_G(\mathbf{k}_1) \zeta_G(\mathbf{k}_2) \zeta_G(\mathbf{k}_3)$$

Propagate ζ to the CMB VS the toy-model $a^G_{\ell m} + \beta_1 g_\ell c^F_{\ell m}$

The final result is a (nearly) one-to-one correspondence between the phenomenological model and a class of theoretical models

$$g_\ell a^G_{\ell_1 m_1} (a^G_{\ell_2 m_2} W_{\ell_2})(a^G_{\ell_3 m_3} W_{\ell_3}) \longleftrightarrow \frac{\pi}{6} \int_0^\infty dr_3 r_3^2 G_\ell(r_{dec}, r_3) \zeta^G_{\ell_1 m_1}(r_3) \zeta^F_{\ell_2 m_2}(r_3) \zeta^F_{\ell_3 m_3}(r_3)$$

$$W_\ell(r, r_j) = \int_0^\infty dk k^2 W(k) j_\ell(kr) j_\ell(kr_j) \qquad \zeta^F_{\ell m}(r) = \int_0^\infty dr_j r_j^2 W_\ell(r, r_j) \zeta^G_{\ell m}(r_j)$$

What about primordial non-Gaussianity?

$$a^G_{\ell m} + \beta_1 g_\ell c^F_{\ell m}$$

$$\beta_1 g_\ell c^F_{\ell m} = \beta_1 g_\ell \sum_{\substack{\ell_i m_i \\ i=1,2,3}} a^G_{\ell_1 m_1} (a^G_{\ell_2 m_2} W_{\ell_2})(a^G_{\ell_3 m_3} W_{\ell_3}) \mathcal{B}^{\ell_1 m_1 \ell_2 m_2 \ell_3 m_3}_{\ell m}$$

↕

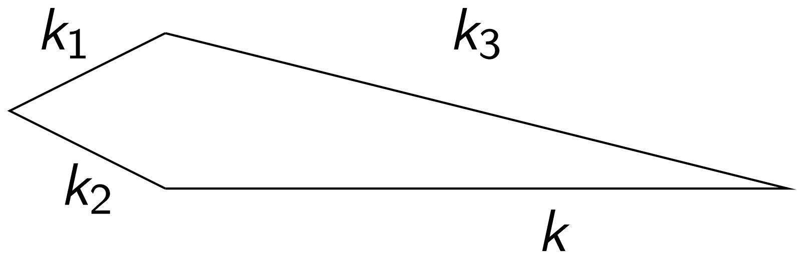
The final result is a (nearly) one-to-one correspondence between Phenomenological model and a class of theoretical models

$$\frac{\pi}{6} \int_0^\infty dr_3 r_3^2 G_\ell(r_{dec}, r_3) \zeta^G_{\ell_1 m_1}(r_3) \zeta^F_{\ell_2 m_2}(r_3) \zeta^F_{\ell_3 m_3}(r_3)$$

$$W_\ell(r, r_j) = \int_0^\infty dk k^2 W(k) j_\ell(kr) j_\ell(kr_j) \qquad \zeta^F_{\ell m}(r) = \int_0^\infty dr_j r_j^2 W_\ell(r, r_j) \zeta^G_{\ell m}(r_j)$$

In search for an inflationary model

The kind of non-Gaussianity (trispectrum) which leads to the phenomenological model we are interested to, might be realized following the approach of Shandera et al. in a two field inflationary scenario (e.g., inflaton+curvaton). It is just a possibility.



Shape of trispectrum:

$k_1, k_2 << k, k_3$

N.B.: even if we allow for f_NL term, thjis would not change our main conclusions

Models for CMB anomalies

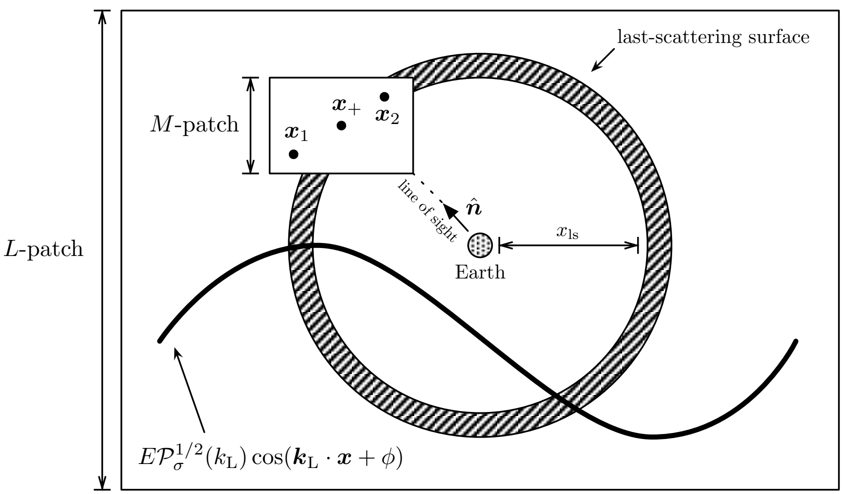
- Main focus on large-scale hemispherical asymmetry $$\frac{\delta T}{T_0} = \frac{\delta T}{T_0}(1 + A\hat{n}\cdot\hat{p})$$

Two broad "families":

i) an explicit breaking of statitstical isotropy <A>≠0

ii) a spontaneous (apparent) breaking of statistical isotropy: large-scale (super-horizon) fluctuations that modulate small scale power

→ *primordial non-Gaussianity of squeezed type.*



Gordon , Hu, Huterer, Crawford '05;
Erickcek, Kamionkowski, Carroll mechanism, '08

Picture from Byrnes et al. 2016

Models for CMB anomalies

- You need to tilt primordial non-Gaussianity to reproduce the large-scale hemispherical asymmetry that is present only for l<60:

$$\mathcal{P}^{\rm obs}(k) \approx \frac{k^3 P(k)}{2\pi^2}\Big(1 + 2A(k)\hat{\mathbf{p}}\cdot\hat{\mathbf{n}} + \cdots\Big) \longrightarrow A(k) = A_{k_0}\left(\frac{k}{k_0}\right)^n$$

best fit n=−0.5
A=0.07
(Aiola et al. 2015)

- ***Typically a squeezed bispectrum with f_NL scale dependent***

- ***Not easy to realize during inflation (e.g. Byrnes et al. 2016).***
  i) if single field

$$\frac{d\ln|f_{\rm NL}|}{d\ln k} = \frac{5}{6f_{\rm NL}}\sqrt{\frac{r}{8}}\frac{M_{\rm P}^3 V'''}{3H^2}$$

→ but then $\eta_\sigma = \dfrac{M_{\rm P}^2 V''}{3H^2}$ >>1, ***breaking near scale-inv.***

ii) Multiple fields: take a second-field subdominant

$$\frac{{\rm d}\ln A}{{\rm d}\ln k} \approx \frac{1}{2}\frac{{\rm d}\ln|f_{\rm NL}(k,k,k)|}{{\rm d}\ln k} \approx \frac{{\rm d}\ln(\mathcal{P}_\sigma/\mathcal{P})}{{\rm d}\ln k} \approx 2\eta_\sigma - (n_s - 1)$$

→ $\eta_\sigma \approx -0.25$

e.g. $W(\phi,\sigma) = V(\phi)\left(1 - \dfrac{1}{2}\dfrac{m_\sigma^2\sigma^2}{M_{\rm P}^4}\right)$ ***But not easy to keep σ subdominat!!***