Multiple -View Imaging in GRAIN

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Coded masks for imaging of neutrino events

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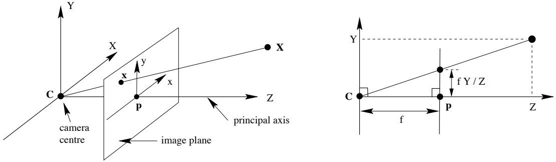
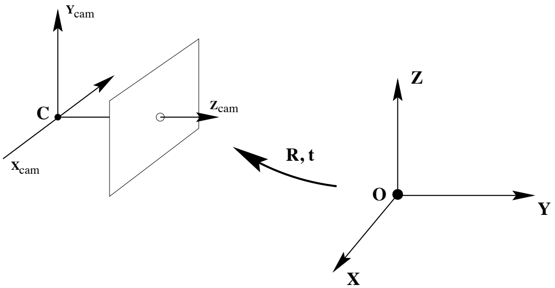
3D to 2D camera projection



Camera Model $\mathbf{x} = P\,\mathbf{X}, \qquad \mathbf{X} \in \mathbb{P}^3 \rightarrow \mathbf{x} \in \mathbb{P}^2$

$$\mathsf{P} = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix} = \begin{bmatrix} \mathbf{P}^{1\mathsf{T}} \\ \mathbf{P}^{2\mathsf{T}} \\ \mathbf{P}^{3\mathsf{T}} \end{bmatrix}$$

Camera projection matrix



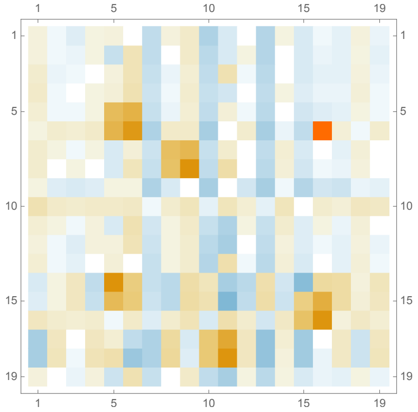
$$P = K\,[R|\mathbf{t}]\,, \qquad \lambda\,\mathbf{x} = P\mathbf{X}, \quad \forall\,\lambda \in \mathbb{R}_{/0}$$

$$R \in SO(3) \text{ and } \mathbb{R}^3 \ni \mathbf{t} = -R\,\mathbf{c}: \qquad P\,\mathbf{C} = P\begin{pmatrix}\mathbf{c}\\1\end{pmatrix} = 0$$

Camera calibration matrix $$K = \begin{pmatrix} f_x & s & x_0 \\ 0 & f_y & y_0 \\ 0 & 0 & 1 \end{pmatrix}$$

Application to a MURA mask

A 19×19 MURA mask, with parameters
$a \to 15, b \to 1.3637, p_m \to 0.11, p_d \to 0.12 \ (cm)$
6 point-like sources (minimal constraint)



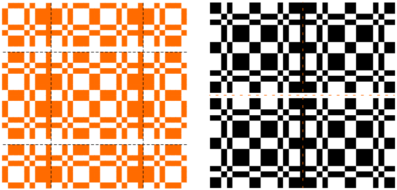
Calibration matrix

$$K = P_{ex}^{1-3} R^{-1} = \begin{pmatrix} 0.281 & 0.223 & 0.236 \\ 0 & 0.560 & -0.402 \\ 0 & 0 & 1 \end{pmatrix}$$

$$R = I_y \; R_z(0.22) \; R_y(0.82) \; R_z(0.12) \qquad \mathbf{c} = (0.33, 1.62, 1.10)$$

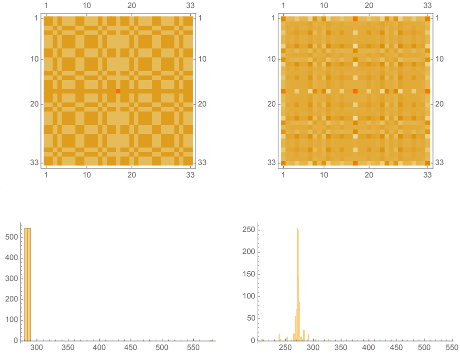
- In first approximation a Coded Mask is a Projective Camera
- Different focal lengths $f_x \neq f_y$
- $(x_0, y_0) \neq \mathbf{0} \leftrightarrow$ image plane Origin $\neq$ principal point
- Non vanishing skew parameter s
- Rotation relative to the world frame
- Traslated camera center

Optimization of the Mosaic Masks



2 × 2 (left) mosaic EVEN side by side and

(right) ODD rotated superimposed 17 × 17 MURA



Autocorrelation function for the EVEN and the

ODD mosaic

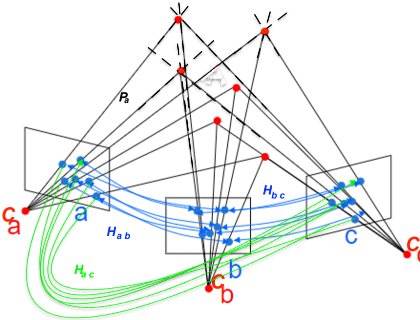
Methods of completing the lost/missed pixels
Valerio Pia thesis

Two - Triple - Many Views

- $P_i : \mathbb{P}^3 \rightarrow \mathbb{P}^2$
- $\lambda_{i\,j}\ \mathbf{x_j} = P_i \mathbf{X}_j, \quad \forall\ \lambda_{i\,j} \in \mathbb{R}_{/0}$
- $P_i = K_i\,[R_i|\mathbf{t_i}] \quad P_i\ \mathbf{C_i} = 0$
- $\mathbf{C_i} = \begin{pmatrix} \mathbf{c_i} \\ 1 \end{pmatrix} = \begin{pmatrix} -R_i^T\ \mathbf{t_i} \\ 1 \end{pmatrix}$
- Transition f. $H_{a\,b} : \mathbb{P}^2 \leftrightarrow \mathbb{P}^2$

$$\begin{cases} H_{a\,a} & = & \mathbb{I} \\ H_{a\,b}\ H_{b\,a} & = & \mathbb{I} \\ H_{a\,b}\ H_{b\,c}\ H_{c\,a} & = & \mathbb{I} \end{cases}$$

Cocycle identities



Pseudo-Inverse and Reconstruction formula in double-view

$$P = K\,[R|\mathbf{t}] \qquad \mathbf{x} = P\,\mathbf{X}$$

$$P^{+} = \begin{bmatrix} R^T \\ \mathbf{t}^T \end{bmatrix} \left(\mathbb{I} - \frac{\mathbf{t} \otimes \mathbf{t}}{1 + |\mathbf{t}|^2} \right) K^{-1} = \begin{bmatrix} \mathbb{I} \\ -\mathbf{c}^T \end{bmatrix} \left(\mathbb{I} - \frac{\mathbf{c} \otimes \mathbf{c}}{1 + |\mathbf{c}|^2} \right) R^T K^{-1}$$

$$\Rightarrow \qquad \mathbf{X} = P^{+}\mathbf{x} + \mu\,\mathbf{C} \qquad \mu \in \mathbb{R}$$

$$\lambda'\,\mathbf{x}' = P'\,\mathbf{X} \quad \lambda' \in \mathbb{R}_{/0}$$

$$\mathbf{x}' \wedge \mathbf{x}' = 0 \Rightarrow \boxed{\mathbf{X} = P^{+}\mathbf{x} + \frac{(P'\,P^{+}\,\mathbf{x}\ \wedge\ \mathbf{x}') \cdot (\mathbf{x}'\ \wedge\ P'\,\mathbf{C})}{(\mathbf{x}'\ \wedge\ P'\,\mathbf{C})^2}\,\mathbf{C}}$$

The Projective Reconstruction Theorem

The triplet $\left(P,\ P',\ \{\mathbf{X}_\alpha\}_{\alpha\geq 8}\right)$ is called a 3D reconstruction of the images $\{\mathbf{x}_\alpha\}$ and $\{\mathbf{x}'_\alpha\}$ if it satisfies the relations

$$\lambda_\alpha\, \mathbf{x}_\alpha = P\, \mathbf{X}_\alpha, \quad \lambda'_\alpha\, \mathbf{x}'_\alpha = \mathbf{P}'\, \mathbf{X}_\alpha$$

Th. A 3D reconstruction is unique up to a homography of $\mathbb{P}^3$.

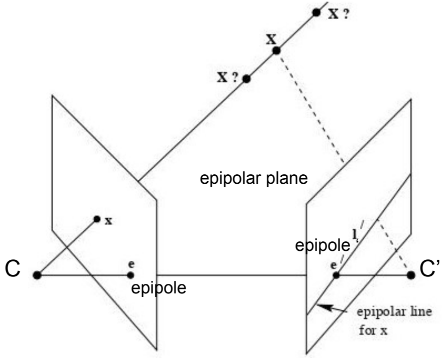
- $\left(\tilde{P},\ \tilde{P}',\ \left\{\tilde{\mathbf{X}}_\alpha\right\}_{\alpha\geq 8}\right)$ is another reconstruction iff
$\tilde{P} = P\, H^{-1}, \quad \tilde{P}' = P'\, H^{-1}, \quad \tilde{\mathbf{X}}_\alpha = H\, \mathbf{X}_\alpha, H : \mathbb{P}^3 \leftrightarrow \mathbb{P}^3 \text{ linear}$
- For any Camera Stereo Rig $P_1, P_2, \ldots$ its canonical form is

$$\tilde{P}_1 = [\mathbb{I}|\mathbf{0}]\,, \tilde{P}_2 = K_2 R_2 \,(K_1 R_1)^{-1}\,[\mathbb{I} \mid \det(K_1)\, K_1 R_2\,(\mathbf{c_1} - \mathbf{c_2})]\,, \ldots$$

$$H = \begin{pmatrix} K_1 R_1 & -K_1 R_1 \mathbf{c}_1 \\ \mathbf{0}^T & 1/\det(K_1) \end{pmatrix}$$

The Fundamental Matrix

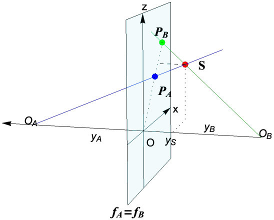
- Epipoles
  $\mathbf{e}' = P' \, \mathbf{C}, \quad \mathbf{e} = P \, \mathbf{C}'$
- $\mathbf{x}' = P' \, \mathbf{X} = P' \, P^{+} \mathbf{x} + \mu \, \mathbf{e}'$
- $\mathbf{l}' = \mathbf{e}' \wedge \mathbf{x}' = \mathbf{e}' \wedge P' \, P^{+} \, \mathbf{x}$
- $\boxed{F = [\mathbf{e}']_{\wedge} \; P' \, P^{+}}$
  rank 2 $\mathbb{P}^2 \to \mathbb{P}^2$, $\det F = 0$
- Corresponding image points
  $\mathbf{x}' \cdot \mathbf{l}' = \boxed{\mathbf{x}'^{T} \; F \, \mathbf{x} = 0}$
- $\sum_{i\,j} x'_i \, F_{i\,j} \, x_j =$
  $\det \begin{pmatrix} P' & \mathbf{x}' & 0 \\ P & 0 & \mathbf{x} \end{pmatrix} = 0$
- $F = [\mathbf{e}']_{\wedge} \, K' \, R' \, K^{-1} = K'^{-T} \, R' \, K^{T} \, [\mathbf{e}]_{\wedge}$



Computing F from a pair of Camera Matrices - Examples I

Equally calibrated c.s $K' = K$

- front-to-front stereo rig:
  $P' = K\ I_z\ [\mathbb{I}, -\mathbf{c}],\ \mathbf{c} = (0,0,c)^T,\ I_z = \mathrm{diag}(1,1,-1)$
  $\mathbf{e}' = c\,(x_0, y_0, 1)^T = c\mathbf{k}_3, \quad F_{ff-z} = [\mathbf{k}_3]_\wedge K I_z K^{-1}$
  $\mathbf{x}'^T F_{ff-z}\mathbf{x} = (y' - y_0)\,x + (y_0 - y)\,x' + (y - y')\,x_0 = 0$
  $\mathbf{X} = \begin{pmatrix} K^{-1}\mathbf{x} \\ \frac{(KI_zK^{-1}\mathbf{x}\wedge\mathbf{x}')\cdot(\mathbf{x}'\wedge\mathbf{k}_3)}{c(\mathbf{x}'\wedge\mathbf{k}_3)^2} \end{pmatrix} \Rightarrow$
  $\mathbf{X}_E = \frac{c(\mathbf{x}'\wedge\mathbf{k}_3)^2 K^{-1}\mathbf{x}}{(KI_zK^{-1}\mathbf{x}\wedge\mathbf{x}')\cdot(\mathbf{x}'\wedge\mathbf{k}_3)} = \frac{c(x'-x_0)}{x-2x_0+x'}K^{-1}\mathbf{x} = \frac{c(y'-y_0)}{y-2y_0+y'}K^{-1}\mathbf{x}$



Computing F from a pair of Camera Matrices - Examples II

- Cameras at right angle:

  $P = K\left[\mathbb{I}, \mathbf{0}\right], \quad P' = K\ R^y_{-\pi/2}\left[\mathbb{I}, -r\ R^y_{\pi/4}\hat{\mathbf{z}}\right]$

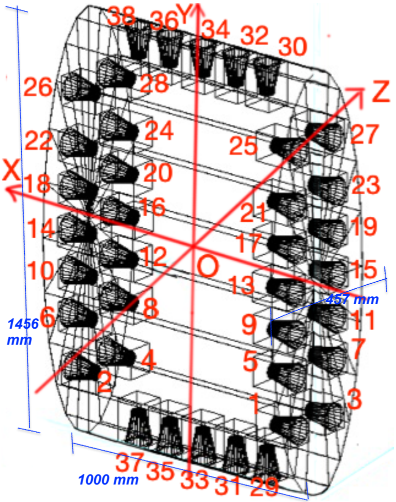
  $\mathbf{e} = r\ K\ R^y_{\pi/4}\hat{\mathbf{z}}, \quad \mathbf{e}' = -r\ K\ R^y_{-\pi/4}\hat{\mathbf{z}}$

$$F_\perp = \left[K\ R^y_{-\pi/4}\ \hat{\mathbf{z}}\right]_\times K\ R^y_{-\pi/2}\ K^{-1} = \begin{pmatrix} 0 & 1 & -y_0 \\ -1 & 0 & x_0 - \alpha \\ y_0 & -\alpha - x_0 & 2\alpha\ y_0 \end{pmatrix}$$

Non anti-symmetric

Horopter: $\mathbf{x}^T \frac{F_\perp + F_\perp^T}{2}\mathbf{x} = 0$

GRAIN as a multiple-view (lens) system



GRAIN

Set of lenses centered at

$$\mathbf{c}_m^{\epsilon_1,\,\epsilon_2,\,\epsilon_3} = \begin{pmatrix} \xi_m^{\epsilon_2,\,\epsilon_3}\ h \\ \psi_m^{\epsilon_1,\,\epsilon_3}\ b \\ \zeta_m^{\epsilon_2,\,\epsilon_3}\ \ell \end{pmatrix} = \begin{pmatrix} \left[(1-\epsilon_3)\,\delta_m - \frac{(-1)^{\epsilon_2}}{2}\epsilon_3\right]\ h \\ -(1-\epsilon_3)\,(-1)^{\epsilon_1}\,\beta_m\ b \\ \left[-(1-\epsilon_3)\,\frac{(-1)^{\epsilon_2}}{2} + \lambda\,\epsilon_3\ m\right]\ \ell \end{pmatrix}$$

for $\epsilon_i = 0,\,1$ and $0 \le |m| \le 3$

$\delta_{-m} = -\delta_m,\ \delta_0 = 0$ and $\beta_{-m} = \beta_m$ Set of Projection Matrices

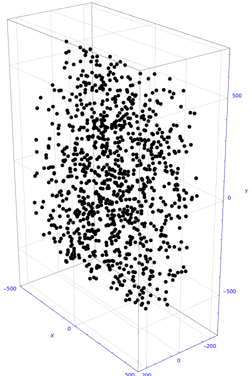
$$P_m^{\epsilon_1\ \epsilon_2\ \epsilon_3} = K\ \left(R_\pi^x\right)^{\epsilon_2(1-\epsilon_3)}\ \left(R^y_{(-1)^{\epsilon_2}\frac{\pi}{2}}\right)^{\epsilon_3}\ \left[\mathbb{I}, -\mathbf{c}_m^{\epsilon_1,\,\epsilon_2,\,\epsilon_3}\right]$$

Fundamental Matrices

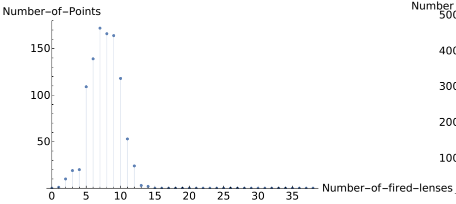
$$F_{m\ m'}^{\epsilon_1\ \epsilon_2\ \epsilon_3\ \epsilon_1'\ \epsilon_2'\ \epsilon_3'} = \left(P_{m'}^{\epsilon_1'\ \epsilon_2'\ \epsilon_3'}\right)^{+\ T}\ \left(P_m^{\epsilon_1\ \epsilon_2\ \epsilon_3}\right)^T\ \left[\mathbf{e}_{m\ m'}^{\epsilon_1\ \epsilon_2\ \epsilon_3\ \epsilon_1'\ \epsilon_2'\ \epsilon_3'}\right]$$

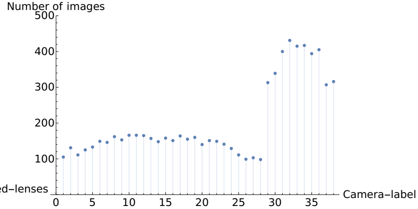
Point-like Light sources in GRAIN and their reconstruction

Joint work with L. Di Noto and M. Vicenzi
Case Study: 1000 selected light source points (randomly/uniformly) in $800 \times 1200 \times 460\ mm^3$ **point Volume** $\subset$ GRAIN
Energy release: 100 MeV per point



Points and Cameras Distribution I



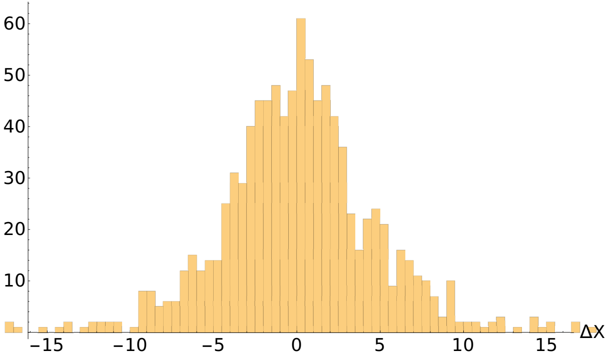


Reconstruction by double-View

- $K = \mathrm{diag}\,(-f, -f, 1)\ \forall m, \epsilon_1, \epsilon_2, \epsilon_3$ of $P_m^{\epsilon_1\,\epsilon_2\,\epsilon_3}$, $f = 100mm$
- Each point is detected/seen by $N \leq 38$ cameras
- The image coordinates are given by the centroids of the photon distribution.
- There are $M = \frac{N!}{2!(N-2)!}$ possible double-views
- Check the consistency conditions $\mathbf{x}^{m'\,T}_{\epsilon_1'\,\epsilon_2'\,\epsilon_3'} \cdot F_{m\,m'}^{\epsilon_1\,\epsilon_2\,\epsilon_3\,\epsilon_1'\,\epsilon_2'\,\epsilon_3'} \cdot \mathbf{x}^m_{\epsilon_1\,\epsilon_2\,\epsilon_3} \approx 0$
- Perform M reconstructions by the 3D formula
- take the mean value of the M possible reconstructions for each coordinate

$$X = \frac{\sum_{i<j}^N X_{ij}}{M},\ Y = \frac{\sum_{i<j}^N Y_{ij}}{M},\ Z = \frac{\sum_{i<j}^N Z_{ij}}{M}$$

Errors in reconstruction I

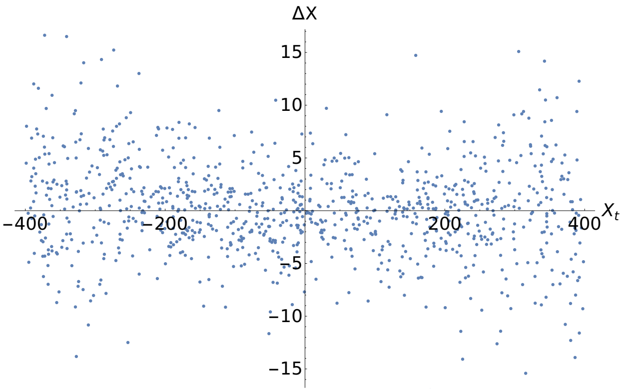


$\Delta X = X_c - X_t$ Distribution

X_c: calculated , X_t: original coordinates

$$\overline{\Delta X} \approx 10^{-2}\text{mm} \quad \overline{\Delta X^2}^{1/2} \approx 5\text{mm}$$

Errors in reconstruction II



ΔX vs X_t (mm)

Errors in computing the corresponding points

$$S = \{\mathsf{x}_1, \ldots, \mathsf{x}_N\}\,,\ S' = \{\mathsf{x}'_1, \ldots, \mathsf{x}'_{N'}\} \to S \times S' = \left\{ \left(\mathsf{x}_\alpha, \mathsf{x}'_\beta \right) \right\}$$

Corresponding points : $S_{cp} = \left\{ \left(\mathsf{x}_\alpha, \mathsf{x}'_\beta \right) : \mathsf{x}'_\beta \; F \; \mathsf{x}_\alpha = 0 \right\}$

Empirical data $0 \leq E_{\alpha,\beta} = |\ \mathsf{x}'_\beta \; F \; \mathsf{x}_\alpha \ | \leq \ \epsilon$

Bounded spread criterion

$$\left(\mathsf{x}_\alpha, \mathsf{x}'_\beta \right) = \left(\mathsf{x}^0_\alpha + \Delta \mathsf{x}, \mathsf{x}'^0_\beta + \Delta \mathsf{x}' \right)$$

$$|\Delta \mathsf{x}| \leq \frac{\epsilon}{|\mathsf{x}'^0_{\bar{\beta}}|\ ||F||},\ |\Delta \mathsf{x}'| \leq \frac{\epsilon}{|\mathsf{x}^0_{\bar{\beta}}|\ ||F||}$$

The Fundamental matrix from data

The set S_{cp} of the corresponding points is known and $\#S_{cp} \geq 8$
$\mathbf{x}'_\alpha F \mathbf{x}_\alpha = 0 \quad \Rightarrow \quad A\mathbf{F} = 0$ with $A \in \mathbb{R}^{n\times 9}$
$\mathrm{rank}\,(A) = 8 \Rightarrow F$ is determined mod. scalar $\neq 0$
$\mathrm{rank}\,(A) = 9$ a least square solution is found by solving

$Min_A||A\mathbf{F}||$ subject to $||\mathbf{F}|| = 1$

Extraction of Camera Matrices from F

- Singular Value Decomposition : $F = U\ D\ V^T$ with $D = \operatorname{diag}(p, q, \epsilon), \epsilon \ll q < p,\ U, V$ orthogonal
- Introduce the matrices $Z = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ and $W = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$
- Compute $S = UZU^T$ and $M = UW^T DV^T$, which satisfy $F = SM$
- An associated to F pair of cameras is $P = [\mathbb{I}|\mathbf{0}], \quad P' = [M|U_{.3}]$

Three view reconstruction

$\lambda_\alpha \, \mathbf{x}_\alpha = P \, \mathbf{X}_\alpha, \quad \lambda'_\alpha \, \mathbf{x}'_\alpha = \mathbf{P}' \, \mathbf{X}_\alpha, \quad \lambda"_\alpha \, \mathbf{x}"_\alpha = \mathbf{P}" \, \mathbf{X}_\alpha$

$$\begin{pmatrix} P & \mathbf{x}_\alpha & 0 & 0 \\ P' & 0 & \mathbf{x}'_\alpha & 0 \\ P" & 0 & 0 & \mathbf{x}"_\alpha \end{pmatrix} \begin{pmatrix} \mathbf{X}_\alpha \\ -\lambda_\alpha \\ -\lambda'_\alpha \\ -\lambda"_\alpha \end{pmatrix} = 0$$

Since a solution exists, then all 7×7 sub-matrices have 0 determinant $\Rightarrow$

$\sum_{i,j,k,q,r=1}^{3} x^i \, x'^j \, x"^k \epsilon_{jqu} \epsilon_{krv} \mathcal{T}_i^{qr} = 0$ 4 independent eq.s

9 trilinear conditions involving the trifocal tensor (27 components)

$\mathcal{T}_i^{qr} = (-1)^{i+1} \det \left(P\hat{i}, P'_q, P"_r \right)$

- 7 triplets of corresponding points completely determine $\mathcal{T}_i^{qr}$
- Known $\mathcal{T}_i^{qr}$, three views allow to reconstruct $\mathbf{X}$
- tri-linear equations hold for corresponding points, lines and mixed point-line ones.

Conclusions I

- A proof of principle for the application and implementation of the coded masks
- Different reconstruction algorithms are in development.
- A coded mask can be treated as a projective camera
- Camera projection , Fundamental matrices and trifocal tensors are common tools in a multiple - view treatment.
- 3D Reconstruction formulas are displayed for generic and special arrangements of cameras.
- Camera projection matrices, fundamental matrices and trifocal tensors can be derived from constructive design data
- Alternatively, they can be derived from calibration methods, exploiting a minimal finite number of empirical data.

Conclusions II

- Optimization methods in the above calculations are already at our disposal.
- Several tests addressed to evaluate the capability of 3D reconstruction point like sources in different regions of GRAIN have been performed.
- Adopting the trifocal tensor approach, point and line sources can be treated at the same foot.
- Generalized methods in presence of more than three view should be developed.
- Accurate reconstruction methods are intended to be developed for lines, performing both analytical and numerical studies.

Determining equation for P

n source points

$$\mathbf{X}_i \leftrightarrow \mathbf{x}_i \;\Leftrightarrow\; \exists\,\lambda_i : \;\lambda_i\,\mathbf{x}_i = \lambda_i\,(x_i,\; y_i,\; w_i)^T = P\,\mathbf{X}_i \quad i = 1,\ldots,n$$

$$\mathbf{x}_i \wedge P\,\mathbf{X}_i = 0 \Leftrightarrow \begin{bmatrix} \mathbf{0}^T & -w_i\,\mathbf{X}_i^T & y_i\,\mathbf{X}_i^T \\ w_i\,\mathbf{X}_i^T & \mathbf{0}^T & -x_i\,\mathbf{X}_i^T \\ -y_i\,\mathbf{X}_i^T & x_i\,\mathbf{X}_i^T & \mathbf{0}^T \end{bmatrix} \begin{pmatrix} P^1 \\ P^2 \\ P^3 \end{pmatrix} = 0$$

$$A = \begin{bmatrix} \mathbf{0}^T & -w_1\,\mathbf{X}_1^T & y_1\,\mathbf{X}_1^T \\ w_1\,\mathbf{X}_1^T & \mathbf{0}^T & -x_1\,\mathbf{X}_1^T \\ \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots \\ \mathbf{0}^T & -w_n\,\mathbf{X}_n^T & y_n\,\mathbf{X}_n^T \\ w_n\,\mathbf{X}_n^T & \mathbf{0}^T & -x_n\,\mathbf{X}_n^T \end{bmatrix} : \qquad A \begin{pmatrix} P^1 \\ P^2 \\ P^3 \end{pmatrix} = 0$$

$A \in \mathbb{R}^{2n\times 12} \quad \det A = 0,\ \operatorname{rank} A = 11$

P is defined by $n \geq 6$ points (modulo a scale factor)

Sorgenti (cm)			Punti Immagine (cm)	
-6.6	-6.6	16.5	-0.44	-0.44
7.92	7.92	15.75	0.66	0.66
-5.28	7.92	$15.$	-0.44	0.66
-2.64	-1.32	14.25	-0.22	-0.11
9.24	1.32	13.5	0.88	0.11
5.28	-5.28	12.75	0.44	-0.55

$$\Rightarrow \qquad A_{ex}, \quad \det A_{ex} \neq 0, \quad A_{ex}\, P_{ex} = 0$$

Solving for P

Direct Linear Algorithm

- Assemble $A \in \mathbb{R}^{2n \times 12}$
- Performe the Singular Valued Decomposition $A = U\ D\ V^T$, with $D = \mathrm{diag}\left(\lambda_{max}, \dots, \lambda_{min}\right) \in \mathbb{R}^{12\times12},\ \lambda_i > 0$
- $P = V_{12^{th}\ \mathrm{column}}$

λ_{min} of $A_{ex} = 0.05042$

$$P_{ex} = \begin{pmatrix} -0.081 & -0.404 & 0.480 & 0.174 \\ 0.209 & -0.839 & -0.534 & 1.851 \\ -1.074 & 0.136 & 1. & -0.920 \end{pmatrix}$$

$$P_{ex}.\mathrm{Sorgenti} \longrightarrow \frac{\Delta\mathrm{Immagini}}{Immagini} = \begin{pmatrix} 0.005 & 0.005 \\ 0.02 & 0.01 \\ 0.01 & 0.02 \\ 0.02 & 0.03 \\ 0.03 & 0.01 \\ 0.003 & 0.01 \end{pmatrix}$$

Optimization

for $n \geq 6$ find the Maximum Likelihood estimate of P by
The Standard Algorithm (Hartley)

1. Estimate P by a linear procedure
2. Normalize source and image points : $\tilde{\mathbf{X}}_i = U\,\mathbf{X}_i, \quad \tilde{\mathbf{x}}_i = T\,\mathbf{x}_i,$

   $$\text{such that } \sum_i \tilde{\mathbf{X}}_i = \mathbf{0},\ \sum_i \tilde{\mathbf{x}}_i = \mathbf{0}\ \frac{\sum_i |\tilde{\mathbf{X}}_i|^2}{n-1} = 3,\ \frac{\sum_i |\tilde{\mathbf{x}}_i|^2}{n-1} = 2$$

3. Generate $A\left(\left\{\tilde{\mathbf{X}}_i\right\}, \{\tilde{\mathbf{x}}_i\}\right)$ and take its normalized eigenvector $\widetilde{\mathbf{p}}_{min} \leftrightarrow \lambda_{min} \neq 0 \ \Rightarrow \ \tilde{P}$
4. Minimize recursively $\min_P \left(\sum_{i=1}^n |\mathbf{x}_i - \tilde{P}\mathbf{X}_i|^2\right)$
5. Back in the original coordinates : $P = T^{-1}\,\tilde{P}\,U$