Probing entanglement and testing Bell inequality violation with $e^+e^- \to \tau^+\tau^-$ at Belle II

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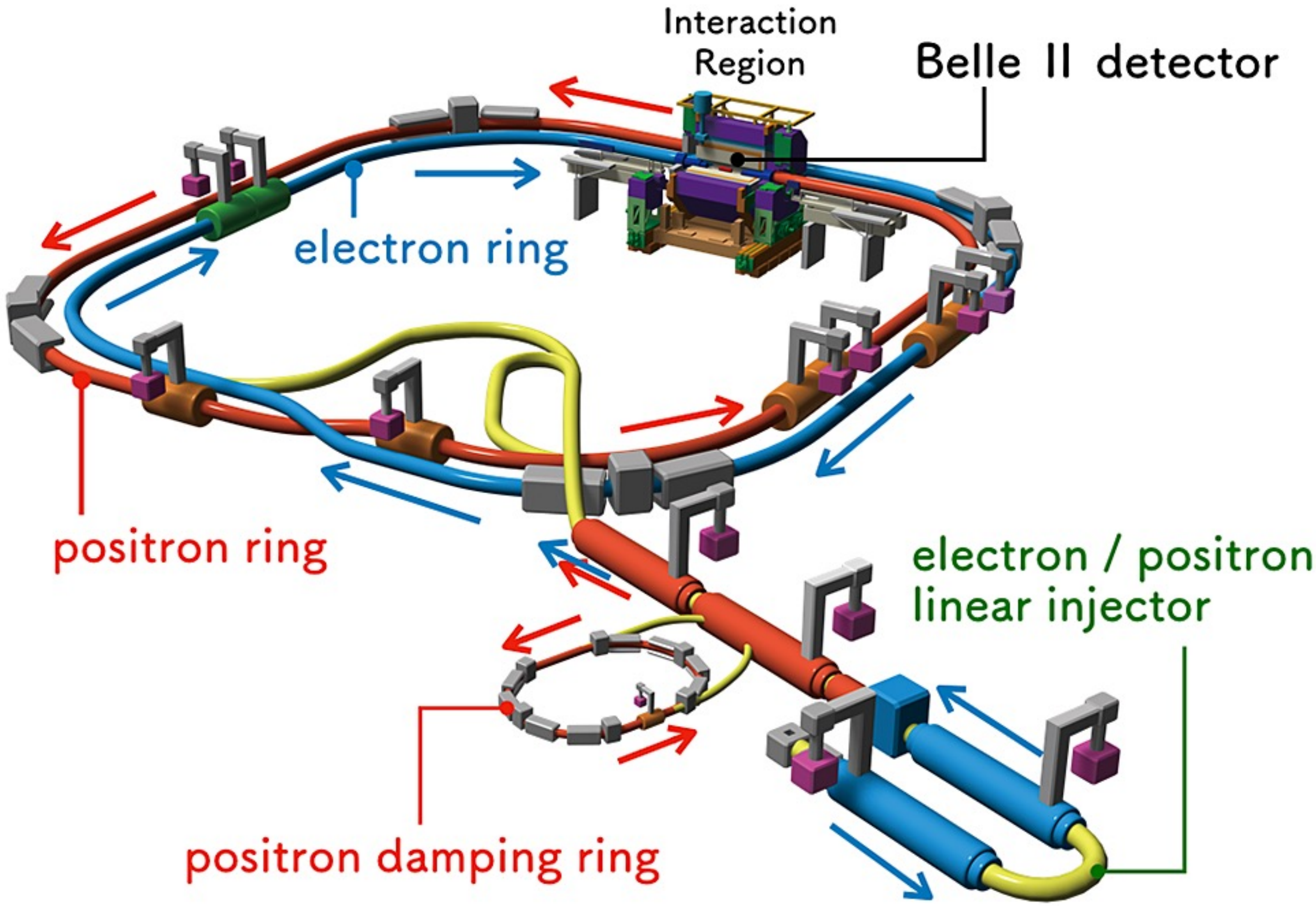
SuperKEKB collider

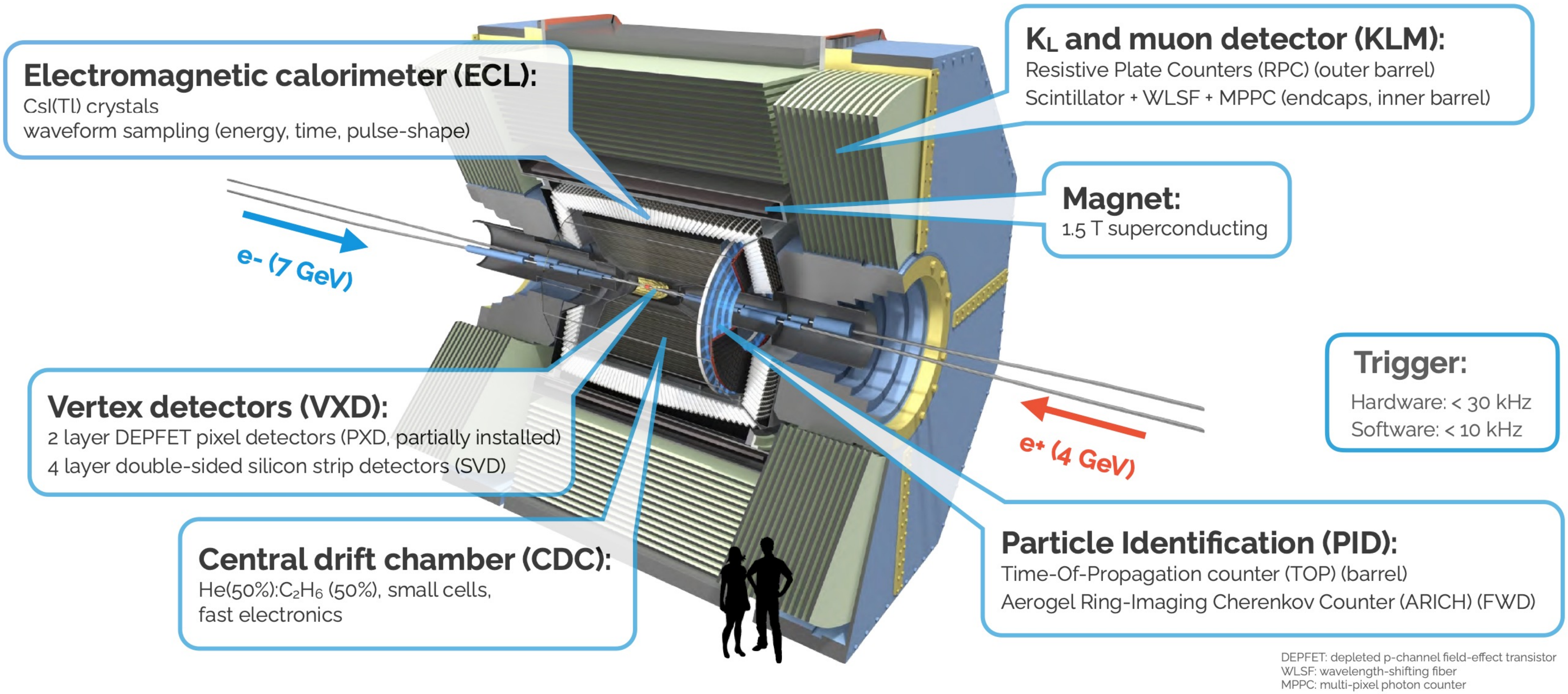
arXiv:1809.01958



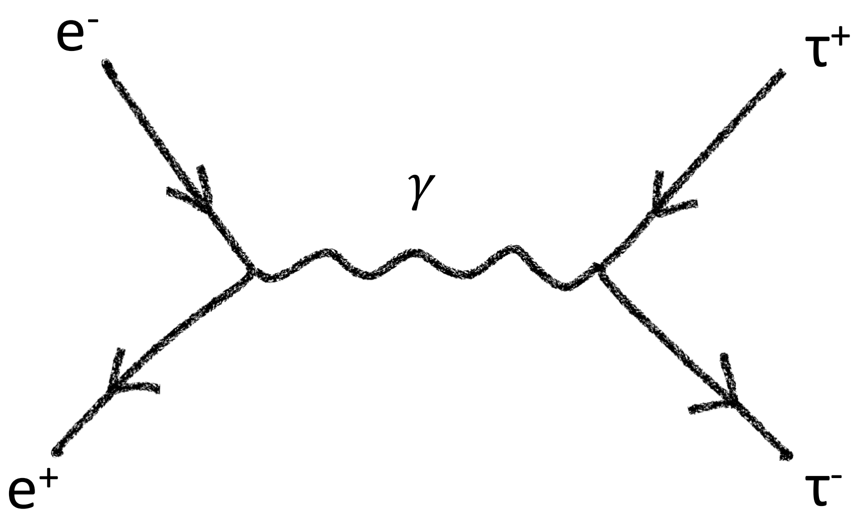
Circumference:	3016m
Beam energy:	7 GeV (e^-), 4 GeV (e^+)
Target luminosity:	$8 \cdot 10^{35}$ cm^{-2}s^{-1}

Belle II detector

arXiv:1011.0352



τ-pair production @ Belle II



Cross section = 0.92 nb

About 700 $\tau^{+}\tau^{-}$ pairs produced per second by SuperKEKB at target luminosity

☞ **SuperKEKB is a τ factory!**

τ-spin correlations in the Standard Model

correlation between spin orientations of τ^+ and τ^-

polarization of τ^+

$$\boxed{\rho} = \frac{1}{4}\left[\mathbb{1}\otimes\mathbb{1} + \sum_i \boxed{\mathrm{B}_i^+}\,(\sigma_i \otimes \mathbb{1}) + \sum_j \boxed{\mathrm{B}_j^-}\,(\mathbb{1}\otimes\sigma_j) + \sum_{i,j} \boxed{\mathrm{C}_{ij}}\,(\sigma_i\otimes\sigma_j)\right]$$

spin density matrix

polarization of τ^-

B^+ and B^- are expected to be zero for the process $e^+e^- \to \tau^+\tau^-$ in the SM

The spin correlation matrix C depends on the scattering angle θ*, the angle between the e^+ and τ^+ in the e^+e^- center-of-mass (CM) frame:

$$\mathrm{C} = c_0 \begin{pmatrix} (4m_\tau^2 - s)\sin^2\theta & 0 & 0 \\ 0 & (4m_\tau^2 + s)\sin^2\theta & 4m_\tau\sqrt{s}\sin\theta\cos\theta \\ 0 & 4m_\tau\sqrt{s}\sin\theta\cos\theta & -4m_\tau^2\sin^2\theta + s\left(\cos^2\theta + 1\right) \end{pmatrix}$$

where $c_0 = 1/\left(4m_\tau^2\sin^2\theta + s\left(1+\cos^2\theta\right)\right)$

The components of C are given in the **helicity frame** { n, r, k }

k: direction of τ^+ momentum in CM frame

r: in e^+- τ^+plane and orthogonal to k, n = r x k

Hadronic τ decays

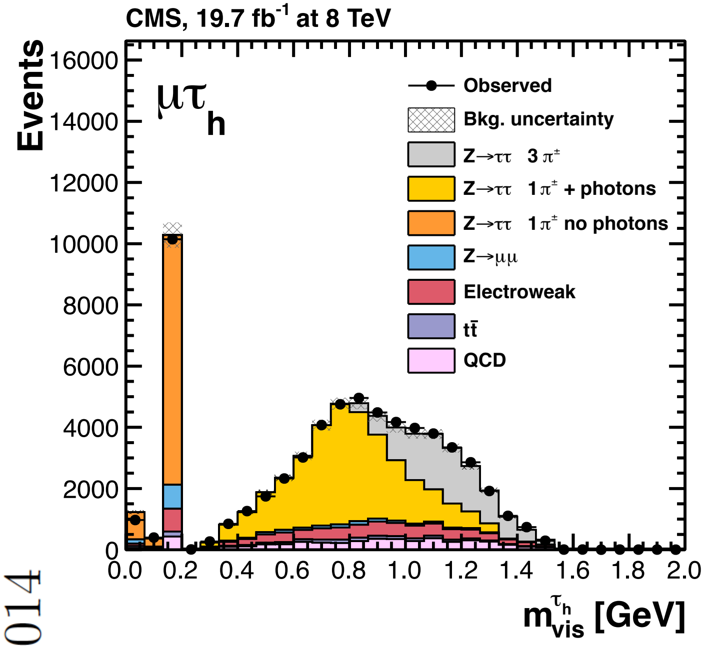
Mass m_τ= 1.78 GeV

Lifetime cτ = 87 μm

Decay Mode	Resonance	BR [%]
$\tau^- \to e^- \overline{\nu}_e \nu_\tau$		17.8
$\tau^- \to \mu^- \overline{\nu}_\mu \nu_\tau$		17.4
$\tau^- \to h^- \nu_\tau$		11.5
$\tau^- \to h^- \pi^0 \nu_\tau$	$\rho(770)$	26.0
$\tau^- \to h^- \pi^0 \pi^0 \nu_\tau$	$a_1(1260)$	10.8
$\tau^- \to h^- h^+ h^- \nu_\tau$	$a_1(1260)$	9.8
$\tau^- \to h^- h^+ h^- \pi^0 \nu_\tau$		4.8
Other hadronic modes		1.8
All hadronic modes		64.8

PDG 2014

JHEP 05 (2014) 104



h^- : about 95% π^- and 5% K^-

Hadronic τ Identification ≅ reconstruction of $\pi^\pm$, $\rho^\pm$, $a_1^\pm$ signatures

Leptonic τ decays not considered, because they are not as well suited for analyses of τ spin correlations as hadronic τ decays

τ polarimeter vector

Comput.Phys.Commun. 64 (1991) 275

Differential decay rate of τ lepton:

Spin averaged matrix element

Spin vector

$$d\Gamma = \frac{\boxed{|\bar{\mathcal{M}}|^2}}{2\mathrm{m}_\tau} (1 - \boxed{\mathrm{h}_\mu} \boxed{\mathrm{s}^\mu}) \, d\mathrm{Lips}$$

Polarimeter vector

This relation holds for all leptonic and hadronic τ decay channels

The issue with leptonic τ decays is that the polarimeter vector is not accessible experimentally, because one would need to reconstruct the **individual** momenta of the two ν produced in each leptonic τ decay [*]

For hadronic τ decays, the polarimeter vector is a function of the momenta of the charged and neutral hadrons produced in these decays

☞ All hadronic τ decays have the same "τ spin analyzing power"

[*] The charged lepton only partially correlated with the polarimeter vector, resulting in a loss of τ spin analyzing power

Analyzed τ decay channels

$\tau^- \to \pi^- \nu_\tau$ "π" channel (BR = 10.8%)

h_μ = momentum of π^-

$\tau^- \to \pi^- \pi^0 \nu_\tau$ "ρ" channel (BR = 25.5%)

ν momentum = $\tau - \Sigma$ pion momenta

$$\mathrm{h}_\mu = -2\gamma_{va} M \,|\, f_2 \,|^2 \frac{\left[2(\boxed{q} \cdot \boxed{N})q - q^2 N\right]}{\omega + \hat{\omega}}$$

difference between π^- and π^0 momenta

$\tau^- \to \pi^- \pi^+ \pi^- \nu_\tau$ "a_1" channel (BR = 9.3%)

No analytic formula available, h_μ based on model for dynamics of hadronic interactions in a_1 decay, which is fitted to data

The decay channel $\tau^- \to \pi^- \pi^0 \pi^0 \nu_\tau$ is not included in the analysis, because we do not know how well Belle II can separately reconstruct the two π^0

☞ The combination of π, ρ, and a_1 decay channels covers 21% of all $\tau^+\tau^-$ pair decays

Measurement of τ spin correlations

Comput.Phys.Commun. 64 (1991) 275

The spin-dependent differential cross section for tau-pair production is given by:

$$d\sigma = |\mathcal{A}|^2 \left(1 - b_\mu^+ \, s_+^\mu - b_\nu^- \, s_-^\nu + c_{\mu\nu} \, s_+^\mu \, s_-^\nu\right) \, d\mathrm{Lips}$$

where $|\mathcal{A}|^2$ denotes the spin-averaged matrix element for the process $e^+e^- \to \tau^+\tau^-$ and $d\mathrm{Lips}$ the Lorentz-invariant phase-space measure

The cross section for the combined process of tau-pair production and decay is:

$$d\sigma = |\mathcal{A}|^2 \, |\bar{\mathcal{M}}|^2 \, |\bar{\mathcal{M}}'|^2 \, \left(1 + \mathbf{B}^+ \cdot \mathbf{h}^+ + \mathbf{B}^- \cdot \mathbf{h}^- + \mathbf{h}^+ \cdot \mathrm{C} \cdot \mathbf{h}^-\right) \, d\mathrm{Lips}$$

where $|\bar{\mathcal{M}}|^2$ and $|\bar{\mathcal{M}}'|^2$ refer to the spin-averaged matrix elements for the decays of τ^+ and τ^-

Acta Phys. Polon. B 15 (1984) 115

Using this relation, we determine the elements of the polarization vectors B^+ and B^- and the elements of the spin correlation matrix C by an unbinned maximum-likelihood (ML) fit, with the likelihood function:

$$\mathcal{L} = \prod_i \left(1 + \mathbf{B}^+ \cdot \mathbf{h}_i^+ + \mathbf{B}^- \cdot \mathbf{h}_i^- + \mathbf{h}_i^+ \cdot \mathrm{C} \cdot \mathbf{h}_i^-\right)$$

where the product extends over all events i in the $e^+e^- \to \tau^+\tau^-$ sample

Monte Carlo study

200mio $e^+e^- \to \tau^+\tau^-$ Monte Carlo (MC) events generated for $\sqrt{s} = 10.579$ GeV using MadGraph with leading order matrix elements.
The τ lepton decays are simulated with PYTHIA8 [*].

This MC sample corresponds to about half of the data already published by Belle II and less than 1% of the data expected by the end of the experiment
Phys.Rev.D 102 (2020) 111101 **arXiv:1809.01958**

Experimental resolutions are simulated by "smearing" MC-truth values by Gaussian distributions. The resolution parameters are taken from the Belle II detector technical design report **arXiv:1011.0352**

☞ Numerical values given in appendix

Simulated events are analyzed at MC-truth and on "reconstruction" level, i.e. after smearing the events and reconstructing the momenta of the ν produced in the τ decays

[*] we also tried TauDecay and KKMC (with a special version of TAUOLA used by Belle II) and observed good agreement between all three

Kinematic reconstruction

The τ polarimeter vectors need to be computed in the restframes of τ^+ and τ^-

☞ We need to reconstruct the full event kinematics, in particular the momenta of the ν produced in the τ decays

The event reconstruction is performed in two stages:

① By solving a set of analytic equations, using 2 τ mass constraints, 2 ν mass constraints, and the 4-momentum of the initial e^+e^- pair to solve for the 8 components of the two 4-momentum vectors of the ν and $\bar{\nu}$

Phys.Rev.D 107 (2023) 093002

The two-fold sign ambiguity of the analytic equations is resolved by choosing the solution more compatible with transverse impact parameters (π, ρ) or the τ decay vertex (a_1)

Phys.Lett.B 313 (1993) 458

② The solution obtained in the 1st stage is refined by a kinematic fit, which employs the transverse impact parameters, τ decay vertices, and the knowledge of experimental resolutions to improve the event reconstruction

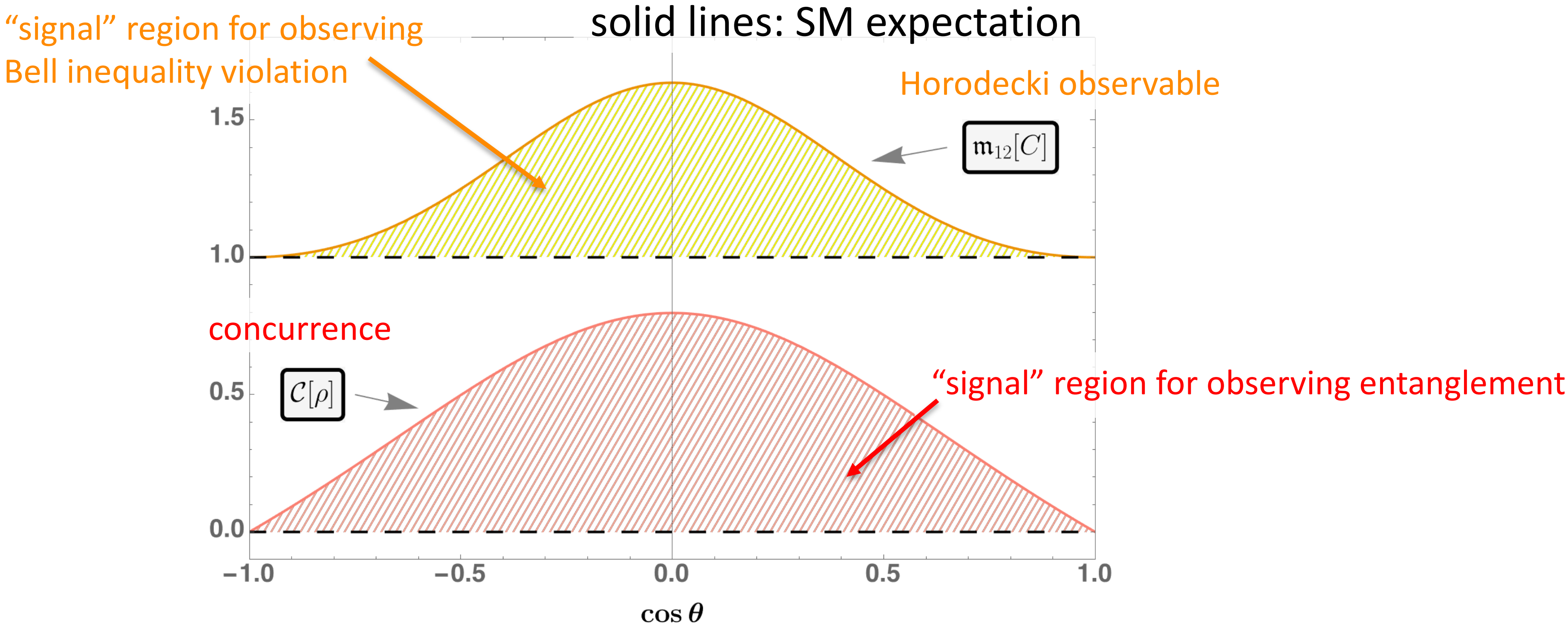
arXiv:1805.06988 **CMS-TS-2011-021**

Observables

We use two observables, C[ρ] and m_{12}[C], to probe entanglement and Bell inequality violation ☞ formal definition of observables in backup

Both observables are functions of the spin correlation matrix C

As C depends on the scattering angle θ^*, both observables depend on θ^*:

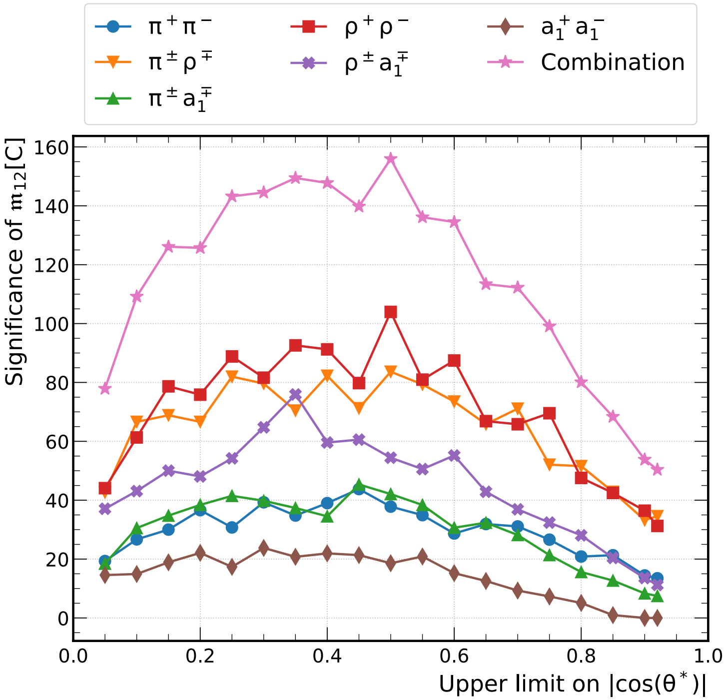


☞ Observation of entanglement and Bell inequality violation helped by selecting events in which τ leptons are produced perpendicular to beam axis

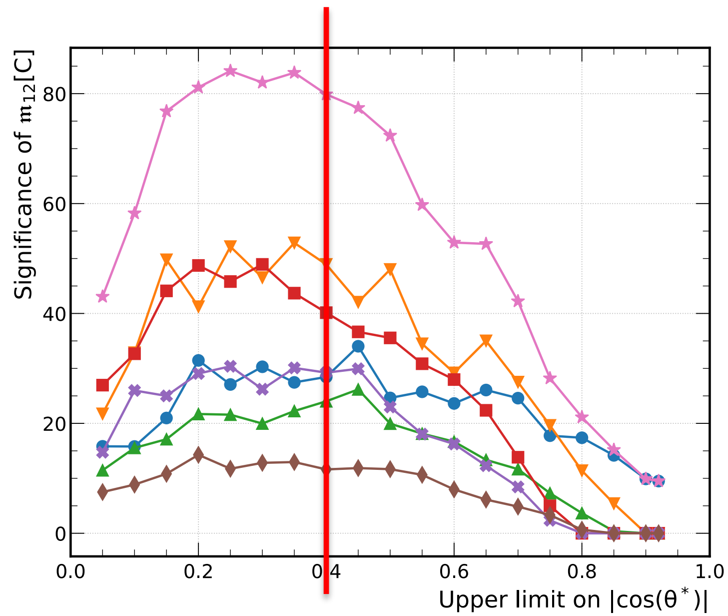
Optimization of cut on cos(θ*)

Horodecki observable $m_{12}[C]$

MC-truth level



Reconstruction level



Significance computed as $(m_{12}[C] - 1)/\delta m_{12}[C]$, with (statistical) uncertainty $\delta m_{12}[C]$ estimated by bootstrapping

☞ Combination of π, ρ, and a_1 decay channels improves significance by about a factor 3 on reconstruction level, compared to $\pi^+\pi^-$ channel

☞ Choose cut $|\cos(\theta^*)| < 0.4$ to enhance significance

Results

MC-truth level

Decay channel	$\mathcal{C}[\rho]$	$\mathfrak{m}_{12}[C]$
$\pi^+\pi^-$	0.7087 ± 0.0054	1.462 ± 0.012
$\pi^\pm\rho^\mp$	0.7090 ± 0.0022	1.466 ± 0.006
$\pi^\pm a_1^\mp$	0.6695 ± 0.0034	1.370 ± 0.011
$\rho^+\rho^-$	0.7095 ± 0.0017	1.467 ± 0.005
$\rho^\pm a_1^\mp$	0.6711 ± 0.0025	1.378 ± 0.006
$a_1^+ a_1^-$	0.6328 ± 0.0051	1.282 ± 0.013
All channels	0.6947 ± 0.0011	1.430 ± 0.003

Reconstruction level

Decay channel	$\mathcal{C}[\rho]$	$\mathfrak{m}_{12}[C]$
$\pi^+\pi^-$	0.6379 ± 0.0059	1.399 ± 0.014
$\pi^\pm\rho^\mp$	0.6332 ± 0.0022	1.279 ± 0.006
$\pi^\pm a_1^\mp$	0.6145 ± 0.0042	1.271 ± 0.011
$\rho^+\rho^-$	0.6106 ± 0.0021	1.227 ± 0.006
$\rho^\pm a_1^\mp$	0.5974 ± 0.0029	1.219 ± 0.007
$a_1^+ a_1^-$	0.6111 ± 0.0089	1.240 ± 0.021
All channels	0.6169 ± 0.0012	1.255 ± 0.003

☞ Bell inequality violation expected to be observed with significance of about 80 standard deviations in 200mio $e^+e^- \to \tau^+\tau^-$ events

Observation of entanglement expected to be easier than observation of Bell inequality violation

☞ Experimental resolution expected to degrade sensitivity by about a factor 2

Effects not included in MC study

Non-Gaussian tails of experimental resolutions

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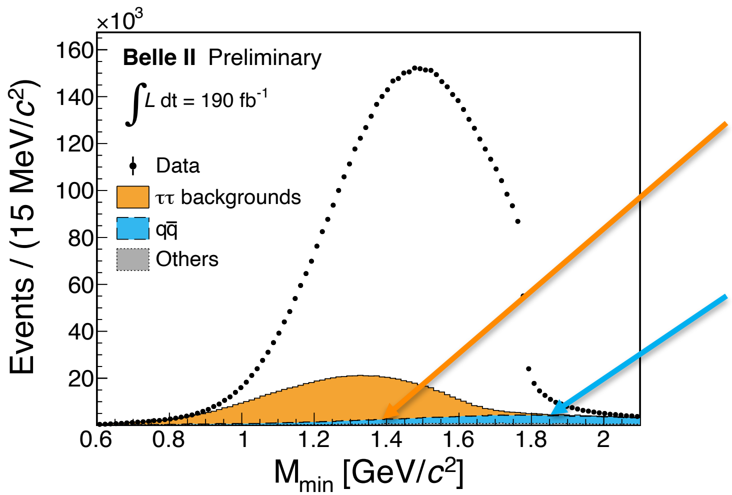


Backgrounds

- Misreconstruction of τ decay channels, due to detector inefficiencies, spurious photons,...
- $e^+e^- \to q\bar{q}$
- $\gamma\gamma \to \mathrm{hadrons}$ overlay background

Dominant background expected to be due to τ decay channel misreconstruction

This type of background needs to be simulated with the full Belle II detector simulation, based on GEANT4

Systematic uncertainties

☞ We expect these effects to have only a moderate effect on the sensitivity to observe entanglement and Bell inequality violation at Belle II

Summary

- The prospects for detecting entanglement and Bell inequality violation has been studied using the process $e^+e^- \rightarrow \tau^+\tau^-$ at Belle II

- The spin orientations of τ leptons are measured using τ polarimeter vectors in a combination of π, ρ, and a_1 decay channels

- Compared to the decay channel $\pi^+\pi^-$, the combination of π, ρ, and a_1 decay channels increases the sensitivity of the analysis by about a factor 3

- Assuming a dataset of 200mio $e^+e^- \rightarrow \tau^+\tau^-$ events, we expect entanglement and Bell inequality violation to be observed with a significance of about 80 standard deviations

  In total, 50 billion $e^+e^- \rightarrow \tau^+\tau^-$ events expected to be recorded until the end of the Belle II experiment

- We expect effects not simulated in our MC study, such as non-Gaussian tails of experimental resolutions, background contributions, and systematic uncertainties, to degrade the sensitivity by only a moderate amount

Backup

Formal definition of observables

Concurrence C[ρ] Rev. Mod. Phys. 81 (2009) 865

$$\mathcal{C}[\rho] = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\} \in [0,1]$$

where λ_i are the eigenvalues, in decreasing order, of the matrix

$$R = \sqrt{\sqrt{\rho}\tilde{\rho}\sqrt{\rho}} \quad \text{with} \quad \tilde{\rho} = (\sigma_2 \otimes \sigma_2)\rho^*(\sigma_2 \otimes \sigma_2)$$

C[ρ] > 0 signals entanglement

Horodecki observable m_{12}[C] Phys. Lett. A 200 (1995) 340

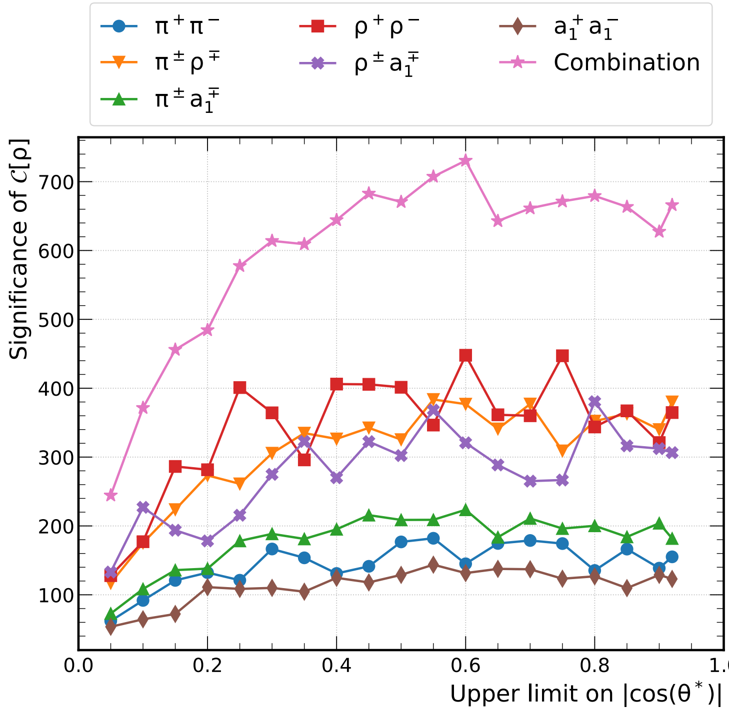
$$\mathfrak{m}_{12}[\mathrm{C}] = m_1 + m_2$$

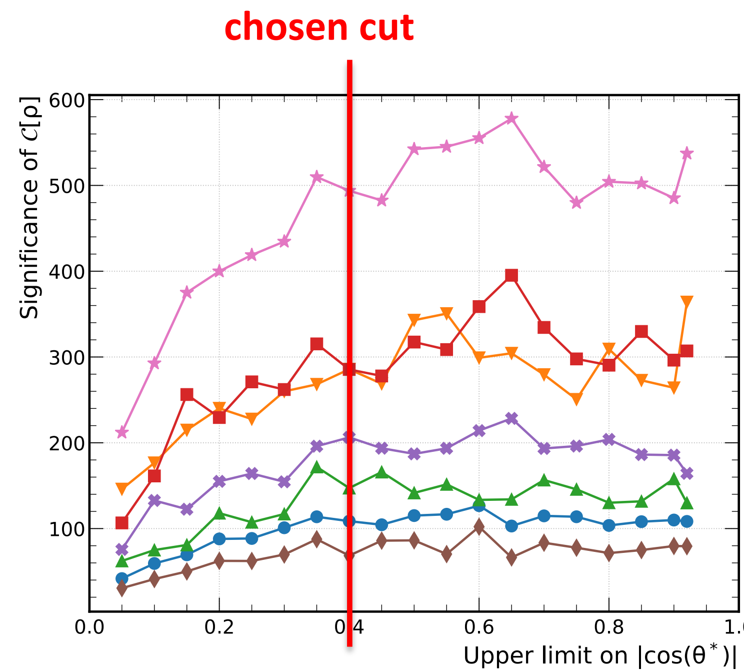
where $m_1 \geq m_2 \geq m_3$ are the eigenvalues of the matrix $\mathrm{C}^T\,\mathrm{C}$

m_{12}[C] > 1 signals Bell inequality violation

Optimization of cut on cos(θ*)

Concurrence C[ρ]





Significance computed as C[ρ] /δC[σ], with (statistical) uncertainty δC[ρ] estimated by bootstrapping

☞ Entanglement easier to observe than Bell inequality violation (cf. slide 13)

☞ Cut |cos(θ*)| < 0.4 yields close to optimal sensitivity for C[ρ] as well as m_{12}[C]

Resolutions used in MC study

Charged hadrons

Quantity	Resolution
$1/p_{\mathrm{T}}$	$2\times10^{-5}\mathrm{GeV}^{-1}$
θ	3×10^{-4}
ϕ	3×10^{-4}
d_{xy}	$20\ \mu\mathrm{m}$
d_{z}	$20\ \mu\mathrm{m}$

Photons

Quantity	Resolution
E: c_0	$0.166\ \mathrm{GeV}^{1/2}$
E: c_1	$0.011\ \mathrm{GeV}$
θ	1.7×10^{-3}
ϕ	1.7×10^{-3}

Event vertex

Quantity	Resolution
x	$10\ \mu\mathrm{m}$
y	$10\ \mu\mathrm{m}$
z	$20\ \mu\mathrm{m}$

$\tau^+\tau^-$ system

Quantity	Resolution
p_{x}	$0.01\ \mathrm{GeV}$
p_{y}	$0.01\ \mathrm{GeV}$
p_{z}	$0.1\ \mathrm{GeV}$
mass	$0.1\ \mathrm{GeV}$

smearing of $\tau^+\tau^-$ system simulates the effect of beam-energy spread and beamstrahlung

Belle II detector resolution

arXiv:1011.0352

Component	Type	Configuration	Readout	Performance
Beam pipe	Beryllium double-wall	Cylindrical, inner radius 10 mm, 10 μm Au, 0.6 mm Be, 1 mm coolant (paraffin), 0.4 mm Be		
PXD	Silicon pixel (DEPFET)	Sensor size: 15×100 (120) mm^2 pixel size: 50×50 (75) μm^2 2 layers: 8 (12) sensors	10 M	impact parameter resolution $\sigma_{z_0} \sim 20\ \mu$m (PXD and SVD)
SVD	Double sided Silicon strip	Sensors: rectangular and trapezoidal Strip pitch: 50(p)/160(n) - 75(p)/240(n) μm 4 layers: 16/30/56/85 sensors	245 k	
CDC	Small cell drift chamber	56 layers, 32 axial, 24 stereo r = 16 - 112 cm - 83 $\leq z \leq$ 159 cm	14 k	$\sigma_{r\phi} = 100\ \mu$m, $\sigma_z = 2$ mm $\sigma_{p_t}/p_t = \sqrt{(0.2\% p_t)^2 + (0.3\%/\beta)^2}$ $\sigma_{p_t}/p_t = \sqrt{(0.1\% p_t)^2 + (0.3\%/\beta)^2}$ (with SVD) $\sigma_{dE/dx} = 5\%$
TOP	RICH with quartz radiator	16 segments in ϕ at $r \sim 120$ cm 275 cm long, 2 cm thick quartz bars with 4x4 channel MCP PMTs	8 k	$N_{p.e.} \sim 20$, $\sigma_t = 40$ ps K/π separation : efficiency $> 99\%$ at $< 0.5\%$ pion fake prob. for $B \to \rho\gamma$ decays
ARICH	RICH with aerogel radiator	4 cm thick focusing radiator and HAPD photodetectors for the forward end-cap	78 k	$N_{p.e.} \sim 13$ K/π separation at 4 GeV/c: efficiency 96% at 1% pion fake prob.
ECL	CsI(Tl) (Towered structure)	Barrel: $r = 125$ - 162 cm End-cap: $z =$ -102 cm and +196 cm	6624 1152 (F) 960 (B)	$\frac{\sigma E}{E} = \frac{0.2\%}{E} \oplus \frac{1.6\%}{\sqrt[4]{E}} \oplus 1.2\%$ $\sigma_{pos} = 0.5$ cm$/\sqrt{E}$ (E in GeV)
KLM	barrel: RPCs	14 layers (5 cm Fe + 4 cm gap) 2 RPCs in each gap	θ: 16 k, ϕ: 16 k	$\Delta\phi = \Delta\theta = 20$ mradian for K_L ~ 1 % hadron fake for muons
	end-caps: scintillator strips	14 layers of $(7-10) \times 40$ mm^2 strips read out with WLS and G-APDs	17 k	$\Delta\phi = \Delta\theta = 10$ mradian for K_L $\sigma_p/p = 18\%$ for 1 GeV/c K_L

SuperKEKB machine parameters

arXiv:1809.01958

		KEKB		SuperKEKB		
		LER (e+)	HER (e-)	LER (e+)	HER (e-)	Units
Beam energy	E	3.5	8.0	4.0	7.007	GeV
Circumference	C	3016.262		3016.315		m
Half crossing angle	θ_x	$0\ (11^{(*)})$		41.5		mrad
Piwinski angle	ϕ_{Piw}	0	0	24.6	19.3	rad
Horizontal emittance	ε_x	18	24	3.2 (1.9)	4.6 (4.4)	nm
Vertical emittance	ε_y	150	150	8.64	12.9	pm
Coupling		0.83	0.62	0.27	0.28	%
Beta function at IP	β_x^*/β_y^*	1200/5.9	1200/5.9	32/0.27	25/0.30	mm
Horizontal beam size	σ_x^*	147	170	10.1	10.7	μm
Vertical beam size	σ_y^*	940	940	48	62	nm
Horizontal betatron tune	ν_x	45.506	44.511	44.530	45.530	
Vertical betatron tune	ν_y	43.561	41.585	46.570	43.570	
Momentum compaction	α_p	3.3	3.4	3.20	4.55	10^{-4}
Energy spread	σ_ε	7.3	6.7	7.92(7.53)	6.37(6.30)	10^{-4}
Beam current	I	1.64	1.19	3.60	2.60	A
Number of bunches	n_b	1584		2500		
Particles/bunch	N	6.47	4.72	9.04	6.53	10^{10}
Energy loss/turn	U_0	1.64	3.48	1.76	2.43	MeV
Long. damping time	τ_z	21.5	23.2	22.8	29.0	msec
RF frequency	f_{RF}	508.9		508.9		MHz
Total cavity voltage	V_c	8.0	13.0	9.4	15.0	MV
Total beam power	P_b	$\sim$3	$\sim$4	8.3	7.5	MW
Synchrotron tune	ν_s	-0.0246	-0.0209	-0.0245	-0.0280	
Bunch length	σ_z	$\sim$7	$\sim$7	6.0 (4.7)	5.0 (4.9)	mm
Beam–beam parameter	ξ_x/ξ_y	0.127/0.129	0.102/0.090	0.0028/0.088	0.0012/0.081	
Luminosity	L	2.108×10^{34}		8×10^{35}		cm^{-2}s^{-1}
Integrated luminosity	$\int L$	1.041		50		ab^{-1}

Alternative measurement techniques

- **Expectation value** **Phys. Rev. D 107 (2023) 093002**

$$\mathrm{C}_{ij} = -9 \left\langle \mathbf{h}_i^+ \, \mathbf{h}_j^- \right\rangle$$ where $i, j \in \{n, r, k\}$ and the expectation value is computed as average over the events in the MC sample

- **Double-differential cross section** **Nucl. Phys. B 690 (2004) 81**

$$\frac{1}{\sigma} \frac{d\sigma}{d\cos\theta_i^+ \, d\cos\theta_j^-} = \frac{1}{4} \left(1 - \mathrm{C}_{ij} \, \cos\theta_i^+ \, \cos\theta_j^-\right)$$

where $\cos\theta_i^+ = \mathbf{h}^+ \cdot \hat{e}_i \; (\cos\theta_j^+ = \mathbf{h}^+ \cdot \hat{e}_j)$ with $i, j \in \{n, r, k\}$

- **Single-differential cross section** **JHEP 12 (2015) 026**

$$\frac{1}{\sigma} \frac{d\sigma}{d\xi_{ij}} = \frac{1}{2} \left(1 - \mathrm{C}_{ij} \, \xi_{ij}\right) \ln\left(\frac{1}{|\xi|}\right)$$ with $\xi_{ij} = \cos\theta_i^+ \, \cos\theta_j^-$

- **Forward/backward asymmetry** **Eur. Phys. J. C 82 (2022) 66**

$$A_{ij} = \frac{N(\cos\theta_i^+ \, \cos\theta_j^- > 0) - N(\cos\theta_i^+ \, \cos\theta_j^- < 0)}{N(\cos\theta_i^+ \, \cos\theta_j^- > 0) + N(\cos\theta_i^+ \, \cos\theta_j^- < 0)} = -\frac{1}{4} \, \mathrm{C}_{ij}$$

Comparison of measurement techniques

Method	$\mathcal{C}[\rho]$	$\mathfrak{m}_{12}[\mathrm{C}]$
Exp. value	0.6917 ± 0.0013	1.4237 ± 0.0035
2d distr.	0.6950 ± 0.0012	1.4299 ± 0.0030
1d distr.	0.6915 ± 0.0012	1.4228 ± 0.0030
FB asymm.	0.6925 ± 0.0018	1.4303 ± 0.0048
ML fit	0.6947 ± 0.0011	1.4305 ± 0.0029

☞ Results obtained by all measurement techniques are compatible within the quoted uncertainties

☞ The ML-fit method yields the smallest uncertainties, as expected, but the expectation value and cross section method come close

The restriction to counting “forward” and “backward” events by the forward/backward asymmetry method rather than using the full distribution in $\mathbf{h}^{+} \cdot \mathbf{h}^{-}$ causes information loss, which results in a loss of sensitivity