Norway grants

NATIONAL SCIENCE CENTRE POLAND

Three-body Entanglement in Particle Decays

Kazuki Sakurai

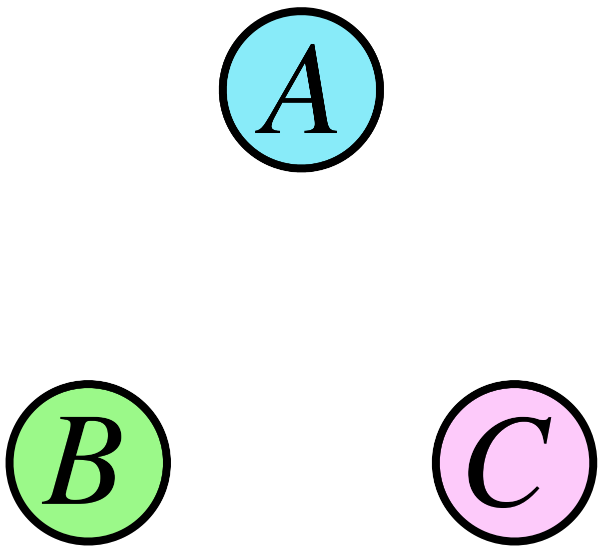
(University of Warsaw)

Based on: KS, Michael Spannowsky [2310.01477]

2023/11/9, Quantum Observables for Collider Physics @ GGI, Florence

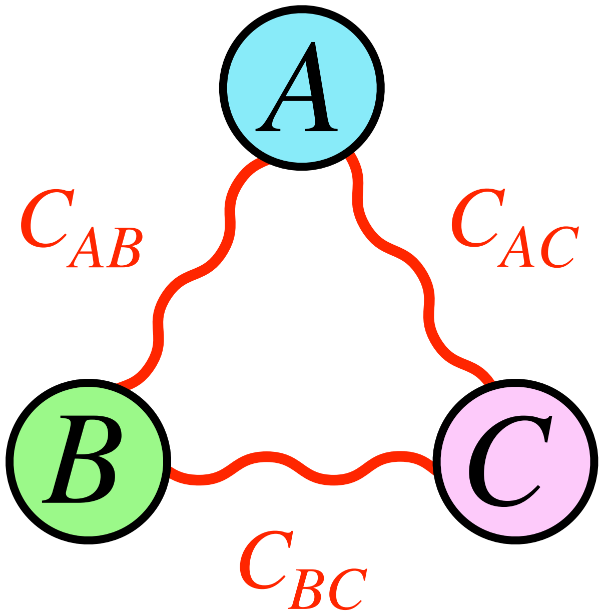
Digest

3-particle entanglement has a much richer structure that 2-PE !



Digest

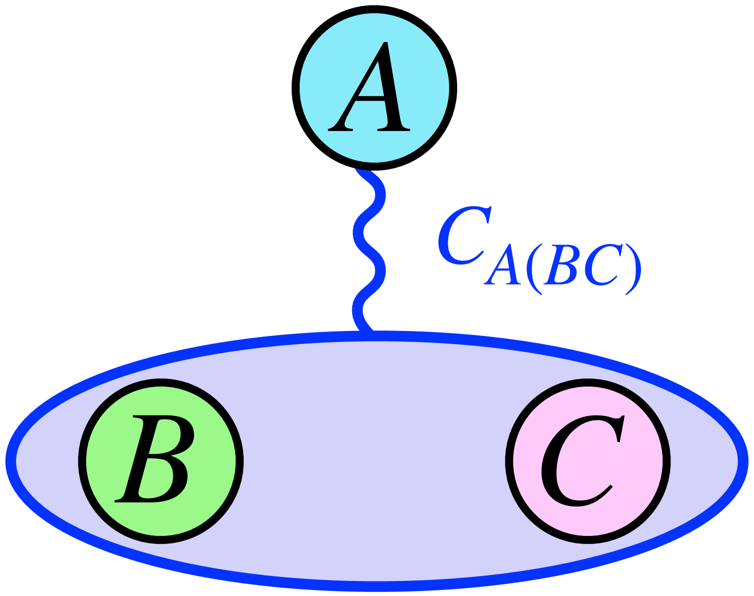
3-particle entanglement has a much richer structure that 2-PE !



- Ent. btw 2-individual particles

Digest

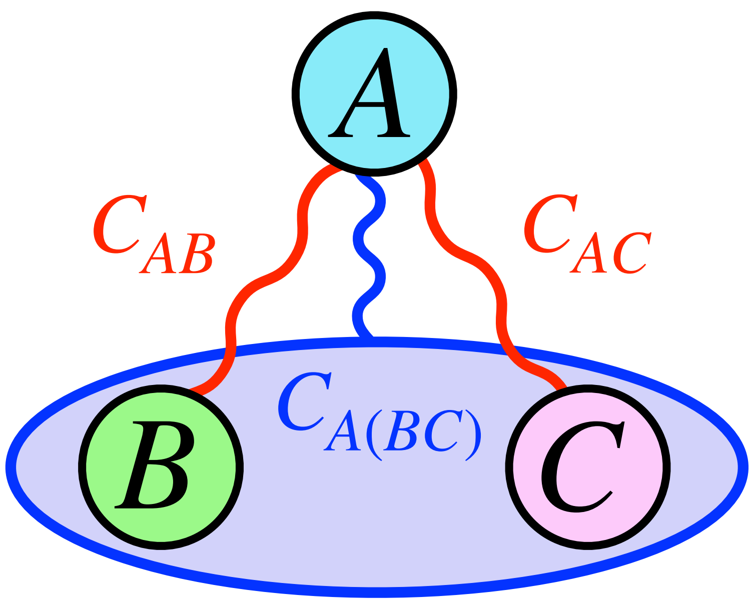
3-particle entanglement has a much richer structure that 2-PE !



- Ent. btw 2-individual particles
- Ent. btw one-to-other

Digest

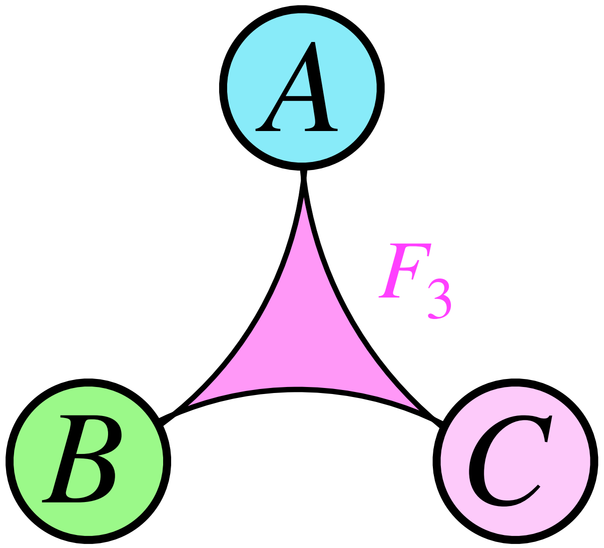
3-particle entanglement has a much richer structure that 2-PE !



- Ent. btw 2-individual particles
- Ent. btw one-to-other
- "Monogamy" $C^2_{A(BC)} \geq C^2_{AB} + C^2_{AC}$

Digest

3-particle entanglement has a much richer structure that 2-PE !

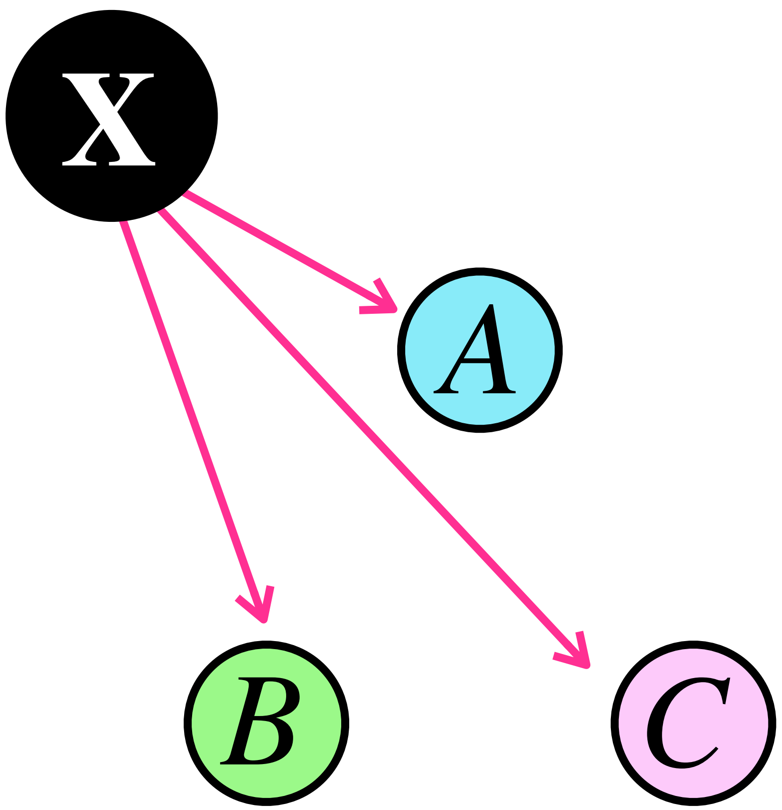


- Ent. btw 2-individual particles
- Ent. btw one-to-other
- "Monogamy" $C^2_{A(BC)} \geq C^2_{AB} + C^2_{AC}$
- "Genuine" 3-particle entanglement F_3
  (non-separable even partially)

  ~~$|\psi\rangle_A \otimes (|00\rangle_{BC} + |11\rangle_{BC})$~~

Digest

3-particle entanglement has a much richer structure that 2-PE !



- Ent. btw 2-individual particles
- Ent. btw one-to-other
- “Monogamy” $C^2_{A(BC)} \geq C^2_{AB} + C^2_{AC}$
- “Genuine” 3-particle entanglement F_3
  (non-separable even partially)

$$\cancel{|\psi\rangle_A \otimes (|00\rangle_{BC} + |11\rangle_{BC})}$$

3-body decay: $X \to ABC$

explore all possible Lorentz invariant interactions

Entanglement

Entanglement monotone: non-negative and non-increasing function under LOCC.

Ex.) **Concurrence** [for 2 qubit system]

$$\mathcal{C}[\rho] = \max(0, \eta_1 - \eta_2 - \eta_3 - \eta_4) \in [0, 1]$$

$\eta_1 \geq \eta_2 \geq \eta_3 \geq \eta_4$ are eigenvalues of $\sqrt{\rho\tilde{\rho}}$ with $\tilde{\rho} \equiv (\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)$.

*) LOCC: Local Operation and Classical Communication

Entanglement

Entanglement monotone: non-negative and non-increasing function under LOCC.

Ex.) **Concurrence** [for 2 qubit system]

$$\mathcal{C}[\rho] = \max(0, \eta_1 - \eta_2 - \eta_3 - \eta_4) \in [0, 1]$$

$\eta_1 \geq \eta_2 \geq \eta_3 \geq \eta_4$ are eigenvalues of $\sqrt{\rho\tilde{\rho}}$ with $\tilde{\rho} \equiv (\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)$.

- For a pure state $\psi\rangle \in \mathscr{H}_A \otimes \mathscr{H}_B$, the concurrence can be computed as

$$\mathcal{C}[|\psi\rangle] = \sqrt{2(1 - \mathrm{Tr}\rho_B^2)}\,, \qquad \rho_B \equiv \mathrm{Tr}_A|\psi\rangle\langle\psi|$$

*) LOCC: Local Operation and Classical Communication

Entanglement

Entanglement monotone: non-negative and non-increasing function under LOCC.

Ex.) **Concurrence** [for 2 qubit system]

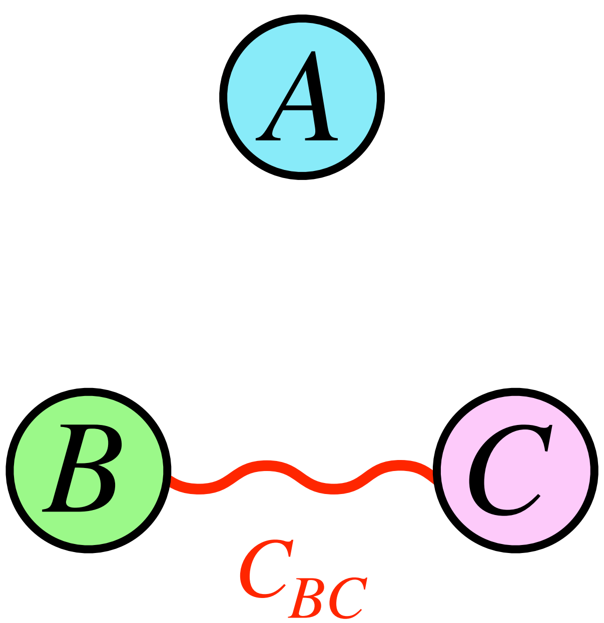
$$\mathcal{C}[\rho] = \max(0, \eta_1 - \eta_2 - \eta_3 - \eta_4) \in [0, 1]$$

$\eta_1 \geq \eta_2 \geq \eta_3 \geq \eta_4$ are eigenvalues of $\sqrt{\rho\tilde{\rho}}$ with $\tilde{\rho} \equiv (\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)$.

How to compute the entanglement btw. 2-individual qubits?

$$|\Psi\rangle = \sum_{a,b,c} c_{abc} \cdot |a\rangle_A \otimes |b\rangle_B \otimes |c\rangle_C$$

$a, b, c \in [0, 1]$



Entanglement

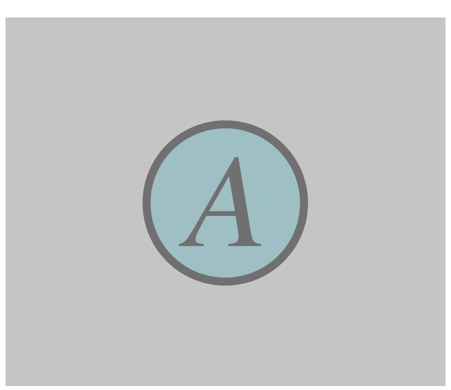
Entanglement monotone: non-negative and non-increasing function under LOCC.

Ex.) **Concurrence** [for 2 qubit system]

$$\mathcal{C}[\rho] = \max(0, \eta_1 - \eta_2 - \eta_3 - \eta_4) \in [0, 1]$$

$\eta_1 \geq \eta_2 \geq \eta_3 \geq \eta_4$ are eigenvalues of $\sqrt{\rho\tilde{\rho}}$ with $\tilde{\rho} \equiv (\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)$.

How to compute the entanglement btw. 2-individual qubits?

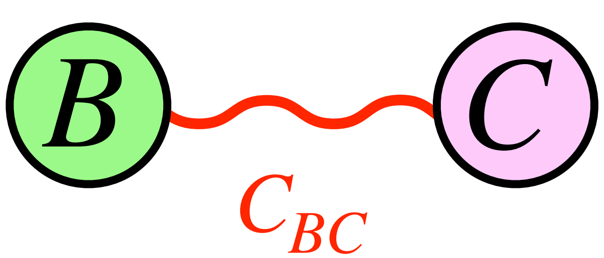


$$|\Psi\rangle = \sum_{a,b,c} c_{abc} \cdot |a\rangle_A \otimes |b\rangle_B \otimes |c\rangle_C$$

$$a, b, c \in [0, 1]$$

trace out A

$$\Rightarrow \quad \rho_{BC} = \mathrm{Tr}_A |\Psi\rangle\langle\Psi|$$



Entanglement

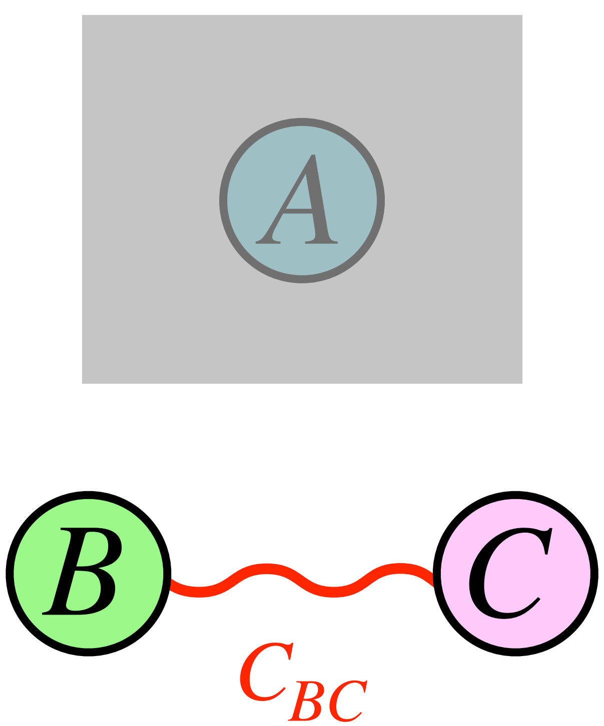
Entanglement monotone: non-negative and non-increasing function under LOCC.

Ex.) **Concurrence** [for 2 qubit system]

$$\mathcal{C}[\rho] = \max(0, \eta_1 - \eta_2 - \eta_3 - \eta_4) \in [0, 1]$$

$\eta_1 \geq \eta_2 \geq \eta_3 \geq \eta_4$ are eigenvalues of $\sqrt{\rho\tilde{\rho}}$ with $\tilde{\rho} \equiv (\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)$.

How to compute the entanglement btw. 2-individual qubits?



$$|\Psi\rangle = \sum_{a,b,c} c_{abc} \cdot |a\rangle_A \otimes |b\rangle_B \otimes |c\rangle_C$$

$$a, b, c \in [0, 1]$$

trace out A

$$\Rightarrow \rho_{BC} = \mathrm{Tr}_A |\Psi\rangle\langle\Psi|$$

Entanglement

Entanglement monotone: non-negative and non-increasing function under LOCC.

Ex.) **Concurrence** [for 2 qubit system]

$$\mathcal{C}[\rho] = \max(0, \eta_1 - \eta_2 - \eta_3 - \eta_4) \in [0, 1]$$

$\eta_1 \geq \eta_2 \geq \eta_3 \geq \eta_4$ are eigenvalues of $\sqrt{\rho\tilde{\rho}}$ with $\tilde{\rho} \equiv (\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)$.

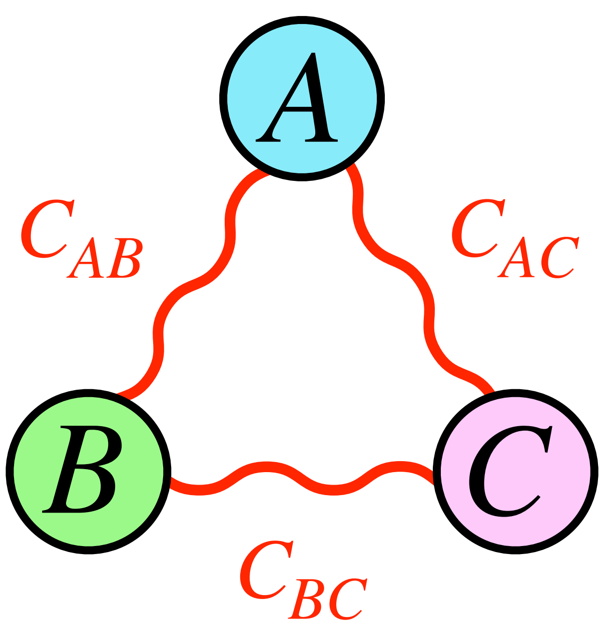
How to compute the entanglement btw. 2-individual qubits?

$$|\Psi\rangle = \sum_{a,b,c} c_{abc} \cdot |a\rangle_A \otimes |b\rangle_B \otimes |c\rangle_C$$

$a, b, c \in [0, 1]$

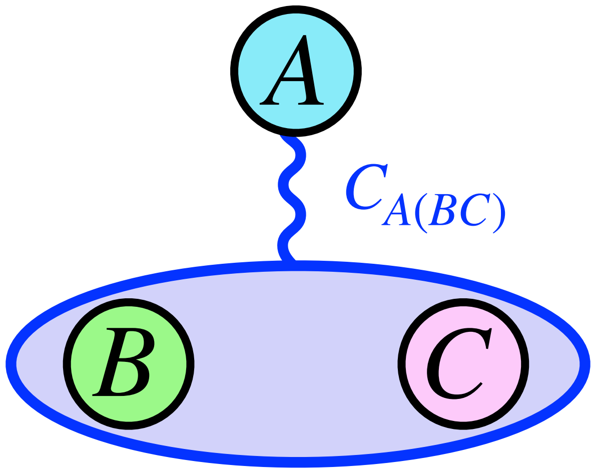
trace out A

$$\Rightarrow \rho_{BC} = \mathrm{Tr}_A |\Psi\rangle\langle\Psi|$$



C_{AB}, C_{BC}, C_{AC}

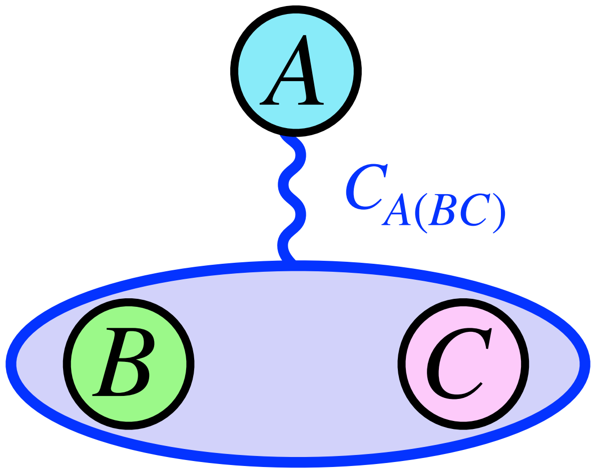
How to compute the ent. btw. A and BC composite system?



$$|\Psi\rangle = \sum_{a,b,c} c_{abc} \cdot |a\rangle_A \otimes |b\rangle_B \otimes |c\rangle_C$$

$$a, b, c \in [0, 1]$$

How to compute the ent. btw. A and BC composite system?



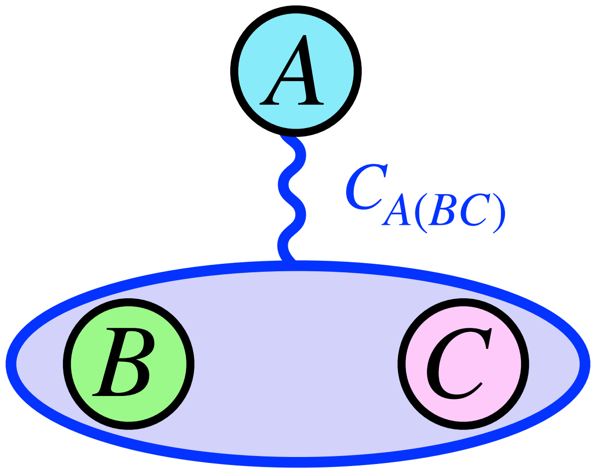
$$|\Psi\rangle = \sum_{a,b,c} c_{abc} \cdot |a\rangle_A \otimes |b\rangle_B \otimes |c\rangle_C$$

$$a, b, c \in [0, 1]$$

- For a pure state $\Psi\rangle \in \mathscr{H}_A \otimes \mathscr{H}_{BC}$, the concurrence can be computed as

$$\mathcal{C}[|\Psi\rangle] = \sqrt{2(1 - \mathrm{Tr}\rho^2_{BC})} \qquad \rho_{BC} \equiv \mathrm{Tr}_A |\Psi\rangle\langle\Psi|$$

How to compute the ent. btw. A and BC composite system?



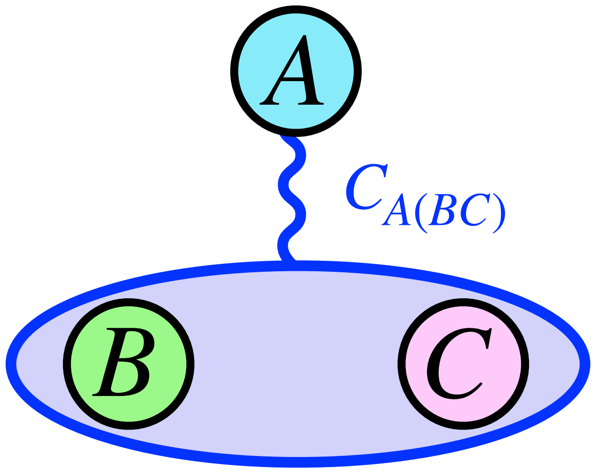
$$|\Psi\rangle = \sum_{a,b,c} c_{abc} \cdot |a\rangle_A \otimes |b\rangle_B \otimes |c\rangle_C$$

$$a, b, c \in [0, 1]$$

- For a pure state $\Psi\rangle \in \mathscr{H}_A \otimes \mathscr{H}_{BC}$, the concurrence can be computed as

$$\mathcal{C}_{A(BC)} \equiv \mathcal{C}[|\Psi\rangle] = \sqrt{2(1 - \mathrm{Tr}\rho_{BC}^2)} \qquad \rho_{BC} \equiv \mathrm{Tr}_A |\Psi\rangle\langle\Psi|$$

How to compute the ent. btw. A and BC composite system?

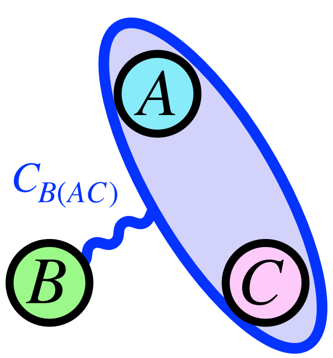


$$|\Psi\rangle = \sum_{a,b,c} c_{abc} \cdot |a\rangle_A \otimes |b\rangle_B \otimes |c\rangle_C$$

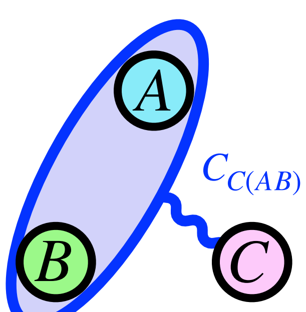
$$a, b, c \in [0, 1]$$

- For a pure state $\Psi\rangle \in \mathscr{H}_A \otimes \mathscr{H}_{BC}$, the concurrence can be computed as

$$\mathcal{C}_{A(BC)} \equiv \mathcal{C}[|\Psi\rangle] = \sqrt{2(1 - \mathrm{Tr}\rho^2_{BC})} \qquad \rho_{BC} \equiv \mathrm{Tr}_A|\Psi\rangle\langle\Psi|$$



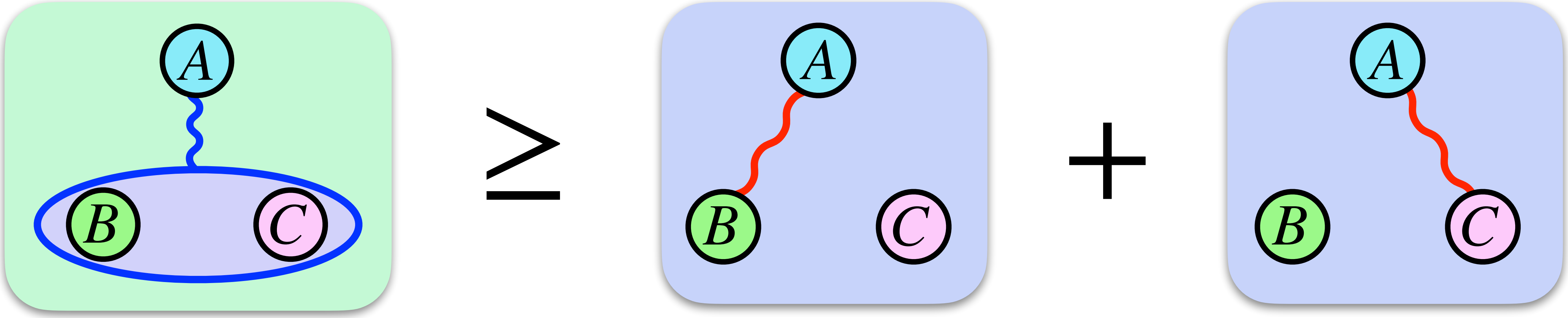
$$\mathcal{C}_{B(AC)} \equiv \mathcal{C}[|\Psi\rangle] = \sqrt{2(1 - \mathrm{Tr}\rho^2_{AC})} \qquad \rho_{BC} \equiv \mathrm{Tr}_A|\Psi\rangle\langle\Psi|$$



$$\mathcal{C}_{C(AB)} \equiv \mathcal{C}[|\Psi\rangle] = \sqrt{2(1 - \mathrm{Tr}\rho^2_{AB})} \qquad \rho_{AB} \equiv \mathrm{Tr}_C|\Psi\rangle\langle\Psi|$$

Monogamy

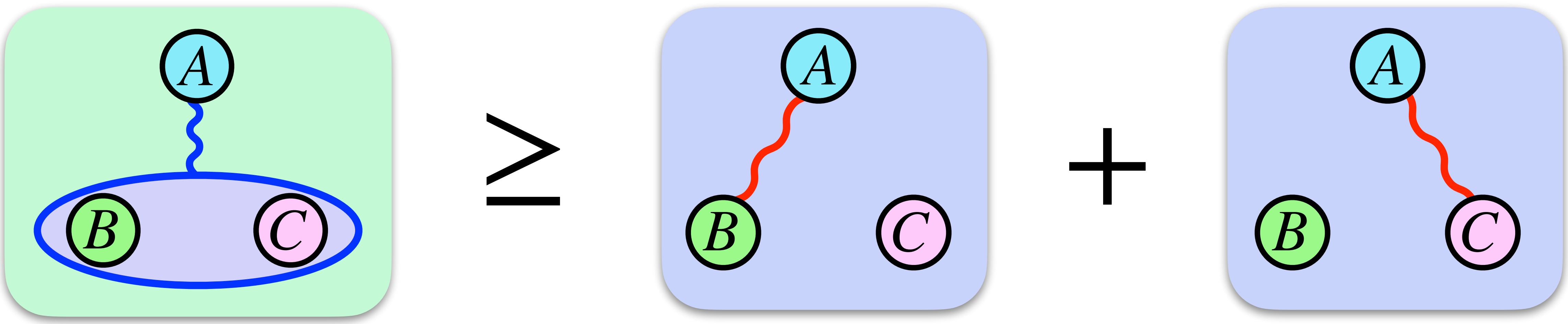
- A-(BC) entanglement limits A-B and A-C entanglements



[Coffman, Kundu, Wootters ’99]

Monogamy

- A-(BC) entanglement limits A-B and A-C entanglements

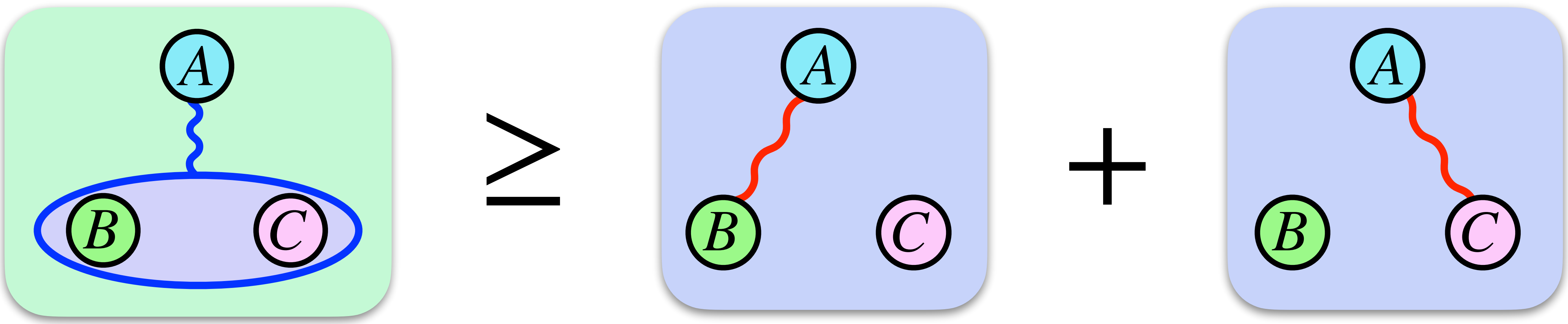


- Coffman-Kundu-Wootters (CKW) monogamy inequality [Coffman, Kundu, Wootters '99]

$$C^2_{\mathrm{A(BC)}} \geq C^2_{\mathrm{AB}} + C^2_{\mathrm{AC}}$$

Monogamy

- A-(BC) entanglement limits A-B and A-C entanglements



- Coffman-Kundu-Wootters (CKW) monogamy inequality [Coffman, Kundu, Wootters ’99]

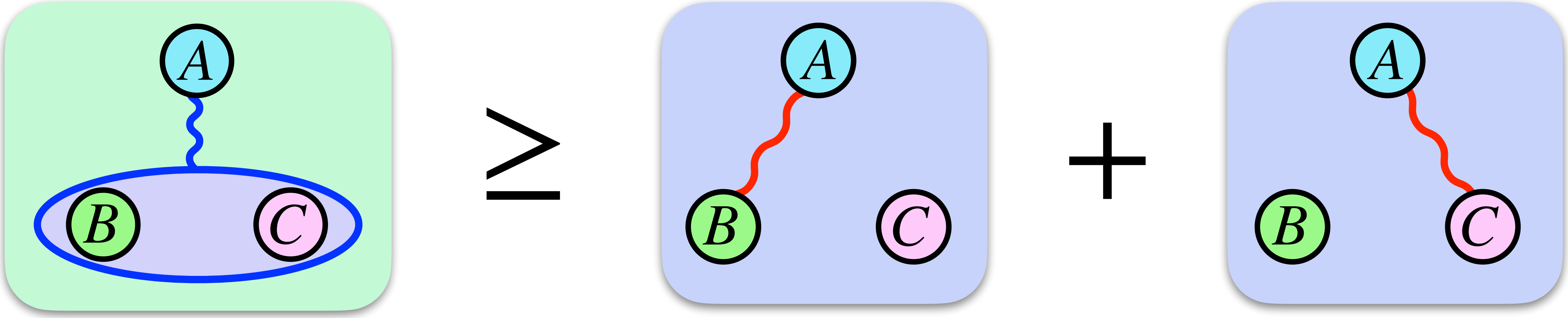
$$C^2_{A(BC)} \geq C^2_{AB} + C^2_{AC}$$

$$C^2_{B(AC)} \geq C^2_{BA} + C^2_{BC}$$

$$C^2_{C(AB)} \geq C^2_{CA} + C^2_{CB}$$

Monogamy

- A-(BC) entanglement limits A-B and A-C entanglements



- Coffman-Kundu-Wootters (CKW) monogamy inequality [Coffman, Kundu, Wootters '99]

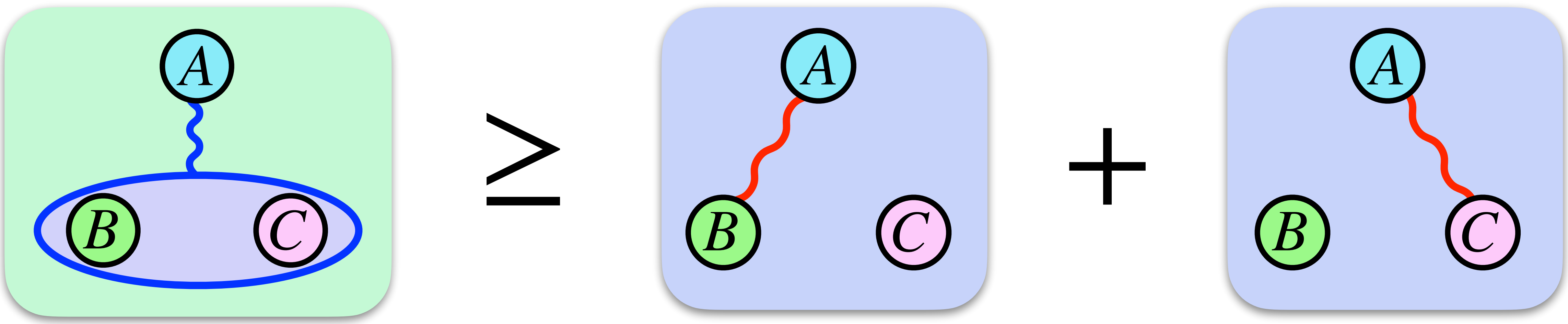
$$C^2_{A(BC)} \geq C^2_{AB} + C^2_{AC}$$

$$C^2_{B(AC)} \geq C^2_{BA} + C^2_{BC}$$

$$C^2_{C(AB)} \geq C^2_{CA} + C^2_{CB}$$

Monogamy

- A-(BC) entanglement limits A-B and A-C entanglements



- Coffman-Kundu-Wootters (CKW) monogamy inequality [Coffman, Kundu, Wootters '99]

$$C^2_{A(BC)} \geq C^2_{AB} + C^2_{AC}$$

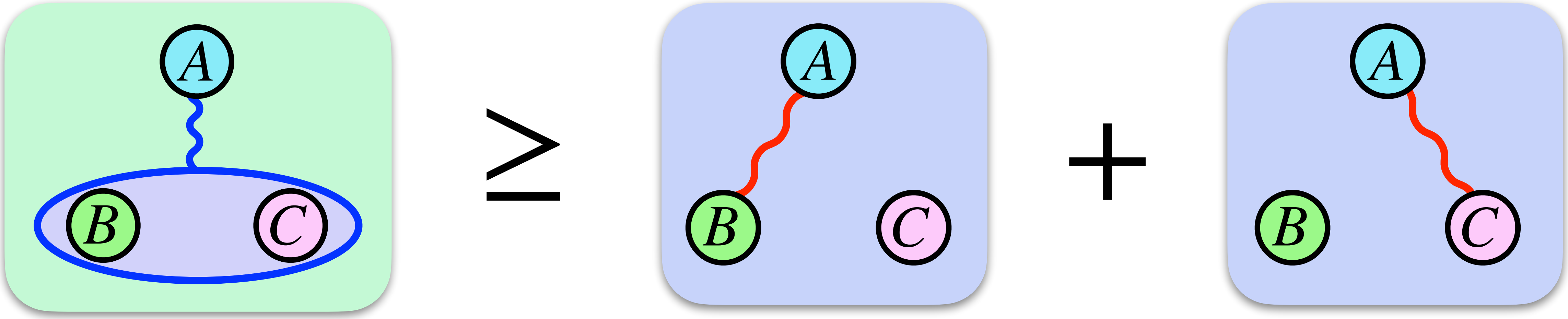
$$C^2_{B(AC)} \geq C^2_{BA} + C^2_{BC}$$

$$C^2_{C(AB)} \geq C^2_{CA} + C^2_{CB}$$

$$C^2_{A(BC)} + C^2_{B(AC)} \geq C^2_{C(AB)}$$

Monogamy

- A-(BC) entanglement limits A-B and A-C entanglements



- Coffman-Kundu-Wootters (CKW) monogamy inequality [Coffman, Kundu, Wootters '99]

$$C^2_{A(BC)} \geq C^2_{AB} + C^2_{AC}$$

$$C^2_{B(AC)} \geq C^2_{BA} + C^2_{BC}$$

$$C^2_{C(AB)} \geq C^2_{CA} + C^2_{CB}$$

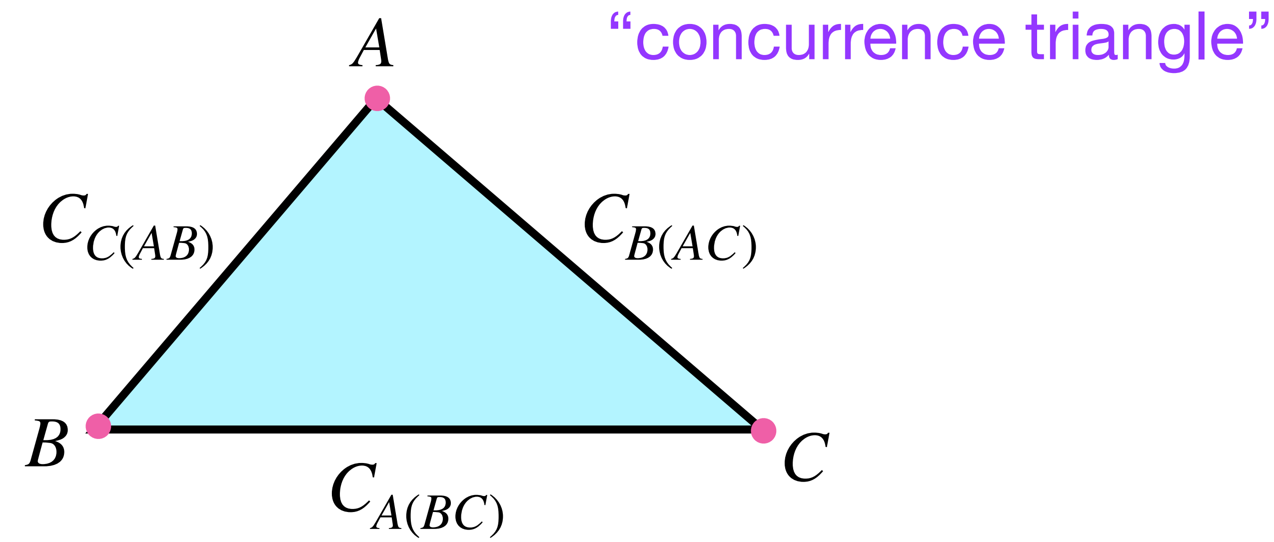
$$C^2_{A(BC)} + C^2_{B(AC)} \geq C^2_{C(AB)}$$

$$\Downarrow$$

$$C_{A(BC)} + C_{B(AC)} \geq C_{C(AB)}$$

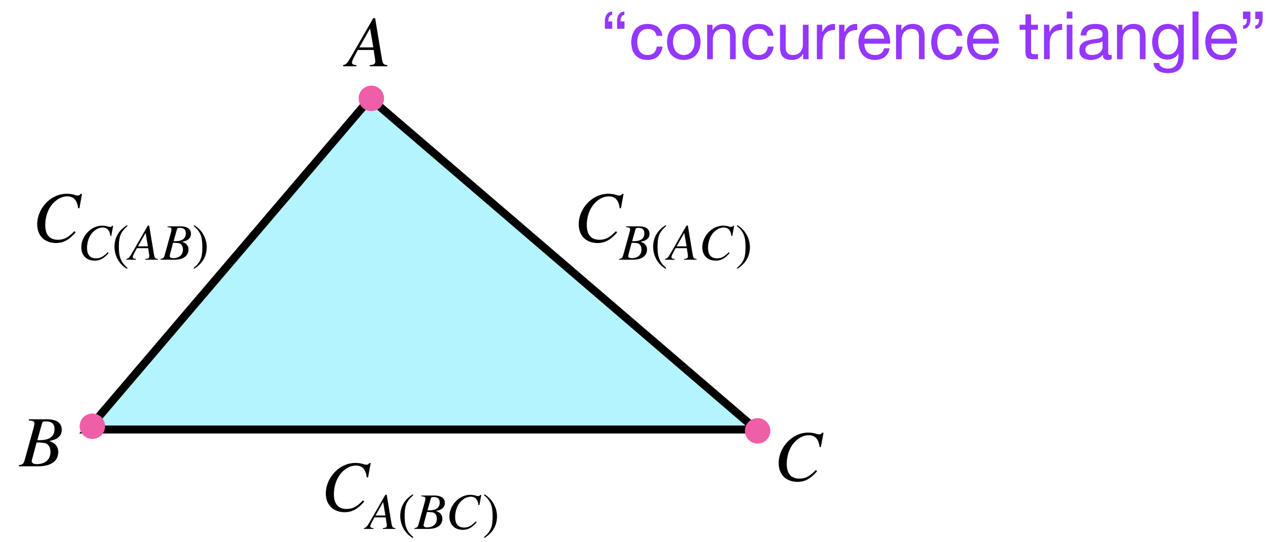
$$C_{A(BC)} + C_{B(AC)} \geq C_{C(AB)}$$

$\Rightarrow$

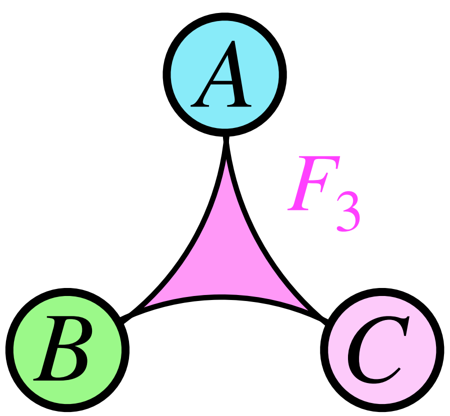


$$C_{A(BC)} + C_{B(AC)} \geq C_{C(AB)}$$

$\Rightarrow$



Genuine Multi-particle Entanglement (GME) measure: [Dur, Vidal, Cirac '00, Ma, Chen, Chen, Spengler, Gabriel, Huber '11, Xie, Eberly '21]



GME should satisfy the following properties:

(1) vanish for all product and biseparable states $\Rightarrow$ unseparable even partially

(2) positive for all non-biseparable states

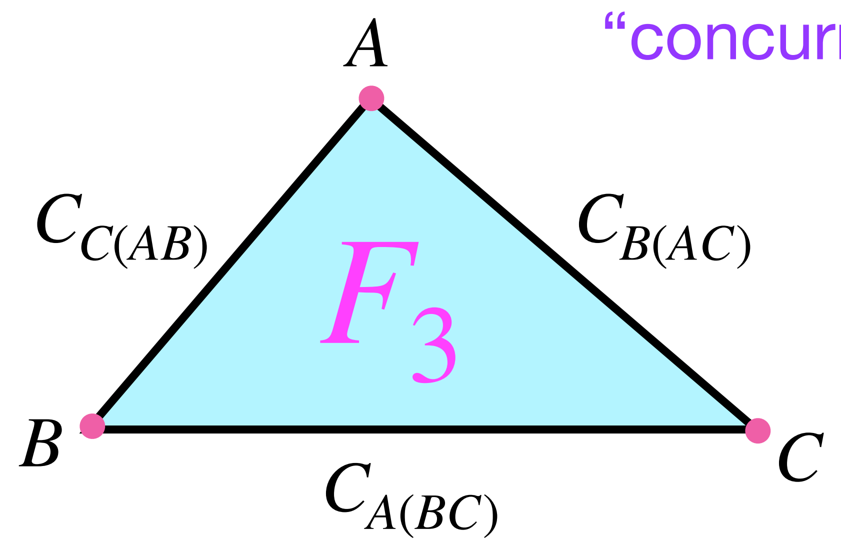
(3) not increase under LOCC

$$|\psi\rangle_A \otimes (|00\rangle_{BC} + |11\rangle_{BC})$$

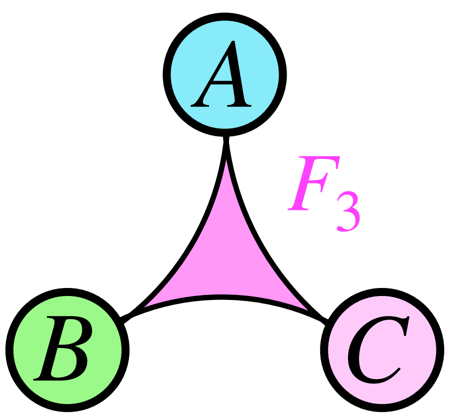
$$\Rightarrow F_3 = 0$$

$$C_{A(BC)} + C_{B(AC)} \geq C_{C(AB)} \quad \Rightarrow$$

“concurrence triangle”



Genuine Multi-particle Entanglement (GME) measure: [Dur, Vidal, Cirac ’00, Ma, Chen, Chen, Spengler, Gabriel, Huber ’11, Xie, Eberly ’21]



GME should satisfy the following properties:

(1) vanish for all product and biseparable states $\Rightarrow$ unseparable even partially

(2) positive for all non-biseparable states

(3) not increase under LOCC

$$|\psi\rangle_A \otimes (|00\rangle_{BC} + |11\rangle_{BC})$$
$$\Rightarrow F_3 = 0$$

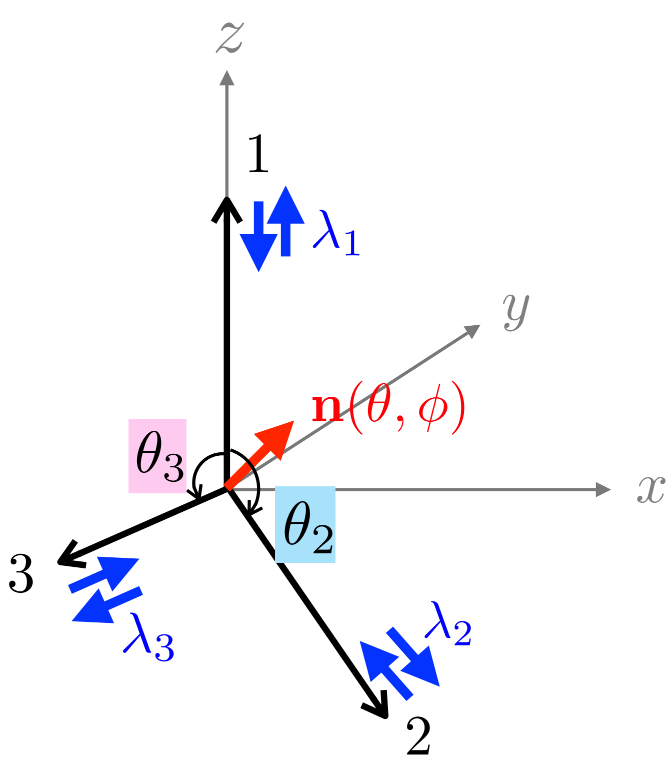
The **area** of the “concurrence triangle” satisfies (1), (2), (3) ! [Jin, Tao, Gui, Fei, Li-Jost, Qiao (2023)]

$$F_3 \equiv \left[\frac{16}{3} Q \left(Q - \mathcal{C}_{A(BC)}\right)\left(Q - \mathcal{C}_{B(AC)}\right)\left(Q - \mathcal{C}_{C(AB)}\right)\right]^{\frac{1}{2}}$$

$$Q \equiv \frac{1}{2}\left[\mathcal{C}_{A(BC)} + \mathcal{C}_{B(AC)} + \mathcal{C}_{C(AB)}\right]$$

3-body decay: $0 \to 123$

[KS, M.Spannowsky 2310.01477]



Assumptions:

- all particles have spin 1/2
- all final particles 1,2,3 are massless

Kinematics:

- rest frame of the initial particle 0
- p_1 is in the z-axis
- decay is in the x-z plane

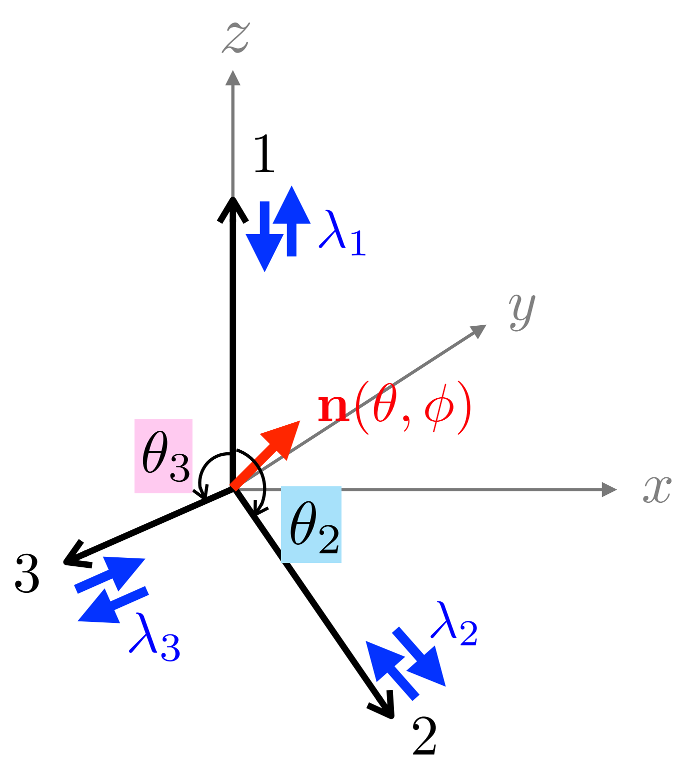
$$p_1^\mu = p_1(1, 0, 0, 1)$$
$$p_2^\mu = p_2(1, \sin\theta_2, 0, \cos\theta_2)$$
$$p_3^\mu = p_3(1, -\sin\theta_3, 0, \cos\theta_3)$$

$\mathbf{n}(\theta, \phi)$: polarisation of initial spin

$\lambda_1, \lambda_2, \lambda_3 \in (+, -)$: helicities of 1,2,3

3-body decay: $0 \to 123$

[KS, M.Spannowsky 2310.01477]



Assumptions:

- all particles have spin 1/2
- all final particles 1,2,3 are massless

Kinematics:

- rest frame of the initial particle 0
- p_1 is in the z-axis
- decay is in the x-z plane

$$p_1^\mu = p_1(1, 0, 0, 1)$$
$$p_2^\mu = p_2(1, \sin\theta_2, 0, \cos\theta_2)$$
$$p_3^\mu = p_3(1, -\sin\theta_3, 0, \cos\theta_3)$$

$\mathbf{n}(\theta, \phi)$: polarisation of initial spin

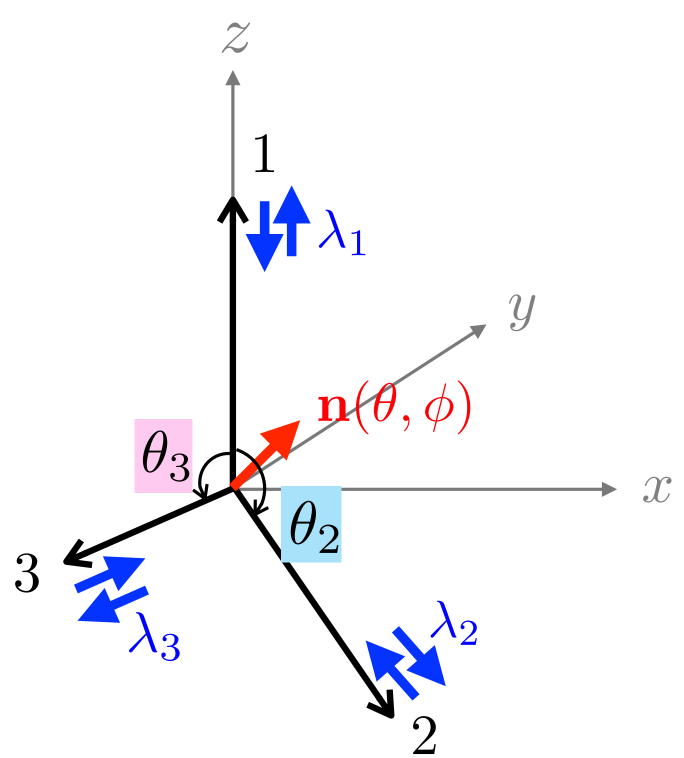
$\lambda_1, \lambda_2, \lambda_3 \in (+, -)$: helicities of 1,2,3

initial state

$$|\mathbf{n}(\theta, \phi)\rangle$$

3-body decay: $0 \to 123$

[KS, M.Spannowsky 2310.01477]



Assumptions:

- all particles have spin 1/2
- all final particles 1,2,3 are massless

Kinematics:

- rest frame of the initial particle 0
- p_1 is in the z-axis
- decay is in the x-z plane

$$p_1^\mu = p_1(1,0,0,1)$$
$$p_2^\mu = p_2(1,\sin\theta_2,0,\cos\theta_2)$$
$$p_3^\mu = p_3(1,-\sin\theta_3,0,\cos\theta_3)$$

$\mathbf{n}(\theta,\phi)$: polarisation of initial spin

$\lambda_1,\lambda_2,\lambda_3 \in (+,-)$: helicities of 1,2,3

amplitude

$$\hat{\mathbf{1}} = \sum_{\lambda_1,\lambda_2,\lambda_3} |\lambda_1,\lambda_2,\lambda_3\rangle\langle\lambda_1,\lambda_2,\lambda_3| \qquad \mathcal{M}^{\mathbf{n}}_{\lambda_1,\lambda_2,\lambda_3} = \langle\lambda_1,\lambda_2,\lambda_3|\mathbf{n}(\theta,\phi)\rangle$$

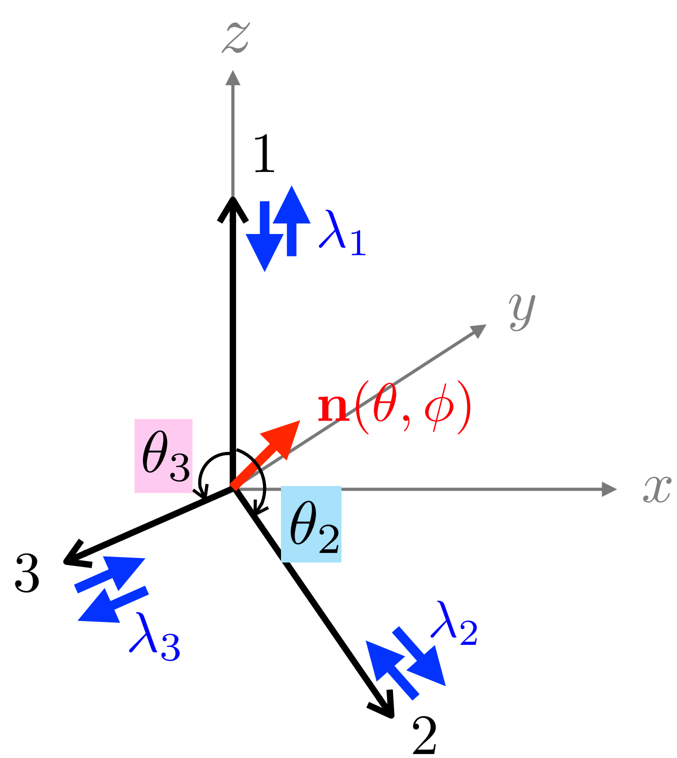
initial state

$$|\mathbf{n}(\theta,\phi)\rangle = \sum_{\lambda_1,\lambda_2,\lambda_3} \mathcal{M}^{\mathbf{n}}_{\lambda_1,\lambda_2,\lambda_3}|\lambda_1,\lambda_2,\lambda_3\rangle + \cdots$$

final state

3-body decay: $0 \to 123$

[KS, M.Spannowsky 2310.01477]



Assumptions:

- all particles have spin 1/2
- all final particles 1,2,3 are massless

Kinematics:

- rest frame of the initial particle 0
- p_1 is in the z-axis
- decay is in the x-z plane

$$p_1^\mu = p_1(1, 0, 0, 1)$$
$$p_2^\mu = p_2(1, \sin\theta_2, 0, \cos\theta_2)$$
$$p_3^\mu = p_3(1, -\sin\theta_3, 0, \cos\theta_3)$$

$\mathbf{n}(\theta, \phi)$: polarisation of initial spin

$\lambda_1, \lambda_2, \lambda_3 \in (+, -)$: helicities of 1,2,3

$$\hat{\mathbf{1}} = \sum_{\lambda_1,\lambda_2,\lambda_3} |\lambda_1, \lambda_2, \lambda_3\rangle\langle\lambda_1, \lambda_2, \lambda_3|$$

amplitude

$$\mathcal{M}^{\mathbf{n}}_{\lambda_1,\lambda_2,\lambda_3} = \langle\lambda_1, \lambda_2, \lambda_3|\mathbf{n}(\theta, \phi)\rangle$$

initial state

$$|\mathbf{n}(\theta, \phi)\rangle = \sum_{\lambda_1,\lambda_2,\lambda_3} \mathcal{M}^{\mathbf{n}}_{\lambda_1,\lambda_2,\lambda_3} |\lambda_1, \lambda_2, \lambda_3\rangle = \Psi\rangle \leftarrow$$

pure (entangled) **3-spin state**

Interaction

- Consider **most general** Lorentz invariant 4-fermion interactions

$$\mathcal{L}_{\rm int} = (\bar{\psi}_1 \Gamma_A \psi_0)(\bar{\psi}_3 \Gamma_B \psi_2)$$

$$\Gamma_{A/B} \in \{1, \gamma^5, \gamma^\mu, \gamma^\mu\gamma^5, \sigma^{\mu\nu}\}$$

❖ **Scalar-type**

$$[\bar{\psi}_1 (c_S + i c_A \gamma_5)\psi_0][\bar{\psi}_3 (d_S + i d_A \gamma_5)\psi_2]$$

$$c \equiv c_S + i c_A = e^{i\delta_1}$$
$$d \equiv d_S + i d_A = e^{i\delta_2}$$

❖ **Vector-type**

$$[\bar{\psi}_1 \gamma_\mu (c_L P_L + c_R P_R)\psi_0][\bar{\psi}_3 \gamma^\mu (d_L P_L + d_R P_R)\psi_2]$$

$$P_{R/L} = \frac{1 \pm \gamma^5}{2}$$
$$c_L, c_R, d_L, d_R \in \mathbb{R}$$

❖ **Tensor-type**

$$[\bar{\psi}_1 (c_M + i c_E \gamma_5)\sigma^{\mu\nu}\psi_0][\bar{\psi}_3 (d_M + i d_E \gamma_5)\sigma_{\mu\nu}\psi_2]$$

$$c \equiv c_M + i c_E = e^{i\omega_1}$$
$$d \equiv d_M + i d_E = e^{i\omega_2}$$

[KS, M.Spannowsky 2310.01477]

Scalar

$$\mathcal{L}_{\rm int} = [\bar{\psi}_1(c_S + ic_A\gamma_5)\psi_0][\bar{\psi}_3(d_S + id_A\gamma_5)\psi_2]$$

$$c \equiv c_S + ic_A = e^{i\delta_1}$$
$$d \equiv d_S + id_A = e^{i\delta_2}$$

$$\Rightarrow \quad |\Psi\rangle = \frac{cd}{\sqrt{2}} \cdot e^{i\phi} s_{\frac{\theta}{2}} |- - -\rangle - \frac{cd^*}{\sqrt{2}} \cdot e^{i\phi} s_{\frac{\theta}{2}} |- + +\rangle + \frac{c^* d}{\sqrt{2}} \cdot c_{\frac{\theta}{2}} |+ - -\rangle - \frac{c^* d^*}{\sqrt{2}} \cdot c_{\frac{\theta}{2}} |+ + +\rangle$$

[KS, M.Spannowsky 2310.01477]

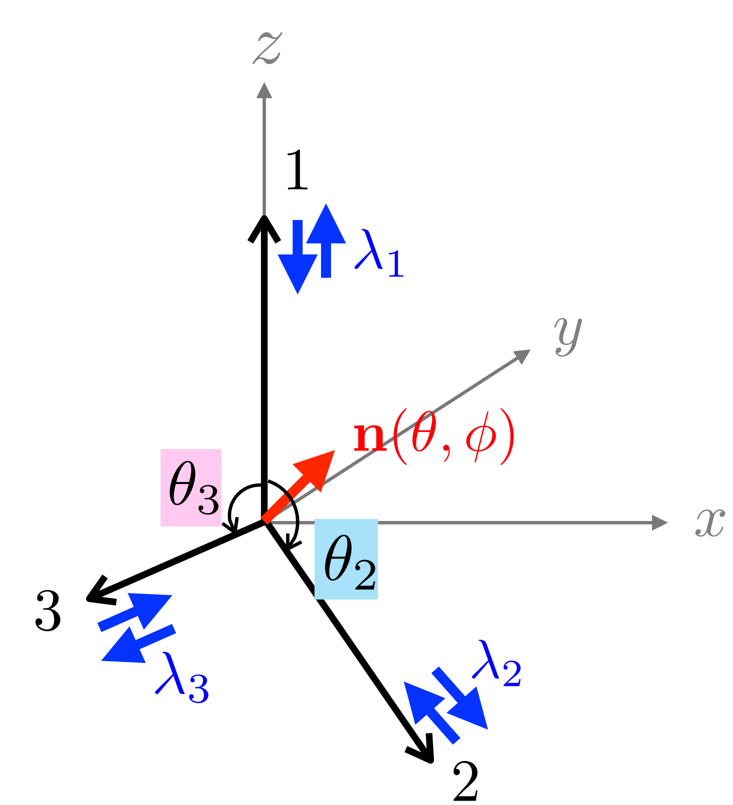
Scalar

$$\mathcal{L}_{\rm int} = \ [\bar{\psi}_1(c_S + ic_A\gamma_5)\psi_0][\bar{\psi}_3(d_S + id_A\gamma_5)\psi_2]$$

$$c \equiv c_S + ic_A = e^{i\delta_1}$$
$$d \equiv d_S + id_A = e^{i\delta_2}$$

$$\Rightarrow \quad |\Psi\rangle = \frac{cd}{\sqrt{2}} \cdot e^{i\phi} s_{\frac{\theta}{2}} |-\,-\,-\rangle - \frac{cd^*}{\sqrt{2}} \cdot e^{i\phi} s_{\frac{\theta}{2}} |-\,+\,+\rangle + \frac{c^*d}{\sqrt{2}} \cdot c_{\frac{\theta}{2}} |+\,-\,-\rangle - \frac{c^*d^*}{\sqrt{2}} \cdot c_{\frac{\theta}{2}} |+\,+\,+\rangle$$

independent of final state momenta θ_2, θ_3



[KS, M.Spannowsky 2310.01477]

Scalar

$$\mathcal{L}_{\rm int} = [\bar\psi_1(c_S + i c_A \gamma_5)\psi_0][\bar\psi_3(d_S + i d_A\gamma_5)\psi_2]$$

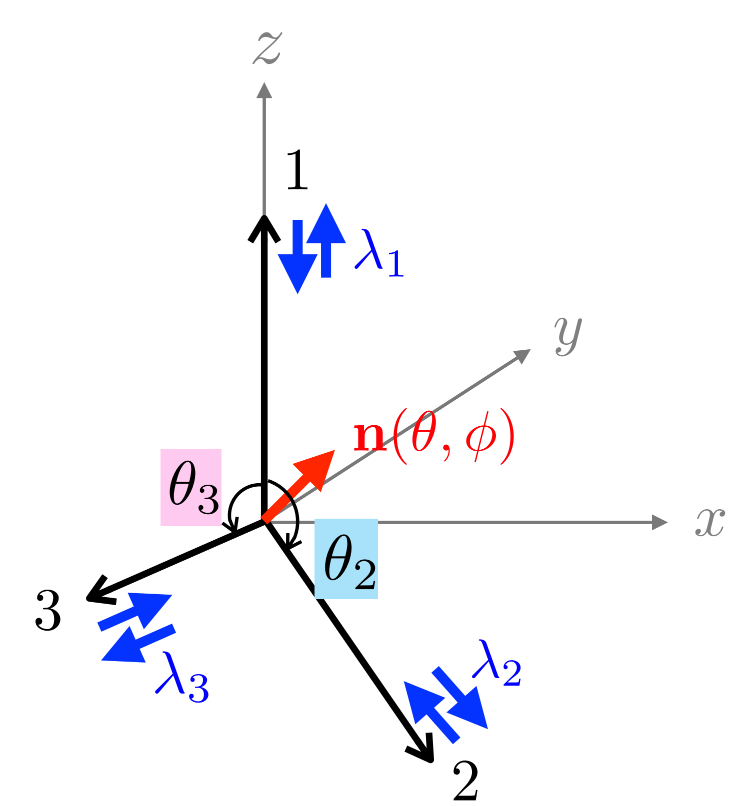
$c \equiv c_S + i c_A = e^{i\delta_1}$
$d \equiv d_S + i d_A = e^{i\delta_2}$

$$\Rightarrow \quad |\Psi\rangle = \frac{cd}{\sqrt{2}} \cdot e^{i\phi} s_{\frac{\theta}{2}} |---\rangle - \frac{cd^*}{\sqrt{2}} \cdot e^{i\phi} s_{\frac{\theta}{2}} |-++\rangle + \frac{c^*d}{\sqrt{2}} \cdot c_{\frac{\theta}{2}} |+--\rangle - \frac{c^*d^*}{\sqrt{2}} \cdot c_{\frac{\theta}{2}} |+++\rangle$$

independent of final state momenta θ_2, θ_3

$$= \left[c e^{i\phi} s_{\frac{\theta}{2}} |-\rangle_1 + c^* c_{\frac{\theta}{2}} |+\rangle_1 \right] \otimes \frac{1}{\sqrt{2}} \left[d|--\rangle_{23} - d^* |++\rangle_{23} \right]$$

bi-separable



[KS, M.Spannowsky 2310.01477]

Scalar

$$\mathcal{L}_{\rm int} = [\bar\psi_1(c_S + ic_A\gamma_5)\psi_0][\bar\psi_3(d_S + id_A\gamma_5)\psi_2]$$

$$c \equiv c_S + ic_A = e^{i\delta_1}$$
$$d \equiv d_S + id_A = e^{i\delta_2}$$

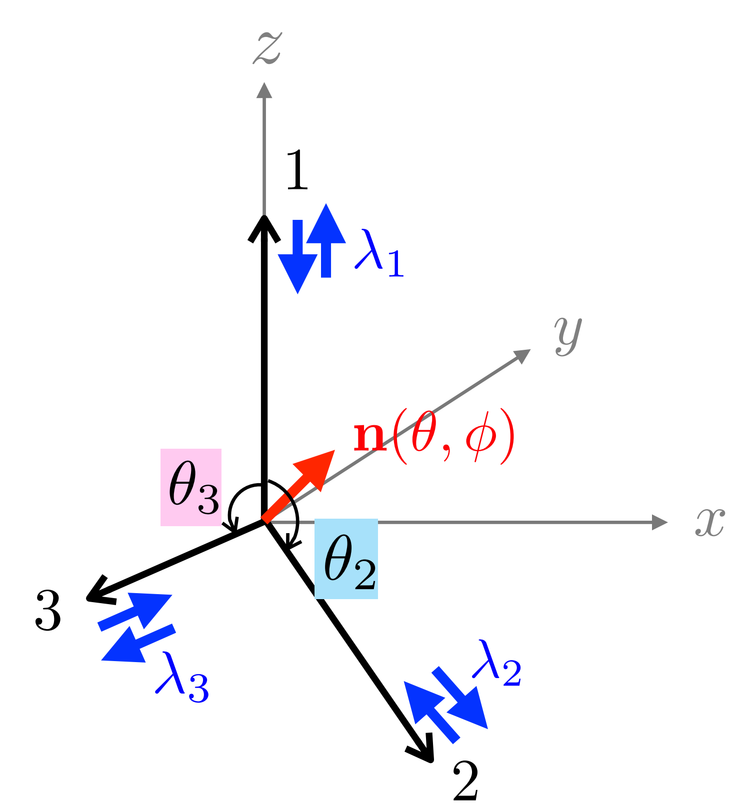
$$\Rightarrow \quad |\Psi\rangle = \frac{cd}{\sqrt{2}} \cdot e^{i\phi} s_{\frac{\theta}{2}} |---\rangle - \frac{cd^*}{\sqrt{2}} \cdot e^{i\phi} s_{\frac{\theta}{2}} |-++\rangle + \frac{c^* d}{\sqrt{2}} \cdot c_{\frac{\theta}{2}} |+--\rangle - \frac{c^* d^*}{\sqrt{2}} \cdot c_{\frac{\theta}{2}} |+++\rangle$$

independent of final state momenta θ_2, θ_3

$$= \left[c e^{i\phi} s_{\frac{\theta}{2}} |-\rangle_1 + c^* c_{\frac{\theta}{2}} |+\rangle_1 \right] \otimes \frac{1}{\sqrt{2}} \left[d|--\rangle_{23} - d^* |++\rangle_{23} \right]$$

bi-separable

$$\Rightarrow F_3 = 0$$



[KS, M.Spannowsky 2310.01477]

Scalar

$$\mathcal{L}_{\rm int} = [\bar{\psi}_1(c_S + i c_A \gamma_5)\psi_0][\bar{\psi}_3(d_S + i d_A \gamma_5)\psi_2]$$

$$c \equiv c_S + i c_A = e^{i\delta_1}$$
$$d \equiv d_S + i d_A = e^{i\delta_2}$$

$$\Rightarrow \quad |\Psi\rangle = \frac{cd}{\sqrt{2}} \cdot e^{i\phi} s_{\frac{\theta}{2}} |---\rangle - \frac{cd^*}{\sqrt{2}} \cdot e^{i\phi} s_{\frac{\theta}{2}} |-++\rangle + \frac{c^* d}{\sqrt{2}} \cdot c_{\frac{\theta}{2}} |+--\rangle - \frac{c^* d^*}{\sqrt{2}} \cdot c_{\frac{\theta}{2}} |+++\rangle$$

independent of final state momenta θ_2, θ_3

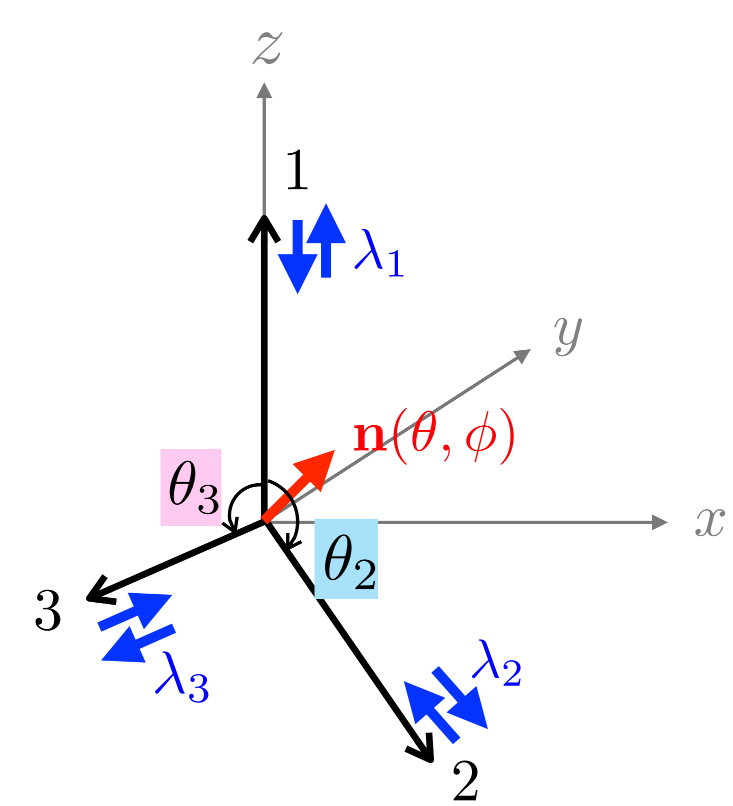
$$= \left[c e^{i\phi} s_{\frac{\theta}{2}} |-\rangle_1 + c^* c_{\frac{\theta}{2}} |+\rangle_1 \right] \otimes \frac{1}{\sqrt{2}} \left[d |--\rangle_{23} - d^* |++\rangle_{23} \right]$$

bi-separable

$$\Rightarrow F_3 = 0$$

✤ **1** is **not entangled** with **2** and **3** in any way:

$$\mathcal{C}_{12} = \mathcal{C}_{13} = \mathcal{C}_{1(23)} = 0$$



[KS, M.Spannowsky 2310.01477]

Scalar

$$\mathcal{L}_{\rm int} = [\bar{\psi}_1(c_S + ic_A\gamma_5)\psi_0][\bar{\psi}_3(d_S + id_A\gamma_5)\psi_2]$$

$$c \equiv c_S + ic_A = e^{i\delta_1}$$
$$d \equiv d_S + id_A = e^{i\delta_2}$$

$$\Rightarrow \quad |\Psi\rangle = \frac{cd}{\sqrt{2}} \cdot e^{i\phi} s_{\frac{\theta}{2}} |---\rangle - \frac{cd^*}{\sqrt{2}} \cdot e^{i\phi} s_{\frac{\theta}{2}} |-++\rangle + \frac{c^* d}{\sqrt{2}} \cdot c_{\frac{\theta}{2}} |+--\rangle - \frac{c^* d^*}{\sqrt{2}} \cdot c_{\frac{\theta}{2}} |+++\rangle$$

independent of final state momenta θ_2, θ_3

$$= \left[c e^{i\phi} s_{\frac{\theta}{2}} |-\rangle_1 + c^* c_{\frac{\theta}{2}} |+\rangle_1 \right] \otimes \frac{1}{\sqrt{2}} \left[d|--\rangle_{23} - d^* |++\rangle_{23} \right]$$

bi-separable

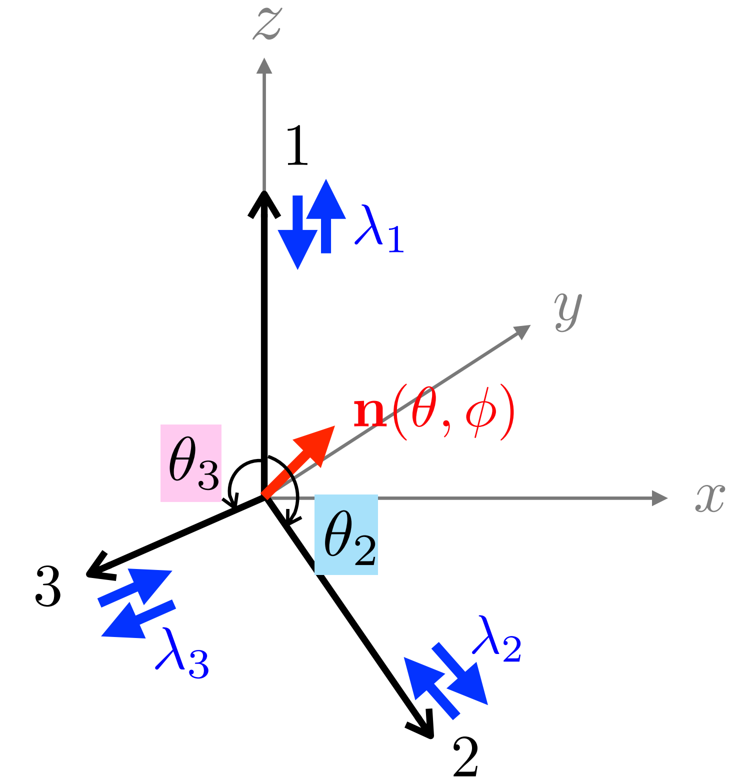
$$\Rightarrow F_3 = 0$$

✤ **1** is **not entangled** with **2** and **3** in any way:

$$\mathcal{C}_{12} = \mathcal{C}_{13} = \mathcal{C}_{1(23)} = 0$$

✤ **2** and **3** are **maximally entangled**

$$\mathcal{C}_{23} = 1$$



[KS, M.Spannowsky 2310.01477]

Scalar

$$\mathcal{L}_{\rm int} = \; [\bar\psi_1(c_S + i c_A\gamma_5)\psi_0][\bar\psi_3(d_S + i d_A\gamma_5)\psi_2]$$

$$c \equiv c_S + i c_A = e^{i\delta_1}$$
$$d \equiv d_S + i d_A = e^{i\delta_2}$$

$$\Rightarrow \; |\Psi\rangle = \frac{cd}{\sqrt{2}} \cdot e^{i\phi} s_{\frac{\theta}{2}} |---\rangle - \frac{cd^*}{\sqrt{2}} \cdot e^{i\phi} s_{\frac{\theta}{2}} |-++\rangle + \frac{c^* d}{\sqrt{2}} \cdot c_{\frac{\theta}{2}} |+--\rangle - \frac{c^* d^*}{\sqrt{2}} \cdot c_{\frac{\theta}{2}} |+++\rangle$$

independent of final state momenta θ_2, θ_3

$$= \left[c e^{i\phi} s_{\frac{\theta}{2}} |-\rangle_1 + c^* c_{\frac{\theta}{2}} |+\rangle_1 \right] \otimes \frac{1}{\sqrt{2}} \left[d|--\rangle_{23} - d^* |++\rangle_{23} \right]$$

bi-separable

$$\Rightarrow F_3 = 0$$

✤ **1** is **not entangled** with **2** and **3** in any way:

$$\mathcal{C}_{12} = \mathcal{C}_{13} = \mathcal{C}_{1(23)} = 0$$

✤ **2** and **3** are **maximally entangled**

$$\mathcal{C}_{23} = 1$$

✤ Due to **monogamy**, **2** and **3** are **maximally entangled** with the rest

$$\mathcal{C}_{2(13)} = \mathcal{C}_{3(12)} = 1$$

Monogamy

$$\mathcal{C}^2_{2(13)} \geq \mathcal{C}^2_{12} + \mathcal{C}^2_{23} \quad (\mathcal{C}^2_{12} = 0,\ \mathcal{C}^2_{23} = 1)$$

$$\mathcal{C}^2_{3(12)} \geq \mathcal{C}^2_{13} + \mathcal{C}^2_{23} \quad (\mathcal{C}^2_{13} = 0,\ \mathcal{C}^2_{23} = 1)$$

[KS, M.Spannowsky 2310.01477]

Vector

$$P_{R/L} = \frac{1 \pm \gamma^5}{2}$$

$$\mathcal{L}_{\text{int}} = [\bar{\psi}_1 \gamma_\mu (c_L P_L + c_R P_R) \psi_0][\bar{\psi}_3 \gamma^\mu (d_L P_L + d_R P_R) \psi_2] \qquad c_L, c_R, d_L, d_R \in \mathbb{R}$$

[KS, M.Spannowsky 2310.01477]

Vector

$$P_{R/L} = \frac{1 \pm \gamma^5}{2}$$

$$\mathcal{L}_{\rm int} = [\bar{\psi}_1 \gamma_\mu (c_L P_L + c_R P_R) \psi_0][\bar{\psi}_3 \gamma^\mu (d_L P_L + d_R P_R) \psi_2] \qquad c_L, c_R, d_L, d_R \in \mathbb{R}$$

$$\Rightarrow \quad |\Psi\rangle = M_{LL}|-+-\rangle + M_{LR}|--+\rangle + M_{RL}|++-\rangle + M_{RR}|+-+\rangle$$

[KS, M.Spannowsky 2310.01477]

Vector

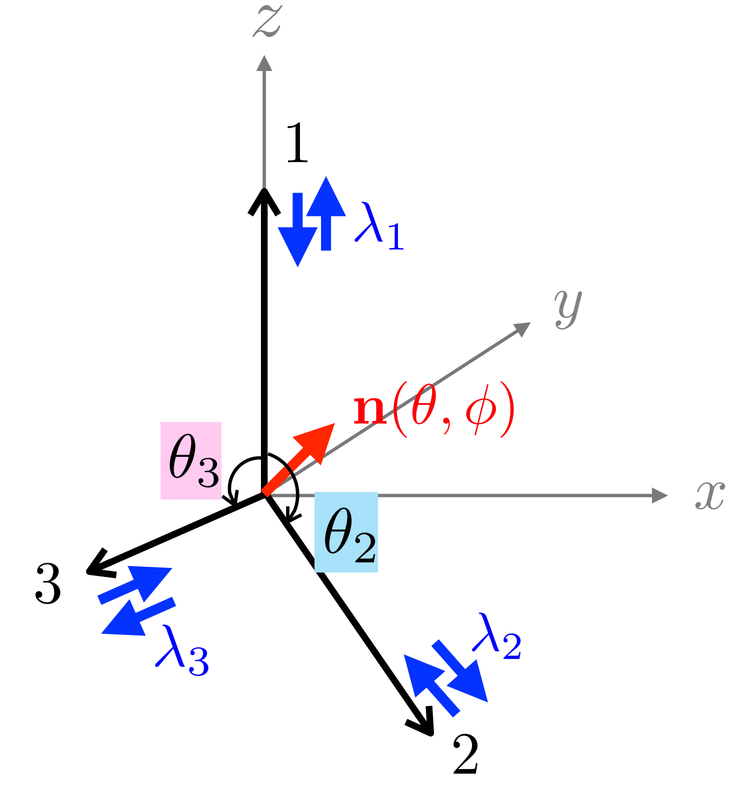
$$P_{R/L} = \frac{1 \pm \gamma^5}{2}$$

$$\mathcal{L}_{\rm int} = [\bar{\psi}_1 \gamma_\mu (c_L P_L + c_R P_R) \psi_0][\bar{\psi}_3 \gamma^\mu (d_L P_L + d_R P_R) \psi_2] \qquad c_L, c_R, d_L, d_R \in \mathbb{R}$$

$$\Rightarrow \; |\Psi\rangle = M_{LL}|-+-\rangle + M_{LR}|--+\rangle + M_{RL}|++-\rangle + M_{RR}|+-+\rangle$$

$$\propto \; c_L d_L s_{\frac{\theta_3}{2}} \left[c_{\frac{\theta}{2}} c_{\frac{\theta_2}{2}} + e^{i\phi} s_{\frac{\theta}{2}} s_{\frac{\theta_2}{2}} \right] |-+-\rangle \; + \; c_L d_R s_{\frac{\theta_2}{2}} \left[c_{\frac{\theta}{2}} c_{\frac{\theta_3}{2}} + e^{i\phi} s_{\frac{\theta}{2}} s_{\frac{\theta_3}{2}} \right] |--+\rangle$$

$$+ \; c_R d_L s_{\frac{\theta_2}{2}} \left[c_{\frac{\theta}{2}} s_{\frac{\theta_3}{2}} - e^{i\phi} s_{\frac{\theta}{2}} c_{\frac{\theta_3}{2}} \right] |++-\rangle \; + \; c_R d_R s_{\frac{\theta_3}{2}} \left[c_{\frac{\theta}{2}} s_{\frac{\theta_2}{2}} - e^{i\phi} s_{\frac{\theta}{2}} c_{\frac{\theta_2}{2}} \right] |+-+\rangle$$



[KS, M.Spannowsky 2310.01477]

Vector

$$P_{R/L} = \frac{1 \pm \gamma^5}{2}$$

$$\mathcal{L}_{\text{int}} = [\bar{\psi}_1 \gamma_\mu (c_L P_L + c_R P_R) \psi_0][\bar{\psi}_3 \gamma^\mu (d_L P_L + d_R P_R) \psi_2] \qquad c_L, c_R, d_L, d_R \in \mathbb{R}$$

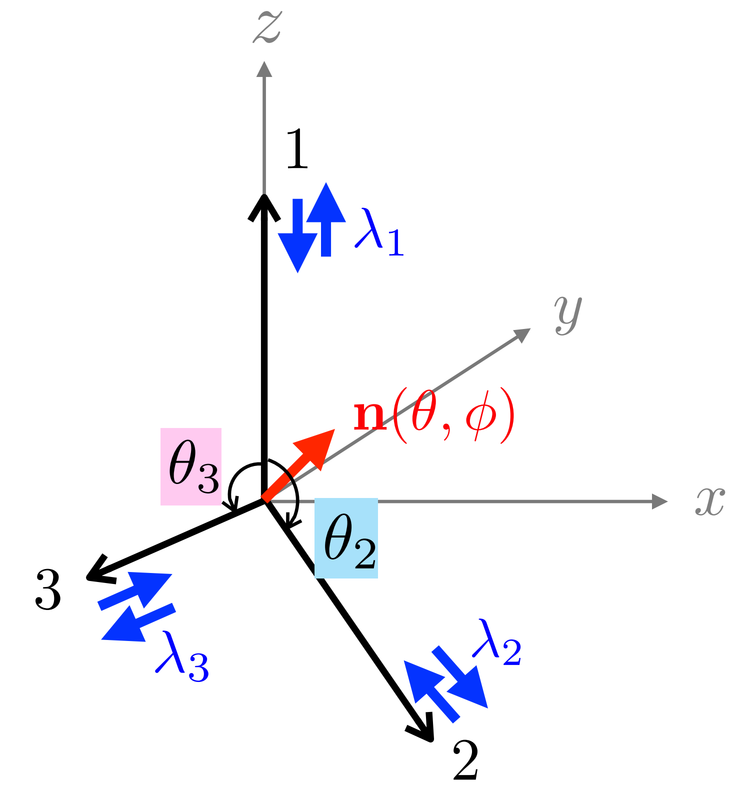
$$\Rightarrow \quad |\Psi\rangle = M_{LL}|-+-\rangle + M_{LR}|--+\rangle + M_{RL}|++-\rangle + M_{RR}|+-+\rangle$$

$$\propto \; c_L d_L s_{\frac{\theta_3}{2}} \left[c_{\frac{\theta}{2}} c_{\frac{\theta_2}{2}} + e^{i\phi} s_{\frac{\theta}{2}} s_{\frac{\theta_2}{2}}\right] |-+-\rangle + c_L d_R s_{\frac{\theta_2}{2}} \left[c_{\frac{\theta}{2}} c_{\frac{\theta_3}{2}} + e^{i\phi} s_{\frac{\theta}{2}} s_{\frac{\theta_3}{2}}\right] |--+\rangle$$

$$+ \; c_R d_L s_{\frac{\theta_2}{2}} \left[c_{\frac{\theta}{2}} s_{\frac{\theta_3}{2}} - e^{i\phi} s_{\frac{\theta}{2}} c_{\frac{\theta_3}{2}}\right] |++-\rangle + c_R d_R s_{\frac{\theta_3}{2}} \left[c_{\frac{\theta}{2}} s_{\frac{\theta_2}{2}} - e^{i\phi} s_{\frac{\theta}{2}} c_{\frac{\theta_2}{2}}\right] |+-+\rangle$$

✤ Individual 2-party entanglement:

$$\mathcal{C}_{12} = \mathcal{C}_{13} = 0\,, \quad \mathcal{C}_{23} = 2|M_{LL}M^*_{LR} + M_{RL}M^*_{RR}|$$



[KS, M.Spannowsky 2310.01477]

Vector

$$P_{R/L} = \frac{1 \pm \gamma^5}{2}$$

$$\mathcal{L}_{\rm int} = [\bar{\psi}_1 \gamma_\mu (c_L P_L + c_R P_R) \psi_0][\bar{\psi}_3 \gamma^\mu (d_L P_L + d_R P_R) \psi_2] \qquad c_L, c_R, d_L, d_R \in \mathbb{R}$$

$$\Rightarrow \quad |\Psi\rangle = M_{LL}|-+-\rangle + M_{LR}|--+\rangle + M_{RL}|++-\rangle + M_{RR}|+-+\rangle$$

$$\propto \quad c_L d_L s_{\frac{\theta_3}{2}} \left[c_{\frac{\theta}{2}} c_{\frac{\theta_2}{2}} + e^{i\phi} s_{\frac{\theta}{2}} s_{\frac{\theta_2}{2}} \right] |-+-\rangle \;+\; c_L d_R s_{\frac{\theta_2}{2}} \left[c_{\frac{\theta}{2}} c_{\frac{\theta_3}{2}} + e^{i\phi} s_{\frac{\theta}{2}} s_{\frac{\theta_3}{2}} \right] |--+\rangle$$

$$+ \quad c_R d_L s_{\frac{\theta_2}{2}} \left[c_{\frac{\theta}{2}} s_{\frac{\theta_3}{2}} - e^{i\phi} s_{\frac{\theta}{2}} c_{\frac{\theta_3}{2}} \right] |++-\rangle \;+\; c_R d_R s_{\frac{\theta_3}{2}} \left[c_{\frac{\theta}{2}} s_{\frac{\theta_2}{2}} - e^{i\phi} s_{\frac{\theta}{2}} c_{\frac{\theta_2}{2}} \right] |+-+\rangle$$

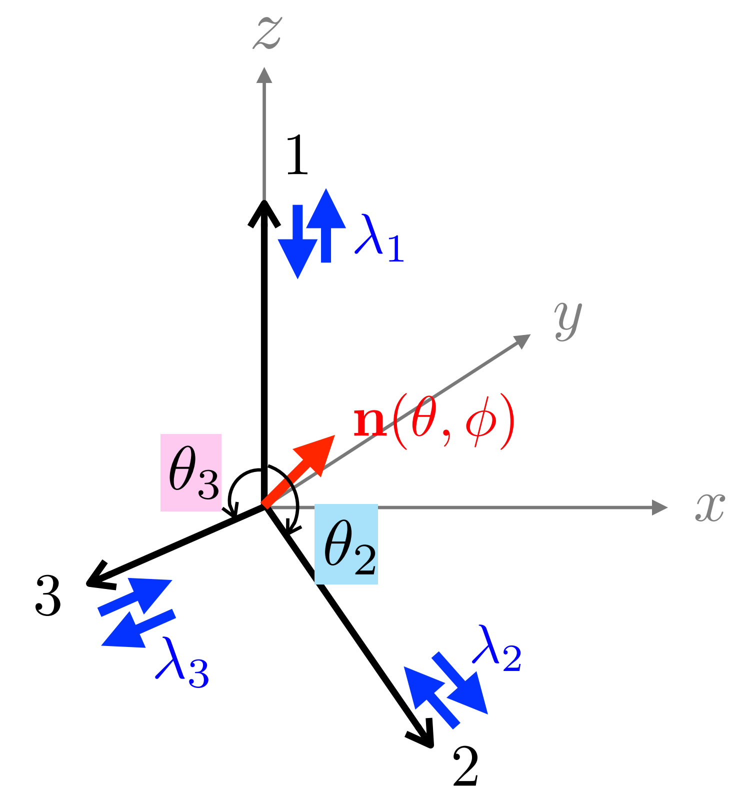
✤ **Individual 2-party entanglement:**

$$\mathcal{C}_{12} = \mathcal{C}_{13} = 0\,, \quad \mathcal{C}_{23} = 2|M_{LL}M_{LR}^* + M_{RL}M_{RR}^*|$$

✤ **one-to-other entanglement:**

$$\mathcal{C}_{2(13)} = \mathcal{C}_{3(12)} = 2\sqrt{(|M_{LL}|^2 + |M_{RL}|^2)\,(|M_{LR}|^2 + |M_{RR}|^2)}$$

$$\mathcal{C}_{1(23)} = 2\big|M_{RR}M_{LL} - M_{LR}M_{RL}\big|$$



[KS, M.Spannowsky 2310.01477]

Vector

$$P_{R/L} = \frac{1 \pm \gamma^5}{2}$$

$$\mathcal{L}_{\rm int} = [\bar{\psi}_1 \gamma_\mu (c_L P_L + c_R P_R) \psi_0][\bar{\psi}_3 \gamma^\mu (d_L P_L + d_R P_R) \psi_2] \qquad c_L, c_R, d_L, d_R \in \mathbb{R}$$

$$\Rightarrow \; |\Psi\rangle = M_{LL}|-+-\rangle + M_{LR}|--+\rangle + M_{RL}|++-\rangle + M_{RR}|+-+\rangle$$

$$\propto \; c_L d_L s_{\frac{\theta_3}{2}} \left[c_{\frac{\theta}{2}} c_{\frac{\theta_2}{2}} + e^{i\phi} s_{\frac{\theta}{2}} s_{\frac{\theta_2}{2}}\right] |-+-\rangle \; + \; c_L d_R s_{\frac{\theta_2}{2}} \left[c_{\frac{\theta}{2}} c_{\frac{\theta_3}{2}} + e^{i\phi} s_{\frac{\theta}{2}} s_{\frac{\theta_3}{2}}\right] |--+\rangle$$

$$+ \; c_R d_L s_{\frac{\theta_2}{2}} \left[c_{\frac{\theta}{2}} s_{\frac{\theta_3}{2}} - e^{i\phi} s_{\frac{\theta}{2}} c_{\frac{\theta_3}{2}}\right] |++-\rangle \; + \; c_R d_R s_{\frac{\theta_3}{2}} \left[c_{\frac{\theta}{2}} s_{\frac{\theta_2}{2}} - e^{i\phi} s_{\frac{\theta}{2}} c_{\frac{\theta_2}{2}}\right] |+-+\rangle$$

✤ **Individual 2-party entanglement:**

$$\mathcal{C}_{12} = \mathcal{C}_{13} = 0\,, \quad \mathcal{C}_{23} = 2|M_{LL}M^*_{LR} + M_{RL}M^*_{RR}|$$

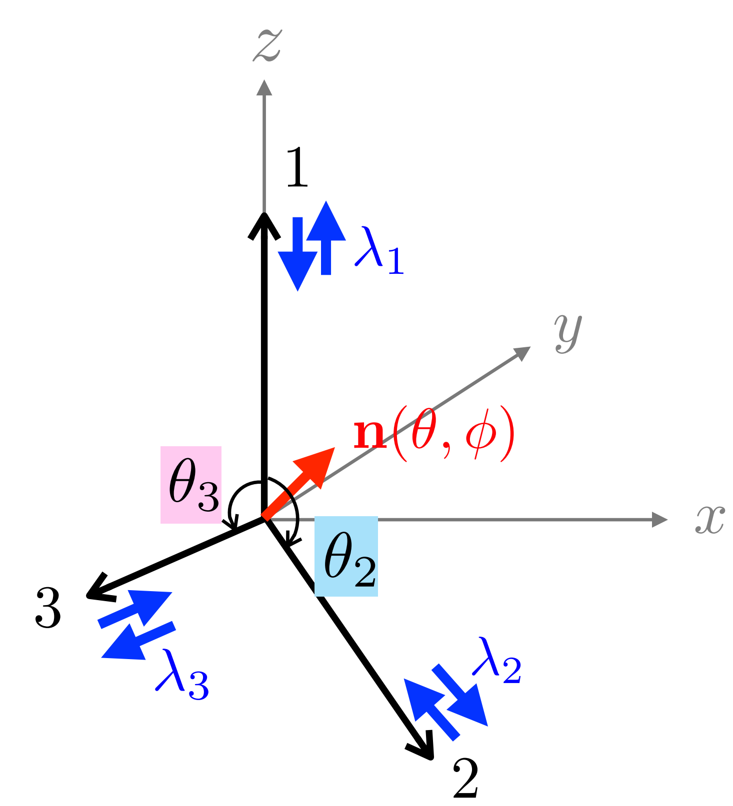
✤ **one-to-other entanglement:**

$$\mathcal{C}_{2(13)} = \mathcal{C}_{3(12)} = 2\sqrt{\left(|M_{LL}|^2 + |M_{RL}|^2\right)\left(|M_{LR}|^2 + |M_{RR}|^2\right)}$$

$$\mathcal{C}_{1(23)} = 2\big|M_{RR}M_{LL} - M_{LR}M_{RL}\big|$$

✤ **Monogamy**

$$M_i \equiv \mathcal{C}^2_{i(jk)} - [\mathcal{C}^2_{ij} + \mathcal{C}^2_{ik}] \;\; \Rightarrow \;\; M_1 = M_2 = M_3 = \mathcal{C}^2_{1(23)} \geq 0$$



[KS, M.Spannowsky 2310.01477]

Vector

$$P_{R/L} = \frac{1 \pm \gamma^5}{2}$$

$$\mathcal{L}_{\rm int} = [\bar{\psi}_1 \gamma_\mu (c_L P_L + c_R P_R) \psi_0][\bar{\psi}_3 \gamma^\mu (d_L P_L + d_R P_R) \psi_2] \qquad c_L, c_R, d_L, d_R \in \mathbb{R}$$

$$\Rightarrow \; |\Psi\rangle = M_{LL}|-+-\rangle + M_{LR}|--+\rangle + M_{RL}|++-\rangle + M_{RR}|+-+\rangle$$

$$\propto \; c_L d_L s_{\frac{\theta_3}{2}} \left[c_{\frac{\theta}{2}} c_{\frac{\theta_2}{2}} + e^{i\phi} s_{\frac{\theta}{2}} s_{\frac{\theta_2}{2}}\right] |-+-\rangle \; + \; c_L d_R s_{\frac{\theta_2}{2}} \left[c_{\frac{\theta}{2}} c_{\frac{\theta_3}{2}} + e^{i\phi} s_{\frac{\theta}{2}} s_{\frac{\theta_3}{2}}\right] |--+\rangle$$

$$+ \; c_R d_L s_{\frac{\theta_2}{2}} \left[c_{\frac{\theta}{2}} s_{\frac{\theta_3}{2}} - e^{i\phi} s_{\frac{\theta}{2}} c_{\frac{\theta_3}{2}}\right] |++-\rangle \; + \; c_R d_R s_{\frac{\theta_3}{2}} \left[c_{\frac{\theta}{2}} s_{\frac{\theta_2}{2}} - e^{i\phi} s_{\frac{\theta}{2}} c_{\frac{\theta_2}{2}}\right] |+-+\rangle$$

- Individual 2-party entanglement:

$$\mathcal{C}_{12} = \mathcal{C}_{13} = 0\,, \quad \mathcal{C}_{23} = 2|M_{LL}M^*_{LR} + M_{RL}M^*_{RR}| \quad \longleftarrow \text{vanish if } d_L d_R = 0$$

- one-to-other entanglement:

$$\mathcal{C}_{2(13)} = \mathcal{C}_{3(12)} = 2\sqrt{\left(|M_{LL}|^2 + |M_{RL}|^2\right)\left(|M_{LR}|^2 + |M_{RR}|^2\right)} \quad \longleftarrow \text{vanish if } c_L c_R = d_L d_R = 0$$

$$\mathcal{C}_{1(23)} = 2\big|M_{RR}M_{LL} - M_{LR}M_{RL}\big| \quad \longleftarrow \text{vanish if } c_L c_R d_L d_R = 0$$

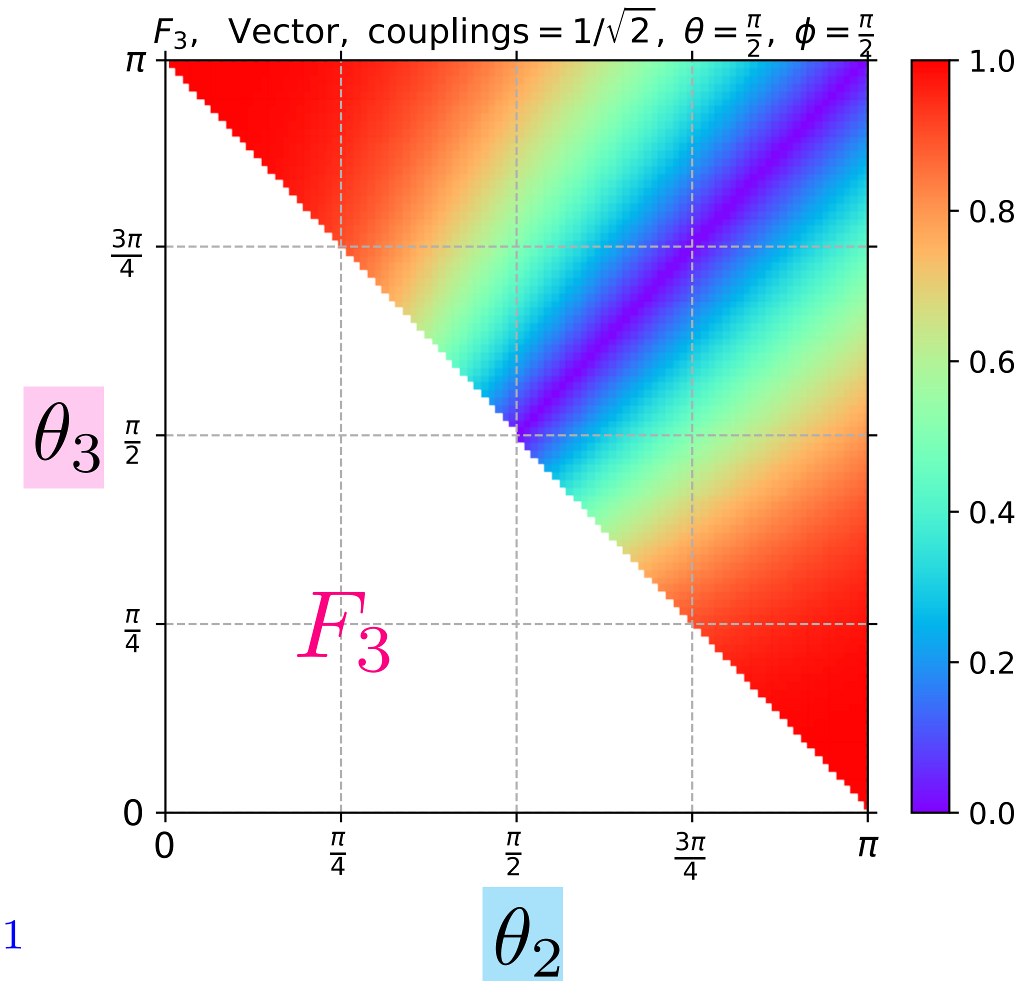
- Monogamy

$$M_i \equiv \mathcal{C}^2_{i(jk)} - [\mathcal{C}^2_{ij} + \mathcal{C}^2_{ik}] \; \Rightarrow \; M_1 = M_2 = M_3 = \mathcal{C}^2_{1(23)} \geq 0$$

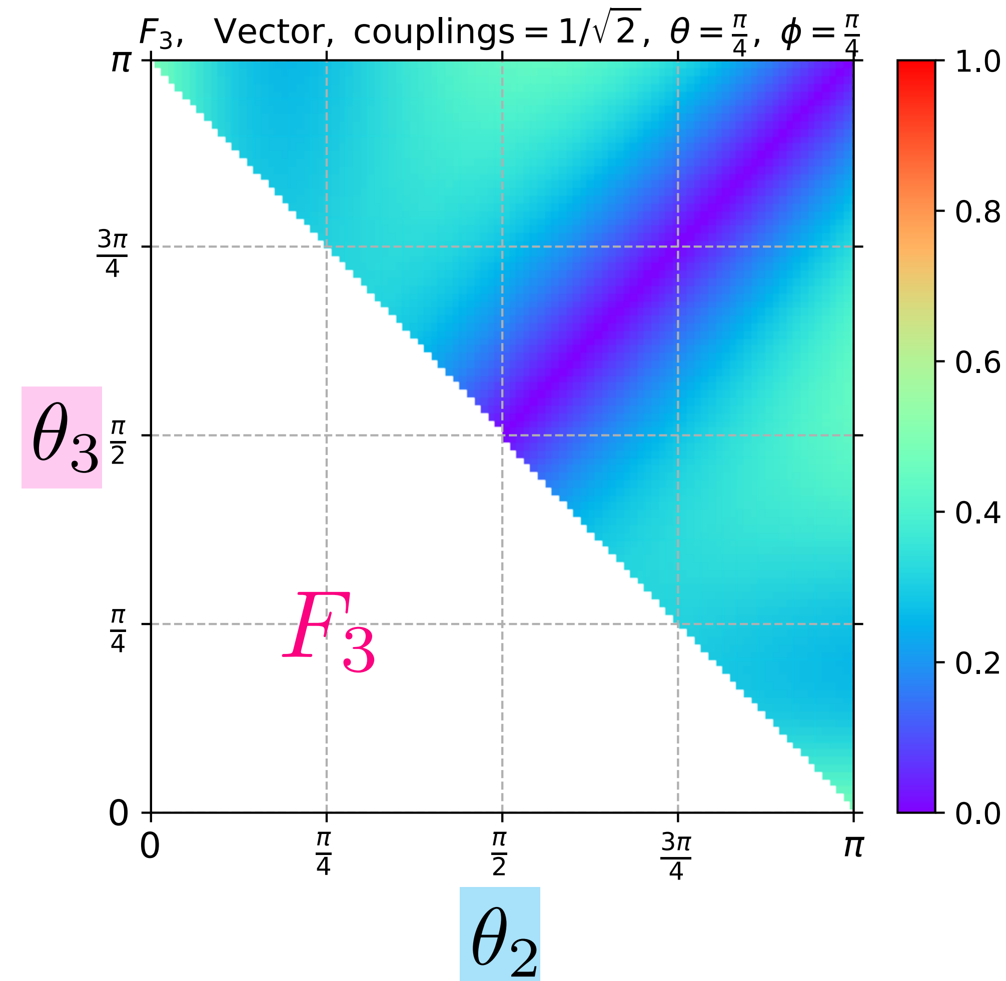
➡ **All** entanglements **vanish** for **weak decays** $c_R = d_R = 0$

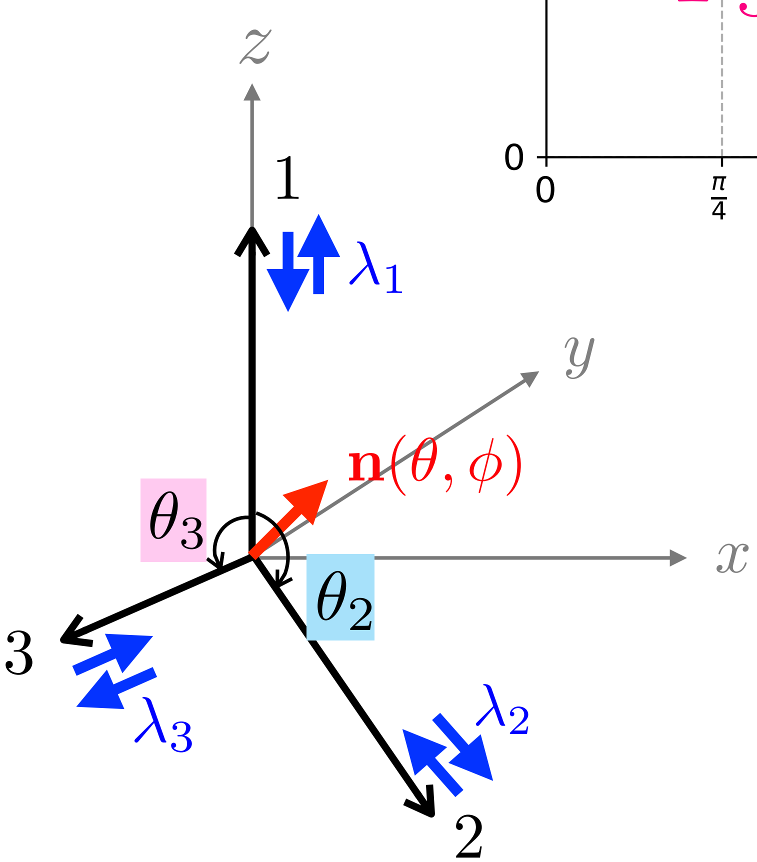
F_3 for Vector

$$\mathbf{n} = \mathbf{e}_y$$

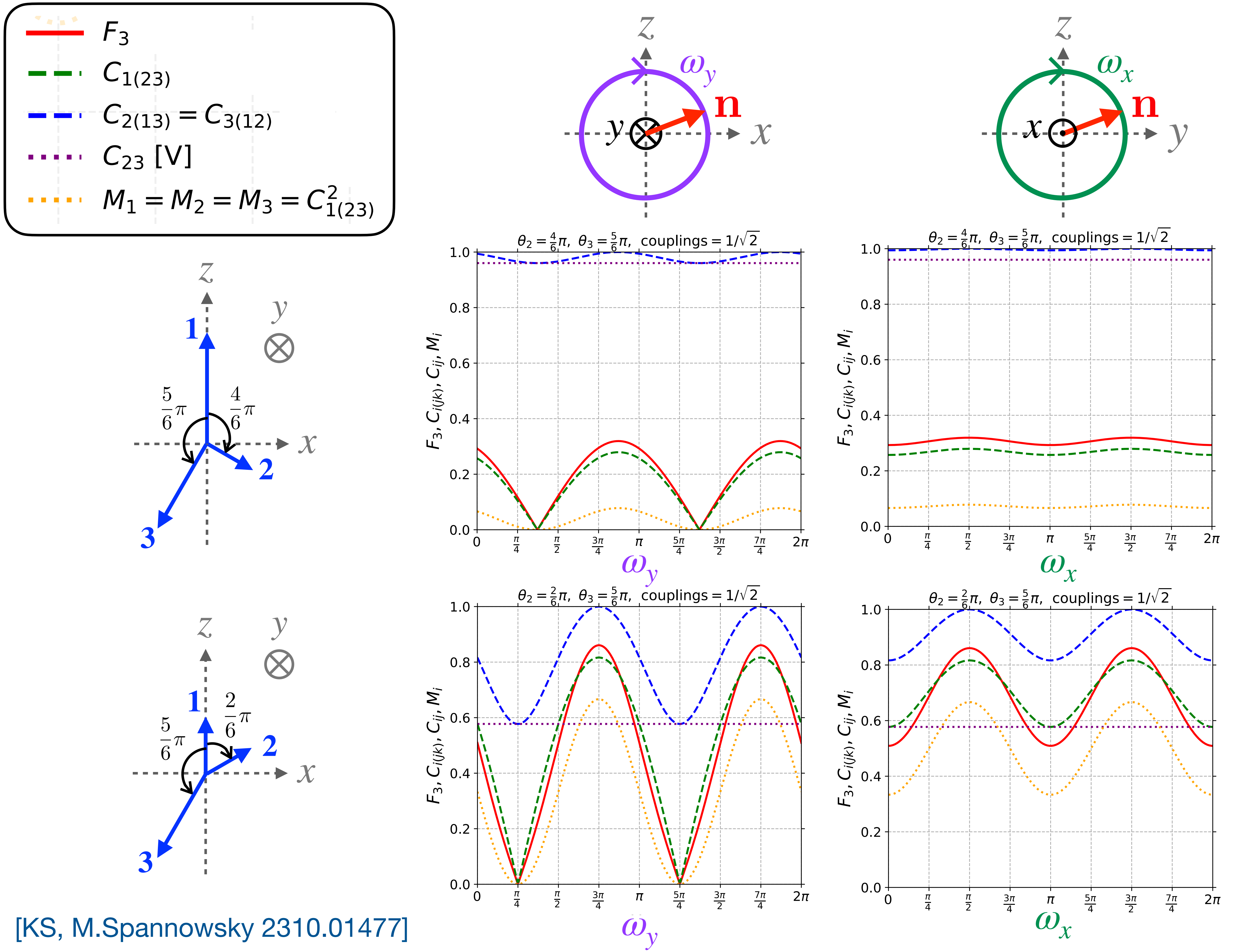


$$\mathbf{n} = \frac{1}{\sqrt{3}}(\mathbf{e}_x + \mathbf{e}_y + \mathbf{e}_z)$$





[KS, M.Spannowsky 2310.01477]



[KS, M.Spannowsky 2310.01477]

[KS, M.Spannowsky
2310.01477]

Tensor

$$\mathcal{L}_{\rm int} = [\bar{\psi}_1(c_M + ic_E\gamma_5)\sigma^{\mu\nu}\psi_0][\bar{\psi}_3(d_M + id_E\gamma_5)\sigma_{\mu\nu}\psi_2]$$

$$c \equiv c_M + ic_E = e^{i\omega_1}$$
$$d \equiv d_M + id_E = e^{i\omega_2}$$

[KS, M.Spannowsky 2310.01477]

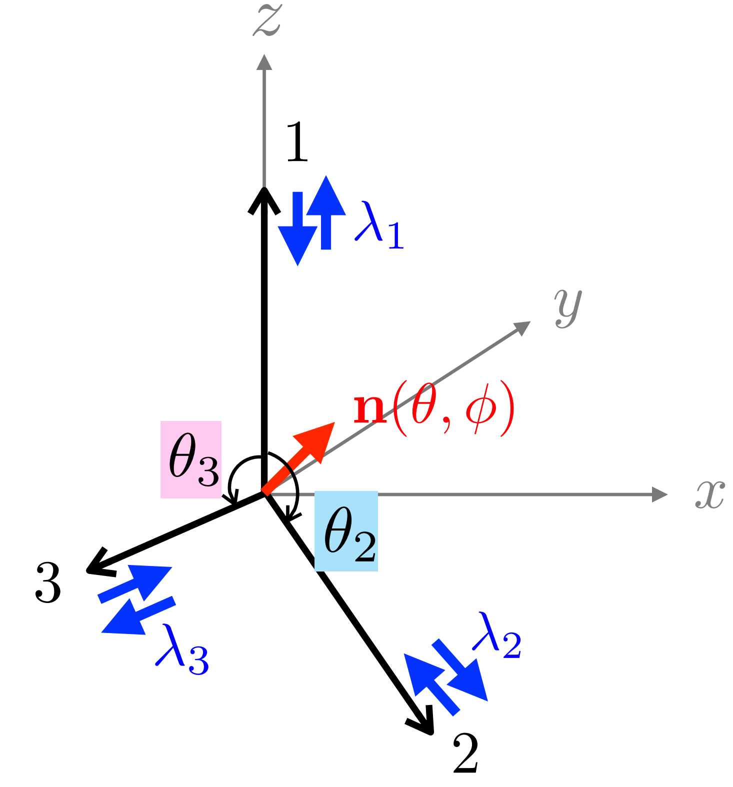
Tensor

$$\mathcal{L}_{\rm int} = [\bar{\psi}_1(c_M + ic_E\gamma_5)\sigma^{\mu\nu}\psi_0][\bar{\psi}_3(d_M + id_E\gamma_5)\sigma_{\mu\nu}\psi_2]$$

$$c \equiv c_M + ic_E = e^{i\omega_1}$$
$$d \equiv d_M + id_E = e^{i\omega_2}$$

$$|\Psi\rangle = M_R|+++\rangle + M_L|---\rangle$$

$$\propto c^* d^* \left[2e^{i\phi}s_{\frac{\theta}{2}}s_{\frac{\theta_2}{2}}s_{\frac{\theta_3}{2}} + c_{\frac{\theta}{2}}s_{\frac{\theta_3-\theta_2}{2}}\right]|+++\rangle + cd\left[-e^{i\phi}s_{\frac{\theta}{2}}s_{\frac{\theta_3-\theta_2}{2}} + 2c_{\frac{\theta}{2}}s_{\frac{\theta_2}{2}}s_{\frac{\theta_3}{2}}\right]|---\rangle$$



[KS, M.Spannowsky 2310.01477]

Tensor

$$\mathcal{L}_{\rm int} = [\bar\psi_1(c_M + i c_E\gamma_5)\sigma^{\mu\nu}\psi_0][\bar\psi_3(d_M + i d_E\gamma_5)\sigma_{\mu\nu}\psi_2]$$

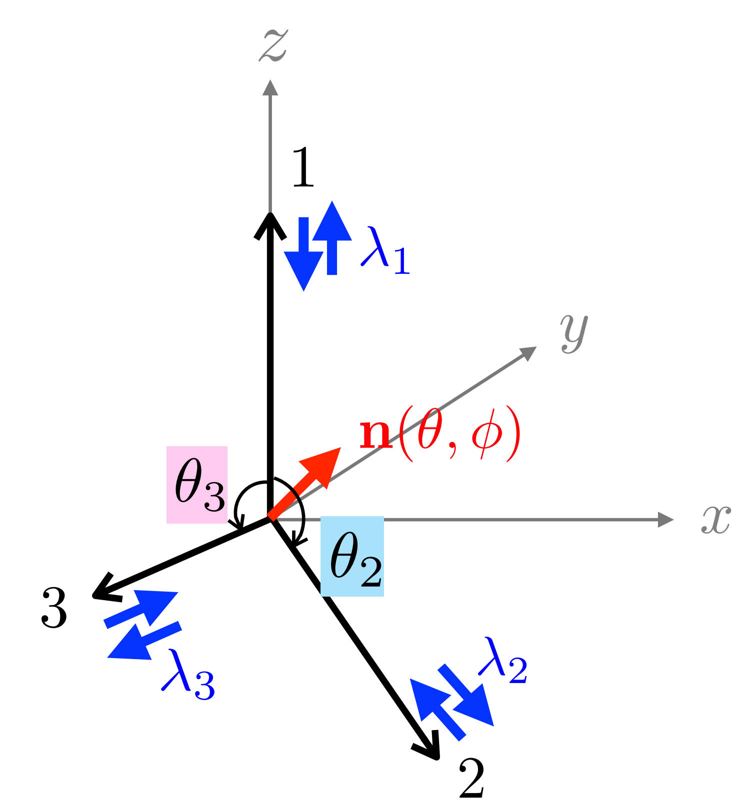
$$c \equiv c_M + i c_E = e^{i\omega_1}$$
$$d \equiv d_M + i d_E = e^{i\omega_2}$$

$$\Rightarrow \quad |\Psi\rangle = M_R|+++\rangle + M_L|---\rangle$$

$$\propto \; c^* d^* \left[2e^{i\phi} s_{\frac{\theta}{2}} s_{\frac{\theta_2}{2}} s_{\frac{\theta_3}{2}} + c_{\frac{\theta}{2}} s_{\frac{\theta_3-\theta_2}{2}}\right]|+++\rangle + cd\left[-e^{i\phi} s_{\frac{\theta}{2}} s_{\frac{\theta_3-\theta_2}{2}} + 2c_{\frac{\theta}{2}} s_{\frac{\theta_2}{2}} s_{\frac{\theta_3}{2}}\right]|---\rangle$$

- $\Psi\rangle$ *interpolates* **product states** and the **maximally entangled** state:

$$(M_R M_L = 0) \qquad |\pm\pm\pm\rangle \longleftrightarrow |{\rm GHZ}\rangle = \frac{1}{\sqrt{2}}(|+++\rangle + |---\rangle) \qquad (M_R = M_L)$$



[KS, M.Spannowsky 2310.01477]

Tensor

$$\mathcal{L}_{\rm int} = [\bar{\psi}_1(c_M + ic_E\gamma_5)\sigma^{\mu\nu}\psi_0][\bar{\psi}_3(d_M + id_E\gamma_5)\sigma_{\mu\nu}\psi_2]$$

$$c \equiv c_M + ic_E = e^{i\omega_1}$$
$$d \equiv d_M + id_E = e^{i\omega_2}$$

$$\Rightarrow \quad |\Psi\rangle = M_R|+++\rangle + M_L|---\rangle$$

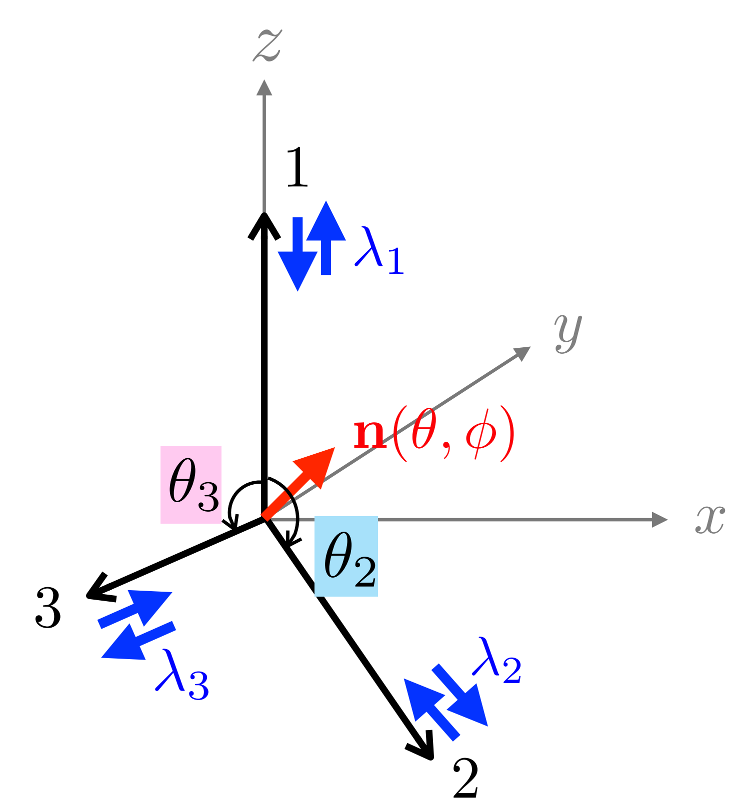
$$\propto \; c^*d^*\,[2e^{i\phi}s_{\frac{\theta}{2}}s_{\frac{\theta_2}{2}}s_{\frac{\theta_3}{2}} + c_{\frac{\theta}{2}}s_{\frac{\theta_3-\theta_2}{2}}]\,|+++\rangle + cd\,[-e^{i\phi}s_{\frac{\theta}{2}}s_{\frac{\theta_3-\theta_2}{2}} + 2c_{\frac{\theta}{2}}s_{\frac{\theta_2}{2}}s_{\frac{\theta_3}{2}}]\,|---\rangle$$

- $\Psi\rangle$ *interpolates* **product states** and the **maximally entangled** state:

$$(M_RM_L = 0) \qquad |\pm\pm\pm\rangle \longleftrightarrow |{\rm GHZ}\rangle = \frac{1}{\sqrt{2}}(|+++\rangle + |---\rangle) \qquad (M_R = M_L)$$

- **No** individual 2-party entanglements:

$$\mathcal{C}_{12} = \mathcal{C}_{13} = \mathcal{C}_{23} = 0$$



[KS, M.Spannowsky 2310.01477]

Tensor

$$\mathcal{L}_{\rm int} = [\bar{\psi}_1(c_M + ic_E\gamma_5)\sigma^{\mu\nu}\psi_0][\bar{\psi}_3(d_M + id_E\gamma_5)\sigma_{\mu\nu}\psi_2]$$

$$c \equiv c_M + ic_E = e^{i\omega_1}$$
$$d \equiv d_M + id_E = e^{i\omega_2}$$

$$\Rightarrow \quad |\Psi\rangle = M_R|+++\rangle + M_L|---\rangle$$

$$\propto c^* d^* [2e^{i\phi} s_{\frac{\theta}{2}} s_{\frac{\theta_2}{2}} s_{\frac{\theta_3}{2}} + c_{\frac{\theta}{2}} s_{\frac{\theta_3-\theta_2}{2}}]\,|+++\rangle + cd\,[-e^{i\phi} s_{\frac{\theta}{2}} s_{\frac{\theta_3-\theta_2}{2}} + 2c_{\frac{\theta}{2}} s_{\frac{\theta_2}{2}} s_{\frac{\theta_3}{2}}]\,|---\rangle$$

✤ $\Psi\rangle$ *interpolates* **product states** and the **maximally entangled** state:

$$(M_R M_L = 0) \qquad |\pm\pm\pm\rangle \longleftrightarrow |{\rm GHZ}\rangle = \tfrac{1}{\sqrt{2}}(|+++\rangle + |---\rangle) \qquad (M_R = M_L)$$

✤ **No** individual 2-party entanglements:

Monogamy is trivial

$$\mathcal{C}_{12} = \mathcal{C}_{13} = \mathcal{C}_{23} = 0 \longrightarrow \mathcal{C}^2_{i(jk)} \geq \mathcal{C}^2_{ij} + \mathcal{C}^2_{ik} = 0$$

[KS, M.Spannowsky 2310.01477]

Tensor

$$\mathcal{L}_{\rm int} = [\bar{\psi}_1(c_M + ic_E\gamma_5)\sigma^{\mu\nu}\psi_0][\bar{\psi}_3(d_M + id_E\gamma_5)\sigma_{\mu\nu}\psi_2]$$

$$c \equiv c_M + ic_E = e^{i\omega_1}$$
$$d \equiv d_M + id_E = e^{i\omega_2}$$

$$\Rightarrow \quad |\Psi\rangle = M_R|+++\rangle + M_L|---\rangle$$

$$\propto \; c^* d^* \,[2e^{i\phi} s_{\frac{\theta}{2}} s_{\frac{\theta_2}{2}} s_{\frac{\theta_3}{2}} + c_{\frac{\theta}{2}} s_{\frac{\theta_3-\theta_2}{2}}]\,|+++\rangle + cd\,[-e^{i\phi} s_{\frac{\theta}{2}} s_{\frac{\theta_3-\theta_2}{2}} + 2c_{\frac{\theta}{2}} s_{\frac{\theta_2}{2}} s_{\frac{\theta_3}{2}}]\,|---\rangle$$

- $\Psi\rangle$ *interpolates* **product states** and the **maximally entangled** state:

$$(M_R M_L = 0) \qquad |\pm\pm\pm\rangle \longleftrightarrow |{\rm GHZ}\rangle = \frac{1}{\sqrt{2}}(|+++\rangle + |---\rangle) \qquad (M_R = M_L)$$

- **No** individual 2-party entanglements:

Monogamy is trivial

$$\mathcal{C}_{12} = \mathcal{C}_{13} = \mathcal{C}_{23} = 0 \longrightarrow \mathcal{C}^2_{i(jk)} \geq \mathcal{C}^2_{ij} + \mathcal{C}^2_{ik} = 0$$

- one-to-other entanglements are **universal**:

$$\mathcal{C}_{1(23)} = \mathcal{C}_{2(13)} = \mathcal{C}_{3(12)} = 2|M_R M_L|$$

$$F_3 = 4|M_R M_L|^2$$

[KS, M.Spannowsky 2310.01477]

Tensor

$$\mathcal{L}_{\rm int} = [\bar{\psi}_1(c_M + ic_E\gamma_5)\sigma^{\mu\nu}\psi_0][\bar{\psi}_3(d_M + id_E\gamma_5)\sigma_{\mu\nu}\psi_2]$$

$$c \equiv c_M + ic_E = e^{i\omega_1}$$
$$d \equiv d_M + id_E = e^{i\omega_2}$$

$$\Rightarrow \quad |\Psi\rangle = M_R|+++\rangle + M_L|---\rangle$$

$$\propto \; c^*d^*\,[2e^{i\phi}s_{\frac{\theta}{2}}s_{\frac{\theta_2}{2}}s_{\frac{\theta_3}{2}} + c_{\frac{\theta}{2}}s_{\frac{\theta_3-\theta_2}{2}}]\,|+++\rangle + cd\,[-e^{i\phi}s_{\frac{\theta}{2}}s_{\frac{\theta_3-\theta_2}{2}} + 2c_{\frac{\theta}{2}}s_{\frac{\theta_2}{2}}s_{\frac{\theta_3}{2}}]\,|---\rangle$$

✤ $\Psi\rangle$ *interpolates* **product states** and the **maximally entangled** state:

$$(M_RM_L = 0) \qquad |\pm\pm\pm\rangle \longleftrightarrow |{\rm GHZ}\rangle = \frac{1}{\sqrt{2}}(|+++\rangle + |---\rangle) \qquad (M_R = M_L)$$

✤ **No** individual 2-party entanglements: **Monogamy is trivial**

$$\mathcal{C}_{12} = \mathcal{C}_{13} = \mathcal{C}_{23} = 0 \longrightarrow \mathcal{C}^2_{i(jk)} \geq \mathcal{C}^2_{ij} + \mathcal{C}^2_{ik} = 0$$

✤ one-to-other entanglements are **universal**:

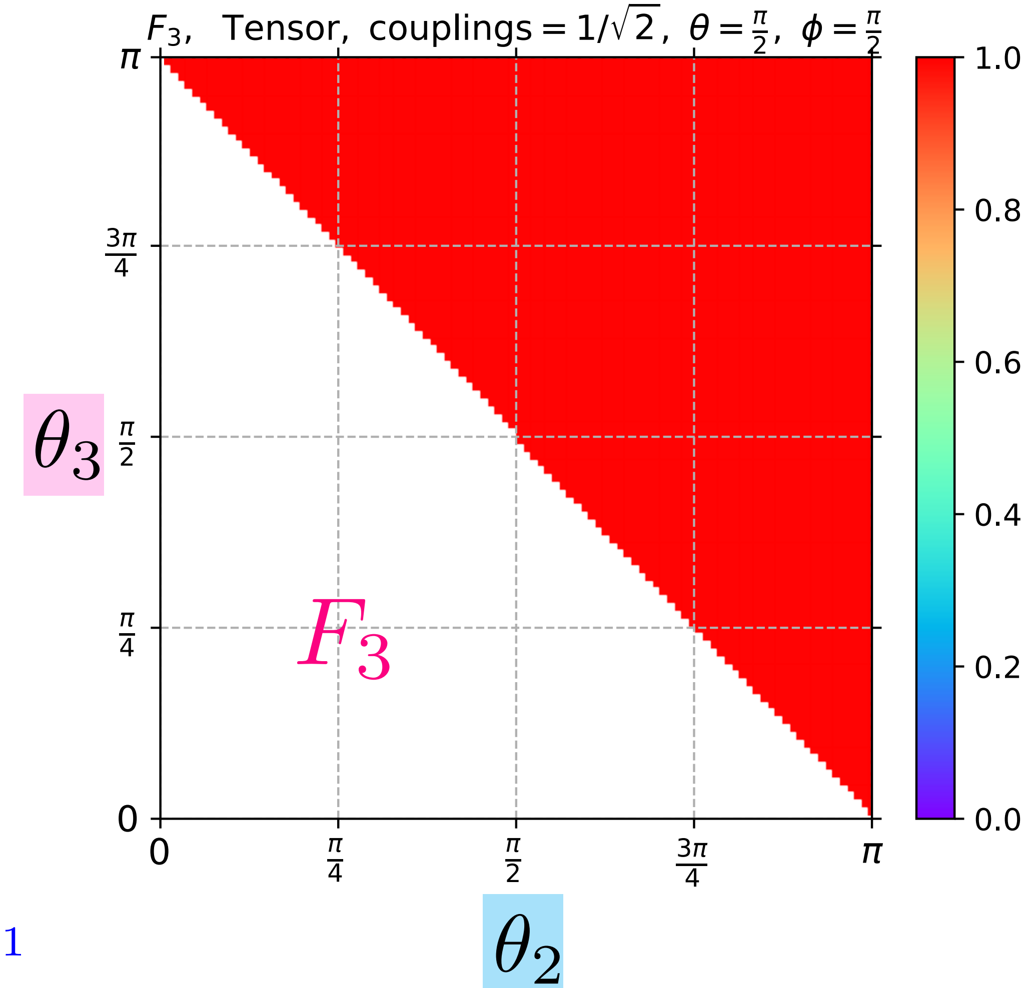
$$\mathcal{C}_{1(23)} = \mathcal{C}_{2(13)} = \mathcal{C}_{3(12)} = 2|M_RM_L|$$

$$F_3 = 4|M_RM_L|^2$$

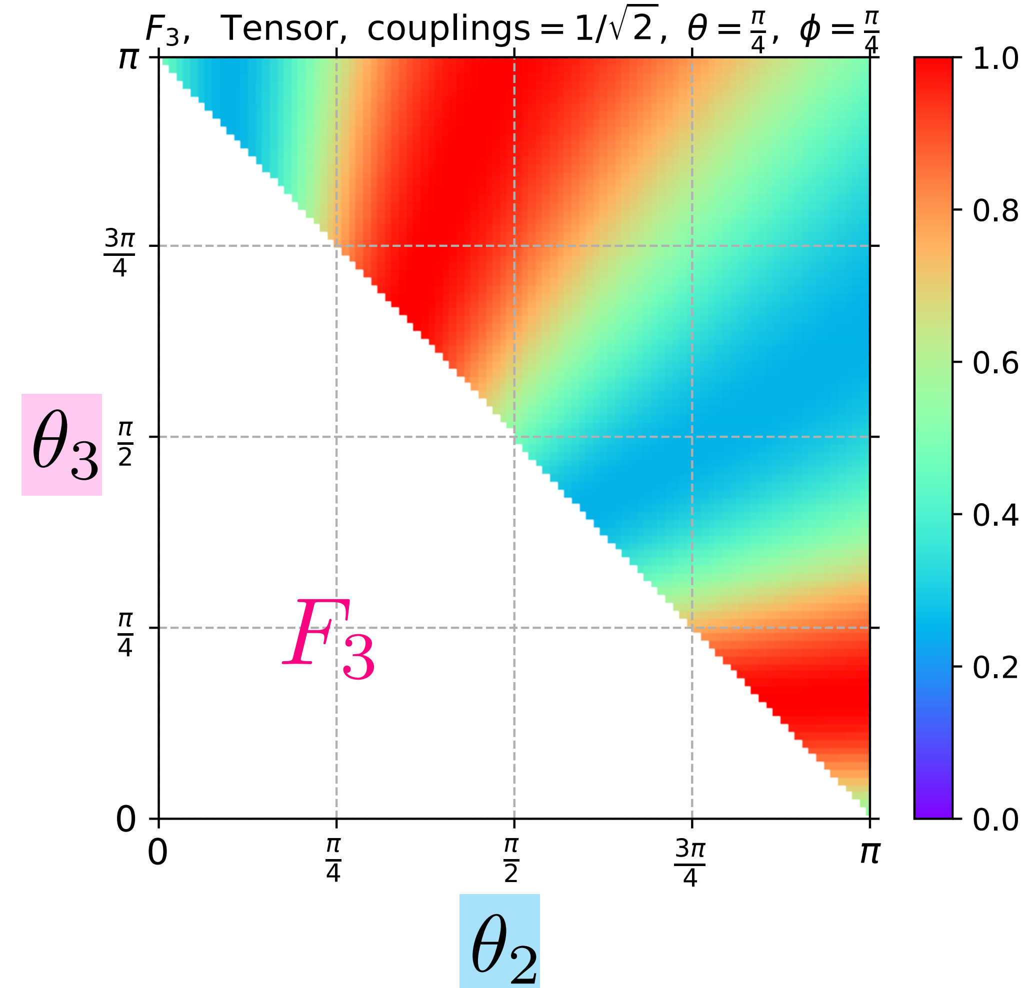
independent of the coupling structure (CP phases) ω_1, ω_2

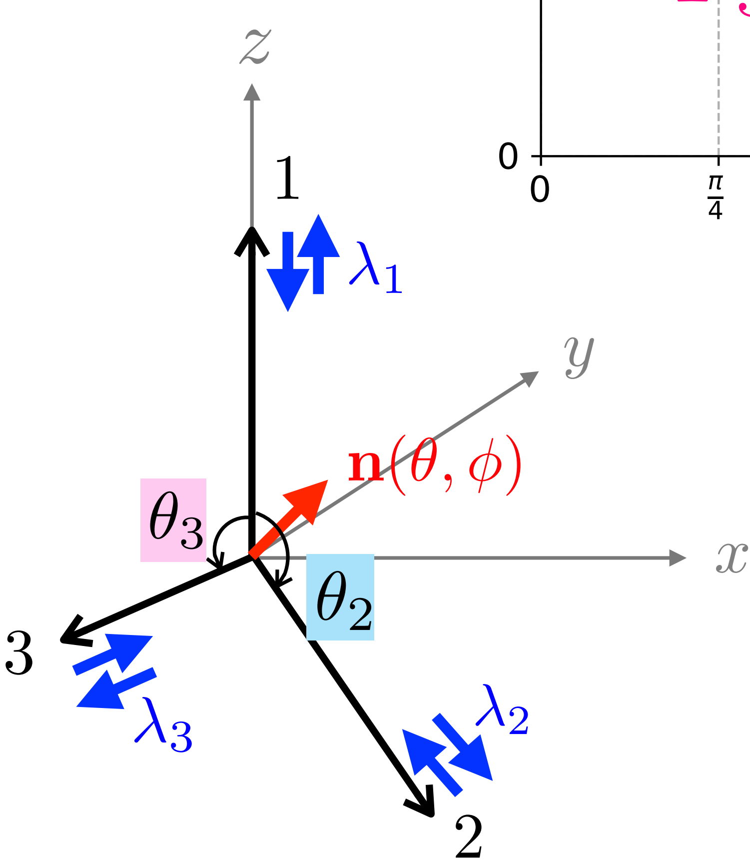
F_3 for Tensor

$$\mathbf{n} = \mathbf{e}_y$$



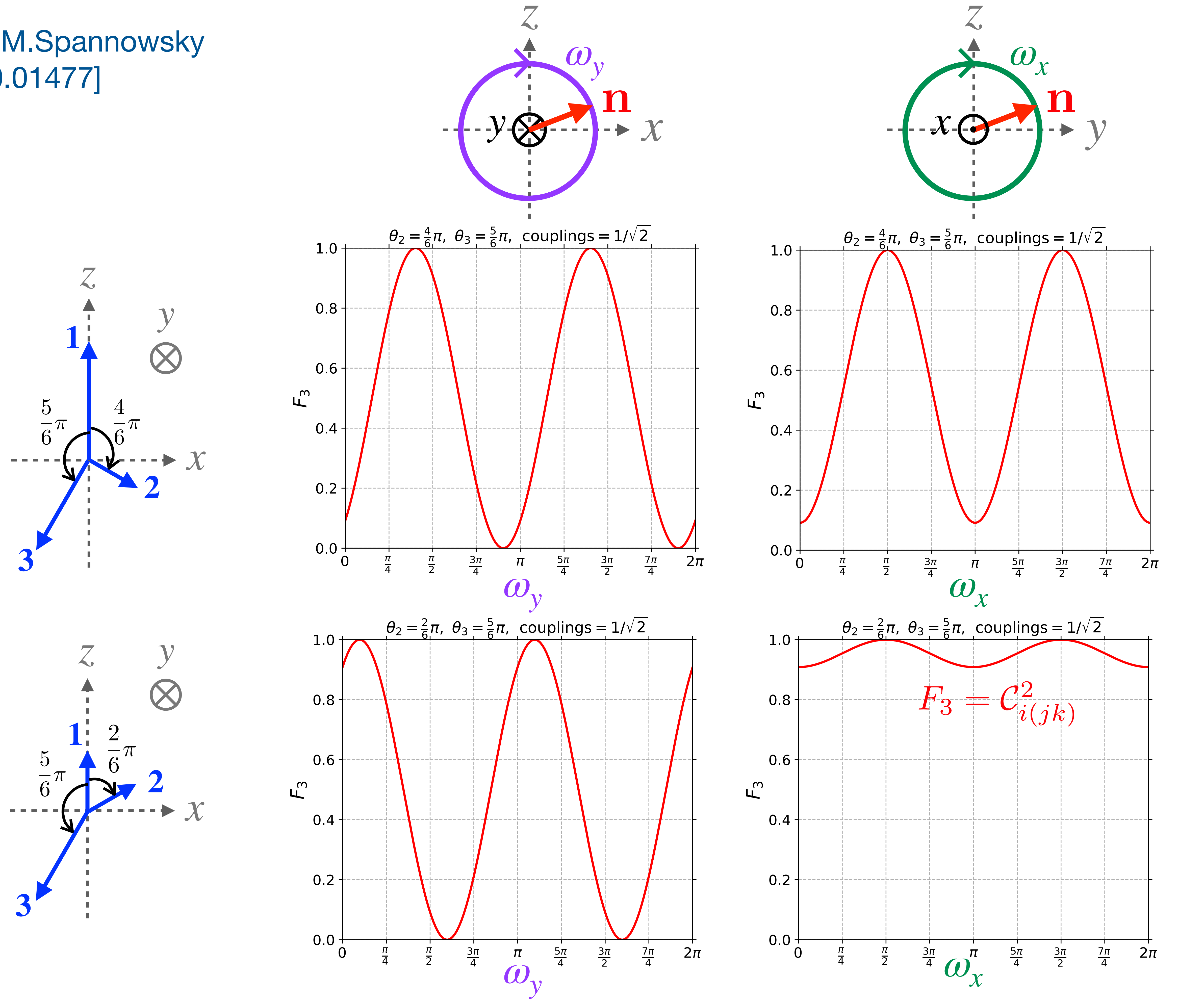
$$\mathbf{n} = \frac{1}{\sqrt{3}}(\mathbf{e}_x + \mathbf{e}_y + \mathbf{e}_z)$$





[KS, M.Spannowsky 2310.01477]

[KS, M.Spannowsky 2310.01477]



Discussion

What to do with it?

- ✤ look for **theories** to **maximise/minimise** the **entanglement** → **I.Low, J.I.Latorre**
- ✤ **measure**/study 3-body entanglements **experimentally** *e.g.* in **hadron decays**

$$\text{e.g.)} \quad \Xi^- \to p\mu^-\mu^-$$

Future directions:

- ✤ Effect of **masses** in the final particles
- ✤ More **spin structures**: $SFFV, VVFF, SFVF_{3/2}, SVVT \cdots$
- ✤ 3-body **non-locality** [Mermin '90, Svetlichny '87]

Mermin ineq: $\langle \mathcal{B}_{\rm M} \rangle_{\rm LR} \le 2 \quad \langle \mathcal{B}_{\rm M} \rangle_{\rm QM} \le 4$

Svetlichny ineq: $\langle \mathcal{B}_{\rm S} \rangle_{\rm HLR} \le 4 \quad \langle \mathcal{B}_{\rm S} \rangle_{\rm QM} \le 4\sqrt{2}$

$$\mathcal{B}_{\rm M} = abc' + ab'c + a'bc - a'b'c'$$

$$\mathcal{B}_{\rm S} = abc + abc' + ab'c + a'bc - a'b'c' - a'b'c - a'bc' - ab'c'$$

KS, Spannowsky, Horodecki, *in progress*

Thank you for listening!

Norway grants

NATIONAL SCIENCE CENTRE POLAND

The research leading to the results presented in this talk has received funding from the Norwegian Financial Mechanism for years 2014-2021, grant nr 2019/34/H/ST2/00707

Understanding the Early Universe:
interplay of theory and collider experiments

Joint research project between the University of Warsaw & University of Bergen

Local **R**eal Hidden Variable theories:

Mermin ineq:

$$P(abc|XYZ) = \sum q_\lambda P_\lambda(a|X) P_\lambda(b|Y) P_\lambda(c|Z) \longrightarrow \langle \mathcal{B}_{\mathrm{M}} \rangle_{\mathrm{LR}} \leq 2 \quad \langle \mathcal{B}_{\mathrm{M}} \rangle_{\mathrm{QM}} \leq 4$$

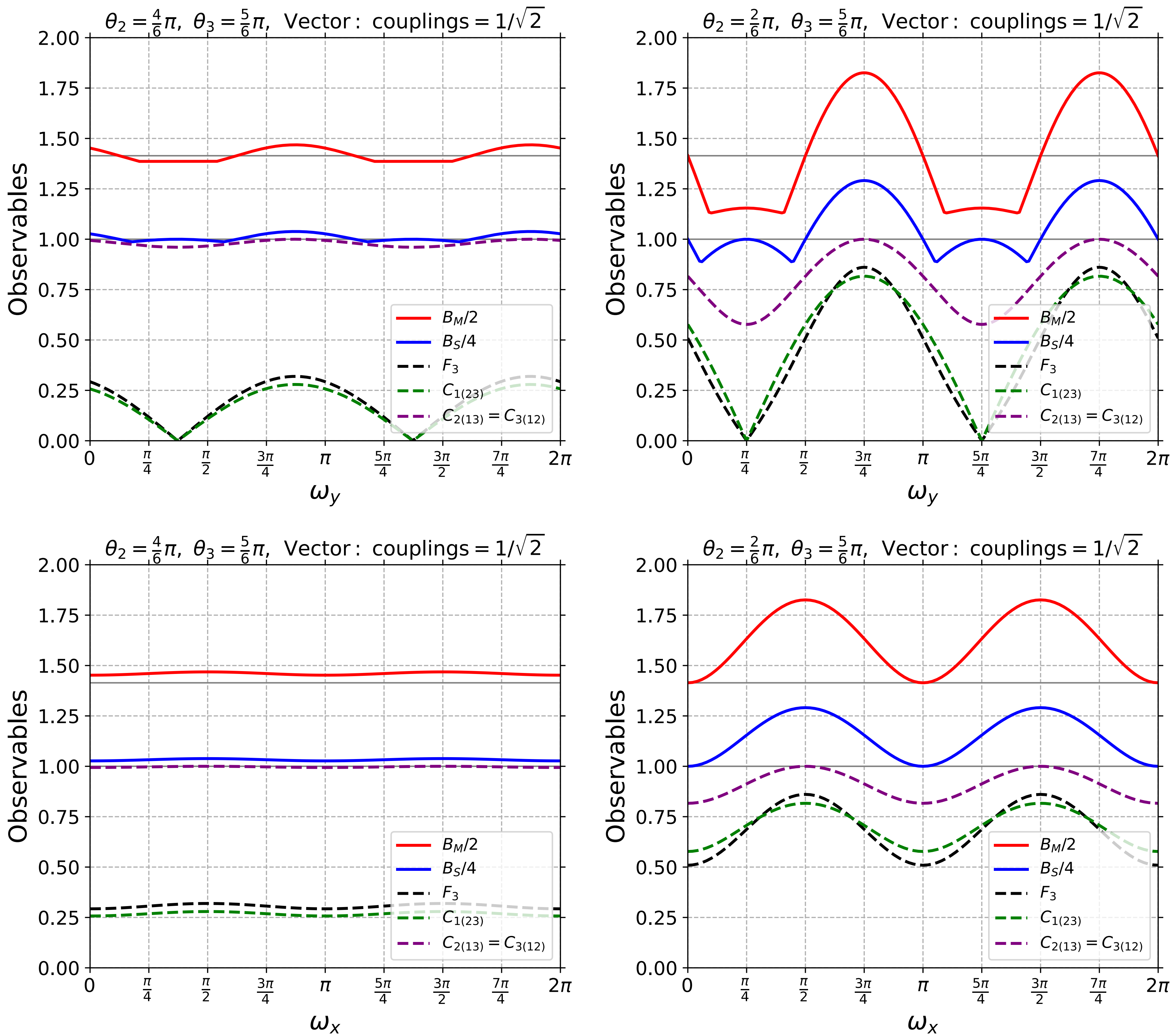
Hybrid (**L**ocal-Nonlocal) **R**eal theories:

$$P(abc|XYZ) = \sum_\lambda q_\lambda P_\lambda(ab|XY) P_\lambda(c|Z) + \sum_\mu q_\mu P_\mu(ac|XZ) P_\mu(b|Y) + \sum_v q_v P_v(bc|YZ) P_v(a|X)$$

$$\longrightarrow \langle \mathcal{B}_{\mathrm{S}} \rangle_{\mathrm{HLR}} \leq 4 \quad \langle \mathcal{B}_{\mathrm{S}} \rangle_{\mathrm{QM}} \leq 4\sqrt{2}$$

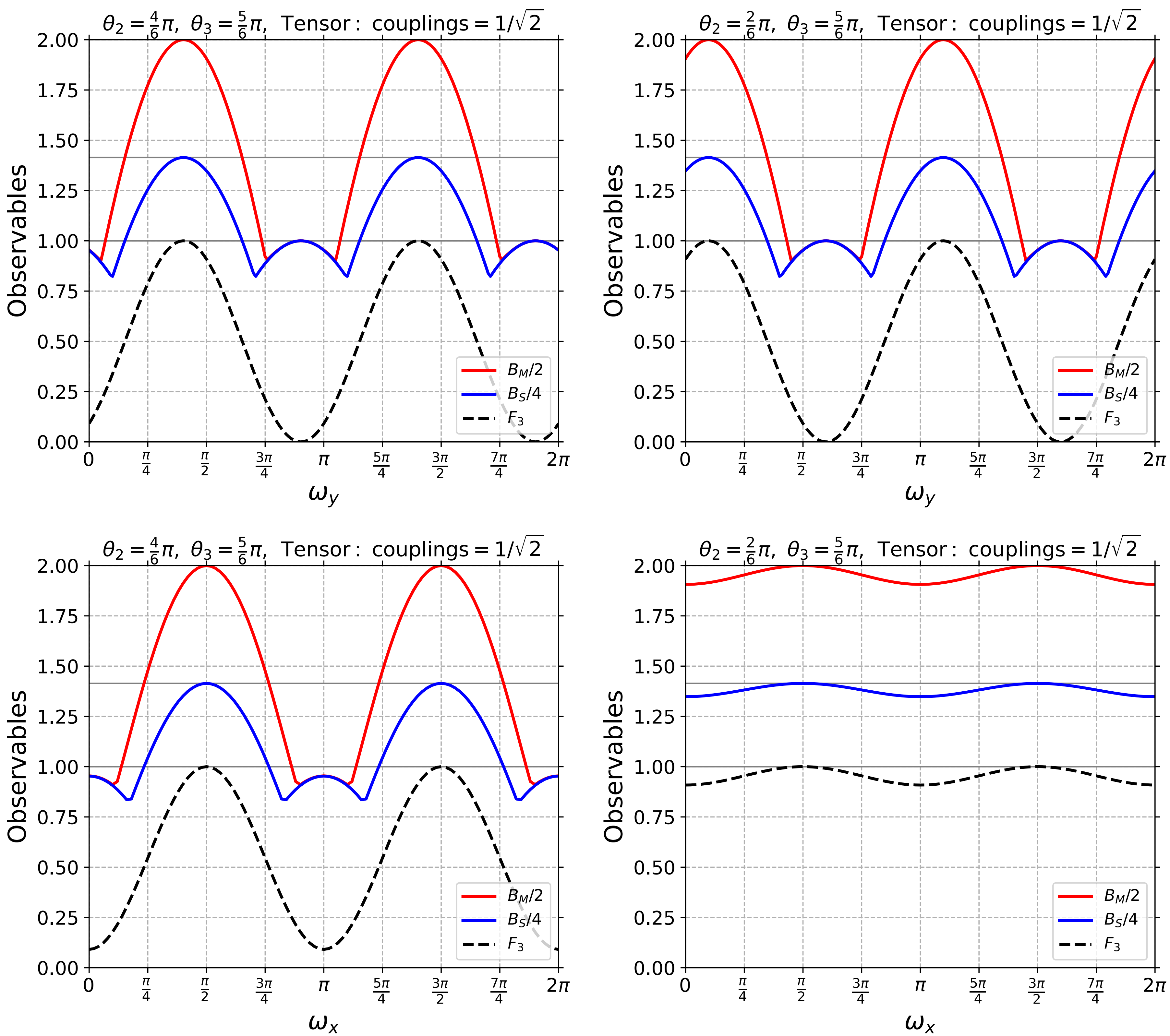
Svetlichny ineq

Nonlocality for Vector



[KS, Spannowsky, Horodecki, *in progress*]

Nonlocality for Tensor



[KS, Spannowsky, Horodecki, *in progress*]