Quantum Observables for Collider Physics

Quantum information & CP measurement in Higgs to tau tau at future lepton colliders

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Mohammad Mahdi Altakach

In collaboration with: **P. Lamba**, **F. Maltoni**, **K. Mawatari** and **K. Sakurai**

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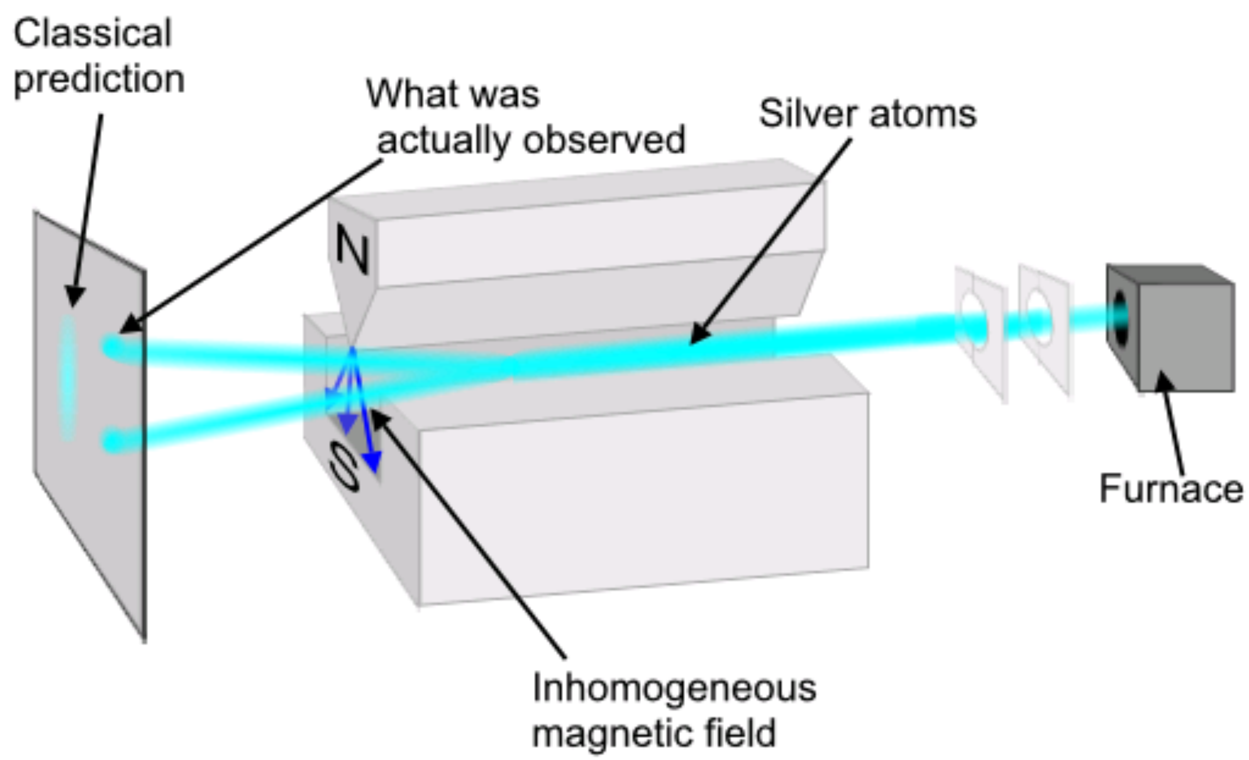
Part I

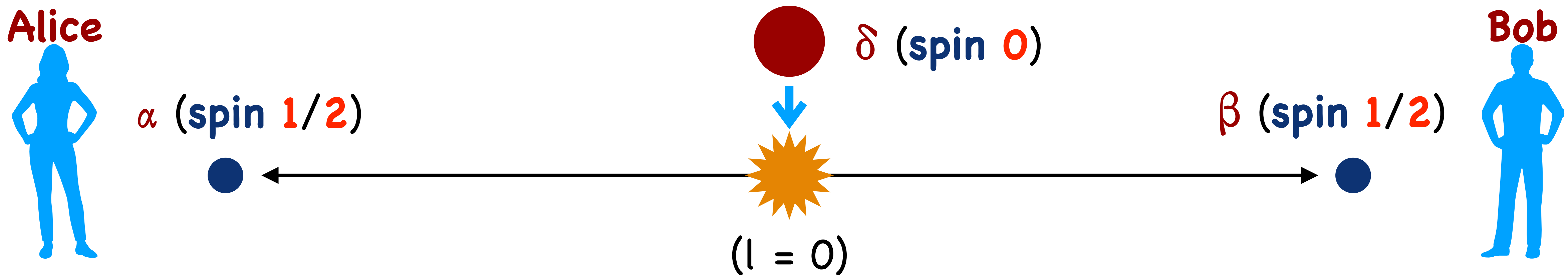
Introduction

Spin

- In **classical mechanics**, the components of **angular momentum** (l_x, l_y, l_z) take continuous real numbers.
- A striking fact, found in the **Stern-Gerlach** experiment, is that the measurement outcome of **spin** component is either **+1** or **-1** (in the **ħ**/**2** unit).



Alice & Bob

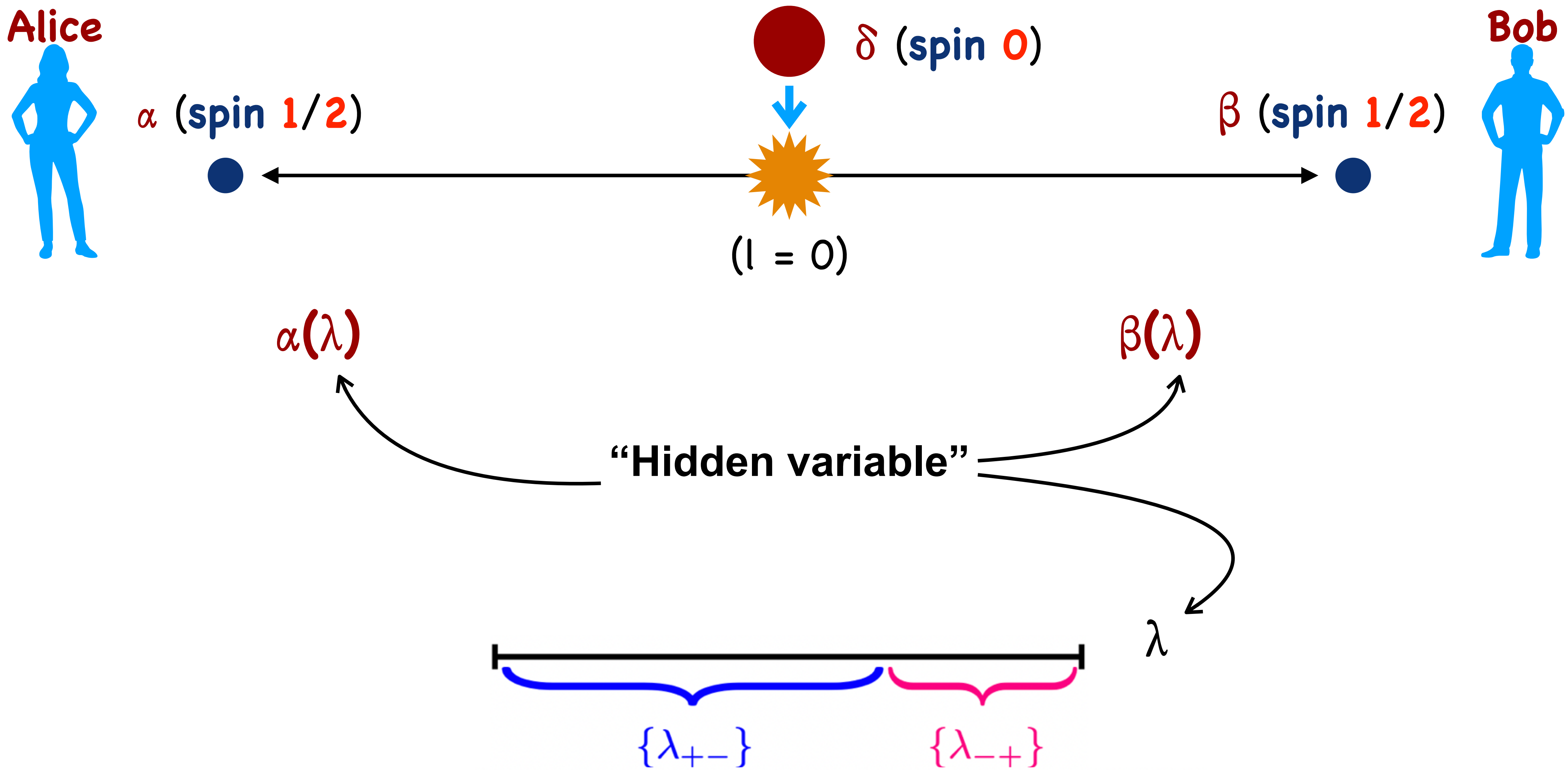


- **Alice** and **Bob** receive particles ***α*** and ***β***, respectively, and measure the **spin *z*-component** of their particles. Repeat the process many times.
- **Alice** and **Bob** will find their results are completely random (**+1** and **-1**, **50**-**50**%).
- Nevertheless, their result is **100**% anti-correlated due to the **angular momentum** conservation. If **Alice**'s result is **+1**, **Bob**'s result is always **-1** and vice versa.

Alice	+	+	-	+	-	-	+	+	+	-	+
Bob	-	-	+	-	+	+	-	-	-	+	-
$S_z^\alpha . S_z^\beta$	-	-	-	-	-	-	-	-	-	-	-

$$\boxed{< S_z^\alpha . S_z^\beta > = -1}$$

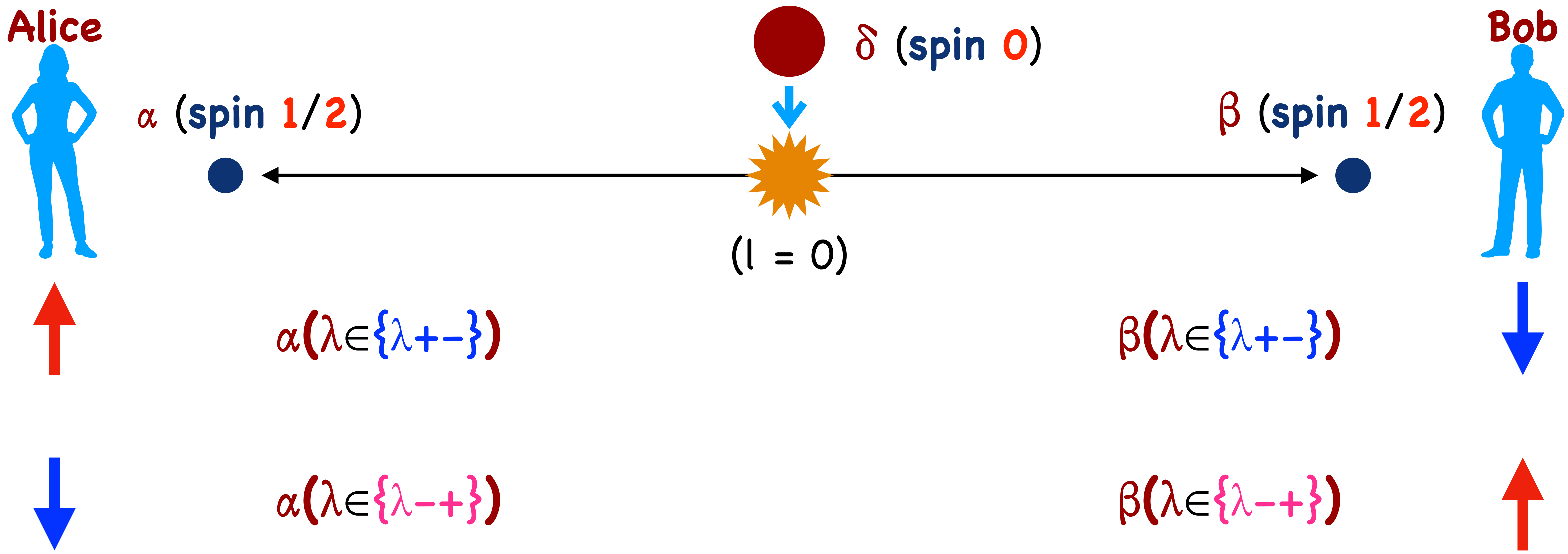
Hidden variable theory

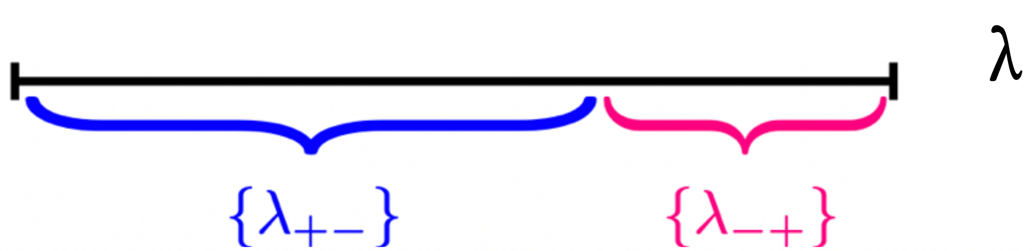


$$P(\lambda \in \{\lambda_{+-}\}) = P(\lambda \in \{\lambda_{-+}\}) = \frac{1}{2}$$

Alice	+	+	-	+	-	-	+	+	+	-	+
Bob	-	-	+	-	+	+	-	-	-	+	-
$S_z^\alpha . S_z^\beta$	-	-	-	-	-	-	-	-	-	-	-

Hidden variable theory

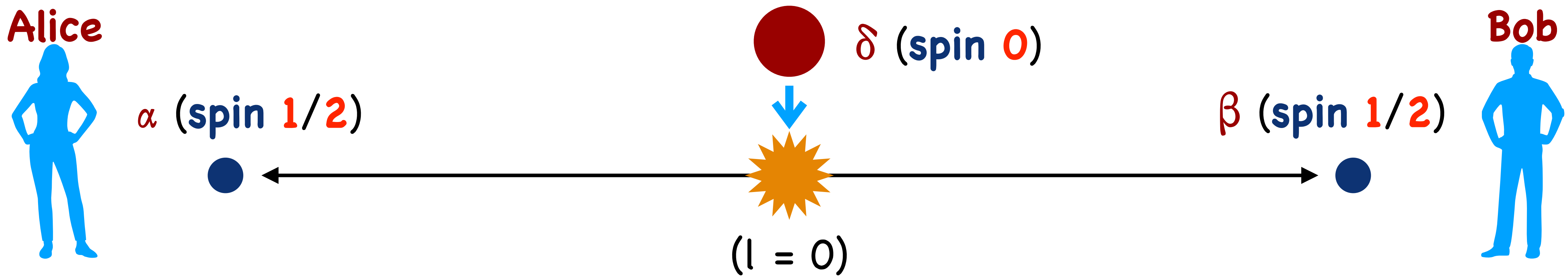




$$P(\lambda \in \{\lambda_{+-}\}) = P(\lambda \in \{\lambda_{-+}\}) = \frac{1}{2}$$

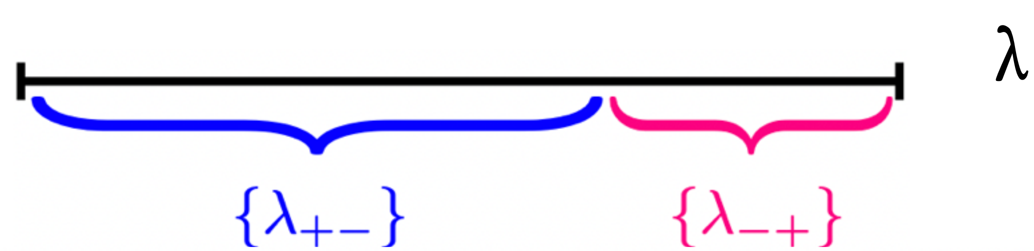
Alice	+	+	-	+	-	-	+	+	+	-	+
Bob	-	-	+	-	+	+	-	-	-	+	-
$S_z^\alpha . S_z^\beta$	-	-	-	-	-	-	-	-	-	-	-

Hidden variable theory



$\alpha(\lambda \in \{\lambda_{+-}\})$ $\beta(\lambda \in \{\lambda_{+-}\})$

$\alpha(\lambda \in \{\lambda_{-+}\})$ $\beta(\lambda \in \{\lambda_{-+}\})$



$$P(\lambda \in \{\lambda_{+-}\}) = P(\lambda \in \{\lambda_{-+}\}) = \frac{1}{2}$$

- Particles have definite properties regardless of the measurement. **(realism)**
- **Alice**'s measurement has no influence on **Bob**'s particle. **(locality)**

Quantum mechanics (QM)

- Although their outcomes are different in each decay, **QM** says the **state** of the particles are exactly the same for all decays:

$$\left|\Psi^{(0,0)}\right\rangle \doteq \frac{\overset{\alpha}{|}{+}\overset{\beta}{-}\rangle_z - |-+\rangle_z}{\sqrt{2}}$$

up to a phase $e^{i\theta}$

- Before the measurements, particles have no definite **spin**. Outcomes are undetermined.

(no **realism**)

- At the moment when Alice makes her measurement, the state collapses into:

$$|\Psi\rangle \xrightarrow{\text{Alice's measurement}} \begin{cases} |+,-\rangle_z & \cdots \text{ Alice finds } S_z[\alpha] = +1 \\ |-,+\rangle_z & \cdots \text{ Alice finds } S_z[\alpha] = -1 \end{cases}$$

Bob's outcome is completely determined (before his measurement) and **100**% anti-correlated with **Alice**'s.

(non-**local**)

Entanglement

- The origin of this bizarre feature is **entanglement**.

general
$$|\Psi\rangle \doteq c_{11}|++\rangle_z + c_{12}|+-\rangle_z + c_{21}|-+\rangle_z + c_{22}|--\rangle_z$$

separable
$$|\Psi_{\text{sep}}\rangle \doteq \left[c_1^\alpha|+\rangle_z + c_2^\alpha|-\rangle_z\right] \otimes \left[c_1^\beta|+\rangle_z + c_2^\beta|-\rangle_z\right]$$

entangled
$$|\Psi_{\text{ent}}\rangle \cancel{\doteq} \left[c_1^\alpha|+\rangle_z + c_2^\alpha|-\rangle_z\right] \otimes \left[c_1^\beta|+\rangle_z + c_2^\beta|-\rangle_z\right]$$

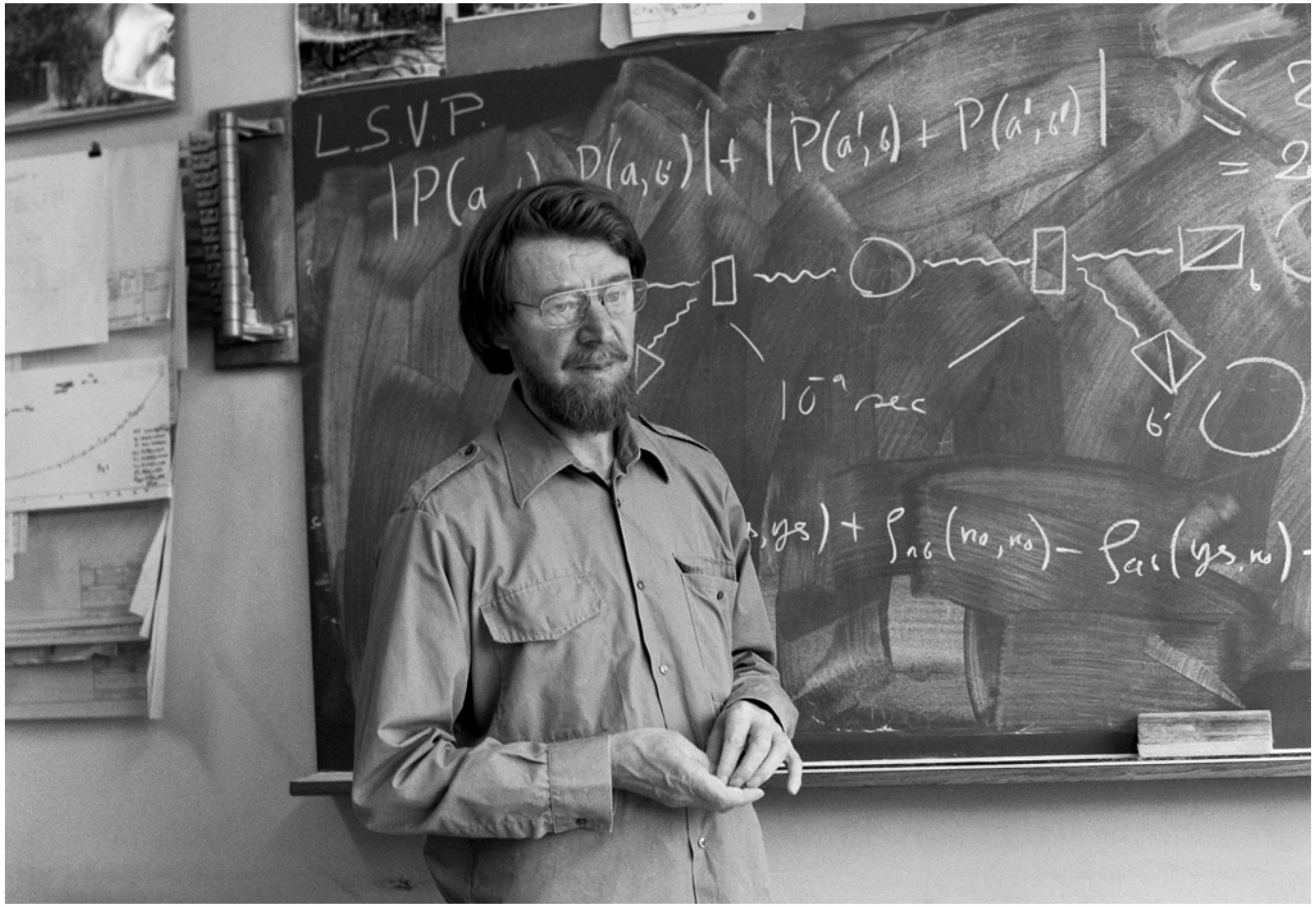
entangled
$$|\Psi^{(0,0)}\rangle \doteq \frac{|+-\rangle_z - |-+\rangle_z}{\sqrt{2}}$$

Bob's measurement collapses the **state** of $\boldsymbol{\beta}$ to $|+\rangle_z$ or $|-\rangle_z$ but does not influence the **state** of $\boldsymbol{\alpha}$.

Bell inequalities

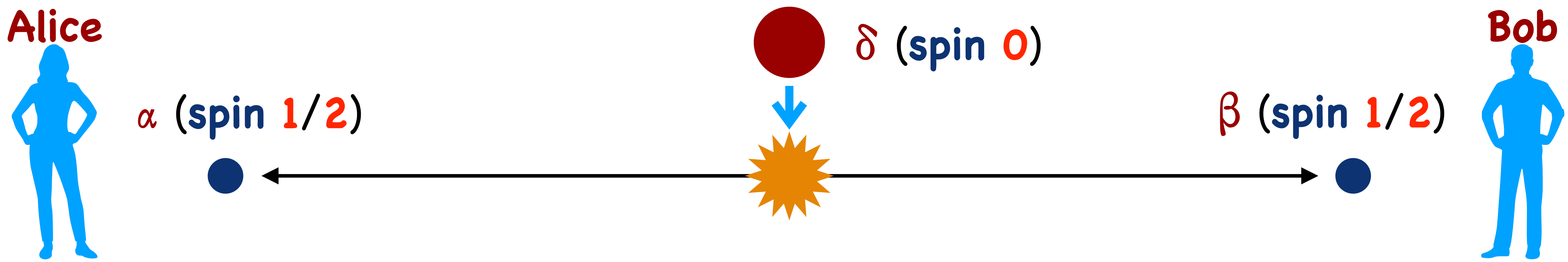
- It seems difficult to experimentally discriminate **QM** and **general hidden variable theories**.

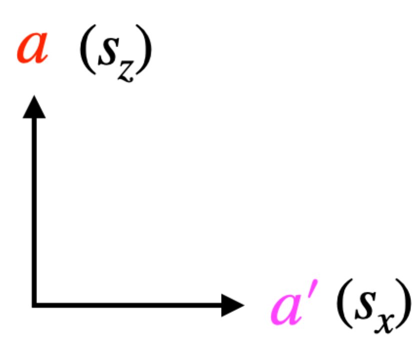
- **John Bell** (**1964**) derived simple inequalities that can discriminate **QM** from any **local-real hidden variable theories**: **Bell inequalities**.

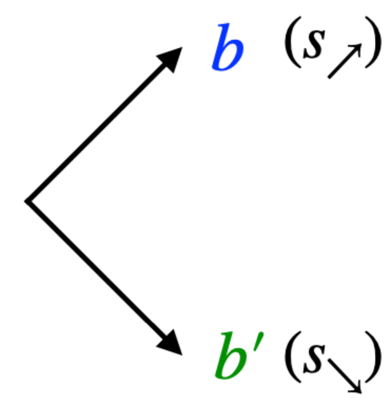


John Bell and his famous theorem in 1982 (Image: CERN)

Bell inequalities







- The experiment consists of **4** sessions:
  1. **Alice** and **Bob** measure $s_a[\alpha]$ and $s_b[\beta]$, respectively. Repeat the measurement many times and calculate $\langle s_a . s_b \rangle$.
  2. Repeat (1) but for ***a*** and ***b'***.
  3. Repeat (1) but for ***a'*** and ***b***.
  4. Repeat (1) but for ***a'*** and ***b'***.
- Finally we construct:

$$R_{\mathrm{CHSH}} \equiv \frac{1}{2} \left| \langle s_a s_b \rangle - \langle s_a s_{b'} \rangle + \langle s_{a'} s_b \rangle + \langle s_{a'} s_{b'} \rangle \right|$$

One can show in **hidden variable theories** that: $R_{CHSH} \leq 1$
[Clauser, Horne, Shimony, Holt, 1969].

Bell inequalities

- In **QM**, for: $$|\Psi^{(0,0)}\rangle \doteq \frac{|+-\rangle_z - |-+\rangle_z}{\sqrt{2}}$$

- One can show: $$\langle s_a s_b\rangle = \langle \Psi^{(0,0)}|s_a s_b|\Psi^{(0,0)}\rangle = (\hat{\mathbf{a}}\cdot\hat{\mathbf{b}})$$

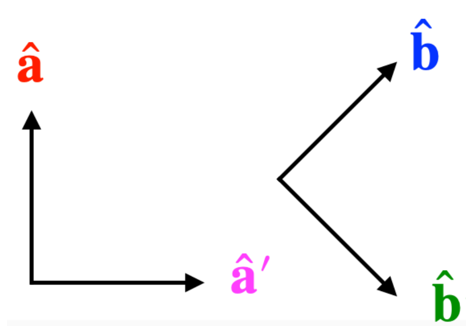
- Therefore:

$$R_{\mathrm{CHSH}} = \frac{1}{2}\left|\langle s_a s_b\rangle - \langle s_a s_{b'}\rangle + \langle s_{a'} s_b\rangle + \langle s_{a'} s_{b'}\rangle\right|$$

$$= \frac{1}{2}\left|\underbrace{(\hat{\mathbf{a}}\cdot\hat{\mathbf{b}})}_{\frac{1}{\sqrt{2}}} - \underbrace{(\hat{\mathbf{a}}\cdot\hat{\mathbf{b}}')}_{-\frac{1}{\sqrt{2}}} + \underbrace{(\hat{\mathbf{a}}'\cdot\hat{\mathbf{b}})}_{\frac{1}{\sqrt{2}}} + \underbrace{(\hat{\mathbf{a}}'\cdot\hat{\mathbf{b}}')}_{\frac{1}{\sqrt{2}}}\right|$$

$$= \sqrt{2}$$

violates the upper bound of **hidden variable theories**!



Part II

Spin 1/2 biparticle system

Density operator

Probability of having $|\psi_1\rangle$

- For a statistical ensemble $\{\{p_1 : |\psi_1\rangle\}, \{p_2 : |\psi_2\rangle\}, \{p_3 : |\psi_3\rangle\}, \ldots\}$, we define the **density operator/matrix**:

$$\hat{\rho} \equiv \sum_k p_k |\Psi_k\rangle\langle\Psi_k| \qquad \rho_{ab} \equiv \langle e_a|\hat{\rho}|e_b\rangle$$

$$0 \leq p_k \leq 1$$

$$\sum_k p_k = 1$$

$$\langle e_a|e_b\rangle = \delta_{ab}$$

- **Density matrices** satisfy the conditions:

$$\hat{\rho}^\dagger = \hat{\rho}$$

$$\mathrm{Tr}\,\hat{\rho} = 1$$

$$\hat{\rho} \text{ is positive definite, that is } \forall|\varphi\rangle;\ \langle\varphi|\hat{\rho}|\varphi\rangle \geq 0.$$

- The expectation of an observable **Ô** is calculated by:

$$\langle\hat{O}\rangle = \mathrm{Tr}\left[\hat{O}\hat{\rho}\right]$$

Spin 1/2 biparticle system

- The spin system of ***α*** and ***β*** particles has **4** independent bases:

$$\boxed{\big(\,|e_1\rangle, |e_2\rangle, |e_3\rangle, |e_4\rangle\big) = \big(|++\rangle, |+-\rangle, |-+\rangle, |--\rangle\big)}$$

3x3
matrix

- ==> ρ_{ab} is a **4**x**4** matrix (hermitian, Tr=**1**). It can be expanded as

$$\boxed{\rho = \frac{1}{4}\big(1 \otimes 1 + B_i \,.\, \sigma_i \otimes 1 + \bar{B}_i . 1 \otimes \sigma_i + C_{ij} \,.\, \sigma_i \otimes \sigma_j\big), \qquad B_i, \bar{B}_i, C_{ij} \in \mathbb{R}}$$

- For the **spin** operators $\hat{s}^\alpha$ and $\hat{s}^\beta$:

$$\boxed{\langle \hat{s}_i^\alpha \rangle = Tr[\hat{s}_i^\alpha \hat{\rho}] = B_i \quad \langle \hat{s}_i^\beta \rangle = Tr[\hat{s}_i^\beta \hat{\rho}] = \bar{B}_i \quad \langle \hat{s}_i^\alpha \hat{s}_j^\beta \rangle = Tr[\hat{s}_i^\alpha \hat{s}_j^\beta \hat{\rho}] = C_{ij}}$$

spin-spin
correlation

Entanglement

- If the **state** is **separable** (not **entangled**):

$$\boxed{\rho = \sum_k p_k \rho_k^\alpha \otimes \rho_k^\beta, \quad 0 \le p_k \le 1 \text{ and } \sum_k p_k = 1}$$

- Then, a modified **matrix** by the partial transpose:

$$\boxed{\rho^{T_\beta} = \sum_k p_k \rho_k^\alpha \otimes [\rho_k^\beta]^T}$$

is also a physical **density matrix**, i.e. **Tr**=1 and non-negative.

- For biparticle systems, **entanglement** $\iff \rho^{T_\beta}$ to be non-positive. [Peres-Horodecki (1996,1997)].

- A simple sufficient condition for **entanglement** is:

$$\boxed{E \equiv C_{11} + C_{22} - C_{33} > 1}$$ [Eur. Phys. J. C 82, 666 (2022)]

Part III

Higgs to tau tau @ lepton colliders

$H \to \tau^+\tau^-$

- Generic $H\tau\tau$ interaction:

$$\mathscr{L}_{int} = -\frac{m_\tau}{v_{SM}}\kappa H\bar{\Psi}_\tau(\cos\delta + i\gamma_5\sin\delta)\Psi_\tau$$

$$\rho_{mn,\bar{m}\bar{n}} = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & e^{-i2\delta} & 0 \\ 0 & e^{-i2\delta} & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

SM: $(\kappa, \delta) = (1,0)$

$$B_i = \bar{B}_i = 0,\ C_{ij} = \begin{pmatrix} \cos 2\delta & \sin 2\delta & 0 \\ -\sin 2\delta & \cos 2\delta & 0 \\ 0 & 0 & -1 \end{pmatrix},\ E = 2cos2\delta + 1$$

Estimation of C_{ij}

- Let's suppose a **spin 1/2** particle ***α*** is **at rest** and spinning in the **s** direction.
- ***α*** decays into a measurable particle **l_α** and the rest **X**: $\alpha \longrightarrow l_\alpha + (X)$
- The decay distribution is generally given by : $\frac{d\Gamma}{d\Omega} \propto 1 + x_\alpha(\hat{I}_\alpha . s)$

$x \in [-1, 1]$ is called spin-analysing power and depends on the decay
$x = 1$ for $\tau^\pm \to \pi^\pm \nu$

Unit direction vector of l_α measured at the rest frame of α

- One can show for ***α***+***β*** -> [l_α + (X)] + [l_β + (X)]:

$$\boxed{\langle \hat{s}_i^\alpha \hat{s}_j^\beta \rangle = -9.\langle \hat{I}_i^\alpha \hat{I}_j^\beta \rangle}$$

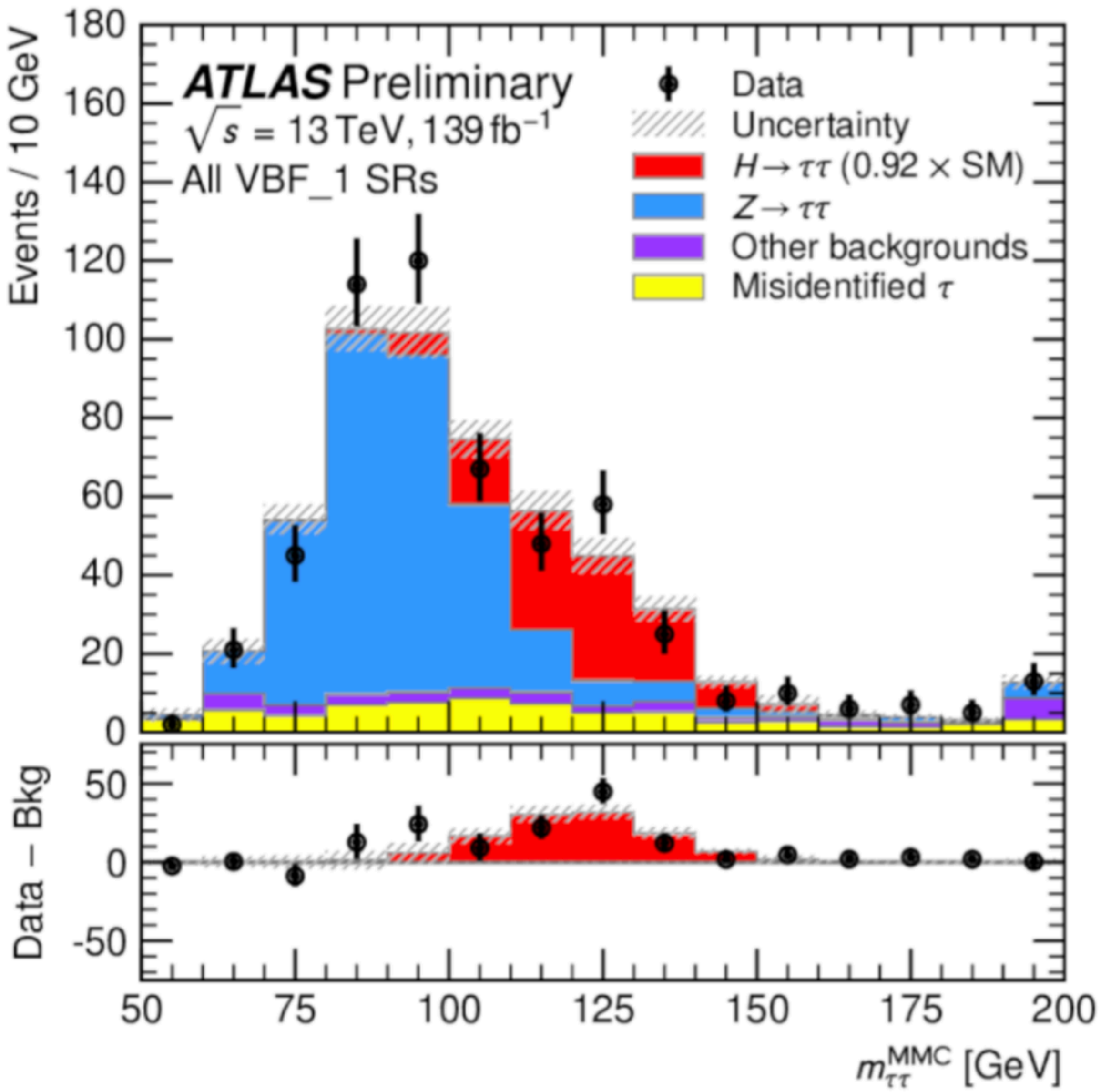
measurable at colliders, but

needs to reconstruct the α (β) rest frames

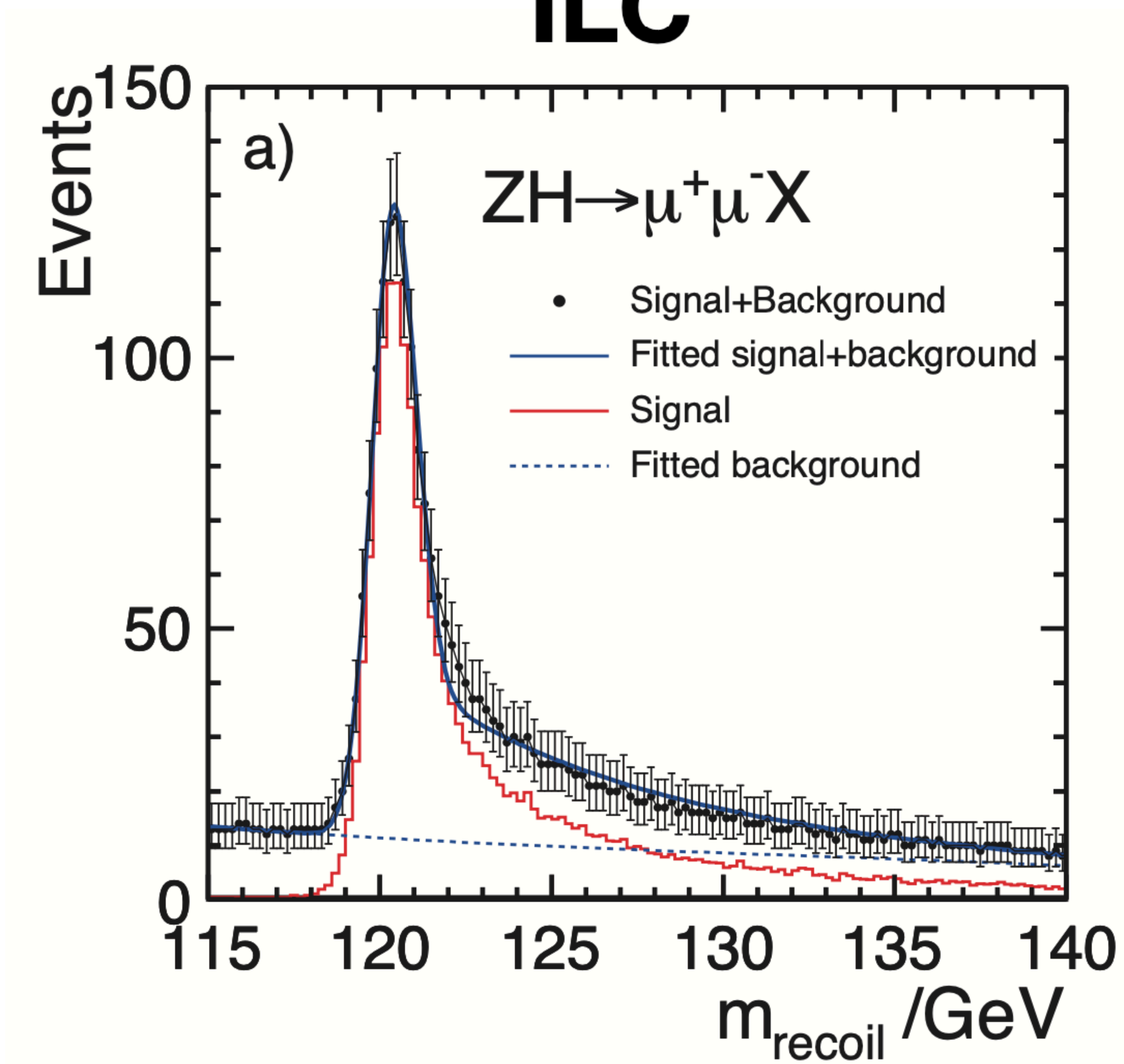
Why lepton colliders?

- Background $Z/\gamma \to \tau^+\tau^-$ is much smaller at **lepton** colliders
- We need to reconstruct each τ rest frame to measure $\hat{I}$. This is challenging at **hadron** colliders since partonic CoM energy is unknown for each event

LHC

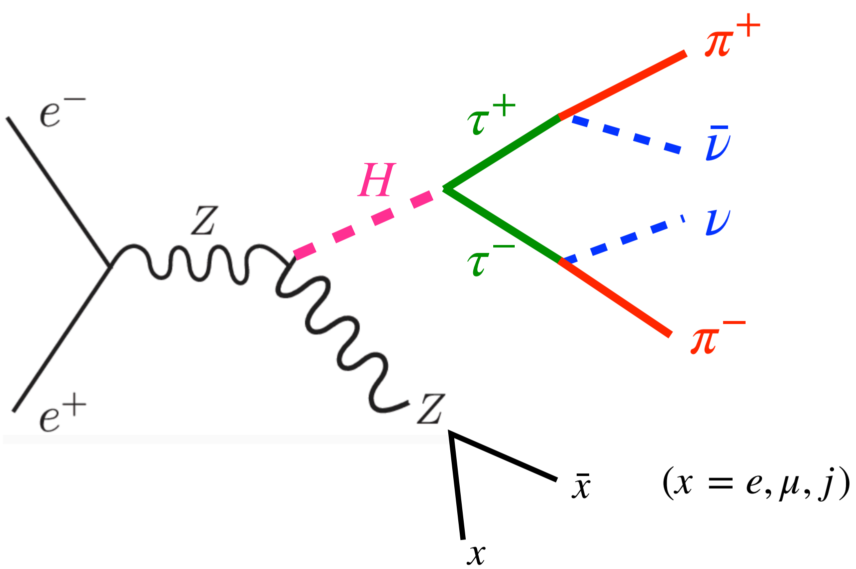


ILC



Simulation

Event selection: $|M_{\text{recoil}} - 125\,\text{GeV}| < 5\,\text{GeV}$



	ILC	FCC-ee
energy (GeV)	250	240
luminosity (ab^{-1})	3	5
beam resolution e^+ (%)	0.18	0.83×10^{-4}
beam resolution e^- (%)	0.27	0.83×10^{-4}
$\sigma(e^+e^- \to HZ)$ (fb)	240.1	240.3
# of signal $(\sigma \cdot \text{BR} \cdot L \cdot \epsilon)$	385	663
# of background $(\sigma \cdot \text{BR} \cdot L \cdot \epsilon)$	20	36

$e^+e^- \to Z + (Z^*/\gamma^*) \to f\bar{f} + \tau^+\tau^-$

- SM events $(\kappa, \delta) = (1{,}0)$ were generated with **Madgraph5**
- We incorporate the detector effect by smearing energies of visible particles

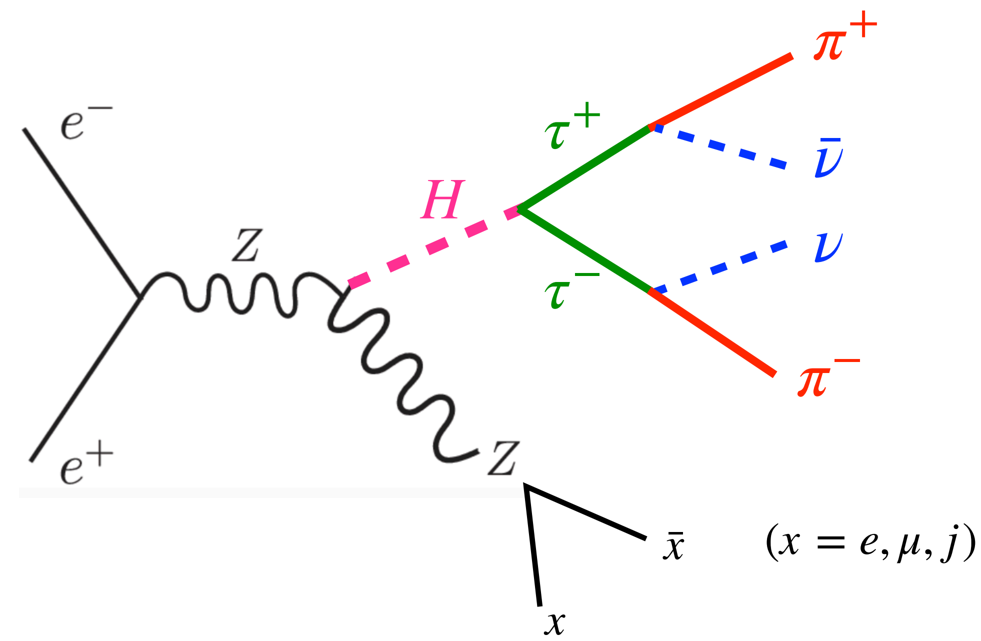
$$E^{true} \to E^{obs} = (1 + \sigma_E \,.\, \omega)\,.\,E^{true} \qquad \sigma_E = 0.03$$

Random number from a normal distribution

- We perform **100 pseudo-experiment** to estimate the statistical uncertainties

Solving kinematical constraints

- To determine the tau momenta, we have to reconstruct the unobserved neutrino momenta: $(p_x^{\nu}, p_y^{\nu}, p_z^{\nu}), (p_x^{\bar{\nu}}, p_y^{\bar{\nu}}, p_z^{\bar{\nu}})$.

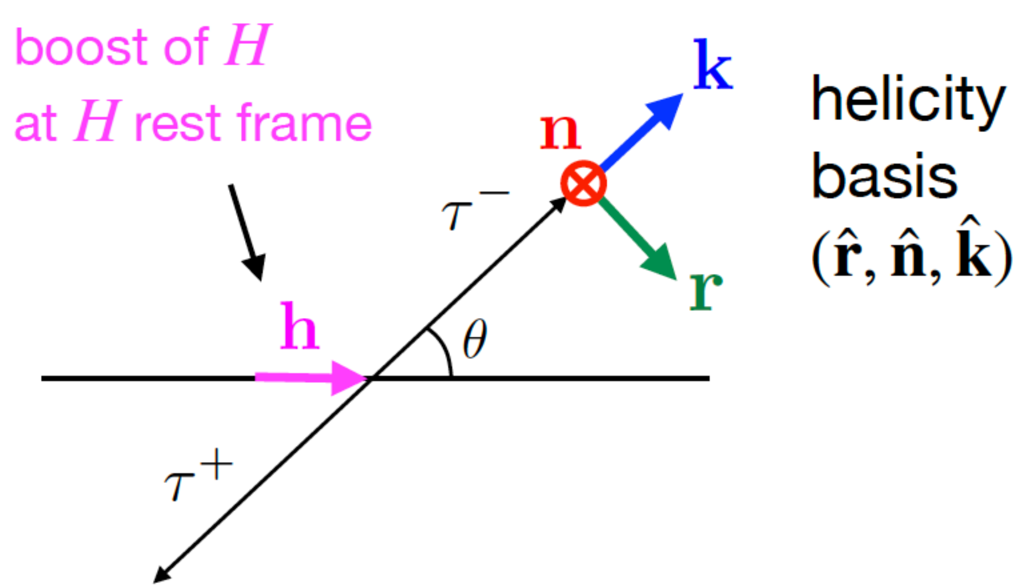


- **6** unknowns can be constrained by **2** mass-shell conditions and **4** energy-momentum conservation:

$$\boxed{\begin{aligned} m_\tau^2 &= (p_{\tau^+})^2 = (p_{\pi^+} + p_{\bar{\nu}}) \\ m_\tau^2 &= (p_{\tau^-})^2 = (p_{\pi^-} + p_{\nu}) \\ (p_{ee} - p_Z)^\mu &= p_H^\mu = \left[(p_{\pi^-} + p_\nu) + (p_{\pi^+} + p_{\bar{\nu}})\right]^\mu \end{aligned}}$$

- With the reconstructed momenta, we define $(\hat{\boldsymbol{r}}, \hat{\boldsymbol{n}}, \hat{\boldsymbol{k}})$ basis at the Higgs rest frame.

$$\boxed{\begin{aligned} &C_{ij} = \langle \hat{s}_i^{(\tau^-)} \hat{s}_j^{(\tau^+)} \rangle = -9.\langle \hat{I}_i^- \hat{I}_j^+ \rangle \\ &(i, j = r, n, k) \end{aligned}}$$



Results

	ILC	FCC-ee
C_{ij}	$\begin{pmatrix} -0.600 \pm 0.210 & 0.003 \pm 0.125 & 0.020 \pm 0.149 \\ 0.003 \pm 0.125 & -0.494 \pm 0.190 & 0.007 \pm 0.128 \\ 0.048 \pm 0.174 & 0.0007 \pm 0.156 & 0.487 \pm 0.193 \end{pmatrix}$	$\begin{pmatrix} -0.559 \pm 0.143 & -0.010 \pm 0.095 & -0.014 \pm 0.122 \\ -0.010 \pm 0.095 & -0.494 \pm 0.139 & -0.002 \pm 0.111 \\ 0.012 \pm 0.124 & 0.020 \pm 0.105 & 0.434 \pm 0.134 \end{pmatrix}$
E_k	-1.057 ± 0.385	-0.977 ± 0.264
R^*_{CHSH}	0.769 ± 0.189	0.703 ± 0.134

SM values:

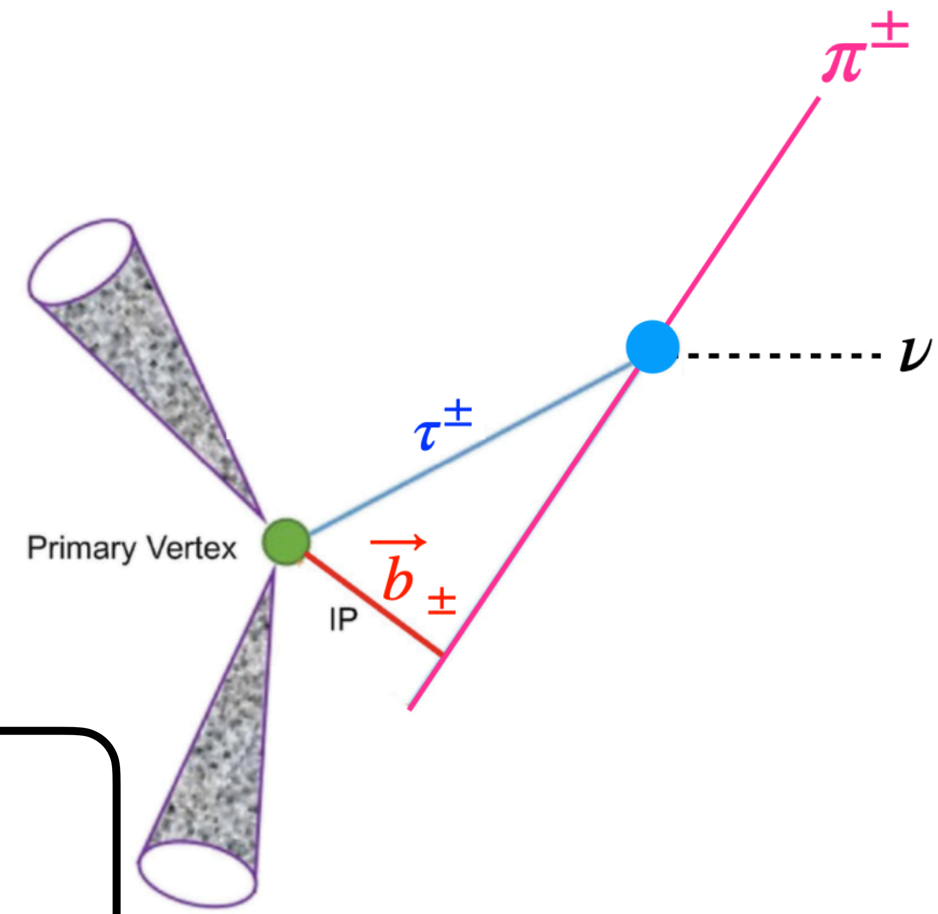
$$C_{ij} = \begin{pmatrix} 1 & & \\ & 1 & \\ & & -1 \end{pmatrix}$$

$E = 3$ **Entanglement** $\Longrightarrow E > 1$

$R_{\text{CHSH}} = \sqrt{2} \simeq 1.414$ **Bell-nonlocal** $\Longrightarrow R_{\text{CHSH}} > 1$

Impact parameter (IP)

- We use the information of the **impact parameter** $\vec{b}_{\pm}$ measurement of $\pi^{\pm}$ to "correct" the observed energies of $\tau^{\pm}$ and Z decay products.
- We check whether the reconstructed τ momenta are consistent with the measured impact parameters.
- We construct the likelihood function and search for the most likely τ momenta.



$$\boxed{E_{\alpha}(\delta_{\alpha}) = (1 + \sigma_{\alpha}^{E} \cdot \delta_{\alpha}) \cdot E_{\alpha}^{\text{obs}}}$$

$$\vec{b}_{+} = |\vec{b}_{+}| \left(\sin^{-1}\Theta_{+} \cdot \vec{e}_{\tau^{+}} - \tan^{-1}\Theta_{+} \cdot \vec{e}_{\pi^{+}}\right)$$

$$\vec{\Delta}_{b_{+}}^{i}(\{\delta\}) \equiv \vec{b}_{+} - |\vec{b}_{+}| \left(\sin^{-1}\Theta_{+}^{i}(\{\delta\}) \cdot \vec{e}_{\tau^{+}}^{\,i}(\{\delta\}) - \tan^{-1}\Theta_{+}^{i}(\{\delta\}) \cdot \vec{e}_{\pi^{+}}\right)$$

$$L_{\pm}^{i}(\{\delta\}) = \frac{[\Delta_{b_{\pm}}^{i}(\{\delta\})]_{x}^{2} + [\Delta_{b_{\pm}}^{i}(\{\delta\})]_{y}^{2}}{\sigma_{b_{T}}^{2}} + \frac{[\Delta_{b_{\pm}}^{i}(\{\delta\})]_{z}^{2}}{\sigma_{b_{z}}^{2}}$$

$$\boxed{L^{i}(\{\delta\}) = L_{+}^{i}(\{\delta\}) + L_{-}^{i}(\{\delta\})}$$

Results

	ILC	FCC-ee
C_{ij}	$\begin{pmatrix} 0.830 \pm 0.176 & 0.020 \pm 0.146 & -0.019 \pm 0.159 \\ -0.034 \pm 0.160 & 0.981 \pm 0.1527 & -0.029 \pm 0.156 \\ -0.001 \pm 0.158 & -0.021 \pm 0.155 & -0.729 \pm 0.140 \end{pmatrix}$	$\begin{pmatrix} 0.925 \pm 0.109 & -0.011 \pm 0.110 & 0.038 \pm 0.095 \\ -0.009 \pm 0.110 & 0.929 \pm 0.113 & 0.001 \pm 0.115 \\ -0.026 \pm 0.122 & -0.019 \pm 0.110 & -0.879 \pm 0.098 \end{pmatrix}$
E_k	2.567 ± 0.279 > 5σ	2.696 ± 0.215 > 5σ
R_{CHSH}	1.103 ± 0.163	1.276 ± 0.094 ~ 3σ

SM values: $C_{ij} = \begin{pmatrix} 1 & & \\ & 1 & \\ & & -1 \end{pmatrix}$

$E = 3$ **Entanglement** $\Longrightarrow E > 1$

$R_{\mathrm{CHSH}} = \sqrt{2} \simeq 1.414$ **Bell-nonlocal** $\Longrightarrow R_{\mathrm{CHSH}} > 1$

The superiority of **FCC-ee** over **ILC** is due to a better **beam resolution**

	ILC	FCC-ee
energy (GeV)	250	240
luminosity (ab^{-1})	3	5
beam resolution e^+ (%)	0.18	$0.83 \cdot 10^{-4}$
beam resolution e^- (%)	0.27	$0.83 \cdot 10^{-4}$

CP measurement

- Under **CP**, the spin correlation matrix transforms: $C \xrightarrow{CP} C^T$
- This can be used for a **model-independent** test of **CP** violation. We define:

$$A \equiv (C_{rn} - C_{nr})^2 + (C_{nk} - C_{kn})^2 + (C_{kr} - C_{rk})^2 \geq 0$$

Observation of $A \neq 0$ immediately confirms **CP** violation

- From our simulation, we observe:

$$A = \begin{cases} 0.168 \pm 0.131 & \text{(ILC)} \\ 0.081 \pm 0.061 & \text{(FCC-ee)} \end{cases}$$

Consistent with the absence of **CPV**

CP measurement

- This model independent bounds can be translated to the constraint on the **CP**-phase δ:

$$\mathscr{L}_{\text{int}} \propto H\bar{\psi}_\tau(\cos\delta + i\gamma_5 \sin\delta)\psi_\tau \;\Rightarrow\; C_{ij} = \begin{pmatrix} \cos 2\delta & \sin 2\delta & 0 \\ -\sin 2\delta & \cos 2\delta & 0 \\ 0 & 0 & -1 \end{pmatrix} \;\Rightarrow\; A(\delta) = 4\sin^2 2\delta$$

- Focusing on the region near $|\delta| = 0$, we find the 1-σ bounds:

$$\delta < \begin{cases} 7.9^\circ & \text{(ILC)} \\ 5.4^\circ & \text{(FCC-ee)} \end{cases}$$

- Other studies:

$$\Delta\delta \sim 11.5^o \quad \text{(HL-LHC)}$$ [Hagiwara, Ma, Mori 2016]

$$\Delta\delta \sim 4.3^o \quad \text{(ILC)}$$ [Jeans and G. W. Wilson 2018]

Part IV

Summary

Summary

- High energy tests of **entanglement** and **Bell inequality** has recently attracted an attention.
- We investigated feasibility of **quantum** property tests @ **ILC** and **FCC-ee**.
- **Quantum tests** require a precise reconstruction of the τ rest frames and **IP** information is crucial to achieve this.
- Spin correlation is sensitive to **CP**-phase and we can measure the **CP**-phase as a byproduct of the quantum property measurement.

	Entanglement	Bell-inquality	CP-phase
FCC-ee	$> 5\sigma$	$\sim 3\sigma$	7.9°
ILC	$> 5\sigma$		5.4°