Entanglement and Bell inequalities violation in H $\rightarrow$ ZZ with anomalous coupling

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Main goals

1. Develop an analytical strategy for testing entanglement and ensuring Bell ineq. violation of ρ_{VV} in $X \to VV$ considering CP-conserving vertices.
2. Apply it for ρ_{ZZ} in the decay chain $H \to ZZ \to \ell_1^+ \ell_1^- \ell_2^+ \ell_2^-$ with an anomalous CP-conserving coupling.

Preliminaries

- Bipartite quantum system:

$$\rho = \sum_i p_i \left|\psi_i\right\rangle \left\langle\psi_i\right|, \quad \left|\psi_i\right\rangle \in \mathcal{H}_A \otimes \mathcal{H}_B, \text{ with } dim\, \mathcal{H}_{A(B)} = d = 3.$$

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- CGLMP Bell inequality (A_1, A_2 and B_1, B_2 observables acting respectively on $\mathcal{H}_A$ and $\mathcal{H}_B$):

$$I_3(P(A_i = k, B_j = l)) = \left\langle\mathcal{O}_{Bell}(U_{A_i}, U_{B_j})\right\rangle_\rho \leq 2.$$

Diboson state from spin-0 particle decay

General scalar state [Barr, Caban, Rembieliński (2023)]

$$\left|\psi_{VV}^{\text{scalar}}\right\rangle = g_{\mu\nu}(k,p) e_{\lambda}^{\mu}(k)\, e_{\sigma}^{\nu}(p)\, |(k,\lambda);(p,\sigma)\rangle$$

where

$$g_{\mu\nu} = \eta_{\mu\nu} + \frac{c}{k\cdot p}\left(k_\mu\, p_\nu + p_\mu\, k_\nu\right), \quad c \in \mathbb{R}$$

$$e(q) = \left[e_{\lambda}^{\mu}(q)\right] = \begin{pmatrix} \frac{\mathbf{q}^T}{m} \\ \mathbb{1} + \frac{\mathbf{q}\otimes\mathbf{q}^T}{m(m+q_0)} \end{pmatrix} V^T, \quad V = \frac{1}{\sqrt{2}} \begin{pmatrix} -1 & i & 0 \\ 0 & 0 & \sqrt{2} \\ 1 & i & 0 \end{pmatrix}.$$

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For $k^\mu = (\omega_1, 0, 0, |\mathbf{k}|)$ and $p^\nu = (\omega_2, 0, 0, -|\mathbf{k}|)$:

$$\left|\psi_{VV}^{\text{scalar}}\right\rangle = \frac{1}{\sqrt{2+\kappa^2}}\left[|+,-\rangle - \kappa\,|0,0\rangle + |-,+\rangle\right], \quad \kappa = \beta + c\,(\beta - 1/\beta)$$

$$\beta = \frac{M^2 - (m_1^2 + m_2^2)}{2 m_1 m_2} \implies c = 0 \text{ corresponds to SM case}$$

Relation to vertex structure

The amplitude for the general Lorentz invariant, CPT conserving coupling of a (pseudo)scalar and two vector bosons is [Godbole, Miller, Mühlleitner (2007)]:

$$\begin{aligned}\mathcal{A}_{\lambda\sigma}(k,p) \quad &\propto \quad \big[v_1\eta_{\mu\nu} + v_2(k+p)_\mu(k+p)_\nu + \\ &\qquad v_3\varepsilon_{\alpha\beta\mu\nu}(k+p)^\alpha(k-p)^\beta\big]\, e^\mu_\lambda(k)\, e^\nu_\sigma(p)\end{aligned}$$

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Comparing with the general structure of our state and using $e^\mu_\lambda(q)q_\mu = 0$:

$$g_{\mu\nu} = \eta_{\mu\nu} + \frac{c}{k\cdot p}\,(k_\mu\, p_\nu + p_\mu\, k_\nu) \implies c = \frac{v_2}{v_1}(k\cdot p)$$

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Physical interpretation of the parameter c!

(From now on we consider $v_3 = 0$, i.e. only CP-conserving couplings.)

Ensemble of events

For an ensemble of events, we need to average over possible configurations:

$$\rho_{VV}(c) = \int dm_1 dm_2 \mathcal{P}_c(m_1, m_2)\, \rho(m_1, m_2, c), \quad \rho = \left|\psi_{VV}^{\text{scalar}}\right\rangle \left\langle\psi_{VV}^{\text{scalar}}\right|$$

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For arbitrary c we use the diff. cross section of the $X \to VV \to f_1 \bar{f}_1 f_2 \bar{f}_2$ decay:

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega_1\, d\Omega_2} = \left(\frac{3}{4\pi}\right)^2 \mathrm{Tr}\left\{\rho_{VV}(c)\left(\Gamma_1^T \otimes \Gamma_2^T\right)\right\}, \quad \Gamma_i \text{ decay matrices.}$$

Integrating w.r.t. Ω_i and differentiating w.r.t. m_i:

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In general, the PDF is obtained via the diff. cross section of the process in hand.
For $H \to ZZ \to \ell_1^+\ell_1^-\ell_2^+\ell_2^-$ with anomalous couplings [Zagoskin, Korchin (2016)]:

$$\frac{1}{\sigma}\frac{d\sigma}{dm_1\,dm_2} = N\,\frac{\lambda^{1/2}(M^2, m_1^2, m_2^2)\,m_1^3\,m_2^3}{D(m_1)\,D(m_2)}\,[2+\kappa^2],$$

with N a normalisation constant and

$$\begin{aligned}
\lambda(x,y,z) &= x^2+y^2+z^2-2(xy+xz+yz), \\
D(m) &= (m^2-m_V^2)^2+(m_V\Gamma_V)^2.
\end{aligned}$$

Once $\mathcal{P}_c(m_1, m_2)$ is identified, the complete density matrix (experimentally determined via Quantum Tomography [Ashby-Pickering, Barr, Wierzchucka (2023)], [AB (2023)]) is

$$\rho_{VV}(c) = \frac{1}{2a+b} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \boxed{a} & 0 & \boxed{-d} & 0 & \boxed{a} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \boxed{-d} & 0 & \boxed{b} & 0 & \boxed{-d} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \boxed{a} & 0 & \boxed{-d} & 0 & \boxed{a} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

where a, b, d are polynomials on c with coefficients given by the integrals

$$I(n) = \int_{0 \le m_1 + m_2 \le M} dm_1\, dm_2\, \frac{\mathcal{P}_c(m_1, m_2)}{2 + \kappa^2}\, \beta^n, \quad n = -2, -1, 0, 1, 2.$$

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For $M = m_H$, $m_V = m_Z$ and $\Gamma_V = \Gamma_Z$:

$$\left.\begin{array}{lcc} a_Z & \simeq & 2989.76 \\ b_Z & \simeq & 9431.55 + 12883.6c + 4983.07c^2 \\ d_Z & \simeq & 4819.07 + 2752.19c \end{array}\right\} \Longrightarrow \begin{array}{c}\text{Entanglement} \\ \text{(Peres-Horodecki)}\end{array}$$

Entanglement and Bell ineq. violation

We quantise the entanglement via the logarithmic negativity:

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Strategy 1 is modified accordingly to introduce the c dependence.
In strategy 2 we choose unitary matrices **ensuring** a violation of I_3 for all values of $\kappa \iff c$:

$$\begin{array}{cc} U_{A_1} = U_V(0), & U_{A_2} = U_V(\frac{\pi}{2}) \\ U_{B_1} = U_V(\frac{\pi}{4}), & U_{B_2} = U_V(-\frac{\pi}{4}) \end{array}, \quad U_V = \begin{pmatrix} \cos\frac{t}{2} & 0 & \sin\frac{t}{2} \\ 0 & 1 & 0 \\ -\sin\frac{t}{2} & 0 & \cos\frac{t}{2} \end{pmatrix}.$$

Allowed values for κ and c?

The ranges for κ as a function of c are:

$$
\begin{aligned}
c \in (-\infty, -1) &\implies \kappa \in (-\infty, 1) \\
c = -1 &\implies \kappa \in [0, 1] \\
c \in (-1, -1/2) &\implies \kappa \in (2\sqrt{-c(c+1)}, \infty) \\
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- **Experimental bounds** [CMS collaboration (2019)]:

$$|c| \leq c_{HZZ}^{\max} \simeq 0.23 \implies \kappa \in [1, \infty).$$

- **Theoretical bounds (perturbative unitarity bounds)** [Dahiya, Dutta, Islam (2016)]:

$$c \in (-\infty, \infty) \implies \text{ no restriction over } \kappa.$$

Logarithmic Negativity

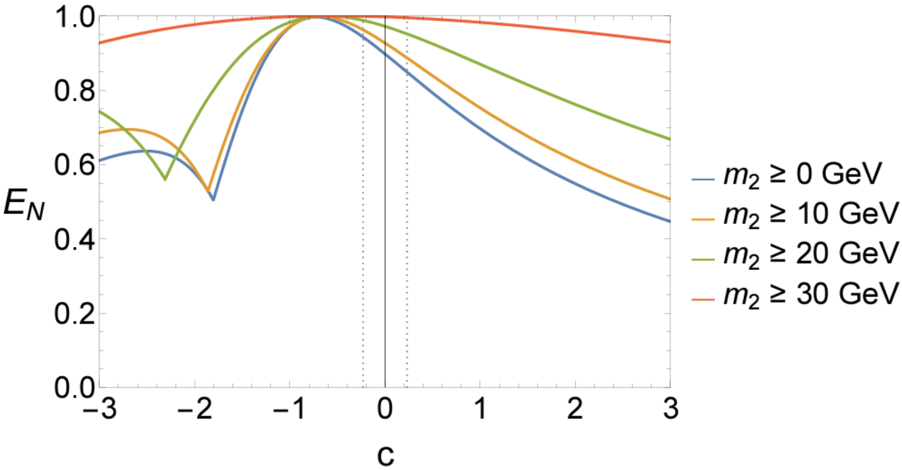

Figure 1: Logarithmic negativity $E_N(c)$ for different cuts on the off-shell mass m_2. Vertical dotted lines delimit the allowed range for c in $H \to ZZ$.

Bell ineq. violation

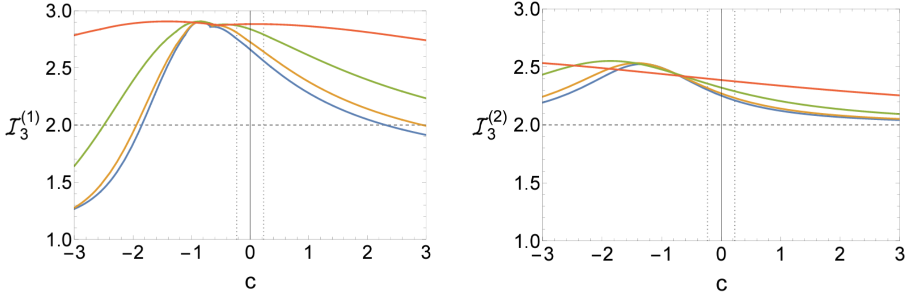

Figure 2: Maximal value of $\mathcal{I}_3$ for the different optimisation strategies as a function of c. Vertical dotted lines delimit the allowed range for c in $H \to ZZ$.

Resistance to noise

Minimal mixture for which ρ_{noise} stops violating Bell inequality:

$$\rho_{\text{noise}} = \lambda\, \rho_{VV}(c) + (1-\lambda)\,\frac{1}{9}\,\mathbb{1}_9(\text{ or } \rho_{BG}), \quad \lambda_{\min} = \frac{2}{\max\{\mathcal{I}_3^{(1)}, \mathcal{I}_3^{(2)}\}}.$$

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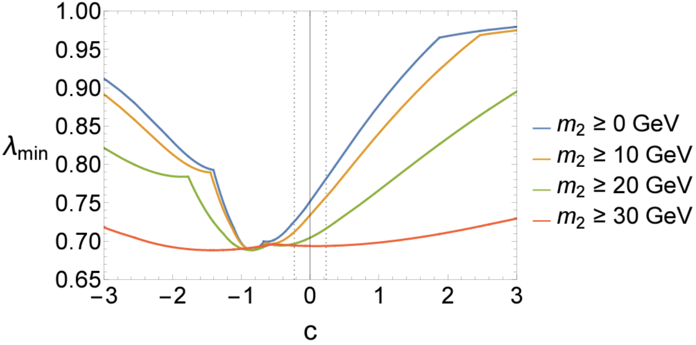

Figure 3: Resistance to noise for different cuts on the off-shell mass m_2 as a function of c. Vertical dotted lines delimit the allowed range for c in $H \to ZZ$.

Conclusions

- The density matrix of an ensemble of diboson scalar states in terms of the most general Lorentz-invariant and CPT conserving couplings can be determined via analytical methods.
- Peres-Horodecki criterion remains as a necessary and sufficient condition for entanglement.
- New optimisation strategies ensuring the violation of Bell inequalities (highly non-trivial for entangled mixed states) is presented.
- In particular, ρ_{ZZ} in the decay $H \to ZZ \to \ell_1^+ \ell_1^- \ell_2^+ \ell_2^-$ violates Bell inequalities for any value of the anomalous coupling c.

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Thank you for listening!