Exploring Bell inequalities and quantum entanglement in vector boson scattering

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Quantum Observables for Collider Physics

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Outline

- VBS processes
- S matrix and ρ density matrix formalism
- Entanglement quantifiers
- Numerical results
- Prospects at colliders
- Possible BSM analysis
- Summary

Why vector boson scattering?

Sensible to the deepest structure of the EW interactions in the SM

precise cancellation of potentially large contributions $\rightarrow$ unitarity restoration

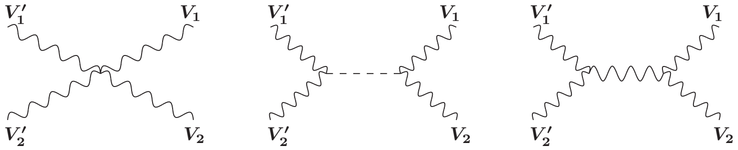
Higgs Mechanism dynamics

Active program of ATLAS and CMS [Covarelli *et al.* (2102.10991) Buarque Franzosi *et al.* (2106.01393)]

Suitable observable for New Physics looking for aTGC, aQGC and new Higgs interactions.

My previous works in the HEFT context: 1-loop renormalization, unitarity, matching with UV theories

$$V_1'(p_1', s_1') + V_2'(p_2', s_2') \rightarrow V_1(p_1, \mathbf{s_1}) + V_2(p_2, \mathbf{s_2})$$



VBS in Quantum Information context

Different **qubit** (photon $\{+,-\}$) and **qutrit** ($W^{\pm}$, Z $\{+,0,-\}$) final state bipartite systems

2⊗2

$W^+W^- \to \gamma\gamma$

Also $t\bar{t}$ production
[Afik *et al.*; Fabbrichesi *et al.*; Severi *et al.*; Aoude *et al.*; Aguilar-Saavedra *et al.*; Dong *et al.*]
$H \to \tau\tau, \gamma\gamma$
[Fabbrichesi *et al.*; Altakach *et al.*]

3⊗2

$W^{\pm}\gamma \to W^{\pm}\gamma$
$W^+W^- \to Z\gamma$
$W^{\pm}Z \to W^{\pm}\gamma$

Also single-top [Aguilar-Saavedra]

3⊗3

$\gamma\gamma \to W^+W^-$, $W^{\pm}\gamma \to W^{\pm}Z$, $W^+W^- \to W^+W^-$
$W^{\pm}W^{\pm} \to W^{\pm}W^{\pm}$, $W^+W^- \to ZZ$, $W^{\pm}Z \to W^{\pm}Z$
$Z\gamma \to W^+W^-$, $ZZ \to W^+W^-$, $ZZ \to ZZ$

Also $H \to WW$, ZZ and diboson production from fermions
[Barr *et al.*; Aguilar-Saavedra *et al.*; Ashby-Pickering *et al.*; Fabbrichesi *et al.*]

Investigate quantum entanglement with highest possible energy at colliders

S matrix and ρ density matrix formalism

Spin density matrix of the final state polarizations $|f\rangle = |s_1\rangle \otimes |s_2\rangle$
$\rho = |f\rangle\langle f|$

QFT: $\langle f|S|i\rangle = i(2\pi)^4\delta^{(4)}(p'_1 + p'_2 - p_1 - p_2)\mathcal{M}(V'_1 V'_2 \to V_1 V_2)$

Time-evolution of this density matrix through the S matrix operator:

$$\langle s_1\, s_2|\rho|\tilde{s}_1\, \tilde{s}_2\rangle = \frac{1}{|\overline{\mathcal{M}}|^2}\mathcal{M}_{s_1,s_2}\mathcal{M}^\dagger_{\tilde{s}_1,\tilde{s}_2}$$

Useful parametrization for quantum tomography [Ashby-Pickering *et al.* (2209.13990)]

$$\rho = \frac{1}{d_1 d_2} I_{d_1 d_2} + \frac{1}{2d_2}\sum_{i=1}^{d_1^2-1} \mathbf{A_i}\lambda_i^{(d_1)} \otimes I_{d_2} + \frac{1}{2d_1}\sum_{j=1}^{d_2^2-1} \mathbf{B_j} I_{d_1} \otimes \lambda_j^{(d_2)} + \frac{1}{4}\sum_{i=1}^{d_1^2-1}\sum_{j=1}^{d_2^2-1} \mathbf{C_{ij}}\lambda_i^{(d_1)} \otimes \lambda_j^{(d_2)}$$

Goal of this first step VBS analysis

- Theoretical predictions for VBS quantum properties.
- Locate relevant kinematical regions [cos(θ), $\sqrt{S}$] where quantum mechanical measurements might be performed.
- Determination of related quantities from simulations is postponed for future dedicated analysis.

Plan:

Define quantifiers related to entanglement detection and test Bell inequality.
They require the full knowledge of ρ.
Compute the scattering amplitudes $\mathcal{M}(S, \theta)$ of VBS processes.

Entanglement vs separability

convex combination of direct product states $\rho_{\rm sep} = \sum_n p_n \rho_n^{(V_1)} \otimes \rho_n^{(V_2)}$?

Peres-Horodecki or PPT Criterion: $\langle s_1\, s_2|\rho^{\rm T_2}|\tilde{s}_1\, \tilde{s}_2\rangle = \langle s_1\, \tilde{s}_2|\rho|\tilde{s}_1\, s_2\rangle$

necessary and sufficient conditions for entanglement
in 2⊗2 and 3⊗2 systems

Negativity $\mathcal{N}[\rho] = \frac{1}{2}\sum_k |\lambda_k^{\rm T_2}| - \lambda_k^{\rm T_2}$

For 3⊗3 case is just sufficient (special H decays [Aguilar-Saavedra *et al.* (2209.14033; 2209.13441)])

$\rho^2 = \rho$

Others entanglement quantifiers for pure states: $\rho_{red\,1} = {\rm Tr}_2[\rho] = \sum_{s_2}\langle s_2|\rho|s_2\rangle$

Entropy of Entanglement $\mathcal{S}_{\rm EE}[\rho] = -{\rm Tr}[\rho_{red}\log\rho_{red}] = -\sum \lambda^{red}\log(\lambda^{red})$

Concurrence $\mathcal{C}[\rho] = \sqrt{2(1 - {\rm Tr}[(\rho_{red})^2])}$

Separable state ↔ vanishing quantifiers
⇒ Restrictions over correlation matrix $\mathbf{C_{ij}}$

Entangled ⊃ Bell-nonlocal: test both phenomena at high-energy

Testing Bell inequalities

Bell inequality $\mathcal{I} \leq 2$ can be violated in QFT

2⊗2 case: optimal Clauser-Horne-Shimony-Holt operator

$$\mathcal{B}_{CHSH} = \vec{a}_1 \cdot \vec{\sigma} \otimes (\vec{b}_1 - \vec{b}_2) \cdot \vec{\sigma} + \vec{a}_2 \cdot \vec{\sigma} \otimes (\vec{b}_1 + \vec{b}_2) \cdot \vec{\sigma}$$

$$\Rightarrow \quad \boldsymbol{\mathcal{I}_2} = \underset{\vec{a}_i, \vec{b}_i}{\mathrm{Max}} \left\{ \mathrm{Tr}[\rho \cdot \mathcal{B}_{CHSH}] \right\} = 2\sqrt{r_1 + r_2}$$

r_1, r_2: two largest eigenvalues of $C^{\mathrm{T}} C$

Cirelson bound: $\mathcal{I}_2 \leq 2\sqrt{2}$ (MaxEnt)

[Lett. Math. Phys. 4 (1980)]

3⊗2 case: generalized CHSH operator (not optimal) [Caban *et al.* (0801.3200); Barr (2106.01377)]

$$\vec{S} = \left(\frac{\lambda_1 + \lambda_6}{\sqrt{2}}, \frac{\lambda_2 + \lambda_7}{\sqrt{2}}, \frac{\lambda_3 + \sqrt{3}\lambda_8}{2} \right)$$

$$\mathcal{B}^{gen}_{CHSH} = \vec{n}_1 \cdot \vec{S} \otimes (\vec{n}_2 - \vec{n}_4) \cdot \vec{\sigma} + \vec{n}_3 \cdot \vec{S} \otimes (\vec{n}_2 + \vec{n}_4) \cdot \vec{\sigma}$$

$$\Rightarrow \quad \boldsymbol{\mathcal{I}_{3\otimes 2}} = \underset{\vec{n}_i}{\mathrm{Max}} \left\{ \mathrm{Tr}[\rho \cdot \mathcal{B}^{gen}_{CHSH}] \right\} = 2\sqrt{\tilde{r}_1 + \tilde{r}_2}$$

$\tilde{r}_1, \tilde{r}_2$: two largest eigenvalues of $\tilde{C}^{\mathrm{T}} \tilde{C}$

Testing Bell inequalities for 3⊗3 case

Collins-Gisin-Linden-Massar-Popescu (CGLMP) $\rightarrow$ optimal

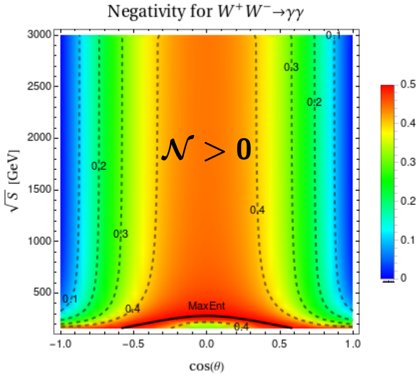
From a suitable Bell operator, corresponding to H decays [Barr (2106.01377)] here, an optimization through rotation matrices $U_k = e^{-iS_z\alpha_k} e^{-iS_y\beta_k}$ is performed

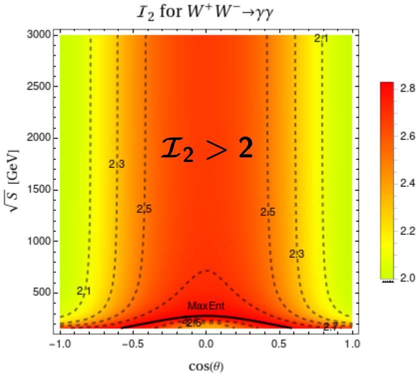
$$\hat{\mathcal{B}}^{xy}_{CGLMP} = -\frac{2}{\sqrt{3}}\left(S_x \otimes S_x + S_y \otimes S_y\right) + \lambda_4 \otimes \lambda_4 + \lambda_5 \otimes \lambda_5$$

$$\Rightarrow \quad \boldsymbol{\mathcal{I}_3} = \underset{\vec{\alpha}_i,\vec{\beta}_i}{\mathrm{Max}}\left\{\mathrm{Tr}[\rho \cdot (U_1 \otimes U_2)^\dagger \cdot \hat{\mathcal{B}}^{xy}_{CGLMP} \cdot (U_1 \otimes U_2)]\right\}$$

maximum is $1 + \sqrt{11/3} \approx 2.915$ but not achieved by MaxEnt (≈ 2.873)
Acin *et al.* [PRA 65 (2002)]

Analytic treatment with illustrative purpose: A_i, B_j, $\mathbf{C_{ij}}$, $\mathcal{N}$, $\mathcal{S}_{\rm EE}$ and $\mathcal{I}_2$





$$S|_{\rm MaxEnt} = \frac{12(1-\cos^2(\theta))M_{\rm W}^2}{1+3\cos^2(\theta)} \qquad \mathcal{I}_2 = 2\sqrt{2 - \frac{256}{D_{\gamma\gamma}^2}\left(S - 3M_{\rm W}^2\right)^2\left((1+3c^2)S - 12(1-c^2)M_{\rm W}^2\right)^2}$$

Caveat [Fabrichessi *et al.* (2208.11723)]

Requires polarization measurements of final photons (not currently in ATLAS nor CMS)

Proposals for CP properties in $H \to \gamma\gamma$ and LHCb measurements in b-baryon decays

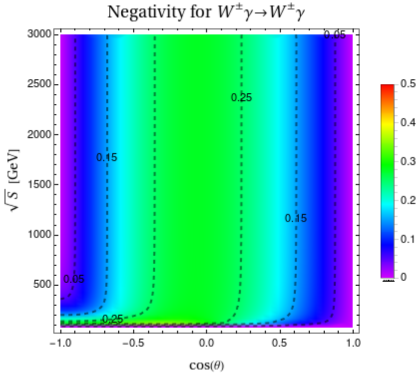
$3\otimes 2$ processes: entanglement

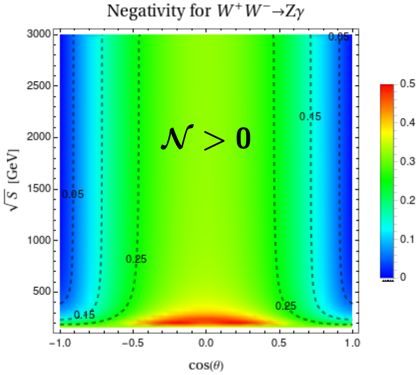
Analytic results for $W^{\pm}\gamma \to W^{\pm}\gamma$

Separable at $\sqrt{S} = M_{\rm W}$ or $\cos\theta = 1$

$W^{\pm}Z \to W^{\pm}\gamma$ similar to $W^{+}W^{-} \to Z\gamma$

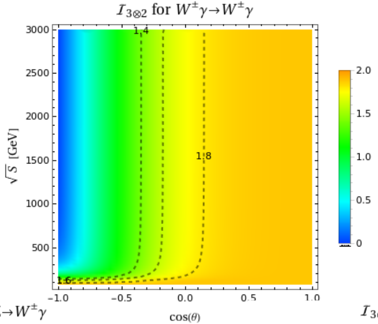
MaxEnt in part of the red region

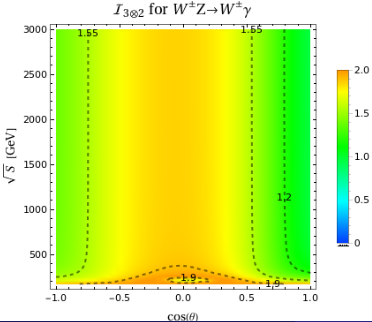


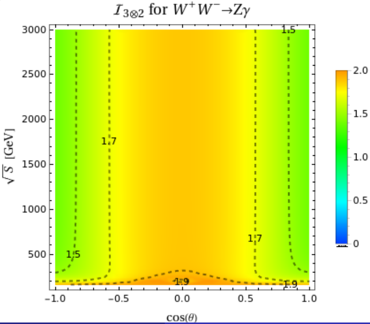


Similar pattern for $\mathcal{S}_{\rm EE}$

3⊗2 processes: Bell inequality







$$\underline{\mathcal{I}_{3\otimes2} \leq 2}$$

$\mathcal{B}^{gen}_{CHSH}$ diminished by vanishing spin [Caban *et al.* (0801.3200); Barr (2106.01377)]

No optimization for $\mathcal{B}_{3\otimes2}$ (as far as I know)

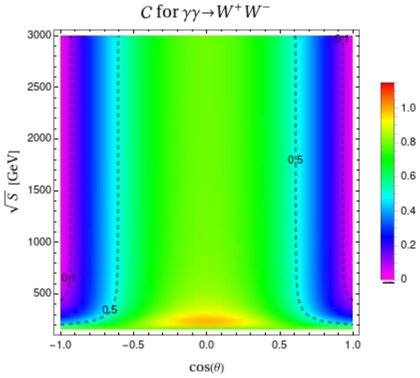
Entanglement not sufficient

Concurrence for 3⊗3 case

All processes have $\mathcal{C} > 0 \Rightarrow$ entangled final state

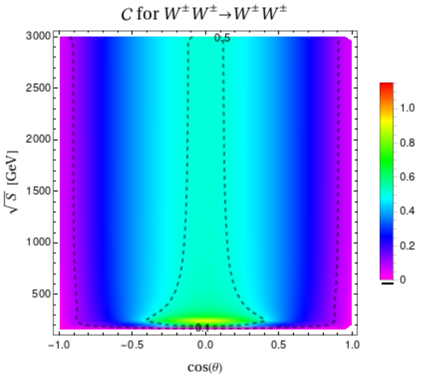
$\gamma\gamma \to W^+W^-$

Analytical results



$W^\pm W^\pm \to W^\pm W^\pm$

Experimental VBS golden channel



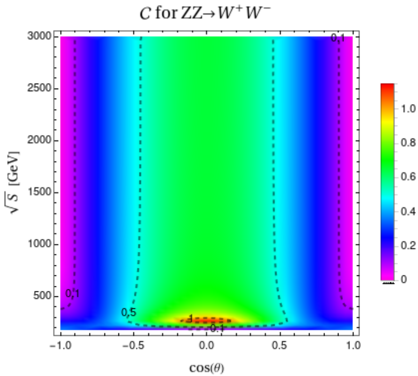
Maxima $\sim$ central lower energy region and similar pattern for $\mathcal{S}_{\text{EE}}$

Concurrence for 3⊗3 processes

<u>$ZZ \to W^+W^-$</u>

Maximal Concurrence $\sim 2/\sqrt{3}$

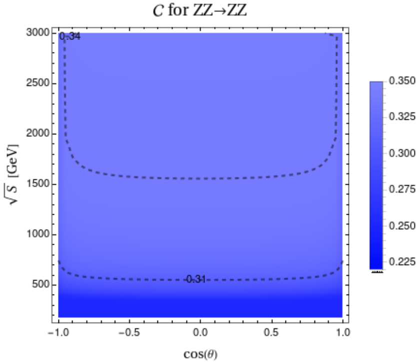
in red region



<u>$ZZ \to ZZ$</u>

Homogeneous distribution ~ 0.3

'less entangled'



Similar pattern for $\mathcal{S}_{\text{EE}}$

Optimization through rotation U matrices for each point of phase space.

$\gamma\gamma \to W^+W^-$

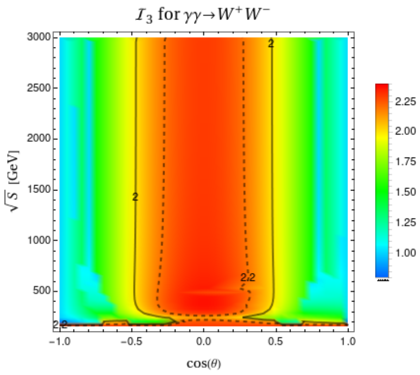
$\mathcal{I}_3 > 2$ for all energies

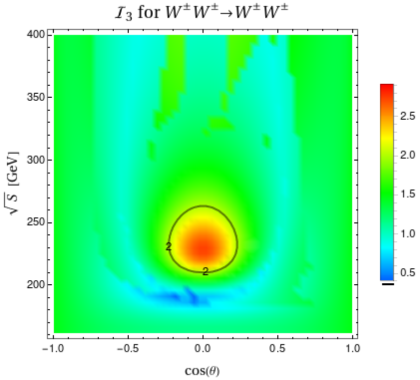
Maximum ~ 2.38

$W^\pm W^\pm \to W^\pm W^\pm$

Reduced region for $\mathcal{I}_3 > 2$

Maximum ~ 2.71



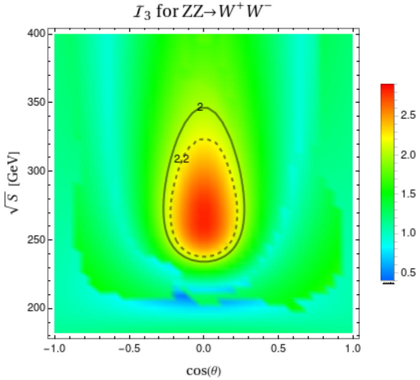


Bell inequality for 3⊗3 processes

$ZZ \to W^+W^-$

Reduced region for $\mathcal{I}_3 > 2$

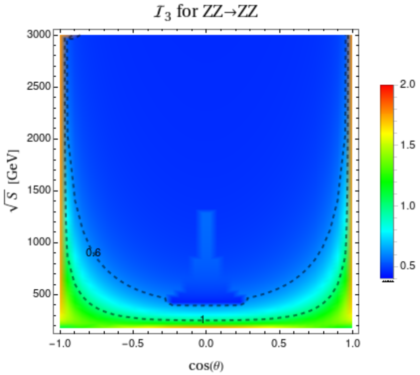
Maximum ~ 2.82



$ZZ \to ZZ$

$\mathcal{I}_3$ **lower** than 2

Maximum ~ 1.76

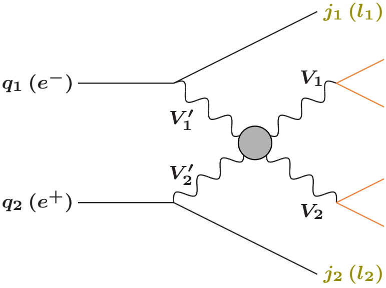


Broader regions for testing Bell inequality?

$\Rightarrow$ optimization through a more appropriate Bell operator [Aguilar-Saavedra *et al.* 2209.13441]

Prospects at colliders

$$pp \rightarrow (V_1 V_2)_{\text{leptonic decay}} + j_1 + j_2 \quad \text{for the LHC}$$
$$e^- e^+ \rightarrow (V_1 V_2)_{\text{leptonic decay}} + l_1 + l_2 \quad \text{for future electron-positron colliders}$$



All final state were observed (except diphoton)

σ_{tot} and σ_{fid} @13 TeV [Covarelli (PoS LHCP2021)]]

Typical VBS cuts: $m_{jj} > 500$ GeV and $\Delta\eta_{jj} > 4$

Mixed states in complete process (no control $|i\rangle$)

$$\rho = \sum \omega_{|i\rangle}^{q_1 q_2} \rho^{|f\rangle}$$

luminosity of q_i and V_i' in $\omega_{|i\rangle}^{q_1 q_2}$ using EWA/EPA [Dawson (1985); Weizsacker-Williams (1934)]

Significance with Monte-Carlo simulation (uncertainties, backgrounds, etc)

Next step: quantum tomography in the relevant regions
reconstruct ρ from data with dedicated analysis

The non-linear HEFT or EChL

- Symmetries: Lorentz, EW gauge $SU(2)_L \times U(1)_Y$ and EW Chiral $SU(2)_L \times SU(2)_R$ (based on ChPT of QCD)

- Light degrees of freedom and building blocks as in the SM:

  Higgs boson is a $SU(2)$ **singlet**, in contrast to the (linear) SM!
  EW gauge bosons $\Rightarrow \hat{W}_\mu = g W_\mu^a \tau^a/2,\ \hat{B}_\mu = g' B_\mu \tau^3/2,\ \hat{W}_{\mu\nu},\ \hat{B}_{\mu\nu}$.
  EW GBs π^a **transform non-linearly** under the EW Chiral symmetry

  $$\text{but } U = \exp\left(\frac{i\pi^a\tau^a}{v}\right) \text{ that transforms linearly } U \to g_L U g_R^\dagger$$

  $$\Rightarrow D_\mu U = \partial_\mu U + i\hat{W}_\mu U - iU\hat{B}_\mu \text{ and } \mathcal{V}_\mu = (D_\mu U)U^\dagger$$

  Our assumptions: fermionic interactions as in SM.

- Based on a **derivative expansion** $\leftrightarrow$ Chiral expansion (powers of p).

  Derivatives and masses are soft scales of the EFT with power counting $\mathcal{O}(p) \Rightarrow \mathcal{L}$ organized in terms of operators $\mathcal{O}(p^2)$, $\mathcal{O}(p^4)$, ...

  $$\text{bosonic sector } \mathcal{L}_{EChL} = \mathcal{L}_2 + \mathcal{L}_4 + \ldots$$

- Convenient for strongly interacting UV completions (in contrast to SMEFT)

Constraining the HEFT

New Physics in VBS:

$$\begin{aligned}
\mathcal{L}_2 &= \frac{1}{2}\partial_\mu H \partial^\mu H - V(H) - \frac{1}{2g^2}\mathrm{Tr}\left[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}\right] - \frac{1}{2g'^2}\mathrm{Tr}\left[\hat{B}_{\mu\nu}\hat{B}^{\mu\nu}\right] \\
&+ \frac{v^2}{4}\left(1 + 2\mathbf{a}\frac{H}{v} + \mathbf{b}\left(\frac{H}{v}\right)^2 + \dots\right)\mathrm{Tr}\left[D_\mu U^\dagger D^\mu U\right] \\
\mathcal{L}_4 &= \mathbf{a_1}\mathrm{Tr}\left[U\hat{B}_{\mu\nu}U^\dagger \hat{W}^{\mu\nu}\right] + i\mathbf{a_2}\mathrm{Tr}\left[U\hat{B}_{\mu\nu}U^\dagger[\mathcal{V}^\mu, \mathcal{V}^\nu]\right] - i\mathbf{a_3}\mathrm{Tr}\left[\hat{W}_{\mu\nu}[\mathcal{V}^\mu, \mathcal{V}^\nu]\right] \\
&+ \mathbf{a_4}\mathrm{Tr}\left[\mathcal{V}_\mu\mathcal{V}_\nu\right]\mathrm{Tr}\left[\mathcal{V}^\mu\mathcal{V}^\nu\right] + \mathbf{a_5}\mathrm{Tr}\left[\mathcal{V}_\mu\mathcal{V}^\mu\right]\mathrm{Tr}\left[\mathcal{V}_\nu\mathcal{V}^\nu\right] \\
&- \mathbf{a_{HBB}}\frac{H}{v}\mathrm{Tr}\left[\hat{B}_{\mu\nu}\hat{B}^{\mu\nu}\right] - \mathbf{a_{HWW}}\frac{H}{v}\mathrm{Tr}\left[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}\right] + \dots
\end{aligned}$$

Different kinematics with respect to the SM:

$$\begin{aligned}
i\Gamma^{\mu\nu\rho\sigma}_{WWZZ} &= -ig^2 c_{\mathrm{w}}^2\left(2g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho}\right) \\
&+ i\frac{g^4}{c_{\mathrm{w}}^2}\left(2(2c_{\mathrm{w}}^2\mathbf{a_3} + \mathbf{a_5})g^{\mu\nu}g^{\rho\sigma} - (2c_{\mathrm{w}}^2\mathbf{a_3} - \mathbf{a_4})(g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho})\right)
\end{aligned}$$

$\Rightarrow$ angular distributions (sensitivity to ρ) could also change

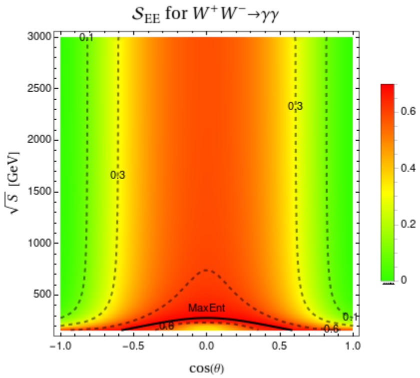
Summary

- VBS analysis from a new perspective
- Relevant kinematical regions for quantum measurements at sub-process level
- Final bosons result entangled after scattering in all processes (even MaxEnt in some cases)
- Testing Bell inequality ($\mathcal{I} > 2$) in reduced regions:
  $W^+W^- \to \gamma\gamma$ requires photon polarization measurement
  $W^\pm\gamma$, $Z\gamma$ final states and $ZZ \to ZZ$ not optimized Bell operator
  $\gamma\gamma \to W^+W^-$ and $ZZ \to W^+W^-$ most promising
- Guide to dedicated experimental searches
- Nest step:
  quantum tomography by means of Monte-Carlo simulation
  HEFT analysis
  optimize $\mathcal{I}_{3\otimes 2}$ and $\mathcal{I}_3$ quantifiers for VBS

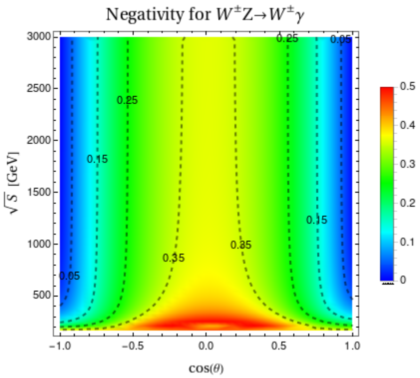
Backup slides

Additional plots

2⊗2

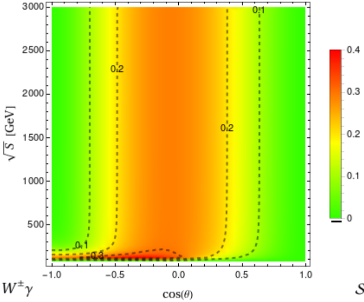


3⊗2

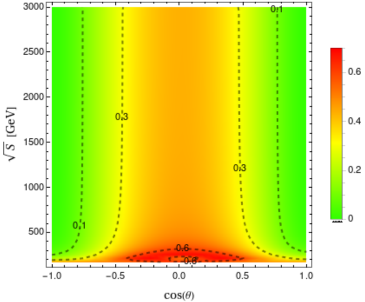


3⊗2 processes: Entropy of Entanglement

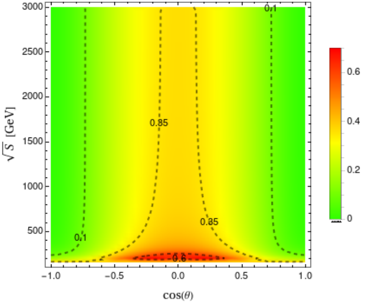
$\mathcal{S}_{\rm EE}$ for $W^{\pm}\gamma \to W^{\pm}\gamma$



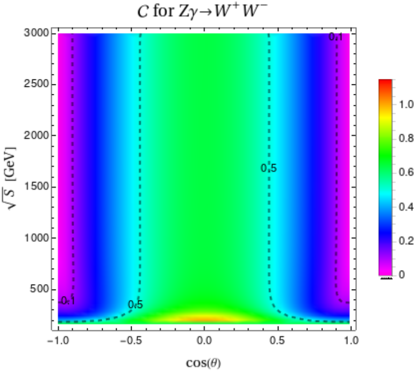
$\mathcal{S}_{\rm EE}$ for $W^{\pm}Z \to W^{\pm}\gamma$

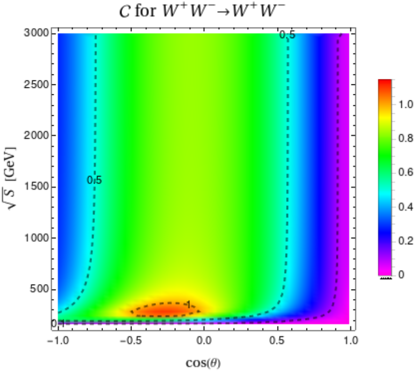


$\mathcal{S}_{\rm EE}$ for $W^{+}W^{-} \to Z\gamma$

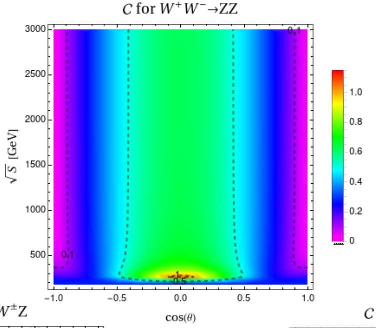


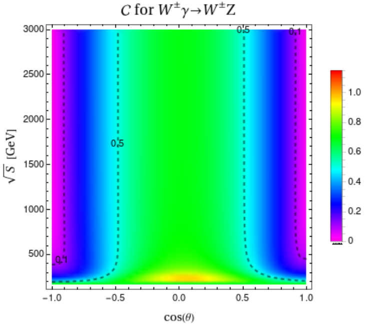
3⊗3 processes: Concurrence

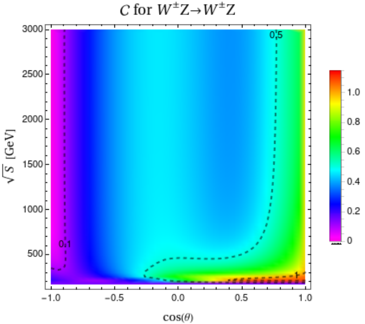




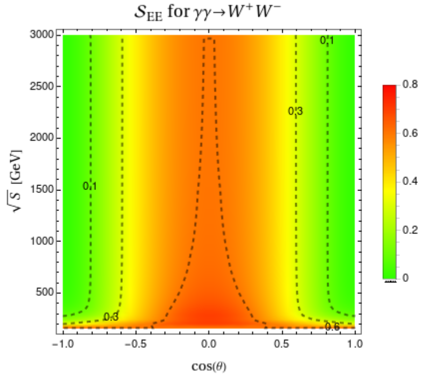
3⊗3 processes: Concurrence



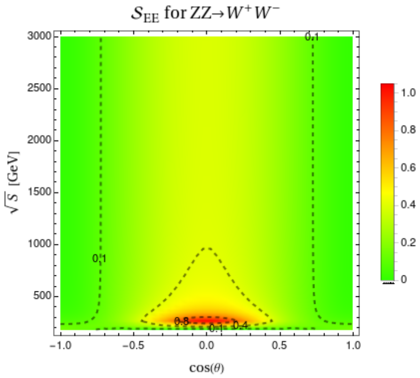


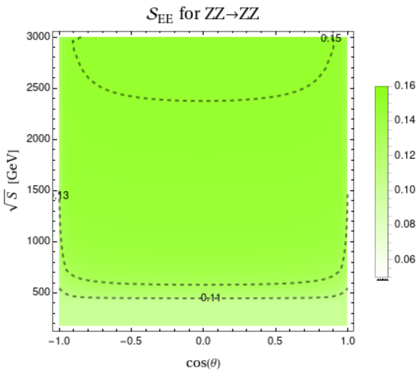


3⊗3 processes: Entropy of Entanglement



3⊗3 processes: Entropy of Entanglement





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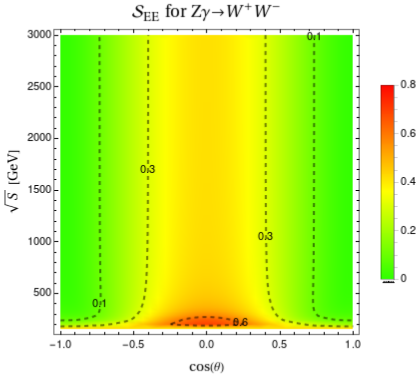
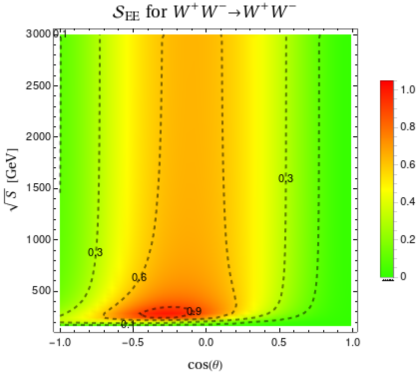
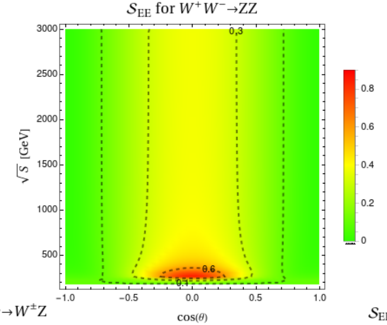
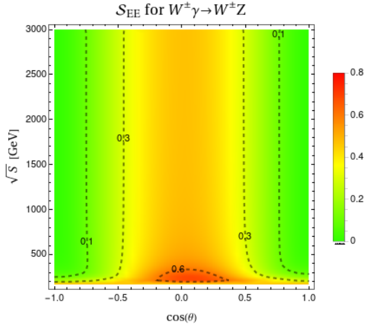
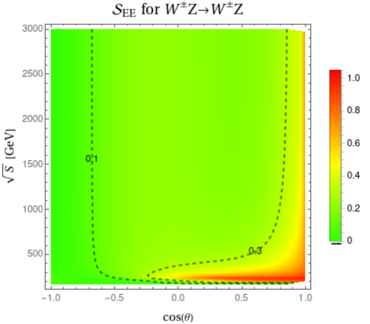




3⊗3 processes: Entropy of Entanglement







More about quantifiers

The Maximally Entangled two-qubits ($d = 2$) and two-qutrits ($d = 3$) **pure states** are defined as $|\Psi_{\rm MaxEnt}\rangle = \frac{1}{\sqrt{d}} \sum_{k=1}^{d} |k\rangle \otimes |k\rangle$ for which

$\mathcal{N}_{\rm MaxEnt} = \frac{d-1}{2}$, $\mathcal{S}_{\rm EE}^{\rm MaxEnt} = \log(d)$ and $\mathcal{C}_{\rm MaxEnt} = \sqrt{\frac{2(d-1)}{d}}$ Eltschka *et al.* [PRA 91 (2015)]

For **mixed states**, concurrence as (hard) optimization over pure sates of all possible decompositions: $\mathcal{C}_{\rm mix} = \inf\{ \sum p_i \mathcal{C}_i^{\rm pure} \}$

$\Rightarrow$ lower bound $\mathcal{C}_{\rm mix}^2 \geq C_2 = 2max\{0, {\rm Tr}[\rho^2] - {\rm Tr}[\rho_{red\,1}^2], {\rm Tr}[\rho^2] - {\rm Tr}[\rho_{red\,2}^2]\}$

Mintert *et al.* [PRL 92 (2007)]

and upper bound $\mathcal{C}_{\rm mix}^2 \leq 2min\{1 - {\rm Tr}[\rho_{red\,1}^2], 1 - {\rm Tr}[\rho_{red\,2}^2]\}$

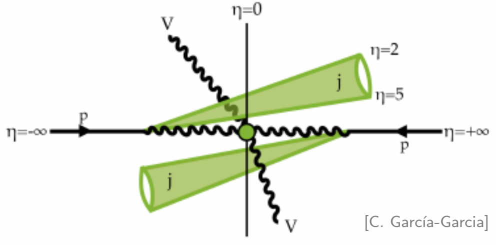
Zhang *et al.* [PRA 78 (2008)]

Effective W Approximation (EWA) Dawson [Nucl. Phys. B 249 no. 42 (1985)]

- W's and Z's considered as partons inside the proton.
  Generalization of the Weiszäcker-Williams approximation for photons.
- They are emitted collinearly from the fermions (quarks) with probability functions $f_V(\hat{x})$ and then scatter on-shell.
- Factorization using a sort of PDFs

$$\sigma(pp \to (V_1 V_2 \to V_3 V_4) + X) =$$

$$\sum_{i,j} \int\int dx_1 dx_2 f_{q_i}(x_1) f_{q_j}(x_2) \int\int d\hat{x}_1 d\hat{x}_2 f_{V_1}(\hat{x}_1) f_{V_2}(\hat{x}_2) \hat{\sigma}(V_1 V_2 \to V_3 V_4)$$



More about the EWA

The most accurate EWA expression in our setup is the Dawson's Improved

$$f^{Improved}_{V_T}(\hat{x}) = \frac{C_V^2 + C_A^2}{8\pi^2 \hat{x}}\eta \left[\frac{-\hat{x}^2}{1 + M_V^2/(4E^2(1-\hat{x}))} + \frac{2\hat{x}^2(1-\hat{x})}{M_V^2/E^2 - \hat{x}^2} + \left\{ \hat{x}^2 + \frac{\hat{x}^4(1-\hat{x})}{(M_V^2/E^2 - \hat{x}^2)^2}\left(2 + \frac{M_V^2}{E^2(1-\hat{x})}\right) - \frac{\hat{x}^2}{(M_V^2/E^2 - \hat{x}^2)^2}\frac{M_V^4}{2E^4} \right\} \log\left(1 + \frac{4E^2(1-\hat{x})}{M_V^2}\right) + \hat{x}^4 \left(\frac{2-\hat{x}}{M_V^2/E^2 - \hat{x}^2}\right)^2 \log\frac{\hat{x}}{2-\hat{x}} \right]$$

with $C_{V(A)}$ the vector(axial) couplings Vqq, $\hat{x}$ the fraction of quark energy $E = \frac{\sqrt{s_{qq}}}{2}$ carried by V and $\eta \equiv \left(1 - \frac{M_V^2}{\hat{x}^2 E^2}\right)^{1/2}$

In the limit $M_V \ll E$ (LLA) $\Rightarrow$

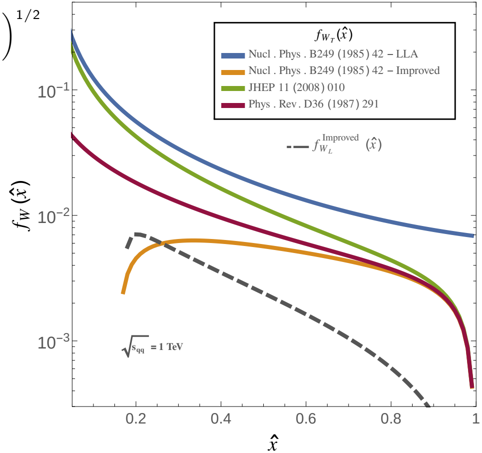
$$f^{LLA}_{V_T}(\hat{x}) = \frac{C_V^2 + C_A^2}{8\pi^2 \hat{x}}\left[\hat{x}^2 + 2(1-\hat{x})\right]\log\left(\frac{4E^2}{M_V^2}\right)$$

Among different f_V

In the high $\hat{x}$ region: similar results.

In the low $\hat{x}$ region: differ quite a lot.

Dawson's Improved gets correct $\sigma(pp \to V_1 V_2 + jj)$ in low m_{VV} region (most events here).



More about the EWA

The most accurate EWA expression in our setup is the Dawson's Improved

$$f_{V_T}^{Improved}(x) = \frac{C_V^2 + C_A^2}{8\pi^2 x}\left[\frac{-x^2}{1 + M_V^2/(4E^2(1-x))} + \frac{2x^2(1-x)}{M_V^2/E^2 - x^2} + \left\{x^2 + \frac{x^4(1-x)}{(M_V^2/E^2 - x^2)^2}\left(2 + \frac{M_V^2}{E^2(1-x)}\right) - \frac{x^2}{(M_V^2/E^2 - x^2)^2}\frac{M_V^4}{2E^4}\right\}\log\left(1 + \frac{4E^2(1-x)}{M_V^2}\right) + x^4\left(\frac{2-x}{M_V^2/E^2 - x^2}\right)^2 \log\frac{x}{2-x}\right]\eta$$

$$f_{V_L}^{Improved}(x) = \frac{C_V^2 + C_A^2}{\pi^2}\frac{1-x}{x}\frac{\eta}{(1+\eta)^2}\left\{\frac{1 - x - M_V^2/(8E^2)}{1 - x + M_V^2/(4E^2)} - \frac{M_V^2}{4E^2}\frac{1 + 2(1-x)^2}{1 - x + M_V^2/(4E^2)}\frac{1}{M_V^2/E^2 - x^2} - \frac{M_V^2}{4E^2}\frac{x^2}{2(1-x)(x^2 - M_V^2/E^2)^2}\left[(2-x)^2\log\frac{x}{2-x} - \left(\left(x - \frac{M_V^2}{E^2 x}\right)^2 - (2(1-x) + x^2)\right)\log\left(1 + \frac{4E^2(1-x)}{M_V^2}\right)\right] - \frac{M_V^2}{8E^2}\frac{x}{\sqrt{x^2 - M_V^2/E^2}}\left[\frac{2}{x^2 - M_V^2/E^2} + \frac{1}{1-x}\right]\left[\log\frac{2 - x - \sqrt{x^2 - M_V^2/E^2}}{2 - x + \sqrt{x^2 - M_V^2/E^2}} - \log\frac{x - \sqrt{x^2 - M_V^2/E^2}}{x + \sqrt{x^2 - M_V^2/E^2}}\right]\right\}$$

with $C_{V(A)}$ the vector(axial) couplings Vqq, x the fraction of quark energy $E = \frac{\sqrt{s_{qq}}}{2}$ carried by V and $\eta \equiv \left(1 - \frac{M_V^2}{x^2 E^2}\right)^{1/2}$

In the limit $M_V \ll E$ (LLA) $\Rightarrow$

$$f_{V_T}^{LLA}(x) = \frac{C_V^2 + C_A^2}{8\pi^2 x}\left[x^2 + 2(1-x)\right]\log\left(\frac{4E^2}{M_V^2}\right)$$

$$f_{V_L}^{LLA}(x) = \frac{C_V^2 + C_A^2}{\pi^2}\frac{1-x}{x}$$