Testing entanglement and Bell inequalities in H→ZZ

In collaboration with

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Quantum Observables for Collider Physics

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EXCELENCIA SEVERO OCHOA

Quantum Entanglement

- Entanglement is perhaps the aspect of quantum mechanics that shows the greatest departure from classical conceptions

- 1935: a strange phenomenon of quantum mechanics, questioning the completeness of the theory

Einstein, Podolsky, and Rosen 1935

Schrödinger 1935

- 1964: Bell realised that entanglement leads to experimentally testable deviations of quantum mechanics from classical physics

Bell 1964

- With the emergence of quantum information theory, entanglement was recognized as a resource, enabling tasks like quantum cryptography, quantum teleportation or measurement based quantum computation: a *threat* became an *opportunity*

- Worth mentioning: the problem of classifying and quantifying the entanglement of general multipartite systems is still an open problem

Jose Ignacio Latorre talk

O. Gühne, G.Tóth 2009

Bell inequalities

The physical consequence of entanglement that departs from classical intuition is the violation of Bell inequalities, an impossible result in any local-realistic ("classical") theory of nature.

CHSH

Clauser, Horne, Shimony and Holt, 1969

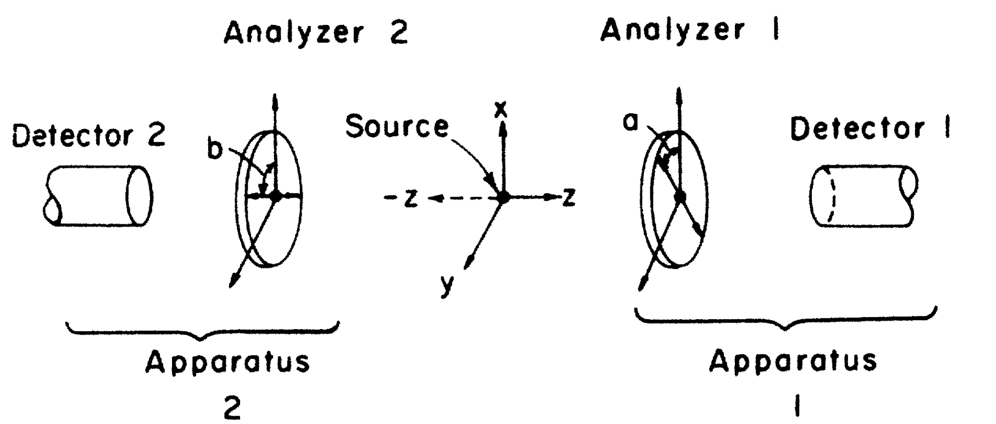


FIG. 1. Scheme considered for a discussion of objective local theories. A source emitting particle pairs is viewed by two apparatuses. Each apparatus consists of an analyzer and an associated detector. The analyzers have parameters, *a* and *b* respectively, which are externally adjustable. In the above example, *a* and *b* represent the angles between the analyzer axes and a fixed reference axis.

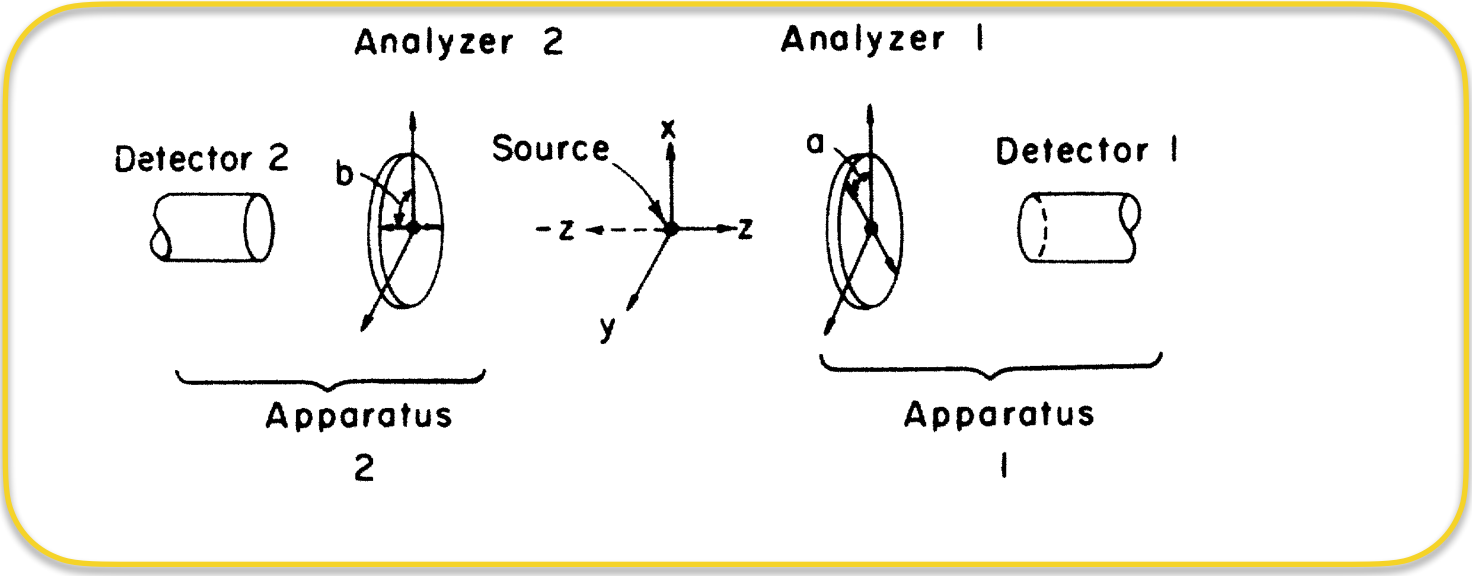
From Clauser, Horne, PRD 10 (1974) 526

Alice (Bob) chooses to measure certain (bi-valued) observables, A, A' (B, B')

Then, classically,

$$\left|\langle AB\rangle - \langle AB'\rangle + \langle A'B\rangle + \langle A'B'\rangle\right| \leq 2$$

Bell inequalities in vector boson Higgs decays

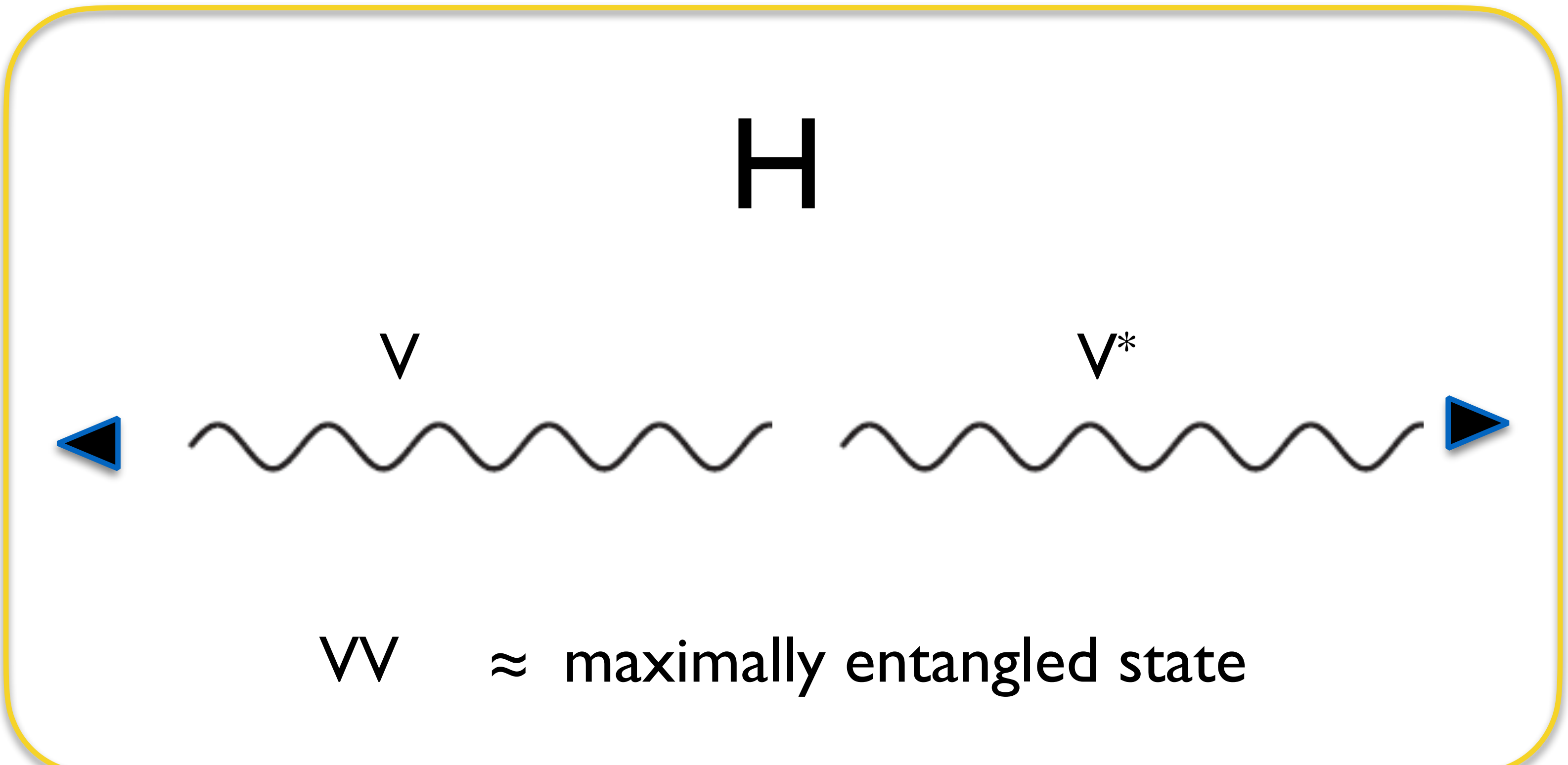


Alan Barr talk

Alexander Bernal talk

Theo Maurin talk

Luca Marzola talk



A.J. Barr, 2022

A.J. Barr, P. Caban, J. Rembieliński, 2022

J.A. Aguilar-Saavedra, 2022

J. A. Aguilar-Saavedra, A. Bernal, J. A. Casas, J. M. Moreno 2022

R. Ashby-Pickering, A.J. Barr, A. Wierzchuck 2022

A. Bernal ,P. Caban, J. Rembieliński, 2023

F. Fabbri, J. Howarth, T. Maurin 2023

M. Fabbrichesi, R. Floreanini, E. Gabrielli, L. Marzola 2023

R. Aoude, E. Madge, F. Maltoni, L. Mantani 2023

Exploring Bell inequalities in H ⟶ ZZ

Based on:

PHYSICAL REVIEW D **107,** 016012 (2023)

Testing entanglement and Bell inequalities in $H \to ZZ$

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We discuss quantum entanglement and violation of Bell inequalities in the $H \to ZZ$ decay, in particular when the two Z-bosons decay into light leptons. Although such process implies an important suppression of the statistics, this is traded by clean signals from a "quasi maximally entangled" system, which makes it very promising to check these crucial phenomena at high energy. In this paper we devise a novel framework to extract from $H \to ZZ$ data all significant information related to this goal, in particular spin correlation observables. In this context we derive sufficient and necessary conditions for entanglement in terms of only two parameters. Likewise, we obtain a sufficient and improved condition for the violation of Bell-type inequalities. The numerical analysis shows that with a luminosity of $L = 300\ \mathrm{fb}^{-1}$ entanglement can be probed at $> 3\sigma$ level. For $L = 3\ \mathrm{ab}^{-1}$ (HL-LHC) entanglement can be probed beyond the 5σ level, while the sensitivity to a violation of the Bell inequalities is at the 4.5σ level.

DOI: 10.1103/PhysRevD.107.016012

- Let $(m_{Z_1},\ m_{Z_2})$ the invariant masses for a *particular event:*

  In the CM reference, z-axis along Z_1 momentum $\vec{k}$

  $|\vec{k}|$ fixed by $(m_{Z_1}, m_{Z_2},\ m_H)$

- J_z - and parity - conservation imply $|\psi_{ZZ}\rangle = \dfrac{1}{\sqrt{2+\beta^2}} \left(|{+}{-}\rangle - \beta\,|00\rangle + |{-}{+}\rangle\right)$

- From the Lorentz structure of SM HZZ vertex, $\propto \eta_{\mu\nu} H Z^\mu Z^\nu$

$$|\psi_{ZZ}\rangle = \eta_{\mu\nu}\ e^\mu_\sigma(m_1,\vec{k})\ e^\nu_\lambda(m_2,-\vec{k})\ |\vec{k},\sigma\rangle_A |-\vec{k},\lambda\rangle_B$$

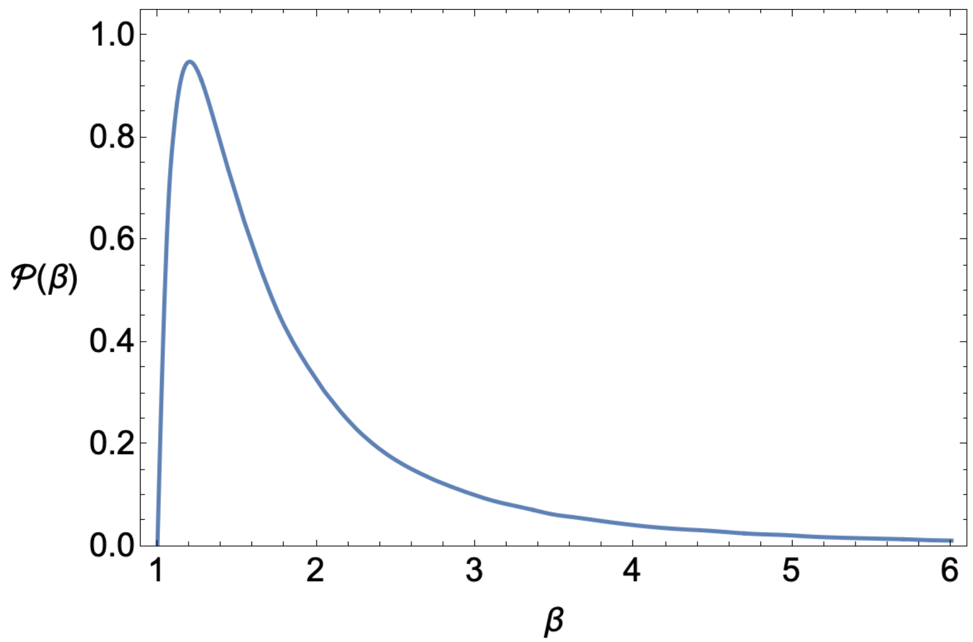
$$e^\mu_\sigma(m,\vec{k}) = \begin{pmatrix} 0 & \frac{|\vec{k}|}{m} & 0 \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{i}{\sqrt{2}} & 0 & \frac{i}{\sqrt{2}} \\ 0 & -\frac{\sqrt{|\vec{k}|^2+m^2}}{m} & 0 \end{pmatrix}$$

One obtains

$$\beta = 1 + \frac{m_H^2 - (m_1+m_2)^2}{2m_1 m_2}$$

Since $|\vec{k}| \geq 0$, then $\beta \geq 1$

Exploring Bell inequalities in $H \longrightarrow ZZ$



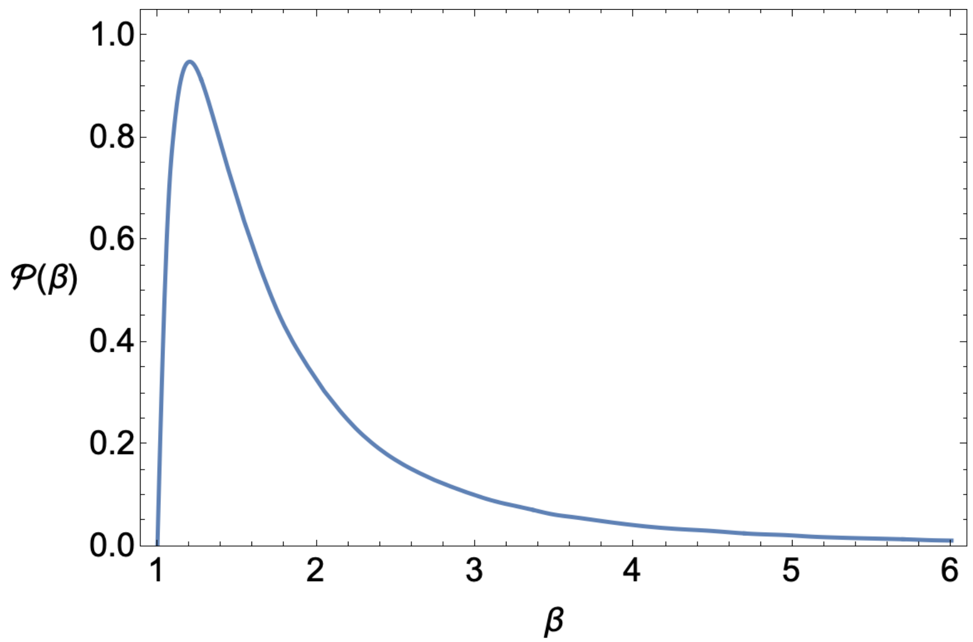
The quantum ZZ state is a mixed state, shaped by the kinematics

$$\rho = \int d\beta \; \mathcal{P}(\beta)\rho_\beta$$

$$\rho_\beta = \frac{1}{2+\beta^2}\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -\beta & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\beta & 0 & \beta^2 & 0 & -\beta & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -\beta & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$(|++\rangle, |+0\rangle, |+-\rangle, |0+\rangle, |00\rangle, |0-\rangle, |-+\rangle, |-0\rangle, |--\rangle)$$

Exploring Bell inequalities in H ⟶ ZZ



The quantum ZZ state is a mixed state, shaped by the kinematics

$$\rho = \int d\beta\ \mathcal{P}(\beta)\rho_\beta$$

The ***numerical*** probability $\mathcal{P}(\beta)$ obtained with the Monte Carlo agrees (~ few %) with the ***analytical*** one obtained by phase space analysis of 3 body decay $H \to Z\ell^+\ell^-$

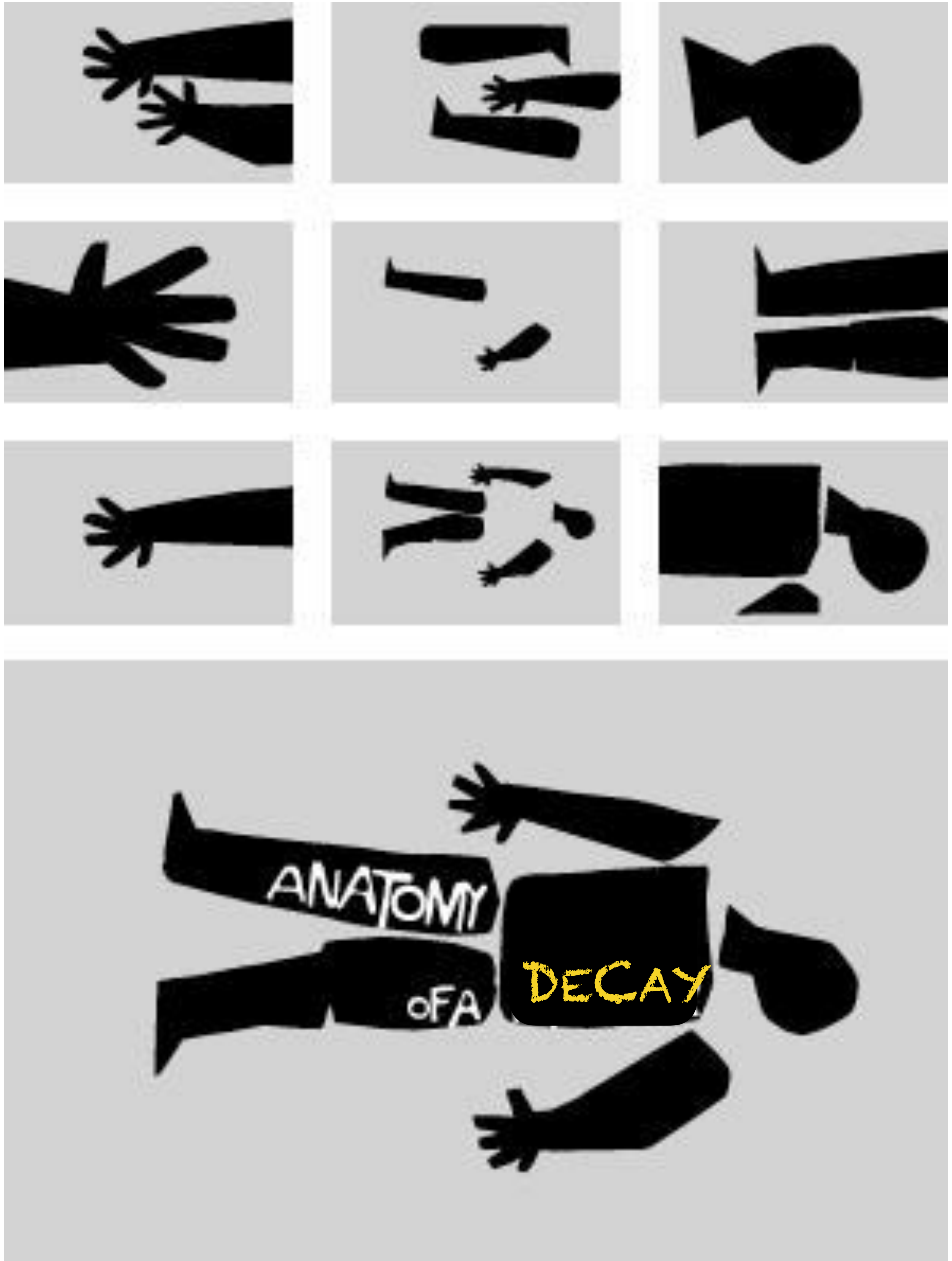
$$\rho = \frac{1}{2+w^2}\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -y & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -y & 0 & w^2 & 0 & -y & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -y & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Notice that

$$y \neq w$$

because

$$\langle\beta^2\rangle \neq \langle\beta\rangle^2$$



QUANTUM TOMOGRAPHY

Reconstructing a quantum state using several measurements on an ensamble of identical states

i) Choose (convenient) a basis for ρ (symmetries, etc)

ii) Express the experimental measurements as functions of the expansion coefficients

Exploring Bell inequalities in H $\longrightarrow$ ZZ

- A convenient way to parametrize the 9 × 9 spin density-operator of the two vector bosons is to use the basis of irreducible tensor operators $\{T^{L_1}_{M_1} \otimes T^{L_2}_{M_2}\}$

$$T^{L_1}_{M_1}, T^{L_2}_{M_2} \in \{\mathbb{1}_3; T^1_1, T^1_0, T^1_{-1}; T^2_2, T^2_1, T^2_0, T^2_{-1}, T^2_{-2}\} \qquad \mathrm{Tr}\left\{T^L_M \; (T^L_M)^\dagger\right\} = 3,$$

T^L_M vs Gell-Mann matrices

$$T^1_1 = \sqrt{\frac{3}{2}}\begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{pmatrix}, \quad T^1_0 = \sqrt{\frac{3}{2}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \quad T^1_{-1} = \sqrt{\frac{3}{2}}\begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}.$$

$$\begin{aligned} T^2_{\pm 2} &= \frac{2}{\sqrt{3}}(T^1_{\pm 1})^2, \\ T^2_{\pm 1} &= \sqrt{\frac{2}{3}}\left[T^1_{\pm 1}T^1_0 + T^1_0 T^1_{\pm 1}\right], \\ T^2_0 &= \frac{\sqrt{2}}{3}\left[T^1_1 T^1_{-1} + T^1_{-1}T^1_1 + 2(T^1_0)^2\right] \end{aligned}$$

$$\rho = \frac{1}{9}\left[\mathbb{1}_3 \otimes \mathbb{1}_3 + A^1_{LM}\; T^L_M \otimes \mathbb{1}_3 + A^2_{LM}\; \mathbb{1}_3 \otimes T^L_M + C_{L_1 M_1 L_2 M_2}\; T^{L_1}_{M_1} \otimes T^{L_2}_{M_2}\right]$$

8 +8+64

80 components

$$\rho = \frac{1}{9}\left[\mathbb{1}_3 \otimes \mathbb{1}_3 + A^1_{LM}\; T^L_M \otimes \mathbb{1}_3 + A^2_{LM}\; \mathbb{1}_3 \otimes T^L_M + C_{L_1 M_1 L_2 M_2}\; T^{L_1}_{M_1} \otimes T^{L_2}_{M_2}\right]$$

The differential $ZZ \to \ell_1^+ \ell_1^- \ell_2^+ \ell_2^-$ cross section is given by

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega_1 d\Omega_2} = \left(\frac{3}{4\pi}\right)^2 \mathrm{T}r\left\{\rho\,(\Gamma_1 \otimes \Gamma_2)^T\right\}$$

with Γ, the decay density matrix of a Z boson into $\ell^+\ell^-$, given by

$$\Gamma = \frac{1}{4}\begin{pmatrix} 1+\cos^2\theta - 2\eta_\ell\cos\theta & \frac{1}{\sqrt{2}}(\sin 2\theta - 2\eta_\ell \sin\theta)e^{i\varphi} & (1-\cos^2\theta)e^{i2\varphi} \\ \frac{1}{\sqrt{2}}(\sin 2\theta - 2\eta_\ell \sin\theta)e^{-i\varphi} & 2\sin^2\theta & -\frac{1}{\sqrt{2}}(\sin 2\theta + 2\eta_\ell \sin\theta)e^{i\varphi} \\ (1-\cos^2\theta)e^{-i2\varphi} & -\frac{1}{\sqrt{2}}(\sin 2\theta + 2\eta_\ell \sin\theta)e^{-i\varphi} & 1+\cos^2\theta - 2\eta_\ell\cos\theta \end{pmatrix} \qquad \eta_\ell$$

Using

$$\mathrm{Tr}\left\{\mathbb{1}_3\;\Gamma^T\right\} = 2\sqrt{\pi}\;Y_0^0(\theta,\varphi), \quad \mathrm{Tr}\left\{T^1_M\;\Gamma^T\right\} = -\sqrt{2\pi}\eta_\ell\;Y_1^M(\theta,\varphi), \quad \mathrm{Tr}\left\{T^2_M\;\Gamma^T\right\} = \sqrt{\frac{2\pi}{5}}\;Y_2^M(\theta,\varphi)$$

We can *very easily* extract

$$A^j_{LM} \;\equiv\; \int \frac{1}{\sigma}\frac{d\sigma}{d\Omega_1 d\Omega_2} Y_L^M(\Omega_j)\, d\Omega_j \qquad\qquad C_{L_1 M_1 L_2 M_2} \;\equiv\; \int \frac{1}{\sigma}\frac{d\sigma}{d\Omega_1 d\Omega_2} Y_{L_1}^{M_1}(\Omega_1) Y_{L_2}^{M_2}(\Omega_2)\, d\Omega_1 d\Omega_2$$

$$\rho = \frac{1}{9}\left[\mathbb{1}_3 \otimes \mathbb{1}_3 + A^1_{LM}\, T^L_M \otimes \mathbb{1}_3 + A^2_{LM}\, \mathbb{1}_3 \otimes T^L_M + C_{L_1 M_1 L_2 M_2}\, T^{L_1}_{M_1} \otimes T^{L_2}_{M_2}\right]$$

The differential $ZZ \to \ell_1^+ \ell_1^- \ell_2^+ \ell_2^-$ cross section is given by

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega_1 d\Omega_2} = \left(\frac{3}{4\pi}\right)^2 \mathrm{T}r\left\{\rho\left(\Gamma_1 \otimes \Gamma_2\right)^T\right\}$$

This Quantum Tomography Method can be generalized to other processes !!

A. Bernal, arXiv 2310.10838

Alternative method:

R. Ashby-Pickering, A.J. Barr, A. Wierzchuck 2022

$$A^j_{LM} \equiv \int \frac{1}{\sigma}\frac{d\sigma}{d\Omega_1 d\Omega_2} Y^M_L(\Omega_j) d\Omega_j \qquad C_{L_1 M_1 L_2 M_2} \equiv \int \frac{1}{\sigma}\frac{d\sigma}{d\Omega_1 d\Omega_2} Y^{M_1}_{L_1}(\Omega_1) Y^{M_2}_{L_2}(\Omega_2) d\Omega_1 d\Omega_2$$

Density matrix parameterization

We get

$$\rho = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{1}{6}\left(\sqrt{2}A^1_{2,0}+2\right) & 0 & \frac{1}{3}C_{2,1,2,-1} & 0 & \frac{1}{3}C_{2,2,2,-2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{1}{3}C_{2,1,2,-1} & 0 & \frac{1}{3}\left(1-\sqrt{2}A^1_{2,0}\right) & 0 & \frac{1}{3}C_{2,1,2,-1} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{1}{3}C_{2,2,2,-2} & 0 & \frac{1}{3}C_{2,1,2,-1} & 0 & \frac{1}{6}\left(\sqrt{2}A^1_{2,0}+2\right) & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}.$$

With the constraint

$$\frac{1}{\sqrt{2}}A^1_{2,0} + 1 = C_{2,2,2,-2}$$

We do not impose this relation when extracting the coefficients. It could be used:

- A way to estimate the uncertainties in the experimental determination of the density matrix

- To improve the determination of the independent coefficients and thereby improve the precision in the measurement of the entanglement observables.

Entanglement in H $\longrightarrow$ ZZ

- A generic, bipartite, quantum system is entangled iff

$$\rho_{\rm ent} \neq \rho_{\rm sep} = \sum_n p_n \, \rho_n^A \otimes \rho_n^B$$

with $p_n > 0$

- Checking separability is not , in general, an easy task

- Peres-Horodecki *sufficient* condition: $\rho = \rho_{i\mu,j\nu}$ non-separable if

A. Peres 1996

P Horodecki 1997

$$\rho^{T_2} = \rho_{i\nu,j\mu}$$ has at least a negative eigenvalue

Entanglement in $H \longrightarrow ZZ$

Generic spin-density matrix with vanishing third-component,

$$\rho = \begin{pmatrix} 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&a&0&b&0&c&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&b^*&0&d&0&f&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&c^*&0&f^*&0&g&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0 \end{pmatrix} \qquad \rho^{T_2} = \begin{pmatrix} 0&0&0&0&0&0&0&0&c\\ 0&0&0&0&0&b&0&0&0\\ 0&0&a&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&f&0\\ 0&0&0&0&d&0&0&0&0\\ 0&b^*&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&g&0&0\\ 0&0&0&f^*&0&0&0&0&0\\ c^*&0&0&0&0&0&0&0&0 \end{pmatrix}$$

Eigenvalues:

$$a, d, g, \pm|b|, \pm|c|, \pm|f|$$

- Therefore, for $b \neq 0,\ c \neq 0$ or $f \neq 0$ the density matrix is entangled
- Notice that if $b = c = f = 0$ the density matrix is separable

  A noteworthy example beyond a two-qubit system, where, thanks to an underlying symmetry, the Peres-Horodecki condition for entanglement is not just *sufficient*, but also *necessary.*

- This result is relevant for both SM and BSM $H \to ZZ$ & $H \to WW$ density matrices

Entanglement in H ⟶ ZZ

In our particular case :

$$\rho = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{6}\left(\sqrt{2}A^1_{2,0}+2\right) & 0 & \frac{1}{3}C_{2,1,2,-1} & 0 & \frac{1}{3}C_{2,2,2,-2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{3}C_{2,1,2,-1} & 0 & \frac{1}{3}\left(1-\sqrt{2}A^1_{2,0}\right) & 0 & \frac{1}{3}C_{2,1,2,-1} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{3}C_{2,2,2,-2} & 0 & \frac{1}{3}C_{2,1,2,-1} & 0 & \frac{1}{6}\left(\sqrt{2}A^1_{2,0}+2\right) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

The quantum system is entangled IFF

$$C_{2,1,2,-1} \neq 0 \qquad \text{or} \qquad C_{2,2,2,-2} \neq 0$$

- BELL INEQUALITIES ?

Technical issue:

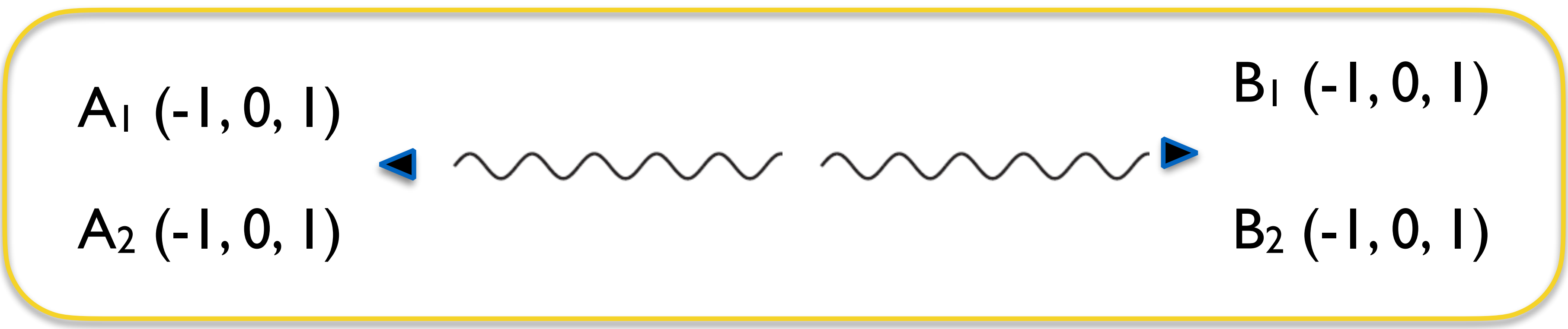
We are dealing with "qutrits" (- , 0, +)

The optimal inequalities are not CHSH but CGLMP

Bell inequalities in H→ZZ

Collins, Gisin, Linden, Massar, Popescu, 2002

- $$I_3 = P(A_1 = B_1) + P(B_1 = A_2 + 1) + P(A_2 = B_2) + P(B_2 = A_1)$$
  $$- \left[P(A_1 = B_1 - 1) + P(B_1 = A_2) + P(A_2 = B_2 - 1) + P(B_2 = A_1 - 1)\right] \le 2$$



($A_{1,2}$, $B_{1,2}$) chosen to optimize I_3

- The inequality can be written in terms of a Bell operator

$$I_3 = \mathrm{Tr}\left\{\rho\, \mathcal{O}_{\mathrm{Bell}}\right\}$$

$$\mathcal{O}_{\mathrm{Bell}} = \mathcal{O}_{\mathrm{Bell}}(U_{A_1}, U_{A_2}, U_{B_1}, U_{B_2})$$

Optimal Bell operator

Collins, Gisin, Linden, Massar, Popescu, 2002

- For the maximally entangled pure state, computational basis

$$|\psi'\rangle = \frac{1}{\sqrt{3}}(|11\rangle + |22\rangle + |33\rangle)$$

← invariant under $U \otimes U^*$

$$\mathcal{O}'_{\rm Bell} = \frac{4}{3\sqrt{3}}(T_1^1 \otimes T_1^1 + T_{-1}^1 \otimes T_{-1}^1) + \frac{2}{3}\,(T_2^2 \otimes T_2^2 + T_{-2}^2 \otimes T_{-2}^2)$$

Acin, Durt, Gisin, Latorre 2002

- Can be mapped into the pure singlet state (β = 1)

$$|\psi_s\rangle = \frac{1}{\sqrt{3}}(|+-\rangle - |00\rangle + |-+\rangle)$$

$$|\psi_s\rangle \to U O_A \otimes U^* \, |\psi_s\rangle$$

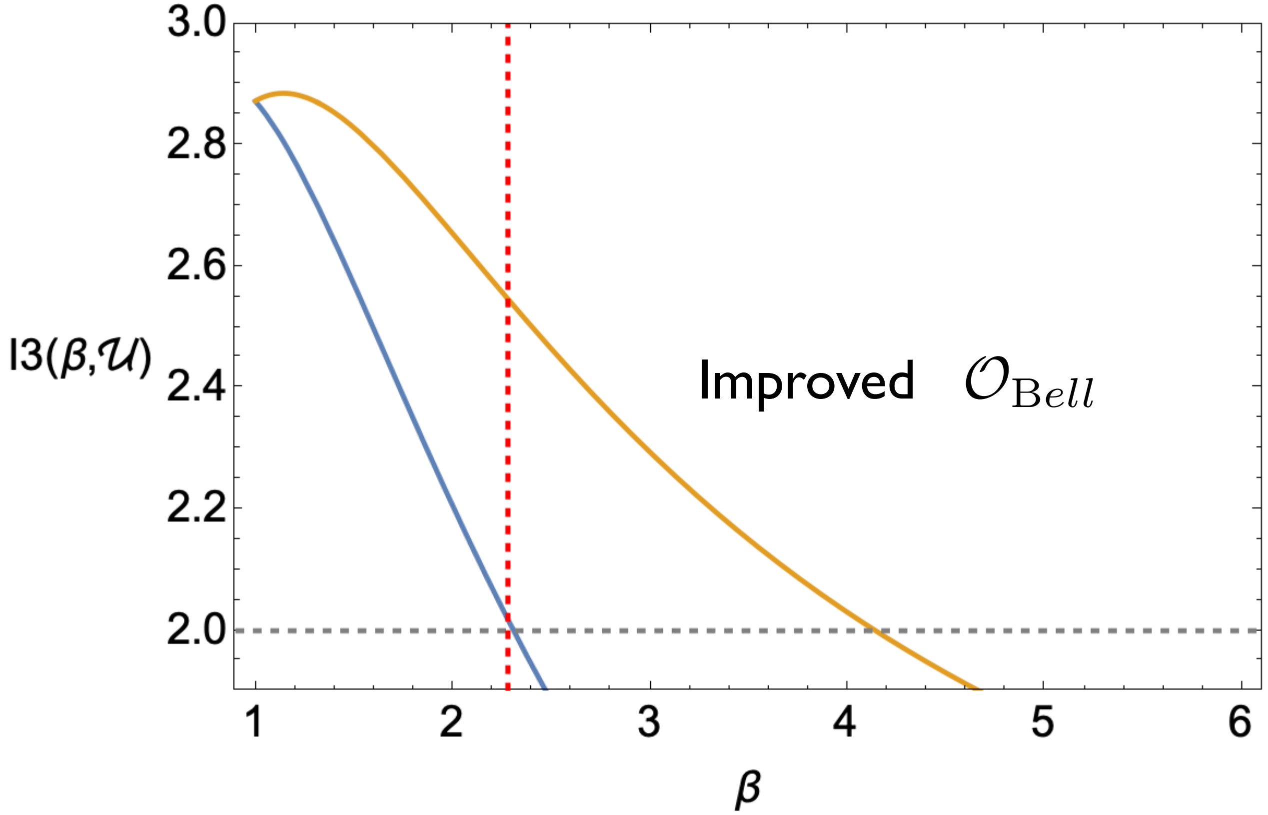
$$O_A = \begin{pmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

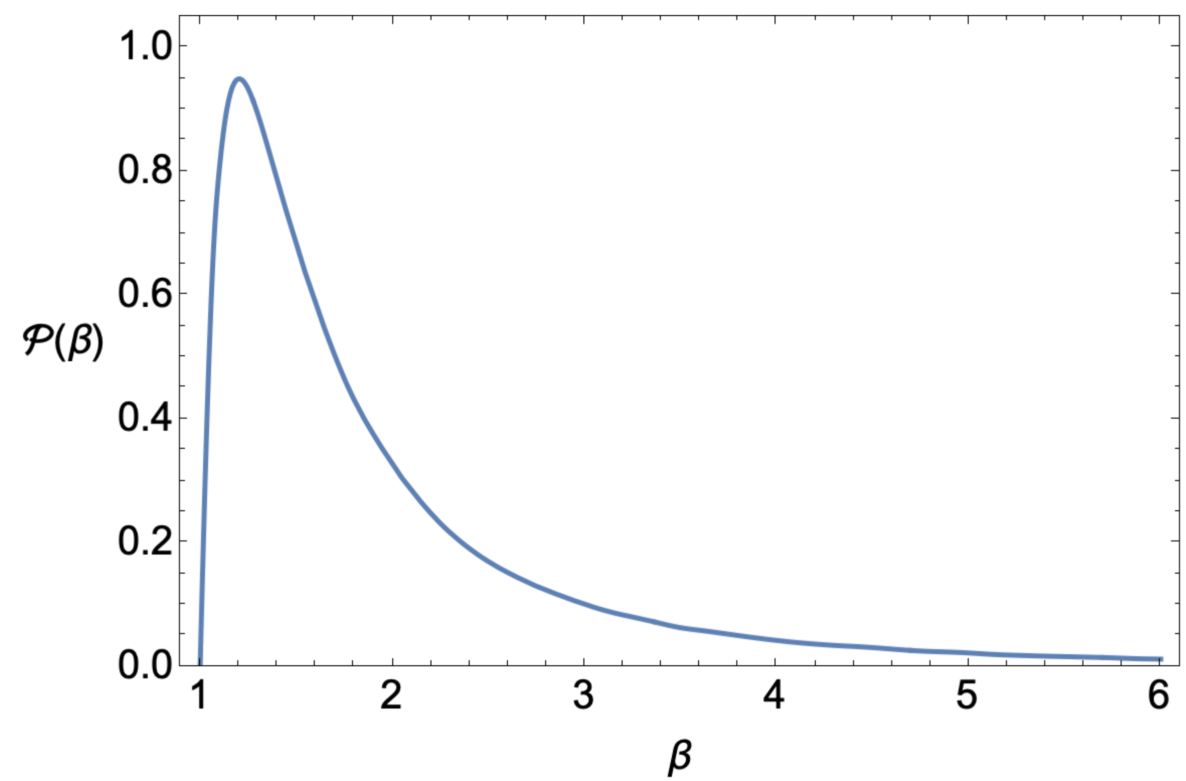
- U can be chosen to optimize the Bell operator

$$U_0 = \begin{pmatrix} \frac{1}{2} & -\frac{1}{\sqrt{2}} & \frac{1}{2} \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{\sqrt{2}} & \frac{1}{2} \end{pmatrix}$$

Optimal Bell operator

- Using the improved version of $\mathcal{O}_{\rm Bell}$ for β ≠ 0





Sizeable improvement in the k-momentum peak region

- In terms of spin polarization and spin correlations:

$$I_3 = \frac{1}{36}\left(18 + 16\sqrt{3} - \sqrt{2}\left(9 - 8\sqrt{3}\right) A^1_{2,0} - 8\left(3 + 2\sqrt{3}\right) C_{2,1,2,-1} + 6\, C_{2,2,2,-2}\right)$$

Numerical results

We have generated

$$pp \to H \to ZZ^* \to 4\ell$$ BR 1.24 x 10^{-4}

using MadGraph and implementing our analysis in $e^+e^-\mu^+\mu^-$ final state

Some technical details:

- Axis orientation: $\hat{z}$ along $\vec{k}_Z$, $\hat{x}$ in the production plane
- Cross section NNNL order is 48.61 pb at a centre-of-mass energy of 13 TeV (6.02 fb in the specific final state)
- Lepton detection efficiency: 0.7 (ie, overall 0.25)
- Luminosity: 300 fb^{-1} (3. ab^{-1}) for LHC Runs 2+3 (HL-LHC) respectively
- Stat. uncertainty in the observables is determined by performing 10^3 pseudo-experiments.
- Results are presented with / without cuts in m_{Z_2}

Numerical results

LHC Runs 2+3

300 fb^{-1}

	min m_{Z_2}			
	0	10 GeV	20 GeV	30 GeV
N	450	418	312	129
$C_{2,1,2,-1}$	-0.98 ± 0.31	-0.97 ± 0.33	-1.05 ± 0.38	-1.06 ± 0.61
$C_{2,2,2,-2}$	0.60 ± 0.37	0.64 ± 0.38	0.74 ± 0.43	0.82 ± 0.63
I_3	2.66 ± 0.46	2.67 ± 0.49	2.82 ± 0.57	2.88 ± 0.89

Entanglement ~ 2 σ

~~Bell~~ < 2 σ

Numerical results

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I_3	2.66 ± 0.46	2.67 ± 0.49	2.82 ± 0.57	2.88 ± 0.89

Entanglement ~ 2 σ

~~Bell~~ < 2 σ

HL - LHC

3 ab^{-1}

	min m_{Z_2}			
	0	10 GeV	20 GeV	30 GeV
N	4500	4180	3120	1290
$C_{2,1,2,-1}$	-0.95 ± 0.10	-1.00 ± 0.10	-1.04 ± 0.12	-1.04 ± 0.19
$C_{2,2,2,-2}$	0.60 ± 0.12	0.64 ± 0.12	0.74 ± 0.14	0.83 ± 0.20
I_3	2.63 ± 0.15	2.71 ± 0.16	2.81 ± 0.18	2.84 ± 0.28

Entanglement ~ 5 σ

~~Bell~~ ~ 4.5 σ

If New Physics in HZZ: consequences?

- At lowest other the Standard Model HZZ vertex is modified as:

$$V^{\mu\nu}_{HZZ} = \frac{igm_Z}{\cos\theta_W}\left[a\, g_{\mu\nu} + b\,\frac{p_\mu p_\nu}{m_Z^2} + c\,\epsilon_{\mu\nu\alpha\beta}\frac{p^\alpha k^\beta}{m_Z^2}\right]$$

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- The previous results can be generalized

$$|\psi_{ZZ}\rangle \equiv |\psi_{ZZ}\rangle\,(k_Z, b, c)$$

$$\rho \equiv \rho\;(b, c)$$

Aoude, Madge, Maltoni, Mantani 2023
Fabbrichesi, Floreanini, Gabrielli, Marzola, 2023

- The entanglement analysis that we have presented remains valid

- The optimal Bell operator will depend on (b,c)

Bernal, Caban, Rembieliński 2023

Conclusions

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- Relevant aspects

  - Improving Quantum Tomography (careful choice of ρ basis, etc)
  - Optimizing Bell operators

THANKS