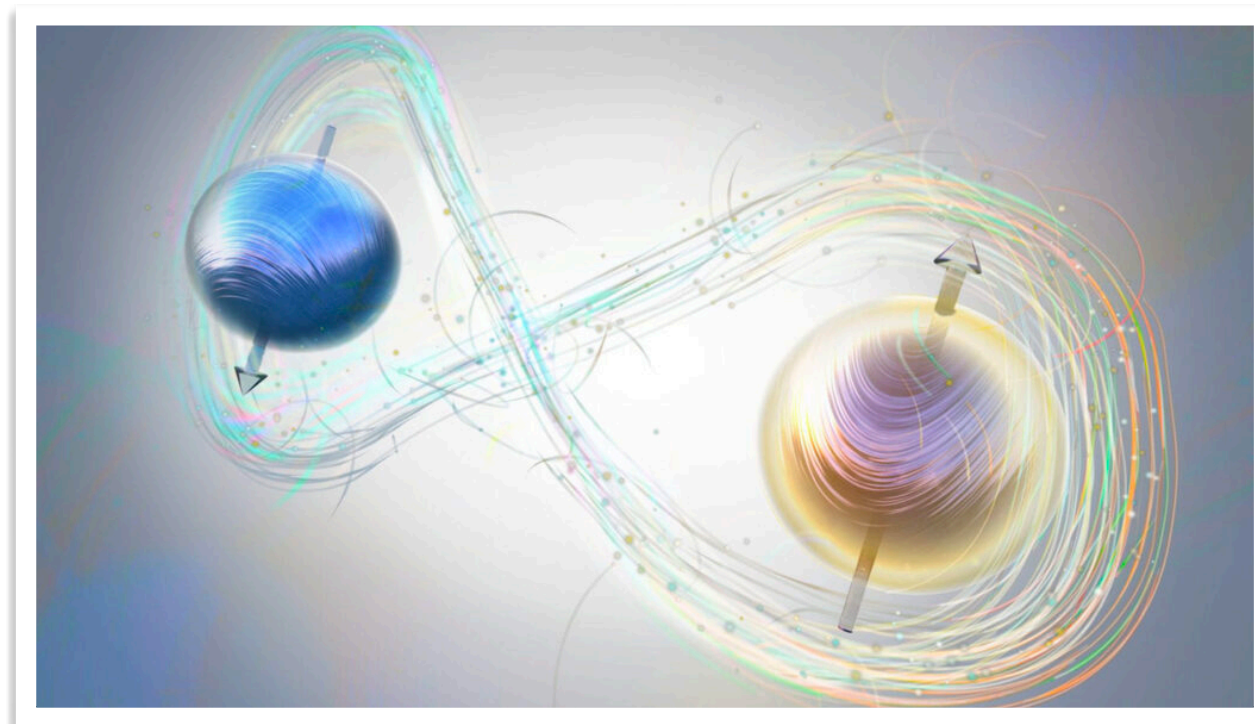# Entanglement and Bell's Inequalities with Boosted Top Quarks
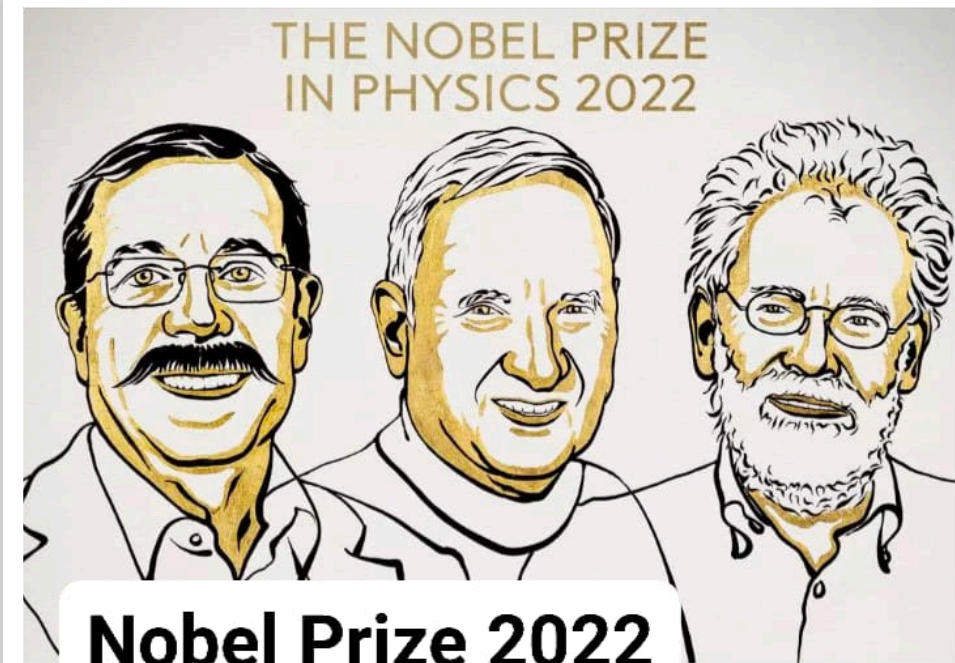
Dorival Gonçalves

arXiv:2305.07075: Z. Dong, DG, KC Kong, A. Navarro

Quantum Observables for Collider Physics, GGI - Nov 7, 2023

THE NOBEL PRIZE
IN PHYSICS 2022

Nobel Prize 2022

Nobel Prize in Physics awarded to Aspect, Clauser and Zeilinger for work in quantum mechanics

The Royal Swedish Academy of Sciences has decided to award the 2022 Nobel Prize in Physics to Alain Aspect, John F Clauser and Anton Zeilinger, according to an official tweet. They have been awarded the Nobel Prize for experiments with entangled photons, establishing the violation of Bell inequalities and pioneering quantum information science.

🔵 QI theory provides a set of tools designed to probe the inner behavior of QM. While these phenomena have been widely tested at low energies, their study at higher energy scales has not be undertaken
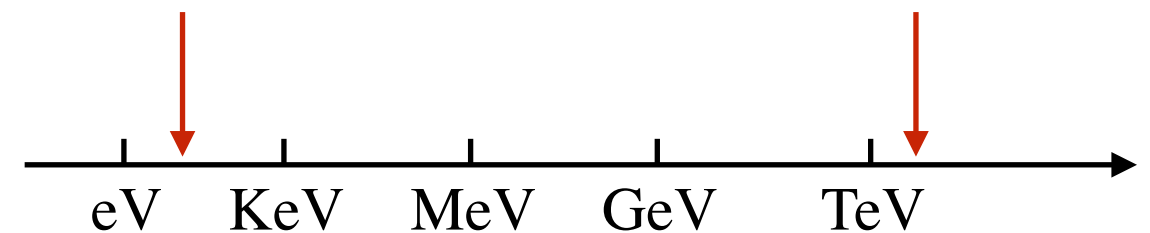
Motivation

🔵 LHC can provide a unique environment to study entanglement and violation of Bell's inequalities at the highest energy available today
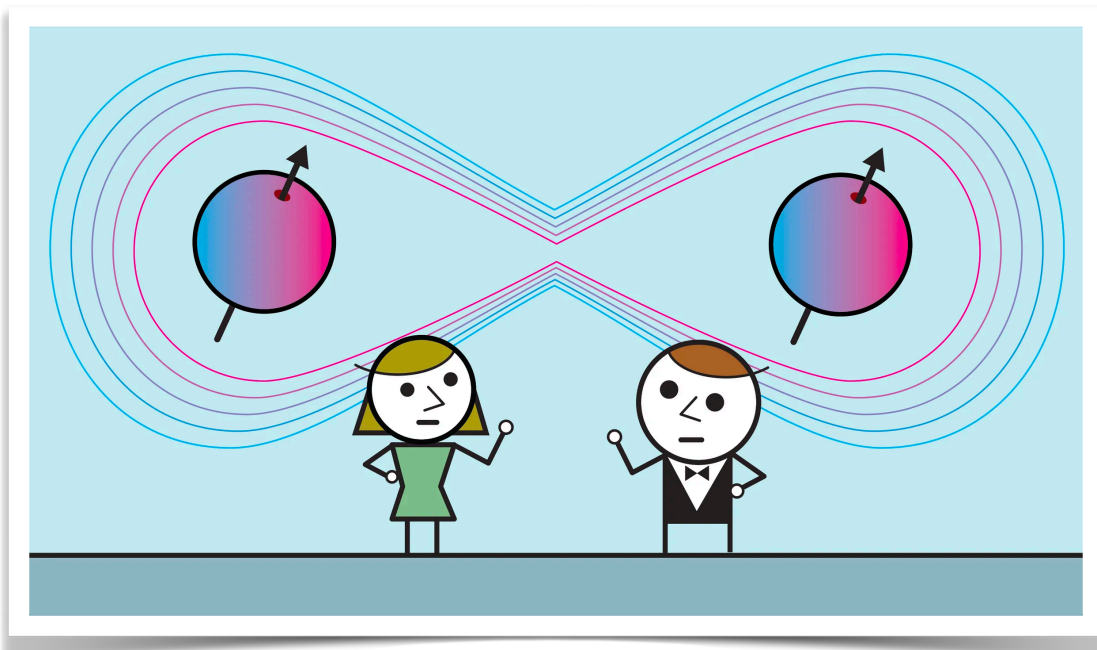
Typical entanglement experiment with photons

LHC

eV   KeV   MeV   GeV   TeV

🔵 Top quark pair production is an optimal candidate for these studies

Afik, Nova '20; Fabbrichesi, Floreanini, Panizzo '21; Severi, Boschi, Maltoni, Sioli '21

Saavedra, Casas '22; Severi, Vryonidou 22

Dong, DG, Kong, Navarro '23; Han, Low, Wu '23

See talk by Jay Howarth

Top quark as a one qubit system

- As a spin-1/2 particle, the most general spin density matrix for the top quark is

$$\rho = \frac{\mathbb{I} + B_i \sigma_i}{2}$$

- Characterized by three parameters: $B_i = \langle \sigma_i \rangle = tr(\sigma_i \rho)$

➡ However, the top quark decays. In general, spin information could be lost by hadronization or spin decorrelation effects
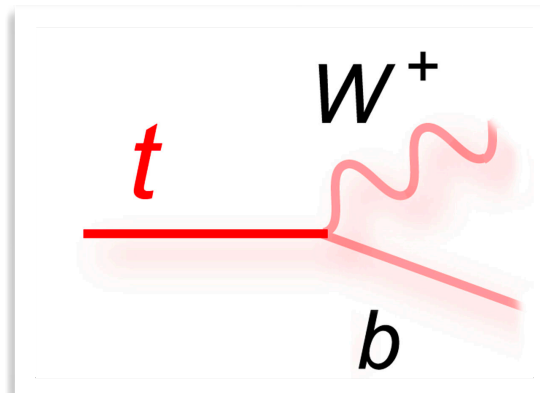
Top Quark is Special

🔵 Decays before it hadronizes or its spin flips

$$\tau_{top} \approx 5 \times 10^{-25} s$$

$$\tau_{had} \approx 2 \times 10^{-24} s$$
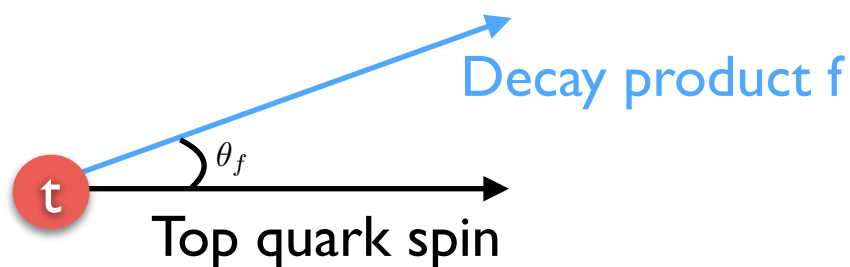
$$\tau_{flip} \approx 10^{-21} s$$

🔵 Top polarization directly observable via angular distributions of its decay products

$$\frac{1}{\Gamma_f}\frac{d\Gamma_f}{d\cos\theta_f} = \frac{1}{2}(1 + \omega_f \cos\theta_f)$$

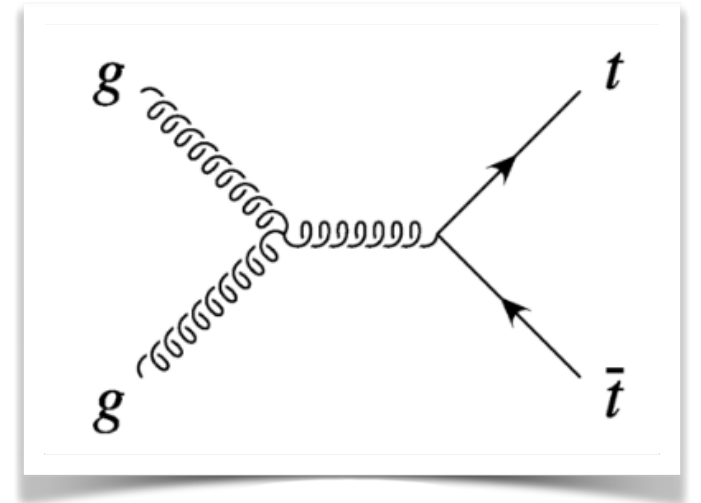|	$l^+, \bar{d}$	b	$\bar{\nu}, u$
ω_f	1	-0.4	-0.3

Spin analyzing power: maximum for charged leptons and down-type quarks

● The most general two-qubit system can be represented by

$$\rho = \frac{\mathbb{I}\otimes\mathbb{I}+\left(B_i\,\sigma_i\otimes\mathbb{I}+\bar{B}_i\,\mathbb{I}\otimes\sigma_i\right)+C_{ij}\,\sigma_i\otimes\sigma_j}{4}$$

● Characterized by 15 parameters parameters: B_i, $\bar{B}_i$, and C_{ij}

$B_i = \langle \sigma_i \otimes \mathbb{I} \rangle$

$\bar{B}_i = \langle \mathbb{I} \otimes \sigma_i \rangle$ $\longrightarrow$ Polarizations

$C_{ij} = \langle \sigma_i \otimes \sigma_j \rangle \longrightarrow$ Spin correlations

● P and CP invariance under $t\bar{t}$ production $\rightarrow B_i = \bar{B}_i = 0$ and $C_{ij} = C_{ji}$

● Things further simplify in the helicity basis: only non-vanishing parameters are the diagonal terms C_{ii} and one off-diagonal term $C_{12} \simeq C_{21}$ Bernreuther, Heisler, Si '15
Frederix, Tsinikos, Vitos '21

🔵 We are probing the spin correlations of top and anti-top since the Tevatron era and continue to study them at the LHC in different forms: relevant precision observable
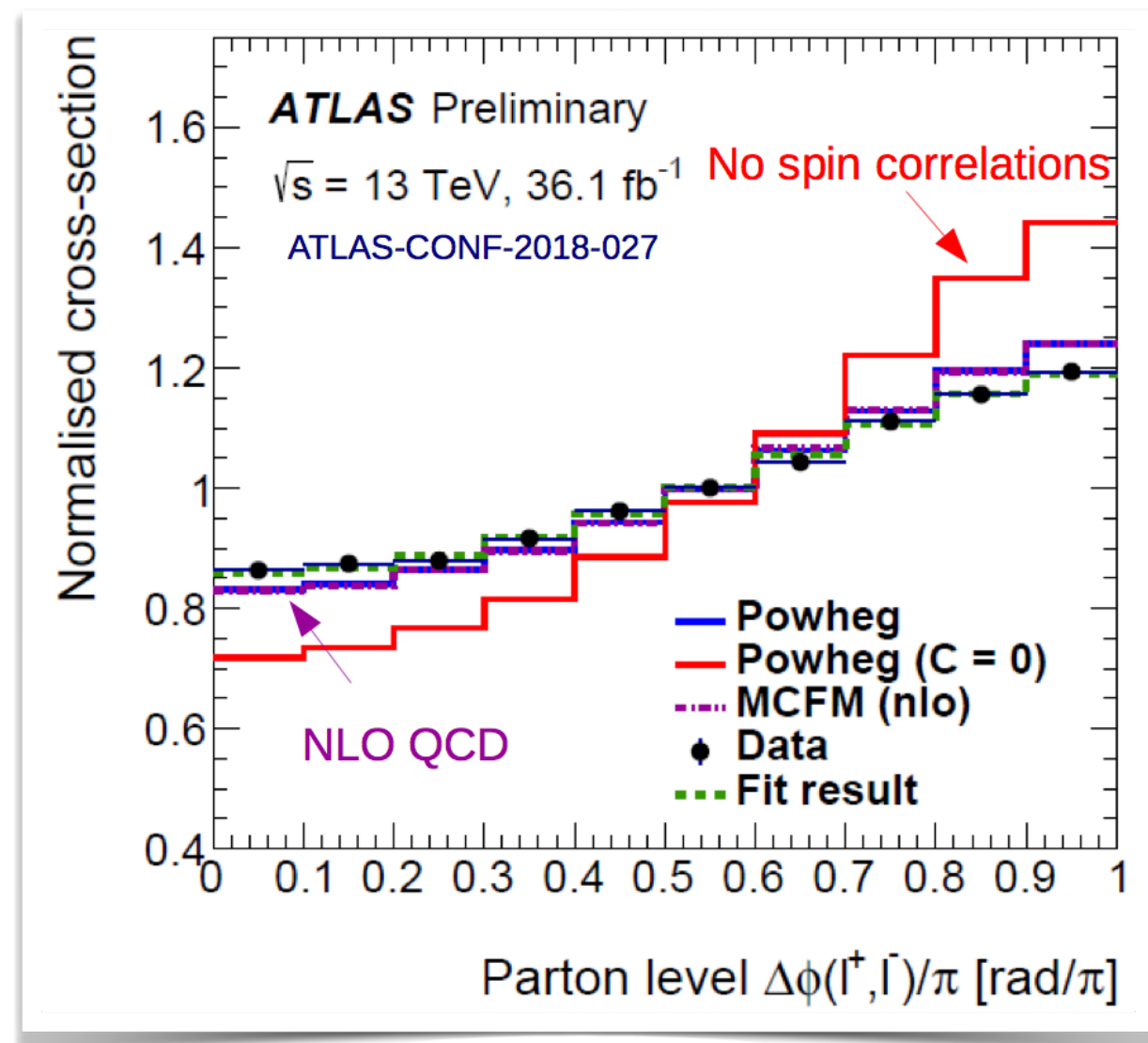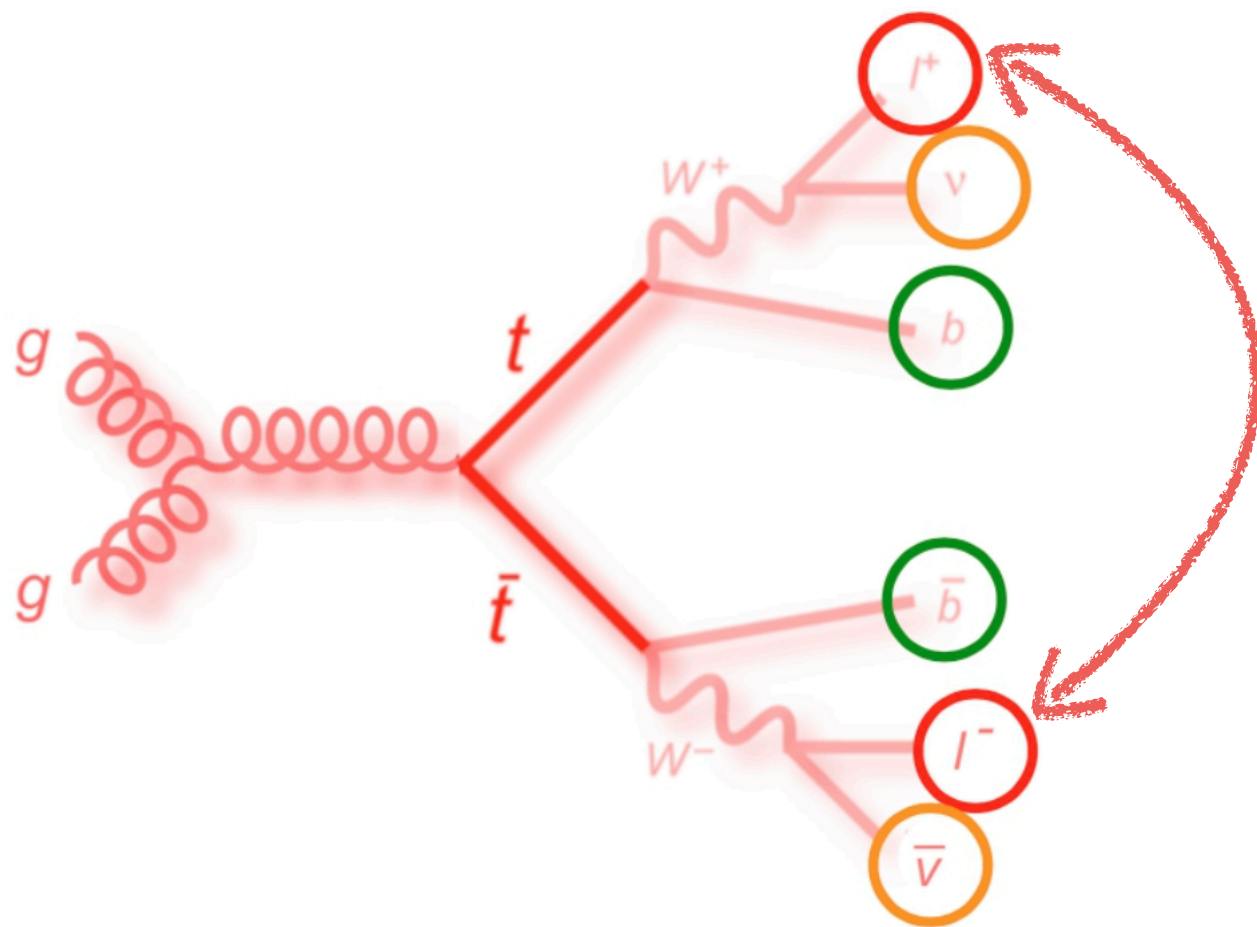
Parke, Mahlon '95

🔵 We are probing the spin correlations of top and anti-top since the Tevatron era and continue to study them at the LHC in different forms: relevant precision observable
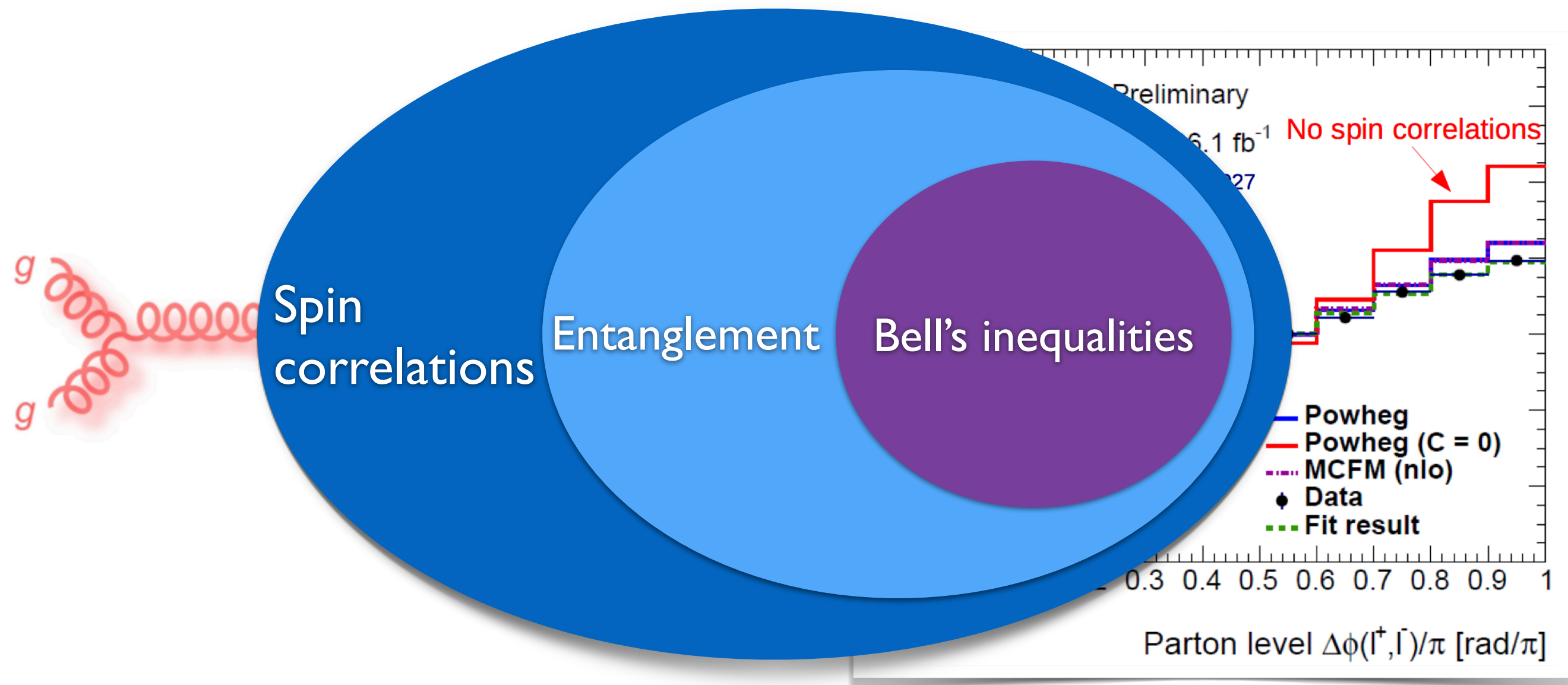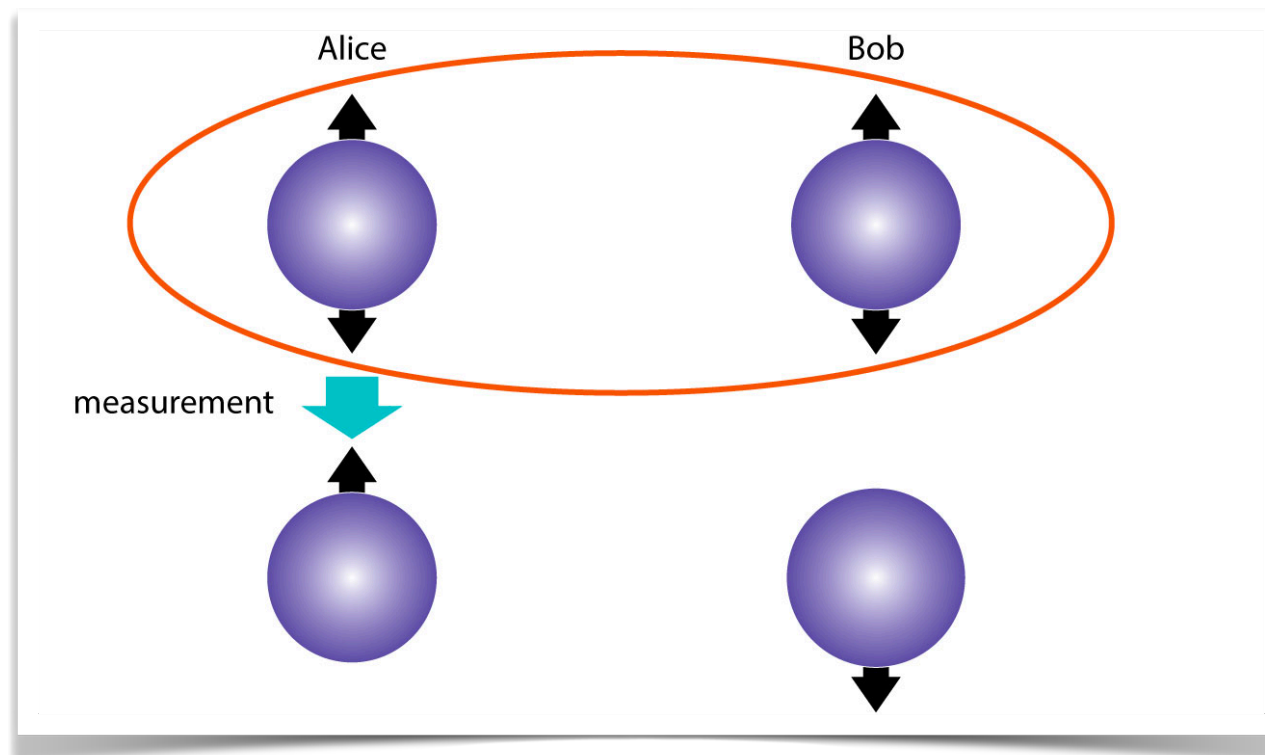

Parke, Mahlon '95

Quantum Entanglement

🔵 A quantum state of two subsystems A and B is separable when its density matrix ρ can be expressed as a convex sum

$$\rho = \sum_i p_i \rho_A^i \otimes \rho_B^i$$

➡️ If the state is not separable, it is named *entangled*

Measurement in one subsystem immediately affect the other, even if they are causally disconnected

Quantum Entanglement

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➡️ If the state is not separable, it is named *entangled*

- The Peres-Horodecki criterion provides a necessary and sufficient condition for entanglement in two-qubit systems:

Peres '96; Horodecki '97

Take the transpose of indices associated only to Bob (or Alice)

$$\rho^{T2} = \sum_i p_i \rho_A^i \otimes (\rho_B^i)^T$$

For a separable system, ρ^{T2} results in a non-negative operator

➡️ If ρ^{T2} displays at least one negative eigenvalue, the system is entangled

Quantum Entanglement

Since for the $t\bar{t}$ system $B_i = \bar{B}_i = 0$ and $C_{ij} = C_{ji}$ (P and CP) and $C_{13} \simeq C_{23} \simeq 0$ (helicity basis), we obtain some simple sufficient conditions for entanglement

Afik, Nova '20

Fabbrichesi, Floreanini, Panizzo '21

Severi, Boschi, Maltoni, Sioli '21

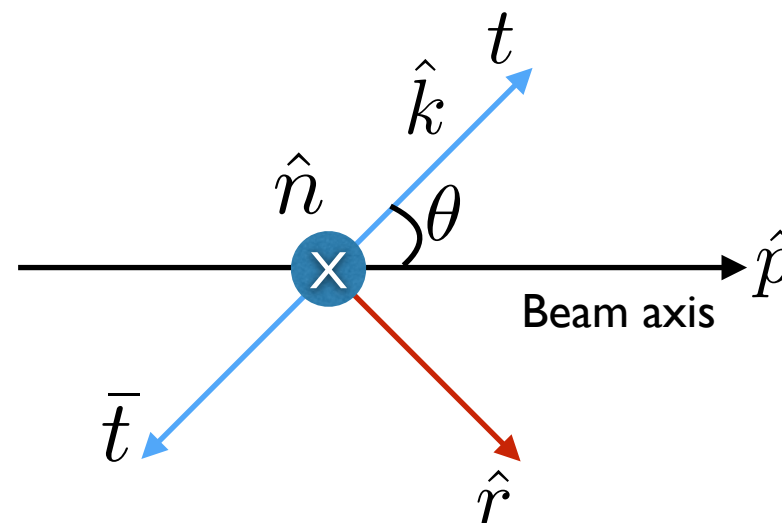Saavedra, Casas '22

$$|C_{kk} + C_{rr}| - C_{nn} - 1 > 0$$

$$|C_{kk} - C_{rr}| + C_{nn} - 1 > 0$$

Helicity basis

$\hat{k}$ = top quark direction

$\hat{r} = \text{sign}(\cos\theta)(\hat{p} - \cos\theta\hat{k})/\sin\theta$

$\hat{n} = \hat{k} \times \hat{r}$

$$\frac{1}{\sigma}\frac{d^2\sigma}{d\cos\theta_a^i \, d\cos\bar{\theta}_b^j} = \frac{1}{4}\left(1 + \beta_a\beta_b C_{ij}\cos\theta_a^i\cos\bar{\theta}_b^j\right)$$

Quantum Entanglement

Since for the $t\bar{t}$ system $B_i = \bar{B}_i = 0$ and $C_{ij} = C_{ji}$ (P and CP) and $C_{13} \simeq C_{23} \simeq 0$ (helicity basis), we obtain some simple sufficient conditions for entanglement
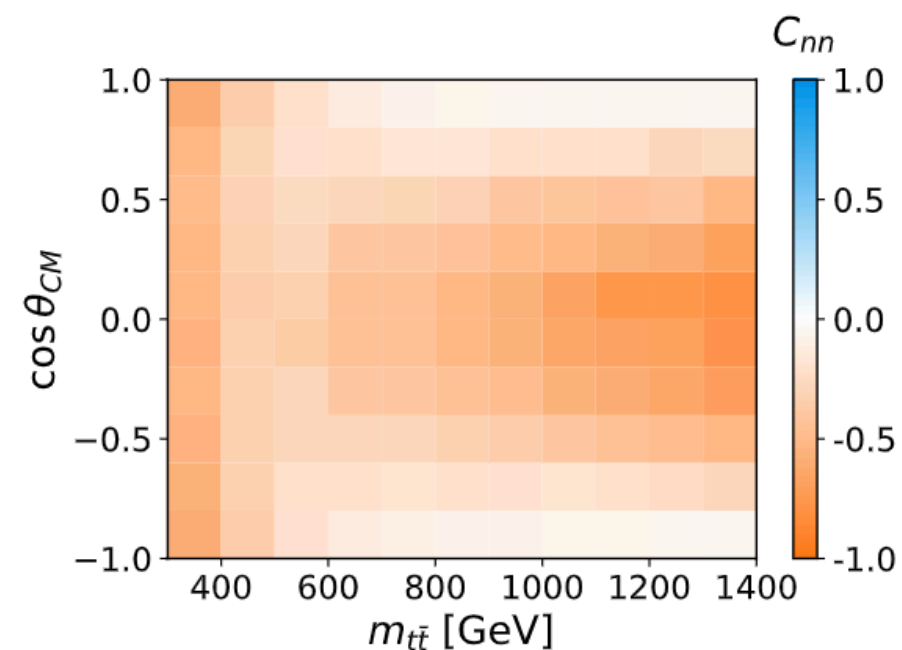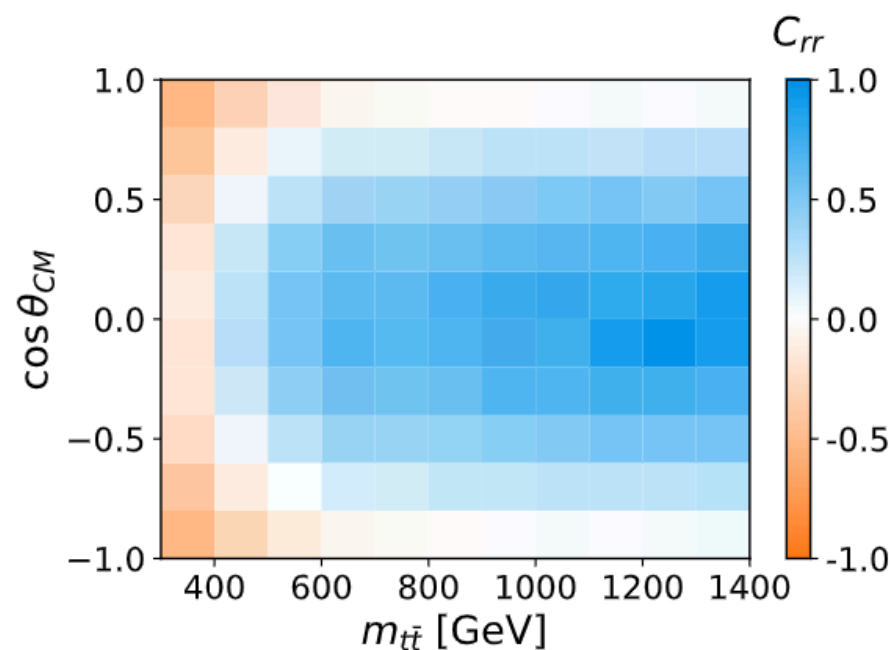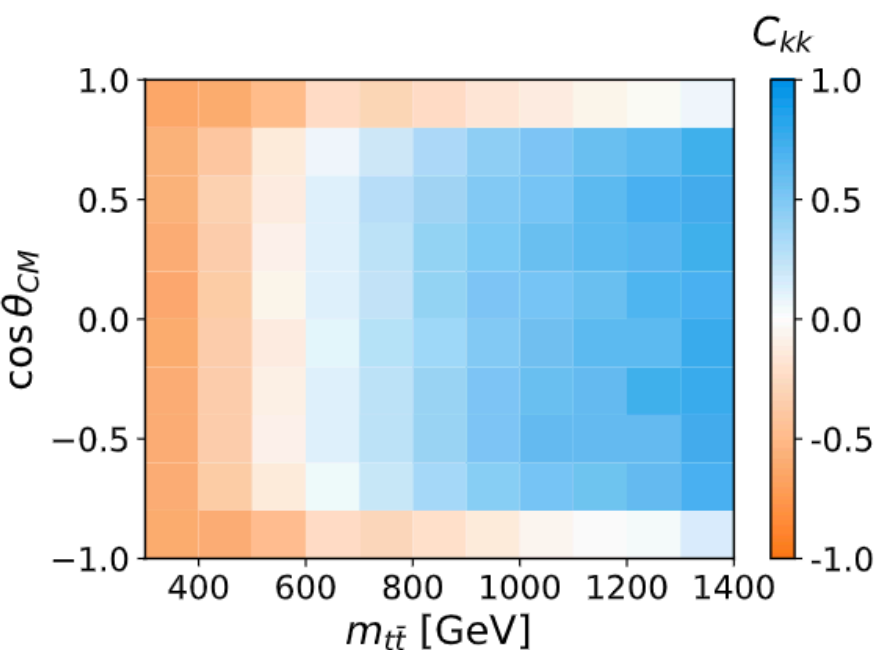
Afik, Nova '20

Fabbrichesi, Floreanini, Panizzo '21

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Saavedra, Casas '22

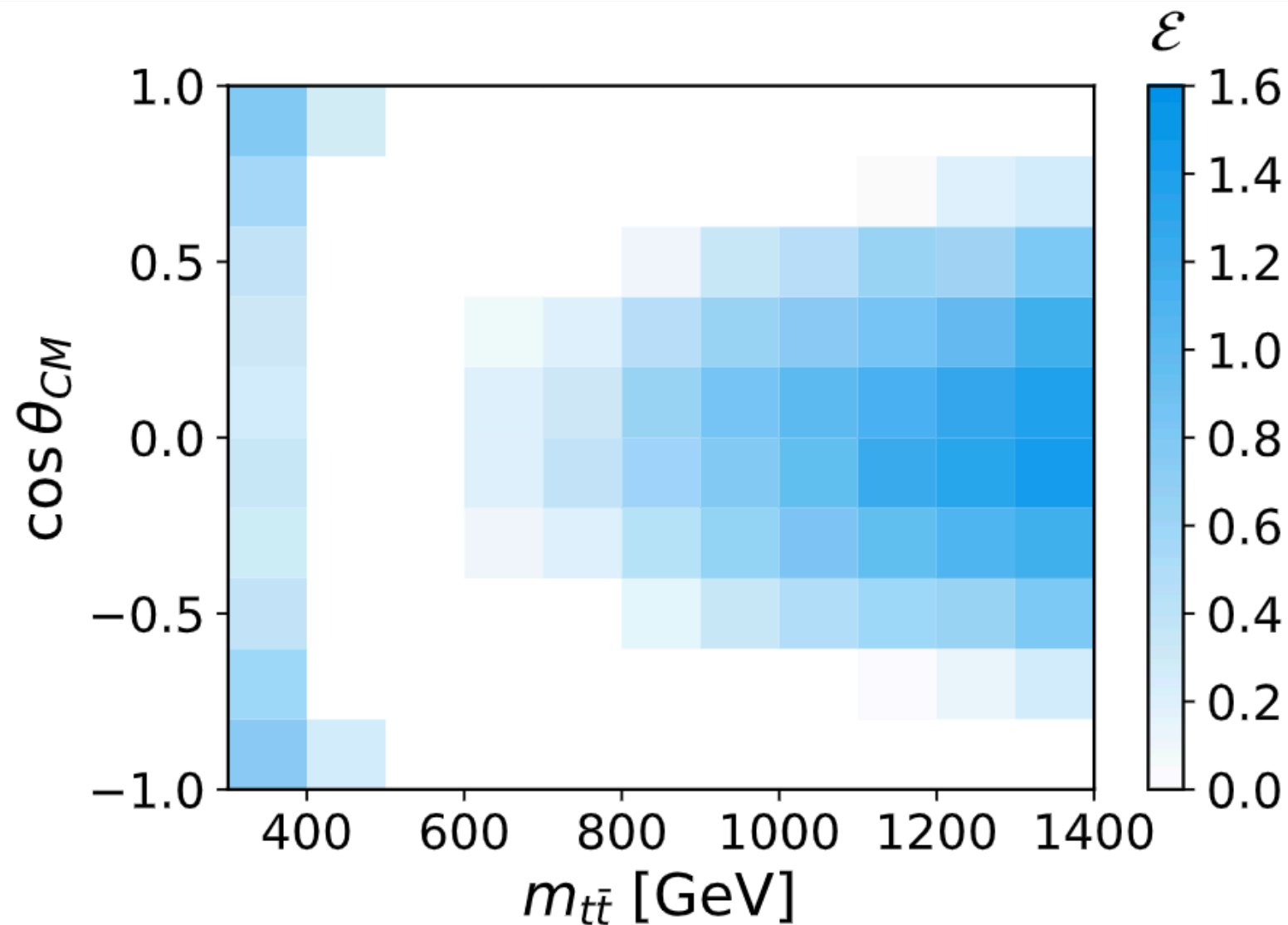$$|C_{kk} + C_{rr}| - C_{nn} - 1 > 0$$

$$|C_{kk} - C_{rr}| + C_{nn} - 1 > 0$$

$$\mathcal{E} \equiv |C_{kk} + C_{rr}| - C_{nn} - 1 > 0$$

Quantum Entanglement

$$\mathcal{E} \equiv |C_{kk} + C_{rr}| - C_{nn} - 1 > 0$$

➡ Top pair displays entanglement in two disconnected regimes:
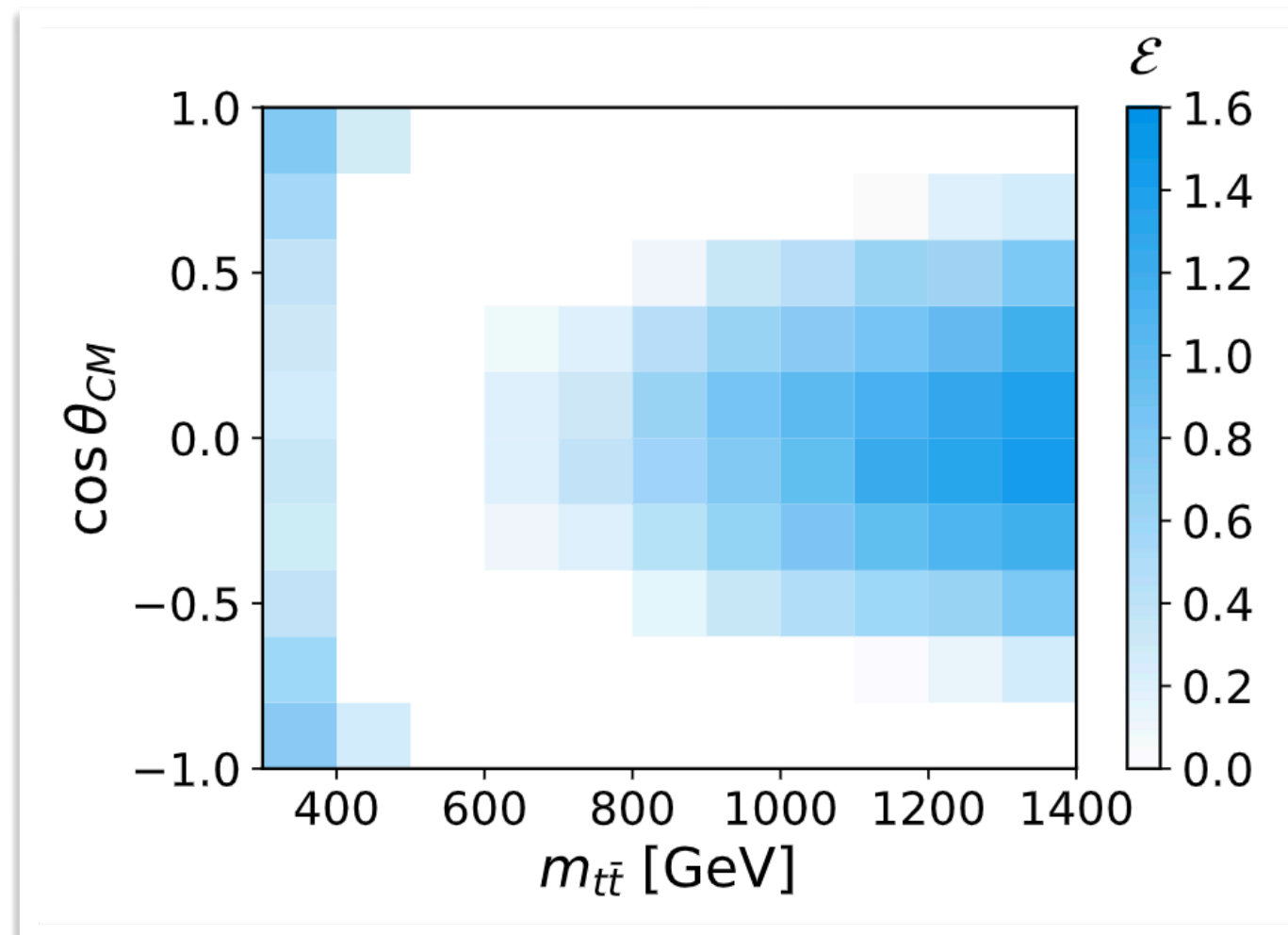**threshold** and **boosted** regions

Dong, DG, Kong, Navarro '23

⬤ i) Semi-leptonic top pair has an event rate higher than that of the dileptonic process (+easier reconstruction), making it a more effective probe for the high-energy regime

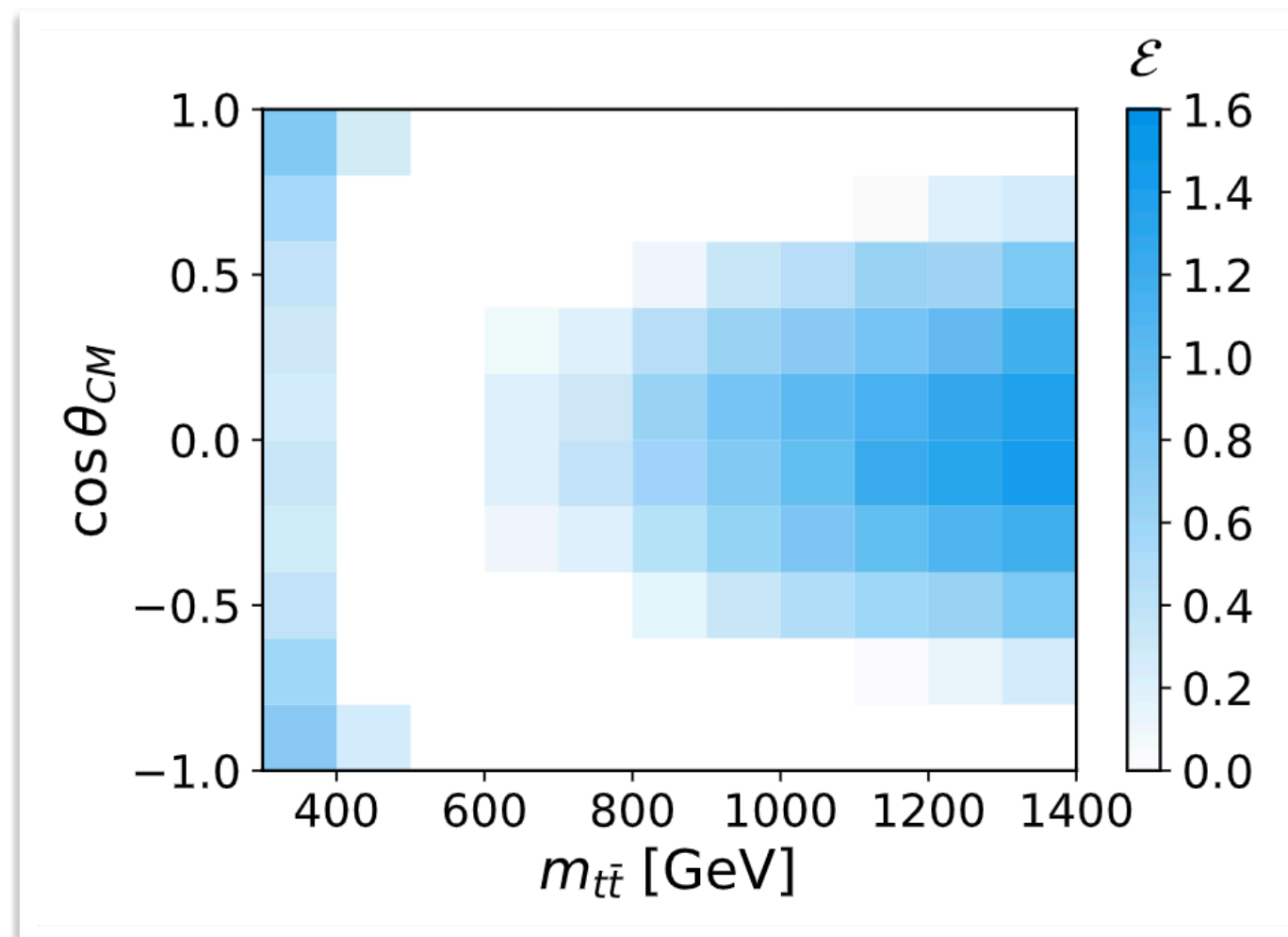$$\mathcal{E} \equiv |C_{kk} + C_{rr}| - C_{nn} - 1 > 0$$

Dong, DG, Kong, Navarro '23

ii) While boosted region is typically well modeled, threshold regime displays relevant "threshold corrections"

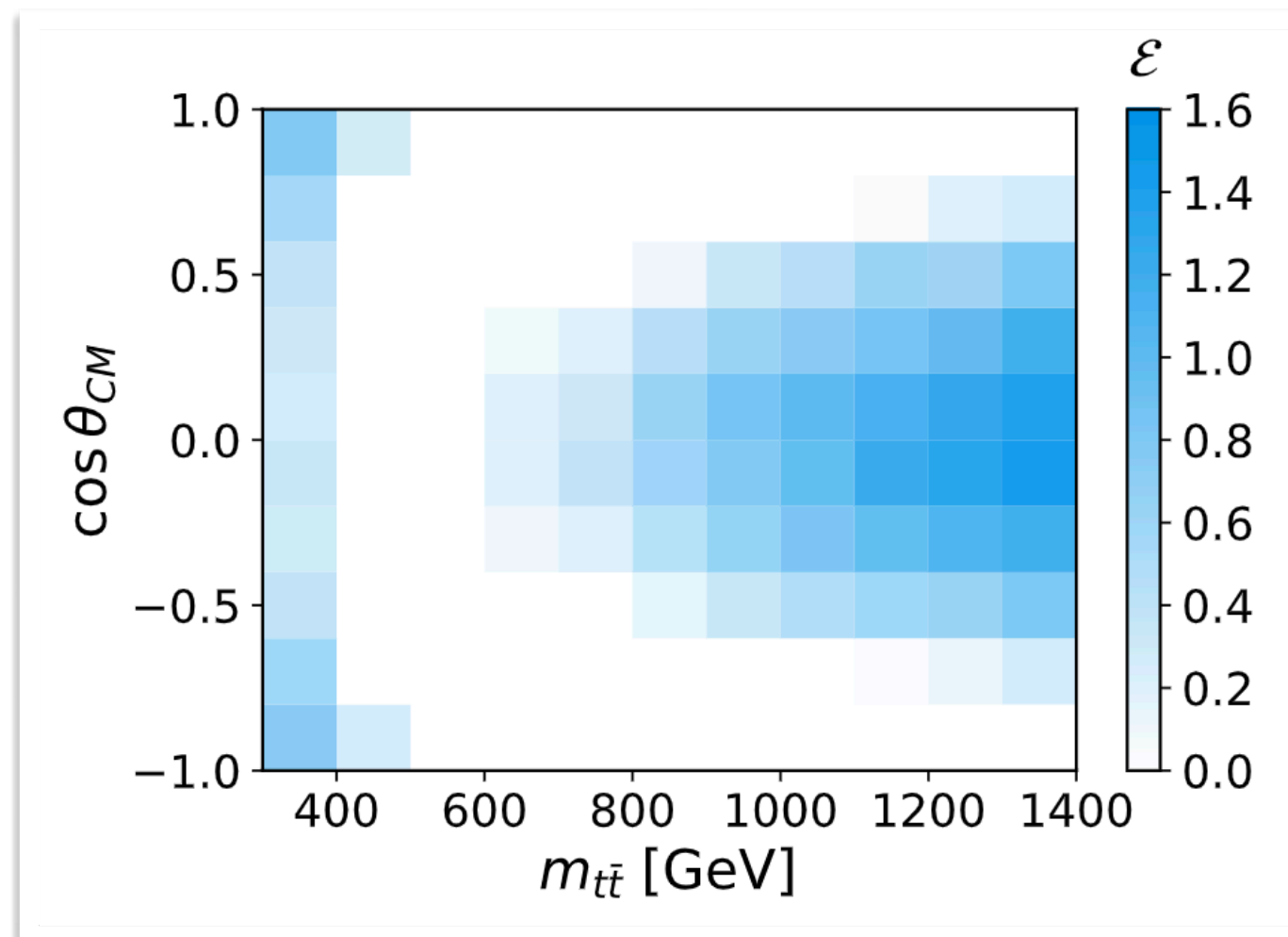$$\mathcal{E} \equiv |C_{kk} + C_{rr}| - C_{nn} - 1 > 0$$

Dong, DG, Kong, Navarro '23

🔵 iii) Boosted top quark regime nicely matches with both entanglement and CHSH probes. High transverse momentum results in large top tagging efficiency and small fake rate for top tagging (e.g., HEPTopTagger) Plehn, Spannowsky, Takeuchi, Zerwas '10
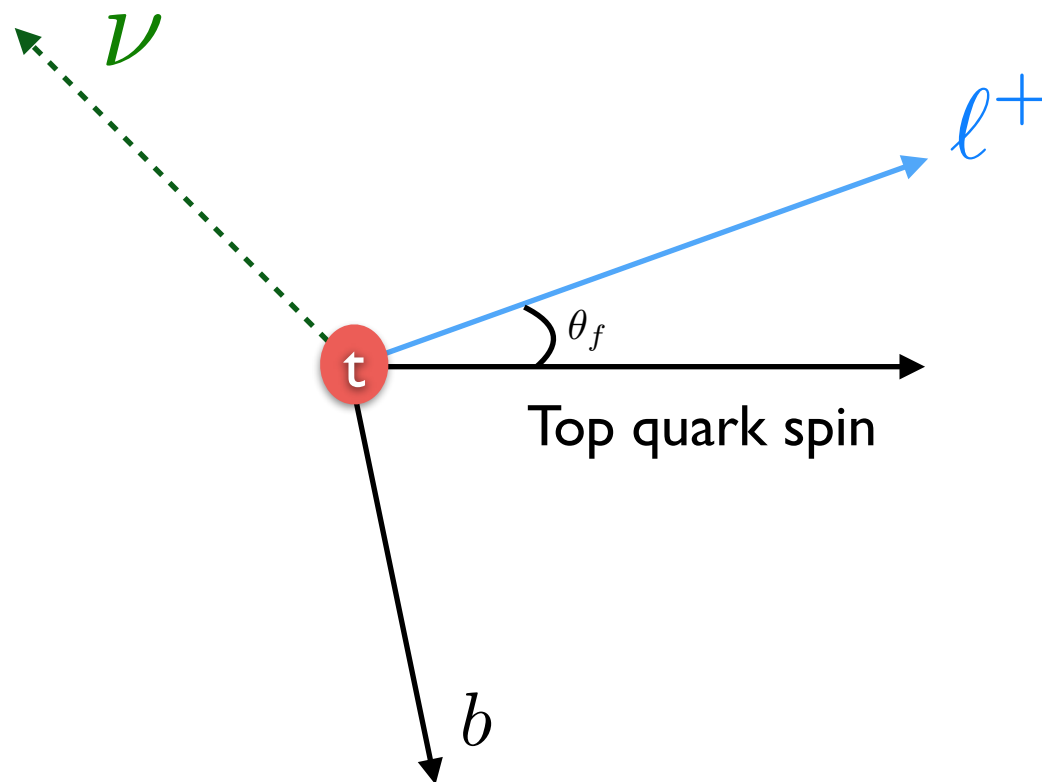
$$\mathcal{E} \equiv |C_{kk} + C_{rr}| - C_{nn} - 1 > 0$$

Dong, DG, Kong, Navarro '23

iv) In the high-energy regime, boosted techniques can be employed to tag the hadronic top and efficiently identify the optimal hadronic direction

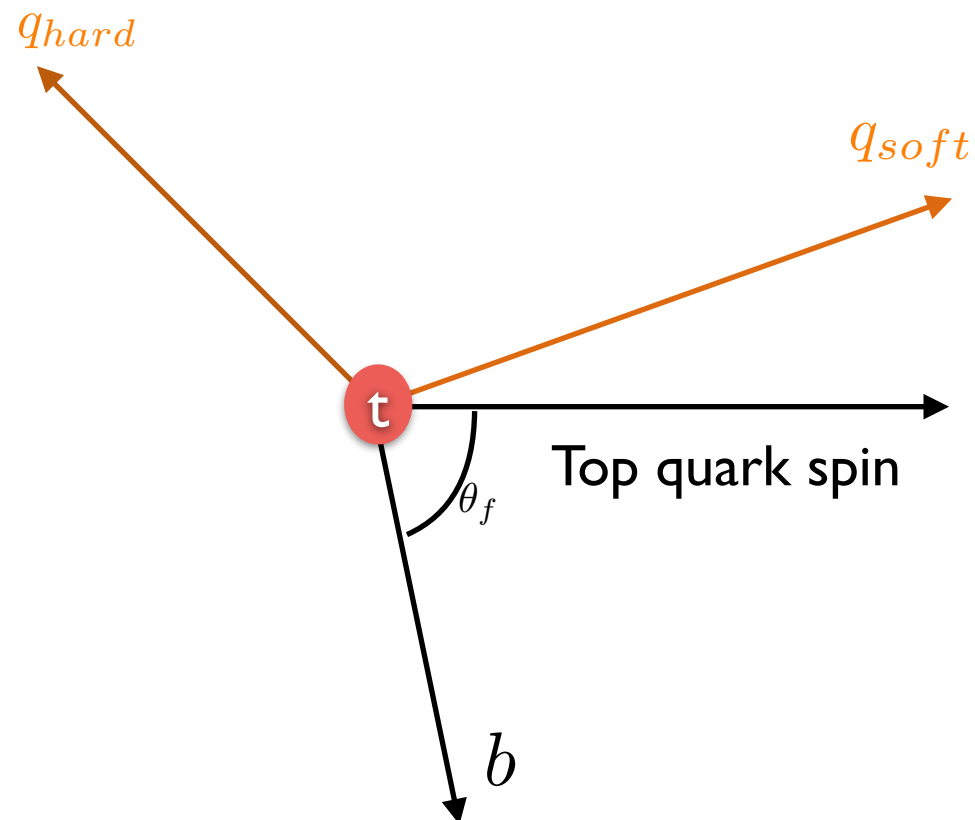$$\frac{1}{\Gamma_f}\frac{d\Gamma_f}{d\cos\theta_f} = \frac{1}{2}(1 + \mathbf{1.0}\ \cos\theta_f)$$

iv) In the high-energy regime, boosted techniques can be employed to tag the hadronic top and efficiently identify the optimal hadronic direction

$$\frac{1}{\Gamma_f}\frac{d\Gamma_f}{d\cos\theta_f} = \frac{1}{2}(1 \ \textbf{-0.4}\ \cos\theta_f)$$

Four reasons to study the boosted semileptonic tt final state

iv) In the high-energy regime, boosted techniques can be employed to tag the hadronic top and efficiently identify the optimal hadronic direction

q_{hard}

q_{soft}

θ_f

t

Top quark spin

b

$$\frac{1}{\Gamma_f}\frac{d\Gamma_f}{d\cos\theta_f} = \frac{1}{2}(1 + \mathbf{0.5}\,\cos\theta_f)$$

Jezabek '94

iv) In the high-energy regime, boosted techniques can be employed to tag the hadronic top and efficiently identify the optimal hadronic direction
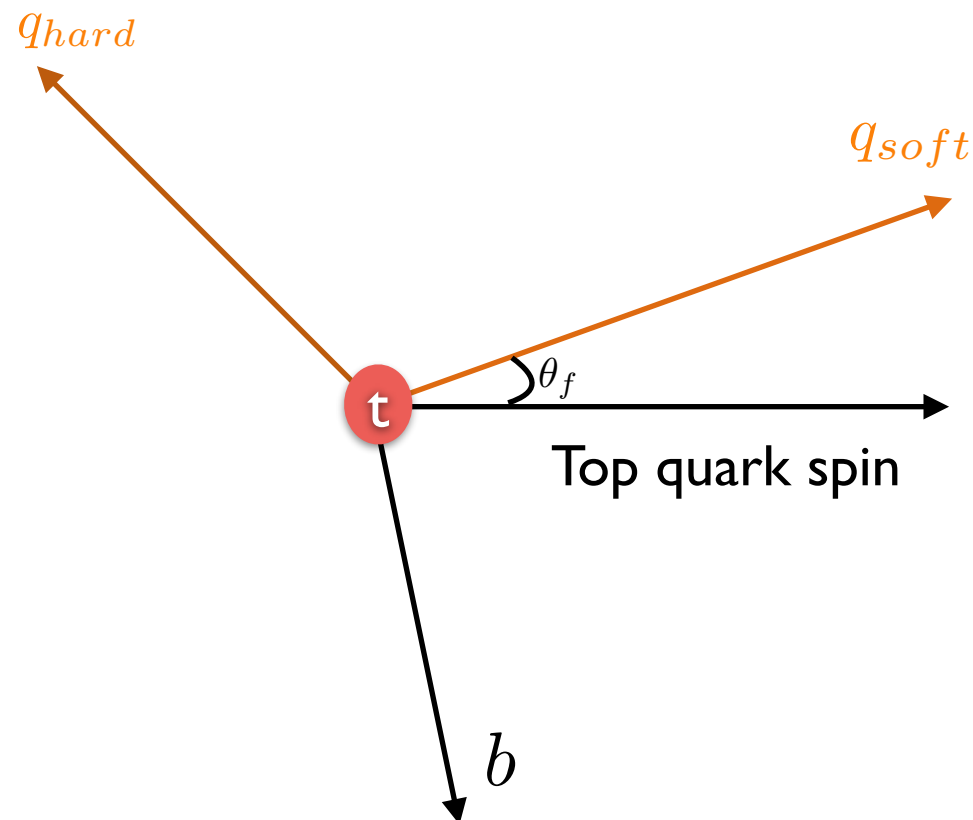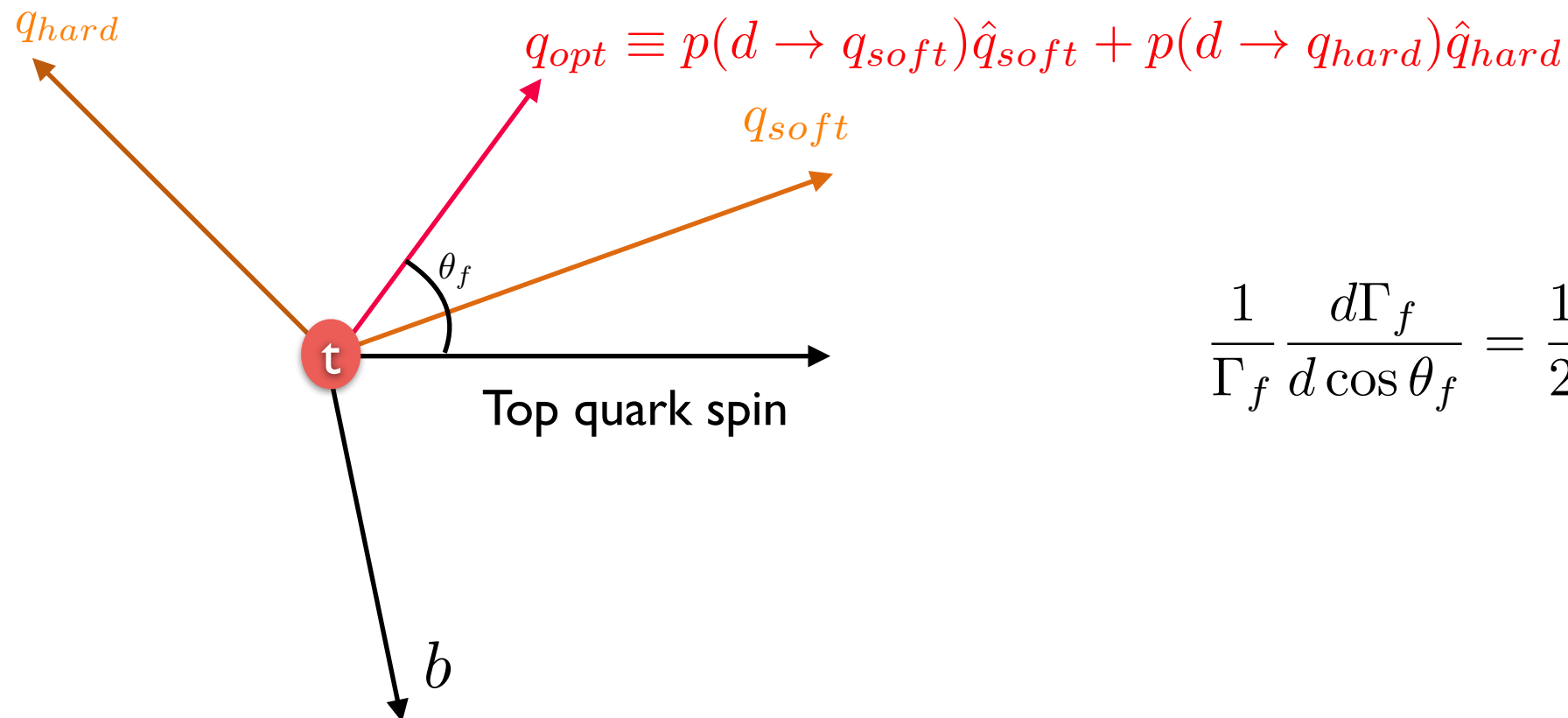
$$q_{opt} \equiv p(d \to q_{soft})\hat{q}_{soft} + p(d \to q_{hard})\hat{q}_{hard}$$

q_{hard}

q_{soft}

θ_f

t

Top quark spin

b

$$\frac{1}{\Gamma_f}\frac{d\Gamma_f}{d\cos\theta_f} = \frac{1}{2}(1 + \textbf{0.64}\cos\theta_f)$$

Tweedie '14

Analysis

🔵 Some key ingredients for the analysis:

- Boosted top tagging (HEPTopTagger)
  Plehn, Spannowsky, Takeuchi, Zerwas '10

- Lorentz Boost Network: i) determines which of the subjects is b-tagged and ii) reconstructs leptonic top

- Proxy for down-quark: optimal hadronic polarimeter
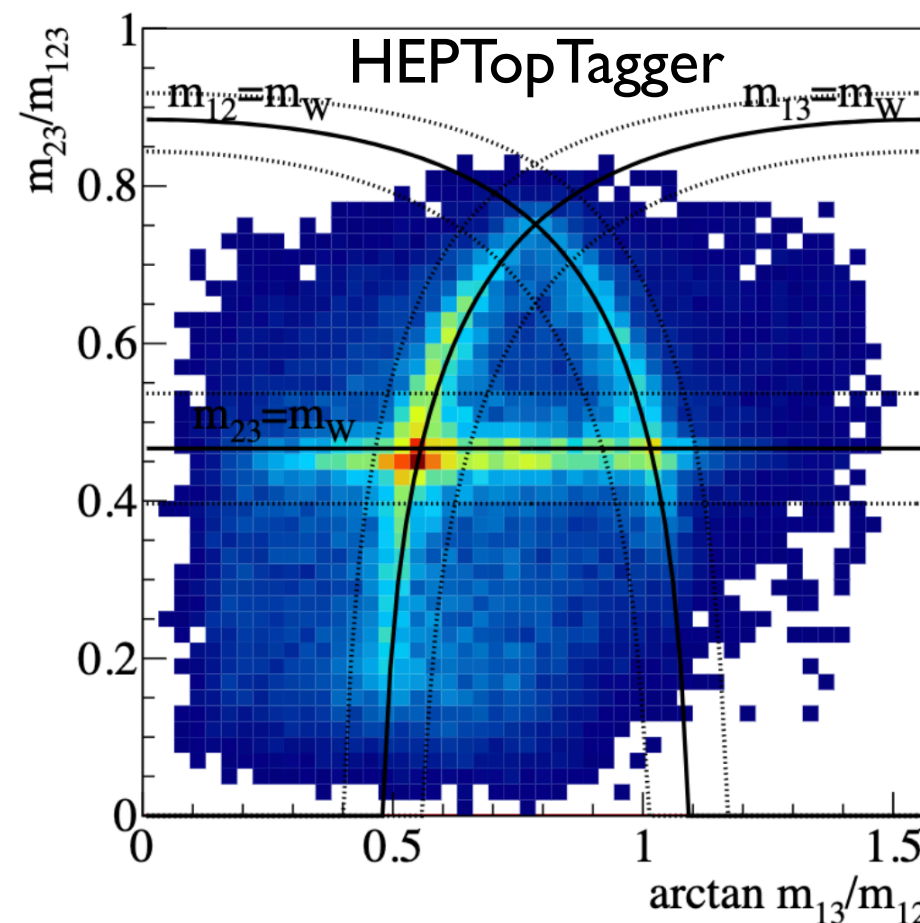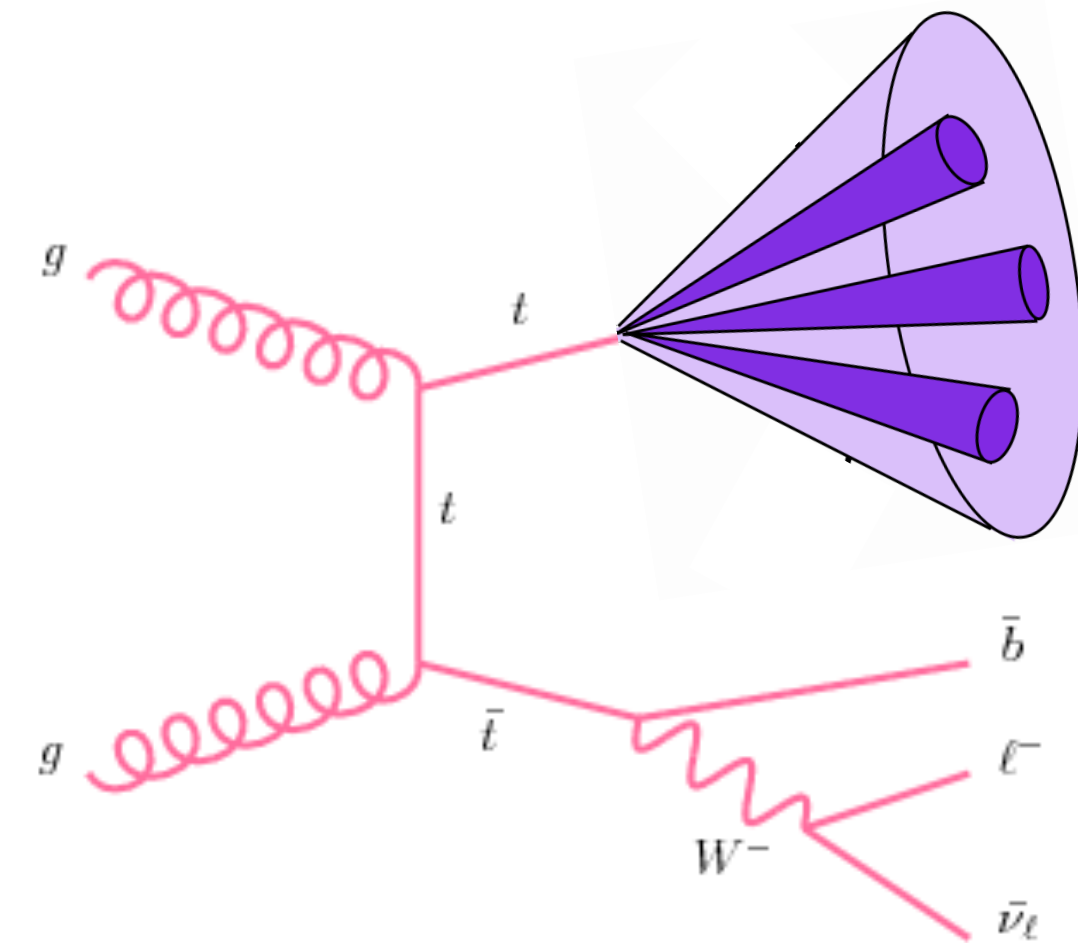
🔵 Entanglement: $\mathcal{E} \equiv |C_{kk} + C_{rr}| - C_{nn} - 1 > 0$
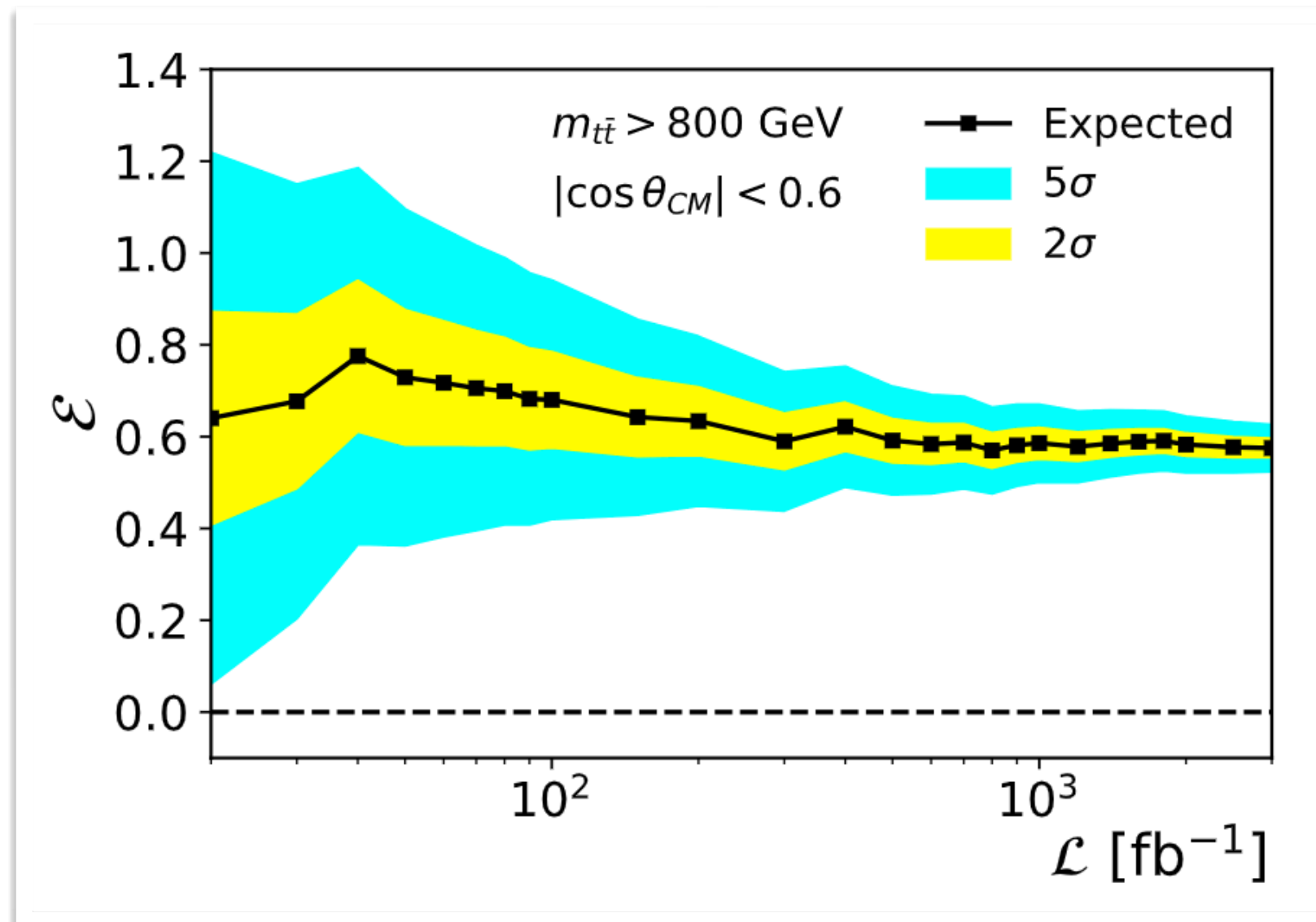
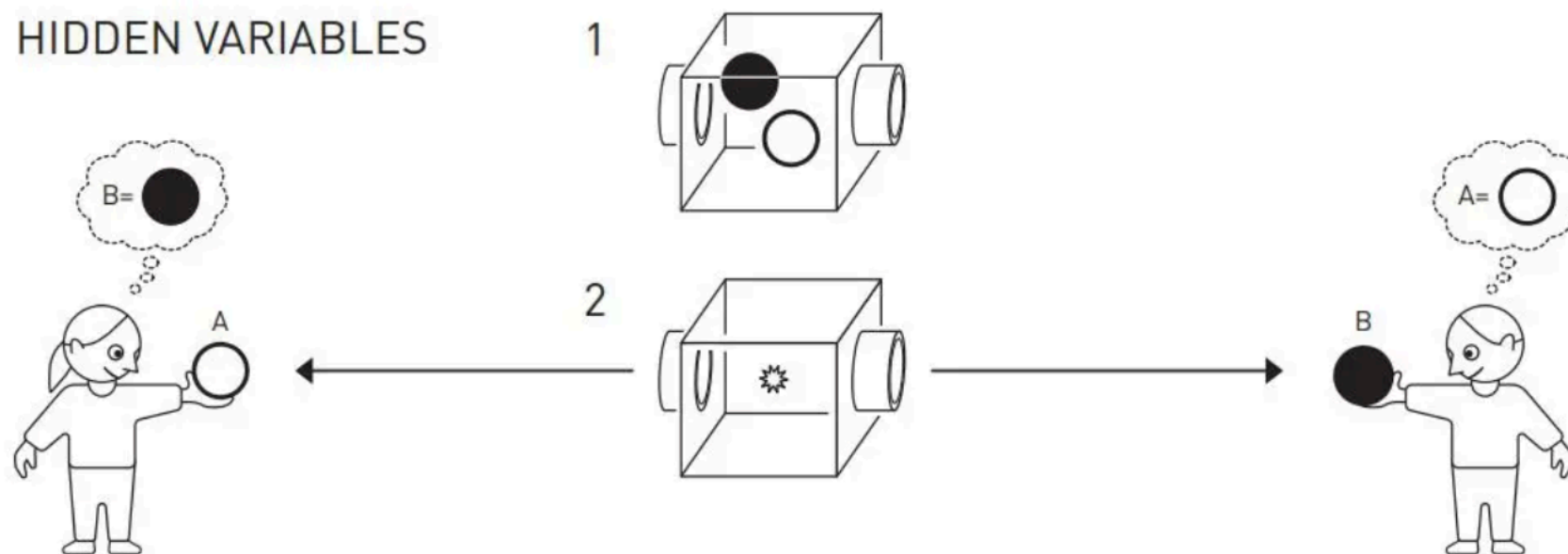Dong, DG, Kong, Navarro '23

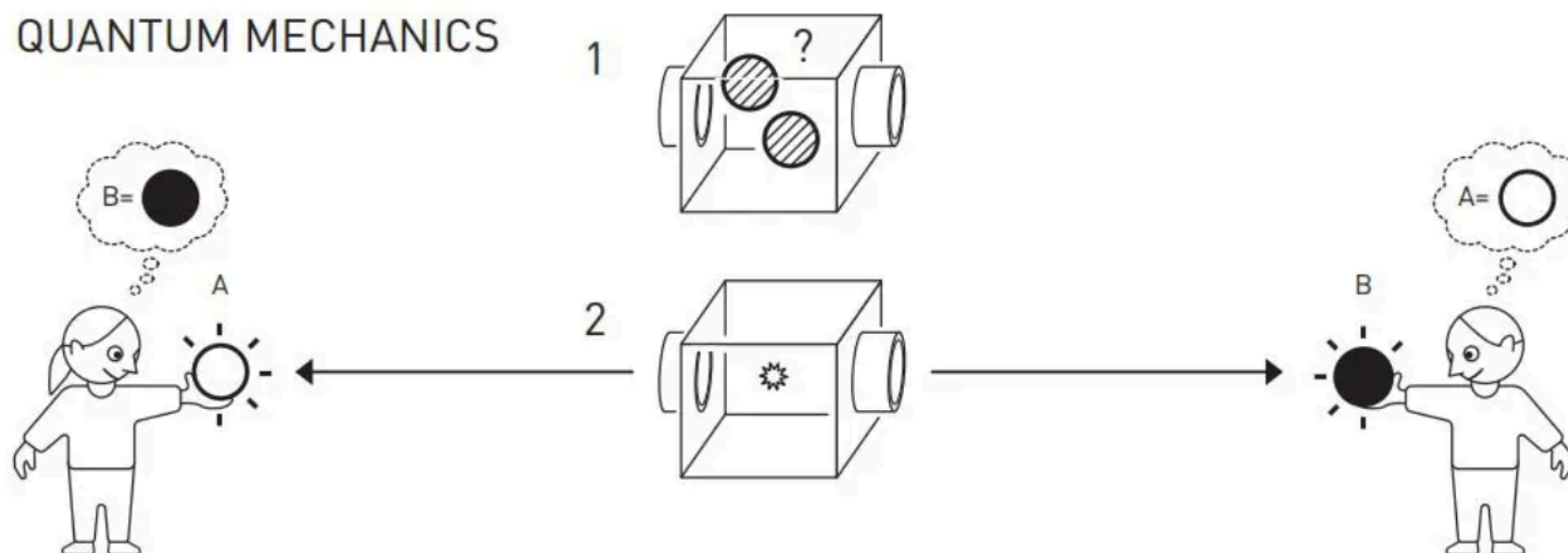➡️ Current dataset can probe entanglement at 5-sigma level

Bell's Inequalities

©Johan Jamestad/The Royal Swedish Academy of Sciences

Bell's Inequalities

- Violation of Bell-type inequalities demonstrates that there is no hidden variable theory capable of encoding the generated entanglement. QM cannot be explained by classical laws

- Bell's inequality can be distilled in a simpler form: CHSH inequality

Clauser, Horne, Shimony, Holt '69

$$|\langle A_1 B_1 \rangle + \langle A_2 B_1 \rangle + \langle A_1 B_2 \rangle - \langle A_2 B_2 \rangle| \leq 2$$

Observer A

Observer B
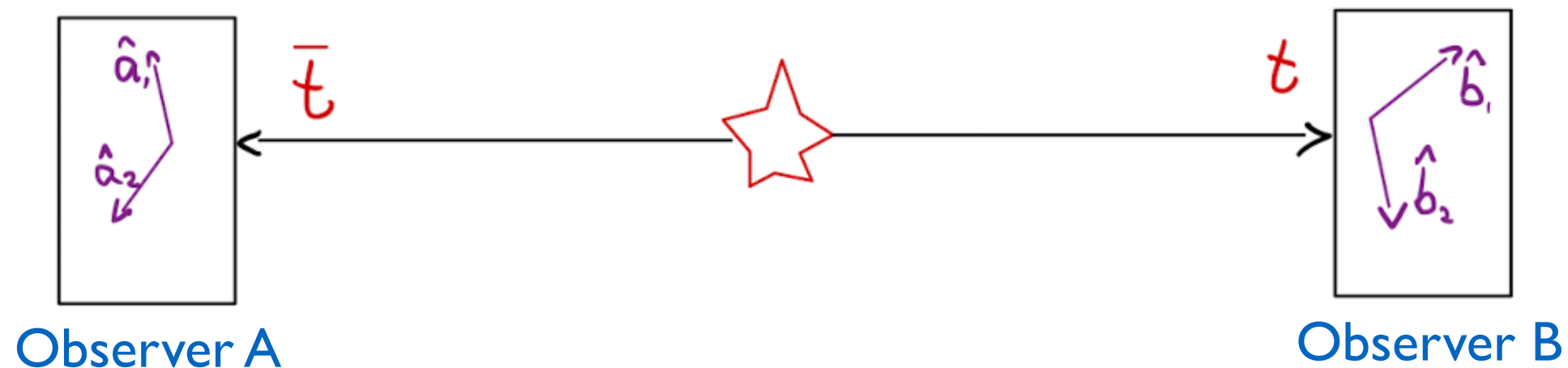
Bell's Inequalities

Bell/CHSH inequalities:
$$\mathcal{B}_1 \equiv |C_{rr} - C_{nn}| - \sqrt{2} > 0$$
$$\mathcal{B}_2 \equiv |C_{kk} + C_{rr}| - \sqrt{2} > 0$$

Afik, Nova '20

Severi, Boschi, Maltoni, Sioli '21

Saavedra, Casas '22

Han, Low, Wu '23

Dong, DG, Kong, Navarro '23

➡ Bell/CHSH violation studies well match **boosted** top pair searches
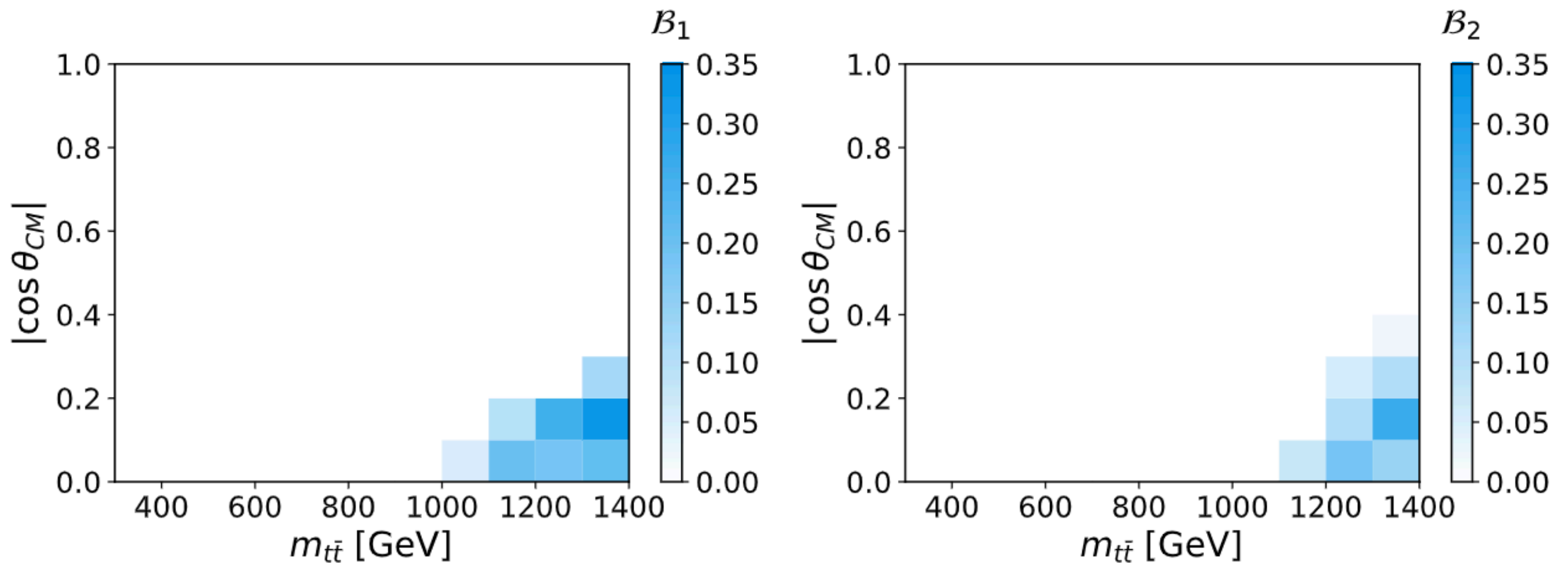
Bell/CHSH inequalities:
$$\mathcal{B}_1 \equiv |C_{rr} - C_{nn}| - \sqrt{2} > 0$$
$$\mathcal{B}_2 \equiv |C_{kk} + C_{rr}| - \sqrt{2} > 0$$

Afik, Nova '20

Severi, Boschi, Maltoni, Sioli '21
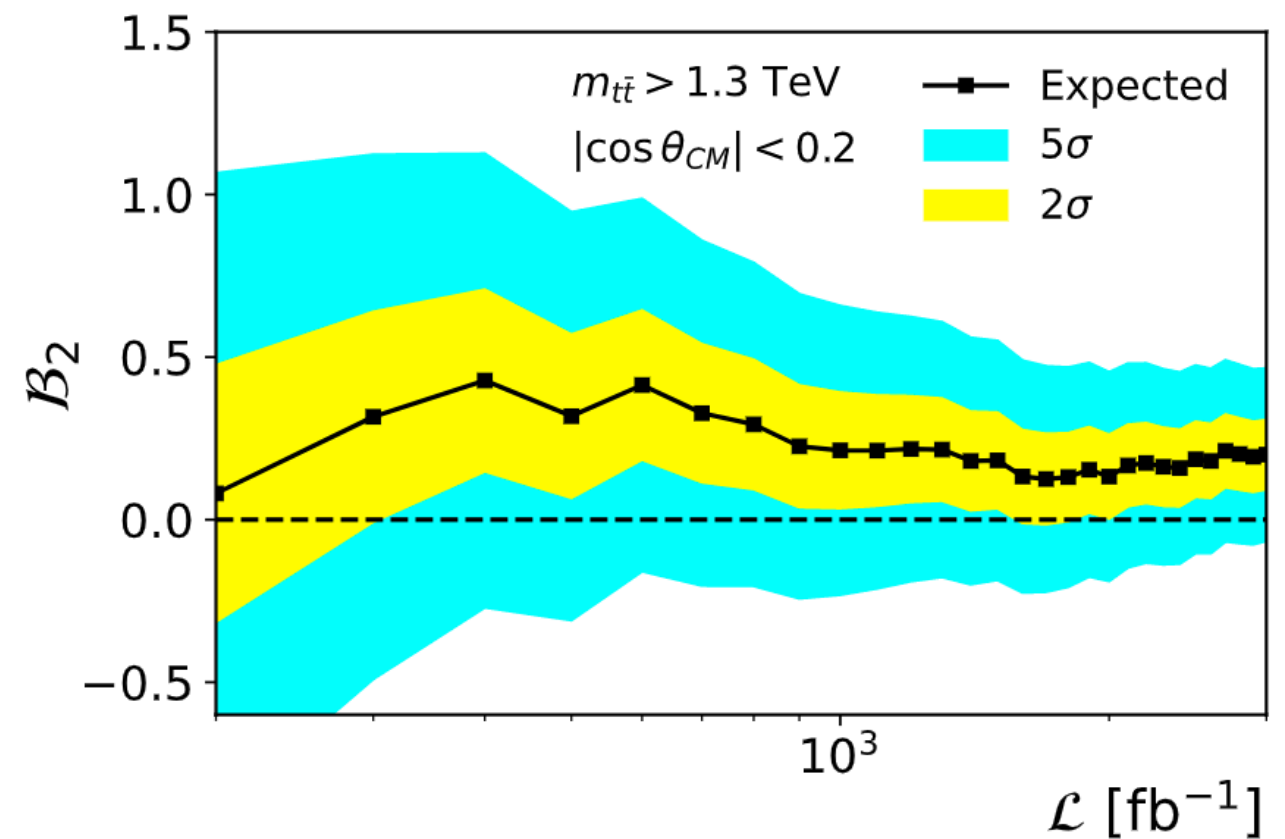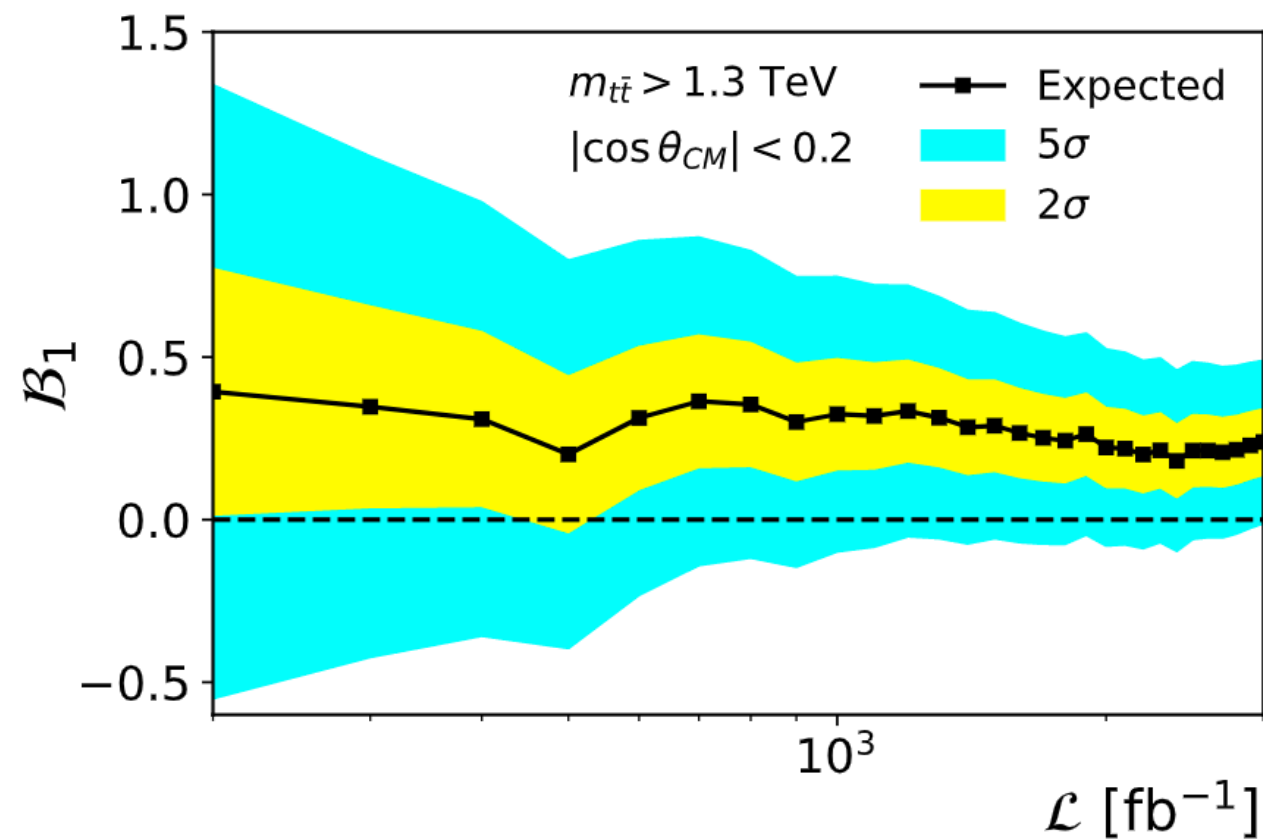
Saavedra, Casas '22

Dong, DG, Kong, Navarro '23

CHSH loopholes in top pair production

Measurement of loophole free Bell violation is more delicate than probing entanglement

Severi, Boschi, Maltoni, Sioli '21; Afik, Nova '22
See talk by Marco Fabbrichesi

🔵 **Free-will loophole**: There is no external intervention in choosing measurements. ❌
However, directions of final state leptons and q_{opt} are randomized ✔️

🔵 **Causally disconnected** at the boosted regime (statistical level) ✔️ ❌

🔵 **Detection loophole**: We examine a subset of events, potentially biasing results ❌

➡️ It is not surprising that the LHC can only probe weak violation of Bell's inequalities. LHC wasn't designed for this. In fact, it took decades to prepare loophole-free setups, with the first measurement in 2015-2018, leading to the 2022 Nobel Prize

We are just at the beginning

Observation of quantum entanglement in top-quark pair production using $p\,p$ collisions of $\sqrt{s} = 13$ TeV with the ATLAS detector

See talk by James W. Howarth

28th September 2023

The ATLAS Collaboration

We report the highest-energy observation of entanglement so far in top–antitop quark events produced at the Large Hadron Collider, using a proton–proton collision data set with a centre-of-mass energy of $\sqrt{s} = 13$ TeV and an integrated luminosity of 140 fb^{-1}. Spin entanglement is detected from the measurement of a single observable D, inferred by the angle between the charged leptons in their parent top- and antitop-quark rest frames. The observable is measured on a narrow interval around the top-quark–antitop-quark production threshold, where the entanglement detection is expected to be significant. The entanglement observable is measured in a fiducial phase-space with stable particles. The entanglement witness is measured to be $D = -0.547 \pm 0.002$ (stat.) ± 0.021 (syst.) for $340 < m_{t\bar{t}} < 380$ GeV. The large spread in predictions from several mainstream event generators indicates that modelling this property is challenging. The predictions depend in particular on the parton-shower algorithm used. The observed result is more than five standard deviations from a scenario without entanglement and hence constitutes the first observation of entanglement in a pair of quarks, and the observation of entanglement at the highest energy to date.

Summary

- LHC provides a unique opportunity to study special quantum correlations, such as entanglement and violation of Bell's inequalities, at high energy scales

- Top pair production at the LHC forms a two-qubit system: quantum tomography

- Boosted semileptonic top pair searches well match entanglement and CHSH studies:
  - Higher event rate + easier reconstruction
  - Boosted top tagging aid subject and light quark matching
  - Optimal hadronic polarimeter

- Promising measurements:
  - Entanglement can be probed with current LHC dataset for semileptonic tops in boosted regime
  - Bell/CHSH violation may be probed at 4-5 sigma level at the HL-LHC
  - Bell violation results can be statistically boosted by ATLAS+CMS combination and combination with dileptonic tops

Work in collaboration with

Zhongtian Dong (KU)

KC Kong (KU)

Alberto Navarro (OSU)