Quantum Information with Electroweak and Higgs Bosons

Alan Barr

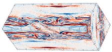
Department of Physics, University of Oxford

Quantum Observables for Collider Physics, GGI

6th November 2023

AJB, Phys.Lett.B 825 (2022) 136866 — 2106.01377 [hep-ph]
AJB, P. Caban, J.Rembieliński — 2204.11063 [quant-ph]
R.Ashby-Pickering, AJB, A.Wierzchucka — 2209.13990 [quant-ph]

Interesting physics $\neq$ 'new' physics $\neq$ beyond-SM physics



ON THE COVER

Heating of Magnetically Dominated Plasma by Alfvén-Wave Turbulence

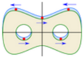
February 14, 2022

Three-dimensional kinetic simulation of the onset of relativistic wave turbulence in the collision of two magnetic shear waves. Selected for a Viewpoint in *Physics*.

Joonas Nättilä and Andrei M. Beloborodov

Phys. Rev. Lett. **128**, 075101 (2022)

Issue 7 Table of Contents | More Covers



Physics NEWS AND COMMENTARY

A Quantized Surprise from Fermi Surface Topology

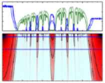
February 16, 2022

The quantized conductance of a two-dimensional electron gas can reflect its Fermi surface topology.

Synopsis on:

C. L. Kane

Phys. Rev. Lett. **128**, 076801 (2022)



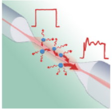
EDITORS' SUGGESTION

Chaotic Diffusion in Delay Systems: Giant Enhancement by Time Lag Modulation

Laminar chaotic diffusion is found in systems with delayed nonlinearity, accompanied by a reduction of the effective dimensionality.

Tony Albers, David Müller-Bender, Lukas Hille, and Günter Radons

Phys. Rev. Lett. **128**, 074101 (2022)



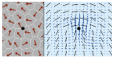
EDITORS' SUGGESTION

Collective Radiative Dynamics of an Ensemble of Cold Atoms Coupled to an Optical Waveguide

An ensemble of cold atoms is coherently coupled in a controlled way to a tapered optical fiber, demonstrating collective effects in this system.

Riccardo Pennetta *et al.*

Phys. Rev. Lett. **128**, 073601 (2022)



Physics NEWS AND COMMENTARY

Extending and Contracting Cells

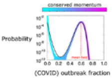
February 15, 2022

Cell-substrate interactions explain a difference in behavior between individual cells and tissues on a surface.

Synopsis on:

Andrew Killeen, Thibault Bertrand, and Chiu Fan Lee

Phys. Rev. Lett. **128**, 078001 (2022)



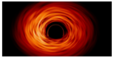
EDITORS' SUGGESTION

Outbreak Size Distribution in Stochastic Epidemic Models

An analytical approach to stochastic epidemic models shows that the statistics of extreme outbreaks depend on an infinite number of minimum-action paths, and that extreme outbreaks define a new class of rare processes for discrete-state stochastic systems.

Jason Hindes, Michael Assaf, and Ira B. Schwartz

Phys. Rev. Lett. **128**, 078301 (2022)



Physics NEWS AND COMMENTARY

Illuminating Black Holes through Turbulent Heating

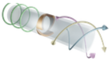
February 14, 2022

Predictions indicate that it should be possible to directly identify how turbulence heats a given black hole's plasma from the spectrum of that plasma's radiation.

Viewpoint on:

Joonas Nättilä and Andrei M. Beloborodov

Phys. Rev. Lett. **128**, 075101 (2022)



Physics NEWS AND COMMENTARY

Waves in a Solid Imitate Twisted Light

February 11, 2022

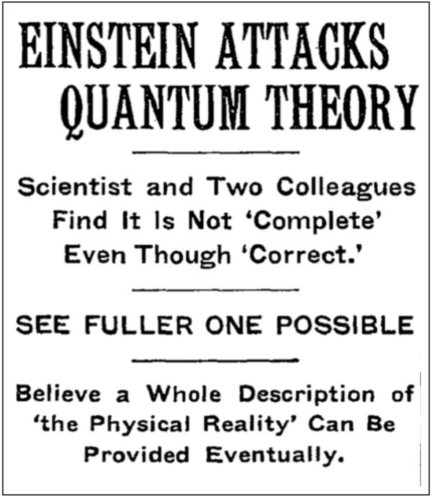
Waves of vibration moving through the walls of a pipe can carry orbital angular momentum that could be used for several purposes, according to new theoretical work.

Focus story on:

G. J. Chaplain, J. M. De Ponti, and R. V. Craster

Phys. Rev. Lett. **128**, 064301 (2022)

Some of the old problems are amongst the deepest. . .


EINSTEIN ATTACKS QUANTUM THEORY

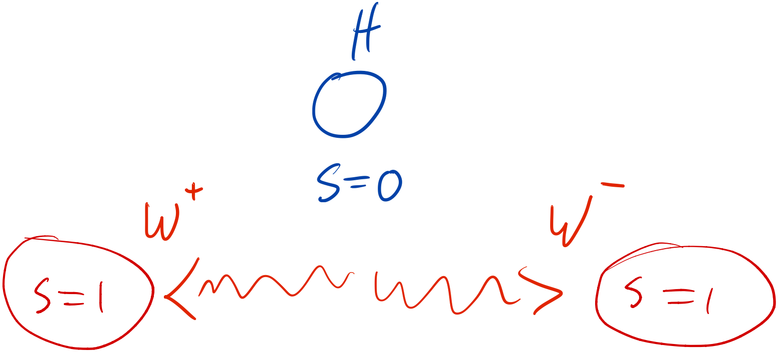
Scientist and Two Colleagues Find It Is Not 'Complete' Even Though 'Correct.'

SEE FULLER ONE POSSIBLE

Believe a Whole Description of 'the Physical Reality' Can Be Provided Eventually.

New York Times, May 4 **1935**, reporting on Einstein-Podolsky-Rosen paper, "*Can Quantum-Mechanical Description of Physical Reality Be Considered Complete*"

A Higgs boson decay



≈maximally entangled spin state

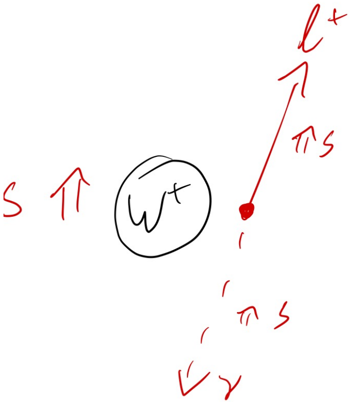
Qu**tr**its

$$|\psi_s\rangle = \frac{1}{\sqrt{3}}\Big(|+\rangle\, |-\rangle - |0\rangle\, |0\rangle + |-\rangle\, |+\rangle \Big)$$

- This is a maximally entangled state of two **qutrits**
- $|\psi\rangle_{AB} \in (\mathbb{C}^{\mathbf{3}})^2$
- Basis for each qutrit $\{0, 1, 2\}$

[On the board: qutrits vs 3-state systems]

Particles with weak decays are their own polarimeters



The charged leptons are emitted in directions that strongly depend on the spin of the parent W boson

More precisely

The probability density function for a $W^{\pm}$ boson to emit its charged lepton in the $\hat{\mathbf{n}}$ direction is:

$$p(\ell^{\pm}_{\hat{\mathbf{n}}};\rho) = \frac{3}{4\pi}\,\mathrm{tr}(\rho\,\Pi_{\pm,\hat{\mathbf{n}}})\,,$$

where ρ is the W boson spin density matrix and the projection operator is

$$\Pi_{\pm,\hat{\mathbf{n}}} \equiv |\pm_{\hat{\mathbf{n}}}\rangle\langle\pm_{\hat{\mathbf{n}}}|$$

observe decay $\Longleftrightarrow$ measure spin

Weak gauge bosons measure their own spin

SU(2) weak force is chiral: $\gamma^\mu(1-\gamma^5)$

W boson

$$W^+ \to \ell_R^+ + \nu_L$$
$$W^- \to \ell_L^- + \bar{\nu}_R$$

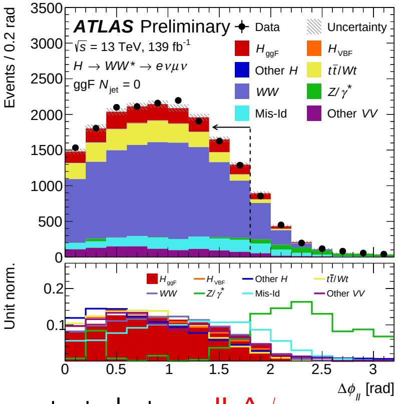
Decay of a $W^\pm$ boson is equivalent to a projective (von Neumann) quantum measurement of its spin along the axis of the emitted lepton

Z boson

Z bosons also have spin-sensitive decays

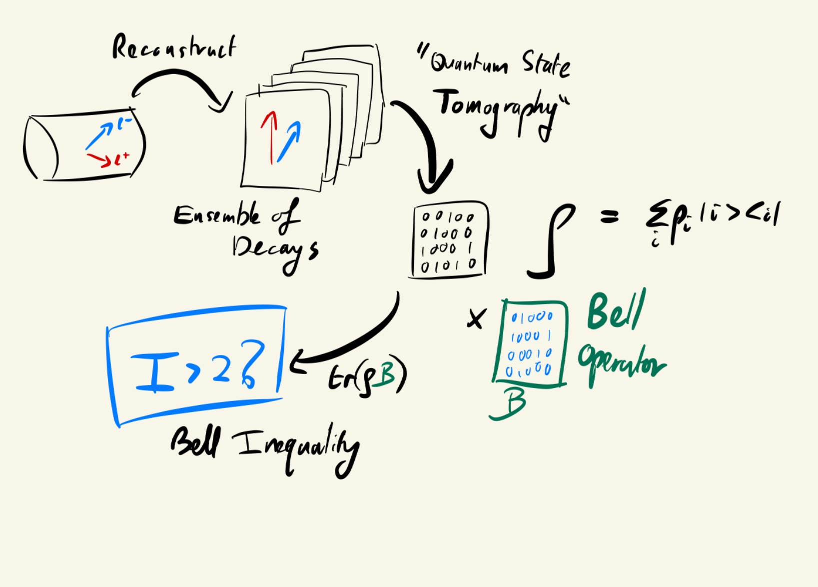
Left, right couplings determined by electroweak mixing
Equivalent to a non-projective quantum measurement

$\ell^+\ell^-$ azimuthal correlations observed in $H \to W^+W^-$

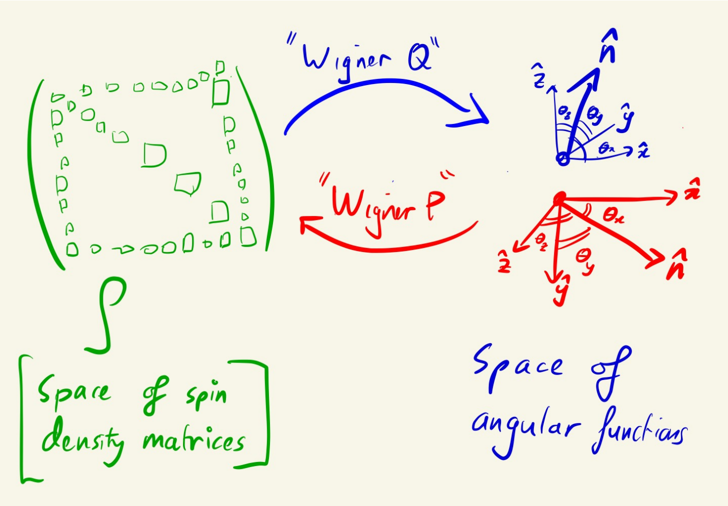


- Higgs signal concentrated at small $\Delta\phi_{\ell\ell}$
- Used e.g. in discovery searches

Observables and density matries



Reconstruct
e⁻
e⁺
Ensemble of Decays
"Quantum State Tomography"
ρ = Σ_i p_i |i><i|
×
Bell Operator
B
I > 2?
Tr(ρB)
Bell Inequality



Also true for e.g. $W^{\pm}$, Z^0, t , τ

Transforming between the spaces

The Wigner-Weyl formalism for spin

Operator $\to$ function

$$\Phi_A^Q(\hat{\mathbf{n}}) = \langle \hat{\mathbf{n}} | A | \hat{\mathbf{n}} \rangle$$

Wigner Q symbols

Function $\to$ operator

$$A = \frac{2j+1}{4\pi} \int \mathrm{d}\Omega_{\hat{\mathbf{n}}} \, |\hat{\mathbf{n}}\rangle \, \Phi_A^P(\hat{\mathbf{n}}) \, \langle \hat{\mathbf{n}}| \, ,$$

Wigner P symbols

Parameterise ρ

Symmetrically for qutrits in terms of the Gell-Mann matrices λ_i

Single vector boson

$$\rho = \frac{1}{3} I_3 + \sum_{i=1}^{8} a_i \lambda_i,$$

a_i : 8 real parameters $(3^2 - 1)$

- Generalised Gell-Mann matrices $\lambda_i^{(d)}$ exist for any spin
- For spin-half ($d = 2$) they are the Pauli matrices and we get the Bloch sphere
- For $d = 3$ they are the eight generators of SU(3)

Other parameterisations, e.g. Cartesian Tensors are good alternatives

Parameterise ρ – bipartite system

Symmetrically for qutrits in terms of the Gell-Mann matrices λ_i

Single vector boson

$$\rho = \frac{1}{3} I_3 + \sum_{i=1}^{8} a_i \lambda_i,$$

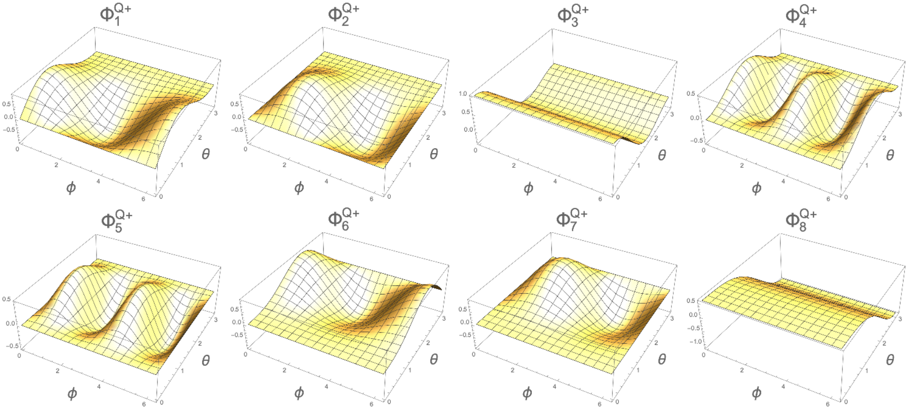
a_i : 8 real parameters $(3^2 - 1)$

Two vector bosons

$$\rho = \frac{1}{9} I_3 \otimes I_3 + \sum_{i=1}^{8} a_i \, \lambda_i \otimes \frac{1}{3} I_3 + \sum_{j=1}^{8} b_j \, \frac{1}{3} I_3 \otimes \lambda_j + \sum_{i,j=1}^{8} c_{ij} \, \lambda_i \otimes \lambda_j,$$

8+8+64 = 80 real parameters $(9^2 - 1)$

Angular distributions for each parameter



Wigner Q functions for the eight Gell-Mann matrices

Extracting the parameters experimentally

Parameters of ρ are the experimentally-measurable classical averages of the Wigner P functions

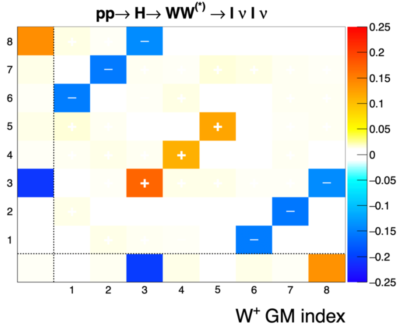
$$\hat{a}_i = \frac{1}{2}\left\langle \Phi_i^P(\hat{\mathbf{n}}_1)\right\rangle_{\rm av}$$

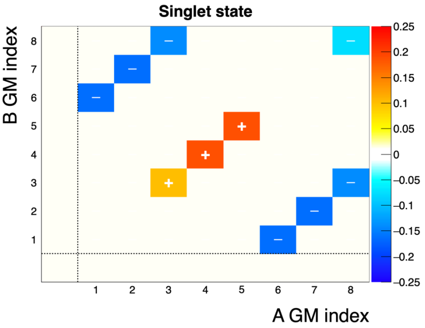
$$\hat{b}_i = \frac{1}{2}\left\langle \Phi_i^P(\hat{\mathbf{n}}_2)\right\rangle_{\rm av}$$

$$\hat{c}_{ij} = \frac{1}{4}\left\langle \Phi_i^P(\hat{\mathbf{n}}_1)\,\Phi_j^P(\hat{\mathbf{n}}_2)\right\rangle_{\rm av}$$

Quantum State Tomography example

Higgs boson decays





Density matrix parameters from simulated Higgs boson decays to vector bosons (Madgraph, no background)

\begin{interlude}

discovernorthernireland.com

Belfast City Council has declined to name a street after one of Northern Ireland's most eminent scientists.

Belfast-born, John Stewart Bell who died in 1990, is regarded as one of the 20th Century's greatest physicists.

The council received an application to name a street in Titanic Quarter after Mr Bell.

However, the council rejected the proposal as it has "traditionally avoided using the names of people" when deciding on street names.

Only two streets in the city have been named after individuals since the 1960s: Prince Edward Park in 1962 and Prince Andrew Park in 1987.

Titanic Quarter Ltd had applied to name a currently unnamed street beside Belfast Metropolitan College as John Bell Crescent.

Bell was born in Belfast in 1928 to a family from a poor background.

He was the only one of his siblings to stay at school over the age of 14, and his family could not afford to send him to one of the city's grammar schools.

Instead he attended Belfast Technical High School, now Belfast Metropolitan College, and then entered Queen's University.

BBC news 19 February 2015

\end{interlude}

Testing a Bell Inequality

The CGLMP Qutrit inequality

Collins Gisin Linden Massar Popescu (2002)

The optimal Bell inequality for pairs of **qutrits**

CGLMP function

$$
\begin{aligned}
\mathcal{I}_3 = P(A_1 = B_1) + P(B_1 = A_2 + 1) \\
+ P(A_2 = B_2) + P(B_2 = A_1) \\
- P(A_1 = B_1 - 1) - P(B_1 = A_2) \\
- P(A_2 = B_2 - 1) - P(B_2 = A_1 - 1).
\end{aligned}
$$

$P(A_i = B_j + k)$ is the probability that A_i and B_j differ by k mod 3

CGLMP limits?

In a local realist theory

$$\mathcal{I}_3 \leq 2$$

In QM

$$\mathcal{I}_3^{\mathrm{QM}} \leq 1 + \sqrt{11/3} \approx 2.9149$$

In QM for a maximally entangled state

$$\mathcal{I}_3^{\mathrm{QM,singlet}} \leq 4/(6\sqrt{3} - 9) \approx 2.8729$$

Testing the CGLMP inequality

Knowing elements of ρ calculate

$$\mathcal{I}_3 = \mathrm{tr}(\rho\,\mathcal{B}^{xy}_{\mathrm{CGLMP}})$$

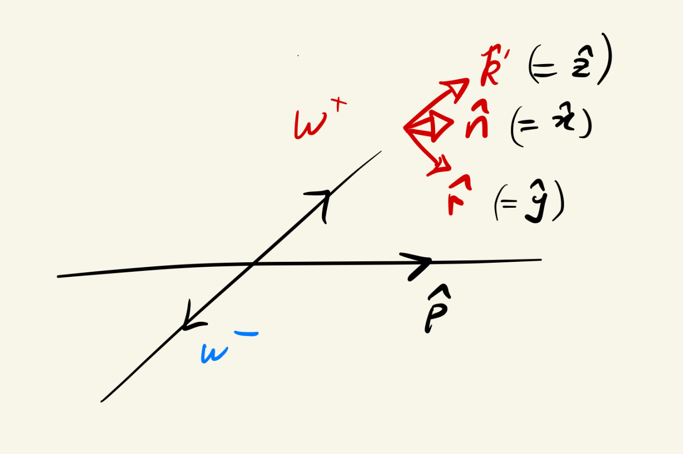
where the CGLMP operator is

$$\mathcal{B}^{xy}_{\mathrm{CGLMP}} = -\frac{2}{\sqrt{3}}\,(S_x \otimes S_x + S_y \otimes S_y) + \lambda_4 \otimes \lambda_4 + \lambda_5 \otimes \lambda_5$$

where

$$S_x = \frac{1}{\sqrt{2}}(\lambda_1 + \lambda_6) \quad \text{and} \quad S_y = \frac{1}{\sqrt{2}}(\lambda_2 + \lambda_7).$$

Coordinates



Angular distributions are measured in the rest-frame of the parent gauge boson

Measuring Bell expectation values directly

CGLMP (qutrit) inequality from data for *WW*

$$\mathcal{I}_3 = \mathrm{tr}(\rho \mathcal{B}^{xy}_{\mathrm{CGLMP}}) = \frac{8}{\sqrt{3}} \left\langle \xi_x^+ \xi_x^- + \xi_y^+ \xi_y^- \right\rangle_{\mathrm{av}} + 25 \left\langle \left((\xi_x^+)^2 - (\xi_y^+)^2 \right) \left((\xi_x^-)^2 - (\xi_y^-)^2 \right) \right\rangle_{\mathrm{av}} + 100 \left\langle \xi_x^+ \xi_y^+ \xi_x^- \xi_y^- \right\rangle_{\mathrm{av}}$$

Is this Bell inequality violated in data?

$$\mathcal{I}_3 \leq 2?$$

$H \to W^+W^-$ simulated results

In idealised, numerical simulation of $H \to W^+W^-$, with finite width effects and relativistic effects:

$$\mathcal{I}_3 \approx 2.6$$

Doing for real & doing better?

Bell operator optimisation

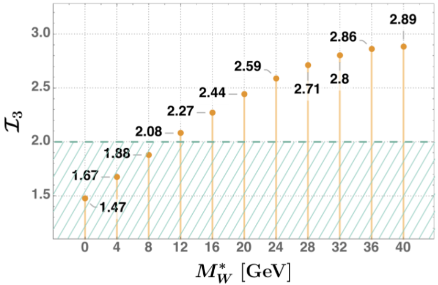
- Optimal Bell operator not known in the general case.
- Use freedom in measurement observables to perform independent unitary transformations on each side of the experiment

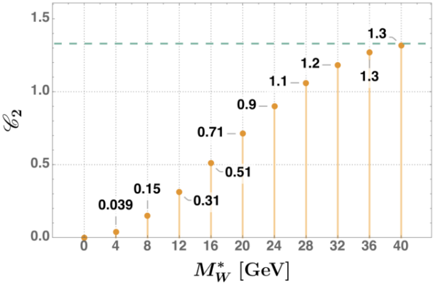
$$\mathcal{B} \longrightarrow (U \otimes V)^\dagger \cdot \mathcal{B} \cdot (U \otimes V)$$

- U, V independent 3×3 unitary matrices, optimised for each kinematic process

Fabbrichesi et al. 2302.00683

H$\longrightarrow WW^*$





Optimised Bell Operator

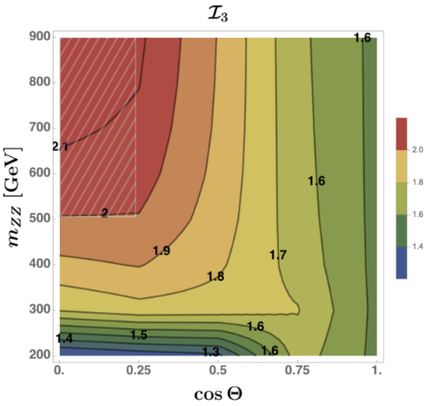
> 2?

Bound on the concurrence

> 0?

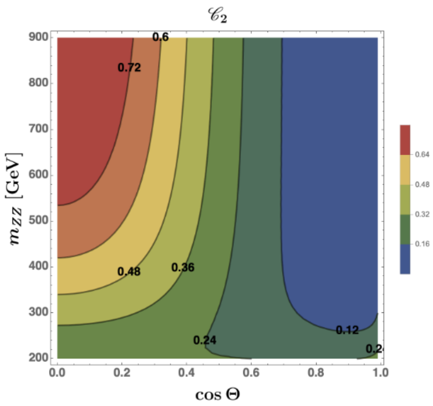
Fabbrichesi et al. 2302.00683

pp⟶ ZZ



Optimised Bell Operator

$> 2?$

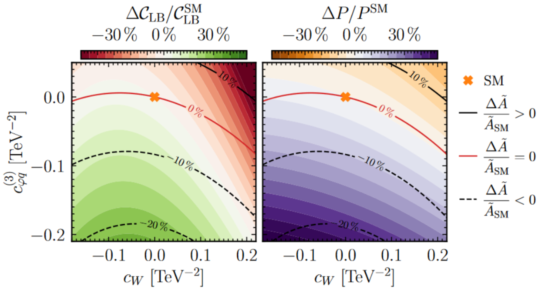


Bound on the concurrence

$> 0?$

Fabbrichesi et al. 2302.00683

Searching Beyond the Standard Model?

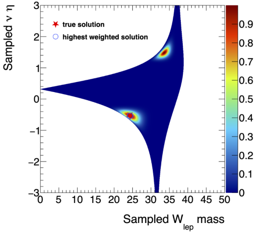


- Production of $W\pm/Z$ pairs at pp, e^+e^-
- Quantum spin observables complementary probes of Wilson coefficients/EFT
- Offer increased sensitivity to certain operators

Aoude, Madge, Maltoni, Mantani *Probing new physics through entanglement in diboson production* 2307.09675

Semi-leptonic $h \to WW^* \to \ell^- \bar{\nu} c \bar{s}$

Semi-leptonic channel allows neutrino weighting reconstruction
Charm tagging allows identification of spin from angular distribution of hadronic W



Lumi [fb^{-1}]	$\langle \mathcal{B}^{zx}_{\mathrm{CGLMP}} \rangle$		(idealised)	Signif. (idealised)
139	2.45	$\pm$	0.25 (0.18)	1.8 (2.5)
300	2.45	$\pm$	0.17 (0.12)	2.65 (3.75)
3000	2.45	$\pm$	0.05 (0.04)	9.0 (11.25)

Fabbri, Howarth, Maurin *Isolating semi-leptonic $H \to WW^*$ decays for Bell inequality tests* 2307.13783

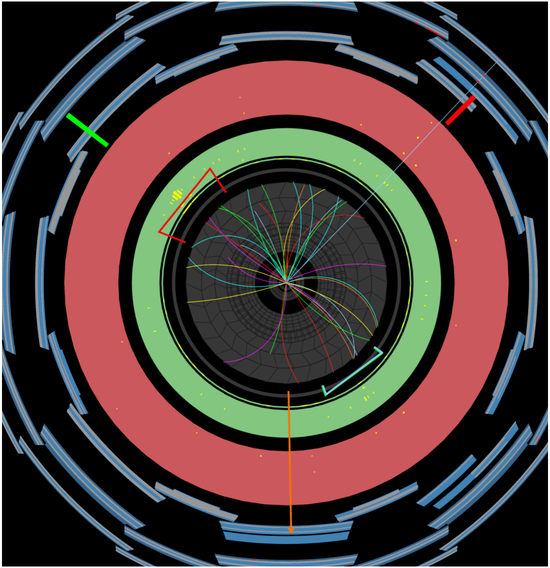
Lots of other interesting work in this area, including:

- Aguilar-Saavedra, Bernal, Casas, Moreno *Testing entanglement and Bell inequalities in* $H \to ZZ$ 2209.13441
- Aguilar-Saavedra, *Laboratory-frame tests of quantum entanglement in* $H \to WW$, 2209.14033
- Fabbrichesi, Floreanini, Gabrielli, Marzola *Stringent bounds on HWW and HZZ anomalous couplings with quantum tomography at the LHC* 2304.02403
- Morales *Exploring Bell inequalities and quantum entanglement in vector boson scattering* 2306.17247
- Bi, Cao, Cheng, Zhang *New observables for testing Bell inequalities in* W *boson pair production* 2307.14895
- Bernal, *Quantum tomography of helicity states for general scattering processes* 2310.10838

Summary

- Weak decays are wonderful quantum probes
- Quantum spin self measurement via chiral weak decays
- Spin density matrix can be reconstructed from angular distributions (‘quantum state tomography’)
- Expect entanglement and even Bell inequality violation
- Expect in Higgs decays but also Drell-Yan processes
- Improved bounds on EFTs and new anomalous HWW and HZZ couplings using quantum information toolbox

EXTRAS



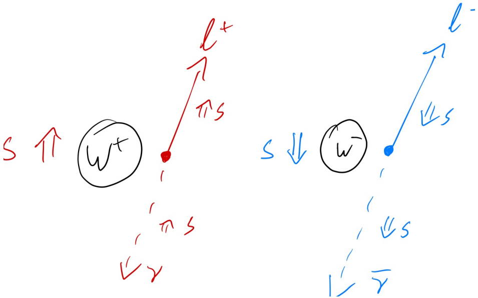
In case you're curious

The CGLMP operator is[1]

$$\mathcal{B}^{xy}_{\mathrm{CGLMP}} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 2 & 0 & 0 \\ 0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 \\ 0 & 0 & 2 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

[1] after a minor tweak – see 2106.01377

Getting the directions right



- ℓ^+ is emitted preferentially along spin direction (of W^+)
  ℓ^- is emitted preferentially against spin direction (of W^-)
- For $H \to W^+W^-$ the $W^\pm$ spins are in different directions
- So the two leptons prefer to go in the same direction as each other

Spin in the $H \to W^+W^-$ decay

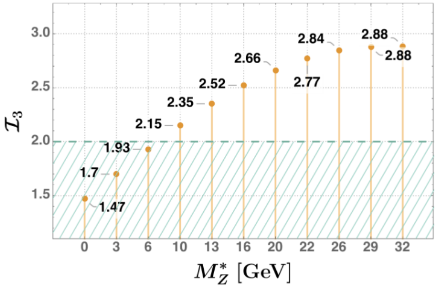
The Higgs boson is a scalar, while $W^\pm$ bosons are massive vector bosons, each with three possible spin states (qutrits)

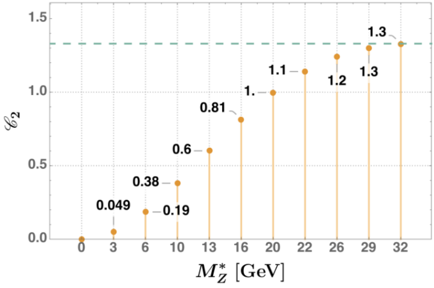
- $H \to W^+W^-$ decays produce pairs of W bosons with zero total angular momentum
- In the narrow-width and non-relativistic approximations:

$$|\psi_s\rangle = \frac{1}{\sqrt{3}}\big(\,|+\rangle\,|-\rangle - |0\rangle\,|0\rangle + |-\rangle\,|+\rangle\,\big)$$

This is a maximally entangled state of two qutrits

$H \longrightarrow ZZ^*$



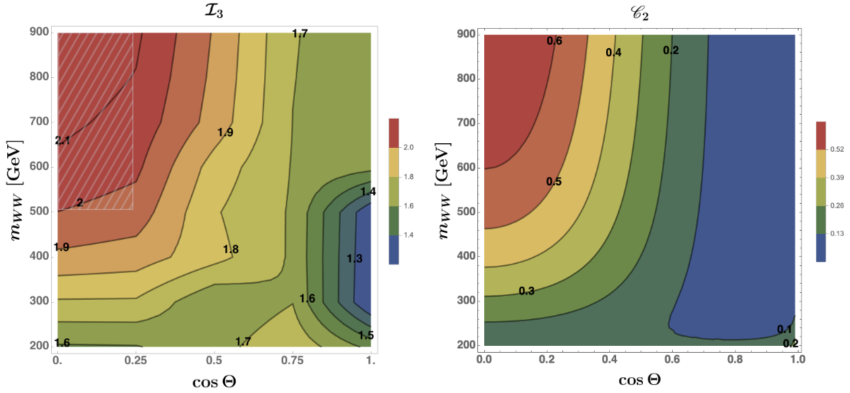


Optimised Bell Operator > 2?

Bound on the concurrence > 0?

Fabbrichesi et al. 2302.00683

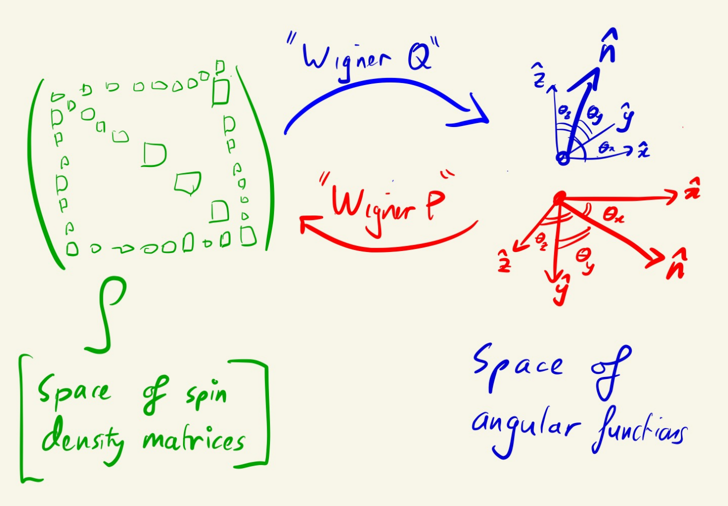
pp $\longrightarrow$ *WW*



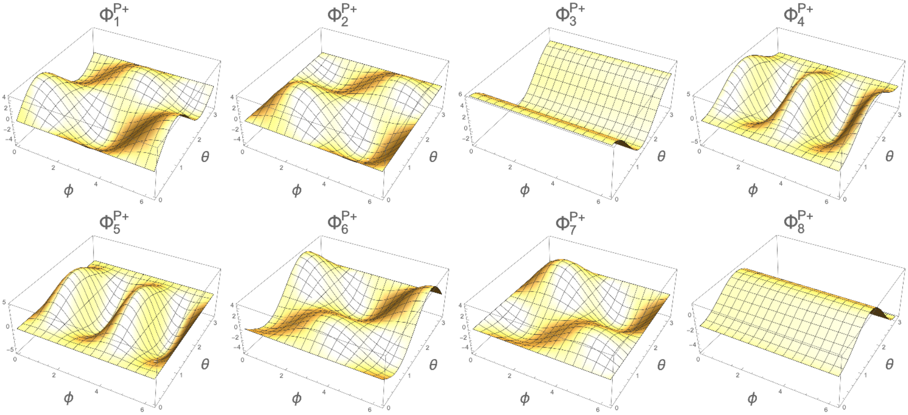
Optimised Bell Operator
$> 2?$

Bound on the concurrence
$> 0?$

Fabbrichesi et al. 2302.00683


"Wigner Q"
"Wigner P"
Space of spin density matrices
Space of angular functions

To get back to the density matrix



Wigner *P* functions for the eight Gell-Mann matrices

Operator	Coefficient	Definition	95 % CL bounds
two-fermion operators			
$\mathcal{O}_{\varphi u}$	$c_{\varphi u}$	$i(\varphi^\dagger \overleftrightarrow{D}_\mu \varphi)(\bar{u}\gamma^\mu u)$	$[-0.17, 0.14]$
$\mathcal{O}_{\varphi d}$	$c_{\varphi d}$	$i(\varphi^\dagger \overleftrightarrow{D}_\mu \varphi)(\bar{d}\gamma^\mu d)$	$[-0.07, 0.09]$
$\mathcal{O}_{\varphi q}^{(1)}$	$c_{\varphi q}^{(1)}$	$i(\varphi^\dagger \overleftrightarrow{D}_\mu \varphi)(\bar{q}\,\gamma^\mu\, q)$	$[-0.06, 0.22]$
$\mathcal{O}_{\varphi q}^{(3)}$	$c_{\varphi q}^{(3)}$	$i(\varphi^\dagger \overleftrightarrow{D}_\mu\, \tau_I \varphi)(\bar{q}\,\gamma^\mu\, \tau^I q)$	$[-0.21, 0.05]$
$\mathcal{O}_{\varphi e}$	$c_{\varphi e}$	$i(\varphi^\dagger \overleftrightarrow{D}_\mu \varphi)(\bar{e}\gamma^\mu e)$	$[-0.21, 0.26]$
$\mathcal{O}_{\varphi l}^{(1)}$	$c_{\varphi l}^{(1)}$	$i(\varphi^\dagger \overleftrightarrow{D}_\mu \varphi)(\bar{l}\gamma^\mu l)$	$[-0.11, 0.13]$
$\mathcal{O}_{\varphi l}^{(3)}$	$c_{\varphi l}^{(3)}$	$i(\varphi^\dagger \overleftrightarrow{D}_\mu \tau_I \varphi)(\bar{l}\gamma^\mu \tau^I l)$	$[-0.21, 0.05]$
bosonic operators			
$\mathcal{O}_W$	c_W	$\varepsilon_{IJK} W^I_{\mu\nu} W^{J,\nu\rho} W^{K,\mu}_\rho,$	$[-0.18, 0.22]$
$\mathcal{O}_{\varphi W}$	$c_{\varphi W}$	$\left(\varphi^\dagger \varphi - \frac{v^2}{2}\right) W_I^{\mu\nu} W^I_{\mu\nu}$	$[-0.15, 0.30]$
$\mathcal{O}_{\varphi B}$	$c_{\varphi B}$	$\left(\varphi^\dagger \varphi - \frac{v^2}{2}\right) B_{\mu\nu} B^{\mu\nu}$	$[-0.11, 0.11]$
$\mathcal{O}_{\varphi WB}$	$c_{\varphi WB}$	$(\varphi^\dagger \tau_I \varphi) B^{\mu\nu} W^I_{\mu\nu}$	$[-0.17, 0.27]$
$\mathcal{O}_{\varphi D}$	$c_{\varphi D}$	$(\varphi^\dagger D^\mu \varphi)^\dagger (\varphi^\dagger D_\mu \varphi)$	$[-0.52, 0.43]$
four-fermion operator			
$\mathcal{O}_{ll}$	c_{ll}	$(\bar{l}\gamma_\mu l)(\bar{l}\gamma^\mu l)$	$[-0.16, 0.02]$

Aoude, Madge, Maltoni, Mantani *Probing new physics through entanglement in diboson production* 2307.09675