

# Quantum Information with Top Quarks

Y. Afik, JRMdN, EPJ Plus 136, 907 (2021)

Y. Afik, JRMdN, Quantum 6, 820 (2022)

Y. Afik, JRMdN, PRL 130, 221801 (2023)

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Quantum Observables for Collider Physics, Firenze, 06/11/2023



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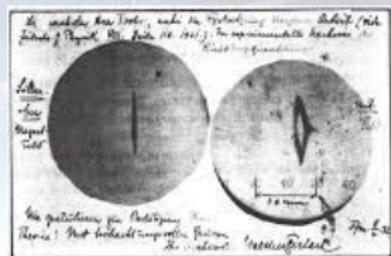
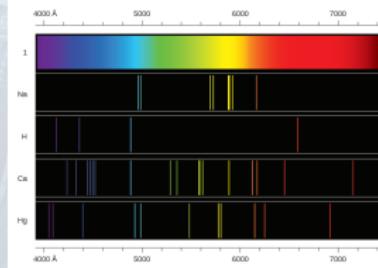
# Outline

- Quantum Theory.
  - Quantum discord.
  - Entanglement.
  - Steering.
  - Bell nonlocality.
  - Quantum tomography.
- Top quark physics.
  - $t\bar{t}$  Kinematics.
  - $t\bar{t}$  Spin Quantum State.
  - LO QCD  $t\bar{t}$  Production.
- Quantum Tops.
- Phenomenological Analysis.
- Conclusions and outlook.

# Part I: Quantum Theory

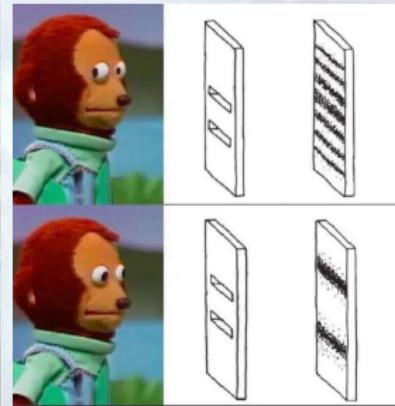
# Quantum Theory: Quantization

- Quantum Mechanics was originally named after observation of quantized values in several systems:
  - Electromagnetic radiation (Black-body/Photoelectric effect)
  - Electron orbits (Atomic spectra)
  - Angular momentum (Stern-Gerlach)



# Quantum Theory: Copenhagen Interpretation

- Copenhagen interpretation of Quantum Mechanics:
  - Particles described in terms of waves  $\Psi \rightarrow$  Superposition of quantum states.
  - Outcomes of measurements: Observable eigenvalues  $\rightarrow$  Quantization.
  - Probabilities of outcomes encoded in  $|\Psi|^2 \rightarrow$  Quantum Interference.
  - Wave-function collapses by measurement process: Quantum state projected to measured states.



# Quantum vs. Classical

- Quantum Mechanics: Particles are in superposition of states → Probabilistic description of measurements.
- Classical Mechanics can also describe random outputs using classical probability distributions (noise, experimental variations...).
- Is God *just* playing dice with the Universe? →

# Quantum vs. Classical

- Quantum Mechanics: Particles are in superposition of states → Probabilistic description of measurements.
- Classical Mechanics can also describe random outputs using classical probability distributions (noise, experimental variations...).
- Is God *just* playing dice with the Universe? → God is well beyond a mere croupier!
- Quantum Correlations=Correlations not accounted by classical probabilistic theories.



# Quantum State

- **Pure state** → Wave function  $|\Psi\rangle$

①  $|\Psi\rangle = \sum_n \alpha_n \cdot |\phi_n\rangle$ ,  $\langle \Psi | \Psi \rangle = \sum_n |\alpha_n|^2 = 1$

② Coherent mixture of quantum states →  $\alpha_n$  are amplitudes

③ Expectation values:  $\langle A \rangle = \langle \Psi | A | \Psi \rangle = \sum_{n,m} \alpha_m^* \overline{\alpha_n} \langle \phi_m | A | \phi_n \rangle$

- **Mixed state** → Generalization to density matrix  $\rho$

①  $\rho = \sum_n p_n \cdot |\phi_n\rangle \langle \phi_n|$ ,  $\text{tr} \rho = \sum_n p_n = 1$

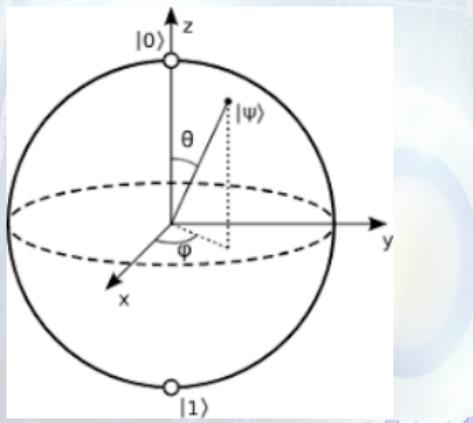
② Incoherent mixture of quantum states →  $p_n$  are probabilities

③ Expectation values:  $\langle A \rangle = \text{tr}(\rho A) = \sum_n p_n \langle \phi_n | A | \phi_n \rangle$



# Qubits: Pure state

- Qubit: Two-level quantum system  $|0\rangle, |1\rangle \rightarrow$  Most simple quantum system.
- Paradigmatic example of qubit: spin-1/2 particle.
- General wave function:  $|\Psi\rangle = \cos \frac{\theta}{2} |0\rangle + \sin \frac{\theta}{2} e^{i\phi} |1\rangle \equiv |\hat{n}\rangle$
- Unit vector  $\hat{n} = [\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta]$  labels quantum states.
- Any state is eigenstate of spin projection:  $\sigma \cdot \hat{n} |\hat{n}\rangle = |\hat{n}\rangle \rightarrow$  Surface of Bloch sphere.



# Qubits: Density matrix

- General density matrix ( $2 \times 2$ ) for 1 qubit  $\rightarrow$  3 parameters  $B_i$ :

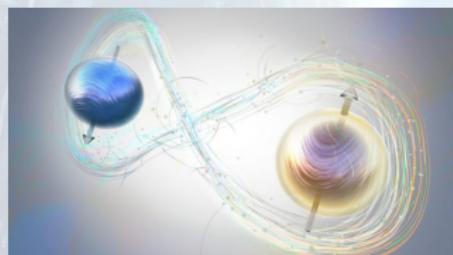
$$\rho = \frac{1 + \sum_i B_i \sigma^i}{2}, \quad \mathbf{B} = \text{tr}[\boldsymbol{\sigma} \rho], \quad |\mathbf{B}| \leq 1$$

- Two qubits  $\rightarrow$  Most simple example of quantum correlations.
- General density matrix ( $4 \times 4$ ) for 2 qubits  $\rightarrow$  15 parameters  $B_i^\pm, C_{ij}$

$$\rho = \frac{\mathbb{1} + \sum_i (B_i^+ \sigma^i \otimes \mathbb{1} + B_i^- \mathbb{1} \otimes \sigma^i) + \sum_{i,j} C_{ij} \sigma^i \otimes \sigma^j}{4}$$

- Polarization vectors  $\mathbf{B}^\pm$  and correlation matrix  $\mathbf{C}$ :

$$B_i^+ = \langle \sigma^i \otimes \mathbb{1} \rangle, \quad B_i^- = \langle \mathbb{1} \otimes \sigma^i \rangle, \quad C_{ij} = \langle \sigma^i \otimes \sigma^j \rangle$$



# Quantum Discord

- Classically, two equivalent expressions for mutual information of bipartite system A and B (Alice and Bob):

$$I(A, B) = H(A) + H(B) - H(A, B) = H(A) - H(A|B)$$

$$H(A, B) = - \sum_{x,y} p(x, y) \log_2 p(x, y)$$

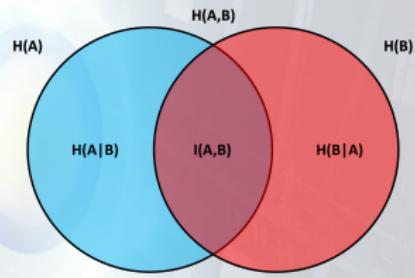
$$H(A|B) = \sum_y p(y) H(A|B = y)$$

- Quantum mechanics can introduce a "discord" between both expressions:

PRL 88, 017901 (2001)

$$\mathcal{D}(A, B) \equiv H(B) - H(A, B) + H(A|B) \neq 0$$

- Most basic form of quantum correlations!
- Quantum Discord is asymmetric  
 $\mathcal{D}(A, B) \neq \mathcal{D}(B, A)$



# Quantum Discord: Classical states

- OK...but where is the Physics here? → Only classical states have zero discord!

$$\rho_{\text{class}} = \sum_{n,m} p_{n,m} |n\rangle \otimes |m\rangle \langle n| \otimes \langle m|$$

- $|n\rangle, |m\rangle$  form an *orthonormal* basis for  $A, B$
- $p_{n,m}$  are purely classical probabilities of being in state  $|n\rangle \otimes |m\rangle$ !
- Two qubits → Tails and heads with two spin directions!

$$\begin{aligned}\rho_{\text{class}} &= p_{++} |\mathbf{n}_A\rangle \otimes |\mathbf{n}_B\rangle \langle \mathbf{n}_A| \otimes \langle \mathbf{n}_B| \\ &+ p_{+-} |\mathbf{n}_A\rangle \otimes |-\mathbf{n}_B\rangle \langle \mathbf{n}_A| \otimes \langle -\mathbf{n}_B| \\ &+ p_{-+} |-\mathbf{n}_A\rangle \otimes |\mathbf{n}_B\rangle \langle -\mathbf{n}_A| \otimes \langle \mathbf{n}_B| \\ &+ p_{--} |-\mathbf{n}_A\rangle \otimes |-\mathbf{n}_B\rangle \langle -\mathbf{n}_A| \otimes \langle -\mathbf{n}_B|\end{aligned}$$

# Quantum Discord: Two qubits

- How do we translate classical into quantum?

# Quantum Discord: Two qubits

- How do we translate classical into quantum?
- Shannon entropy  $\rightarrow$  Von Neumann entropy ( $p_n \geq 0$ ,  $\rho$  eigenvalues)

$$H(A, B) \rightarrow H(\rho) = - \sum_n p_n \log_2 p_n$$

$$H(A) \rightarrow H(\rho_A), H(B) \rightarrow H(\rho_B), \rho_{A,B} = \text{Tr}_{B,A}\rho$$

- Conditional probability  $\rightarrow$  Conditional state  $\rho_{A|B}$  = One-qubit state after Bob's spin measurement along  $\hat{\mathbf{n}}$ :

$$H(A|B) = p_{\hat{\mathbf{n}}} H(\rho_{\hat{\mathbf{n}}}) + p_{-\hat{\mathbf{n}}} H(\rho_{-\hat{\mathbf{n}}})$$

$$\rho_{\hat{\mathbf{n}}} = \frac{\Pi_{\hat{\mathbf{n}}}^B \rho \Pi_{\hat{\mathbf{n}}}^B}{p_{\hat{\mathbf{n}}}} = \frac{1 + \mathbf{B}_{\hat{\mathbf{n}}}^+ \cdot \sigma}{2}, \quad \mathbf{B}_{\hat{\mathbf{n}}}^+ = \frac{\mathbf{B}^+ + \mathbf{C} \cdot \hat{\mathbf{n}}}{1 + \hat{\mathbf{n}} \cdot \mathbf{B}^-}, \quad p_{\hat{\mathbf{n}}} = \frac{1 + \hat{\mathbf{n}} \cdot \mathbf{B}^-}{2}$$

- Genuine quantumness  $\rightarrow$  Minimization over all spin directions to exclude quantization effects:

$$\mathcal{D}(A, B) = H(\rho_B) - H(\rho) + \min_{\hat{\mathbf{n}}} p_{\hat{\mathbf{n}}} H(\rho_{\hat{\mathbf{n}}}) + p_{-\hat{\mathbf{n}}} H(\rho_{-\hat{\mathbf{n}}}) \neq 0$$

# Entanglement

- What if we generalize the previous idea? → Separability:

$$\rho_{\text{sep}} = \sum_{n,m} p_{n,m} |n\rangle \otimes |m\rangle \langle n| \otimes \langle m|$$

- $|n\rangle, |m\rangle$  not necessarily *orthonormal* now!
- Any classically correlated state (classical probability) is separable.
- Entanglement: Non-separability of a bipartite quantum state.



Separable



Non-Separable

# Entanglement: Two qubits

- Two qubits: Separability=Positive  $P$ -representation  $P(\mathbf{n}_A, \mathbf{n}_B) \geq 0$ :

$$\rho = \int d\Omega_A d\Omega_B P(\mathbf{n}_A, \mathbf{n}_B) |\mathbf{n}_A \mathbf{n}_B\rangle \langle \mathbf{n}_A \mathbf{n}_B|, \quad \int d\Omega_A d\Omega_B P(\mathbf{n}_A, \mathbf{n}_B) = 1$$

- Classical spins pointing at directions  $\mathbf{n}_A, \mathbf{n}_B$ !
- Separability=Purely classical correlations

$$C_{ij} = \langle \sigma^i \otimes \sigma^j \rangle = \int d\Omega_A d\Omega_B P(\mathbf{n}_A, \mathbf{n}_B) n_A^i n_B^j$$

- Entanglement=NO positive  $P$ -representation  $\rightarrow$  Genuine non-classical!



# Steering: Two qubits

- Steering: Original conception of Schrödinger of EPR paradox=Quantum Mechanics+Locality → Only well-defined in 2007! ([PRL 98, 140402 \(2007\)](#))
- Alice post-measurement state described by local-hidden states:

$$\tilde{\rho}_{\hat{n}} = \Pi_{\hat{n}}^B \rho \Pi_{\hat{n}}^B = \int d\lambda p(1|\hat{n}\lambda) p(\lambda) \rho_B(\lambda)$$

- If not → Bob can “steer” quantum state of Alice → **Steering**.

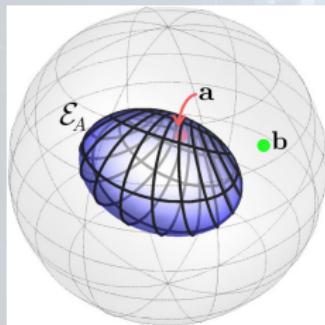


# Steering Ellipsoid

- Alice post-measurement state: same as for quantum discord.

$$\rho_{\hat{n}} = \frac{\tilde{\rho}_{\hat{n}}}{\text{Tr} \tilde{\rho}_{\hat{n}}} = \frac{1 + \mathbf{B}_{\hat{n}}^+ \cdot \sigma}{2}, \quad \mathbf{B}_{\hat{n}}^+ = \frac{\mathbf{B}^+ + \mathbf{C} \cdot \hat{n}}{1 + \hat{n} \cdot \mathbf{B}^-}$$

- Set of conditional polarizations  $\mathbf{B}_{\hat{n}}^+$  describes an ellipsoid.
- Steering ellipsoid: Fundamental QI object, containing all information about the system.
- Similar for Bob  $\rightarrow$  Steering: also asymmetric between Alice and Bob.



PRL 113, 020402 (2014)

# Bell Nonlocality: Two qubits

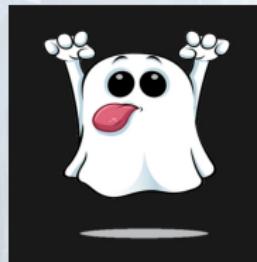
- Local realism: Joint Alice and Bob measurements  $M_A, M_B$  accounted by local hidden-variable model

$$p(a, b|M_A M_B) = \int d\lambda p(a|M_A \lambda)p(b|M_B \lambda)p(\lambda)$$

- Local realism holds if Bell inequality is satisfied. Two qubits → **CHSH inequality** ( $\mathbf{a}_i, \mathbf{b}_i$  spin axes of measurements  $M_A, M_B$ )

$$|\mathbf{a}_1^T \mathbf{C} (\mathbf{b}_1 - \mathbf{b}_2) + \mathbf{a}_2^T \mathbf{C} (\mathbf{b}_1 + \mathbf{b}_2)| \leq 2$$

- Stronger condition than entanglement → "Spooky action at distance"
- See more on Marco's talk later!



# Hierarchy of Quantum Correlations

- Steering and Discord can be asymmetric between Alice and Bob.
- Bell Nonlocality and Entanglement are always symmetric.
- Quantum Hierarchy:

*Bell Nonlocality ⊂ Steering ⊂ Entanglement ⊂ Discord*



# Quantum Tomography: Two qubits

- **Quantum Tomography:** Reconstruction of quantum state from measurement of a set of observables.
- Quantum tomography → Evaluation of ALL quantum observables.
- Most general density matrices for 1, 2 qubits:

$$\rho = \frac{1 + \sum_i B_i \sigma^i}{2}, \quad \rho = \frac{1 + \sum_i (B_i^+ \sigma^i + B_i^- \bar{\sigma}^i) + \sum_{i,j} C_{ij} \sigma^i \bar{\sigma}^j}{4}$$

- One-qubit quantum tomography=Measurement of 3 parameters, polarization vector  $\mathbf{B}$ :  $B_i = \langle \sigma^i \rangle$
- Two-qubit quantum tomography=Measurement of 15 parameters, polarization vectors  $\mathbf{B}^\pm$  and correlation matrix  $\mathbf{C}$ .



# Quantum Experiments

- Quantum correlations observed in a wide variety of systems

## Experimental long-lived entanglement of two macroscopic objects

Brian Jakschard, Alexander Koschella & Eugene S. Polzik

Institute of Physics and Astronomy, University of Aarhus, 8000 Aarhus, Denmark

Entanglement is considered to be one of the most profound features of quantum mechanics<sup>1,2</sup>. An entangled state of a system consisting of two subsystems cannot be described as a product of the quantum states of the two subsystems<sup>3,4</sup>. In this article we show that systems consisting of large objects can be entangled and local. It is generally believed that entanglement is usually confined in systems consisting of a small number of microscopic particles. Here we demonstrate experimentally the entanglement of two macroscopic objects, each consisting of a caesium gas sample containing about  $10^{13}$  atoms. Entanglement is generated via

Measurement of the Entanglement of Two Superconducting Qubits via State Tomography

Matthew Doherty,<sup>1</sup> R. Amerigo, Endrődi C., Márkusz, N. Kita,<sup>1</sup> Iñaki Larralde,<sup>1</sup> R. McMullan,<sup>1</sup>

Matthew Norley,<sup>1</sup> E. M. Wong,<sup>1</sup> A. H. Orlandi,<sup>1</sup> John R. Moriarty,<sup>2</sup>

Demonstration of quantum entanglement is key research in quantum computation along with a nonlocal correlation of two states, requires complete measurement of states in a single basis, or two different bases. Single qubit operations and bipartite coupling between two separate systems are not enough to generate entanglement. We demonstrate that entanglement can be generated by density matrix estimation at entanglement fidelity up to 87%. Our results demonstrate a high degree of entanglement of the systems, indicating that larger implementation is within reach.

Entangling Macroscopic Diamonds at Room Temperature

S. C. Ian,<sup>1</sup> M. F. Tigray,<sup>1</sup> R. J. Jennings,<sup>1</sup> R. W. Landwehr,<sup>1</sup> S.-M. Jin,<sup>1,2</sup>

J. Chapman,<sup>1</sup> R. Asheberg,<sup>1</sup> C. F. Roos,<sup>1</sup> S. Segal,<sup>1</sup> D. J. Wineland,<sup>1</sup> D. L. Matsukevich,<sup>1</sup>

Quantum entanglement of the motion of macroscopic solid bodies has implications both for particle technologies and fundamental studies of the boundary between the quantum and classical regimes. We report the entanglement of two macroscopic diamonds, which are very structured but internally soft and with the ratio environment. By generating mutual entanglement between diamond states in two spatially separated, reference-free frames of motion, entanglement of the motion of the diamonds is demonstrated. Our results indicate that, as expected, we showed that the quantum state of the classical superimposition state with 98% probability, with a limited confidence.

nature

LETTERS

Vol 440(20 April 2006) doi:10.1038/nature04627

LETTERS

nature  
physics

PUBLISHED ONLINE 14 OCTOBER 2002 | DOI:10.1038/NPHYS1044

## Experimental determination of entanglement with a single measurement

S. P. Walborn<sup>1</sup>, P. H. Souto Ribeiro<sup>1</sup>, L. Davidovich<sup>1</sup>, F. Mintert<sup>1,2</sup> & A. Buchleitner<sup>1</sup>

LETTER

https://doi.org/10.1038/wl1586-018-01958x

## Stabilized entanglement of massive mechanical oscillators

C. E. Oetkeisen-Korppi<sup>1</sup>, E. Damgaard<sup>1</sup>, J.-M. Pirkkalainen<sup>1</sup>, M. Ajor<sup>2</sup>, A. A. Clerk<sup>2</sup>, F. Massel<sup>2</sup>, M. J. Woolley<sup>2</sup> & M. A. Sillanpää<sup>2</sup>

nature  
physics

PUBLISHED ONLINE 15 AUGUST 2016 | DOI:10.1038/NPHYS3603

ARTICLES

## Observation of quantum Hawking radiation and its entanglement in an analogue black hole

Jeff Steinhauer

## Demonstration of entanglement-by-measurement of solid-state qubits

Wolfgang Pfaff<sup>1</sup>, Tim H. Taminiau<sup>1</sup>, Lucio Robledo<sup>1</sup>, Hannes Bernien<sup>1</sup>, Matthew Markham<sup>2</sup>, Daniel J. Twitchin<sup>2</sup> & Ronald Hanson<sup>2\*</sup>

PRL 99, 131802 (2007)

PHYSICAL REVIEW LETTERS

week ending

28 SEPTEMBER 2007

## Measurement of Einstein-Podolsky-Rosen-Type Flavor Entanglement in $Y(4S) \rightarrow B^0 \bar{B}^0$ Decays

VOLUME 79

7 JULY 1997

NUMBER 1

## Generation of Einstein-Podolsky-Rosen Pairs of Atoms

E. Hagley,<sup>1</sup> X. Maître,<sup>1</sup> G. Nogues,<sup>1</sup> C. Westbrook,<sup>1</sup> M. Brune,<sup>1</sup> J.-M. Raimond,<sup>1</sup> and S. Haroche,<sup>1</sup>  
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24 rue Lhomond, F-75231 Paris Cedex 05, France  
(Received 6 March 1997)

Pairs of atoms have been prepared in an entangled state of the Einstein-Podolsky-Rosen type.

PHYSICAL REVIEW LETTERS 122, 113602 (2019)

Editor's Suggestion | Featured in Physics

## On-Demand Semiconductor Source of Entangled Photons Which Simultaneously Has High Fidelity, Efficiency, and Indistinguishability

Hui Wang,<sup>1,2</sup> Hai Hu,<sup>1,2</sup> Y.-H. Chung,<sup>1,2</sup> Jian Qin,<sup>1,2</sup> Xiaoxia Yang,<sup>1,2</sup> J.-P. Li,<sup>1,2</sup> R.-Z. Liu,<sup>1,2</sup> H.-S. Zhong,<sup>1,2</sup>  
Y.-M. He,<sup>1,2</sup> Xing Ding,<sup>1,2</sup> Y.-H. Deng,<sup>1,2</sup> Qing Du,<sup>1,2</sup> Y.-H. Huo,<sup>1,2</sup> Sven Höfflin,<sup>1,2</sup>  
Chao-Yang Lu,<sup>1,2</sup> and Jian-Wei Pan<sup>1,2</sup>

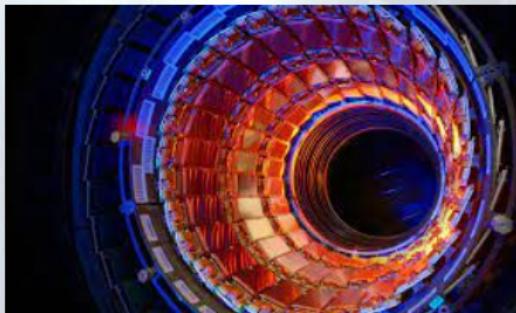
- Unfortunately, not many measurements in High-Energy Physics:

- Bell violation in B-mesons: [PRL 99, 131802 \(2007\)](#),
- Leggett-Garg violation in neutrinos: [PRL 117, 050402 \(2016\)](#)

- Entanglement has never been measured between a pair of quarks :(

# Quantum High-Energy Colliders?

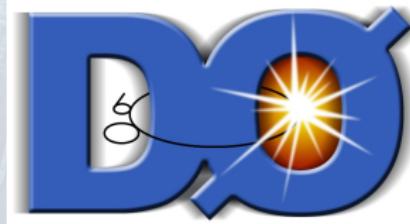
- Standard Model is a Relativistic Quantum Field Theory = Special Relativity + Quantum Mechanics.
- Naively, Quantum Correlations should be easily studied via Standard Model...right? → Not so fast!
  - Momentum measurement → Decoherence
  - Lack of control of internal d.o.f. in initial state → Decoherence
  - Most relevant observables in colliders: cross-sections, lifetimes... → Classical probabilistic objects
- Even measuring quantum interference in colliders is challenging: [PRD 105, 096012 \(2022\)](#)



# Part II: Top Quark Physics

# Who Top Quarks?

- Top quark is the most massive fundamental particle known to exist ( $m_t c^2 \approx 173$  GeV).
- First discovered by the D0 and CDF collaborations at the Tevatron in 1995.
- Top quarks produced in top-antitop ( $t\bar{t}$ ) pairs through QCD or Electroweak processes.



# Why Top Quarks?

- Large Width  $\Gamma_t \sim 1$  GeV  $\rightarrow$  Very short lifetime  $\tau = 1/\Gamma_t \sim 10^{-25}$ s
- Tops decay before
  - Hadronisation  $\sim 10^{-23}$ s.
  - Spin-decorrelation  $\sim 10^{-21}$ s.
- $\rightarrow$  NO DECOHERENCE OR RANDOMIZATION!
- Rotational invariance in  $t\bar{t}$  rest frames  $\rightarrow t\bar{t}$  spins measured from decay products.
- Measurements by D0 and CDF (Tevatron), ATLAS and CMS (LHC)  
 $\rightarrow$  Well-established technique!



# Top pair kinematics

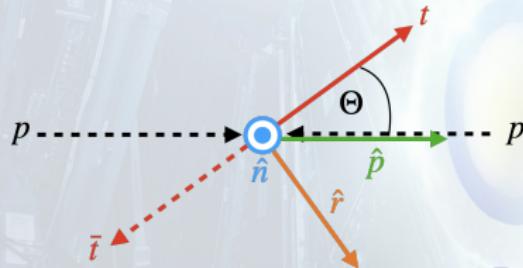
- $t\bar{t}$  pair kinematically described by invariant mass  $M_{t\bar{t}}$  and top direction  $\hat{k}$  in c.m. frame

$$\begin{aligned} k_t^\mu &= (k_t^0, \mathbf{k}), k_{\bar{t}}^\mu = (k_{\bar{t}}^0, -\mathbf{k}) \\ M_{t\bar{t}}^2 &\equiv s_{t\bar{t}} \equiv (k_t + k_{\bar{t}})^2 \end{aligned}$$

- Invariant mass is simply related to top c. m. velocity  $\beta$

$$M_{t\bar{t}} = \frac{2m_t}{\sqrt{1 - \beta^2}} \rightarrow \beta = 0 \rightarrow M_{t\bar{t}} = 2m_t$$

- Threshold production:  $M_{t\bar{t}} = 2m_t \approx 346$  GeV



# Top pair Quantum State

- How to translate HEP features to Quantum Information language?
- $t\bar{t}$  spins described by production spin density matrix  $R(M_{t\bar{t}}, \hat{k})$ :

$$R = \tilde{A} + \sum_i \left( \tilde{B}_i^+ \sigma^i + \tilde{B}_i^- \bar{\sigma}^i \right) + \sum_{i,j} \tilde{C}_{ij} \sigma^i \bar{\sigma}^j$$

- Quantum state in experiment: Momentum measurements + Average over events  $\rightarrow$  Genuine density-matrix description!

- Proper spin density matrix  $\rho(M_{t\bar{t}}, \hat{k}) = \frac{R(M_{t\bar{t}}, \hat{k})}{\text{tr} [R(M_{t\bar{t}}, \hat{k})]}$

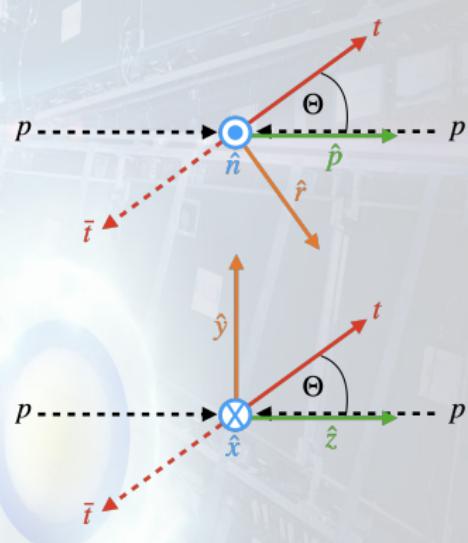


Don Alfonso re d'Castela  
re Toledo re Leon  
Rey Benito de Castilla

Esta es la primera canticula de los reyes  
Santa Anna, emperatriz de los reyes  
que nune de seu filio.

# Basis Selection

- Different basis for computing spin polarization and spin correlations characterizing the quantum state.
- Helicity basis:  $\{\hat{k}, \hat{r}, \hat{n}\}$ :
  - $\hat{k}$  - top direction in  $t\bar{t}$  c.m. frame.
  - $\hat{p}$  - beam direction ( $\cos\Theta = \hat{k} \cdot \hat{p}$ ).
  - $\hat{r} = (\hat{p} - \cos\Theta\hat{k})/\sin\Theta$ .
  - $\hat{n} = \hat{r} \times \hat{k}$ .
  - Study of individual  $t\bar{t}$  production with **fixed energy and direction**.
- Beam basis:  $\{\hat{x}, \hat{y}, \hat{z}\}$ :
  - $\hat{z}$  along beam axis.
  - $\hat{x}, \hat{y}$  transverse directions to beam.
  - Fixed in space: no change with  $\hat{k}$ .
  - Study of **total integrated quantum state**.

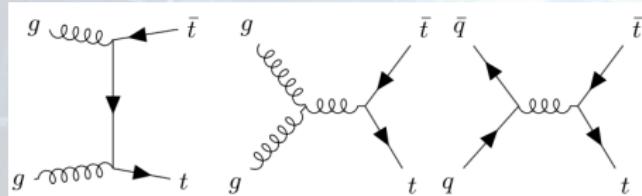


# LO QCD Elementary Process

- Illustrative example: QCD analytical LO calculation.
  - Analytical results.
  - NLO corrections are small.
  - Building blocks of actual high-energy processes.
- Most elementary QCD processes:

$$q + \bar{q} \rightarrow t + \bar{t}, \quad q = u, d \dots$$
$$g + g \rightarrow t + \bar{t}$$

- Each initial state  $I = q\bar{q}, gg$  gives rise to quantum state  $\rho^I(M_{t\bar{t}}, \hat{k})$



# LO QCD Realistic

- No free quarks or gluons  $\rightarrow$  Hadrons: Bound states of quarks and gluons (partons).
- LHC, Tevatron:  $p p$ ,  $p \bar{p}$  collisions at high c.m. energies  $\sqrt{s}$ .

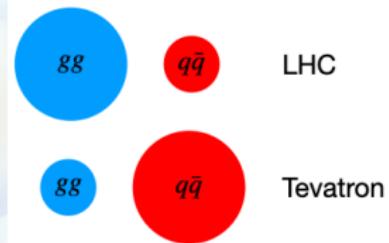
$$p + p \rightarrow \dots \rightarrow t + \bar{t} \quad \text{LHC}$$

$$p + \bar{p} \rightarrow \dots \rightarrow t + \bar{t} \quad \text{Tevatron}$$

- Quantum state depends now on c.m. energy  $\sqrt{s}$ :

$$\rho(M_{t\bar{t}}, \hat{k}) = \sum_{I=q\bar{q}, gg} w_I(M_{t\bar{t}}, \sqrt{s}) \rho^I(M_{t\bar{t}}, \hat{k})$$

- Total QCD process: *Incoherent* sum of elementary QCD processes with probability  $w_I$   $\rightarrow$  PDF-dependence encoded in  $w_I$ !
- QCD Input:  $w_I(M_{t\bar{t}}, \sqrt{s})$ ,  $\rho^I(M_{t\bar{t}}, \hat{k}) \rightarrow$  QI  
Output: Textbook problem of *convex sum* of quantum states!



# LO QCD Quantum State

- Most general 2-qubit density matrix (15 parameters):

$$\rho(M_{t\bar{t}}, \hat{k}) = \frac{1 + \sum_i (B_i^+ \sigma^i + B_i^- \bar{\sigma}^i) + \sum_{i,j} C_{ij} \sigma^i \bar{\sigma}^j}{4}$$

- $CP$ -invariance of Standard Model  $\rightarrow \mathbf{B}^+ = \mathbf{B}^-, \mathbf{C}^T = \mathbf{C}$
- LO QCD  $\rightarrow$ 
  - ①  $\rho(M_{t\bar{t}}, \hat{k})$  is a T-state (unpolarized)  $\rightarrow \mathbf{B}^\pm = 0$
  - ② Spin along  $\hat{n}$  is uncorrelated to other directions
- Only 4 parameters in SM LO QCD:  $C_{kk}, C_{rr}, C_{nn}, C_{kr}$

$$\mathbf{B}^\pm = 0, \quad \mathbf{C} = \begin{bmatrix} C_{kk} & C_{kr} & 0 \\ C_{kr} & C_{rr} & 0 \\ 0 & 0 & C_{nn} \end{bmatrix}$$

# Part III: Quantum Tops

# Two-qubit Quantum Criteria

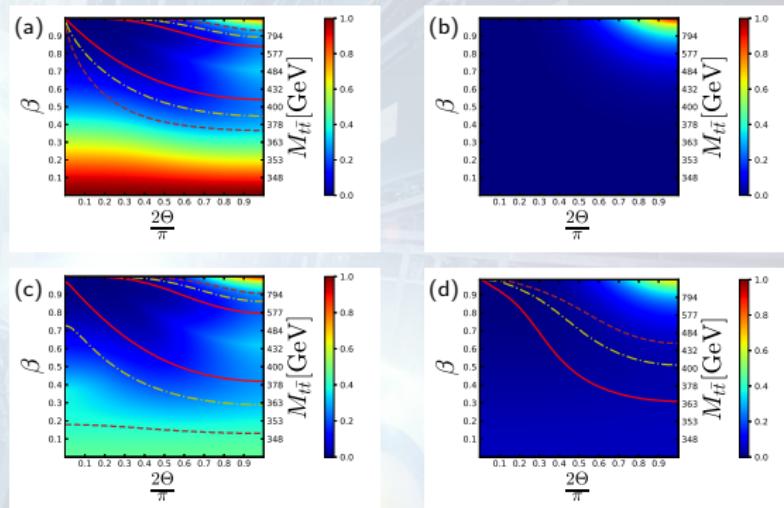
- In general, evaluation of all quantum correlations is a complicated problem (discord and steering).
- However, due to the simple form of  $\rho(M_{t\bar{t}}, \hat{k})$  in SM LO QCD:
  - ① Quantum Discord: Analytical ( $T$ -states).
  - ② Entanglement: Concurrence  $0 \leq \mathcal{C}[\rho] \leq 1$ ,  $\mathcal{C}[\rho] > 0$  iff  $\rho$  entangled:

$$\mathcal{C}[\rho] = \max(\Delta, 0), \quad \Delta \equiv \frac{-C_{nn} + |C_{kk} + C_{rr}| - 1}{2}$$

- ③ Steerability iff  $\int d\hat{\mathbf{n}} \sqrt{\hat{\mathbf{n}}^T \mathbf{C}^T \mathbf{C} \hat{\mathbf{n}}} > 2\pi$  ( $T$ -state).
- ④ CHSH violation iff  $\mu_1 + \mu_2 > 1$  ( $\mu_{1,2}$  largest eigenvalues of  $\mathbf{C}^T \mathbf{C}$ ).

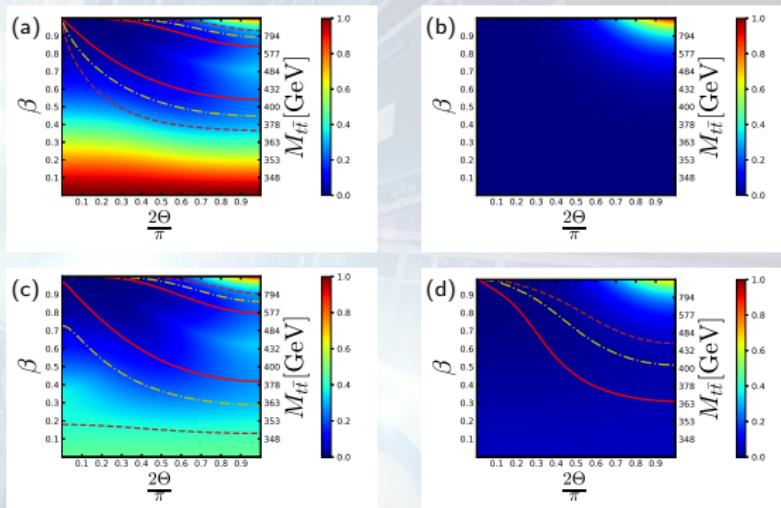
# $t\bar{t}$ Quantum Correlations

- a)  $gg \rightarrow t\bar{t}$  Discord.
- b)  $q\bar{q} \rightarrow t\bar{t}$  Discord.
- c) LHC  $\rho(M_{t\bar{t}}, \hat{k})$  Discord.
- d) Tevatron  $\rho(M_{t\bar{t}}, \hat{k})$  Discord.
  - Solid red, dashed-dotted yellow, dashed brown: Critical boundaries of entanglement, steerability, and Bell nonlocality  $\rightarrow$  Quantum Hierarchy respected!



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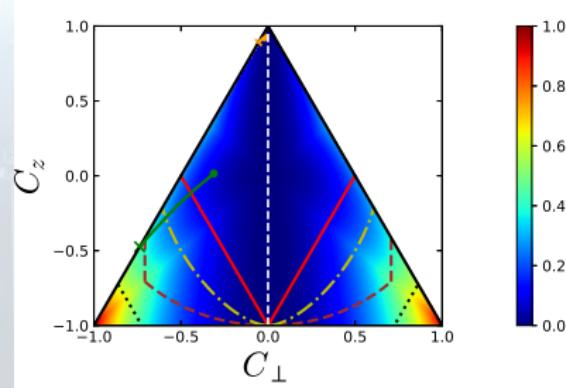
Full picture of quantum correlations in  $t\bar{t}$ .

# Total Quantum State

- Realistic measurement: Average over many different processes.
- Total quantum state: Events in window  $[2m_t, M_{t\bar{t}}]$

$$\rho(M_{t\bar{t}}) \equiv \frac{1}{\sigma(M_{t\bar{t}})} \int_{2m_t}^{M_{t\bar{t}}} dM \int d\Omega \frac{d\sigma}{dM d\Omega} \rho(M, \hat{k})$$

- Intuitively: Total quantum state = Sum of  $t\bar{t}$  quantum states weighted with the differential cross-section.
- Rotational invariance around beam axis  $\rightarrow$  Correlation matrix diagonal in beam basis  
 $C_{ij} = C_i \delta_{ij}$ ,  $C_x = C_y = C_\perp \rightarrow$  2D dependence on  $C_\perp, C_z$ .
- Green: LHC.  
Orange: Tevatron.
- Cross:  $\beta = 0$ ; Circle:  $\beta = 1$ .



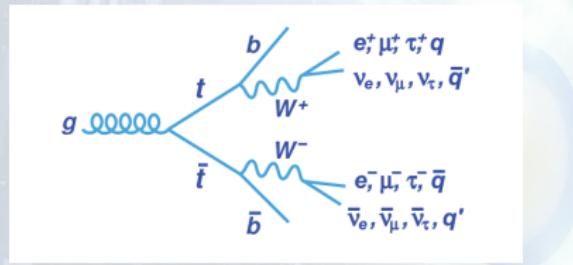
# Part IV: Phenomenological Analysis

# Top pair Quantum Tomography

- $\rho(M_{t\bar{t}}) \rightarrow \underline{\text{Two qubit quantum state}} \rightarrow \text{Quantum tomography} = \text{Measurement of spin polarizations and spin correlations.}$
- Spin polarizations  $\mathbf{B}^\pm$  and spin correlation matrix  $\mathbf{C}$  extracted from cross-section  $\sigma_{\ell\bar{\ell}}$  of dileptonic decay

$$\frac{1}{\sigma_{\ell\bar{\ell}}} \frac{d\sigma_{\ell\bar{\ell}}}{d\Omega_+ d\Omega_-} = \frac{1}{(4\pi)^2} \left[ 1 + \mathbf{B}^+ \cdot \hat{\ell}_+ - \mathbf{B}^- \cdot \hat{\ell}_- - \hat{\ell}_+ \cdot \mathbf{C} \cdot \hat{\ell}_- \right]$$

- $\hat{\ell}_\pm$ : lepton directions in each top (antitop) rest frames.



# Entanglement in $t\bar{t}$ production at LHC $\sqrt{s} = 13$ TeV

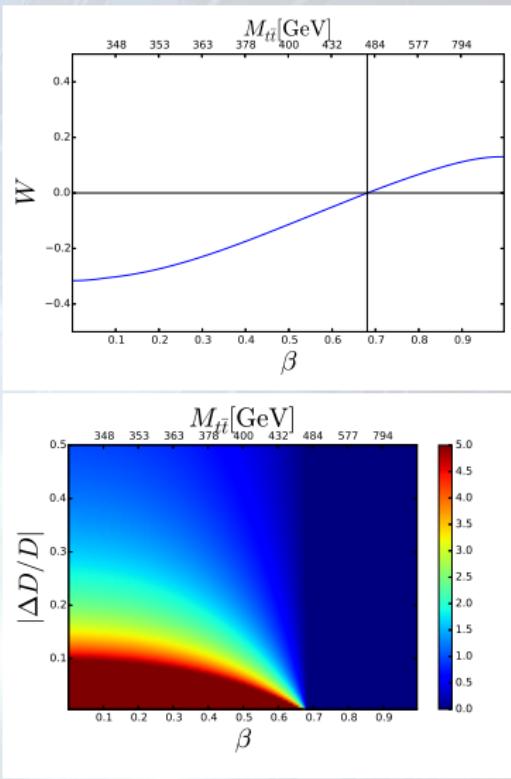
- Entanglement witness

$$W = D + 1/3 < 0, D \equiv \text{tr } \mathbf{C}/3$$

- $D$  directly measurable from decay cross-sections:

$$\frac{1}{\sigma} \frac{d\sigma}{d \cos \varphi} = \frac{1}{2} (1 - D \cos \varphi)$$

- Entanglement detection from one single magnitude!  $\rightarrow$  No need for Quantum Tomography!
- Entanglement only close to threshold  $\rightarrow$  See James talk tomorrow!
- Entanglement also available at high- $p_T$ :  
[PRL 127, 161801 \(2021\)](#), [EPJC 82, 285 \(2022\)](#)



# Discord and Steering

- Normalized dileptonic cross-section → Angular probability distribution:

$$p(\hat{\ell}_+, \hat{\ell}_-) = \frac{1}{\sigma_{\ell\bar{\ell}}} \frac{d\sigma_{\ell\bar{\ell}}}{d\Omega_+ d\Omega_-} = \frac{1 + \mathbf{B}^+ \cdot \hat{\ell}_+ - \mathbf{B}^- \cdot \hat{\ell}_- - \hat{\ell}_+ \cdot \mathbf{C} \cdot \hat{\ell}_-}{(4\pi)^2}$$

- Direct one-qubit tomography of  $\rho_{A,B}, \rho_{\hat{\mathbf{n}}}$  from Bloch vectors  $\mathbf{B}^\pm, \mathbf{B}_{\hat{\mathbf{n}}}^\pm$ :

$$p(\hat{\ell}_\pm) = \int d\Omega_\mp p(\hat{\ell}_+, \hat{\ell}_-) = \frac{1 \pm \mathbf{B}^\pm \cdot \hat{\ell}_\pm}{4\pi}$$

$$p(\hat{\ell}_\pm | \hat{\ell}_\mp = \mp \hat{\mathbf{n}}) = \frac{p(\hat{\ell}_\pm, \hat{\ell}_\mp = \mp \hat{\mathbf{n}})}{p(\hat{\ell}_\mp = \mp \hat{\mathbf{n}})} = \frac{1 \pm \mathbf{B}_{\hat{\mathbf{n}}}^\pm \cdot \hat{\ell}_\pm}{4\pi}$$

- Actual discord → Evaluated from minimization over  $\hat{\mathbf{n}}$ .
- Measurement of  $\mathbf{B}_{\hat{\mathbf{n}}}^\pm$  → Reconstruction of  $t, \bar{t}$  steering ellipsoids.
- Highly-challenging measurements in conventional setups → Natural implementation in colliders!

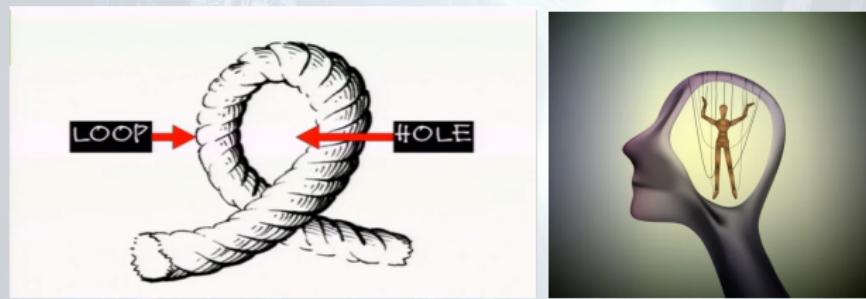
# New Physics Witnesses

- Approximate  $CP$ -invariance of Standard Model  $\rightarrow \mathbf{C} = \mathbf{C}^T, \mathbf{B}^+ = \mathbf{B}^-$   
 $\rightarrow$  Symmetric discord and steering!
- Therefore: Discord and/or Steering asymmetry  $\rightarrow$  New Physics!
- New physics witnesses: Symmetry protected observables by SM, only non-zero for New Physics:
  - $\Delta\mathcal{D}_{t\bar{t}} \equiv \mathcal{D}_t - \mathcal{D}_{\bar{t}}$
  - Asymmetries in ellipsoid centers and/or semiaxes.
- No SM contribution to New Physics witnesses!



# Bell Test Loopholes in a Collider Experiment

- Loopholes: Experimental tests of Bell's inequality may not fulfill all hypotheses.
- Collider experiment:
  - Free-will loophole: Spin measurement directions should be independent from hidden-variables. → Not even single-detection events from Alice and Bob!
  - Detection loophole: Only a subset of events selected for measurement → Bias!
- Quite natural: Colliders were not designed to test Bell's Inequality!
- See Marco talk later!



# Conclusions and outlook

- Quantum Information theory  $\longleftrightarrow$  High-Energy Physics.  
Interdisciplinary, huge potential and great interest!
- QI perspective:
  - ① Highest-energy observation of entanglement ever!  
See James talk tomorrow
  - ② Genuinely relativistic, exotic symmetries and interactions, fundamental nature  $\rightarrow$  Frontier of known Physics!
  - ③ Highly-demanding measurements naturally implemented at LHC.
- HEP perspective:
  - ① Quantum Tomography: Novel experimental tool.
  - ② QI techniques can inspire new approaches for searching New Physics:  
See Luca and Claudio talks tomorrow
- Extension to  $e^+e^-$  colliders: Spin of initial state can be controlled!  
 $\rightarrow$  Manipulation of qubits? Quantum gates?
- Adaptation to  $\tau$  leptons, qutrits  $W^\pm, Z^0 \rightarrow$  See other talks

Thank You

