An X-ray survey of wave function collapse signal

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on behalf of the VIP-2 collaboration

High Precision X-Ray Measurements 2023

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LNF, INFN

The LNGS laboratories environment

The experiments are performed in the low-background environment of the underground Gran Sasso National Laboratory of INFN:

- overburden corresponding to a minimum thickness of 3100 m w.e.
- the muon flux is reduced by almost six orders of magnitude, n flux of three oom.
- the main background source consists of γ-radiation produced by long-lived γ-emitting primordial isotopes and their decay products.

QT & dynamical reduction

- Why the quantum properties (superposition) do not carry over to the macro-world?
- The mechanism at the basis of the transition from Quantum to Classical behavior is not embedded in QT
- Superposition principle is a consequence of the linearity of the Schroedinger equation, which has to break down at a certain scale.
- Phenomenological dynamical models of w. f. collapse (Dios, Ghirardi, Rimini, Weber, Pearle, Adler, Penrose, Karojhazi, Lukacs, Milburn, Bassi ...): progressive reduction of the superposition, proportional to the increase of the mass of the system under consideration.

Dynamical reduction, the idea

modify the Schroedinger dynamics in one capable to describe the collapse, preserving QM at microscopic scale:

1) Non linear ;

2) Stochastic ;

3) Von Neumann reduction:

$$|\psi\rangle = \frac{|+\rangle + |-\rangle}{\sqrt{2}} \quad \longrightarrow \quad \rho = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

We want this state to evolve in:

$$\{50\% \ |+\rangle \ , \ 50\% \ |-\rangle\} \quad \longrightarrow \quad \rho = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Axioms of QM:

1) every physical system is associated to a Hilbet space, observables are self-adjoint operators, possible measurement outcomes are:

$$O\,|o_n\rangle \quad = \quad o_n\,|o_n\rangle$$

2) time evolution is governed by the Schördinger equation

$$i\hbar\frac{d}{dt}|\psi(t)\rangle \quad = \quad H\,|\psi(t)\rangle$$

first order in t → deterministic

$$|\psi(t)\rangle \quad = \quad U(t,t_0)\,|\psi(t_0)\rangle$$

linear →
superposition principle

3) probability of getting a measurement outcome o_n at time t :

$$P[o_n] \quad = \quad |\langle o_n|\psi(t)\rangle|^2$$

4) wavepacket reduction principle (WPR):

$$|\psi(t)\rangle \ \text{before measurement} \quad \longrightarrow \quad |o_n\rangle \ \text{after measurement}$$

genuinely probabilistic, stochastic

non-linear

May collapse emerge from space-time uncertainty?

- Decoherence means destruction of interference -> diminishes coherent dispersion

  large dispersion of an observable - Quantum ; small dispersion - Classical

- Decoherence induce classicality in quantum systems

- Decoherence of various observables can be correlated or anticorrelated

  e.g. decoherence of local energy induces decoherence of position of massive objects

- But Nature does not tell us which observable is the primary, to induce decoherence on the others and, hence, classicality

If local time would be affected by uncertainty -> decoherence

Diosi, L. (2005), Braz. J. Phys. 35, 260, Diosi, L., and B. Lukacs (1987), Annalen der Physik 44, 488, Diosi, L. (1987), Physics Letters A 120, 377, A. Bassi et al.,Rev. Mod. Phys. 85,471

Initial state of a quantum system is a superposition of two eigenstates of total Hamiltonian

$$|\psi\rangle = c_1|\varphi_1\rangle + c_2|\varphi_2\rangle$$

time evolution

$$|\psi(t)\rangle = c_1 \exp(-i\hbar^{-1}E_1 t)|\varphi_1\rangle + c_2 \exp(i\hbar^{-1}E_2 t)|\varphi_2\rangle$$

Let us add an uncertainty to the time $t \rightarrow t + \delta t$

and assume that is distributed Gaussian, with zero mean, and dispersion which is proportional to the mean time, $\mathbf{M}[(\delta t)^2] = \tau t$ then the density matrix evolves as:

$$\begin{aligned} & \rho(t) \equiv \mathbf{M}\left[|\psi(t)\rangle\langle\psi(t)|\right] = \\ = \; & |c_1|^2|\varphi_1\rangle\langle\varphi_1| + |c_2|^2|\varphi_2\rangle\langle\varphi_2| + \\ + \; & \{c_1^\star c_2 \exp(i\hbar^{-1}\Delta E t)\mathbf{M}\left[\exp(i\hbar^{-1}\Delta E \delta t)\right]|\varphi_2\rangle\langle\varphi_1| + \\ + \; & \quad \text{h.c.} \quad \} \, . \end{aligned}$$

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The off-diagonal terms decay in time !

$$\begin{aligned}
&\rho(t) \equiv \mathbf{M}\left[|\psi(t)\rangle\langle\psi(t)|\right] = \\
= \; & |c_1|^2|\varphi_1\rangle\langle\varphi_1| + |c_2|^2|\varphi_2\rangle\langle\varphi_2| + \\
& \xcancel{+ \; \{c_1^* c_2 \exp(i\hbar^{-1}\Delta E t)\mathbf{M}\left[\exp(i\hbar^{-1}\Delta E\delta t)\right]|\varphi_2\rangle\langle\varphi_1| + \\
+ \quad \text{h.c.} \quad \} \; .}
\end{aligned}$$

$$\boxed{\begin{gathered}
\mathbf{M}\left[\exp(i\hbar^{-1}\Delta E\delta t)\right] = e^{-t/t_D} \\
t_D = \frac{\hbar^2}{\tau}\frac{1}{(\Delta E)^2}
\end{gathered}}$$

Time uncertainty and decoherence

The time evolution for the density matrix

$$\hat{\rho}(t+\tau)=\exp\left[\frac{-i\hat{H}\tau}{\hbar}\right]\hat{\rho}(t)\exp\left[\frac{i\hat{H}\tau}{\hbar}\right]$$

Described by the von Neumann equation

$$\frac{d\rho}{dt}=-i\hbar^{-1}[H,\rho]$$

turns to

$$\frac{d\rho}{dt}=-i\hbar^{-1}[H,\rho]-\frac{1}{2}\tau\hbar^{-2}[H,[H,\rho]]$$

G. J. Milburn Prys. Rev. A 44 5401 (1991)

local time uncertainty means uncertainty of the local gravitational potential

$$\Delta\tau = \int_P d\tau = \int_P \frac{1}{c}\sqrt{g_{00}}dx^0$$

In the Newtonian limit $g_{00} = 1 + \frac{2\phi}{c^2}$

What if the gravitational potential should not be quantized ???

QM requires an absolute indeterminacy of the gravitational field.

Diosi and Lukacs [Ann. Phys. 44, 488 (1987)] same arguments of [N. Bohr and L. Rosenfeld, K. Dan. Vidensk. Selsk., Mat.-Fys. Medd. 12, 1 (1933)]

I.E. the gravitational potential is a c-number stochastic variable, whose mean value is to be identified with the classical Newtonian potential.

Master equation

$$\tau_{\mathbf{rr}'} = \text{const} \times \frac{G\hbar}{|\mathbf{r}-\mathbf{r}'|}c^{-4}$$

the local time correlation is extremely small

substituted in the master equation

$$\frac{d\rho}{dt} = -i\hbar^{-1}[H,\rho] - \frac{1}{2}\hbar^{-2}\sum_{\mathbf{r},\mathbf{r}'}\tau_{\mathbf{rr}'}[H_{\mathbf{r}},[H_{\mathbf{r}'},\rho]]$$

yields

$$\begin{aligned}\frac{d\rho}{dt} = & - i\hbar^{-1}[H,\rho] \\ & - \frac{G}{2}\hbar^{-1}\int\int\frac{d\mathbf{r}d\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|}[f(\mathbf{r}),[f(\mathbf{r}'),\rho]]\end{aligned}$$

Master equation

Denote the configuration coordinates (classical and spin) of the dynamical system by X. The corresponding mass density at the point r is $f(\mathbf{r}|X)$

Given the coordinate eigenstate |x> we have $f(\boldsymbol{r}|X)\delta(X'-X) \equiv \langle X'|\hat{f}(\boldsymbol{r})|X\rangle$

So if one introduces the damping time:

$$[\tau_d(X,X')]^{-1} = \frac{G}{2\hbar}\int\int d^3r\, d^3r' \times \frac{[f(\mathbf{r}|X) - f(\mathbf{r}|X')][f(\mathbf{r}'|X) - f(\mathbf{r}'|X')]}{|\mathbf{r}-\mathbf{r}'|}$$

the master equation becomes

$$\langle X|\dot{\hat{\rho}}(t)|X'\rangle = (-\mathrm{i}/\hbar)\langle X|[\hat{H}_0, \hat{\rho}(t)]|X'\rangle$$

$$-[\tau_d(X,X')]^{-1}\langle X|\hat{\rho}(t)|X'\rangle\ .$$

Energy decoherence

$$\langle X|\dot{\hat{\rho}}(t)|X'\rangle = (-\mathrm{i}/\hbar)\langle X|[\hat{H}_0,\hat{\rho}(t)]|X'\rangle$$

$$\boxed{-[\tau_d(X,X')]^{-1}\langle X|\hat{\rho}(t)|X'\rangle\ .}$$

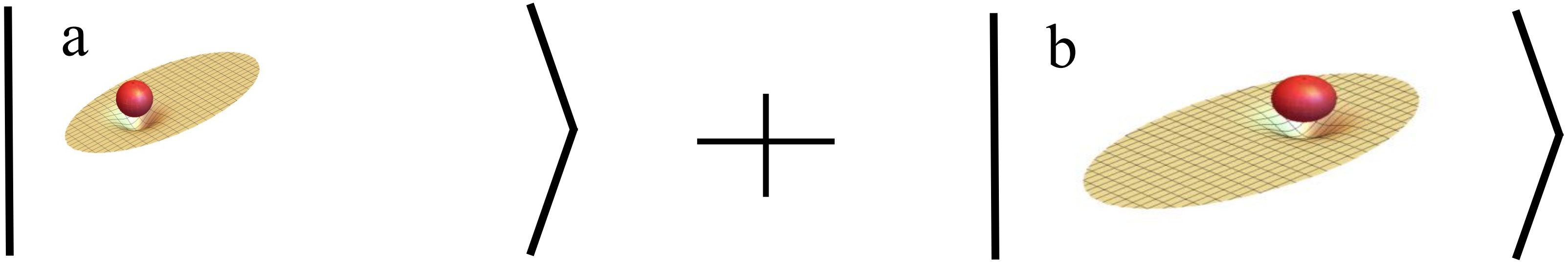
$$[\tau_d(X,X')]^{-1} = \frac{G}{2\hbar}\int\int d^3r\ d^3r' \ \times \frac{[f(\mathbf{r}|X)-f(\mathbf{r}|X')][f(\mathbf{r}'|X)-f(\mathbf{r}'|X')]}{|\mathbf{r}-\mathbf{r}'|}$$

If the difference between the mass distributions of two states |X> and |X'> in superposition becomes big

damping time becomes small

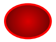
ENERGY DECOHERENCE

Gravity induced collapse



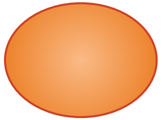
$$\Delta E_{\text{DP}} = 4\pi G \int d\boldsymbol{r} \int d\boldsymbol{r}' \frac{\left[\mu_a(\boldsymbol{r}) - \mu_b(\boldsymbol{r})\right]\left[\mu_a(\boldsymbol{r}') - \mu_b(\boldsymbol{r}')\right]}{|\boldsymbol{r} - \boldsymbol{r}'|}.$$

$$\tau_{\text{DP}} = \frac{\hbar}{\Delta E_{\text{DP}}}$$



Proton: m ≃ 10^{-27} Kg, R ≃ 10^{-15} m

$\tau_{\text{DP}} \simeq 10^{6}$ years



Dust grain: m ≃ 10^{-12} Kg, R ≃ 10^{-5} m

$\tau_{\text{DP}} \simeq 10^{-8}$ s

Gravity induced collapse

The DP theory is parameter-free, but the gravitational self energy difference diverges for point-like particles -> a short-length cutoff R_0 is introduced to regularize the theory.

- **Diósi**: minimum length R_0 limits the spatial resolution of the mass density, a short-length cutoff to regularize the mass density. EG becomes a function of R_0 the larger R_0 the longer the collapse time.
- **Penrose**: solution of the stationary Shroedinger-Newton equation, with R_0 the size of the particle mass density $\mu(\boldsymbol{r}) = m|\psi(\boldsymbol{r},t)|^2$

Direct tests: creating a large superposition of a massive system, to guarantee that decay time is short enough for the collapse to become effective before any kind of external noise disrupts the measurement, matter-wave interferometry with macromolecules, phononic states, experiments in space: no gravity ---> more time (MAQRO, CAL, etc..).

Kovachy, T. et al. Quantum superposition at the half-metre scale. Nature 528, 530–533 (2015). Fein, Y. Y. et al. Quantum superposition of molecules beyond 25 kDa. Nature Physics 15, 1242–1245 (2019). Lee, K. C. et al. Entangling macroscopic diamonds at room temperature. Science 334, 1253–1256 (2011).

Testing collapse models by means of Gamma ray spectroscopy

Unavoidable side effect of the collapse: Brownian-like diffusion of the system in space.

Collapse probability is Poissonian in t -> Lindblad dynamics for the statistical operator -> free particle average square momentum increases in time.

A recent general result, see S. Donadi, L. Ferialdi, A. Bassi, "Collapse dynamics are diffusive"
arXiv:2209.09697v1 [quant-ph]

Then charged particles emit spontaneous radiation. We search for spontaneous radiation emission from a germanium crystal and the surrounding materials in the experimental apparatus.

Strategy: simulate the background from all the known emission processes -> perform a Bayesian comparison of the residual spectrum with the theoretical prediction -> look for an eventual signal of collapse

Theoretical prediction

GAMMA RAYS spontaneous emission E > hundreds of keV

- CSL - s. e. photons rate:

$$\frac{d\Gamma}{dE} = N_{atoms} \times \left(N_A^2 + N_A\right) \times \frac{\lambda\hbar e^2}{4\pi^2\varepsilon_0 m_0^2 r_C^2 c^3 E}$$

- DP - s. e. photons rate:

$$\frac{d\Gamma}{dE} = N_{atoms} \times (N_A^2 + N_A) \times \frac{2\,G\,e^2}{3\,\pi^{3/2}\,\epsilon_0\,c^3\,R_0^3\,E}$$

Bassi - Donadi

In range ΔE = (1 - 4)MeV electrons are relativistic, only the contribution of protons (N) is considered.

λ - collapse strength

r_C - correlation length

see e. g. S. L. Adler, JPA 40, (2007) 2935, Adler, S.L.; Bassi, A.; Donadi, S., JPA 46, (2013) 245304.

R_0 - size of the particle mass density. See e.g. Diósi, L. J. Phys. Conf. Ser. 442, 012001 (2013)., Penrose, R. Found. Phys. 44, 557–575 (2014).

The experimental setup

The experimental apparatus is based on a coaxial p-type high purity germanium detector (HPGe):

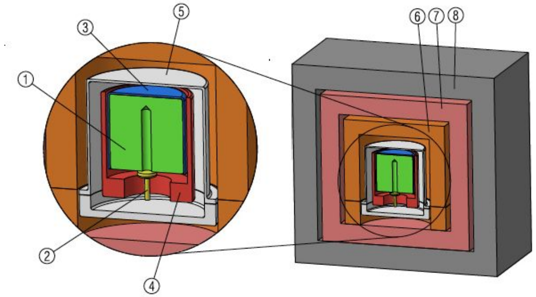
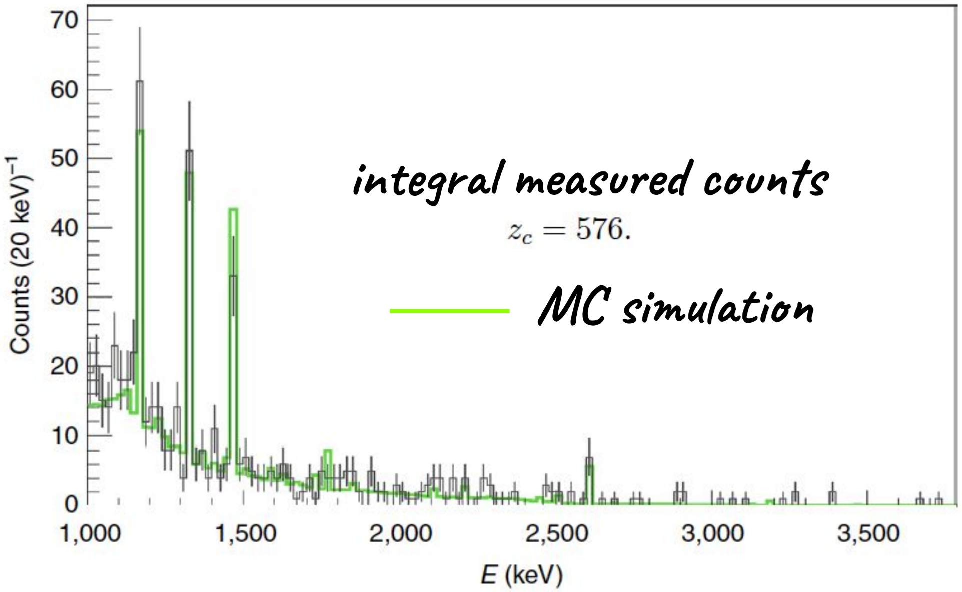


Figure 1: *Schematic representation of the experimental setup: 1 - Ge crystal, 2 - Electric contact, 3 - Plastic insulator, 4 - Copper cup, 5 - Copper end-cup, 6 - Copper block and plate, 7 - Inner Copper shield, 8 - Lead shield.*

- Exposure 124 kg · day, m_{Ge} ~ 2kg
- passive shielding: inner - electrolytic copper, outer - lead
- on the bottom and on the sides 5 cm thick borated polyethylene plates give a partial reduction of the neutron flux
- an airtight steel housing encloses the shield and the cryostat, flushed with boil-off nitrogen to minimize the presence of radon.

Measured spectrum and background simulation

The experimental apparatus is characterised, through a validated MC code, based on the GEANT-4 software library. The background is due to emission of residual radio-nuclides:



integral measured counts

$$z_c = 576.$$

- the activities are measured for each component
- the MC simulation accounts for:

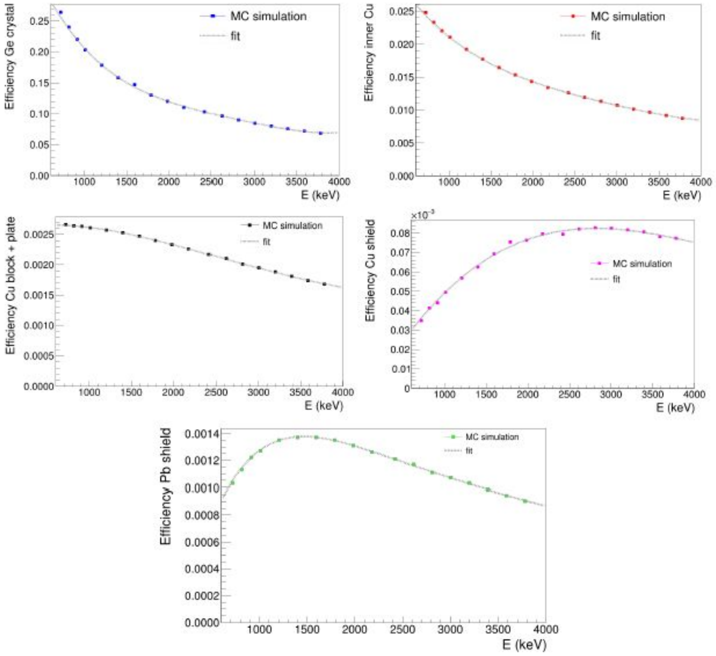
1. emission probabilities and decay schemes for each radio-nuclide in each material
1. photons propagation and interactions
2. detection efficiencies.

The simulation describes 88% of the spectrum:

$$z_{b,ij} = \frac{m_i\, A_{ij}\, T\, N_{rec,ij}}{N_{ij}}, \qquad z_b = \sum_{i,j} z_{b,ij} = 506.$$

Lower bound on R_0
expected signal contribution

The expected number of photons spontaneously emitted by the nuclei of all the materials of the detector are obtained weighting the theoretical rate for the detection efficiencies:



- 10^8 photons generated for each energy for each material
- efficiency functions are obtained by polinomial fits $\epsilon_i(E) = \sum_{j=0}^{c_i} \xi_{ij} E^j$
- the expected signal contribution is:

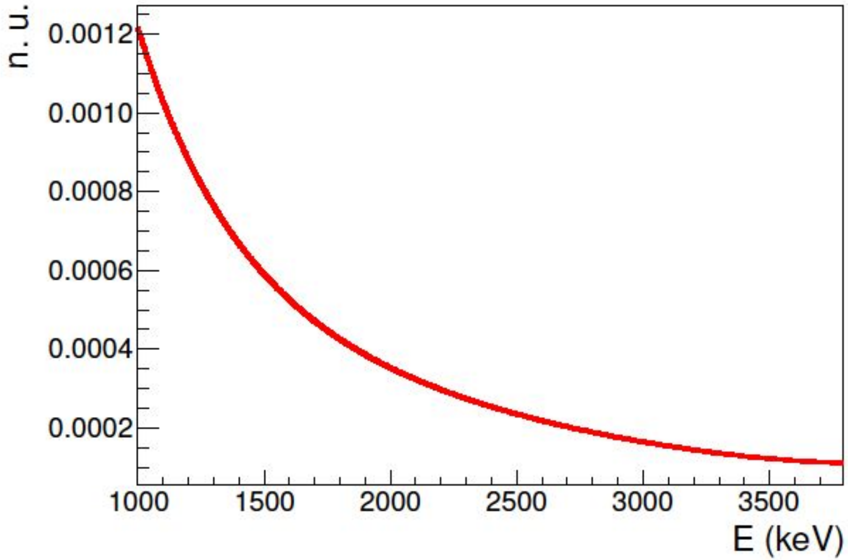
$$z_s(R_0) = \sum_i \int_{\Delta E} \frac{\mathrm{d}\Gamma_t}{\mathrm{d}E}\bigg|_i T\epsilon_i(E)\,\mathrm{d}E = \frac{a}{R_0^3}$$

with $a = 1.8\ 10^{-29}$ m^3

Lower bound on R_0

expected signal contribution

The expected signal of spontaneously emitted photons by the nuclei of all the materials of the detector is obtained weighting the theoretical rate for the detection efficiencies:



Energy distribution of the expected signal, resulting from the sum of the emission rates of all the materials, weighted for the eciency functions.

The area of the distribution is normalised to the unity (n. u.)

Lower bound on R_0
pdf of R_0

z_c is distributed according to a Poissonian $p(z_c|\Lambda_c) = \frac{\Lambda_c^{z_c} e^{-\Lambda_c}}{z_c!}$ with $\Lambda_c(R_0) = \Lambda_b + \Lambda_s(R_0)$

The pdf of R_0 is then given by probability inversion: $\tilde{p}(\Lambda_c(R_0)|p(z_c|\Lambda_c(R_0))) = \frac{p(z_c|\Lambda_c(R_0)) \cdot \tilde{p}_0(\Lambda_c(R_0))}{\int_D p(z_c|\Lambda_c(R_0)) \cdot \tilde{p}_0(\Lambda_c(R_0))\, d[\Lambda_c(R_0)]}$

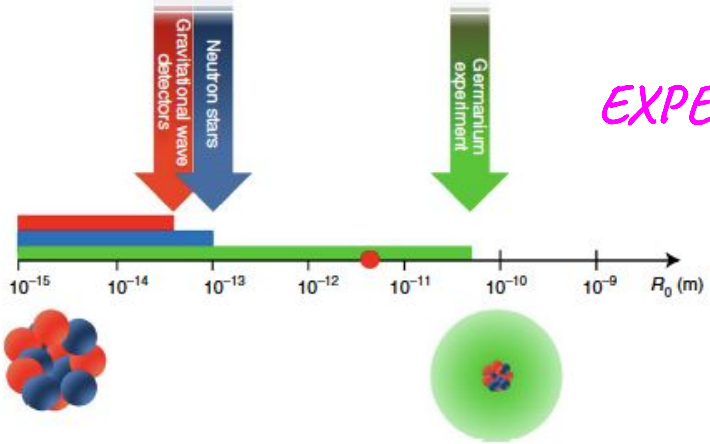
The prior $\tilde{p}_0(\Lambda_c(R_0)) = \theta(\Lambda_c^{max} - \Lambda_c(R_0))$ accounts for previous limits from gravitational wave detectors and neutron stars data analyses [Phys. Rev. D 95, 084054 (2017), Phys. Rev. Lett. 123, 080402 (2019)].

$$\tilde{P}(\bar{\Lambda}_c) = \frac{\gamma(z_c + 1, \bar{\Lambda}_c)}{\gamma(z_c + 1, \Lambda_c^{\max})} = 0.95$$

A bound on R_0 is obtained from the cumulative pdf:

$$R_0 > 0.54 \cdot 10^{-10}\ m$$

Lower bound on R_0



EXPERIMENTAL : $R_0 > 0.54 \cdot 10^{10}$ m

If R_0 is the size of the nucleus's wave function as suggested by Penrose, we have to compare the limit with the properties of nuclei in matter.

In a crystal $R^2_0 = \langle u^2 \rangle$ is the mean square displacement of a nucleus in the lattice, which, for the germanium crystal, cooled to liquid nitrogen temperature amounts to:

THEORETICAL EXPECTATION $R_0 = 0.05 \cdot 10^{10}$ m

"Underground test of gravity-related wave function collapse". Nature Physics 17, pages 74–78 (2021)

The future of Gravity-related collapse

Penrose proposal is rouled out in present formulation!

ways out .. generalized models e.g. :

- add dissipation terms to the master equation and stochastic nonlinear Schroedinger equation of the DP theory, to counteract the runaway energy increase,
- non-Markovian correlation function.

$$< \phi(\mathbf{r},t)\,\phi(\mathbf{r}',t') > - < \phi(\mathbf{r},t) >< \phi(\mathbf{r}',t') > \sim \frac{\hbar G}{|\mathbf{r}-\mathbf{r}'|}\,\delta(t-t')$$

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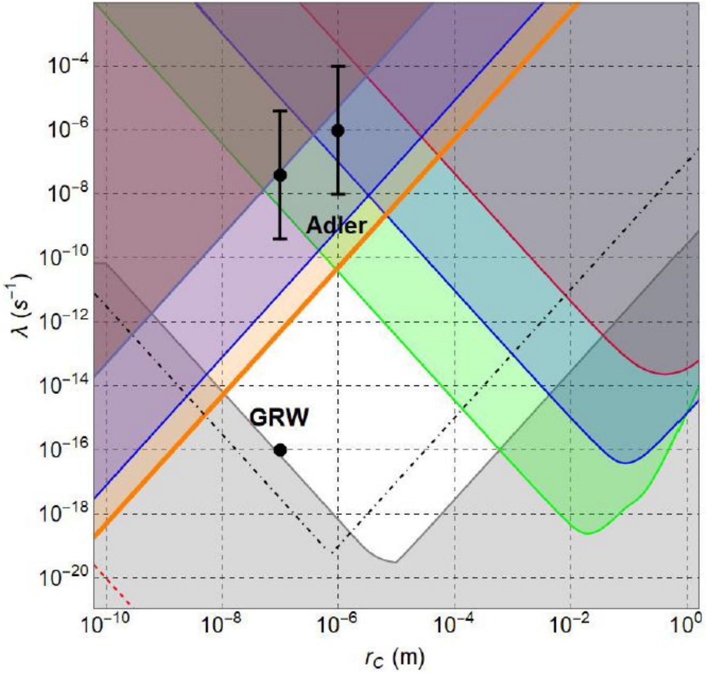
$$< \phi(\mathbf{r},t)\,\phi(\mathbf{r}',t') > - < \phi(\mathbf{r},t) > < \phi(\mathbf{r}',t') > \sim \frac{\hbar G}{|\mathbf{r}-\mathbf{r}'|}\,\xcancel{\delta(t-t')}$$

complex dependence of the S. E. on energy and on the atomic structure is to be considered!

Constraints on the CSL

Similar analysis leads to bounds on the strength and correlation length of the CSL
(Eur. Phys. J. C (2021) 81: 773)



$\lambda/r_C^2 < 52\ m^{-2}\ s^{-1}$

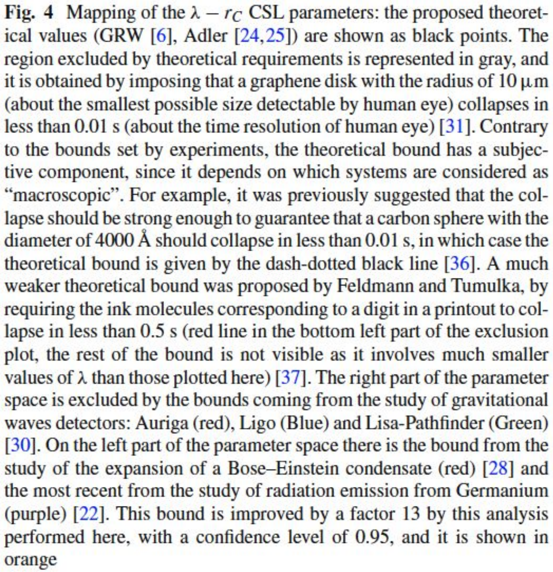


Fig. 4 Mapping of the $\lambda - r_C$ CSL parameters: the proposed theoretical values (GRW [6], Adler [24,25]) are shown as black points. The region excluded by theoretical requirements is represented in gray, and it is obtained by imposing that a graphene disk with the radius of 10 µm (about the smallest possible size detectable by human eye) collapses in less than 0.01 s (about the time resolution of human eye) [31]. Contrary to the bounds set by experiments, the theoretical bound has a subjective component, since it depends on which systems are considered as "macroscopic". For example, it was previously suggested that the collapse should be strong enough to guarantee that a carbon sphere with the diameter of 4000 Å should collapse in less than 0.01 s, in which case the theoretical bound is given by the dash-dotted black line [36]. A much weaker theoretical bound was proposed by Feldmann and Tumulka, by requiring the ink molecules corresponding to a digit in a printout to collapse in less than 0.5 s (red line in the bottom left part of the exclusion plot, the rest of the bound is not visible as it involves much smaller values of λ than those plotted here) [37]. The right part of the parameter space is excluded by the bounds coming from the study of gravitational waves detectors: Auriga (red), Ligo (Blue) and Lisa-Pathfinder (Green) [30]. On the left part of the parameter space there is the bound from the study of the expansion of a Bose–Einstein condensate (red) [28] and the most recent from the study of radiation emission from Germanium (purple) [22]. This bound is improved by a factor 13 by this analysis performed here, with a confidence level of 0.95, and it is shown in orange

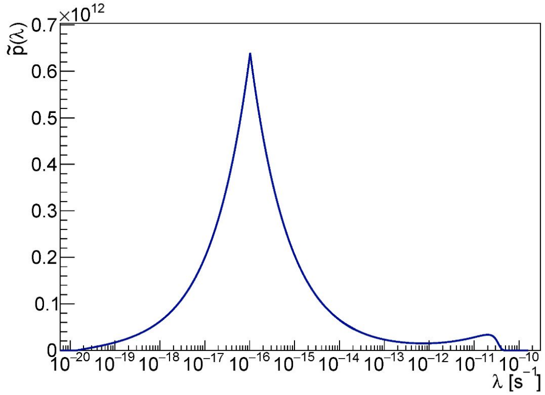
First separate determination of pdfs for λ and r_C

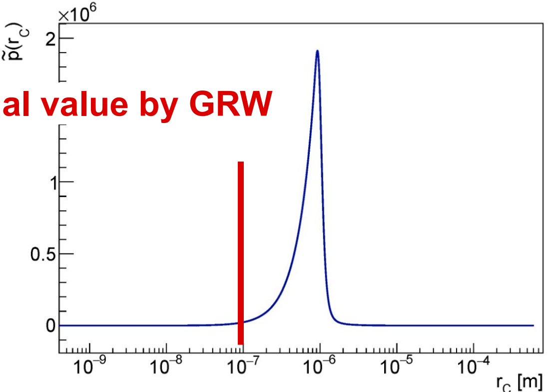
Entropy 2023, 25(2), 295

- *Experimental studies of the spontaneous radiation phenomenon focused so far on the λ/r_C^2 ratio, which regulates the predicted yield -> allow to exclude regions of the (λ-r_C) parameter space.*

- *Combined information from theoretical considerations and other experiments has led to the further exclusion of sectors of the (λ-r_C) plane, characterized by a different functional relation between λ and r_C.*

- *Including this rich prior information in the statistical analysis permits to disentangle the two parameters' probability density functions:*



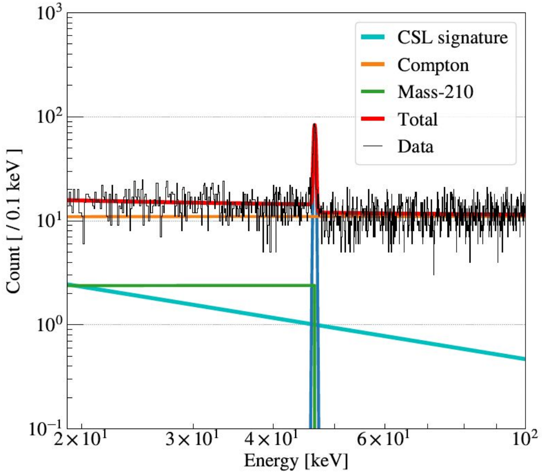


original value by GRW

The future of spontaneous radiation:

from γ-rays to X-rays

MAJORANA DEMONSTRATOR - PHYS. REV. LETT. 129, 080401 (2022)



applying the same S.E. rate above

$$\frac{d\Gamma}{dE} = N_{atoms} \times \left(N_A^2 + N_A\right) \times \frac{\lambda\hbar e^2}{4\pi^2\varepsilon_0 m_0^2 r_C^2 c^3 E}$$

$$\lambda/r_C^2 < 0.494 \pm 0.015\ m^{-2}\ s^{-1}$$

Non-Markovian extension

↓

cutoff frequency

↓

low-energy range is relevant

BUT - In this range S.E. from protons and valence electrons cancels ! ALSO e- start to emit coherently

X-rays spontaneous radiation

the CSL

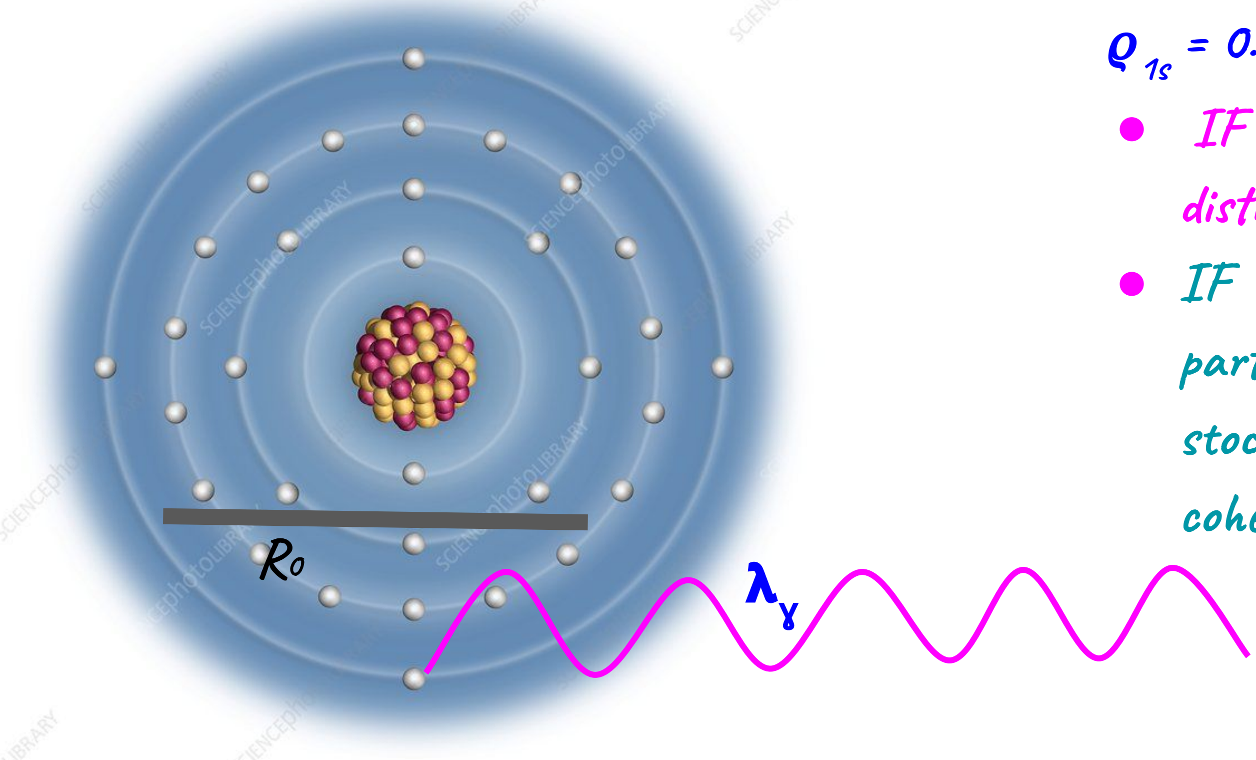
In the low-energy regime, the photon w.l. is comparable to the atomic orbits dimensions:

e.g. $\lambda_{dB}(E=15\ keV) = 0.8\ A$

$\varrho_{1s} = 0.025\ A\ ;\ \varrho_{4p} = 1.5\ A$

- IF λ_{γ} greater than particles distances -> they emit coherently
- IF correlation length greater than particles distances -> the stochastic field vibrates them coherently

CANCELLATION



X-rays spontaneous radiation

the CSL

In the general case:

$$\left.\frac{d\Gamma}{dE}\right|_t^{CSL} = \frac{\hbar\, e^2\, \lambda}{12\, \pi^2\, \epsilon_0\, c^3\, m_0^2\, r_C^2\, E}.$$

X-rays spontaneous radiation

the CSL

In the general case:

$$\left.\frac{d\Gamma}{dE}\right|_t^{CSL} = \frac{\hbar\, e^2\, \lambda}{12\,\pi^2\, \epsilon_0\, c^3\, m_0^2\, r_C^2\, E}\cdot$$

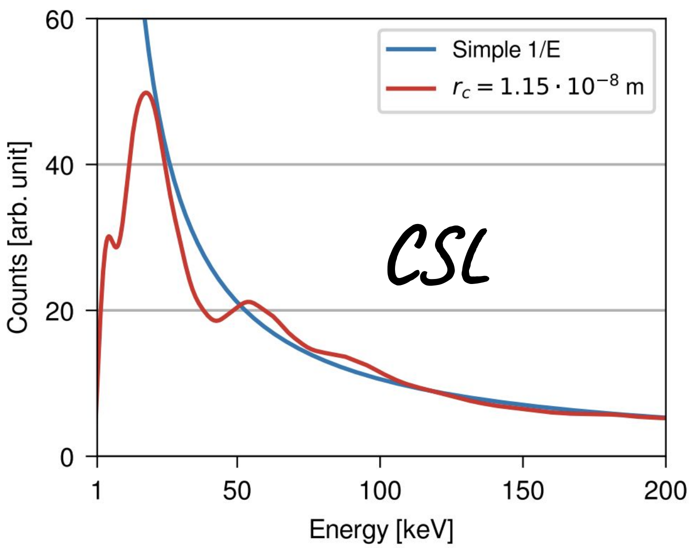
$$\cdot\left\{3\,N_p^2 + 3\,N_e + 2\sum_{o\,o'\text{ pairs}} N_o\, N_{o'}\, \frac{\sin\left[\frac{|\rho_o-\rho_{o'}|\,E}{\hbar\, c}\right]}{\left[\frac{|\rho_o-\rho_{o'}|\,E}{\hbar\, c}\right]}\, e^{-\frac{(\rho_o-\rho_{o'})^2}{4r_C^2}}\left(3-\frac{(\rho_o-\rho_{o'})^2}{2r_C^2}\right)+\right.$$

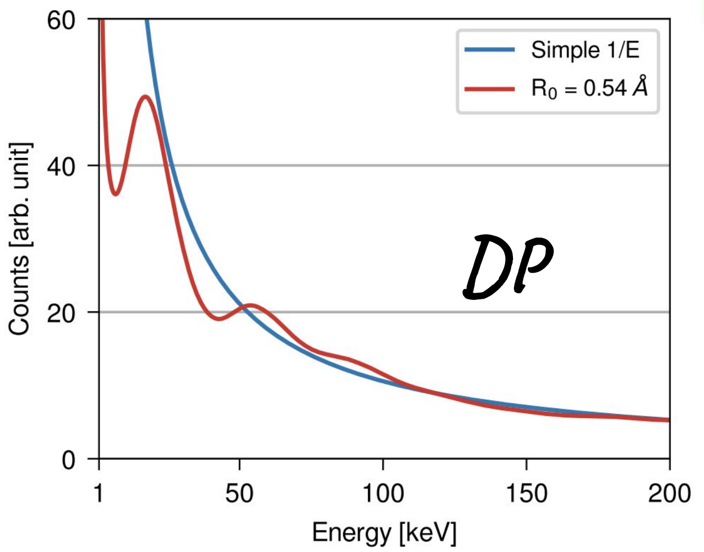
$$+\sum_o N_o\, \frac{\sin\left(\frac{\rho_o\, E}{\hbar\, c}\right)}{\left(\frac{\rho_o\, E}{\hbar\, c}\right)}\cdot\left[(N_o-1)\, e^{-\frac{\rho_o^2}{r_C^2}}\left(3-\frac{2\rho_o^2}{r_C^2}\right)\cos\left(\frac{\rho_o\, E}{\hbar\, c}\right)-\right.$$

$$\left.\left.-2\,N_p\, e^{-\frac{\rho_o^2}{4r_C^2}}\left(3-\frac{\rho_o^2}{2r_C^2}\right)\right]\right\}.$$

at each energy the atomic structure influences the shape of the expected S.E. spectrum

X-rays spontaneous radiation in Ge

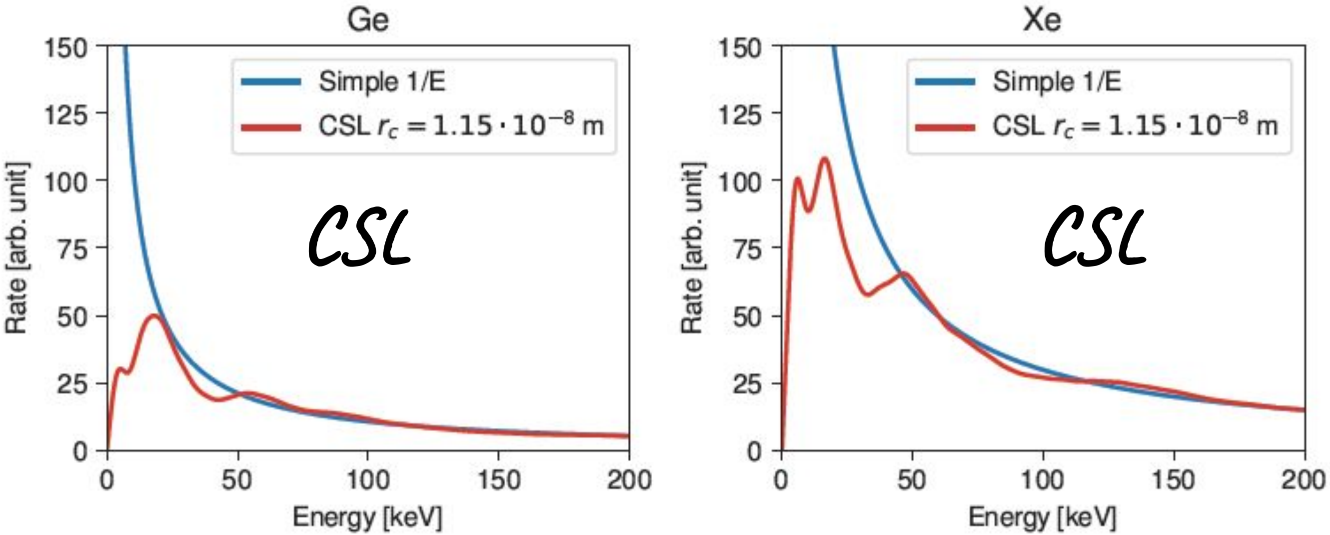




arXiv:2301.09920v1 **[quant-ph]** https://doi.org/10.48550/arXiv.2301.09920

- *at each energy the atomic structure influences the expected S.E. spectrum shape*
- *accurate shape analyses should allow to set much stronger bounds*
- *in principle the S.E. spectrum shape is different for different collapse models.*

X-rays spontaneous radiation in Xe



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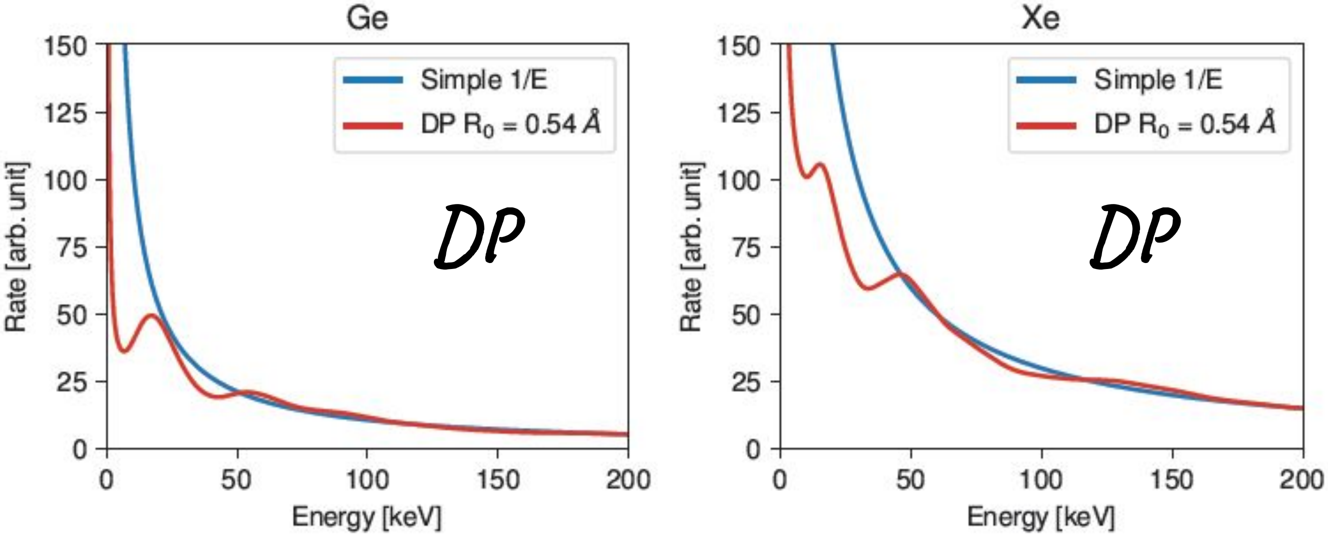

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- *at each energy the atomic structure influences the expected S.E. spectrum shape*
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This work was made possible through the support of
Grant 62099 from the John Templeton Foundation.

Thank you !

Local time uncertainty and decoherence

To generalize the concept for a local time $t_{\mathbf{r}} \to t + \delta t_{\mathbf{r}}$

one defines the correlation $\mathbf{M}[\delta t_{\mathbf{r}} \delta t_{\mathbf{r}'}] = \tau_{\mathbf{r}\mathbf{r}'} t$

Galileo invariant spatial correlation function

If the total Hamiltonian is decomposed in the sum of the local ones

$$\frac{d\rho}{dt} = -i\hbar^{-1}[H,\rho] - \frac{1}{2}\hbar^{-2}\sum_{\mathbf{r},\mathbf{r}'} \tau_{\mathbf{r}\mathbf{r}'}[H_{\mathbf{r}},[H_{\mathbf{r}'},\rho]]$$

The master equation suppresses superpositions of eigenstates of local energy

Reminder .. proper time interval

In special relativity the Minkowski metric is

$$\eta_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

the coordinates of the arbitrary Lorentz frame are $(x^0, x^1, x^2, x^3) = (ct, x, y, z)$

the infinitesimal time-like interval is $ds^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2 = \eta_{\mu\nu} dx^\mu dx^\nu$

due to invariance of the interval, if we consider the coordinates of an instantaneous rest frame

$$ds^2 = c^2 d\tau^2 - dx_\tau^2 - dy_\tau^2 - dz_\tau^2 = c^2 d\tau^2$$

Reminder .. proper time interval

The proper time interval is then the integral on the world-line

$$\Delta\tau = \int_P d\tau = \int \frac{ds}{c} \longrightarrow \Delta\tau = \int_P \frac{1}{c}\sqrt{\eta_{\mu\nu} dx^\mu dx^\nu}$$

In general relativity the analogous expression for the generic metric tensor yields

$$\Delta\tau = \int_P d\tau = \int_P \frac{1}{c}\sqrt{g_{\mu\nu}\ dx^\mu\ dx^\nu}$$

and when constant coordinates are chosen

$$\Delta\tau = \int_P d\tau = \int_P \frac{1}{c}\sqrt{g_{00}} dx^0$$

local time uncertainty means uncertainty of the local grav. potential

$$\Delta\tau = \int_P d\tau = \int_P \frac{1}{c}\sqrt{g_{00}}\,dx^0$$

In the Newtonian limit $g_{00} = 1 + \frac{2\phi}{c^2}$

Here then comes the crucial point ... it is assumed that the gravitational potential should not be quantized

BUT that QM requires an absolute indeterminacy of the gravitational field.

I.E. the gravitational potential is a c-number stochastic variable, whose mean value is to be identified with the classical Newtonian potential.

Then local time fluctuation is related to a fluctuation of the local gravitational potential

$$\delta t_{\mathbf{r}} \equiv \delta \int_0^t dt' g_{00}^{1/2}(\mathbf{r},t') \approx -c^{-2} \int_0^t dt' \Phi(\mathbf{r},t')$$

.. so correlations of local uncertainties of Newtonian gravity can lead to correlation of local time uncertainties.

Can the gravitational field be measured with unlimited precision?

Diosi and Lukacs [Ann. Phys. 44, 488 (1987)] apply the arguments of [N. Bohr and L. Rosenfeld, K. Dan. Vidensk. Selsk., Mat.-Fys. Medd. 12, 1 (1933)]: $\Delta\phi(\mathbf{r},t) = -4\pi G\rho(\mathbf{r},t)$ $\mathbf{g}(\mathbf{r},t) = -\nabla\phi$

The apparatus, obeying QM, is characterized by parameters m, R , T. In realistic measurements only a time-space averaged gravitational field is meaningful

$$\tilde{\mathbf{g}}(\mathbf{r},t) = \frac{1}{VT}\int \mathbf{g}(\mathbf{r}',t')d^3r'dt \qquad \textbf{with} \qquad |\mathbf{r}-\mathbf{r}'| < R \ , \ |t-t'| < T/2$$

The target is a point-like particle (of mass m) at rest at time t=0, immersed in the field g. Detector measures momentum changes. In the time T the momentum gain is $p = m\tilde{\mathbf{g}}T$

$$\delta p = \hbar/R \quad \xrightarrow{\qquad\qquad} \quad \sigma(\tilde{\mathbf{g}}) \sim \frac{\hbar}{mRT}$$

Can the gravitational field be measured with unlimited precision?

It's useless to increase R and T, since this would decrease the error on average field, not on the instantaneous local field of the Newtonian theory. m can be increased, till its own field does not perturb g, i.e. till:

$$\sigma(\tilde{\mathrm{g}}) \sim \frac{\hbar}{m\,RT} \qquad\qquad \delta\tilde{\mathrm{g}}_m \sim \frac{G\,m}{R^2}$$

Given the optimal mass choice then:

$$m_{\mathrm{opt}} \sim \left(\frac{\hbar\,R}{G\,T}\right)^{1/2} \qquad\qquad \sigma(\tilde{\mathrm{g}}) \sim \left(\frac{\hbar\,G}{V\,T}\right)^{1/2}$$

If the limitation is universal then the actual gravitational field is:

$$\mathbf{g}(\mathbf{r},t) = \mathbf{g}_N(\mathbf{r},t) + \mathbf{g}_S(\mathbf{r},t)$$

solution of Poisson Eq.

stochastic fluctuation

Uncorrelated gravitational field fluctuations

It's useless to increase R and T, since this would decrease the error on average field, not on the instantaneous local field of the Newtonian theory. m can be increased, till its own field does not perturb g, i.e. till:

$$\sigma(\tilde{\mathrm{g}}) \sim \frac{\hbar}{m\,R\,T} \qquad \delta\tilde{\mathrm{g}}_m \sim \frac{G\,m}{R^2}$$

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If the limitation is universal then the actual gravitational field is: $\mathbf{g}(\mathbf{r},t) = \mathbf{g}_N(\mathbf{r},t) + \mathbf{g}_S(\mathbf{r},t)$

$$< \tilde{\mathrm{g}}_S >= 0 \qquad ; \qquad < \tilde{\mathrm{g}}_S^2 >= \frac{\hbar\,G}{V\,T}$$

The squared dispersion of the averaged gs is inversely proportional to the space-time cell volume -> hence gs is uncorrelated in time and space

$$\boxed{< \mathbf{g}_S(\mathbf{r},t)\,\mathbf{g}_S(\mathbf{r}',t') >= \hbar\,G\,\delta(\mathbf{r}-\mathbf{r}')\,\delta(t-t')}$$

Gravitational potential as a stochastic variable

In terms of the potential, this can be regarded as a stochastic variable, with momenta:

$$< \phi(\mathbf{r},t) > = \phi_N(\mathbf{r},t)$$

$$< \phi(\mathbf{r},t)\,\phi(\mathbf{r}',t') > - < \phi(\mathbf{r},t) >< \phi(\mathbf{r}',t') > \sim \frac{\hbar\, G}{|\mathbf{r}-\mathbf{r}'|}\,\delta(t-t')$$

The covariance function for the gravitational potential is not dependent on the parameters of the gedanken apparatus (m, T, R), which may suggest universality of the potential intrinsic fluctuation.

Going back to the searched correlation of the local time fluctuation $\mathbf{M}[\delta t_{\mathbf{r}}\delta t_{\mathbf{r}'}] = \tau_{\mathbf{rr}'}t$

$$\delta t_{\mathbf{r}} \equiv \delta\int_0^t dt'\, g_{00}^{1/2}(\mathbf{r},t') \approx -c^{-2}\int_0^t dt'\,\Phi(\mathbf{r},t') \longrightarrow \tau_{\mathbf{rr}'} = \text{const}\times\frac{G\hbar}{|\mathbf{r}-\mathbf{r}'|}c^{-4}$$

Master equation

$$\tau_{\mathbf{r}\mathbf{r}'} = \mathrm{const} \times \frac{G\hbar}{|\mathbf{r} - \mathbf{r}'|} c^{-4}$$

the local time correlation is extremely small

substituted in the master equation

$$\frac{d\rho}{dt} = -i\hbar^{-1}[H,\rho] - \frac{1}{2}\hbar^{-2}\sum_{\mathbf{r},\mathbf{r}'} \tau_{\mathbf{r}\mathbf{r}'}[H_{\mathbf{r}},[H_{\mathbf{r}'},\rho]]$$

yields

$$\begin{aligned}\frac{d\rho}{dt} = & - \quad i\hbar^{-1}[H,\rho] \\ & - \quad \frac{G}{2}\hbar^{-1}\int\int \frac{d\mathbf{r}d\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|}[f(\mathbf{r}),[f(\mathbf{r}'),\rho]]\end{aligned}$$

Master equation

Denote the configuration coordinates (classical and spin) of the dynamical system by X. The corresponding mass density at the point r is $f(\mathbf{r}|X)$

Given the coordinate eigenstate /x> we have $$f(r|X)\delta(X'-X)\equiv\langle X'|\hat{f}(r)|X\rangle$$

So if one introduces the damping time:

$$[\tau_d(X,X')]^{-1}=\frac{G}{2\hbar}\int\int d^3r\;d^3r' \;\times$$
$$\frac{[f(\mathbf{r}|X)-f(\mathbf{r}|X')][f(\mathbf{r}'|X)-f(\mathbf{r}'|X')]}{|\mathbf{r}-\mathbf{r}'|}$$

the master equation becomes

$$\langle X|\dot{\hat{\rho}}(t)|X'\rangle=(-\mathrm{i}/\hbar)\langle X|[\hat{H}_0,\hat{\rho}(t)]|X'\rangle$$
$$-[\tau_d(X,X')]^{-1}\langle X|\hat{\rho}(t)|X'\rangle\;.$$

Energy decoherence

$$\langle X|\dot{\hat{\rho}}(t)|X'\rangle = (-i/\hbar)\langle X|[\hat{H}_0,\hat{\rho}(t)]|X'\rangle$$

$$\boxed{-[\tau_d(X,X')]^{-1}\langle X|\hat{\rho}(t)|X'\rangle\ .}$$

$$[\tau_d(X,X')]^{-1} = \frac{G}{2\hbar}\int\int d^3r\ d^3r'\ \times \frac{[f(\mathbf{r}|X)-f(\mathbf{r}|X')][f(\mathbf{r}'|X)-f(\mathbf{r}'|X')]}{|\mathbf{r}-\mathbf{r}'|}$$

If the difference between the mass distributions of two states |X> and |X’> in superposition becomes big

the corresponding damping time becomes short

the corresponding off-diagonal terms of the density operator vanish

this QM violating phenomenon is ENERGY DECOHERENCE

in Diosi approach.

X-rays spontaneous radiation

the CSL

In the low-energy regime, the photon w.l. is comparable to the atomic orbits dimensions

↓

general expression for the rate applies:

$$\left.\frac{d\Gamma}{dE}\right|_t^{CSL} = N_{atoms} \cdot \frac{\hbar\,\lambda}{6\,\pi^2\,\epsilon_0\,c^3\,m_0^2\,E} \sum_{i,j} \frac{q_i\,q_j}{mi\,m_j} \cdot f_{ij}(\mu) \cdot \frac{sin(b_{ij})}{b_{ij}}$$

$$f_{ij}^k(\mu) := \int ds \int ds' e^{-\frac{(\bar{r}_i-\bar{r}_j+s'-s)^2}{4r_C^2}} \left(\frac{\partial\mu_i(s)}{\partial s^k}\right)\left(\frac{\partial\mu_j(s')}{\partial s'^k}\right)$$

S. Donadi et al., Eur. Phys. J. C (2021) 81: 773

let's consider the simple case of a white CSL:

$$r_C \gg |\bar{\mathbf{r}}_i - \bar{\mathbf{r}}_j| \longrightarrow f_{ij}(\mu) = \frac{3}{2}\frac{m_i m_j}{r_C^2}$$

the stochastic fluctuations ALWAYS vibrate electrons and protons coherently

$$b_{i,j} = \frac{2\pi}{c}\frac{|\mathbf{r}_i - \mathbf{r}_j|}{\lambda_\gamma} \longrightarrow$$

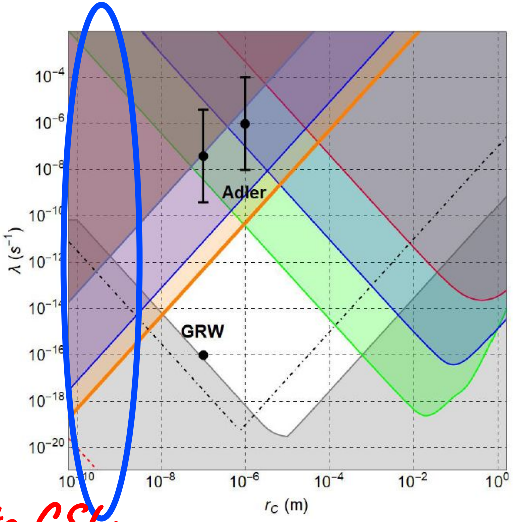
if $\lambda_{dB} << Q_{1s}$

$$\frac{d\Gamma}{dE} = N_{atoms} \times \left(N_A^2 + N_A\right) \times \frac{\lambda\hbar e^2}{4\pi^2\varepsilon_0 m_0^2 r_C^2 c^3 E}$$

X-rays spontaneous radiation

the CSL

Atomic structure range



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$$b_{i,j} = \frac{2\pi}{c}\frac{|\mathbf{r}_i - \mathbf{r}_j|}{\lambda_\gamma} \longrightarrow$$

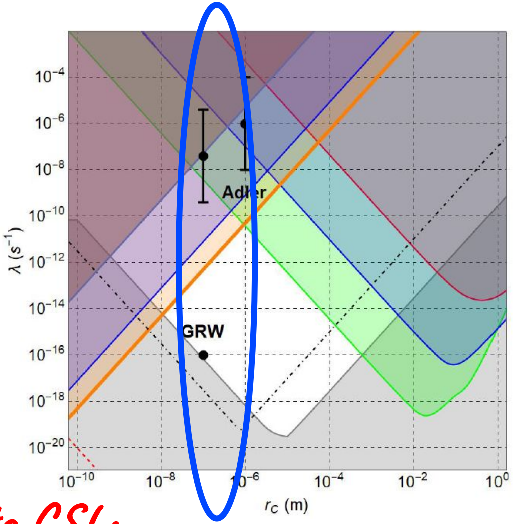
if $\lambda_{dB} << \rho_{1s}$

$$\frac{d\Gamma}{dE} = N_{atoms} \times \left(N_A^2 + N_A\right) \times \frac{\lambda \hbar e^2}{4\pi^2 \varepsilon_0 m_0^2 r_C^2 c^3 E}$$

X-rays spontaneous radiation

the CSL

Present bound



let's consider the simple case of a white CSL:

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the stochastic fluctuations ALWAYS vibrate electrons and protons coherently

$$b_{i,j} = \frac{2\pi}{c}\frac{|\mathbf{r}_i - \mathbf{r}_j|}{\lambda_\gamma} \longrightarrow \text{if } \lambda_{dB} << \rho_{1s} \quad \frac{d\Gamma}{dE} = N_{atoms} \times \left(N_A^2 + N_A\right) \times \frac{\lambda \hbar e^2}{4\pi^2 \varepsilon_0 m_0^2 r_C^2 c^3 E}$$

X-rays spontaneous radiation

the CSL

In the low-energy regime, the photon w.l. is comparable to the atomic orbits dimensions

e.g. $\lambda_{dB}(E=15\ keV) = 0.8\ A$

$\varrho_{1s} = 0.025\ A$; $\varrho_{4p} = 1.5\ A$

↓

general expression for the rate applies:

$$\left.\frac{d\Gamma}{dE}\right|_t^{CSL} = N_{atoms} \cdot \frac{\hbar\,\lambda}{6\,\pi^2\,\epsilon_0\,c^3\,m_0^2\,E} \sum_{i,j} \frac{q_i\,q_j}{mi\,m_j} \cdot f_{ij}(\mu) \cdot \frac{sin(b_{ij})}{b_{ij}}$$

$$f_{ij}^k(\mu) := \int ds \int ds' e^{-\frac{(\bar{r}_i - \bar{r}_j + s' - s)^2}{4 r_C^2}} \left(\frac{\partial \mu_i(s)}{\partial s^k}\right)\left(\frac{\partial \mu_j(s')}{\partial s'^k}\right)$$

non-Markovian CSL is simpler:

$$r_C \gg |\bar{\mathbf{r}}_i - \bar{\mathbf{r}}_j| \longrightarrow f_{ij}(\mu) = 3\frac{m_i m_j}{r_C^2}$$

$$b_{i,j} = \frac{2\pi}{c}\frac{|\mathbf{r}_i - \mathbf{r}_j|}{\lambda_\gamma} \longrightarrow$$

the stochastic fluctuations ALWAYS vibrate electrons and protons coherently

if $\lambda_{dB} > \varrho_{1s}$

electrons and protons emit coherently

X-rays spontaneous radiation

the CSL

both electrons and protons contribute:

$$\left.\frac{d\Gamma}{dE}\right|_t^{CSL} = N_{atoms} \cdot \frac{\hbar\,\lambda}{4\,\pi^2\,\epsilon_0\,c^3\,m_0^2\,r_C^2\,E}\left[\sum_{ip,jp} q_{ip}\,q_{jp} \cdot \frac{sin(2\pi|\bar{\mathbf{r}}_{ip} - \bar{\mathbf{r}}_{jp}|/\lambda_\gamma)}{2\pi|\bar{\mathbf{r}}_i - \bar{\mathbf{r}}_j|/\lambda_\gamma} + \right.$$

nuclear emission

$$+ \sum_{ip,je} q_{ip}\,q_{je} \cdot \frac{sin(2\pi|\bar{\mathbf{r}}_{ip} - \bar{\mathbf{r}}_{je}|/\lambda_\gamma)}{2\pi|\bar{\mathbf{r}}_{ip} - \bar{\mathbf{r}}_{je}|/\lambda_\gamma} +$$

$$+ \sum_{ie,jp} q_{ie}\,q_{jp} \cdot \frac{sin(2\pi|\bar{\mathbf{r}}_{ie} - \bar{\mathbf{r}}_{jp}|/\lambda_\gamma)}{2\pi|\bar{\mathbf{r}}_{ie} - \bar{\mathbf{r}}_{jp}|/\lambda_\gamma} +$$

$$\left. + \sum_{ie,je} q_{ie}\,q_{je} \cdot \frac{sin(2\pi|\bar{\mathbf{r}}_{ie} - \bar{\mathbf{r}}_{je}|/\lambda_\gamma)}{2\pi|\bar{\mathbf{r}}_{ie} - \bar{\mathbf{r}}_{je}|/\lambda_\gamma}\right]$$

X-rays spontaneous radiation

the CSL

both electrons and protons contribute:

$$\frac{d\Gamma}{dE}\bigg|_t^{CSL} = N_{atoms}\cdot\frac{\hbar\,\lambda}{4\,\pi^2\,\epsilon_0\,c^3\,m_0^2\,r_C^2\,E}\left[\sum_{ip,jp} q_{ip}\,q_{jp}\cdot\frac{sin(2\pi|\overline{\mathbf{r}}_{ip}-\overline{\mathbf{r}}_{jp}|/\lambda_\gamma)}{2\pi|\overline{\mathbf{r}}_{i}-\overline{\mathbf{r}}_{j}|/\lambda_\gamma}+\right.$$

$$+\sum_{ip,je} q_{ip}\,q_{je}\cdot\frac{sin(2\pi|\overline{\mathbf{r}}_{ip}-\overline{\mathbf{r}}_{je}|/\lambda_\gamma)}{2\pi|\overline{\mathbf{r}}_{ip}-\overline{\mathbf{r}}_{je}|/\lambda_\gamma}+$$

$$+\sum_{ie,jp} q_{ie}\,q_{jp}\cdot\frac{sin(2\pi|\overline{\mathbf{r}}_{ie}-\overline{\mathbf{r}}_{jp}|/\lambda_\gamma)}{2\pi|\overline{\mathbf{r}}_{ie}-\overline{\mathbf{r}}_{jp}|/\lambda_\gamma}+$$

$$\left.+\sum_{ie,je} q_{ie}\,q_{je}\cdot\frac{sin(2\pi|\overline{\mathbf{r}}_{ie}-\overline{\mathbf{r}}_{je}|/\lambda_\gamma)}{2\pi|\overline{\mathbf{r}}_{ie}-\overline{\mathbf{r}}_{je}|/\lambda_\gamma}\right]$$

electronic emission

X-rays spontaneous radiation

the CSL

both electrons and protons contribute:

$$\left.\frac{d\Gamma}{dE}\right|_t^{CSL} = N_{atoms} \cdot \frac{\hbar\,\lambda}{4\,\pi^2\,\epsilon_0\,c^3\,m_0^2\,r_C^2\,E}\left[\sum_{ip,jp} q_{ip}\,q_{jp} \cdot \frac{sin(2\pi|\bar{\mathbf{r}}_{ip} - \bar{\mathbf{r}}_{jp}|/\lambda_\gamma)}{2\pi|\bar{\mathbf{r}}_i - \bar{\mathbf{r}}_j|/\lambda_\gamma} + \right.$$

$$+ \sum_{ip,je} q_{ip}\,q_{je} \cdot \frac{sin(2\pi|\bar{\mathbf{r}}_{ip} - \bar{\mathbf{r}}_{je}|/\lambda_\gamma)}{2\pi|\bar{\mathbf{r}}_{ip} - \bar{\mathbf{r}}_{je}|/\lambda_\gamma} +$$

$$+ \sum_{ie,jp} q_{ie}\,q_{jp} \cdot \frac{sin(2\pi|\bar{\mathbf{r}}_{ie} - \bar{\mathbf{r}}_{jp}|/\lambda_\gamma)}{2\pi|\bar{\mathbf{r}}_{ie} - \bar{\mathbf{r}}_{jp}|/\lambda_\gamma} +$$

$$\left. + \sum_{ie,je} q_{ie}\,q_{je} \cdot \frac{sin(2\pi|\bar{\mathbf{r}}_{ie} - \bar{\mathbf{r}}_{je}|/\lambda_\gamma)}{2\pi|\bar{\mathbf{r}}_{ie} - \bar{\mathbf{r}}_{je}|/\lambda_\gamma}\right]$$

electrons-protons

coupled emission

X-rays spontaneous radiation

the CSL

both electrons and protons contribute:

$$\frac{d\Gamma}{dE}\bigg|_t^{CSL} = N_{atoms} \cdot \frac{\hbar\,\lambda}{4\,\pi^2\,\epsilon_0\,c^3\,m_0^2\,r_C^2\,E}\Bigg[\sum_{ip,jp} q_{ip}\,q_{jp} \cdot \cancelto{1}{\frac{sin(2\pi|\bar{\mathbf{r}}_{ip}-\bar{\mathbf{r}}_{jp}|/\lambda_\gamma)}{2\pi|\bar{\mathbf{r}}_{i}-\bar{\mathbf{r}}_{j}|/\lambda_\gamma}} +$$

$$+\sum_{ip,je} q_{ip}\,q_{je} \cdot \cancelto{1}{\frac{sin(2\pi|\bar{\mathbf{r}}_{ip}-\bar{\mathbf{r}}_{je}|/\lambda_\gamma)}{2\pi|\bar{\mathbf{r}}_{ip}-\bar{\mathbf{r}}_{je}|/\lambda_\gamma}} +$$

$$+\sum_{ie,jp} q_{ie}\,q_{jp} \cdot \cancelto{1}{\frac{sin(2\pi|\bar{\mathbf{r}}_{ie}-\bar{\mathbf{r}}_{jp}|/\lambda_\gamma)}{2\pi|\bar{\mathbf{r}}_{ie}-\bar{\mathbf{r}}_{jp}|/\lambda_\gamma}} +$$

$$+\sum_{ie,je} q_{ie}\,q_{je} \cdot \cancelto{1}{\frac{sin(2\pi|\bar{\mathbf{r}}_{ie}-\bar{\mathbf{r}}_{je}|/\lambda_\gamma)}{2\pi|\bar{\mathbf{r}}_{ie}-\bar{\mathbf{r}}_{je}|/\lambda_\gamma}}\Bigg]$$

in the limit $\lambda_{dB} >> \varrho_{4p}$

X-rays spontaneous radiation

the CSL

both electrons and protons contribute:

$$\left.\frac{d\Gamma}{dE}\right|_t^{CSL} = N_{atoms} \cdot \frac{\hbar\,\lambda}{4\,\pi^2\,\epsilon_0\,c^3\,m_0^2\,r_C^2\,E}\left[\sum_{ip,jp} q_{ip}\,q_{jp} \cdot \underbrace{\frac{sin(2\pi|\mathbf{\bar{r}}_{ip}-\mathbf{\bar{r}}_{jp}|/\lambda_\gamma)}{2\pi|\mathbf{\bar{r}}_i-\mathbf{\bar{r}}_j|/\lambda_\gamma}}_{\xrightarrow{} 1} + \right.$$

$$+ \sum_{ip,je} q_{ip}\,q_{je} \cdot \underbrace{\frac{sin(2\pi|\mathbf{\bar{r}}_{ip}-\mathbf{\bar{r}}_{je}|/\lambda_\gamma)}{2\pi|\mathbf{\bar{r}}_{ip}-\mathbf{\bar{r}}_{je}|/\lambda_\gamma}}_{\xrightarrow{} 1} +$$

$$+ \sum_{ie,jp} q_{ie}\,q_{jp} \cdot \underbrace{\frac{sin(2\pi|\mathbf{\bar{r}}_{ie}-\mathbf{\bar{r}}_{jp}|/\lambda_\gamma)}{2\pi|\mathbf{\bar{r}}_{ie}-\mathbf{\bar{r}}_{jp}|/\lambda_\gamma}}_{\xrightarrow{} 1} +$$

$$\left. + \sum_{ie,je} q_{ie}\,q_{je} \cdot \underbrace{\frac{sin(2\pi|\mathbf{\bar{r}}_{ie}-\mathbf{\bar{r}}_{je}|/\lambda_\gamma)}{2\pi|\mathbf{\bar{r}}_{ie}-\mathbf{\bar{r}}_{je}|/\lambda_\gamma}}_{\xrightarrow{} 1}\right]$$

in the limit $\lambda_{dB} >> \varrho_{4p}$

$$\downarrow$$

$$\left.\frac{d\Gamma}{dE}\right|_t^{CSL} = N_{atoms} \cdot \frac{\hbar\,e^2\,\lambda}{4\,\pi^2\,\epsilon_0\,c^3\,m_0^2\,r_C^2\,E}\left[N_p^2 - 2 \cdot N_p\,N_e + N_e^2,\right]$$

In neutral matter complete cancellation !

CSL

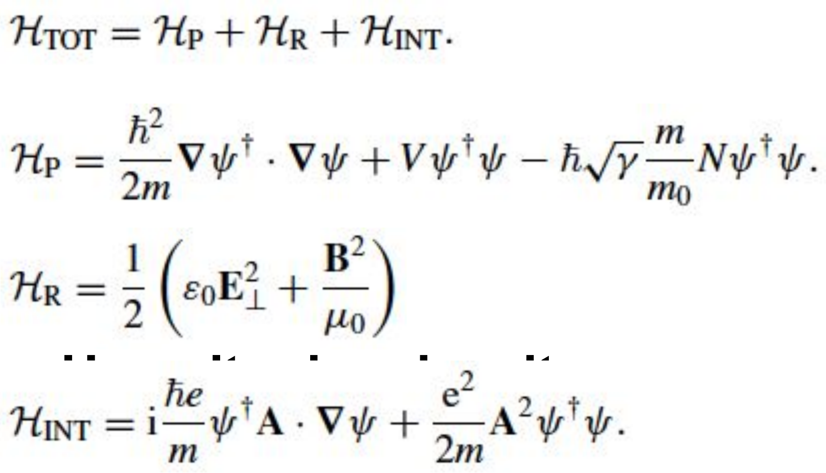
averaged density matrix evolution can be derived from a standard Schroedinger equation with a random Hamiltonian. Such equation does not lead to the state vector reduction, because it is linear, but reproduce the same noise averaged density matrix evolution (photon emission rate ..)

$$H_{\mathrm{TOT}} = H - \hbar\sqrt{\gamma}\sum_{i}\frac{m_j}{m_0}\int N(\mathbf{y},t)\psi_j^{\dagger}(\mathbf{y})\psi_j(\mathbf{y})\,\mathrm{d}^3y$$

$$N(\mathbf{y},t) = \int g(\mathbf{y}-\mathbf{x})\xi_t(\mathbf{x})\,\mathrm{d}^3x,$$

and $\xi_t(\mathbf{x}) = \mathrm{d}W_t(\mathbf{x})/\mathrm{d}t$ is a white noise field (in the simplest case), with correlation function $\mathbb{E}[\xi_t(\mathbf{x})\xi_s(\mathbf{y})] = \delta(t-s)\delta(\mathbf{x}-\mathbf{y}).$

So *N* is a Gaussian noise field, with zero mean, and correlation function: $\mathbb{E}[N(\mathbf{x},t)N(\mathbf{y},s)] = \delta(t-s)F(\mathbf{x}-\mathbf{y}), \quad F(\mathbf{x}) = \frac{1}{(\sqrt{4\pi}r_C)^3}\mathrm{e}^{-\mathbf{x}^2/4r_C^2}.$



$$\mathcal{H}_{\mathrm{TOT}} = \mathcal{H}_{\mathrm{P}} + \mathcal{H}_{\mathrm{R}} + \mathcal{H}_{\mathrm{INT}}.$$

$$\mathcal{H}_{\mathrm{P}} = \frac{\hbar^2}{2m}\nabla\psi^\dagger \cdot \nabla\psi + V\psi^\dagger\psi - \hbar\sqrt{\gamma}\frac{m}{m_0}N\psi^\dagger\psi.$$

$$\mathcal{H}_{\mathrm{R}} = \frac{1}{2}\left(\varepsilon_0\mathbf{E}_\perp^2 + \frac{\mathbf{B}^2}{\mu_0}\right)$$

Th

$$\mathcal{H}_{\mathrm{INT}} = \mathrm{i}\frac{\hbar e}{m}\psi^\dagger\mathbf{A}\cdot\nabla\psi + \frac{\mathrm{e}^2}{2m}\mathbf{A}^2\psi^\dagger\psi.$$

$$\mathcal{H}_{\mathrm{TOT}} = \mathcal{H}_{\mathrm{P}} + \mathcal{H}_{\mathrm{R}} + \mathcal{H}_{\mathrm{INT}}.$$

$$\mathcal{H}_{\mathrm{P}} = \frac{\hbar^2}{2m}\nabla\psi^\dagger \cdot \nabla\psi + V\psi^\dagger\psi - \hbar\sqrt{\gamma}\frac{m}{m_0}N\psi^\dagger\psi.$$

$$\mathcal{H}_{\mathrm{R}} = \frac{1}{2}\left(\varepsilon_0\mathbf{E}_\perp^2 + \frac{\mathbf{B}^2}{\mu_0}\right)$$

$$\mathcal{H}_{\mathrm{INT}} = \mathrm{i}\frac{\hbar e}{m}\psi^\dagger\mathbf{A}\cdot\nabla\psi + \frac{\mathrm{e}^2}{2m}\mathbf{A}^2\psi^\dagger\psi.$$

🟦 **perturbation terms**

the calculation is performed at first order in $\sqrt{\gamma}$ and e.

So the first-order transition amplitude for a charged particle to emit

If the correlation function in time of the collapsing noise is a delta, the expected rate of radiation, as a consequence of the interaction of the non-relativistic particle with the noise filed (*spontaneous radiation*) is:

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}p} = \frac{\lambda \hbar \mathrm{e}^2}{2\pi^2 \varepsilon_0 c^3 m_0^2 r_c^2 p}$$

Emission rate in the non-white noise case

If a general correlation function in time is considered for the collapsing noise:

$$\mathbb{E}[N(\mathbf{x},t)N(\mathbf{y},s)] = f(t-s)F(\mathbf{x}-\mathbf{y})$$

the photon emission rate changes as:

$$\left.\frac{\mathrm{d}\Gamma}{\mathrm{d}p}\right|_{\text{NON-WHITE}} = \frac{1}{2}[\tilde{f}(0)+\tilde{f}(pc)] \times \left.\frac{\mathrm{d}\Gamma}{\mathrm{d}p}\right|_{\text{WHITE}}$$

Second term: the probability of emitting a photon with momentum *p* is proportional to the weight of the Fourier component of the noise corresponding to the frequency $\omega_p = pc$.

The first term is $\left.\frac{\mathrm{d}\Gamma}{\mathrm{d}p}\right|_{\text{NON-WHITE}} = \frac{1}{2}[\tilde{f}(0) + \tilde{f}(pc)] \times \left.\frac{\mathrm{d}\Gamma}{\mathrm{d}p}\right|_{\text{WHITE}}$ nentum

such term is un-physical. It arises because perturbation theory is formally not valid in the large time limit, since the effect of the noise accumulates continuously in time. Such terms disappears when adding higher terms in the perturbative expansion, or the perturbative calculation is “cured” by e.g. confining the noise.

Time correlation functi **chastic noise considered in literature:**

$$f(t-s) = \frac{\Omega_c}{2} e^{-\Omega_c |t-s|}$$

$$\frac{\Omega_c^2}{\Omega_c^2 + \omega^2}$$

whose Fourier transform is

or the Gaussian case:

$$f(t-s) = N e^{-\frac{\Omega_c^2 \, (t-s)^2}{2}}$$

$$e^{-\frac{1}{2}\left(\frac{\omega}{\Omega_c}\right)^2}$$