

Primordial Black Hole formation during the QCD phase transition

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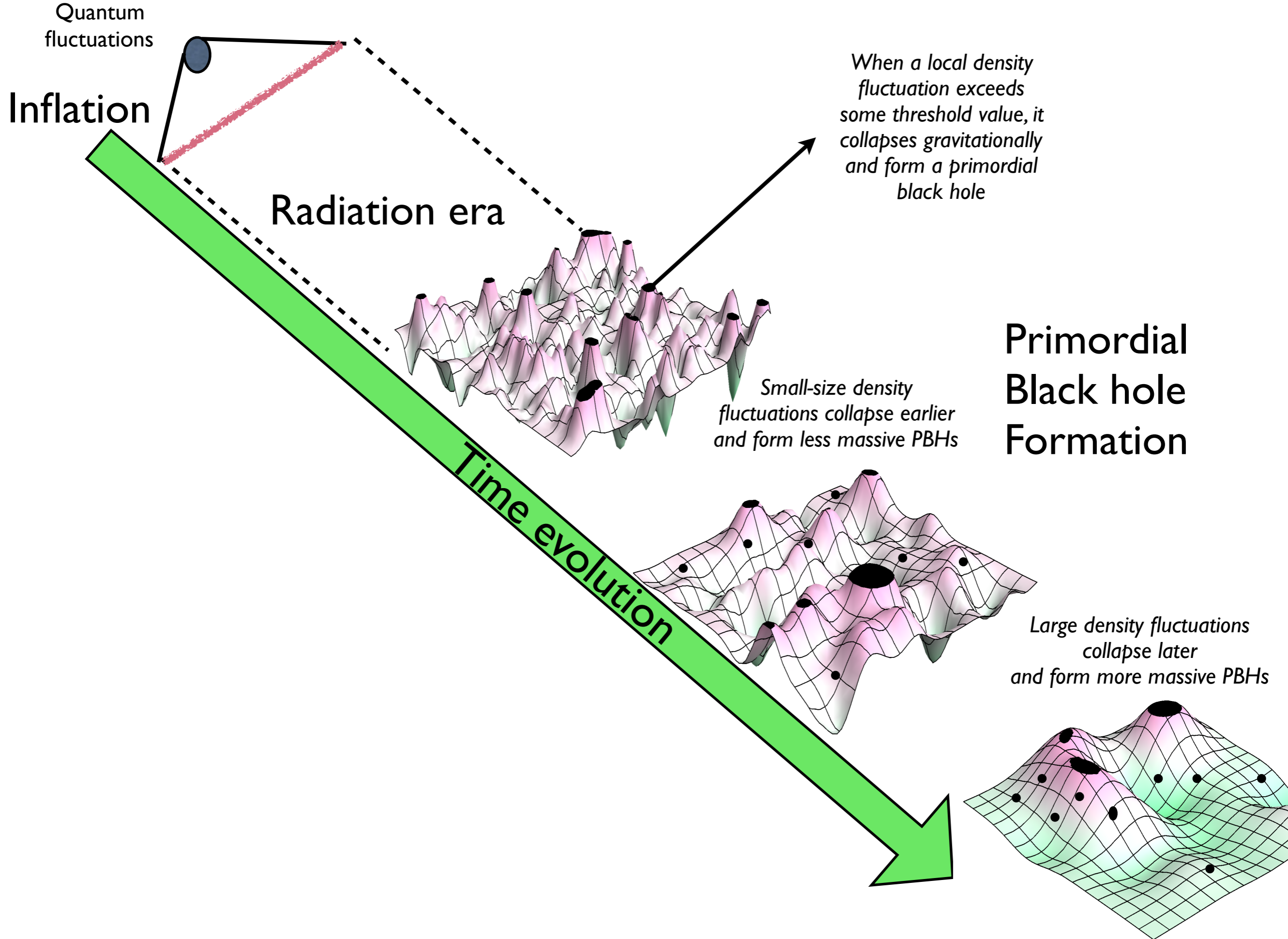
HORIZON 2020 G.A. n. 754496

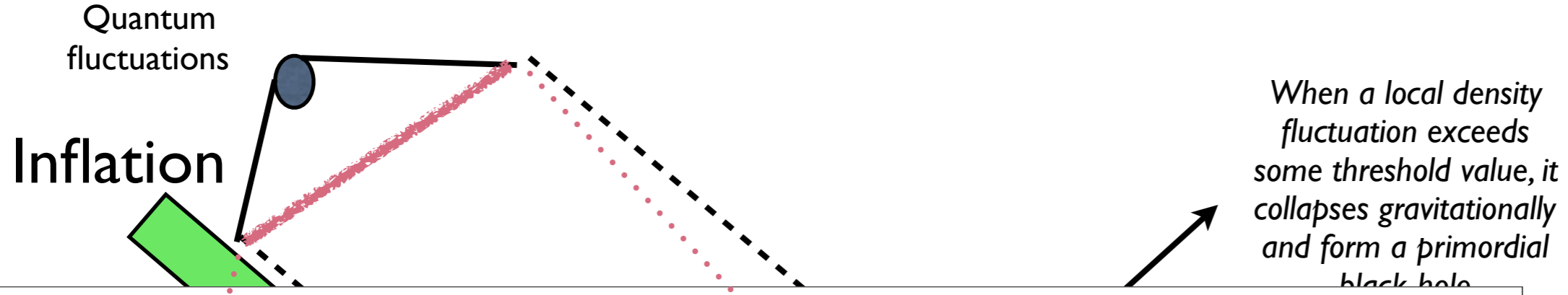
Introduction: a very brief overview

- **Primordial Black Holes (PBHs)** [**Zeldovich & Novikov** (1967), **Hawking** (1971)] could form from the collapse of cosmological perturbation during the radiation dominated era.

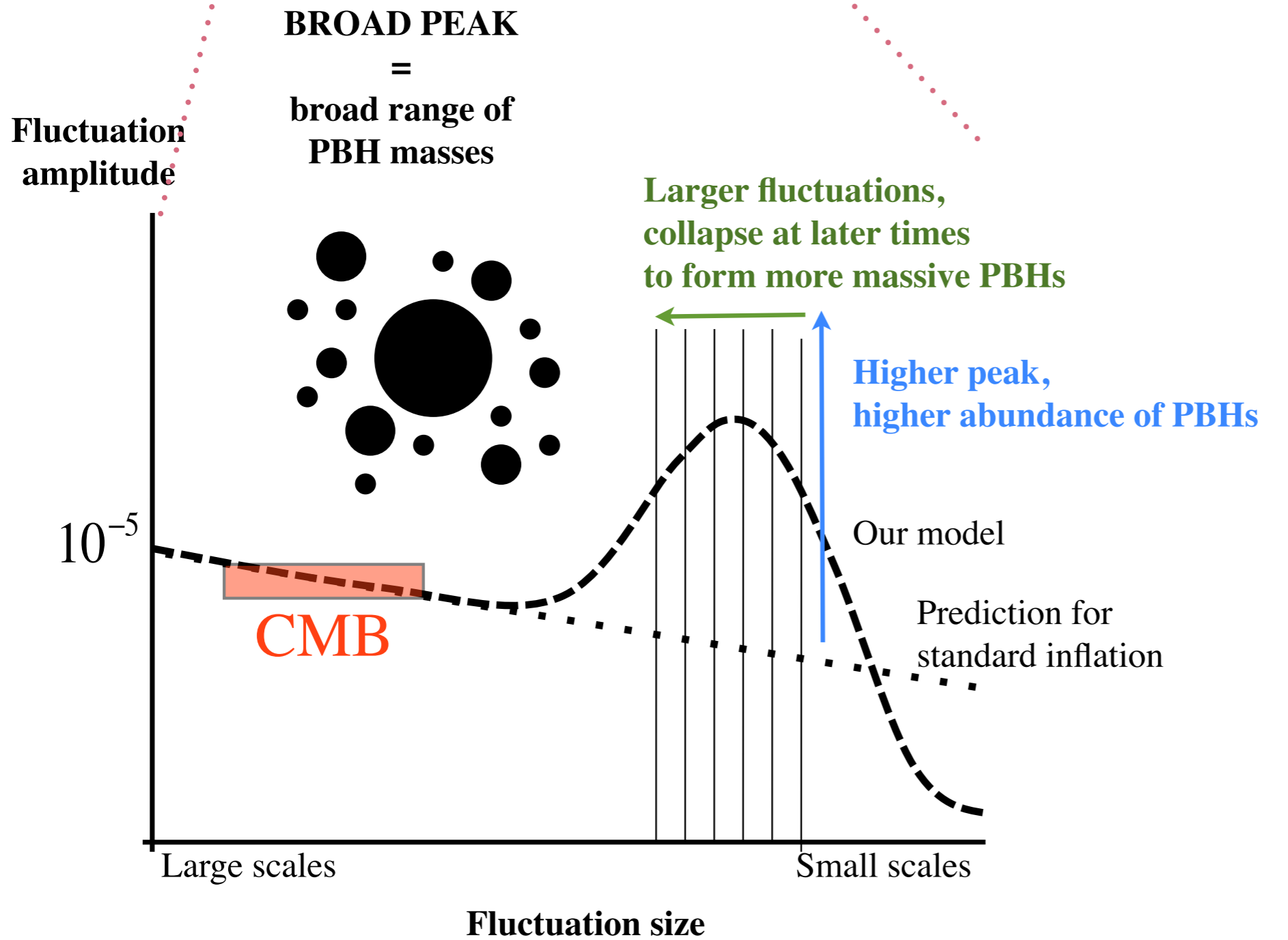
$$p = \frac{\rho}{3}$$

- PBHs could span a large wide range of masses and if not evaporated [BH evaporation **Hawking** (1974)]: PBHs with $M > 10^{15} g$ are interesting candidates for dark matter, intermediate mass black holes and the seeds of supermassive black holes.
- **Numerical hydrodynamical simulations in spherical symmetry** of a cosmological perturbation, characterized by an amplitude δ , have shown:
 - $\delta > \delta_c \Rightarrow$ PBH formation
 - $\delta < \delta_c \Rightarrow$ perturbation bounce
 - $\delta_c \sim c_s^2 \equiv \frac{\partial p}{\partial \rho}$ (**Carr 1975**)



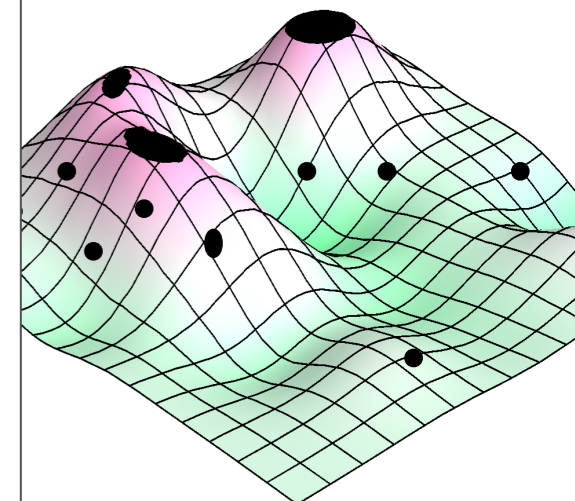


Spectrum of density fluctuations after inflation



Primordial Black hole formation

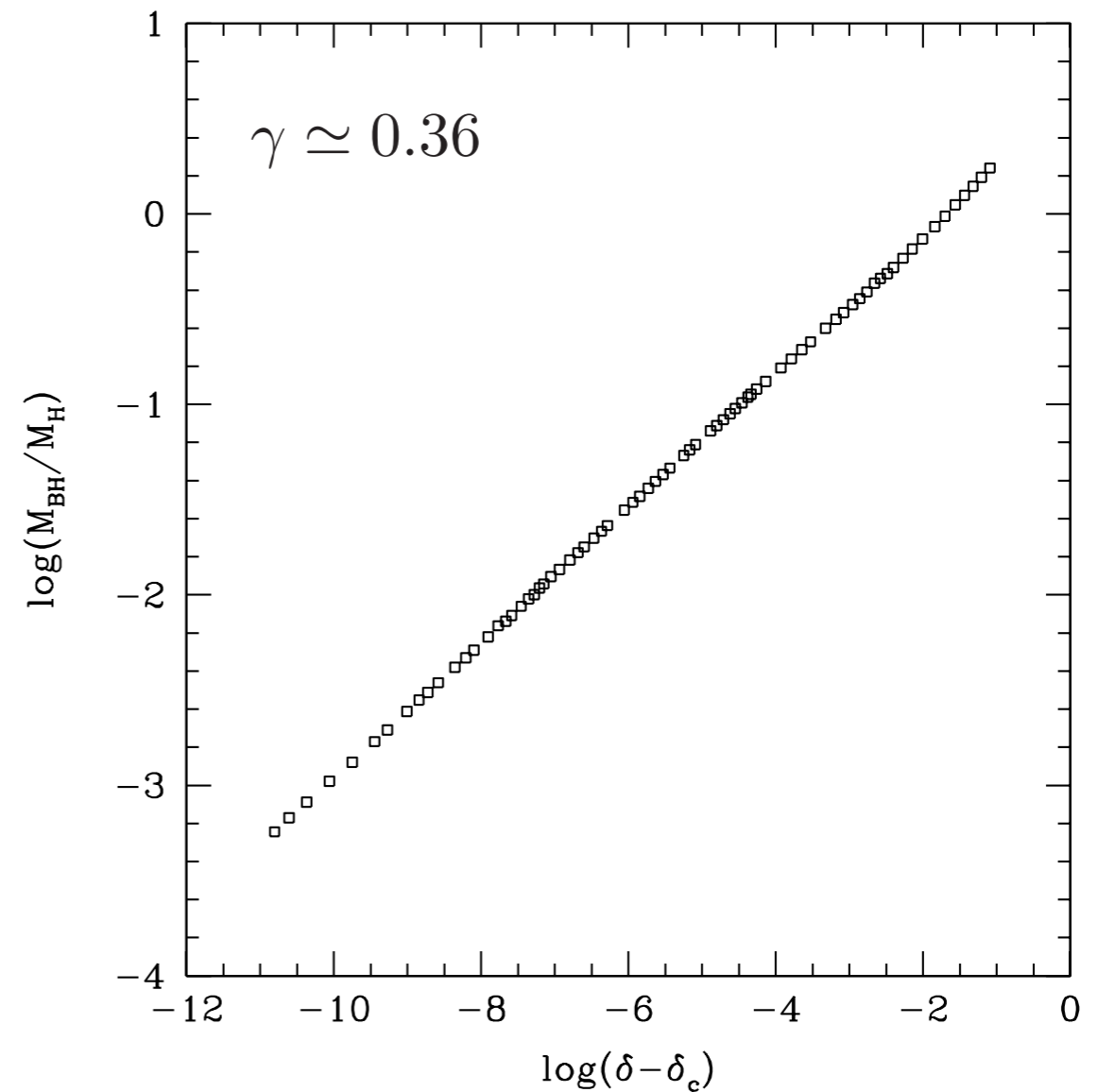
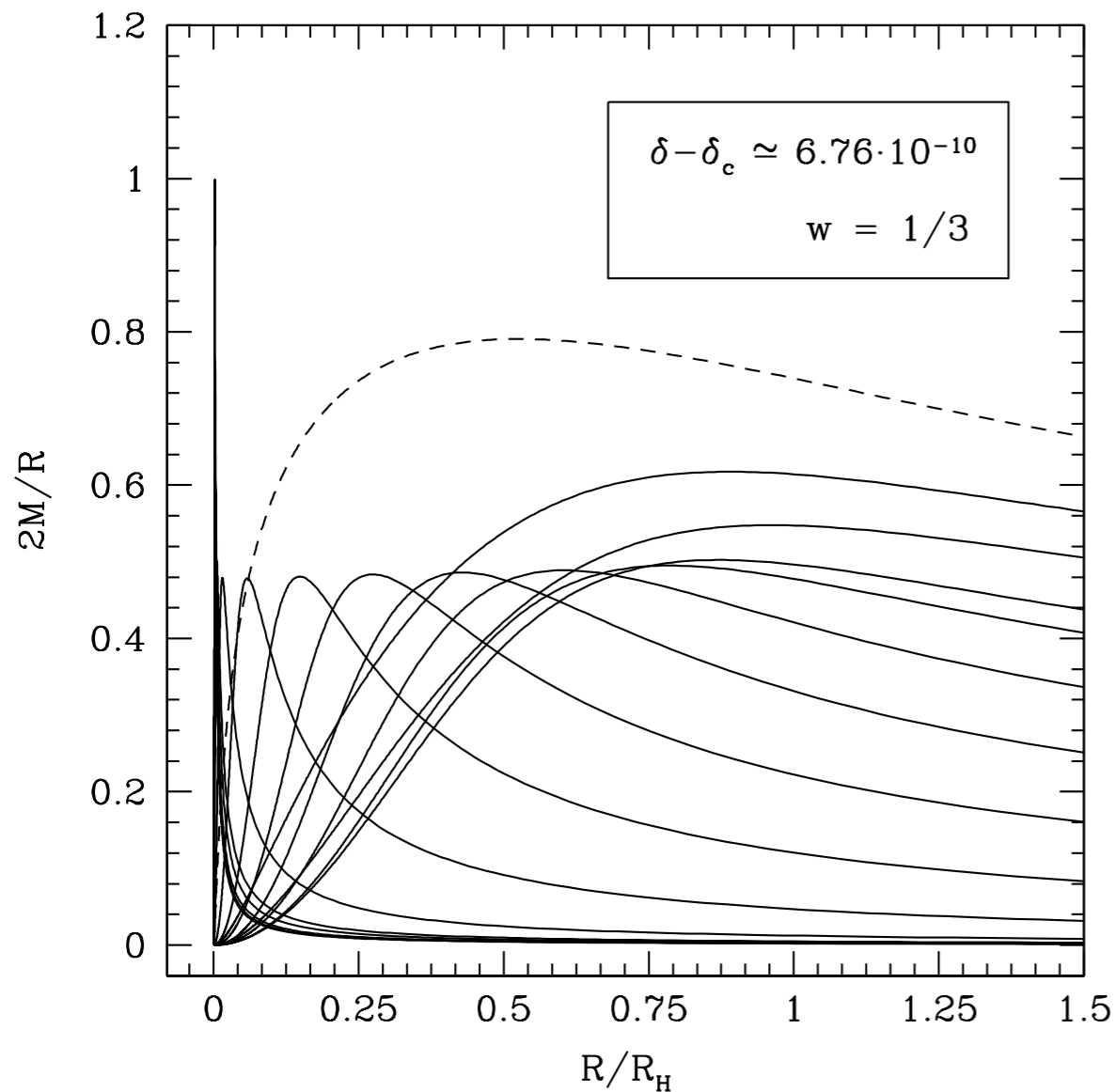
Large density fluctuations collapse later and form more massive PBHs



Numerical Results: PBH formation / mass spectrum

$$R(r, t) = 2M(r, t)$$

$$M_{PBH} = \mathcal{K}(\delta - \delta_c)^\gamma M_H$$



\mathcal{K}, δ_c — shape dependent

Initial conditions: curvature profile

- The asymptotic metric ($t \rightarrow 0$), describing super-horizon cosmological perturbations in the comoving synchronous gauge can be written as:

$$ds^2 \simeq -dt^2 + a^2(t)e^{2\zeta(r)} [dr^2 + r^2 d\Omega^2]$$

- In the “linear regime” of cosmological perturbations, adiabatic perturbations on super horizon scales can be described by a time independent curvature profile using the quasi-homogeneous / gradient expansion approach.

$$\frac{\delta\rho}{\rho_b} = - \left(\frac{1}{aH} \right)^2 \frac{4}{9} \left[\nabla^2 \zeta(r) + \frac{1}{2} (\nabla \zeta(r))^2 \right] e^{-2\zeta(r)}$$

- The perturbation amplitude δ is measured by the peak of the compaction function, corresponding to the excess of mass of the over density.

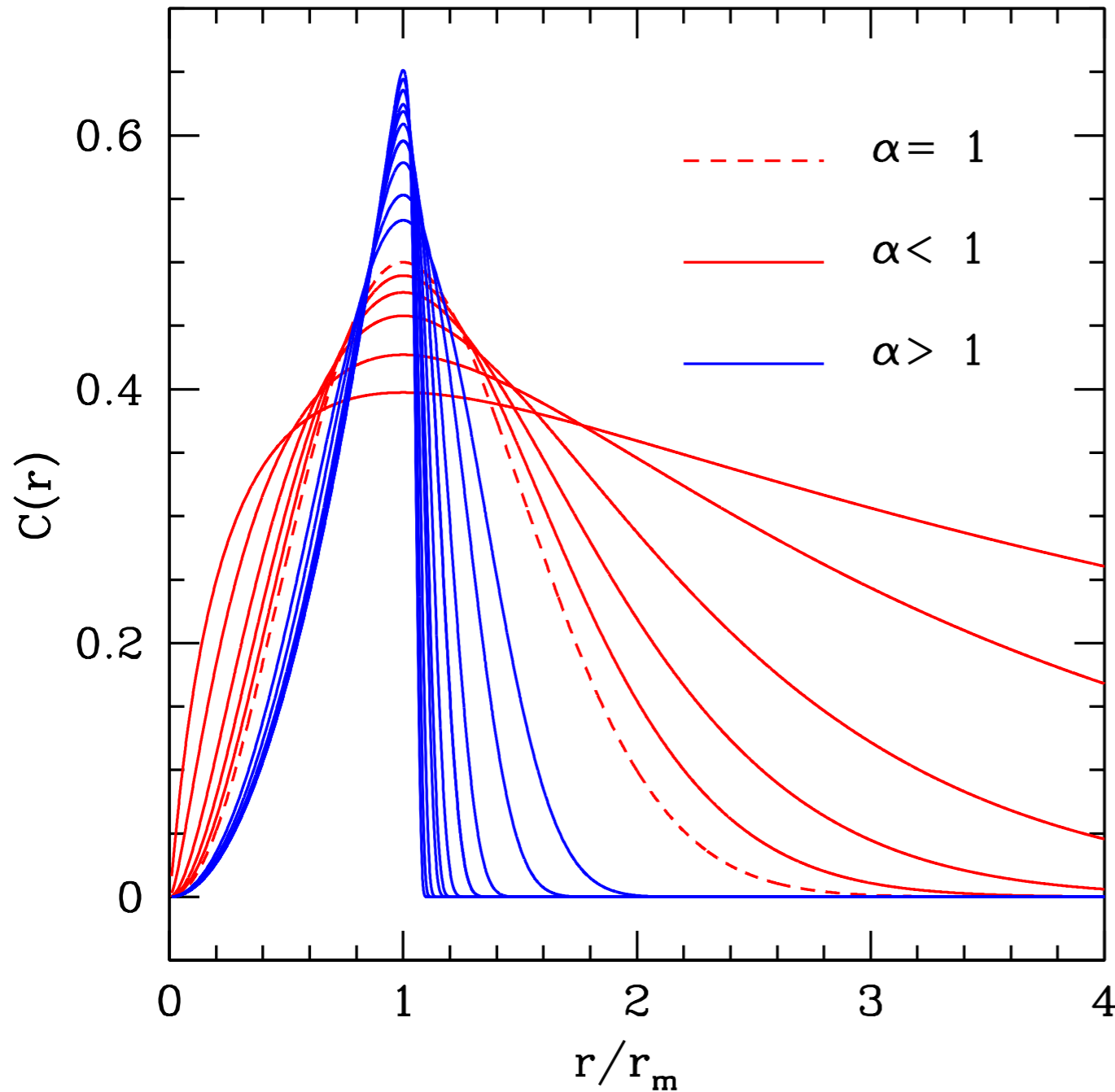
$$\mathcal{C}(r) := \frac{2[M(r, t) - M_b(r, t)]}{R(r, t)} = -\frac{4}{3} \tilde{r} \zeta'(r) \left[1 + \frac{1}{2} \tilde{r} \zeta'(r) \right] \Rightarrow \delta = \delta_G \left[1 - \frac{3}{8} \delta_G \right]$$

Shape parameter

I. Musco - PRD (2019)

$$\mathcal{C}'(r_m) = 0, \quad \Phi_m \equiv -r_m \zeta'(r_m)$$

$$\delta(r_m, t_H) = 3 \frac{\delta \rho}{\rho_b}(r_m, t_H)$$



$$\alpha \equiv -\frac{\mathcal{C}''(r_m)r_m^2}{4\mathcal{C}(r_m)} = \frac{\alpha_G}{(1 - \frac{1}{2}\Phi_m)(1 - \Phi_m)}$$

$$0.4 \leq \delta_c(\alpha) \leq \frac{2}{3}$$

$$\delta_c \simeq \begin{cases} \alpha^{0.047} - 0.50 & 0.1 \lesssim \alpha \lesssim 7 \\ \alpha^{0.035} - 0.475 & 7 \lesssim \alpha \lesssim 13 \\ \alpha^{0.026} - 0.45 & 13 \lesssim \alpha \lesssim 30 \end{cases}$$

IM, De Luca, Franciolini, Riotto - PRD (2021)

PBH Abundance (Peak Theory)

C.Germani, IM - PRL (2019)

• PDF of δ follows a Gaussian distribution:
$$P(\delta) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{\delta^2}{2\sigma^2}\right)$$

$$\sigma^2 = \langle \delta^2 \rangle = \int_0^\infty \frac{dk}{k} \mathcal{P}_\delta(k, r_m) = \left(\frac{2}{3}\right)^4 \int_0^\infty \frac{dk}{k} (kr_m)^4 T^2(k, r_m) W^2(k, r_m) \mathcal{P}_\zeta(k)$$

$$\beta_f \simeq \sqrt{\frac{2}{\pi}} \mathcal{K} \left(\frac{k_*}{a_m H_m} \right)^3 \sigma^\gamma \nu_c^{1-\gamma} \gamma^{\gamma+1/2} e^{-\frac{\nu_c^2}{2}} \quad \nu_c \equiv \frac{\delta_c}{\sigma}$$

• **If** $M_{PBH} \sim 10^{16} g$ are Dark Matter $\Rightarrow \beta_f \simeq 10^{-8} \sqrt{\frac{M_{PBH}}{M_\odot}} \simeq 10^{-16}$

• **Narrow peak:** $\frac{k_*}{\sigma} \gg 1 \Rightarrow \nu_c \simeq 0.22 \sqrt{\frac{k_*}{\sigma \mathcal{P}_0}} \Rightarrow \mathcal{P}_0 \sim 7 \times 10^{-4} \frac{k_*}{\sigma} \gg 10^{-3}$

• **Broad peak:** $\frac{k_*}{\sigma} \ll 1 \Rightarrow \nu_c \simeq 0.46 (\mathcal{P}_0)^{-1/2} \Rightarrow \mathcal{P}_0 \sim 3 \times 10^{-3}$

• **Non linear effects:** $\delta = \delta_G \left[1 - \frac{3}{8} \delta_G \right] \Rightarrow 1.5 \lesssim \frac{\mathcal{P}_{0NL}}{\mathcal{P}_{0L}} = \frac{16 \left(1 - \sqrt{1 - \frac{3}{2} \delta_c} \right)^2}{9 \delta_c^2} \lesssim 4$

S.Young, IM, C.Byrnes JCAP (2019)

De Luca, Franciolini, Kehagias, Peloso, Riotto and Unal (2019)

PBH threshold prescription

Curvature power spectrum \mathcal{P}_ζ



Characteristic overdensity scale $k_* \hat{r}_m$



Characteristic shape parameter α



Threshold δ_c

IM, De Luca, Franciolini, Riotto - PRD (2021)

1. **The power spectrum of the curvature perturbation:** take the primordial power spectrum \mathcal{P}_ζ of the Gaussian curvature perturbation and compute, on superhorizon scales, its convolution with the transfer function $T(k, \eta)$

$$P_\zeta(k, \eta) = \frac{2\pi^2}{k^3} \mathcal{P}_\zeta(k) T^2(k, \eta).$$

2. **The comoving length scale \hat{r}_m** of the perturbation is related to the characteristic scale k_* of the power spectrum P_ζ . Compute the value of $k_* \hat{r}_m$ by solving the following integral equation

$$\int dk k^2 \left[(k^2 \hat{r}_m^2 - 1) \frac{\sin(k \hat{r}_m)}{k \hat{r}_m} + \cos(k \hat{r}_m) \right] P_\zeta(k, \eta) = 0.$$

3. **The shape parameter:** compute the corresponding shape parameter α of the collapsing perturbation, including the correction from the non linear effects, by solving the following equation

$$F(\alpha) [1 + F(\alpha)] \alpha = -\frac{1}{2} \left[1 + \hat{r}_m \frac{\int dk k^4 \cos(k \hat{r}_m) P_\zeta(k, \eta)}{\int dk k^3 \sin(k \hat{r}_m) P_\zeta(k, \eta)} \right]$$

$$F(\alpha) = \sqrt{1 - \frac{2}{5} e^{-1/\alpha} \frac{\alpha^{1-5/2\alpha}}{\Gamma(\frac{5}{2\alpha}) - \Gamma(\frac{5}{2\alpha}, \frac{1}{\alpha})}}.$$

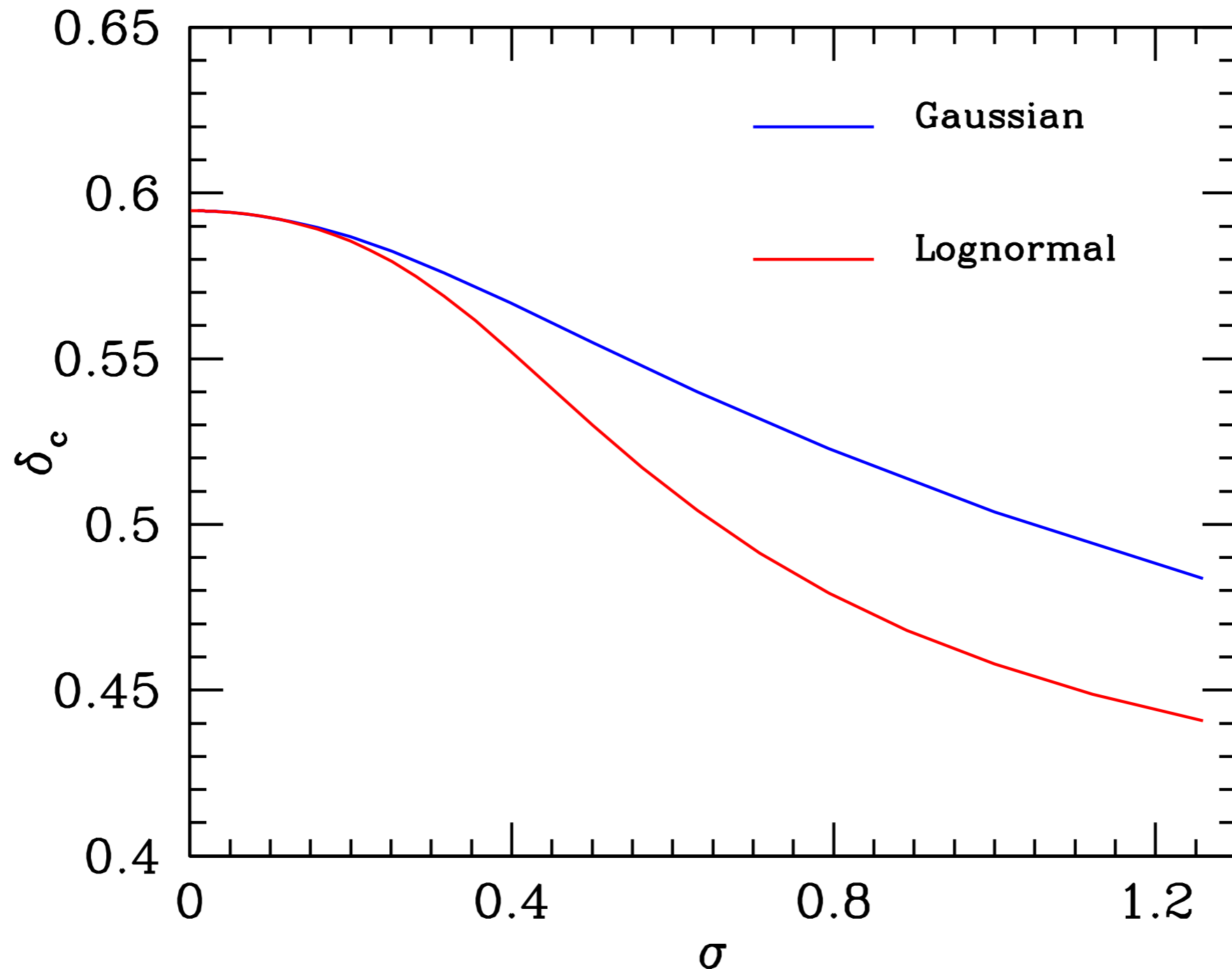
4. **The threshold δ_c :** compute the threshold as function of α , fitting the numerical simulations, at *superhorizon scales*, making a linear extrapolation at horizon crossing ($aHr_m = 1$).

$$\delta_c \simeq \begin{cases} \alpha^{0.047} - 0.50 & 0.1 \lesssim \alpha \lesssim 7 \\ \alpha^{0.035} - 0.475 & 7 \lesssim \alpha \lesssim 13 \\ \alpha^{0.026} - 0.45 & 13 \lesssim \alpha \lesssim 30 \end{cases}$$

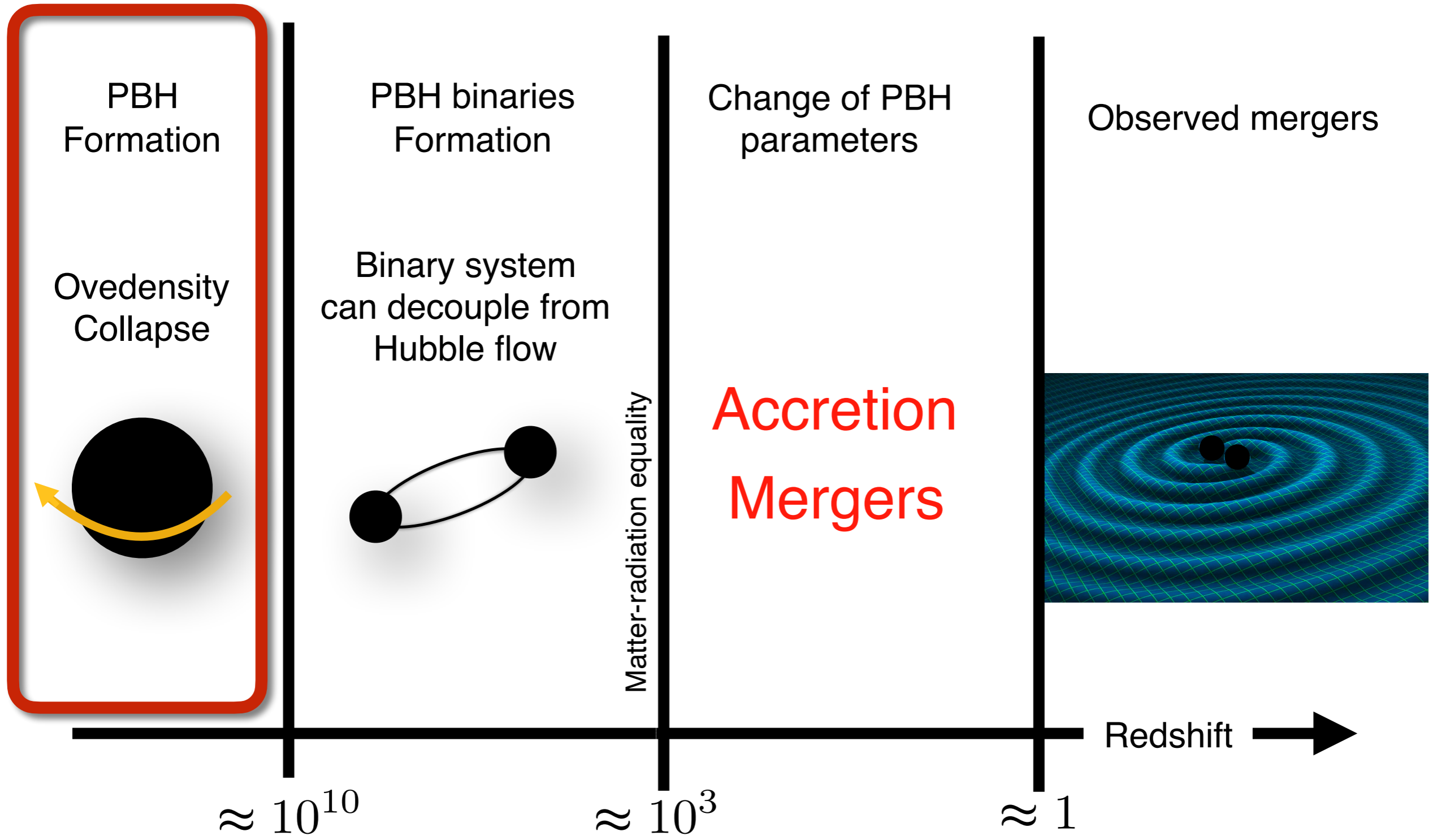
Power Spectrum:

Gaussian: $\mathcal{P}_\zeta(k) = \mathcal{P}_0 \exp [-(k - k_*)^2 / 2\sigma^2]$

Lognormal: $\mathcal{P}_\zeta(k) = \mathcal{P}_0 \exp [-\ln^2 (k/k_*) / 2\sigma^2]$



PBH evolution



Equation of State of the Early Universe

The early Universe goes through 3 main transitions before matter-radiation equality:

- Electroweak phase-transition
- **QCD phase-transition** (crossover)

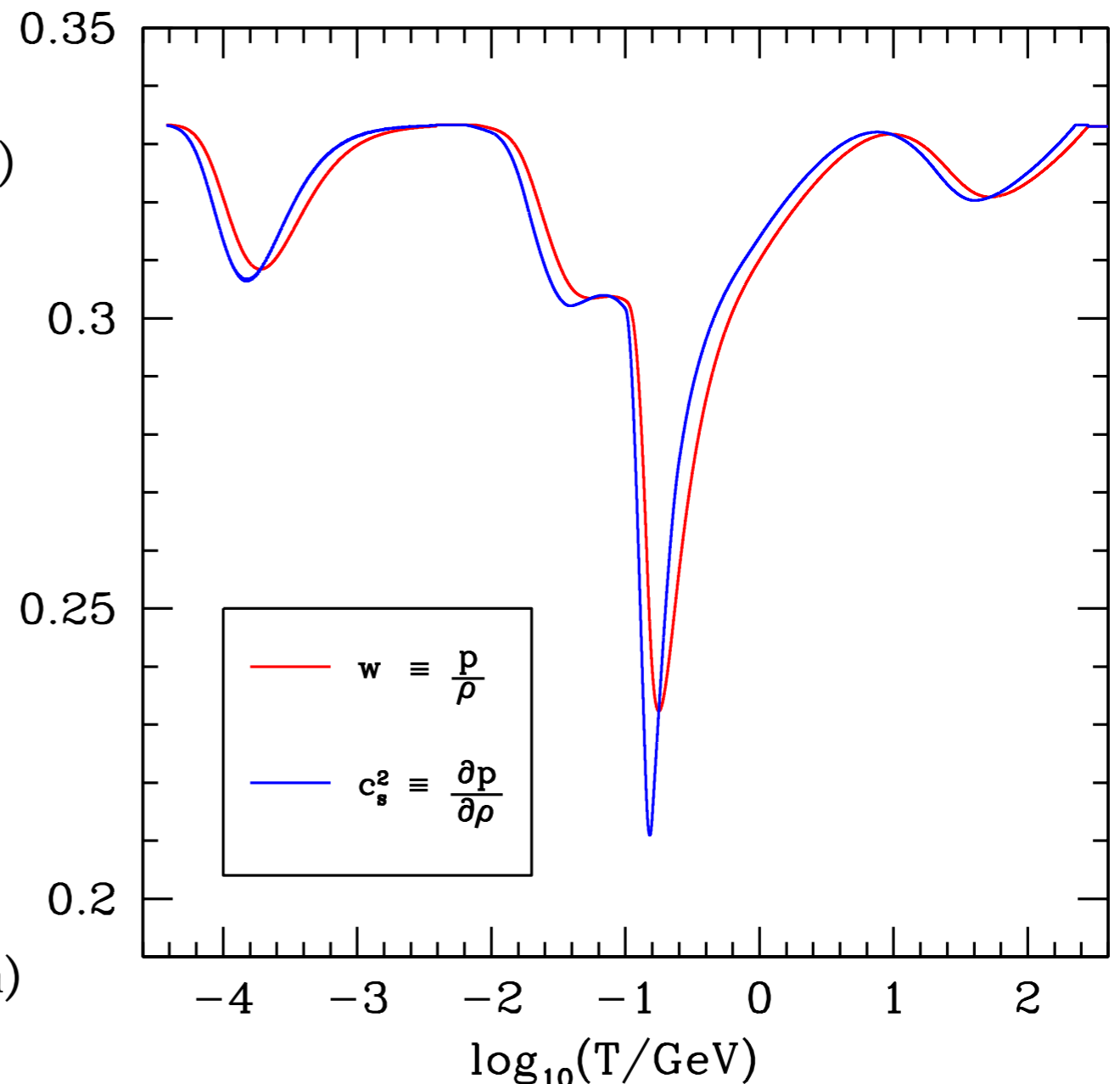
$$\rho = \frac{\pi^2}{30} g_{\text{eff}}(T) T^4 \quad s = \frac{2\pi^2}{45} h_{\text{eff}}(T) T^3$$

$$p = sT - \rho = w(T)\rho$$

$$w(T) = \frac{4h_{\text{eff}}(T)}{3g_{\text{eff}}(T)} - 1$$

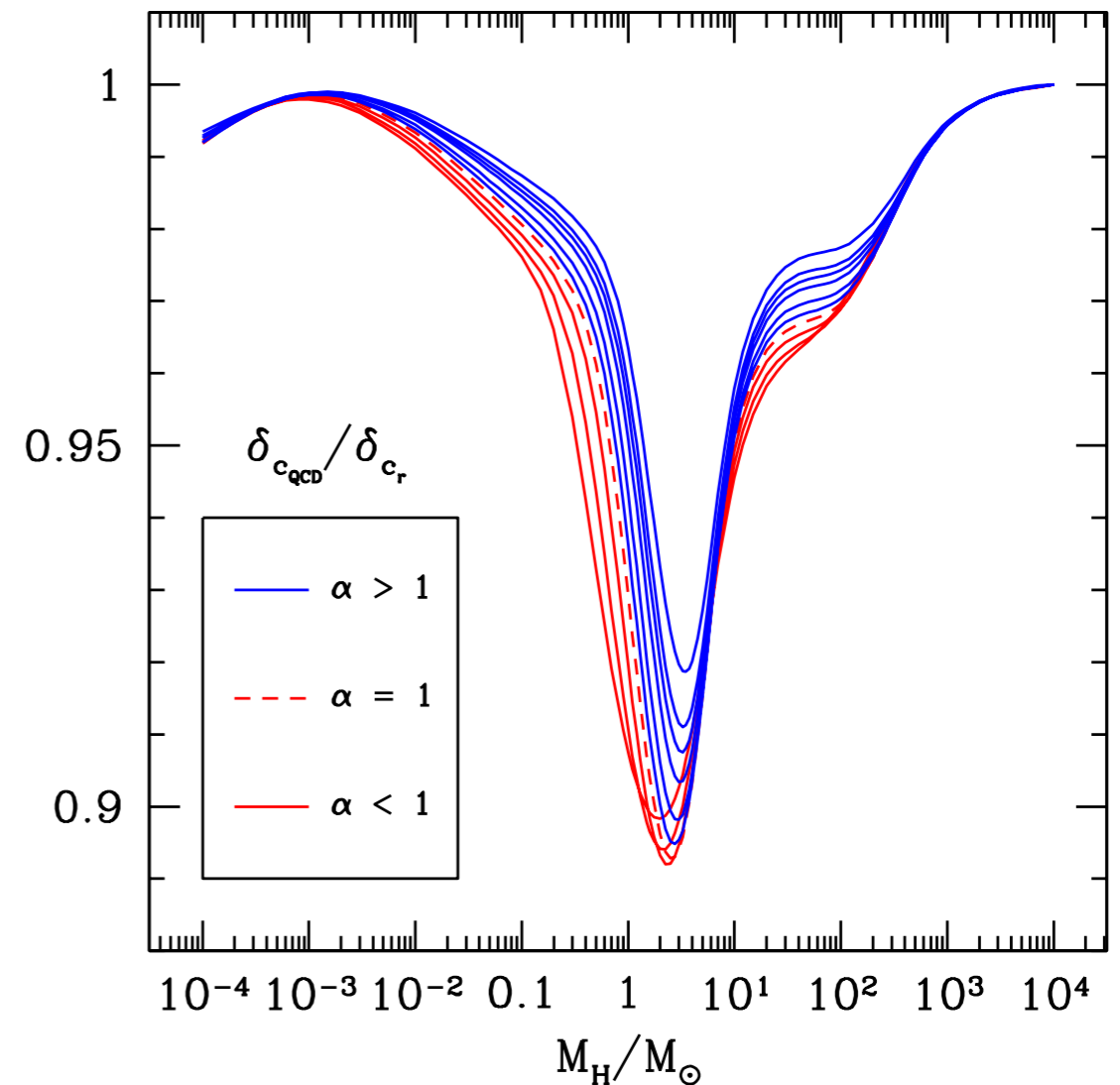
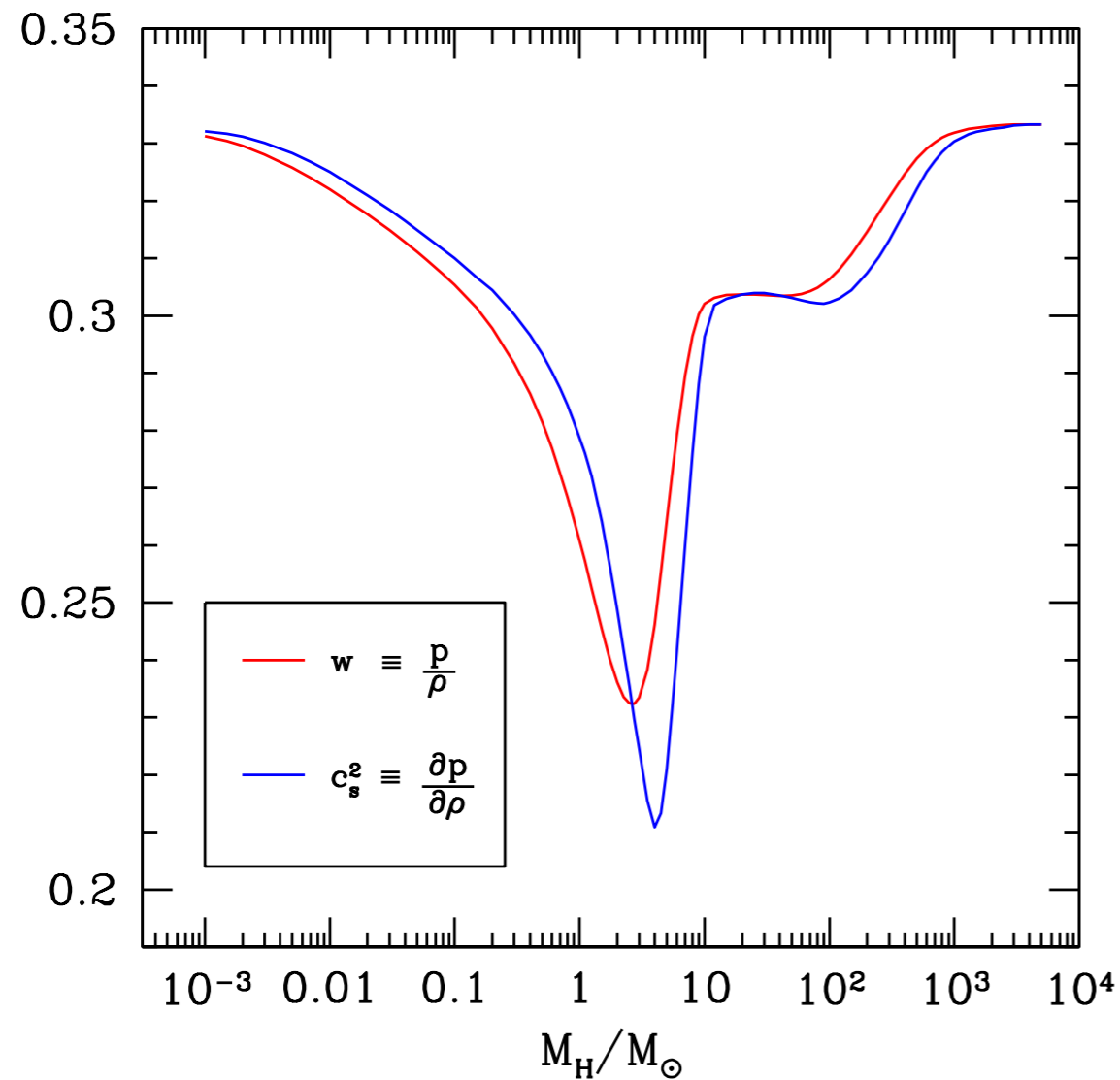
$$c_s^2(T) = \frac{\partial p}{\partial \rho} = \frac{4(4h_{\text{eff}} + Th'_{\text{eff}})}{3(4g_{\text{eff}} + Tg'_{\text{eff}})} - 1$$

- Nucleosynthesis (e+e- annihilation)



PBH Threshold during the QCD

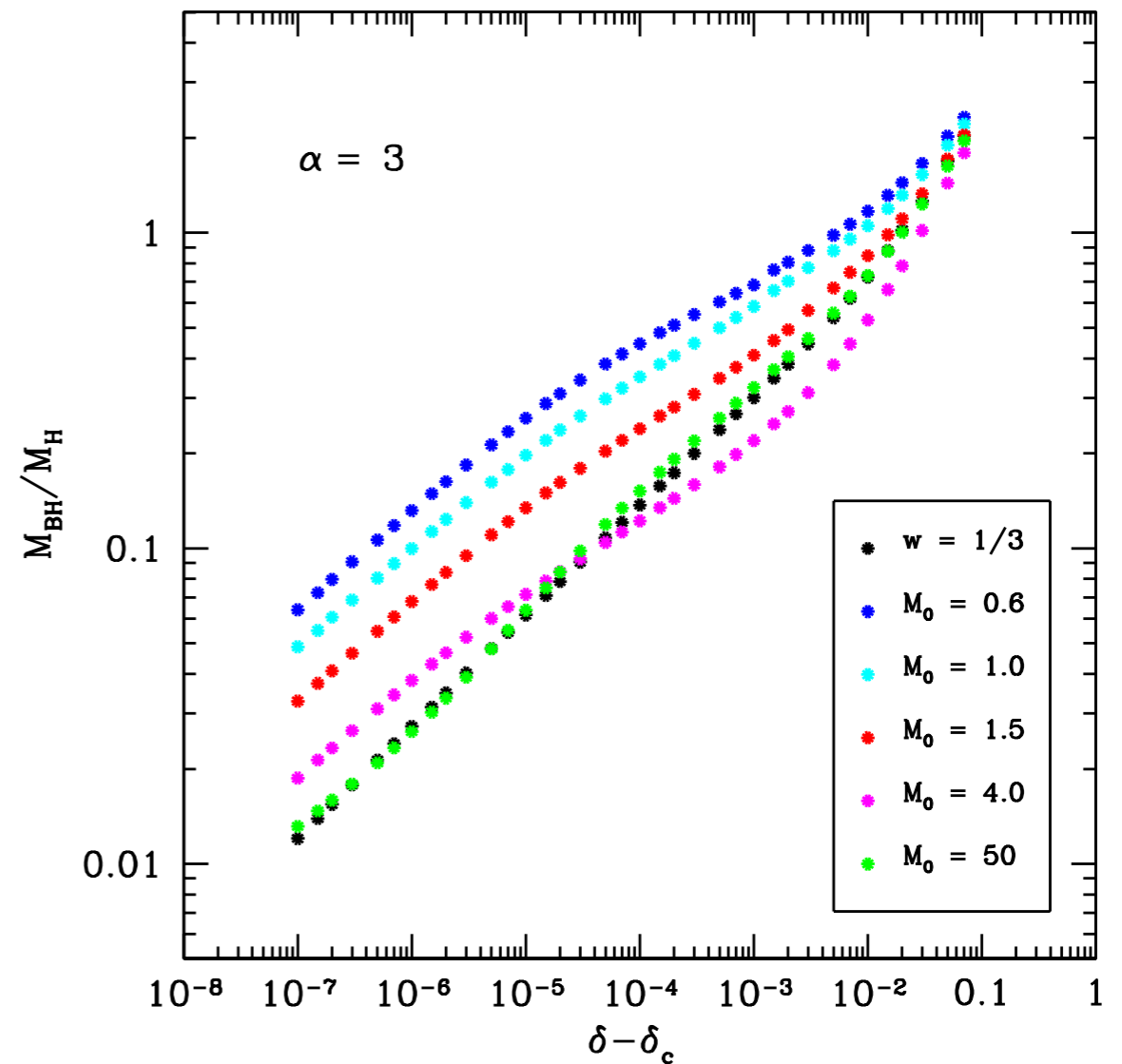
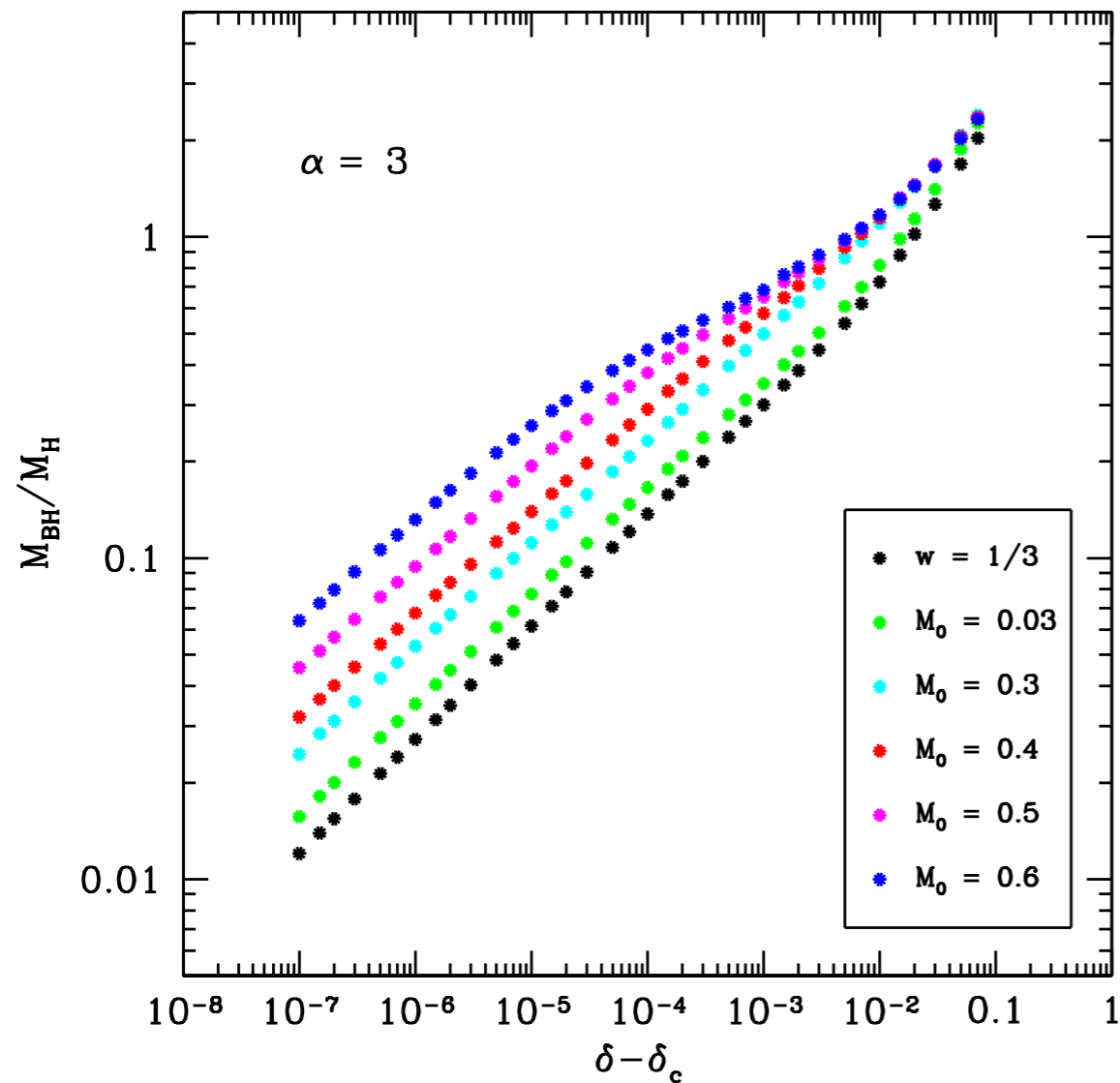
IM, K. Jedamzik, Sam Young - arXiv:2303.07980



Depending on the shape, the threshold for PBH formation during the QCD phase transition is reduced about 10% around the minimum of $w(T)$.

Significant enhancement of PBH formation around the solar mass scale: abundance increased of about $O(3)$ with respect radiation!

PBH scaling law during the QCD



$$M_{PBH} = \mathcal{K}(\delta - \delta_c)^\gamma M_H$$

$$\delta_c(M_H), \gamma(M_H), \mathcal{K}(M_H)$$

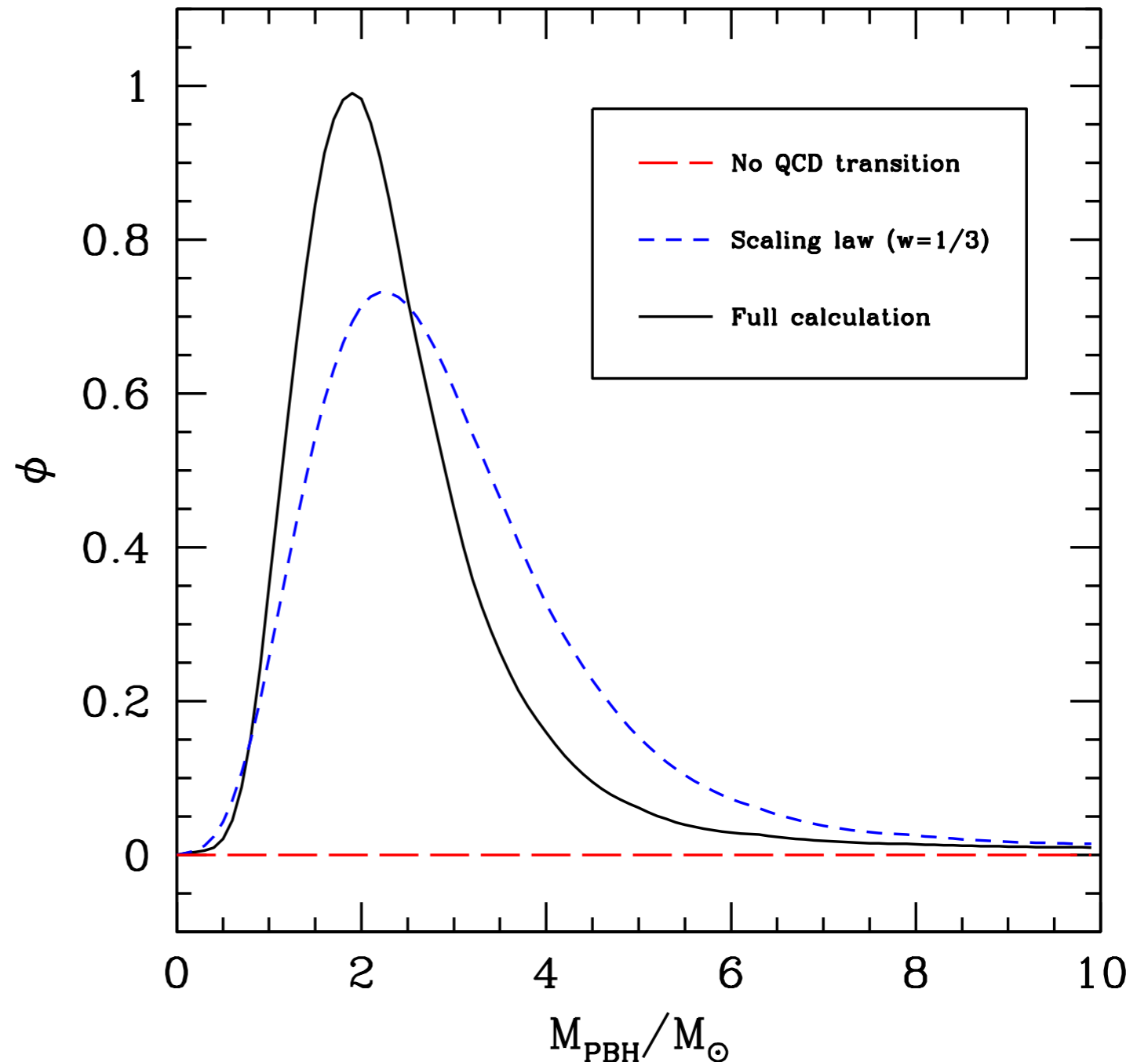
PBH mass function during the QCD

Mass Function $\psi(m_{\text{PBH}})$: fraction of PBHs with mass in the infinitesimal interval of M_{PBH}

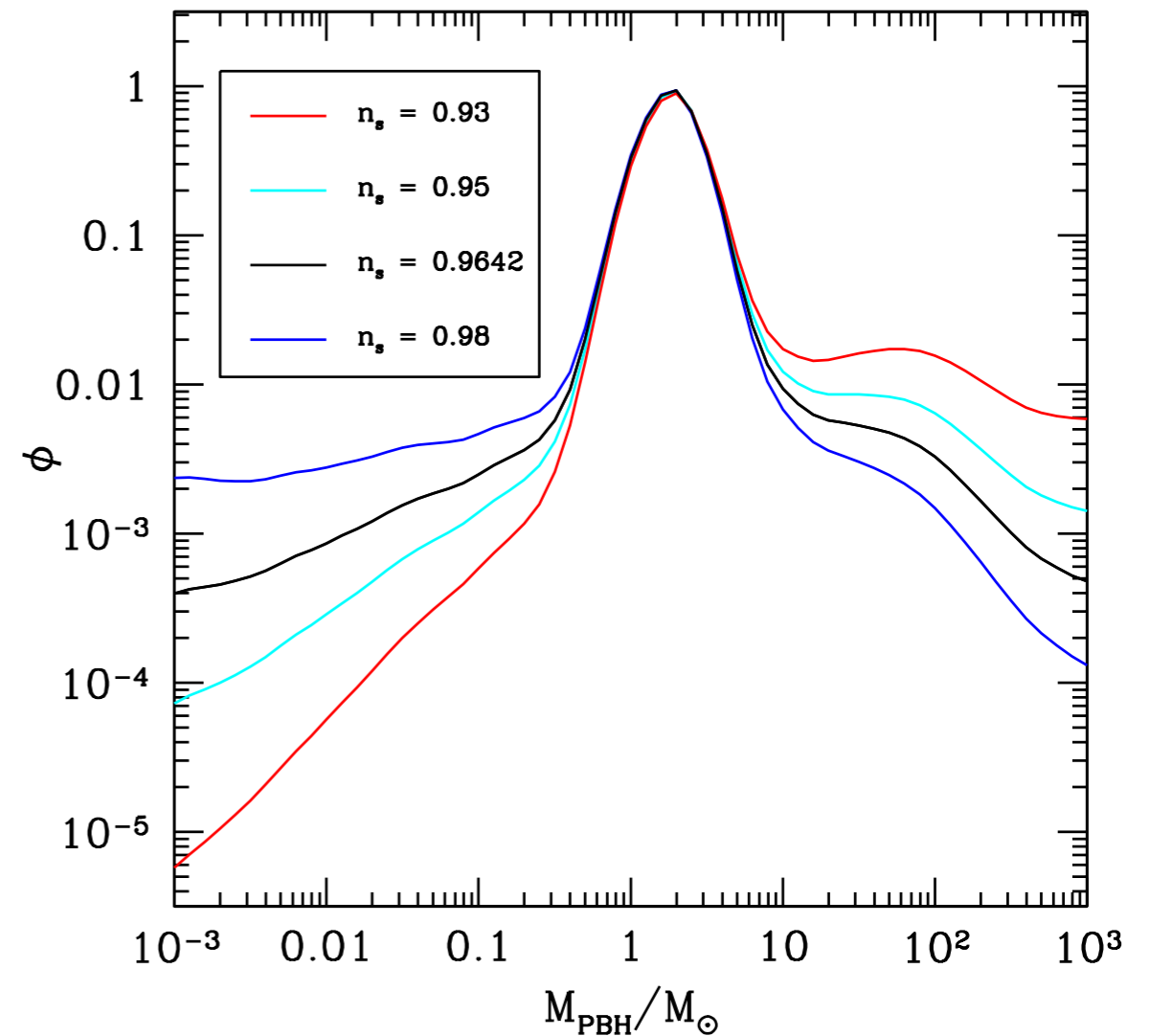
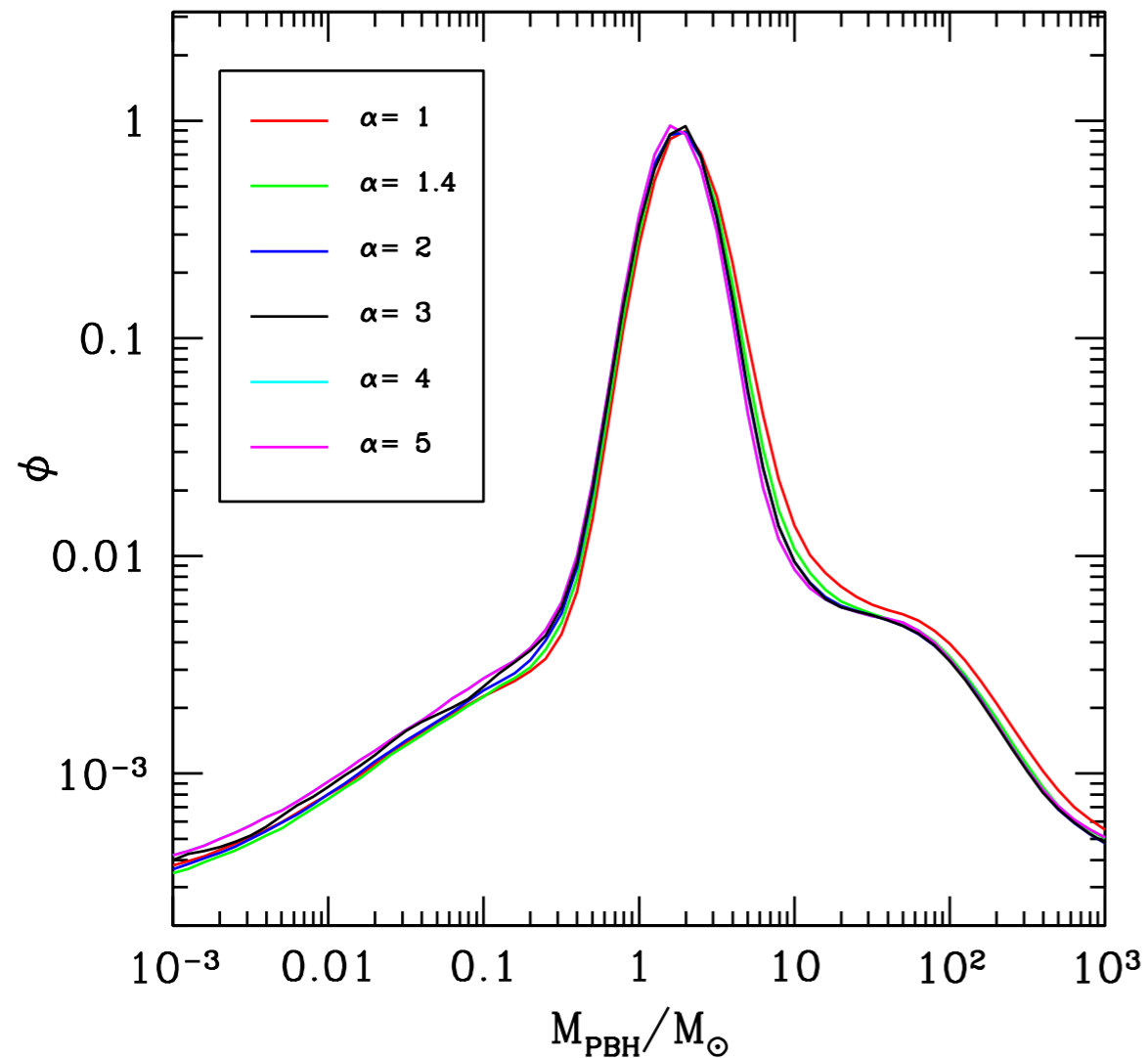
$$\phi(M_{\text{PBH}}) = \frac{1}{\Omega_{\text{PBH}}} \frac{d\Omega_{\text{PBH}}}{dM_{\text{PBH}}}$$

$$\int dm_{\text{PBH}} \phi(m_{\text{PBH}}) = 1$$

- The main effect is given by the modification of the threshold.
- The modified scaling law gives a pile up of PBHs on smaller masses.



PBH mass function during the QCD: shape/tilt dependence

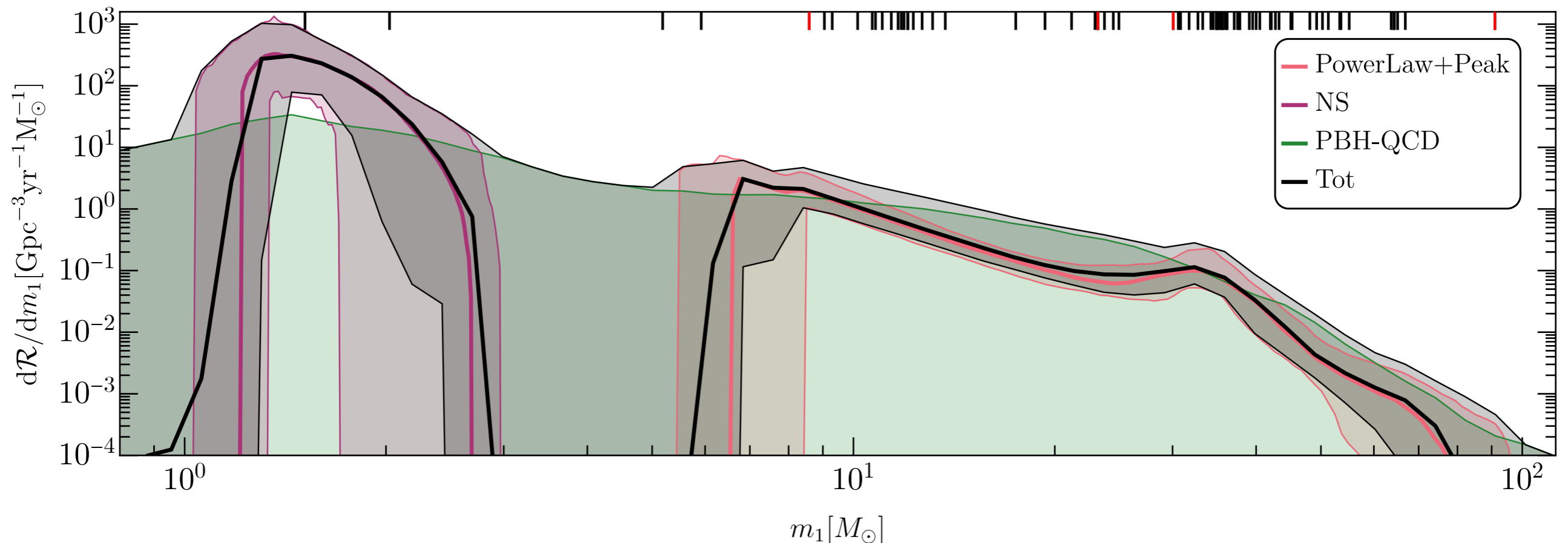


- Given the PBH abundance, the shape does not play a significant role on the mass function (attractor solution)!
- The tilt of the power spectrum does not affect the peak of the mass function.

GWs from PBH mergers

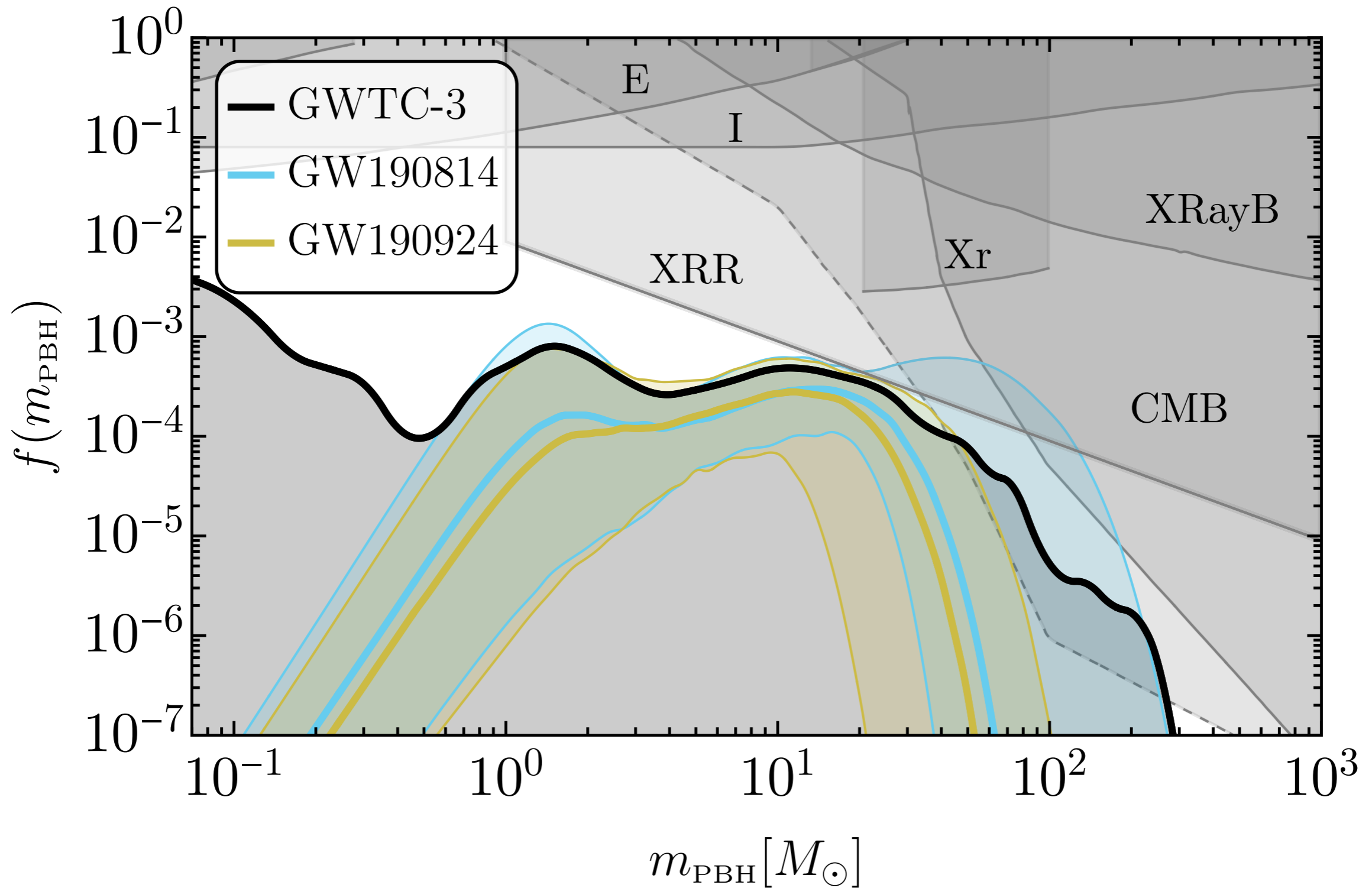
G. Franciolini, IM, P.Pani, A urbano - PRD (2022)

- Making **Bayesian inference analysis** we found that a sub-population of PBHs is compatible with the LVK catalog.
- PBHs give a natural explanation for the events in with BH mass gap: in particular GW190814 falling within the lower mass gap (predictions for O4 and O5).

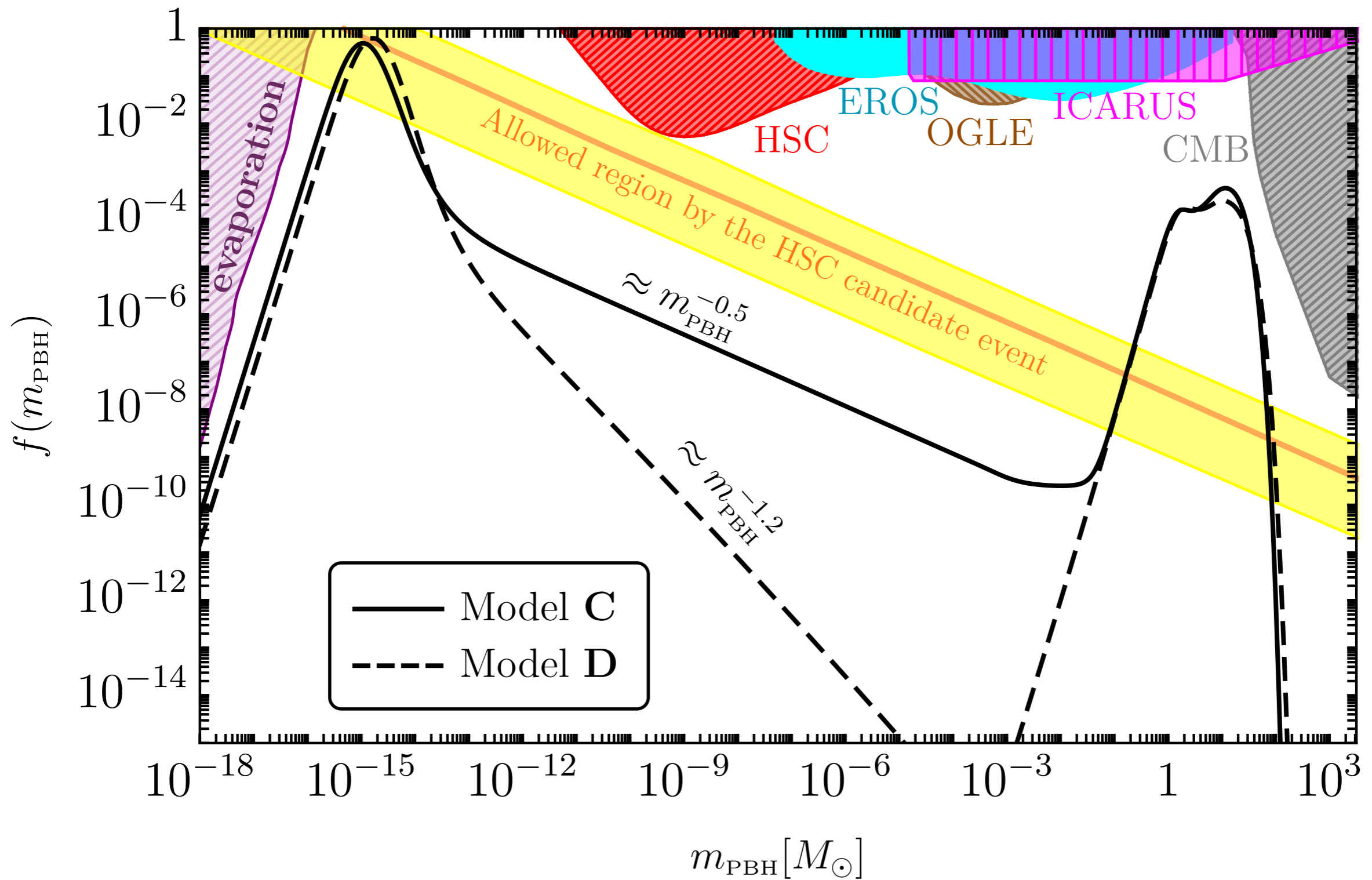


PBH constraints

G. Franciolini, IM, P.Pani, A urbano - arXiv:2209.05959



PBH and Dark Matter



Conclusions

- The non linear threshold for PBH and the mass spectrum could be fully computed from the shape of the power spectrum of cosmological perturbations, making relativistic numerical simulations.
- A softening of the equation of state (QCD) significantly enhances the formation of PBHs, with a mass distribution peaked between 1 and 2 solar masses (the range of heavy NSs and light BHs).
- This could give a sub-population of BH mergers compatible with the LVK catalog, explaining mass gap events as GW190814.
- Our analysis predicts a constraint on the abundance of DM in PBHs formed during the QCD (up to 0.1%), compatible with the current observational constraints.
- A large enough feature of the power spectrum could account for all dark matter in PBHs in the asteroidal mass range (USR inflation models).