Opportunities with B_c semileptonic decays

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LHCb workshop on semileptonic exclusive b → c decays

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based on works in collaboration with
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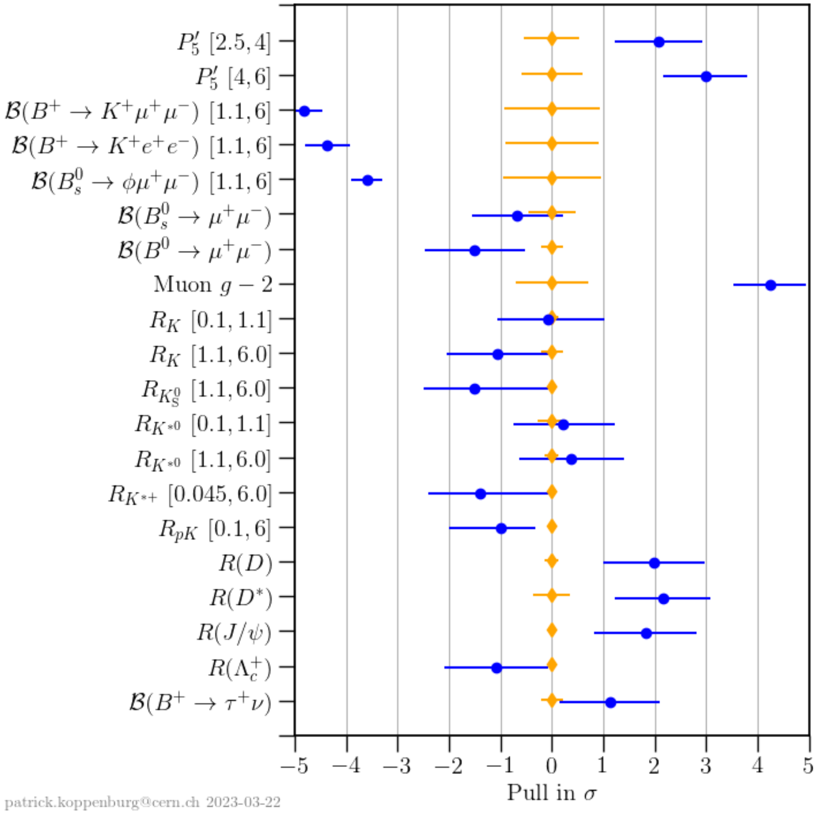
Outline

- Semileptonic $b \to c$ transitions and B_c decays: motivations
- Spin symmetry + NRQCD : relations among FF in the SM and BSM
- Application to $B_c \to J/\psi$ and $B_c \to \eta_c$ form factors
- Application to B_c to P-wave charmonia and insights on X(3872)
- Other semileptonic B_c decays: $c \to s, d$ transitions
- Summary



- ➢ b → c transitions

- Precisely measure $|V_{cb}|$:
  insights on the tension from inclusive/exclusive determinations

- Anomalies shown up in modes induced by b →c $\ell\,\nu_\ell$ transition

  Look for new modes/observables/correlations

- ➢ other quark-level transitions (e.g. c → s,d)

- do anomalies show up?

  Look for new modes/observables/correlations

Semileptonic B_c decays

B_c

- discovered at Tevatron in 1998
- m_{Bc}=6.274.47 +/- 0.27 +/- 0.17 GeV
- τ_{Bc}=0.510 +/- 0.009 ps
- decays weakly
- possible modes: annihilation, b transitions, c transitions (dominant)

Motivations:

1. explore BSM effects
2. $B_c \to$ charmonium: probe the structure of the charmonia produced in the decay

control of theoretical uncertainties in phenomenological analyses requires reliable determination of the hadronic form factors

possibility to exploit NRQCD methods + HQ spin symmetry

Explore BSM effects: SMEFT $\longrightarrow$ systematic extension of the SM

- NP exists at a high scale $\Lambda >> M_W$
- NP gauge group $G \supset SU(3)_C \times SU(2)_L \times U(1)_Y$
- SM gauge fields contained
- SM an effective theory at the scale M_W

Buchmuller et al,NPB 268 (1986) 621
Grzadkowski et al., JHEP 10 (2010) 085

Weinberg operator: ν oscillations

$$\mathcal{L}_{\rm SM} = \mathcal{L}_{\rm SM}^{(4)} + \frac{1}{\Lambda} C_{\nu\nu}^{(5)} Q_{\nu\nu}^{(5)} + \frac{1}{\Lambda^2} \sum_k C_k^{(6)} Q_k^{(6)} + \mathcal{O}\left(\frac{1}{\Lambda^3}\right)$$

NP

$$\mathcal{L}^{kin} + \mathcal{L}^{gauge} + \mathcal{L}^{Higgs} + \mathcal{L}^{Yukawa}$$

accidental symmetries

- violates accidental symmetries
- source of (SM) CP violation
- fermion mass terms

Semileptonic B_c decays: b → c transition

Generalized effective Hamiltonian

SM

$$H_{\text{eff}}^{b\to c\ell\bar{\nu}} = \frac{G_F}{\sqrt{2}} V_{cb} \Big[(1 + \epsilon_V^\ell) \left(\bar{c}\gamma_\mu (1-\gamma_5) b\right) \left(\bar{\ell}\gamma^\mu (1-\gamma_5)\nu_\ell\right)$$
$$+ \epsilon_R^\ell \left(\bar{c}\gamma_\mu (1+\gamma_5) b\right) \left(\bar{\ell}\gamma^\mu (1-\gamma_5)\nu_\ell\right)$$
$$+ \epsilon_S^\ell \left(\bar{c} b\right) \left(\bar{\ell}(1-\gamma_5)\nu_\ell\right)$$
$$+ \epsilon_P^\ell \left(\bar{c}\gamma_5 b\right) \left(\bar{\ell}(1-\gamma_5)\nu_\ell\right)$$
$$+ \epsilon_T^\ell \left(\bar{c}\sigma_{\mu\nu} (1-\gamma_5) b\right) \left(\bar{\ell}\sigma^{\mu\nu} (1-\gamma_5)\nu_\ell\right) \Big]$$

complex
lepton flavour dependent couplings

Semileptonic B_c decays: $b \to c$ transition

Generalized effective Hamiltonian

SM

$$
\begin{aligned}
H_{\rm eff}^{b\to c\ell\bar\nu} = \frac{G_F}{\sqrt{2}} V_{cb}\Big[& (1+\epsilon_V^\ell)\,(\bar c\gamma_\mu(1-\gamma_5)b)\,\big(\bar\ell\gamma^\mu(1-\gamma_5)\nu_\ell\big) \\
& + \epsilon_R^\ell\,(\bar c\gamma_\mu(1+\gamma_5)b)\,\big(\bar\ell\gamma^\mu(1-\gamma_5)\nu_\ell\big) \\
& + \epsilon_S^\ell\,(\bar c b)\,\big(\bar\ell(1-\gamma_5)\nu_\ell\big) \\
& + \epsilon_P^\ell\,(\bar c\gamma_5 b)\,\big(\bar\ell(1-\gamma_5)\nu_\ell\big) \\
& + \epsilon_T^\ell\,(\bar c\sigma_{\mu\nu}(1-\gamma_5)b)\,\big(\bar\ell\sigma^{\mu\nu}(1-\gamma_5)\nu_\ell\big)\Big]
\end{aligned}
$$

larger set of form factors required wrt the SM case

complex
lepton flavour dependent couplings

Semileptonic B_c decays to charmonium

- $B_c \to \eta_c$, J/ψ 1S-wave charmonia $J^{PC}=(0^{--},1^{--})$

Motivations:

1. explore BSM effects

Semileptonic B_c decays to charmonium

- $B_c \to \eta_c$, J/ψ 1S-wave charmonia $J^{PC}=(0^{--},1^{--})$
- $B_c \to \chi_{c0}, \chi_{c1}, \chi_{c2}$, h_c 1P-wave charmonia $J^{PC}=(0^{++}, 1^{++}, 2^{++}, 1^{+-})$
- $B_c \to \chi'_{c0}, \chi'_{c1}, \chi'_{c2}$, h'_c 2P-wave charmonia $J^{PC}=(0^{++}, 1^{++}, 2^{++}, 1^{+-})$

Motivations:

1. explore BSM effects

2. $B_c \to$ charmonium: probe the structure of the charmonia produced in the decay

$\to$ question: can X(3872) be identified with $\chi_{c1}(2P)$?

A few details on X(3872)

X(3872)

- discovered by Belle in 2003, confirmed by CDF, D0, BaBar,...
- in 2015 LHCb: $J^P=1^{++}$ ➡ candidate for identification with $\chi_{c1}(2P)$
- other possible interpretations
  - tetraquark
  - D D^* molecule (proximity to the threshold)
  - ....
- isospin violation disfavours the charmonium interpretation (but phase space suppression is at work)
- the preference of $\psi(2S)\,\gamma$ wrt $J/\psi\,\gamma$ favours the interpretation as $\chi_{c1}(2P)$

look for further information:

does X(3872) fulfill the expectations for the production of $\chi_{c1}(2P)$ in semileptonic B_c decays?

Semileptonic B_c decays to charmonium

SM

NP

$B_c \to \eta_c$:

$$\langle \eta_c(v')|\bar{Q}'\gamma_\mu Q|B_c(v)\rangle = \sqrt{m_P\, m_{B_c}}\,[h_+(w)\,(v+v')_\mu + h_-(w)\,(v-v')_\mu]$$

$$\langle \eta_c(v')|\bar{Q}'Q|B_c(v)\rangle = \sqrt{m_P\, m_{B_c}}\,h_S(w)(1+w)$$

$$\langle \eta_c(v')|\bar{Q}'\sigma_{\mu\nu}Q|B_c(v)\rangle = -i\,\sqrt{m_P\, m_{B_c}}\,h_T(w)\,(v_\mu\, v'_\nu - v_\nu\, v'_\mu)$$

$$w = v\cdot v'$$

$B_c \to J/\psi$:

$$\langle J/\psi(v',\epsilon)|\bar{Q}'\gamma_\mu Q|B_c(v)\rangle = i\,\sqrt{m_V\, m_{B_c}}\,h_V(w)\,\epsilon_{\mu\nu\alpha\beta}\,\epsilon^{*\nu}\,v'^\alpha\,v^\beta$$

$$\langle J/\psi(v',\epsilon)|\bar{Q}'\gamma_\mu\gamma_5\, Q|B_c(v)\rangle = \sqrt{m_V\, m_{B_c}}\,[h_{A_1}(w)\,(1+w)\,\epsilon^*_\mu - h_{A_2}(w)\,(\epsilon^*\cdot v)\,v_\mu - h_{A_3}(w)\,(\epsilon^*\cdot v)\,v'_\mu]$$

$$\langle J/\psi(v',\epsilon)|\bar{Q}'\gamma_5 Q|B_c(v)\rangle = -\sqrt{m_V\, m_{B_c}}\,h_P(w)\,(\epsilon^*\cdot v)$$

$$\langle J/\psi(v',\epsilon)|\bar{Q}'\sigma_{\mu\nu}Q|B_c(v)\rangle = -\sqrt{m_V\, m_{B_c}}\,\epsilon^{\mu\nu\alpha\beta}\,[h_{T_1}(w)\epsilon^*_\alpha(v+v')_\beta + h_{T_2}(w)\epsilon^*_\alpha(v-v')_\beta + h_{T_3}(w)(\epsilon^*\cdot v)v_\alpha v'_\beta]$$

Semileptonic B_c decays to charmonium

$B_c \to \chi_{c0}$:

$$\langle \chi_{c0}(v')|\bar{c}\gamma_\mu\gamma_5 b|B_c(v)\rangle = \sqrt{m_{\chi_{c0}}\, m_{B_c}}\, [g_+(w)(v+v')_\mu + g_-(w)(v-v')_\mu]$$

$$\langle \chi_{c0}(v')|\bar{c}\gamma_5 b|B_c(v)\rangle = \sqrt{m_{\chi_{c0}}\, m_{B_c}}\, g_P(w)$$

$$\langle \chi_{c0}(v')|\bar{c}\sigma_{\mu\nu} b|B_c(v)\rangle = \sqrt{m_{\chi_{c0}}\, m_{B_c}}\, g_T(w)\, \epsilon_{\mu\nu\alpha\beta} v^\alpha v'^\beta$$

$B_c \to h_c$:

$$\langle h_c(v',\epsilon)|\bar{c}\gamma_\mu b|B_c(v)\rangle = \sqrt{m_{h_c}\, m_{B_c}}\, \Big[f_{V_1}(w)\epsilon^*_\mu + (\epsilon^*\cdot v)\big(f_{V_2}(w)(v+v')_\mu + f_{V_3}(w)(v-v')_\mu\big)\Big]$$

$$\langle h_c(v',\epsilon)|\bar{c}\gamma_\mu\gamma_5 b|B_c(v)\rangle = i\sqrt{m_{h_c}\, m_{B_c}}\, f_A(w)\, \epsilon_{\mu\alpha\beta\sigma}\epsilon^{*\alpha} v^\beta v'^\sigma$$

$$\langle h_c(v',\epsilon)|\bar{c} b|B_c(v)\rangle = \sqrt{m_{h_c}\, m_{B_c}}\, (\epsilon^*\cdot v) f_S(w)$$

$$\langle h_c(v',\epsilon)|\bar{c}\sigma_{\mu\nu} b|B_c(v)\rangle = i\sqrt{m_{h_c}\, m_{B_c}}\, \Big[f_{T_1}(w)\big(\epsilon^*_\mu(v+v')_\nu - \epsilon^*_\nu(v+v')_\mu\big) + f_{T_2}(w)\big(\epsilon^*_\mu(v-v')_\nu - \epsilon^*_\nu(v-v')_\mu\big) + f_{T3}(w)\,(\epsilon^*\cdot v)\,(v_\mu v'_\nu - v_\nu v'_\mu)\Big].$$

$B_c \to \chi_{c1}$:

$$\langle \chi_{c1}(v',\epsilon)|\bar{c}\gamma_\mu b|B_c(v)\rangle = i\sqrt{m_{\chi_{c1}}\, m_{B_c}}\, \Big[g_{V_1}(w)\epsilon^*_\mu + (\epsilon^*\cdot v)[g_{V_2}(w)(v+v')_\mu + g_{V_3}(w)(v-v')_\mu]\Big]$$

$$\langle \chi_{c1}(v',\epsilon)|\bar{c}\gamma_\mu\gamma_5 b|B_c(v)\rangle = \sqrt{m_{\chi_{c1}}\, m_{B_c}}\, g_A(w)\, \epsilon_{\mu\alpha\beta\sigma}\epsilon^{*\alpha} v^\beta v'^\sigma$$

$$\langle \chi_{c1}(v',\epsilon)|\bar{c} b|B_c(v)\rangle = i\sqrt{m_{\chi_{c1}}\, m_{B_c}}\, g_S(w)\, (\epsilon^*\cdot v)$$

$$\langle \chi_{c1}(v',\epsilon)|\bar{c}\sigma_{\mu\nu} b|B_c(v)\rangle = \sqrt{m_{\chi_{c1}}\, m_{B_c}}\, \Big[g_{T_1}(w)\big(\epsilon^*_\mu(v+v')_\nu - \epsilon^*_\nu(v+v')_\mu\big) + g_{T_2}(w)\big(\epsilon^*_\mu(v-v')_\nu - \epsilon^*_\nu(v-v')_\mu\big) + g_{T_3}(w)(\epsilon^*\cdot v)(v_\mu v'_\nu - v_\nu v'_\mu)\Big]$$

$B_c \to \chi_{c2}$:

$$\langle \chi_{c2}(v',\eta)|\bar{c}\gamma_\mu b|B_c(v)\rangle = \sqrt{m_{\chi_{c2}}\, m_{B_c}}\, i\, k_V(w)\, \epsilon_{\mu\alpha\beta\sigma}\eta^{*\alpha\tau} v_\tau v^\beta v'^\sigma$$

$$\langle \chi_{c2}(v',\eta)|\bar{c}\gamma_\mu\gamma_5 b|B_c(v)\rangle = \sqrt{m_{\chi_{c2}}\, m_{B_c}}\, \left[k_{A_1}(w)\, \eta^*_{\mu\alpha} v^\alpha + \eta^*_{\alpha\beta} v^\alpha v^\beta \left(k_{A_2}(w) v_\mu + k_{A_3}(w) v'_\mu \right)\right]$$

$$\langle \chi_{c2}(v',\eta)|\bar{c}\gamma_5 b|B_c(v)\rangle = \sqrt{m_{\chi_{c2}}\, m_{B_c}}\, k_P(w)\, \eta^*_{\alpha\beta} v^\alpha v^\beta$$

$$\langle \chi_{c2}(v',\eta)|\bar{c}\sigma_{\mu\nu}\gamma_5 b|B_c(v)\rangle = i\sqrt{m_{\chi_{c2}}\, m_{B_c}}\, \big[k_{T_1}(w)(\eta^{*\alpha}_{\ \mu} v_\alpha v_\nu - \eta^{*\alpha}_{\ \nu} v_\alpha v_\mu) + k_{T_2}(w)(\eta^{*\alpha}_{\ \mu} v_\alpha v'_\nu - \eta^{*\alpha}_{\ \nu} v_\alpha v'_\mu) + k_{T_3}(w)\eta^*_{\alpha\beta} v^\alpha v^\beta (v_\mu v'_\nu - v_\nu v'_\mu)\big]$$

SM

NP

HQ spin symmetry in B_c decays

HQ limit: decoupling of the HQ

- Heavy-light mesons → HQ spin & flavour symmetry
- Heavy-heavy mesons → HQ spin symmetry

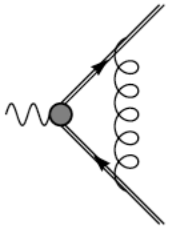
relations among the FF in selected kinematical ranges

Well known example:

FF of weak matrix elements between heavy-light mesons are all described by the Isgur-Wise function

Less known case:

Heavy-heavy meson decays



→ IR divergent for 2 HQs with the same v

- Infrared divergences regulated in the HQ limit by the kinetic energy operator O_π
- O_π breaks flavour symmetry → only spin symmetry

Thacker and Lepage, PRD43 (1991) 196

Systems with heavy quarks: effective theories at work

Heavy-Light mesons:

HQET

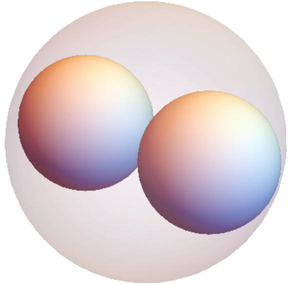
$p_Q = m_Q v + k$ $\quad$ $k \simeq \mathcal{O}(\Lambda_{QCD}) \ll m_Q$

residual momentum
HQET Lagrangian: expansion in k/m_Q

N. Isgur, M.B. Wise, PLB 232 (89) 113
PLB 237 (90) 527

Heavy-Heavy mesons:

NRQCD



non relativistic quarks
relative velocity $\tilde{v}$

NRQCD Lagrangian: expansion in $1/m_Q$
terms further organized: expansion in powers of $\tilde{v}$

different power counting

W.E. Caswell, G.P. Lepage, PLB 167 (86) 437
G.T. Bodwin, E. Braaten, G.P. Lepage, PRD51 (95) 1125

Systems with heavy quarks: effective theories at work

- expansion parameters for a system with 2 Heavy Quarks:
  1. relative HQ 3-velocity (hadron rest-frame) (NRQCD)
  2. inverse HQ mass $1/m_Q$ (HQET)

- HQ field:

$$Q(x) = e^{-im_Q v\cdot x}\psi(x) = e^{-im_Q v\cdot x}\Big(\psi_+(x) + \psi_-(x)\Big)$$

$$\psi_\pm(x) = P_\pm \psi(x) = \frac{1 \pm \not{v}}{2}\psi(x)$$

$$\Rightarrow \quad Q(x) = e^{-im_Q v\cdot x}\left(1 + \frac{i\not{D}_\perp}{2m_Q} + \frac{(-iv\cdot D)}{2m_Q}\frac{i\not{D}_\perp}{2m_Q} + \dots\right)\psi_+(x)$$

$$D_{\perp\mu} = D_\mu - (v\cdot D)v_\mu$$

$$\Rightarrow \quad \mathcal{L}_{QCD} = \bar{\psi}_+(x)\left(iv\cdot D + \frac{(iD_\perp)^2}{2m_Q} + \frac{g}{4m_Q}\sigma\cdot G_\perp + \frac{i\not{D}_\perp}{2m_Q}\frac{(-iv\cdot D)}{2m_Q}(i\not{D}_\perp) + \dots\right)\psi_+(x)$$

Systems with heavy quarks: effective theories at work

power counting in NRQCD

$$\psi_+ \sim \tilde{v}^{3/2}$$

$$D_\perp \sim \tilde{v} \qquad D_t \sim \tilde{v}^2$$

$$E_i = G_{0i} \sim \tilde{v}^3 \qquad B_i = \frac{1}{2}\epsilon_{ijk}G^{jk} \sim \tilde{v}^4$$

$$\mathcal{L}_{QCD} = \bar{\psi}_+(x)\left(iv\cdot D + \frac{(iD_\perp)^2}{2m_Q} + \frac{g}{4m_Q}\sigma\cdot G_\perp + \frac{i\not{D}_\perp}{2m_Q}\frac{(-iv\cdot D)}{2m_Q}(i\not{D}_\perp) + \dots\right)\psi_+(x)$$

$\mathcal{O}(\tilde{v}^2)$ LO

$\mathcal{O}(\tilde{v}^4)$ NLO

$$\mathcal{L}_0 = \bar{\psi}_+(x)\left(iv\cdot D + \frac{(iD_\perp)^2}{2m_Q}\right)\psi_+(x)$$

$$\mathcal{L}_1 = \mathcal{L}_{1,1} + \mathcal{L}_{1,2}$$

Form Factors in the effective theory

$$\langle C|\bar{Q}'\Gamma Q|B_c\rangle$$

$C = \eta_c,\ J/\psi$ $\qquad$ $C = \chi_{c0},\ \chi_{c1},\ \chi_{c2},\ h_c$

I. expand the current:

$$\bar{Q}'(x)\Gamma Q(x) = J_0 + \left(\frac{J_{1,0}}{2m_Q} + \frac{J_{0,1}}{2m_{Q'}}\right) + \left(-\frac{J_{2,0}}{4m_Q^2} - \frac{J_{0,2}}{4m_{Q'}^2} + \frac{J_{1,1}}{4m_Q m_{Q'}}\right)$$

$$J_0 = \bar{\psi}'_+\Gamma\psi_+$$

$$J_{1,0} = \bar{\psi}'_+\Gamma i\overrightarrow{\not{D}}_\perp\psi_+ \qquad J_{0,1} = \bar{\psi}'_+\left(-i\overleftarrow{\not{D}}'_\perp\right)\Gamma\psi_+$$

$$J_{2,0} = \bar{\psi}'_+\Gamma\left(iv\cdot\overrightarrow{D}\right)i\overrightarrow{\not{D}}_\perp\psi_+ \qquad J_{0,2} = \bar{\psi}'_+ i\overleftarrow{\not{D}}'_\perp\left(iv'\cdot\overleftarrow{D}\right)\Gamma\psi_+ \qquad J_{1,1} = \bar{\psi}'_+\left(-i\overleftarrow{\not{D}}'_\perp\right)\Gamma\left(i\overrightarrow{\not{D}}_\perp\right)\psi_+$$

Form Factors in the effective theory

II: exploit spin symmetry:

doubלet of negative parity states: (B_c, B_c^*) → $\mathcal{M}(v) = P_+(v)\,[B_c^{*\mu}\,\gamma_\mu - B_c\,\gamma_5]\;P_-(v)$

$(\eta_c,\; J/\psi)$ → $\mathcal{M}'(v') = P_+(v')\left[\Psi^{*\mu}\gamma_\mu - \eta_c\,\gamma_5\right]P_-(v')$

4-plet of positive parity states $(\chi_{c0,1,2}, h_c)$

$$\mathcal{M}'^{\mu}(v') = P_+(v')\left[\chi_{c2}^{\mu\nu}\,\gamma_\nu + \frac{1}{\sqrt{2}}\,\chi_{c1,\gamma}\,\epsilon^{\mu\alpha\beta\gamma}\,v'_\alpha\,\gamma_\beta + \frac{1}{\sqrt{3}}\,\chi_{c0}\,(\gamma^\mu - v'^\mu) + h_c^\mu\,\gamma_5\right]P_-(v')$$

$$v'_\mu\,\mathcal{M}'^{\mu} = 0$$

analogous for 2P charmonia

Form Factors in the effective theory

III. trace formalism:

$$\langle C|\bar{Q}'\Gamma D_{\mu_1} D_{\mu_2} \ldots Q|B_c\rangle = -\mathrm{Tr}\Big[\mathcal{F}_{\mu\,\mu_1\mu_2\ldots}\bar{\mathcal{M}}'^{\mu}\Gamma\mathcal{M}\Big]$$

↓

universal functions: the same for all the members of the multiplet of final states

relations among the various modes

Form Factors in the effective theory

III. trace formalism: at LO in the HQ expansion all the matrix elements involve a single universal function

$$\langle M'(v')|J_0|M(v)\rangle = -\Xi(w)\, v_\mu\, \mathrm{Tr}\left[\overline{\mathcal{M}}'^{\mu}\, \Gamma\, \mathcal{M}\right]$$

Form Factors in the effective theory

III. trace formalism: At LO in the HQ expansion all the matrix elements involve a single universal function

$$\langle M'(v')|J_0|M(v)\rangle = -\Xi(w)\, v_\mu\, \mathrm{Tr}\left[\overline{\mathcal{M}}'^{\mu}\, \Gamma\, \mathcal{M}\right]$$

$O(1/m_Q)$

$$\langle M'(v')|\bar\psi'_+\, \Gamma\, i\, \overrightarrow{D}_\alpha\, \psi_+|M(v)\rangle = -\mathrm{Tr}\left[\Sigma^{(b)}_{\mu\alpha}\, \overline{\mathcal{M}}'^{\mu}\, \Gamma\, \mathcal{M}\right]$$

$$\langle M'(v')|\bar\psi'_+\, (-i\, \overleftarrow{D}_\alpha)\, \Gamma\, \psi_+|M(v)\rangle = -\mathrm{Tr}\left[\Sigma^{(c)}_{\mu\alpha}\, \overline{\mathcal{M}}'^{\mu}\, \Gamma\, \mathcal{M}\right]$$

$$\Sigma^{(Q)}_{\mu\alpha} = \Sigma^{(Q)}_1\, g_{\mu\alpha} + \Sigma^{(Q)}_2\, v_\mu\, v_\alpha + \Sigma^{(Q)}_3\, v_\mu\, v'_\alpha + \Sigma^{(Q)}_4\, v_\mu\, \gamma_\alpha + \Sigma^{(Q)}_5\, \gamma_\mu\, v_\alpha + \Sigma^{(Q)}_6\, \gamma_\mu\, v'_\alpha + \Sigma^{(Q)}_7\, i\, \sigma_{\mu\alpha}$$

constraints:

$$\Sigma^{(b)}_i(w) - \Sigma^{(c)}_i(w) = 0 \qquad\qquad i = 1, 4, 5, 6, 7$$

$$\Sigma^{(b)}_2(w) - \Sigma^{(c)}_2(w) = \tilde\Lambda\, \Xi\, ,$$

$$\Sigma^{(b)}_3(w) - \Sigma^{(c)}_3(w) = -\tilde\Lambda'\, \Xi(w)\, .$$

III. trace formalism: At LO in the HQ expansion all the matrix elements involve a single universal function

$$\langle M'(v')|J_0|M(v)\rangle = -\Xi(w)\,v_\mu\,\mathrm{Tr}\left[\overline{\mathcal{M}}'^{\mu}\,\Gamma\,\mathcal{M}\right]$$

$O(1/m_Q)$

$$\langle M'(v')|\bar{\psi}'_+\,\Gamma\, i\,\overrightarrow{D}_\alpha\,\psi_+|M(v)\rangle = -\mathrm{Tr}\left[\Sigma^{(b)}_{\mu\alpha}\,\overline{\mathcal{M}}'^{\mu}\,\Gamma\,\mathcal{M}\right]$$
$$\langle M'(v')|\bar{\psi}'_+\,(-i\,\overleftarrow{D}_\alpha)\,\Gamma\,\psi_+|M(v)\rangle = -\mathrm{Tr}\left[\Sigma^{(c)}_{\mu\alpha}\,\overline{\mathcal{M}}'^{\mu}\,\Gamma\,\mathcal{M}\right]$$

$O(1/m_Q)^2$

$$\langle M'(v')|\bar{\psi}'_+\,\Gamma\, i\,\overrightarrow{D}_\alpha\, i\,\overrightarrow{D}_\beta\,\psi_+|M(v)\rangle = -\mathrm{Tr}\left[\Omega^{(b)}_{\mu\alpha\beta}\,\overline{\mathcal{M}}'^{\mu}\,\Gamma\,\mathcal{M}\right]$$
$$\langle M'(v')|\bar{\psi}'_+\, i\,\overleftarrow{D}_\alpha\, i\,\overleftarrow{D}_\beta\,\Gamma\,\psi_+|M(v)\rangle = -\mathrm{Tr}\left[\Omega^{(c)}_{\mu\alpha\beta}\,\overline{\mathcal{M}}'^{\mu}\,\Gamma\,\mathcal{M}\right]$$

constraints:

$$\Omega^{(b)}_{\mu\alpha\beta} - \Omega^{(c)}_{\mu\alpha\beta} = \left(\tilde{\Lambda}\,v_\alpha - \tilde{\Lambda}'\,v'_\alpha\right)\Sigma^{(b)}_{\mu\beta} + \left(\tilde{\Lambda}\,v_\beta - \tilde{\Lambda}'\,v'_\beta\right)\Sigma^{(c)}_{\mu\alpha}$$

Form Factors in the effective theory

other corrections from the expansion of the states
(non-local corrections)

$$\langle M'(v')|i\int \mathrm{d}^4x\, \mathrm{T}\,[J_0(0),\mathcal{L}_1(x)]\,|M(v)\rangle =$$

$$-\frac{1}{4\,m_b}\underbrace{\left(-\frac{i}{2}\right)\mathrm{Tr}\big[\Upsilon^{(b)}_{2\mu\alpha\beta}\,\overline{\mathcal{M}}'^{\mu}\,\Gamma\,P_+\,\sigma^{\alpha\beta}\,\mathcal{M}\big]}_{G^{(b)}}-\frac{1}{2\,m_b^2}\underbrace{\mathrm{Tr}\big[\Upsilon^{(b)}_{1\mu}\,\overline{\mathcal{M}}'^{\mu}\,\Gamma\,\mathcal{M}\big]}_{K^{(b)}},$$

$$\langle M'(v')|i\int \mathrm{d}^4x\, \mathrm{T}\,[J_0(0),\mathcal{L}'_1(x)]\,|M(v)\rangle =$$

$$-\frac{1}{4\,m_c}\underbrace{\left(-\frac{i}{2}\right)\mathrm{Tr}\big[\Upsilon^{(c)}_{2\mu\alpha\beta}\,\overline{\mathcal{M}}'^{\mu}\,\sigma^{\alpha\beta}\,P'_+\,\Gamma\,\mathcal{M}\big]}_{G^{(c)}}-\frac{1}{2\,m_c^2}\underbrace{\mathrm{Tr}\big[\Upsilon^{(c)}_{1\mu}\,\overline{\mathcal{M}}'^{\mu}\,\Gamma\,\mathcal{M}\big]}_{K^{(c)}},$$

→ other universal functions involved

$B_c \to J/\psi$, η_c Form Factors in the effective theory: relations at $O(1/m_Q)$

- relations among the form factors of the same decay mode

$B_c \to J/\psi$

$$h_{T_1}(w) = \frac{1}{2}\Big((1+w)h_{A_1}(w) - (w-1)h_V(w)\Big)$$

$$h_{T_2}(w) = \frac{1+w}{2(m_b+3m_c)}\Big((m_b-3m_c)h_{A_1}(w) + 2m_c(h_{A_2}(w)+h_{A_3}(w)) - (m_b-m_c)h_V(w)\Big)$$

$$h_{T_3}(w) = h_{A_3}(w) - h_V(w)$$

$$h_P(w) = \frac{1}{m_b+3m_c}\Big((1+w)\left(m_b h_{A_1}(w) + 2m_c h_V(w)\right) + \left(-m_b+(w-2)m_c\right)h_{A_2}(w) - \left(w\,m_b+(2w-1)m_c\right)h_{A_3}(w)\Big)$$

$B_c \to \eta_c$

$$h_-(w) = \frac{m_b-m_c}{2(m_b+3m_c)}(1+w)\Big(3h_{A_1}(w) - h_{A_2}(w) - h_{A_3}(w) - 2h_V(w)\Big)$$

$$h_T(w) - h_+(w) = -\frac{m_b+m_c}{2(m_b+3m_c)}(1+w)\Big(3h_{A_1}(w) - h_{A_2}(w) - h_{A_3}(w) - 2h_V(w)\Big)$$

$$h_T(w) - h_S(w) = -\frac{m_b+m_c}{(m_b+3m_c)}\Big(3h_{A_1}(w) - h_{A_2}(w) - h_{A_3}(w) - 2h_V(w)\Big).$$

all related to

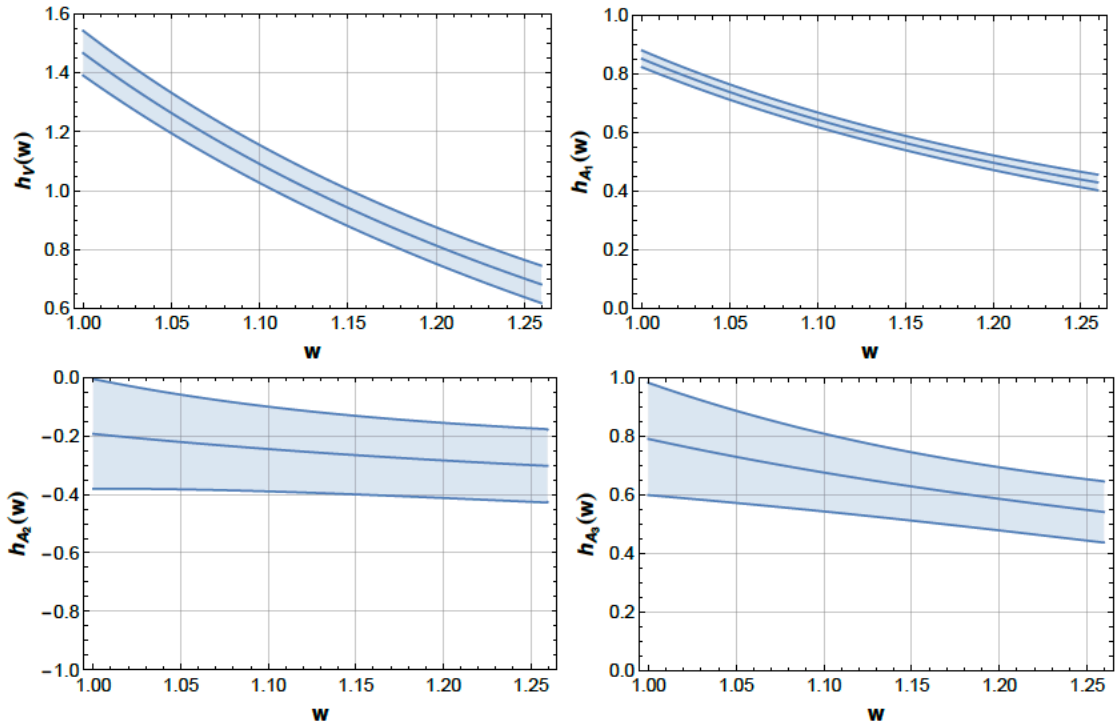
$h_V(w)$
$h_{A1}(w)$ $h_{A2}(w)$ $h_{A3}(w)$

P.Colangelo, F. Loparco, N. Losacco, M. Novoa Brunet, FDF
arXiv:2205.08933,
JHEP09 (2022) 028

Bc →J/ψ, η_c Form Factors in the effective theory: relations at $O(1/m_Q)$

available lattice results

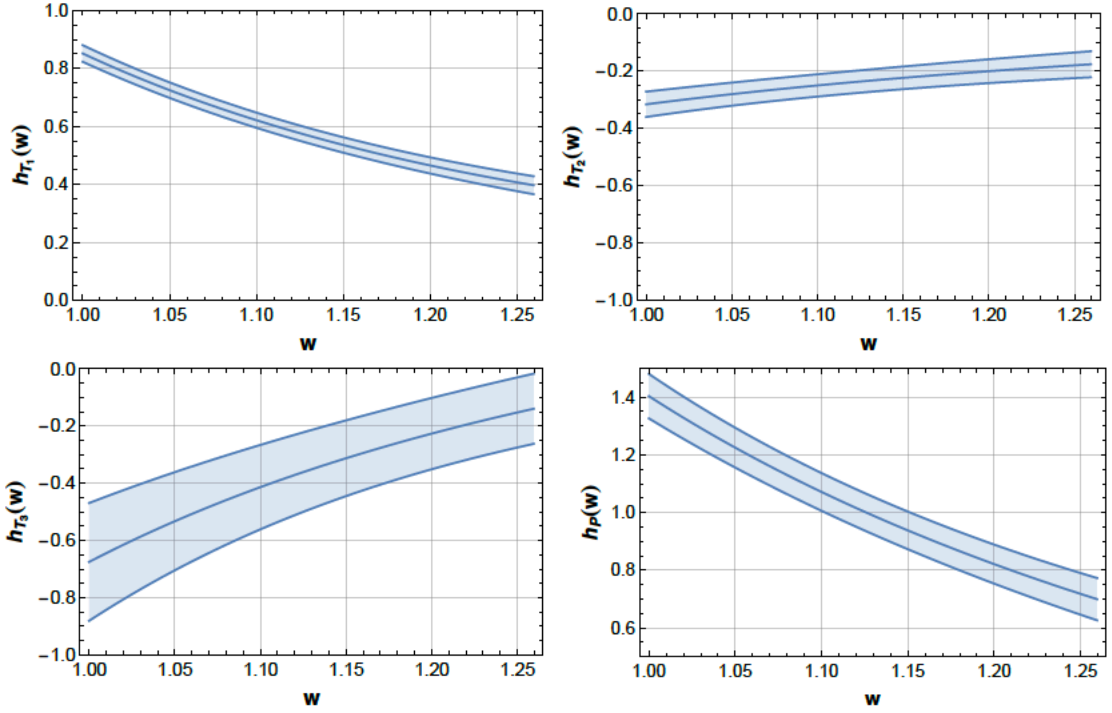
HPQCD Collab.
PRD 102 (2020) 094518
arXiv:2007.06957



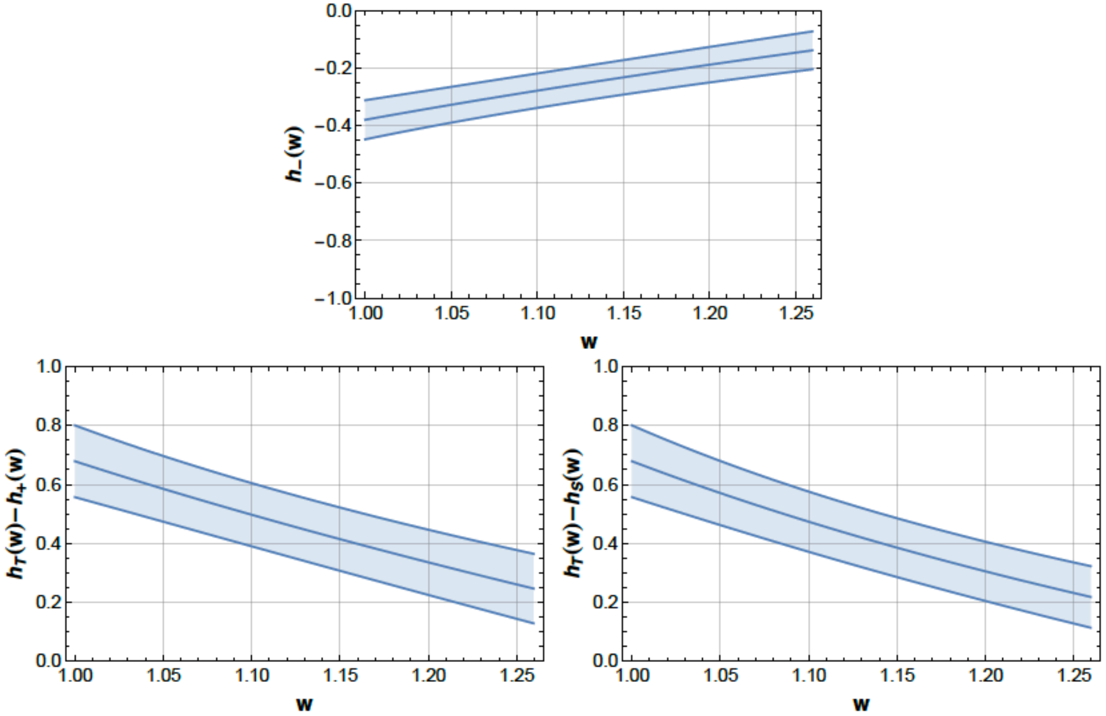
$B_c \to J/\psi, \eta_c$ Form Factors in the effective theory: relations at $O(1/m_Q)$

results

$B_c \to J/\psi$



$B_c \to \eta_c$



P.Colangelo, F. Loparco, N. Losacco, M. Novoa Brunet, FDF
arXiv:2205.08933,
JHEP09 (2022) 028

$B_c \to (\chi_{c0}, \chi_{c1}, \chi_{c2}, h_c)$ Form Factors in the effective theory: relations at $O(1/m_Q)$

- relations among the form factors of the same decay mode

P.Colangelo, F. Loparco, N. Losacco,
M. Novoa Brunet, FDF
PRD 106 (2022) 094005
arXiv:2208.13398

- $B_c \to \chi_{c0}$

$$g_T(w) = -\frac{1}{w+1}\left[2g_-(w) + g_P(w)\right]$$

- $B_c \to \chi_{c1}$

$$g_{T_2}(w) = -\frac{1}{2}\left[g_{V_1}(w) - (1+w)g_A(w)\right]$$

$$g_{T_3}(w) = -\frac{1}{2(w-1)}\left[g_{V_1}(w) + 4g_{V_2}(w)\right] + \frac{1}{2}g_A(w) + \frac{1}{w-1}\left[g_S(w) + g_{T_1}(w)\right]$$

- $B_c \to \chi_{c2}$

$$k_{T_1}(w) = -wk_V(w) + k_{A_2}(w) + wk_{A_3}(w) + k_P(w)$$

$$k_{T_2}(w) = k_V(w) - k_{A_1}(w) - k_{A_2}(w) - wk_{A_3}(w) - k_P(w)$$

$$k_{T_3}(w) = -k_V(w) + k_{A_3}(w)$$

- $B_c \to h_c$

$$f_{T_2}(w) = \frac{1}{2}\left[f_{V_1}(w) + (1+w)f_A(w)\right]$$

$$f_{T_3}(w) = \frac{1}{2(w-1)}\left[f_{V_1}(w) + 4f_{V_2}(w)\right] + \frac{1}{2}f_A(w) - \frac{1}{w-1}\left[f_S(w) - f_{T_1}(w)\right]$$

$B_c \to (\chi_{c0}, \chi_{c1}, \chi_{c2}, h_c)$ Form Factors in the effective theory: relations at $O(1/m_Q)$

- relations among the form factors of pairs of decay modes

- $B_c \to \chi_{c0}$ and $B_c \to \chi_{c1}$

$$(w+1)g_+(w) - (w-1)g_-(w) + g_P(w) =$$
$$\frac{w+1}{\sqrt{6}}\{2g_{V_1}(w) + (w+1)g_{V_2}(w) - (w-1)\left[g_{V_3}(w) + g_A(w)\right] - g_S(w) + 2g_{T_1}(w)\}$$

- $B_c \to h_c$ and $B_c \to \chi_{c1}$

$$f_{V_1}(w) + (w-1)f_A(w) - 2f_{T_1}(w) =$$
$$\sqrt{2}\{g_{V_1}(w) + (w+1)g_{V_2}(w) - (w-1)g_{V_3}(w) - g_S(w)\}$$
$$3f_{V_1}(w) + 2(w+1)f_{V_2}(w) - (w-1)\left[2f_{V_3}(w) - f_A(w)\right] - 2\left[f_S(w) + f_{T_1}(w)\right] =$$
$$\sqrt{2}\{g_{V_1}(w) - (w-1)g_A(w) + 2g_{T_1}(w)\}$$

Bc → (χ_{c0}, χ_{c1}, χ_{c2}, h_c) Form Factors in the effective theory: relations at LO

$$g_+(w) = 0$$

$$g_S(w) = g_{T_1}(w) = 0$$

$$k_{A_2}(w) = k_{T_3}(w) = 0$$

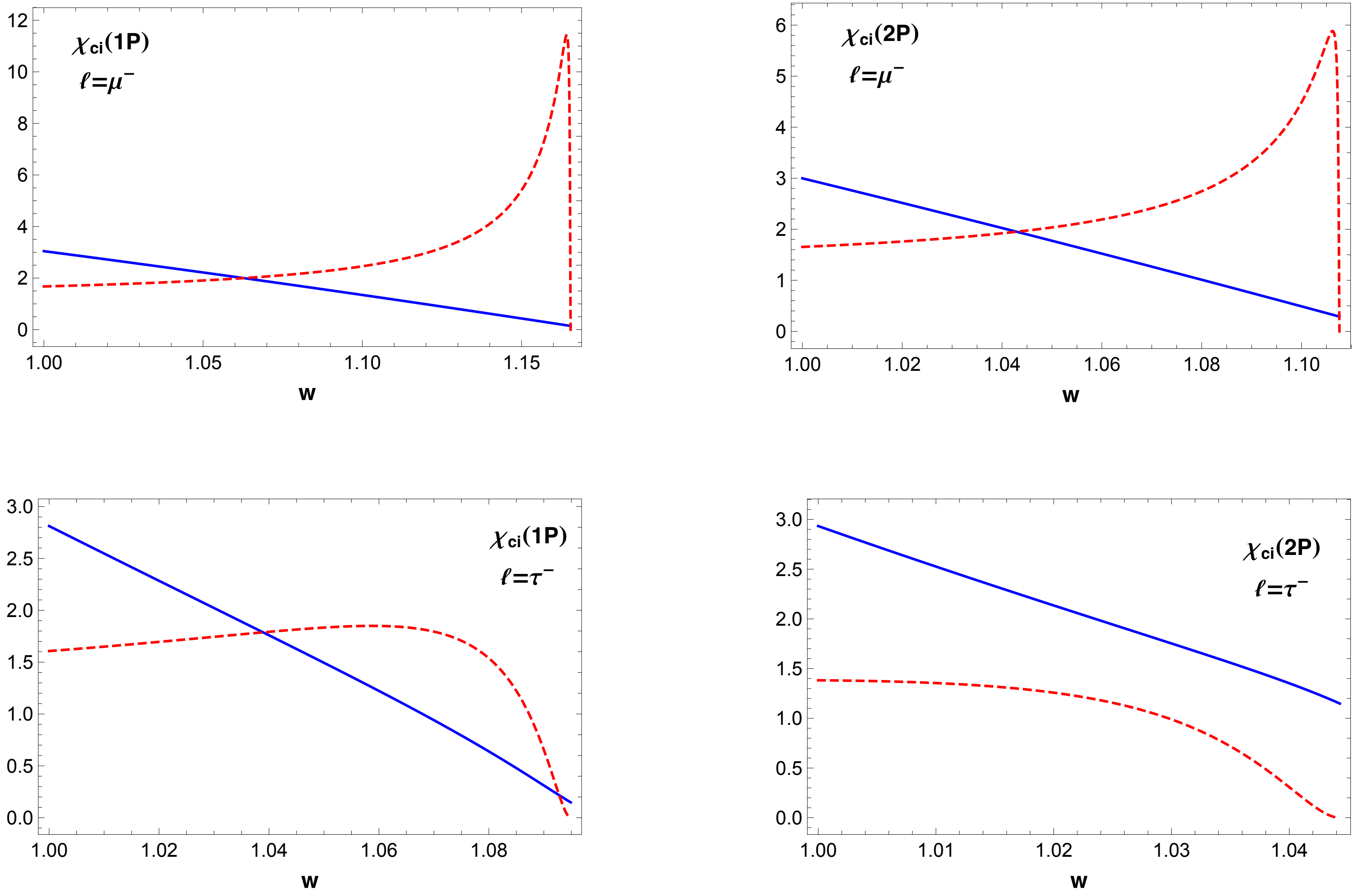
$$f_{V_1}(w) = f_{V_3}(w) = f_A(w) = f_{T_1}(w) = f_{T_2}(w) = 0$$

- χ_{c0}
- χ_{c1}
- χ_{c2}
- h_c

$$\begin{aligned}
\Xi(w) &= \frac{\sqrt{3}}{(w+1)} g_-(w) = -\frac{\sqrt{3}}{(w+1)} g_T(w) = \frac{\sqrt{3}}{(w^2-1)} g_P(w) \\
&= \frac{\sqrt{2}}{(w^2-1)} g_{V_1}(w) = -\frac{2\sqrt{2}}{(w-1)} g_{V_2}(w) = \frac{2\sqrt{2}}{(w+1)} g_{V_3}(w) = \frac{\sqrt{2}}{(w+1)} g_A(w) = \frac{\sqrt{2}}{(w+1)} g_{T_2}(w) \\
&= -k_V(w) = \frac{1}{w+1} k_{A_1}(w) = -k_{A_3}(w) = -k_P(w) = -k_{T_1}(w) = -k_{T_2}(w) \\
&= -f_{V_1}(w) = -f_{V_2}(w) = -\frac{1}{w+1} f_S(w) = f_{T_3}(w)
\end{aligned}$$

$B_c \rightarrow (\chi_{c0}, \chi_{c1}, \chi_{c2}, h_c)$ exploiting FF relations at LO

$$\frac{d\Gamma(B_c \to \chi_{c1} \ell \bar{\nu})/dw}{d\Gamma(B_c \to \chi_{c0} \ell \bar{\nu})/dw} \qquad \frac{d\Gamma(B_c \to \chi_{c2} \ell \bar{\nu})/dw}{d\Gamma(B_c \to \chi_{c1} \ell \bar{\nu})/dw}$$



Solid blue line: $\dfrac{\Gamma(B_c \to \chi_{c1} \ell \bar{\nu}_\ell)}{\Gamma(B_c \to \chi_{c0} \ell \bar{\nu}_\ell)}$

Dashed red line: $\dfrac{\Gamma(B_c \to \chi_{c2} \ell \bar{\nu}_\ell)}{\Gamma(B_c \to \chi_{c1} \ell \bar{\nu}_\ell)}$

Bc $\to$ (χ_{c0}, χ_{c1}, χ_{c2}, h_c) exploiting FF relations at LO

- constraint at LO both in SM and for generic NP

$$2\frac{d\Gamma}{dw}(B_c \to \chi_{c0}\ell\bar{\nu}_\ell) + \frac{d\Gamma}{dw}(B_c \to \chi_{c1}\ell\bar{\nu}_\ell) - \frac{d\Gamma}{dw}(B_c \to \chi_{c2}\ell\bar{\nu}_\ell) = 0.$$

to be satisfied by the three members of the 4-plet

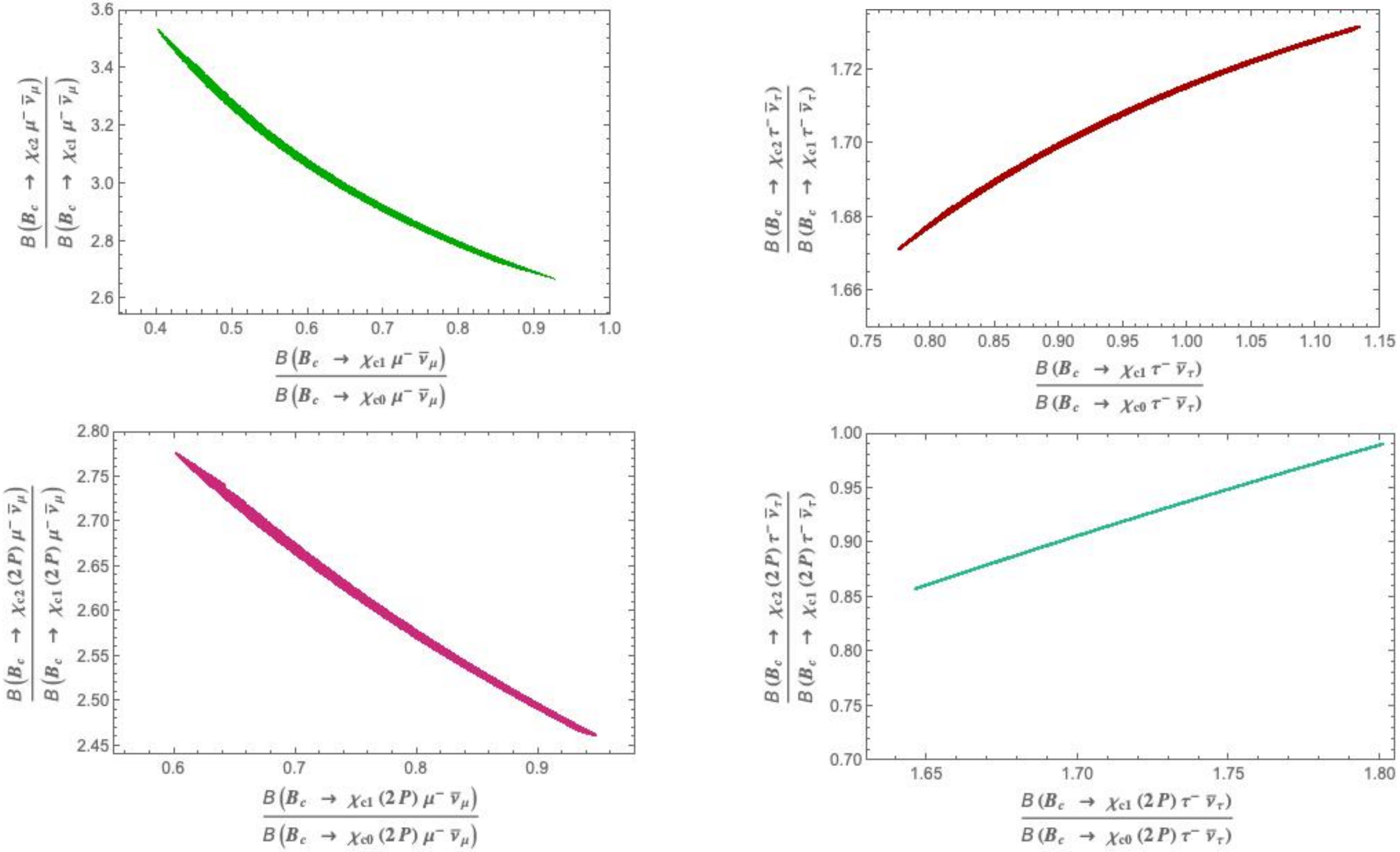
Bc → (χ_{c0}, χ_{c1}, χ_{c2}, h_c) exploiting FF relations at LO

parametrization: $\Xi(w) = \Xi_0 + \Xi_1(w-1) + \Xi_2(w-1)^2$

$\Xi_0 \in [0.1, 1]$, $\Xi_1 \in [-1, 0]$ and $\Xi_2 \in [-1, 1]$

fulfill $\mathcal{B}(B_c^+ \to \chi_{c0}\pi^+) = (2.4^{+0.9}_{-0.8}) \times 10^{-5}$

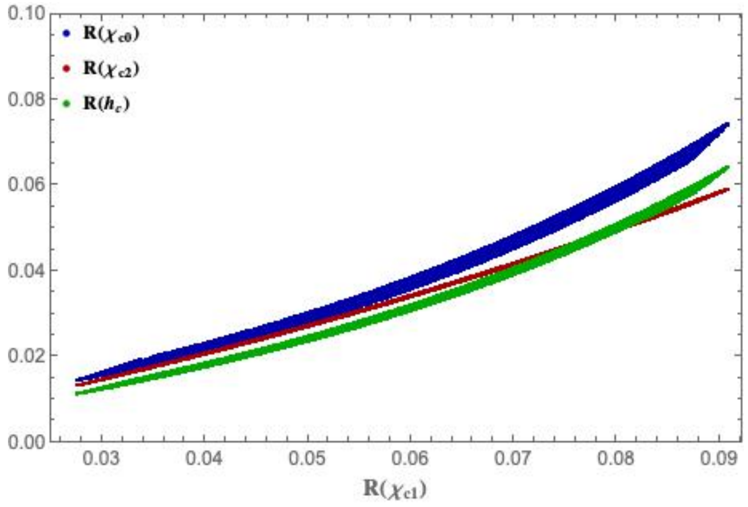
correlations predicted:

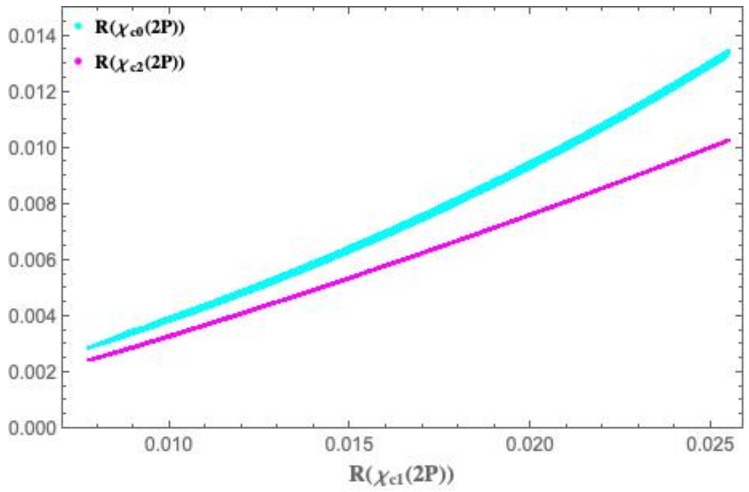


Bc → (χ_{c0}, χ_{c1}, χ_{c2}, h_c) exploiting FF relations at LO

tests of LFU:

$$R(C) = \frac{\Gamma(B_c \to C\tau\bar{\nu}_\tau)}{\Gamma(B_c \to C\mu\bar{\nu}_\mu)}$$





$B_c \to (\chi_{c0}, \chi_{c1}, \chi_{c2}, h_c)$ exploiting FF relations at NLO

At NLO the number of universal functions increase. However:

- they enter in different modes, model independent predictions
- can be used also in other processes
- model independent: tests of direct computations (should satisfy the effective theory predictions)
- Once reliable determinations for a few form factors are available (i.e. by lattice QCD) the others are predicted
- a reduced number of structures contributes close to w=1:

$$\lim_{w\to 1}\frac{1}{\tilde\Gamma}\frac{d\Gamma}{dw}(B_c\to\chi_{c0}\ell\bar\nu_\ell) = 18\,\hat m_\ell^2(\epsilon_b+\epsilon_c)^2\Big[\Sigma^{(b)}_{\chi_{c1},1}(1)\Big]^2$$

$$\lim_{w\to 1}\frac{1}{\tilde\Gamma}\frac{d\Gamma}{dw}(B_c\to\chi_{c1}\ell\bar\nu_\ell) = 12\Big[2(1-r_1)^2+\hat m_\ell^2\Big]\Big[\epsilon_b\Sigma^{(b)}_{\chi_{c1},1}(1)-\epsilon_c\Sigma^{(c)}_{\chi_{c1},1}(1)\Big]^2$$

$$\lim_{w\to 1}\frac{1}{\tilde\Gamma}\frac{d\Gamma}{dw}(B_c\to h_c\ell\bar\nu_\ell) = 6\Big[2(1-r_h)^2+\hat m_\ell^2\Big]\Big[(\epsilon_b-\epsilon_c)\Sigma^{(b)}_{\chi_{c1},1}(1)+2\epsilon_c\Sigma^{(c)}_{\chi_{c1},1}(1)\Big]^2$$

$$\hat m_\ell^2 = \frac{m_\ell^2}{m_{B_c}^2}$$

$$r = m_C/m_{B_c} \qquad C = m_{\chi_{c0}},\ \chi_{c1},\ \chi_{c2},\ h_c$$

$$\epsilon_b = \frac{1}{2m_b} \qquad \epsilon_c = \frac{1}{2m_c}$$

if X(3872) is χ_{c1}(2P) these relations should be fulfilled (hard task…)

Semileptonic B_c decays: c → s,d transitions

$B_c \to B_{s,d}$

$$\langle P(p')|\bar{q}\gamma_\mu Q|B_c(p)\rangle = f_+^{B_c\to P}(q^2)\Big(p_\mu + p'_\mu - \frac{m_{B_c}^2 - m_P^2}{q^2}q_\mu\Big) + f_0^{B_c\to P}(q^2)\frac{m_{B_c}^2 - m_P^2}{q^2}q_\mu \ ,$$

$$\langle P(p')|\bar{q}Q|B_c(p)\rangle = f_S^{B_c\to P}(q^2) \ ,$$

$$\langle P(p')|\bar{q}\sigma_{\mu\nu}Q|B_c(p)\rangle = -i\frac{2f_T^{B_c\to P}(q^2)}{m_{B_c}+m_P}(p_\mu p'_\nu - p_\nu p'_\mu) \ ,$$

$$\langle P(p')|\bar{q}\sigma_{\mu\nu}\gamma_5 Q|B_c(p)\rangle = -\frac{2f_T^{B_c\to P}(q^2)}{m_{B_c}+m_P}\epsilon_{\mu\nu\alpha\beta}\,p^\alpha p'^\beta$$

4 FFs

$B_c \to B^*_{s,d}$

$$\langle V(p',\epsilon)|\bar{q}\gamma_\mu Q|B_c(p)\rangle = -\frac{2V^{B_c\to V}(q^2)}{m_{B_c}+m_V}i\epsilon_{\mu\nu\alpha\beta}\epsilon^{*\nu}p^\alpha p'^\beta,$$

$$\langle V(p',\epsilon)|\bar{q}\gamma_\mu\gamma_5 Q|B_c(p)\rangle = (m_{B_c}+m_V)\Big(\epsilon^*_\mu - \frac{(\epsilon^*\cdot q)}{q^2}q_\mu\Big)A_1^{B_c\to V}(q^2) - \frac{(\epsilon^*\cdot q)}{m_{B_c}+m_V}\Big((p+p')_\mu - \frac{m_{B_c}^2-m_V^2}{q^2}q_\mu\Big)A_2^{B_c\to V}(q^2)$$
$$+(\epsilon^*\cdot q)\frac{2m_V}{q^2}q_\mu A_0^{B_c\to V}(q^2),$$

$$\langle V(p',\epsilon)|\bar{q}\gamma_5 Q|B_c(p)\rangle = -\frac{2m_V}{m_Q+m_q}(\epsilon^*\cdot q)A_0^{B_c\to V}(q^2),$$

$$\langle V(p',\epsilon)|\bar{q}\sigma_{\mu\nu}Q|B_c(p)\rangle = T_0^{B_c\to V}(q^2)\frac{\epsilon^*\cdot q}{(m_{B_c}+m_V)^2}\epsilon_{\mu\nu\alpha\beta}p^\alpha p'^\beta + T_1^{B_c\to V}(q^2)\epsilon_{\mu\nu\alpha\beta}p^\alpha\epsilon^{*\beta} + T_2^{B_c\to V}(q^2)\epsilon_{\mu\nu\alpha\beta}p'^\alpha\epsilon^{*\beta},$$

$$\langle V(p',\epsilon)|\bar{q}\sigma_{\mu\nu}\gamma_5 Q|B_c(p)\rangle = i\,T_0^{B_c\to V}(q^2)\frac{\epsilon^*\cdot q}{(m_{B_c}+m_V)^2}(p_\mu p'_\nu - p_\nu p'_\mu)$$
$$+i\,T_1^{B_c\to V}(q^2)(p_\mu\epsilon^*_\nu - \epsilon^*_\mu p_\nu) + i\,T_2^{B_c\to V}(q^2)(p'_\mu\epsilon^*_\nu - \epsilon^*_\mu p'_\nu)$$

6 FFs

HQ spin symmetry in B_c decays

$$\langle P(v,k)|\bar{q}\gamma_\mu Q|B_c(v)\rangle = 2\sqrt{m_{B_c} m_P}\Big(\Omega_1(y)\ v_\mu + a_0\Omega_2(y)\ k_\mu\Big),$$

$$\langle P(v,k)|\bar{q}Q|B_c(v)\rangle = 2\sqrt{m_{B_c} m_P}\Big(\Omega_1(y) + a_0\Omega_2(y)\ v\cdot k\Big),$$

$$\langle P(v,k)|\bar{q}\sigma_{\mu\nu}Q|B_c(v)\rangle = -2i\sqrt{m_{B_c} m_P}\ a_0\Omega_2(y)\Big(v_\mu k_\nu - v_\nu k_\mu\Big)$$

$$\langle V(v,k,\epsilon)|\bar{q}\gamma_\mu Q|B_c(v)\rangle = 2i\sqrt{m_{B_c} m_V}\ a_0\Omega_2(y)\ \epsilon_{\mu\nu\alpha\beta}\epsilon^{*\nu}\ k^\alpha v^\beta,$$

$$\langle V(v,k,\epsilon)|\bar{q}\gamma_\mu\gamma_5 b|B_c(v)\rangle = 2\sqrt{m_{B_c} m_V}\Big(\epsilon^*_\mu\ (\Omega_1(y) + v\cdot k\ a_0\Omega_2(y)) - (v_\mu - \frac{k_\mu}{m_V})\epsilon^*\cdot k\, a_0\Omega_2(y)\Big),$$

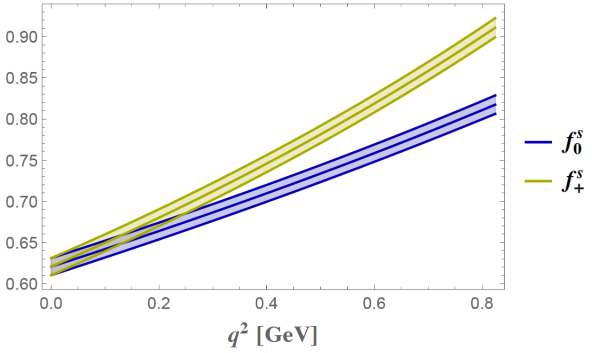
$$\langle V(v,k,\epsilon)|\bar{q}\sigma_{\mu\nu}Q|B_c(v)\rangle = -2\sqrt{m_{B_c} m_V}\Big(\epsilon_{\mu\nu\alpha\beta}\epsilon^{*\alpha}v^\beta\Omega_1(y) + \epsilon_{\mu\nu\alpha\beta}\epsilon^{*\alpha}k^\beta a_0\Omega_2(y)\Big),$$

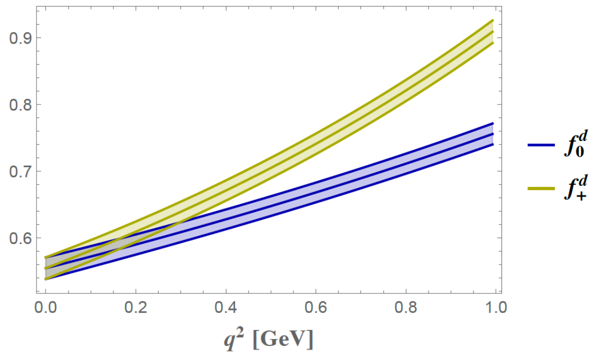
$$\langle V(v,k,\epsilon)|\bar{q}\sigma_{\mu\nu}\gamma_5 Q|B_c(v)\rangle = 2i\sqrt{m_{B_c} m_V}\Big(\epsilon^*_\nu(v_\mu\Omega_1(y) + k_\mu a_0\Omega_2(y)) - \epsilon^*_\mu(v_\nu\Omega_1(y) + k_\nu a_0\Omega_2(y))\Big)$$

all expressed in terms of Ω_1 and Ω_2

HQ spin symmetry in B_c decays

lattice results for f_+ and f_0



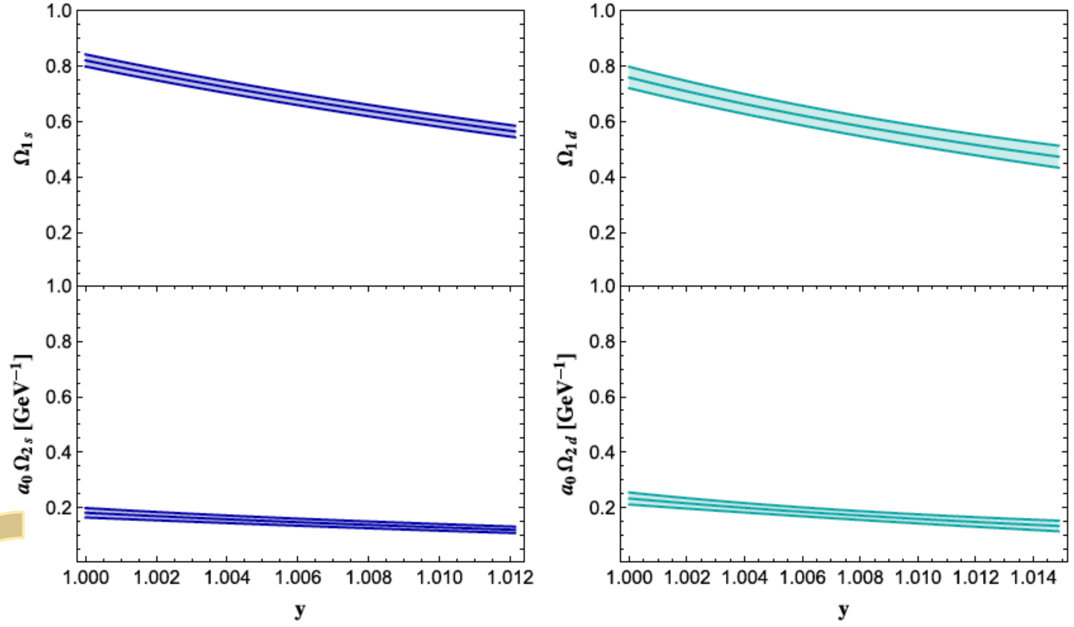


obtained from data in
HPQCD Collab. PRD102 (2020) 014513

translated into Ω_1 and Ω_2:

$$\Omega_1 = \frac{m_{B_c} + m_P}{2q^2\sqrt{m_{B_c} m_P}}\left((m_{B_c} - m_P)^2 (f_0 - f_+) + q^2 f_+\right)$$

$$a_0 \Omega_2 = \frac{1}{2q^2\sqrt{m_{B_c} m_P}}\left((m_{B_c}^2 - m_P^2)(f_+ - f_0) + q^2 f_+\right)$$

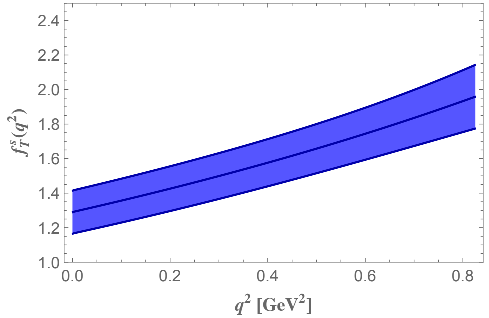


all other FFs derived from these functions

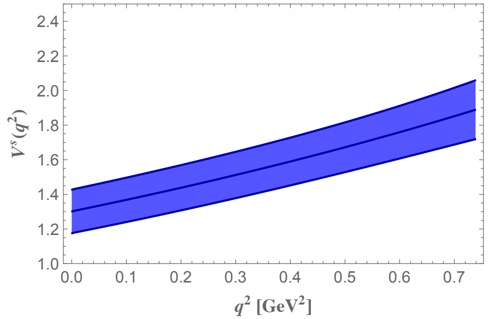
P. Colangelo, F. Loparco, FDF, PRD103 (2021) 075019

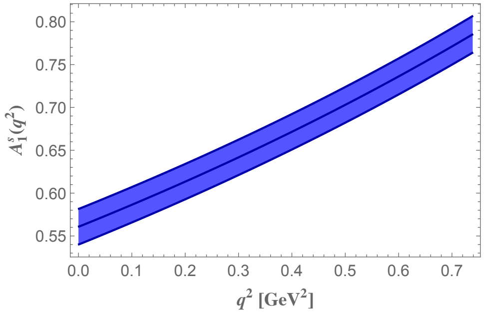
HQ spin symmetry in B_c decays

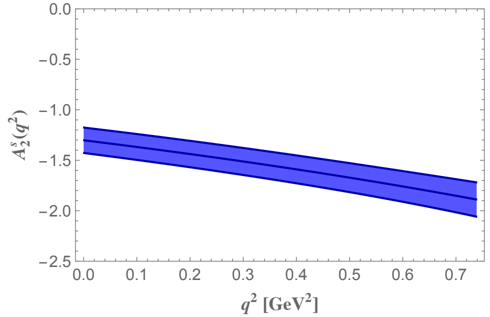
$B_c \to B_s \ell \nu_\ell$

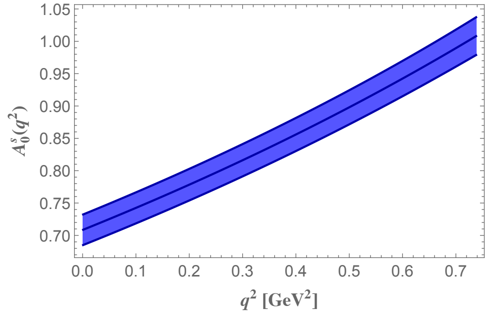


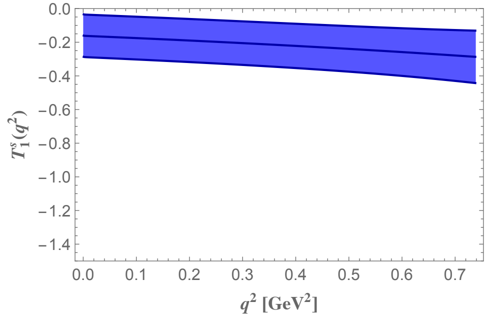
$B_c \to B^*_s \ell \nu_\ell$

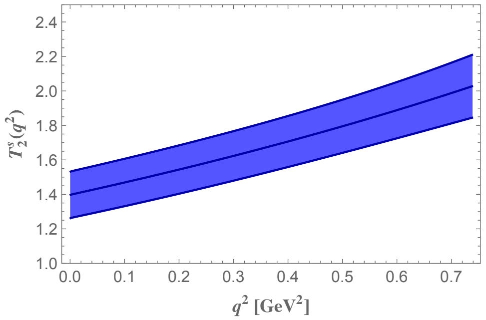












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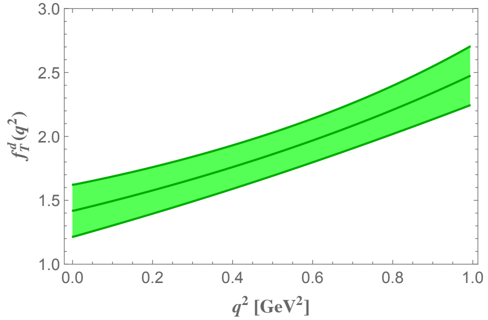
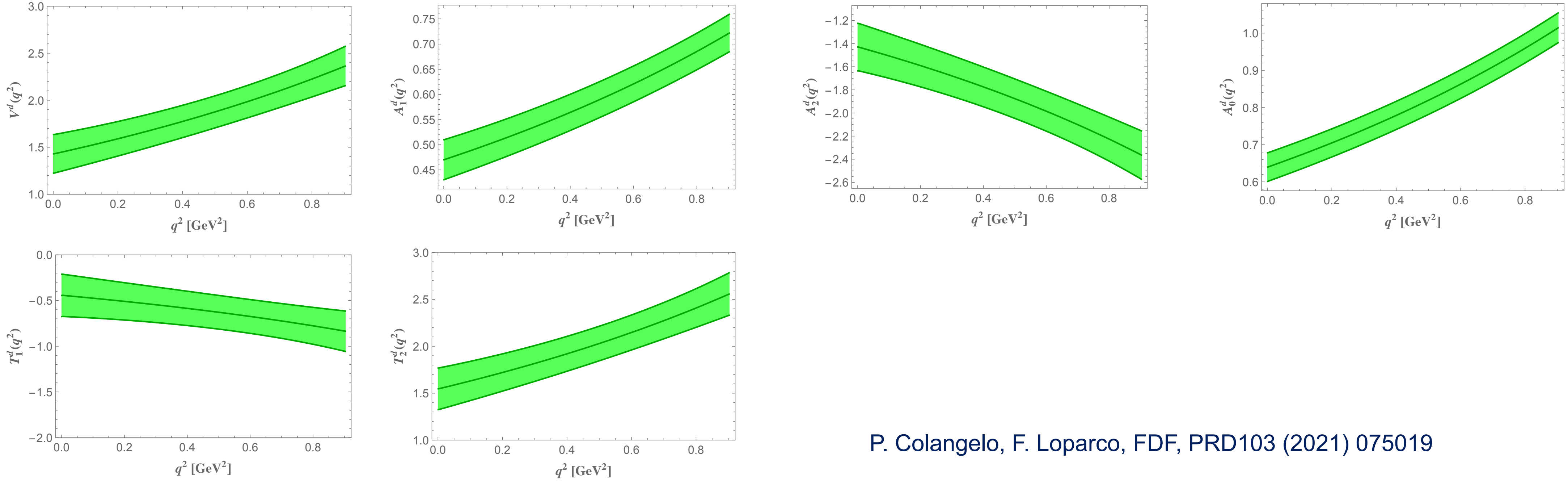
HQ spin symmetry in B_c decays

$B_c \to B_d \ell \nu_\ell$



$B_c \to B^*_d \ell \nu_\ell$



P. Colangelo, F. Loparco, FDF, PRD103 (2021) 075019

Results

$B_c \to B^{(*)}_s \ell \nu_\ell$

$$\mathcal{B}(B_c^+ \to B_s \mu^+ \nu_\mu) = 0.0125\,(4) \left(\frac{|V_{cs}|}{0.987}\right)^2$$

$$\mathcal{B}(B_c^+ \to B_s e^+ \nu_e) = 0.0131\,(4) \left(\frac{|V_{cs}|}{0.987}\right)^2$$

$$\mathcal{B}(B_c^+ \to B_s^* \mu^+ \nu_\mu) = 0.030\,(1) \left(\frac{|V_{cs}|}{0.987}\right)^2$$

$$\mathcal{B}(B_c^+ \to B_s^* e^+ \nu_e) = 0.032\,(1) \left(\frac{|V_{cs}|}{0.987}\right)^2$$

SM branching fractions

$B_c \to B^{(*)}_d \ell \nu_\ell$

$$\mathcal{B}(B_c^+ \to B_d \mu^+ \nu_\mu) = 8.3\,(5) \times 10^{-4} \left(\frac{|V_{cd}|}{0.221}\right)^2$$

$$\mathcal{B}(B_c^+ \to B_d e^+ \nu_e) = 8.7\,(5) \times 10^{-4} \left(\frac{|V_{cd}|}{0.221}\right)^2$$

$$\mathcal{B}(B_c^+ \to B_d^* \mu^+ \nu_\mu) = 20\,(1) \times 10^{-4} \left(\frac{|V_{cd}|}{0.221}\right)^2$$

$$\mathcal{B}(B_c^+ \to B_d^* e^+ \nu_e) = 21\,(1) \times 10^{-4} \left(\frac{|V_{cd}|}{0.221}\right)^2$$

small uncertainty: role of the HQSS relations

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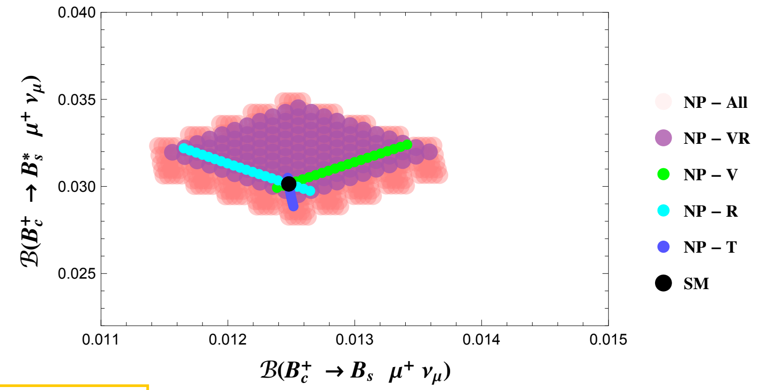
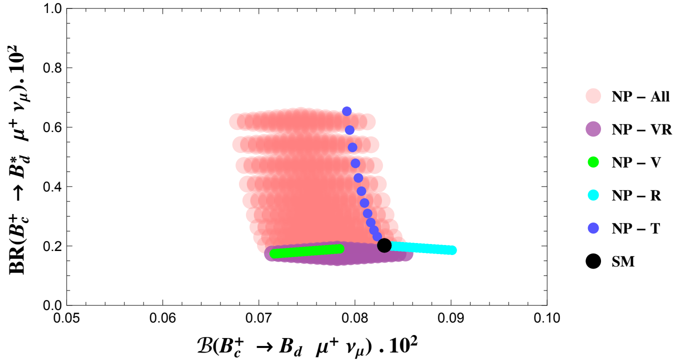
Impact of NP: correlations

NP couplings from
D. Becirevic, F. Jaffredo, A. Penuelas, O. Sumnsari,
JHEP05 (2021) 175

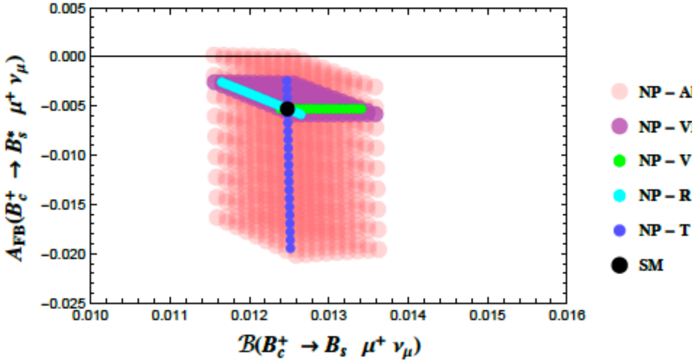
branching ratios

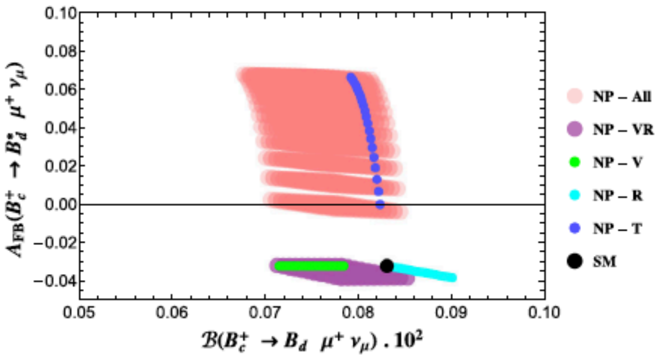


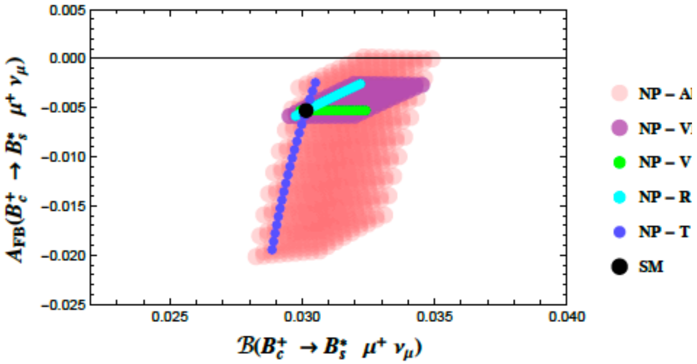


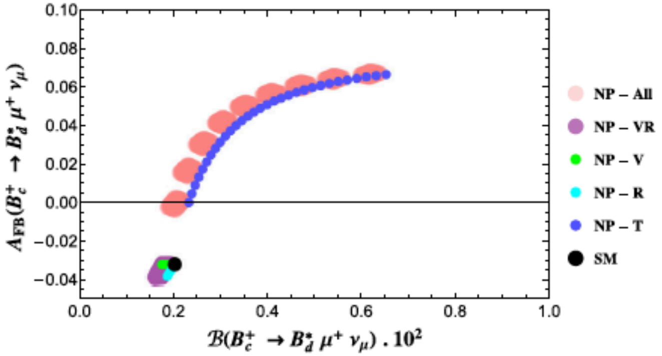
Forward-Backward lepton asymmetry

$$\mathcal{A}_{\mathcal{FB}}(q^2) = \left(\frac{d\Gamma}{dq^2}\right)^{-1} \left[\int_0^1 d\cos\theta \, \frac{d^2\Gamma}{dq^2 d\cos\theta} - \int_{-1}^0 d\cos\theta \, \frac{d^2\Gamma}{dq^2 d\cos\theta}\right]$$









different pattern of correlations:

the presence of R would produce

- anticorrelated BRs
- anticorrelated AFB ($B_c \to B^*_s \ \mu \ \nu_m$) and B($B_c \to B_s \ \mu \ \nu_\mu$)
- correlated AFB ($B_c \to B^*_s \ \mu \ \nu_m$) and B($B_c \to B^*_s \ \mu \ \nu_\mu$)
- large impact of T on AFB
- large impact of T on BR in c →d

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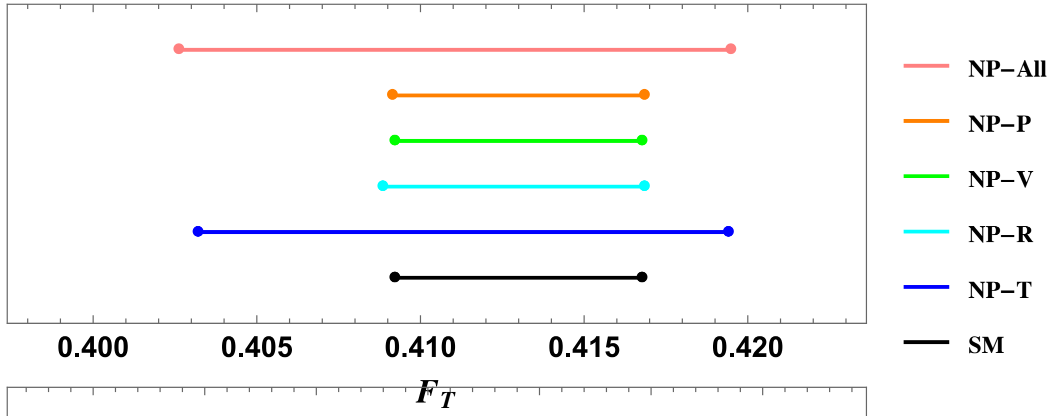
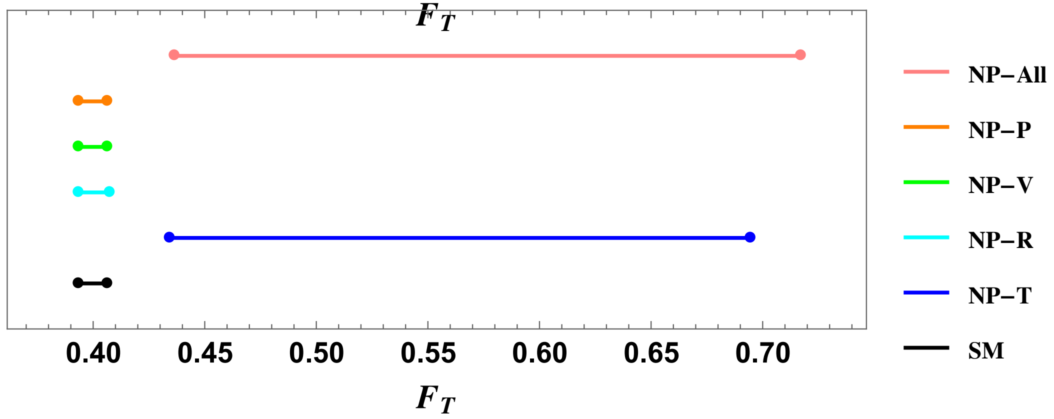
$B_c \to B^*_{s,d}\, \mu\, \nu_\mu$

fraction of transversely polarized $B^*_{s,d}$

$B_c \to B^*_s\, \mu\, \nu_\mu$



$B_c \to B^*_d\, \mu\, \nu_\mu$



$c \to s$: $F_T<0.5$ in SM and NP

$c \to d$: T can reverse the hierarchy

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Conclusions

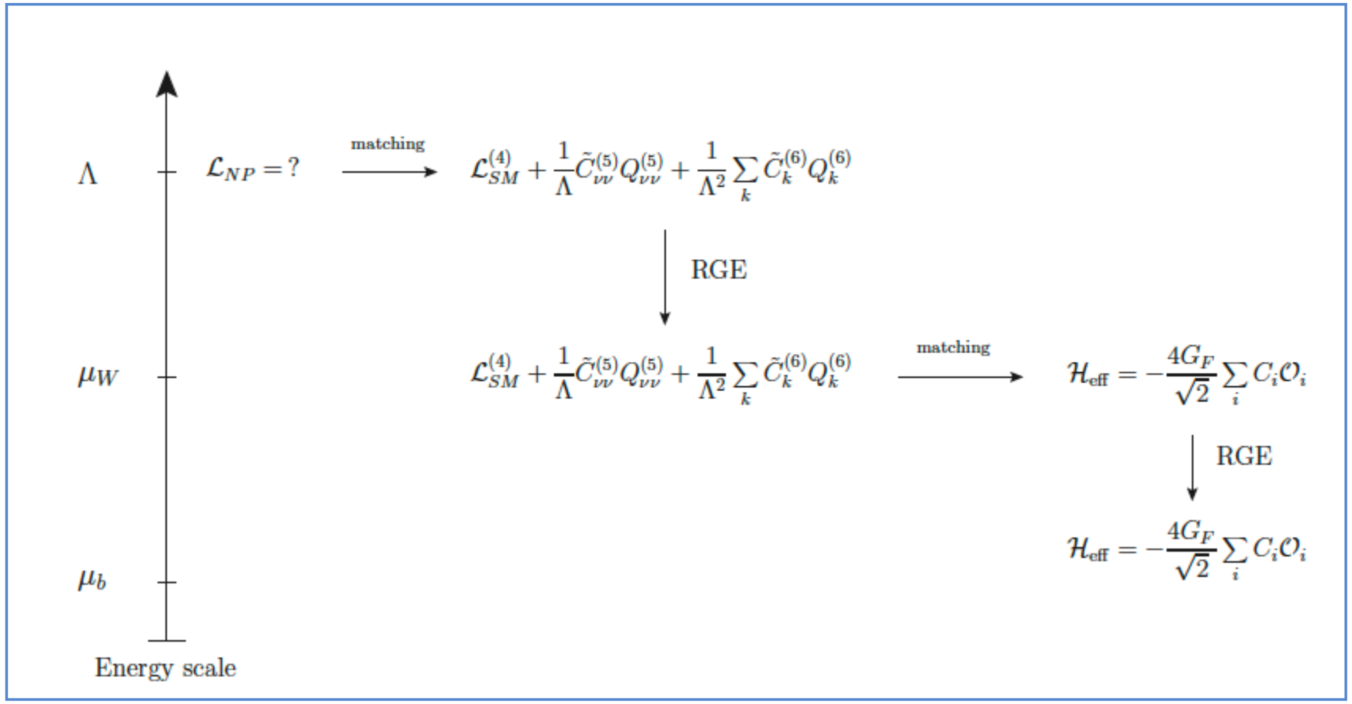
B_c decays represent an interesting testing ground for

- determination of V_{cb}
- flavour anomalies
- probing the structure of the hadrons in the final state

predictions based on NRQCD + HQE

- relations among FFs
- relations to be fulfilled by modes with final hadrons connected by HQSS
- tests of explicit calculations

Explore BSM effects: Systematic extension of the SM



picture from Aebischer, Crivellin, Fael, Greub
JHEP05 (2016)037

→ final evolution
to the scale at which we work

- coefficients in the low energy $\mathcal{H}_{\rm eff}$ related to those at high scale
- relations among coefficients entering in different processes

Semileptonic B_c decays

Impact of NP encoded in the NP couplings ε and in the new operators -> new FF

$B_c \rightarrow P$

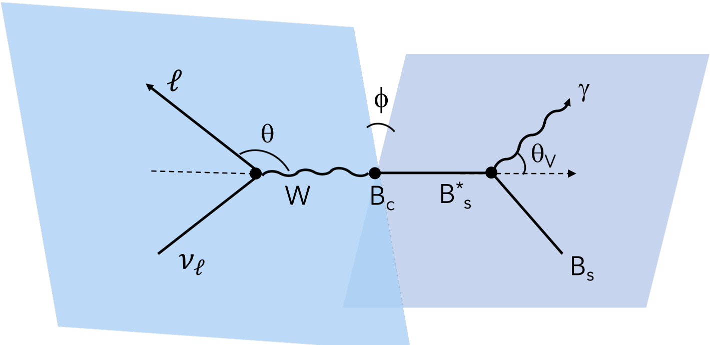
$$\frac{d\Gamma(B_c \to P\bar{\ell}\nu_\ell)}{dq^2} = \frac{G_F^2|V_{CKM}|^2\lambda^{1/2}}{128\, m_{B_c}^3\pi^3 q^2}\left(1-\frac{m_\ell^2}{q^2}\right)^2$$

$$\Bigg\{\left|m_\ell(1+\epsilon_V^\ell+\epsilon_R^\ell)+\frac{q^2\epsilon_S^\ell}{m_Q-m_q}\right|^2(m_{B_c}^2-m_P^2)^2 f_0^2(q^2)$$

$$+\lambda\left[\frac{1}{3}\left|m_\ell(1+\epsilon_V^\ell+\epsilon_R^\ell)f_+(q^2)+\frac{4q^2}{m_{B_c}+m_P}\epsilon_T^\ell f_T(q^2)\right|^2+\frac{2q^2}{3}\left|(1+\epsilon_V^\ell+\epsilon_R^\ell)f_+(q^2)+4\frac{m_\ell}{m_{B_c}+m_P}\epsilon_T^\ell f_T(q^2)\right|^2\right]\Bigg\}$$

Semileptonic B_c decays

$B_c \to V$



$$\frac{d^4\Gamma(B_c \to V(\to P\gamma)\bar{\ell}\nu_\ell)}{dq^2\, d\cos\theta_V\, d\cos\theta\, d\phi} = \mathcal{N}_\gamma |\vec{p}_V| \left(1 - \frac{m_\ell^2}{q^2}\right)^2$$

$$\Big\{ I_{1s} \sin^2\theta_V + I_{1c}(3+\cos 2\theta_V)$$
$$+ (I_{2s} \sin^2\theta_V + I_{2c}(3+\cos 2\theta_V))\cos 2\theta$$
$$+ I_3 \sin^2\theta_V \sin^2\theta \cos 2\phi + I_4 \sin 2\theta_V \sin 2\theta \cos\phi$$
$$+ I_5 \sin 2\theta_V \sin\theta \cos\phi$$
$$+ (I_{6s} \sin^2\theta_V + I_{6c}(3+\cos 2\theta_V))\cos\theta$$
$$+ I_7 \sin 2\theta_V \sin\theta \sin\phi + I_8 \sin 2\theta_V \sin 2\theta \sin\phi$$
$$+ I_9 \sin^2\theta_V \sin^2\theta \sin 2\phi \Big\}$$

$$I_i = |1+\epsilon_V|^2 I_i^{\rm SM} + |\epsilon_R|^2 I_i^{{\rm NP},R} + |\epsilon_P|^2 I_i^{{\rm NP},P} + |\epsilon_T|^2 I_i^{{\rm NP},T} + 2{\rm Re}[\epsilon_R(1+\epsilon_V^*)] I_i^{{\rm INT},R}$$
$$+ 2{\rm Re}[\epsilon_P(1+\epsilon_V^*)] I_i^{{\rm INT},P} + 2{\rm Re}[\epsilon_T(1+\epsilon_V^*)] I_i^{{\rm INT},T}$$
$$+ 2{\rm Re}[\epsilon_R\epsilon_T^*] I_i^{{\rm INT},RT} + 2{\rm Re}[\epsilon_P\epsilon_T^*] I_i^{{\rm INT},PT} + 2{\rm Re}[\epsilon_P\epsilon_R^*] I_i^{{\rm INT},PR}$$

$$I_7 = 2{\rm Im}[\epsilon_R(1+\epsilon_V^*)] I_7^{{\rm INT},R} + 2{\rm Im}[\epsilon_P(1+\epsilon_V^*)] I_7^{{\rm INT},P} + 2{\rm Im}[\epsilon_T(1+\epsilon_V^*)] I_7^{{\rm INT},T}$$
$$+ 2{\rm Im}[\epsilon_R\epsilon_T^*] I_7^{{\rm INT},RT} + 2{\rm Im}[\epsilon_P\epsilon_T^*] I_7^{{\rm INT},PT} + 2{\rm Im}[\epsilon_P\epsilon_R^*] I_7^{{\rm INT},PR},$$

angular coefficient functions depend on NP couplings and FF

$$H_0 = \frac{1}{2m_V(m_{B_c}+m_V)\sqrt{q^2}}\Big((m_{B_c}+m_V)^2(m_{B_c}^2 - m_V^2 - q^2)A_1(q^2) - \lambda(m_{B_c}^2, m_V^2, q^2)A_2(q^2)\Big)$$

$$H_\pm = \frac{(m_{B_c}+m_V)^2 A_1(q^2) \mp \sqrt{\lambda(m_{B_c}^2, m_V^2, q^2)}\,V(q^2)}{m_{B_c}+m_V}$$

$$H_t = -\frac{\sqrt{\lambda(m_{B_c}^2, m_V^2, q^2)}}{\sqrt{q^2}} A_0(q^2)$$

$$H_\pm^{NP} = \frac{1}{\sqrt{q^2}}\Big\{\Big(m_{B_c}^2 - m_V^2 \pm \sqrt{\lambda(m_{B_c}^2, m_V^2, q^2)}\Big)(T_1+T_2) + q^2(T_1 - T_2)\Big\}$$

$$H_L^{NP} = 4\Big\{\frac{\lambda(m_{B_c}^2, m_V^2, q^2)}{m_V(m_{B_c}+m_V)^2}T_0 + 2\frac{m_{B_c}^2 + m_V^2 - q^2}{m_V}T_1 + 4m_V T_2\Big\}$$

HQ spin symmetry in B_c decays

$B_c \to B^{(*)}_a \ell \nu_\ell$ a=s,d

Underlying transition: $c \to s,d$
$m_b >> m_c \to$ the b quark is not deflected $\to$ the velocity of the final meson is the same as in the initial state

momentum transferred to leptons: $q=(m_{Bc}-m_{Ba})v-k$ with $v.k=O(1/m_b)$ *residual momentum*

Matrix elements computed using the trace formalism (as for Heavy-Light)

$$\langle B_a^{(*)}(v,k)|\bar{q}\Gamma Q|B_c(v)\rangle = -\sqrt{m_{B_c} m_{B_a}}\, \mathrm{Tr}\left[\overline{H}_a^{(b)} \Omega_a(v, a_0 k)\Gamma H^{(c\bar{b})}\right]$$

Effective heavy-heavy field

$$H^{c\bar{b}} = \frac{1+\not{v}}{2}[B_c^{*\mu}\gamma_\mu - B_c\gamma_5]\frac{1-\not{v}}{2}$$

Effective heavy-light field

$$H^{\bar{b}} = [B_a^{*\mu}\gamma_\mu - B_a\gamma_5]\frac{1-\not{v}}{2}$$

$$\Omega_a(v, a_0 k) = \Omega_{1a} + \not{k} a_0 \Omega_{2a}$$

Bohr radius for quarkonium

Only two independent functions appear for all the possible Dirac structures Γ