B hadron decays to double charm

at LHCb and B factories

Anton Poluektov

Aix Marseille Univ, CNRS/IN2P3, CPPM, Marseille, France

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Introduction

Many reasons to study double charm decays:

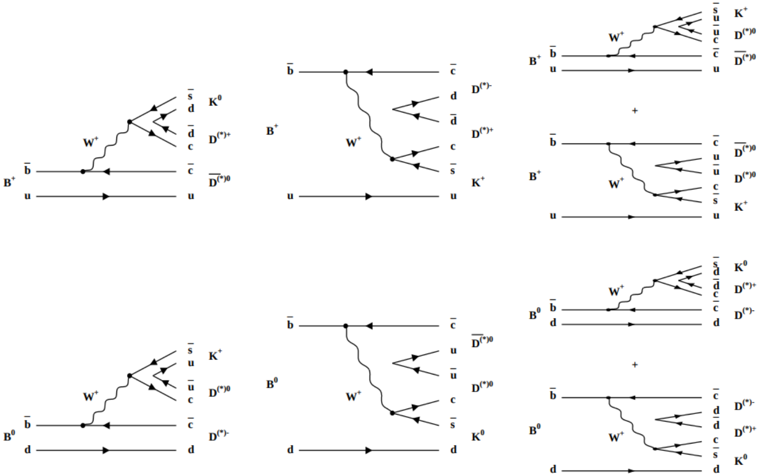
- Background in semileptonic analyses:
  - $B \to DD_s^+ X$ with $D_s^+ \to 3\pi X$: background for $\tau \to 3\pi(\pi^0)\nu$
  - $B \to DD_{(s)} X$ with $D_{(s)} \to \mu X$: background for $\tau \to \mu\nu\nu$
  - $B \to DD_s^+ X$ with $D_s \to \tau\nu$: background for both
- $B \to D^{**}$ formfactors are poorly known, use hadronic $B \to D^{**} X$ modes to constrain them [A. Le Yaouanc, and J.-P. Leroy, P. Roudeau, PRD 105 (2022) 013004]
  - $B \to D^{**}\pi$, $B \to D^{**}\omega$, $B \to D^{**}D_s^+$, $B \to D^{**}D_s^{*+}$, *etc.*
- Hadron spectroscopy, including exotic
  - "Conventional" D, D_s^+, $c\bar{c}$ spectroscopy
  - Significant exotic contributions found in a few analyses, has to be taken into account when constraining SL formfactors from hadronic decays.

This talk:

- What do we know about double charm decays ($\mathcal{B}$, amplitudes)
- What do we need to know and how we can improve our knowledge

Dominant $b \to c\bar{c}s$ transitions:

- External: $B \to D^{(*)} D_s^{(*)-}$
  - $B \to D^{*+} D_s^{(*)-}$
  - $B \to D^{**} D_s^{(*)-}$
- Internal: $B \to (c\bar{c} \to D\overline{D})K$
  - $B \to D^{*+} \overline{D}^{(*)-} K$

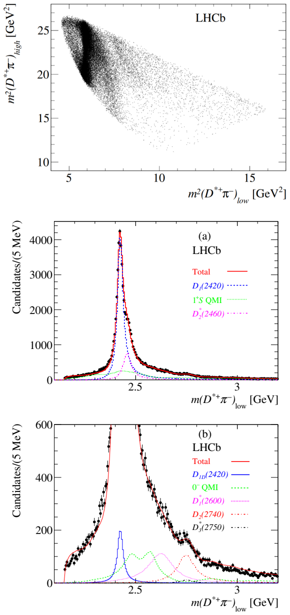


Excited D states

In addition to $L = 0$ D and D^* states, a number of higher excitations decaying to mostly to $D^{(*)}\pi$ (also to $D\pi\pi$)

Resonance	J^P	Mass (MeV)	Width (MeV)	$D\pi$	$D^*\pi$
$D_0^*(2300)$	0^+	2343 ± 10	229 ± 16	seen	
$D_1(2420)$	1^+	2422.1 ± 0.6	31.3 ± 1.9		seen
$D_1(2430)$	1^+	2412 ± 9	314 ± 29		seen
$D_2^*(2460)$	2^+	$2461.1^{+0.7}_{-0.8}$	47.3 ± 0.8		seen
$D_0(2550)$	0^-	2549 ± 19	165 ± 24		seen
$D_1^*(2600)$	1^-	2627 ± 10	141 ± 23	seen	seen
$D_2(2740)$	2^-	2747 ± 6	88 ± 19		seen
$D_3^*(2750)$	3^-	2763.1 ± 3.2	66 ± 5	seen	seen
$D_1^*(2760)$	1^-	2781 ± 22	177 ± 40	seen	
$D(3000)$	$2^+?$	3214 ± 60	186 ± 80		seen

Quantum numbers measured in $B^+ \to D^{*-}\pi^+\pi^+$ analysis [LHCb, PRD 101 (2020) 032005]



Excited D_s^+ states

Narrow states below D^*K threshold

State	Width, MeV	J^P	$D_s^+\gamma$	$D_s^+\pi^0$	$D_s^{*+}\pi^0$	$D_s^+\pi^+\pi^-$
$D_s^*(2112)^+$	< 1.9	1^-	94%	6%		
$D_{s0}^*(2317)^+$	< 3.8	0^+		$\sim$100%		
$D_{s1}(2460)^+$	< 3.5	1^+	$(18 \pm 4)\%$		$(48 \pm 11)\%$	$(4.3 \pm 1.3)\%$

$D^{(*)}K$ states:

State	Width, MeV	J^P	D^0K^+	D^+K^0	$D^{*0}K^+$	$D^{*+}K^0$	Other
$D_{s1}(2536)^+$	0.92 ± 0.05	1^+			$\equiv 1$	~ 0.85	$D_s^+\pi^+\pi^-$
$D_{s2}^*(2573)^+$	16.9 ± 0.7	2^+	seen	seen			
$D_{s1}^*(2590)^+$	89 ± 20	0^-					$D^+K^+\pi^-$
$D_{s1}^*(2700)^+$	122 ± 10	1^-	seen	seen	seen	seen	
$D_{s1}^*(2860)^+$	159 ± 80	1^-	seen				
$D_{s3}^*(2860)^+$	53 ± 10	3^-	seen	seen	seen	seen	

B decays into $D^{*+} + X_c$

Cabibbo-favoured $b \to c\bar{c}s$ transitions with D^{*+} in final state (backgrounds to $R(D^*)$)

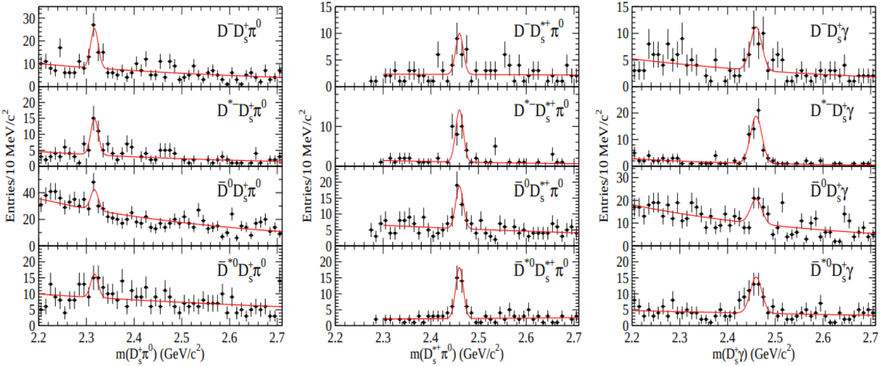
Decay mode		Branching fraction	Reference
$\overline{B}^0 \to D^{*+} D_s^-$	$c\bar{s}$	$(8.0 \pm 1.1) \times 10^{-3}$	[BaBar'06]
$\overline{B}^0 \to D^{*+} D_s^{*-}$		$(1.77 \pm 0.14)\%$	[BaBar'06], [BaBar'05], [LHCb'21]
$\overline{B}^0 \to D^{*+} D_{s0}(2317)^-$		$(1.5 \pm 0.6) \times 10^{-3}$	[BaBar'04]
$\overline{B}^0 \to D^{*+} D_{s1}(2460)^-$		$(9.3 \pm 2.2) \times 10^{-3}$	[BaBar'04]
$\overline{B}^0 \to D^{*+} D_{s1}(2536)^-$		$(0.50 \pm 0.14) \times 10^{-3}$	[Belle'11]
$\overline{B}^0 \to D^{*+} D^0 K^-$		$(2.47 \pm 0.21) \times 10^{-3}$	[BaBar'11], [LHCb'20]
$\overline{B}^0 \to D^{*+} \overline{D}^{*0} K^-$		$(1.06 \pm 0.09)\%$	[BaBar'11]
$B^+ \to D^{*+} \overline{D}^0 K^0$		$(3.8 \pm 0.4) \times 10^{-3}$	[BaBar'11]
$B^+ \to D^{*+} \overline{D}^{*0} K^0$		$(9.2 \pm 1.2) \times 10^{-3}$	[BaBar'11]
$B \to D^{**} D_s^{(*)-}$		$(2.7 \pm 1.2)\%$	[CLEO'00]
$B^+ \to D^{*+} D^- K^+$	$c\bar{c}$	$(0.63 \pm 0.11) \times 10^{-3}$	[BaBar'11], [LHCb'20]
$B^- \to D^{*+} D^- K^-$		$(0.60 \pm 0.13) \times 10^{-3}$	[BaBar'11], [LHCb'20]
$B^- \to D^{*+} D^{*-} K^-$		$(1.32 \pm 0.18) \times 10^{-3}$	[BaBar'11]
$B^0/\overline{B}^0 \to D^{*+} D^- K^0/\overline{K}^0$	$c\bar{c}$, $c\bar{s}$	$(6.4 \pm 0.5) \times 10^{-3}$	[BaBar'11]
$\overline{B}^0 \to D^{*+} D^{*-} \overline{K}^0$		$(8.1 \pm 0.7) \times 10^{-3}$	[BaBar'11]

BaBar: $\mathcal{B}[B \to D^{(*)} D_s(2317, 2460)^+]$

[BaBar'04]

Exclusive reconstruction in $D_s^+\pi^0$, $D_s^+\gamma$ and $D_s^{*+}\pi^0$ final states.

Using sample of 122M $B\overline{B}$ pairs (only 1/4 of the full BaBar stats!)



Belle (II) and LHCb could improve this (also adding $D_{s1}(2460)^+ \to D_s^+\pi^+\pi^-$ mode)

BaBar: $\mathcal{B}[B \to D^{(*)} D_s(2317, 2460)^+]$ [BaBar'04]

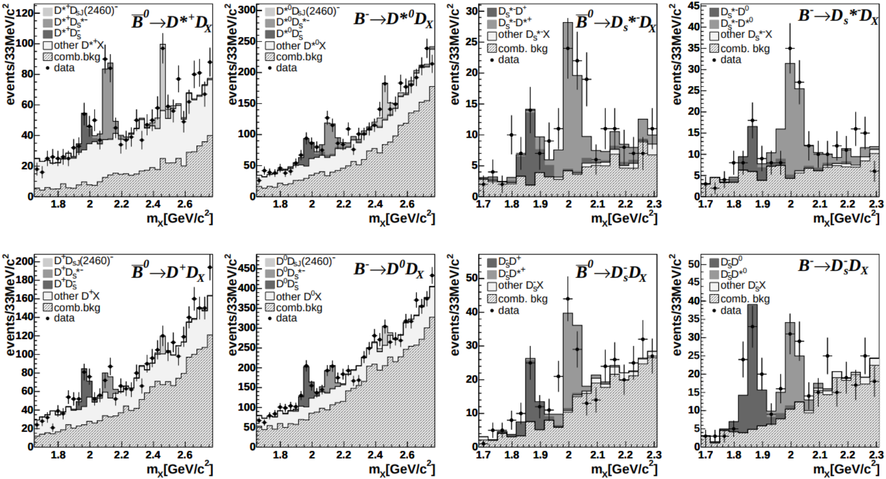
Exclusive reconstruction in $D_s^+\pi^0$, $D_s^+\gamma$ and $D_s^{*+}\pi^0$ final states.

Using sample of 122M $B\overline{B}$ pairs (only 1/4 of the full BaBar stats!)

B mode		$\mathcal{B}(10^{-3})$	Significance
$B^0 \to D_{sJ}^*(2317)^+ D^-$	$[D_s^+\pi^0]$	$1.8 \pm 0.4 \pm 0.3^{+0.6}_{-0.4}$	5.5
$B^0 \to D_{sJ}^*(2317)^+ D^{*-}$	$[D_s^+\pi^0]$	$1.5 \pm 0.4 \pm 0.2^{+0.5}_{-0.3}$	5.2
$B^+ \to D_{sJ}^*(2317)^+ \overline{D}^0$	$[D_s^+\pi^0]$	$1.0 \pm 0.3 \pm 0.1^{+0.4}_{-0.2}$	3.1
$B^+ \to D_{sJ}^*(2317)^+ \overline{D}^{*0}$	$[D_s^+\pi^0]$	$0.9 \pm 0.6 \pm 0.2^{+0.3}_{-0.2}$	2.5
$B^0 \to D_{sJ}(2460)^+ D^-$	$[D_s^{*+}\pi^0]$	$2.8 \pm 0.8 \pm 0.5^{+1.0}_{-0.6}$	4.2
$B^0 \to D_{sJ}(2460)^+ D^{*-}$	$[D_s^{*+}\pi^0]$	$5.5 \pm 1.2 \pm 1.0^{+1.9}_{-1.2}$	7.4
$B^+ \to D_{sJ}(2460)^+ \overline{D}^0$	$[D_s^{*+}\pi^0]$	$2.7 \pm 0.7 \pm 0.5^{+0.9}_{-0.6}$	5.1
$B^+ \to D_{sJ}(2460)^+ \overline{D}^{*0}$	$[D_s^{*+}\pi^0]$	$7.6 \pm 1.7 \pm 1.8^{+2.6}_{-1.6}$	7.7
$B^0 \to D_{sJ}(2460)^+ D^-$	$[D_s^+\gamma]$	$0.8 \pm 0.2 \pm 0.1^{+0.3}_{-0.2}$	5.0
$B^0 \to D_{sJ}(2460)^+ D^{*-}$	$[D_s^+\gamma]$	$2.3 \pm 0.3 \pm 0.3^{+0.8}_{-0.5}$	11.7
$B^+ \to D_{sJ}(2460)^+ \overline{D}^0$	$[D_s^+\gamma]$	$0.6 \pm 0.2 \pm 0.1^{+0.2}_{-0.1}$	4.3
$B^+ \to D_{sJ}(2460)^+ \overline{D}^{*0}$	$[D_s^+\gamma]$	$1.4 \pm 0.4 \pm 0.3^{+0.5}_{-0.3}$	6.0

Belle (II) and LHCb could improve this (also adding $D_{s1}(2460)^+ \to D_s^+\pi^+\pi^-$ mode)

Partial reconstruction: reconstruct only one of D mesons, obtain mass spectrum for the other from B kinematic constraint



Uses $210\,\mathrm{fb}^{-1}$ of BaBar data ($\sim 1/2$ of the full stats)

$\mathcal{B}$ obtained by combining results from two $D_{\rm meas}$.

Decay mode	$D_{\rm meas}$	$N_{\rm fit}$	$\varepsilon(\%)$	$\mathcal{B}(\%)$	Combined $\mathcal{B}(\%)$
$\overline{B}^0 \to D_s^- D^+$	D^+	86 ± 17	3.29 ± 0.16	$0.90 \pm 0.18 \pm 0.14$	$0.64 \pm 0.13 \pm 0.10$
	D_s^-	39 ± 9	1.79 ± 0.12	$(0.74 \pm 0.17 \pm 0.13) \cdot k$	
$\overline{B}^0 \to D_s^{*-} D^+$	D^+	63 ± 19	3.24 ± 0.16	$0.67 \pm 0.20 \pm 0.11$	$0.69 \pm 0.16 \pm 0.09$
	D_s^{*-}	30 ± 9	0.91 ± 0.08	$(1.15 \pm 0.33 \pm 0.26) \cdot k$	
$\overline{B}^0 \to D_s^- D^{*+}$	D^{*+}	48 ± 13	2.86 ± 0.13	$0.57 \pm 0.16 \pm 0.09$	$0.71 \pm 0.13 \pm 0.09$
	D_s^-	68 ± 12	1.63 ± 0.10	$(1.42 \pm 0.26 \pm 0.20) \cdot k$	
$\overline{B}^0 \to D_s^{*-} D^{*+}$	D^{*+}	129 ± 18	2.68 ± 0.09	$1.65 \pm 0.23 \pm 0.19$	$1.68 \pm 0.21 \pm 0.19$
	D_s^{*-}	84 ± 14	0.86 ± 0.05	$(3.38 \pm 0.60 \pm 0.61) \cdot k$	
$B^- \to D_s^- D^0$	D^0	214 ± 28	3.46 ± 0.11	$1.33 \pm 0.18 \pm 0.32$	$0.92 \pm 0.14 \pm 0.18$
	D_s^-	66 ± 10	1.28 ± 0.07	$(1.11 \pm 0.17 \pm 0.17) \cdot k$	
$B^- \to D_s^{*-} D^0$	D^0	160 ± 31	3.71 ± 0.12	$0.93 \pm 0.18 \pm 0.19$	$0.77 \pm 0.15 \pm 0.13$
	D_s^{*-}	26 ± 10	0.64 ± 0.05	$(0.87 \pm 0.33 \pm 0.16) \cdot k$	
$B^- \to D_s^- D^{*0}$	D^{*0}	152 ± 29	2.69 ± 0.10	$1.21 \pm 0.23 \pm 0.20$	$0.76 \pm 0.15 \pm 0.13$
	D_s^-	52 ± 11	1.33 ± 0.07	$(0.82 \pm 0.18 \pm 0.10) \cdot k$	
$B^- \to D_s^{*-} D^{*0}$	D^{*0}	216 ± 33	2.73 ± 0.07	$1.70 \pm 0.26 \pm 0.24$	$1.62 \pm 0.22 \pm 0.18$
	D_s^{*-}	90 ± 15	0.82 ± 0.04	$(2.38 \pm 0.41 \pm 0.31) \cdot k$	
$D_s^- \to \phi\pi^-$	-	-	-	-	$4.58 \pm 0.48 \pm 0.68$
$\overline{B}^0 \to D_{sJ}(2460)^- D^+$	D^+	27 ± 16	3.61 ± 0.27	$0.26 \pm 0.15 \pm 0.07$	
$\overline{B}^0 \to D_{sJ}(2460)^- D^{*+}$	D^{*+}	64 ± 15	2.51 ± 0.15	$0.88 \pm 0.20 \pm 0.14$	
$B^- \to D_{sJ}(2460)^- D^0$	D^0	75 ± 28	3.78 ± 0.24	$0.43 \pm 0.16 \pm 0.13$	
$B^- \to D_{sJ}(2460)^- D^{*0}$	D^{*0}	147 ± 34	2.81 ± 0.14	$1.12 \pm 0.26 \pm 0.20$	

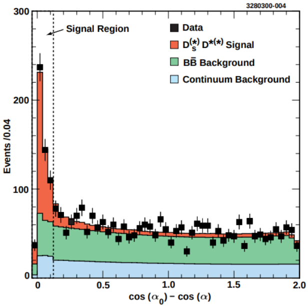
Independent on the assumptions on D_s^* $\mathcal{B}$'s.

Obtain $\mathcal{B}(D_s^- \to \phi\pi^-)$ by combining full and part-rec D_s^-

Partial reconstruction: reconstruct only D_s^+ and π_s from $D^{*+} \to D^0 \pi_s$

Signal selection: angles α (btw. π_s and D_s^+), $\alpha_0 = |\theta_1 - \theta_2|$:

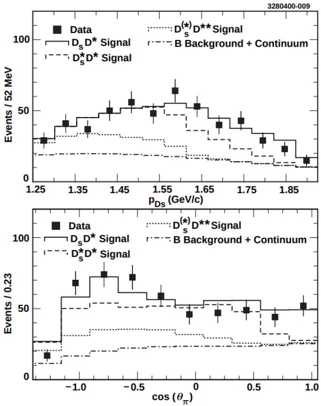
$\theta_{1,2}$: angle btw, π_s/D_s^+ and cone of allowed D^{*-} directions



$\mathcal{B}(B^0 \to D_s^+ D^{*-}) = (1.10 \pm 0.18 \pm 0.11 \pm 0.28)\%$

$\mathcal{B}(B^0 \to D_s^{*+} D^{*-}) = (1.82 \pm 0.37 \pm 0.25 \pm 0.46)\%$

$\mathcal{B}(B^0 \to D_s^{(*)+} \bar{D}^{**0}) = (2.73 \pm 0.78 \pm 0.51 \pm 0.68)\%$



2D fit to separate 3 modes

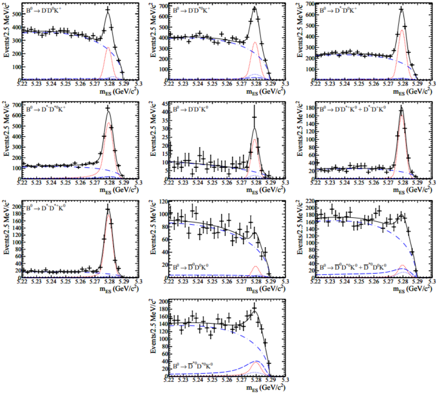
Uses many assumptions (*e.g.* fractions of D^{**} states). LHCb can improve a lot here.

BaBar: $\mathcal{B}(B \to D\bar{D}K)$

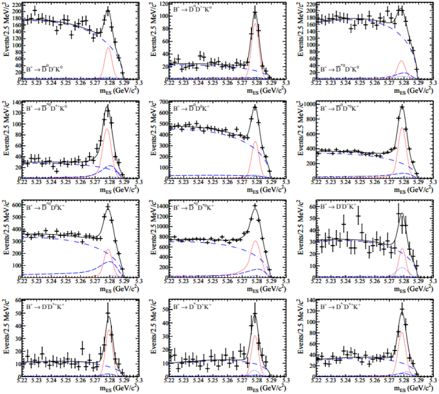
[BaBar'11]

All possible combinations with $D = (D^0, D^+, D^{*0}, D^{*+})$ and $K = (K^+, K^0)$

B^0 decays:



B^+ decays:



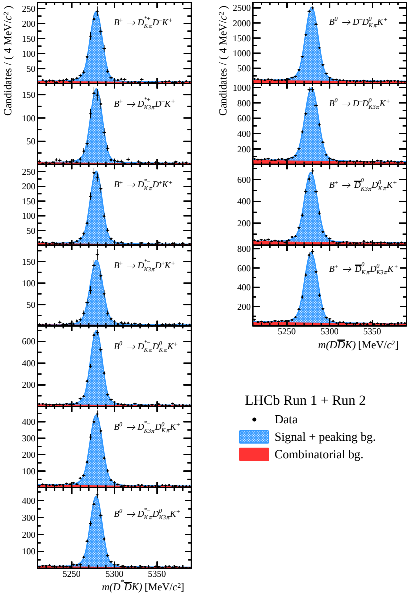
Uses full BaBar data sample ($429\,\text{fb}^{-1}$)

Mode	$\mathcal{B}$, $\times 10^{-4}$	Signif.
B^0 decays through external W-emission amplitudes		
$B^0 \to D^- D^0 K^+$	$10.7 \pm 0.7 \pm 0.9$	8.6σ
$B^0 \to D^- D^{*0} K^+$	$34.6 \pm 1.8 \pm 3.7$	7.6σ
$B^0 \to D^{*-} D^0 K^+$	$24.7 \pm 1.0 \pm 1.8$	12.6σ
$B^0 \to D^{*-} D^{*0} K^+$	$106.0 \pm 3.3 \pm 8.6$	11.4σ
B^0 decays through external+internal W-emission amplitudes		
$B^0 \to D^- D^+ K^0$	$7.5 \pm 1.2 \pm 1.2$	5.1σ
$B^0 \to D^- D^{*+} K^0 + D^{*-} D^+ K^0$	$64.1 \pm 3.6 \pm 3.9$	13.4σ
$B^0 \to D^{*-} D^{*+} K^0$	$82.6 \pm 4.3 \pm 6.7$	12.5σ
B^0 decays through internal W-emission amplitudes		
$B^0 \to \bar{D}^0 D^0 K^0$	$2.7 \pm 1.0 \pm 0.5$	2.3σ
$B^0 \to \bar{D}^0 D^{*0} K^0 + \bar{D}^{*0} D^0 K^0$	$10.8 \pm 3.2 \pm 3.6$	2.2σ
$B^0 \to \bar{D}^{*0} D^{*0} K^0$	$24.0 \pm 5.5 \pm 6.7$	2.2σ
B^+ decays through external W-emission amplitudes		
$B^+ \to \bar{D}^0 D^+ K^0$	$15.5 \pm 1.7 \pm 1.3$	6.6σ
$B^+ \to \bar{D}^0 D^{*+} K^0$	$38.1 \pm 3.1 \pm 2.3$	10.7σ
$B^+ \to \bar{D}^{*0} D^+ K^0$	$20.6 \pm 3.8 \pm 3.0$	3.3σ
$B^+ \to \bar{D}^{*0} D^{*+} K^0$	$91.7 \pm 8.3 \pm 9.0$	7.5σ
B^+ decays through external+internal W-emission amplitudes		
$B^+ \to \bar{D}^0 D^0 K^+$	$13.1 \pm 0.7 \pm 1.2$	8.6σ
$B^+ \to \bar{D}^0 D^{*0} K^+$	$63.2 \pm 1.9 \pm 4.5$	12.5σ
$B^+ \to \bar{D}^{*0} D^0 K^+$	$22.5 \pm 1.0 \pm 1.5$	22.4σ
$B^+ \to \bar{D}^{*0} D^{*0} K^+$	$112.3 \pm 3.6 \pm 12.6$	6.8σ
B^+ decays through internal W-emission amplitudes		
$B^+ \to D^- D^+ K^+$	$2.2 \pm 0.5 \pm 0.5$	2.8σ
$B^+ \to D^- D^{*+} K^+$	$6.3 \pm 0.9 \pm 0.6$	6.7σ
$B^+ \to D^{*-} D^+ K^+$	$6.0 \pm 1.0 \pm 0.8$	5.1σ
$B^+ \to D^{*-} D^{*+} K^+$	$13.2 \pm 1.3 \pm 1.2$	7.4σ

$\longleftarrow$ Signal seen in all combinations (but some are only $\sim 2\sigma$)

Mode	$r \times \tau_{B^+}/\tau_{B^0}$
$\mathcal{B}(B^0 \to D^- D^0 K^+)/\mathcal{B}(B^+ \to \bar{D}^0 D^+ K^0)$	0.74 ± 0.13
$\mathcal{B}(B^0 \to D^- D^{*0} K^+)/\mathcal{B}(B^+ \to \bar{D}^0 D^{*+} K^0)$	0.97 ± 0.15
$\mathcal{B}(B^0 \to D^{*-} D^0 K^+)/\mathcal{B}(B^+ \to \bar{D}^{*0} D^+ K^0)$	1.28 ± 0.32
$\mathcal{B}(B^0 \to D^{*-} D^{*0} K^+)/\mathcal{B}(B^+ \to \bar{D}^{*0} D^{*+} K^0)$	1.24 ± 0.20
$\mathcal{B}(B^0 \to D^- D^+ K^0)/\mathcal{B}(B^+ \to \bar{D}^0 D^0 K^+)$	0.61 ± 0.15
$\dfrac{\mathcal{B}(B^0 \to D^- D^{*+} K^0 + D^{*-} D^+ K^0)}{\mathcal{B}(B^+ \to \bar{D}^0 D^{*0} K^+) + \mathcal{B}(B^+ \to \bar{D}^{*0} D^0 K^+)}$	0.80 ± 0.09
$\mathcal{B}(B^0 \to D^{*-} D^{*+} K^0)/\mathcal{B}(B^+ \to \bar{D}^{*0} D^{*0} K^+)$	0.79 ± 0.12
$\mathcal{B}(B^0 \to \bar{D}^0 D^0 K^0)/\mathcal{B}(B^+ \to D^- D^+ K^+)$	1.28 ± 0.69
$\dfrac{\mathcal{B}(B^0 \to \bar{D}^0 D^{*0} K^0 + \bar{D}^{*0} D^0 K^0)}{\mathcal{B}(B^+ \to D^- D^{*+} K^+) + \mathcal{B}(B^+ \to D^{*-} D^+ K^+)}$	0.94 ± 0.44
$\mathcal{B}(B^0 \to \bar{D}^{*0} D^{*0} K^0)/\mathcal{B}(B^+ \to D^{*-} D^{*+} K^+)$	1.94 ± 0.75

Check isospin symmetry: some ratios up to 2.6σ different from 1



Fully reconstructed charged final states only

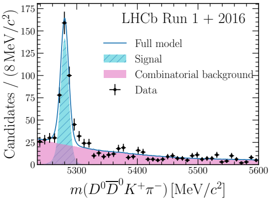
$$\frac{\mathcal{B}(B^+ \to D^{*+}D^-K^+)}{\mathcal{B}(B^+ \to \overline{D}^0 D^0 K^+)} = 0.517 \pm 0.015 \pm 0.013 \pm 0.011,$$

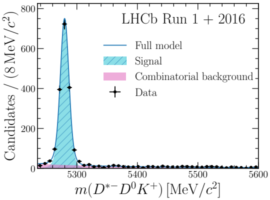
$$\frac{\mathcal{B}(B^+ \to D^{*-}D^+K^+)}{\mathcal{B}(B^+ \to \overline{D}^0 D^0 K^+)} = 0.577 \pm 0.016 \pm 0.013 \pm 0.013,$$

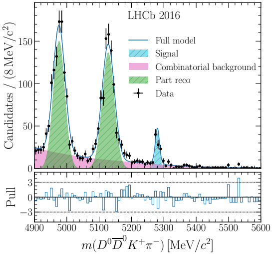
$$\frac{\mathcal{B}(B^0 \to D^{*-}D^0K^+)}{\mathcal{B}(B^0 \to D^- D^0 K^+)} = 1.754 \pm 0.028 \pm 0.016 \pm 0.035,$$

$$\frac{\mathcal{B}(B^+ \to D^{*+}D^-K^+)}{\mathcal{B}(B^+ \to D^{*-}D^+K^+)} = 0.907 \pm 0.033 \pm 0.014,$$

Measure $\mathcal{B}$ of $B^0 \to D^0 \overline{D}^0 K^+ \pi^-$ using $D^{*-} D^0 K^+$ as a reference







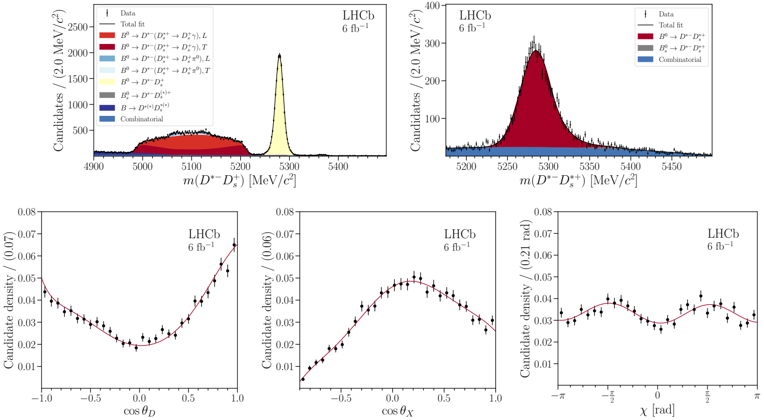
$$\mathcal{B}(B^0 \to D^0 \overline{D}^0 K^+ \pi^-) = (3.50 \pm 0.27 \pm 0.26 \pm 0.30) \times 10^{-4}$$

But large contribution from part-rec $B \to D^{(*)} \overline{D}^{(*)} K^+ \pi^-$ is seen in the spectrum.

Uses 2011+2012+2016 LHCb data. Could further extend to:

- exclusive $D^{*+} \overline{D}^{(*)} K \pi$ final states
- Measure $\mathcal{B}$ in part-rec fits
- Final states with K^0_S

Also interesting to perform amplitude analysis (*e.g.* are "exotic" $D_s^+ \pi$ and DK peaks due to $D^* K^*$ interaction?)

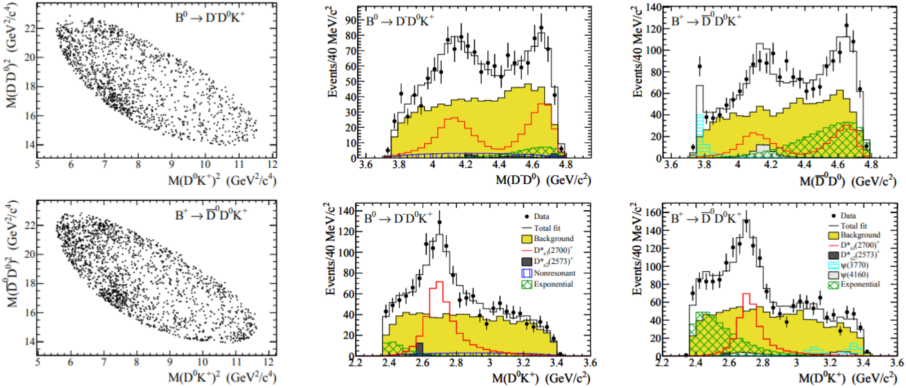


$\mathcal{B}$ and F_L from part-rec fit, remaining params from exclusive $D_s^{*+} \to D_s^+ \gamma$

$$\frac{\mathcal{B}(B^0 \to D^{*-} D_s^{*+})}{\mathcal{B}(B^0 \to D^{*-} D_s^{+})} = 2.045 \pm 0.022 \pm 0.071$$

$$f_L = 0.578 \pm 0.010 \pm 0.011$$

$$\begin{aligned}
|H_0| &= 0.760 \pm 0.007 \pm 0.007, \\
|H_-| &= 0.195 \pm 0.022 \pm 0.032, \\
|H_+| &= 0.620 \pm 0.011 \pm 0.013, \\
\phi_- &= -0.046 \pm 0.102 \pm 0.020 \text{ rad}, \\
\phi_+ &= 0.108 \pm 0.170 \pm 0.051 \text{ rad}.
\end{aligned}$$



Contribution	Modulus	Phase (°)	Fraction (%)
$D^*_{s1}(2700)^+$	1.00	0	$66.7 \pm 7.8^{+3.5}_{-3.8}$
$D^*_{s2}(2573)^+$	$0.031 \pm 0.008 \pm 0.002$	$277 \pm 17^{+6}_{-9}$	$3.2 \pm 1.6^{+0.3}_{-0.4}$
Nonresonant	$1.33 \pm 0.63^{+0.46}_{-0.35}$	$287 \pm 21^{+10}_{-15}$	$10.9 \pm 6.6^{+7.0}_{-4.3}$
Exponential	$6.94 \pm 1.83^{+0.82}_{-0.43}$	$269 \pm 33^{+17}_{-15}$	$9.9 \pm 2.9^{+3.0}_{-3.3}$
Sum			$90.6 \pm 10.7^{+8.4}_{-6.7}$

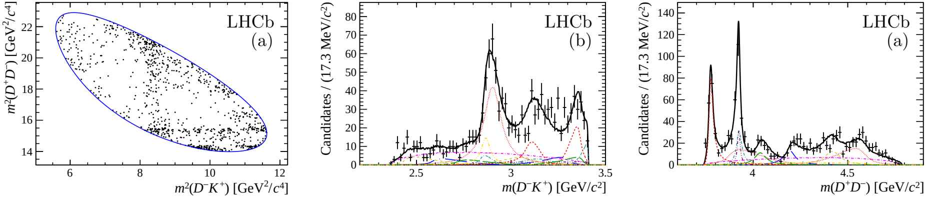
Contribution	Modulus	Phase (°)	Fraction (%)
$D^*_{s1}(2700)^+$	1.00	0	$38.3 \pm 5.0^{+0.8}_{-6.2}$
$D^*_{s2}(2573)^+$	$0.021 \pm 0.010^{+0.009}_{-0.003}$	$267 \pm 30^{+17}_{-13}$	$0.6 \pm 1.1^{+0.4}_{-0.2}$
$\psi(3770)$	$1.40 \pm 0.21^{+0.20}_{-0.24}$	$284 \pm 22^{+26}_{-30}$	$9.0 \pm 3.1^{+0.4}_{-0.8}$
$\psi(4160)$	$0.78 \pm 0.20^{+0.18}_{-0.14}$	$188 \pm 13^{+14}_{-17}$	$6.4 \pm 3.1^{+1.9}_{-2.4}$
Exponential	$16.15 \pm 2.26^{+1.09}_{-1.74}$	$308 \pm 8^{+6}_{-5}$	$44.5 \pm 6.2^{+1.3}_{-2.1}$
Sum			$98.9 \pm 9.2^{+2.5}_{-7.0}$

Large "non-resonant" contributions found

LHCb: $B^+ \to D^+ D^- K^+$ amplitude

[LHCb, PRD 102 (2020) 112003]

$B^+ \to D^+ D^- K^+$ decay



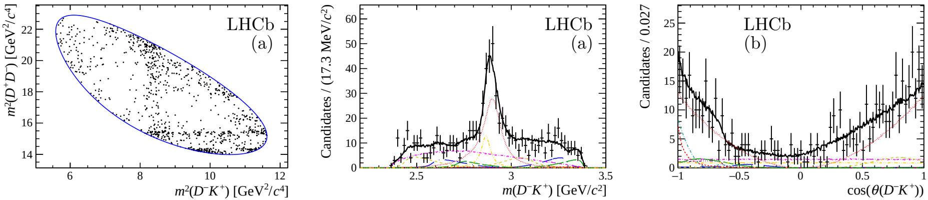
Amplitude analysis including charmonium and $D^- K^+$ resonances

$T_{cs0}(2900)^0$: $M = 2866 \pm 7 \pm 2\,\mathrm{MeV}$, $\Gamma = 57 \pm 12 \pm 4\,\mathrm{MeV}$, $FF = (5.6 \pm 1.4 \pm 0.5)\%$
$T_{cs1}(2900)^0$: $M = 2904 \pm 5 \pm 1\,\mathrm{MeV}$, $\Gamma = 110 \pm 11 \pm 4\,\mathrm{MeV}$, $FF = (30.6 \pm 2.4 \pm 2.1)\%$

Significant contribution from “exotic” structure in $D^- K^+$ (tetraquark or kinematic effect *e.g.* $D^* K^*$ threshold)

Many connections between different $B \to DDK$ modes (same channels, isospin relations, rescattering ...): calls for combined fit for consistent description.

$B^+ \to D^+ D^- K^+$ decay



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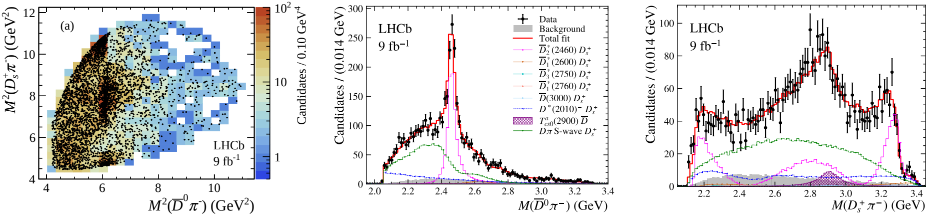
LHCb: $B \to D D_s^+ \pi$ amplitude

[LHCb, arXiv:2212.02717]

Simultaneous analysis of $B^+ \to D^- D_s^+ \pi^+$ and $B^0 \to \bar{D}^0 D_s^+ \pi^-$ decays
Same amplitude if isospin symmetry, mostly $D^{**} \to D\pi$

$B^0 \to \bar{D}^0 D_s^+ \pi^-$:



Scalar resonance-like structures in $D_s^+\pi$ channel:

$T_{c\bar{s}0}(2900)^0$: $M = 2879 \pm 17 \pm 18\,\mathrm{MeV}$, $\Gamma = 153 \pm 28 \pm 20\,\mathrm{MeV}$
$T_{c\bar{s}0}(2900)^{++}$: $M = 2935 \pm 21 \pm 13\,\mathrm{MeV}$, $\Gamma = 143 \pm 38 \pm 25\,\mathrm{MeV}$

No sign of $D_{s0}^*(2317)\pi$ or $D_{s1}(2460)$ isospin partners

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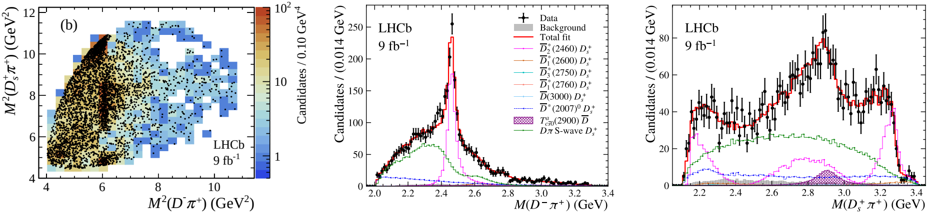

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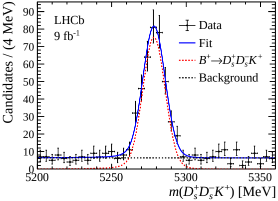
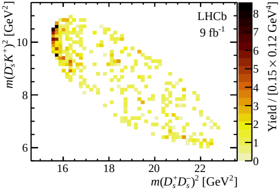
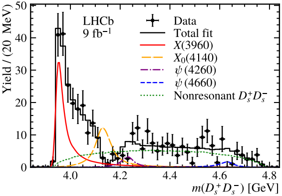
Mostly the internal W emission with $c\bar{c} \to D_s^+ D_s^-$ $\Rightarrow$ suppressed







$$\mathcal{R} = \frac{\mathcal{B}(B^+ \to D_s^+ D_s^- K^+)}{\mathcal{B}(B^+ \to D^+ D^- K^+)} = 0.525 \pm 0.033 \pm 0.027 \pm 0.034.$$

$$\mathcal{B}(B^+ \to D_s^+ D_s^- K^+) = (1.15 \pm 0.07 \pm 0.06 \pm 0.38) \times 10^{-4}.$$

$B \to D_s D_s^* K$ contribution?

Amplitude analysis: mostly $D_s^+ D_s^-$ structures. Dip at 4140 MeV due to $J/\psi\,\phi \to D_s^+ D_s^-$ rescattering?

$\mathcal{B}$ and amplitude structure:

- $B \to D^{**} D_s^{(*)-}$ (incl $D_s^*(2317)^-$ and $D_s^*(2460)^-$)
- $B \to D^{(*)} \overline{D}^{(*)} K\pi$ (incl. K^0 and/or π^0)
- Modes with $\pi^+\pi^- \to K^+K^-$ rescattering, *e.g.* $B \to D^{(*)} D_s^+ K^+ K^-$
- Cabibbo-suppressed $b \to c\bar{c}d$ modes: $D^{(*,**)} \overline{D}^{(*,**)}$

Amplitude analyses:

- Analyses with excited states: harder but doable (> 2 degrees of freedom)
- Can improve for 3-body e.g. $B \to DDK$: complicated because of exotic structures, combined analyses employing isospin symmetries?

Partial reconstruction with Belle-II

- $B \to D\overline{D}$ with one D fully reconstructed and applying kinematic constraint from B (similar to [BaBar'06])

Inclusive measurements?

- Secondary $D\overline{D}$ combinations:
  $D^0\overline{D}^0$, $D^0 D^-$, $D^+ D^-$, $D^0 D_s^-$, $D_s^+ D^-$, $D^{*+}\overline{D}^0$, $D^{*+} D^-$, $D^{*+} D_s^-$, $D^{*+} D^{*-}$, ...
- Constrain the amount of $B \to D\overline{D}X$ decays not measured in exclusive analyses.

B_s^0 decays

The only double charm B_s^0 modes observed so far:

- $B_s^0 \to D_s^+ D_s^-$: $\mathcal{B} = (4.4 \pm 0.5) \times 10^{-3}$ (mostly [LHCb, PRD 87 (2013) 092007])
- $B_s^0 \to D_s^{*+} D_s^- + B_s^0 \to D_s^+ D_s^{*-}$: $\mathcal{B} = (1.4 \pm 0.2)\%$ [LHCb, PRD 93 (2016) 092008]
- $B_s^0 \to D_s^{*+} D_s^{*-}$: $\mathcal{B} = (1.4 \pm 0.2)\%$ [LHCb, PRD 93 (2016) 092008]
- Cabibbo-suppressed:
  - $B_s^0 \to D_s^- D^+$: $\mathcal{B} = (2.8 \pm 0.5) \times 10^{-4}$
  - $B_s^0 \to D_s^- D^{*+}$: $\mathcal{B} = (3.9 \pm 0.8) \times 10^{-4}$
- W-exchange or $D_s^+ D_s^+ \to DD$ rescattering, $B_s^0 \to D^{(*)+} D^{(*)-}$, $\mathcal{B} \sim 10^{-4}$.

Dominant $b \to c\bar{c}s$ transitions for B_s^0 decays **not measured yet**:

- External W: $B_s^0 \to D_s^{(*)+} D_s^{(*)-}$
  - $B_s^0 \to D^{(*)+} D_s^{(*)-} K^0$, $B_s^0 \to D^{(*)0} D_s^{(*)-} K^+$, $B_s^0 \to D^{**0} D_s^{(*)-} K^+$
- Internal W: $B_s^0 \to (c\overline{c} \to D\overline{D})\phi$
  - $B_s^0 \to D^{(*)+} D^{(*)-} \phi$

Λ_b^0 decays

The only Λ_b^0 double-charm decays observed:

- $\Lambda_b^0 \to \Lambda_c^+ D_s^+$: $\mathcal{B} = (1.10 \pm 0.10)\%$ [LHCb, PRL 112 (2014) 202001]
- $\Lambda_b^0 \to \Lambda_c^+ D^-$: $\mathcal{B} = (4.6 \pm 0.6) \times 10^{-4}$ [LHCb, PRL 112 (2014) 202001]

Dominant $b \to c\bar{c}s$ Λ_b^0 decays **not measured yet**:

- External W: $\Lambda_b^0 \to \Lambda_c^{(*)+} D_s^{(*)-}$
  - $\Lambda_b^0 \to \Lambda_c^+ D^{(*,**)} K$
  - $\Lambda_b^0 \to D^{(*)+} D_s^- n$
  - $\Lambda_b^0 \to D^{(*)0} D_s^- p$: $\Lambda_b^0 \to D^0 D_s^- p$ spectrum + part-rec
  - $\Lambda_b^0 \to D^{*+} \pi^- D_s^- p$, $\Lambda_b^0 \to D^{*+} \overline{D}^0 n K^-$ ($\Lambda_b^0 \to D^0 \overline{D}^0 p K^-$), etc.
- Internal W: $\Lambda_b^0 \to (c\overline{c} \to D\overline{D})\Lambda^{(*)}$
  - $\Lambda_b^0 \to D^{(*)} \overline{D}^{(*)} \Lambda^0$
  - $\Lambda_b^0 \to D^{(*)} \overline{D}^{(*)} p K^-$

Conclusion

- Studies of $B \to$ double charm decays is important for many reasons (background to SL decays, FF determination, spectroscopy)
- $\mathcal{B}$ measurements measured systematically mostly by BaBar
  - Based on a part of BaBar dataset! LHCb and Belle II could improve a lot.
  - LHCb: fully-charged final states, single γ, π^0, $K^0_{\rm S}$ — OK. Partial reconstruction of $D_{(s)}$.
  - Belle II: especially the final states with neutrals. Inclusive measurements using B kinematic constraints.
- Many measurements with higher $D_{(s)}$ excitations are still missing (and can be non-negligible)
- Can we do inclusive $\mathcal{B}(B \to D\bar{D}X)$ with LHCb using $D\bar{D}$ displacement?
  - Upper limit on the unmeasured exclusive decays
- Amplitude fits, incl. those beyond simple Dalitz plots: hard but doable.
- CP asymmetry: should be suppressed, but model-indep. test useful for CPV in SL decays.
- B^0_s and Λ^0_b studies