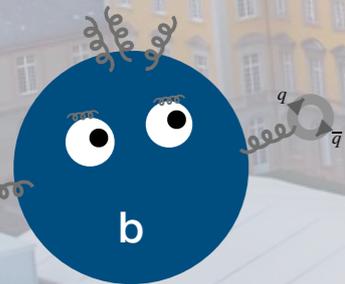


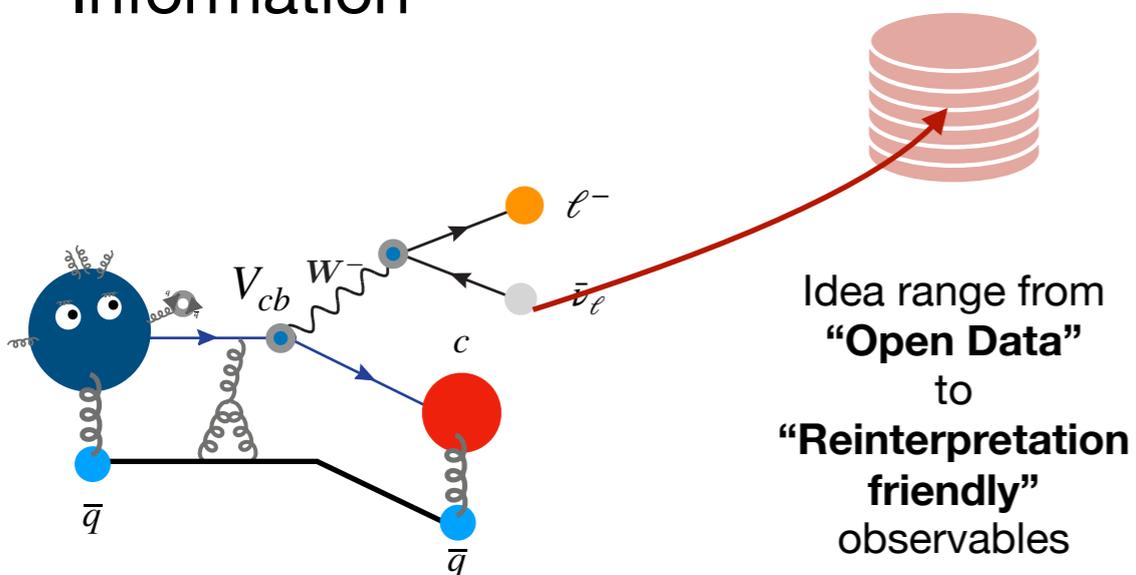
Prospects of Angular Analyses of
 $B \rightarrow D^* \ell \bar{\nu}_\ell$ at the B-Factories

LHCb Open Workshop on Semileptonic Decays

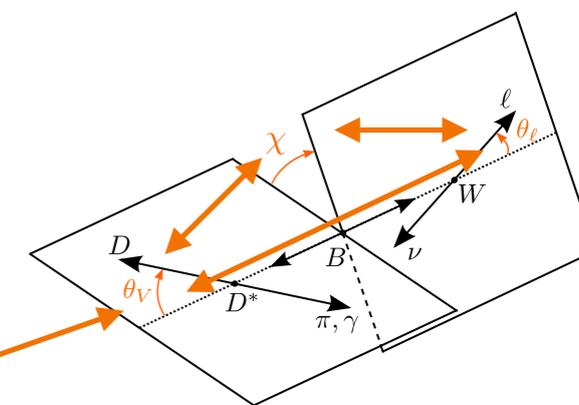
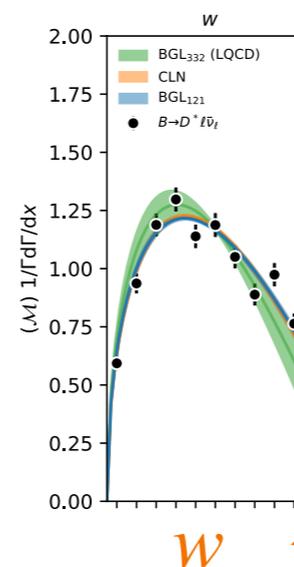


Talk Overview

1. Why we need to do better to preserve our experimental Information

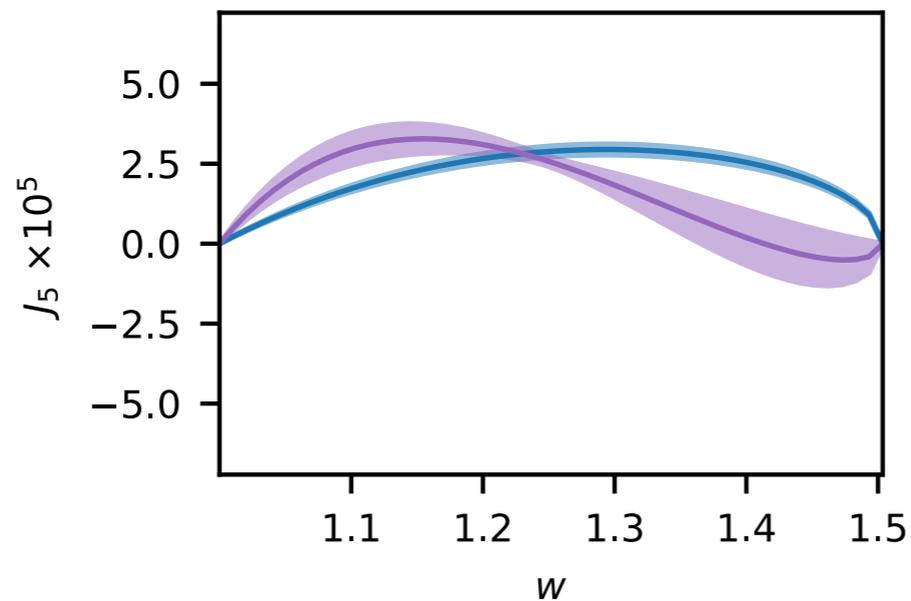


2. From 1D projections to full angular information



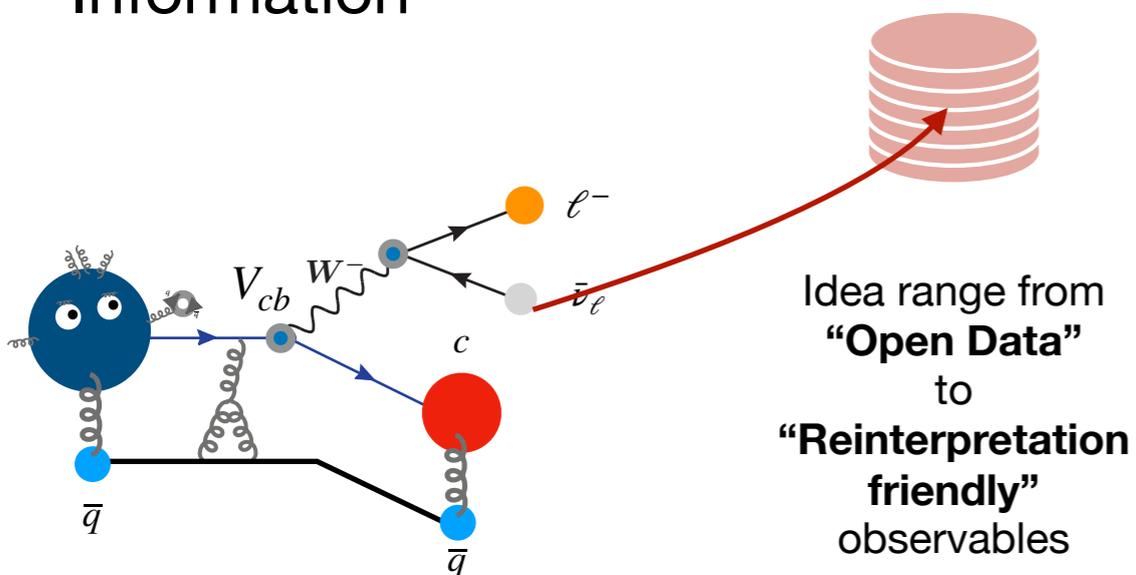
Working around the curse of dimensionality

3. Potential of full angular fits

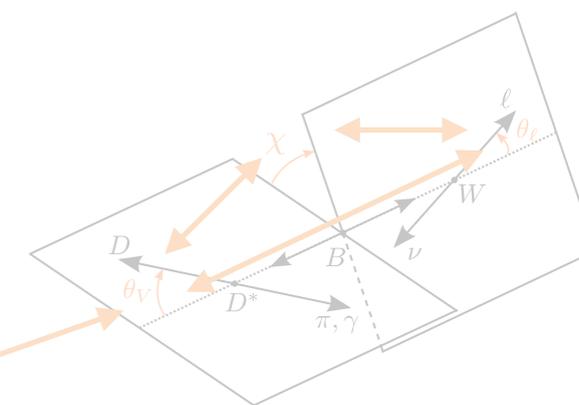
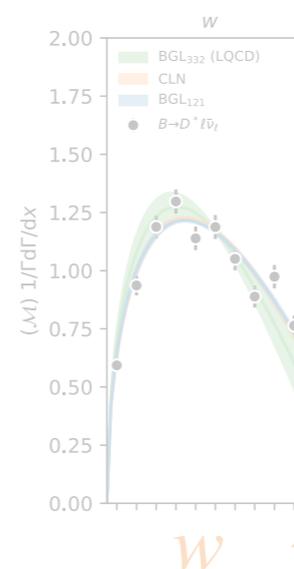


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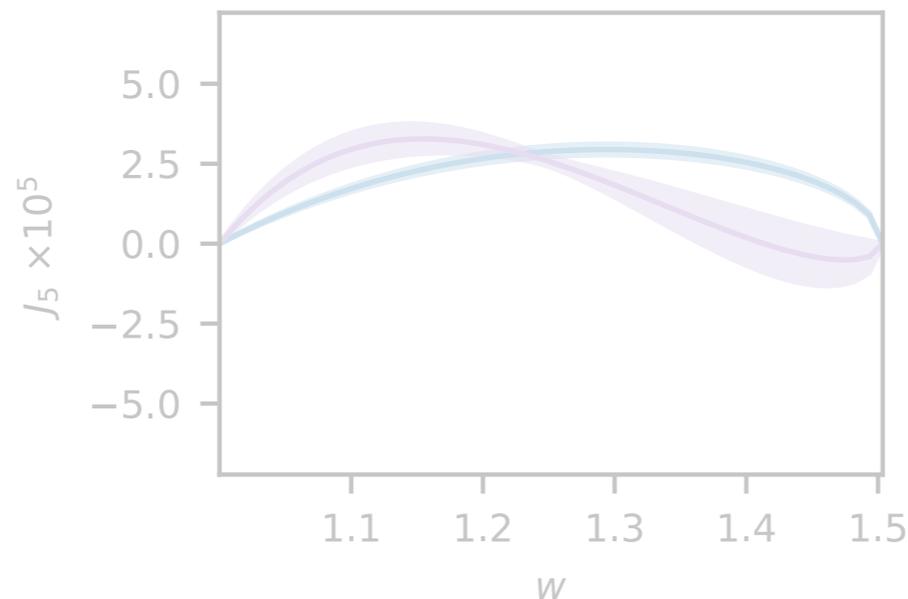


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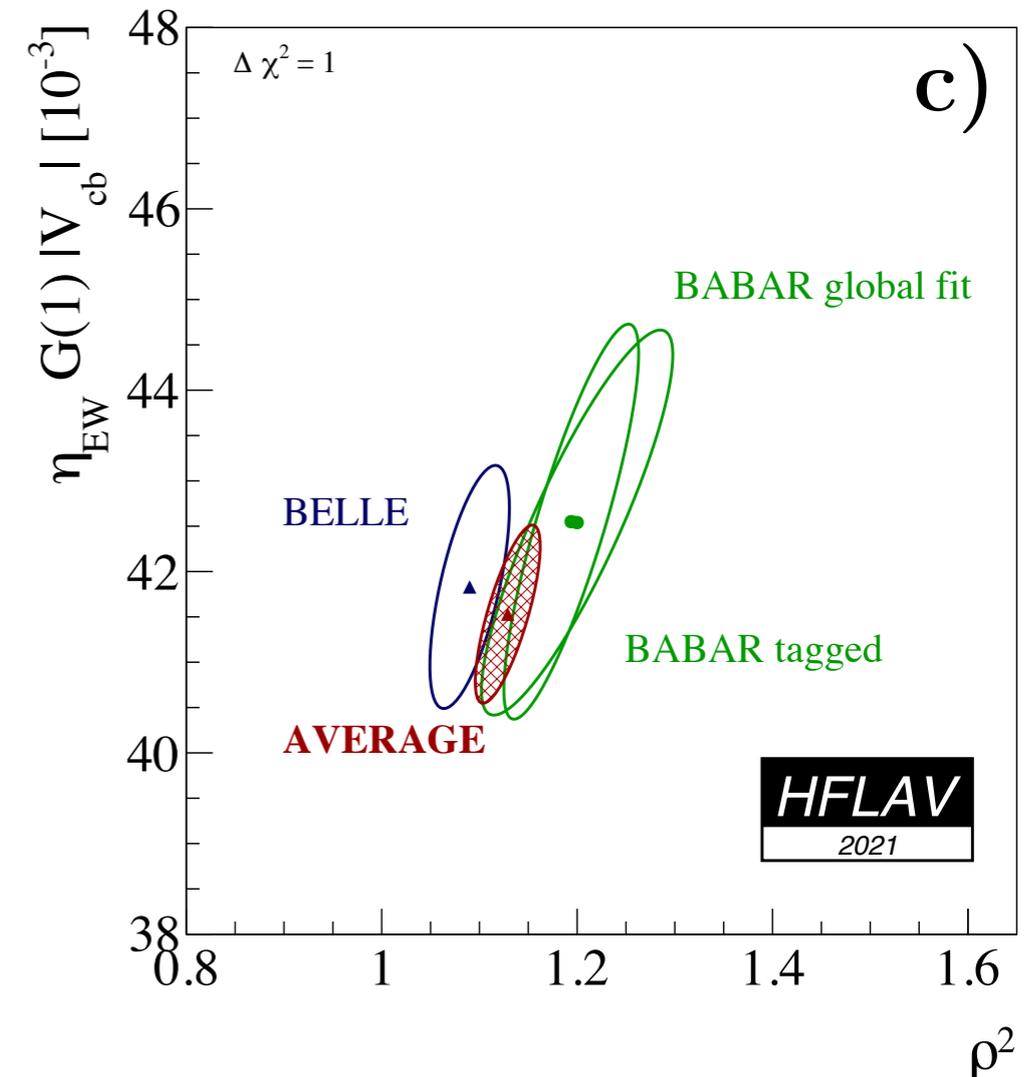
More than a decade of $B \rightarrow D^{(*)} \ell \bar{\nu}_\ell$ is “lost” :-)

For $B \rightarrow D^{(*)} \ell \bar{\nu}_\ell$ traditionally **single form factor** parametrization (Caprini-Lellouch-Neubert, **CLN**) was used. Nucl.Phys. B530 (1998) 153-181

Measurements directly determined the parameters and quoted these with correlations.

Problem: Theory knowledge advances; **today more general parametrization are preferred (BGL, ...)**

Experiment	$\eta_{EW} \mathcal{F}(1) V_{cb} [10^{-3}]$ (rescaled) $\eta_{EW} \mathcal{F}(1) V_{cb} [10^{-3}]$ (published)	ρ^2 (rescaled) ρ^2 (published)
ALEPH [497]	$31.38 \pm 1.80_{\text{stat}} \pm 1.24_{\text{syst}}$ $31.9 \pm 1.8_{\text{stat}} \pm 1.9_{\text{syst}}$	$0.488 \pm 0.226_{\text{stat}} \pm 0.146_{\text{syst}}$ $0.37 \pm 0.26_{\text{stat}} \pm 0.14_{\text{syst}}$
CLEO [501]	$40.16 \pm 1.24_{\text{stat}} \pm 1.54_{\text{syst}}$ $43.1 \pm 1.3_{\text{stat}} \pm 1.8_{\text{syst}}$	$1.363 \pm 0.084_{\text{stat}} \pm 0.087_{\text{syst}}$ $1.61 \pm 0.09_{\text{stat}} \pm 0.21_{\text{syst}}$
OPAL excl [498]	$36.20 \pm 1.58_{\text{stat}} \pm 1.47_{\text{syst}}$ $36.8 \pm 1.6_{\text{stat}} \pm 2.0_{\text{syst}}$	$1.198 \pm 0.206_{\text{stat}} \pm 0.153_{\text{syst}}$ $1.31 \pm 0.21_{\text{stat}} \pm 0.16_{\text{syst}}$
OPAL partial reco [498]	$37.44 \pm 1.20_{\text{stat}} \pm 2.32_{\text{syst}}$ $37.5 \pm 1.2_{\text{stat}} \pm 2.5_{\text{syst}}$	$1.090 \pm 0.137_{\text{stat}} \pm 0.297_{\text{syst}}$ $1.12 \pm 0.14_{\text{stat}} \pm 0.29_{\text{syst}}$
DELPHI partial reco [499]	$35.52 \pm 1.41_{\text{stat}} \pm 2.29_{\text{syst}}$ $35.5 \pm 1.4_{\text{stat}}^{+2.3}_{-2.4_{\text{syst}}}$	$1.139 \pm 0.123_{\text{stat}} \pm 0.382_{\text{syst}}$ $1.34 \pm 0.14_{\text{stat}}^{+0.24}_{-0.22_{\text{syst}}}$
DELPHI excl [500]	$35.87 \pm 1.69_{\text{stat}} \pm 1.95_{\text{syst}}$ $39.2 \pm 1.8_{\text{stat}} \pm 2.3_{\text{syst}}$	$1.070 \pm 0.141_{\text{stat}} \pm 0.153_{\text{syst}}$ $1.32 \pm 0.15_{\text{stat}} \pm 0.33_{\text{syst}}$
Belle [502]	$34.82 \pm 0.15_{\text{stat}} \pm 0.55_{\text{syst}}$ $35.06 \pm 0.15_{\text{stat}} \pm 0.56_{\text{syst}}$	$1.106 \pm 0.031_{\text{stat}} \pm 0.008_{\text{syst}}$ $1.106 \pm 0.031_{\text{stat}} \pm 0.007_{\text{syst}}$
BABAR excl [503]	$33.37 \pm 0.29_{\text{stat}} \pm 0.97_{\text{syst}}$ $34.7 \pm 0.3_{\text{stat}} \pm 1.1_{\text{syst}}$	$1.182 \pm 0.048_{\text{stat}} \pm 0.029_{\text{syst}}$ $1.18 \pm 0.05_{\text{stat}} \pm 0.03_{\text{syst}}$
BABAR D^{*0} [507]	$34.55 \pm 0.58_{\text{stat}} \pm 1.06_{\text{syst}}$ $35.9 \pm 0.6_{\text{stat}} \pm 1.4_{\text{syst}}$	$1.124 \pm 0.058_{\text{stat}} \pm 0.053_{\text{syst}}$ $1.16 \pm 0.06_{\text{stat}} \pm 0.08_{\text{syst}}$
BABAR global fit [509]	$35.45 \pm 0.20_{\text{stat}} \pm 1.08_{\text{syst}}$ $35.7 \pm 0.2_{\text{stat}} \pm 1.2_{\text{syst}}$	$1.171 \pm 0.019_{\text{stat}} \pm 0.060_{\text{syst}}$ $1.21 \pm 0.02_{\text{stat}} \pm 0.07_{\text{syst}}$
Average	$35.00 \pm 0.11_{\text{stat}} \pm 0.34_{\text{syst}}$	$1.121 \pm 0.014_{\text{stat}} \pm 0.019_{\text{syst}}$



Old measurements **cannot be updated** the underlying distributions were not provided but only the result of the fit.

Obviously we should **avoid** this in the future.

The emergence of beyond zero-recoil lattice:

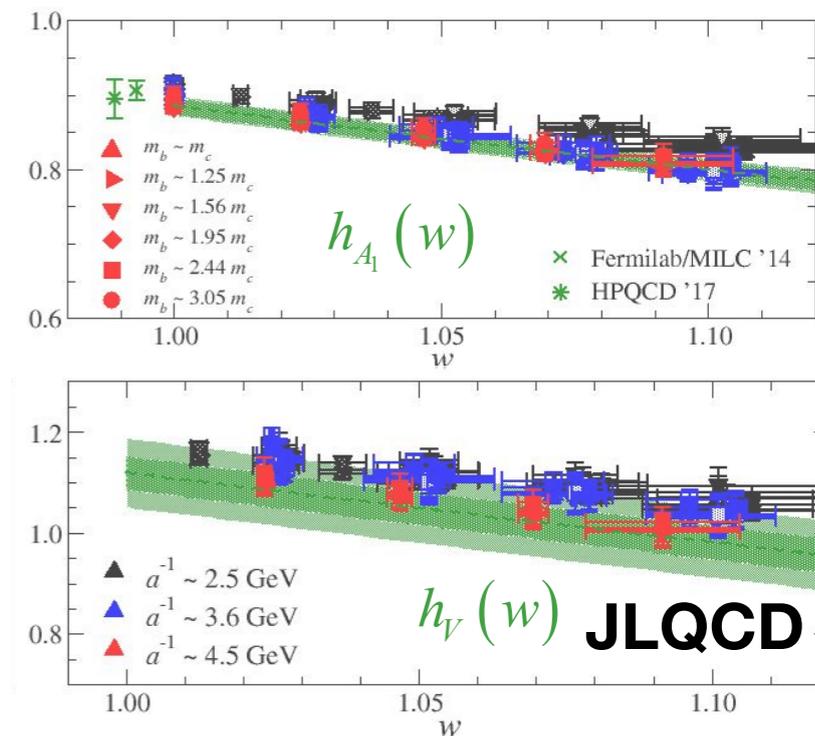
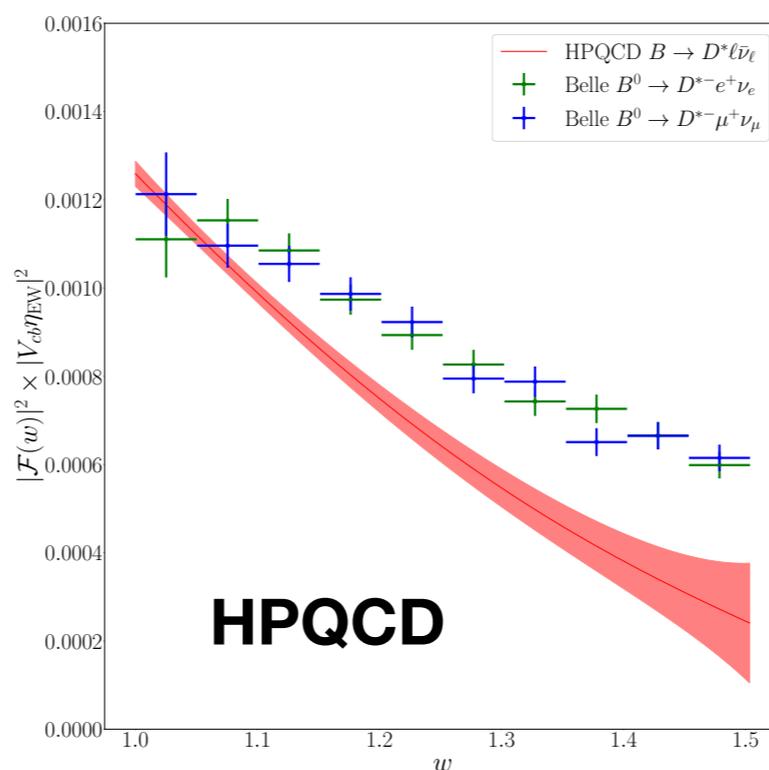
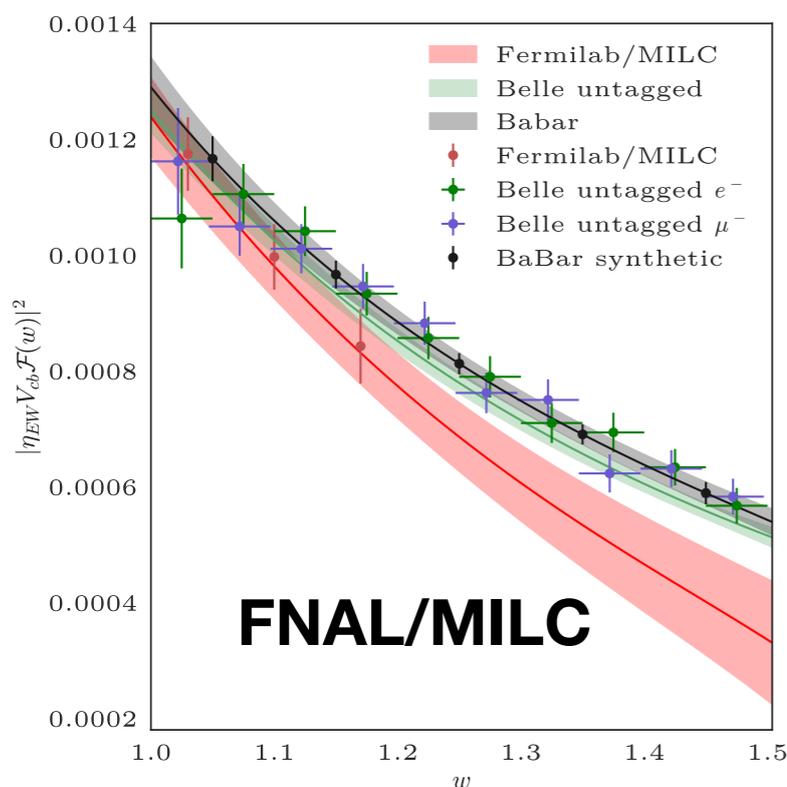
Very exciting times:

A. Bazavov et al. [FNAL/MILC] [Eur. Phys. J. C 82, 1141 (2022), arXiv:2105.14019]

J. Harrison & T.H. Davies [HPQCD] [arXiv:2304.03137 [hep-lat]]

After more than 10 years in the making, we have beyond zero recoil
LQCD predictions for $B \rightarrow D^* \ell \bar{\nu}_\ell$

Three groups: One published, One freshly on arxiv, One preliminary :



Tension with measured shapes ...

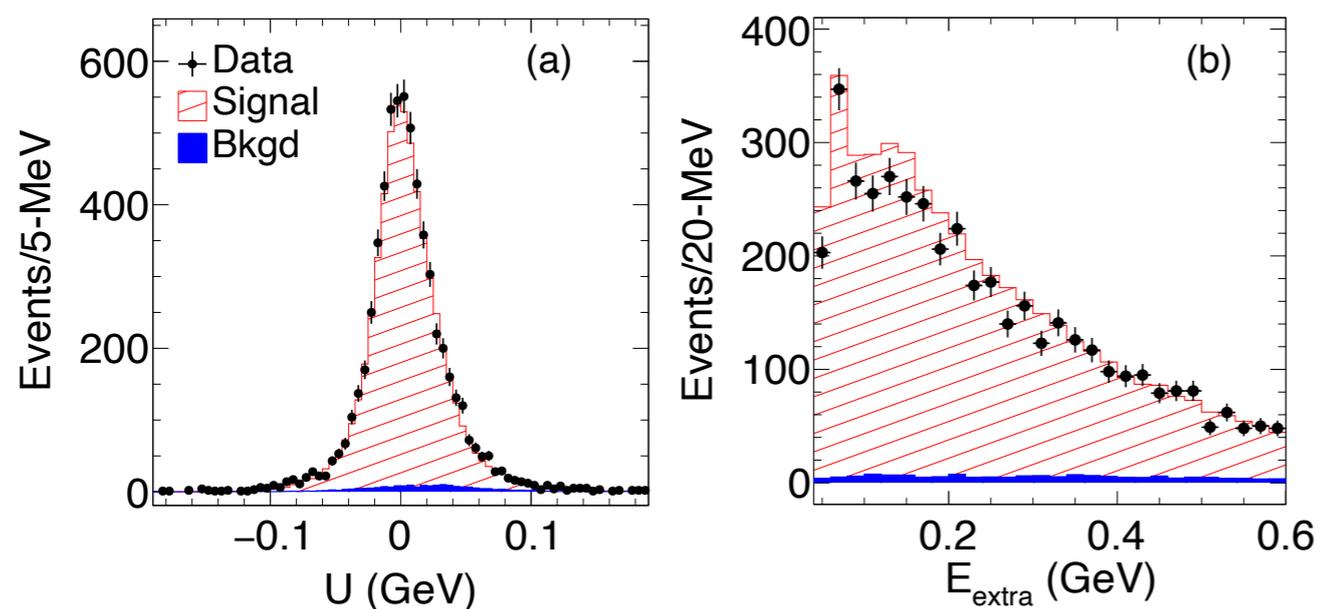
BGL is much better, model independent

So is it ok to just present results with Boyd Grinstein Lebed (**BGL**) ?

BGL looks great:

- it removes the relation between slope and curvature on the leading form factor; **data can pull it.**
- Slope and curvature of the form factor ratios $R_{1/2}$ are not constrained, **data can pull it.**

Beautiful **unbinned 4D fit (!)** from BaBar [Phys. Rev. Lett. 123, 091801 (2019)]



$a_0^f \times 10^2$	$a_1^f \times 10^2$	$a_1^{F_1} \times 10^2$	$a_0^g \times 10^2$	$a_1^g \times 10^2$	$ V_{cb} \times 10^3$
1.29	1.63	0.03	2.74	8.33	38.36
± 0.03	± 1.00	± 0.11	± 0.11	± 6.67	± 0.90

TABLE I. The $N = 1$ BGL expansion results of this analysis, including systematic uncertainties.

ρ_D^2	$R_1(1)$	$R_2(1)$	$ V_{cb} \times 10^3$
0.96 ± 0.08	1.29 ± 0.04	0.99 ± 0.04	38.40 ± 0.84

TABLE II. The CLN fit results from this analysis, including systematic uncertainties.

Truncation Order

Model independence is a **step forward**, but **choices have** to be **made** here as well..

$$g(z) = \frac{1}{P_g(z)\phi_g(z)} \sum_{n=0}^N a_n z^n, \quad f(z) = \frac{1}{P_f(z)\phi_f(z)} \sum_{n=0}^N b_n z^n, \quad \mathcal{F}_1(z) = \frac{1}{P_{\mathcal{F}_1}(z)\phi_{\mathcal{F}_1}(z)} \sum_{n=0}^N c_n z^n,$$

One Problem you face as an experimentalist: where do you truncate?

Truncate too soon:

- Model dependence in extracted result for $|V_{cb}|$?

Truncate too late:

- Unnecessarily increase variance on $|V_{cb}|$?

Is there an **ideal** truncation order?

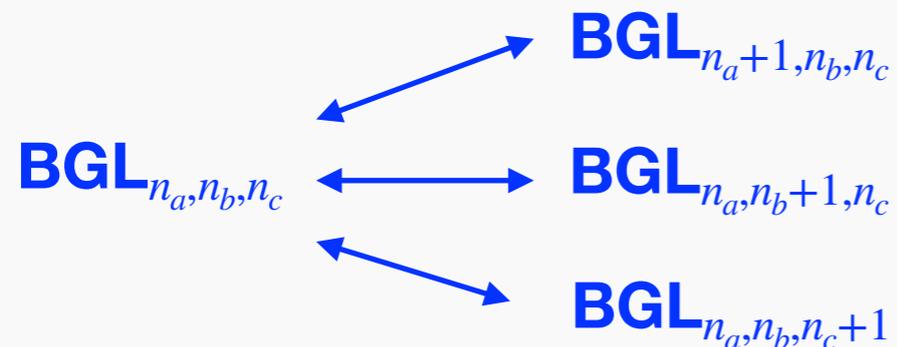
What about **additional constraints**?

Nested Hypothesis Tests or Saturation Constraints

Z. Ligeti, D. Robinson, M. Papucci, FB
[arXiv:1902.09553, PRD100,013005 (2019)]

Use a **nested hypothesis test (NHT)**
to determine optimal truncation order

Challenge nested fits



Test statistics & Decision boundary

$$\Delta\chi^2 = \chi_N^2 - \chi_{N+1}^2 \quad \Delta\chi^2 > 1$$

Distributed like a χ^2 -distribution with 1 dof
(Wilk's theorem)

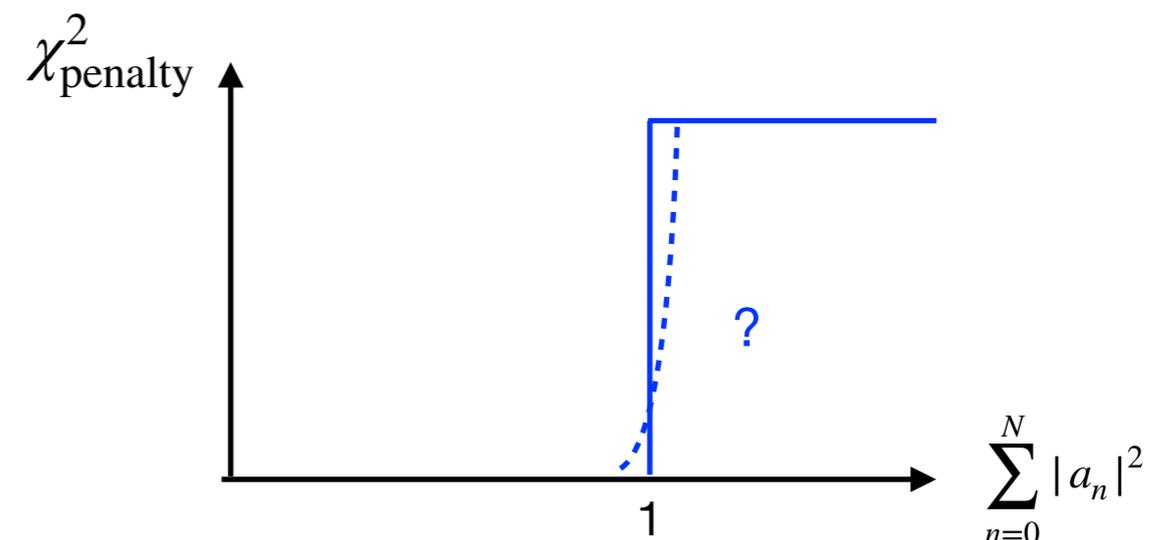
Gambino, Jung, Schacht
[arXiv:1905.08209, PLB]

Constrain contributions
from higher order coefficients
using **unitarity bounds**

$$\sum_{n=0}^N |a_n|^2 \leq 1 \quad \sum_{n=0}^N (|b_n|^2 + |c_n|^2) \leq 1$$

e.g.

$$\chi^2 \rightarrow \chi^2 + \chi_{\text{penalty}}^2$$



Nesting Procedure

Steps:

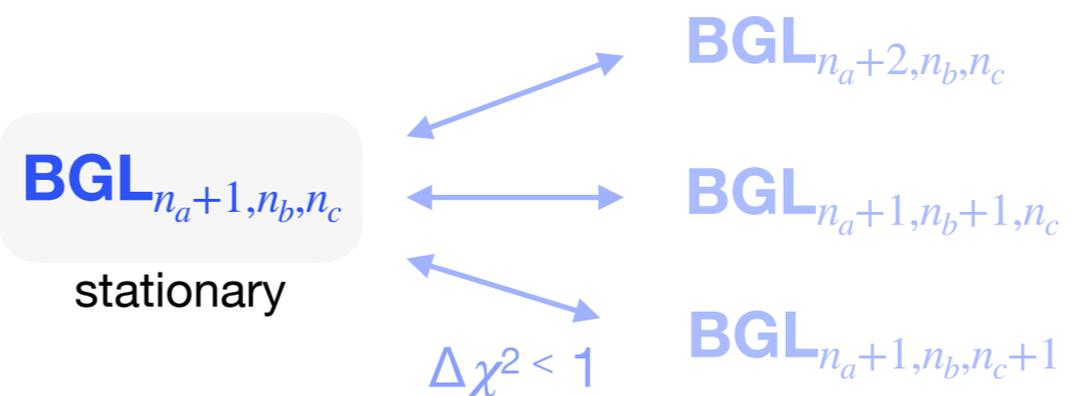
1 Carry out nested fits with one parameter added

2 Accept descendant over parent fit, if $\Delta\chi^2 > 1$

3 Repeat 1 and 2 until you find **stationary** points

4 If multiple **stationary** points remain, choose the one with smallest N , then smallest χ^2

5 Reject scenarios that **produce strong correlations** (= blind directions)



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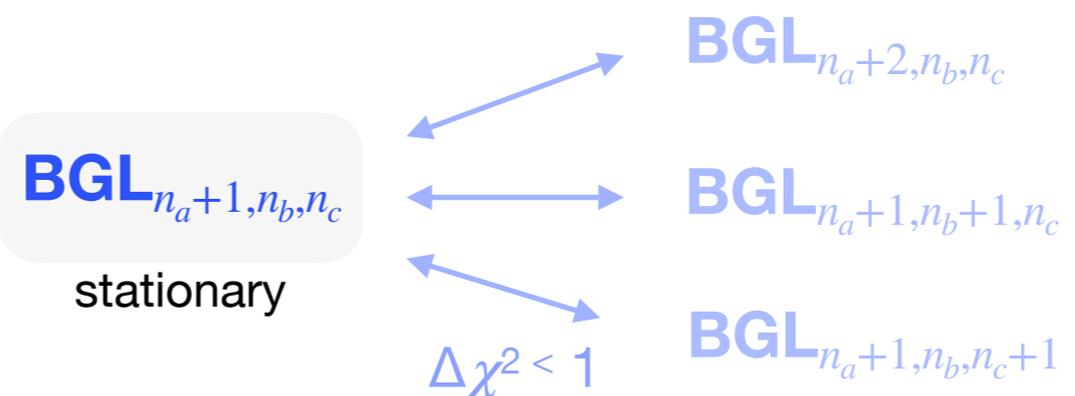
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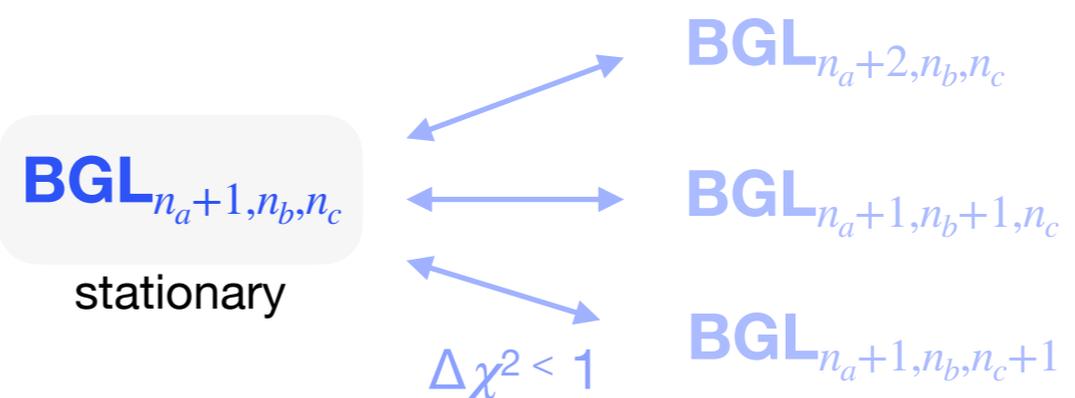
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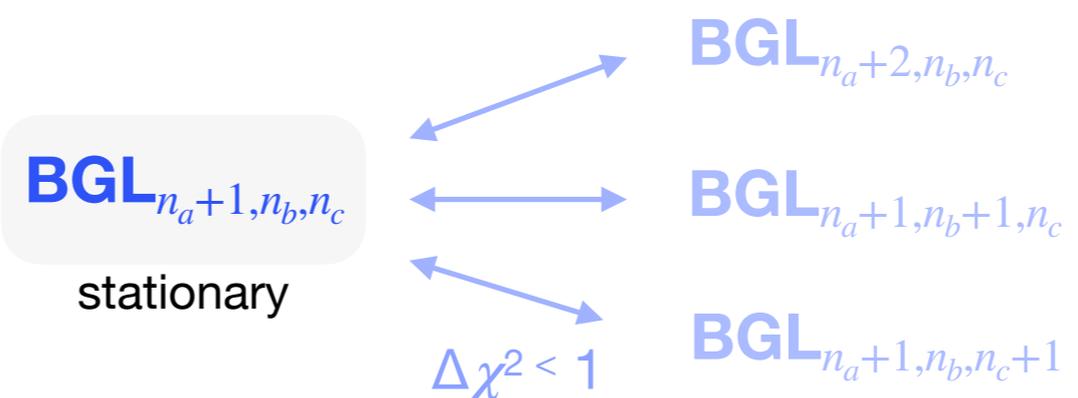
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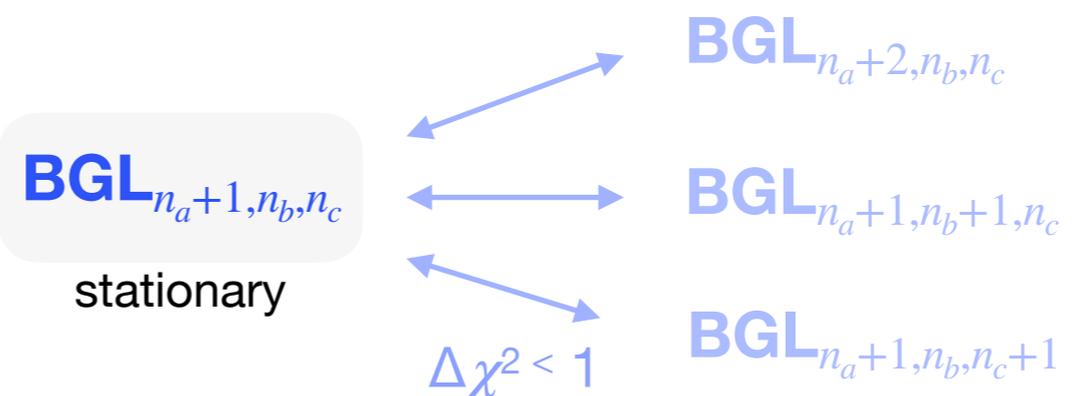
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Toy study to illustrate possible bias

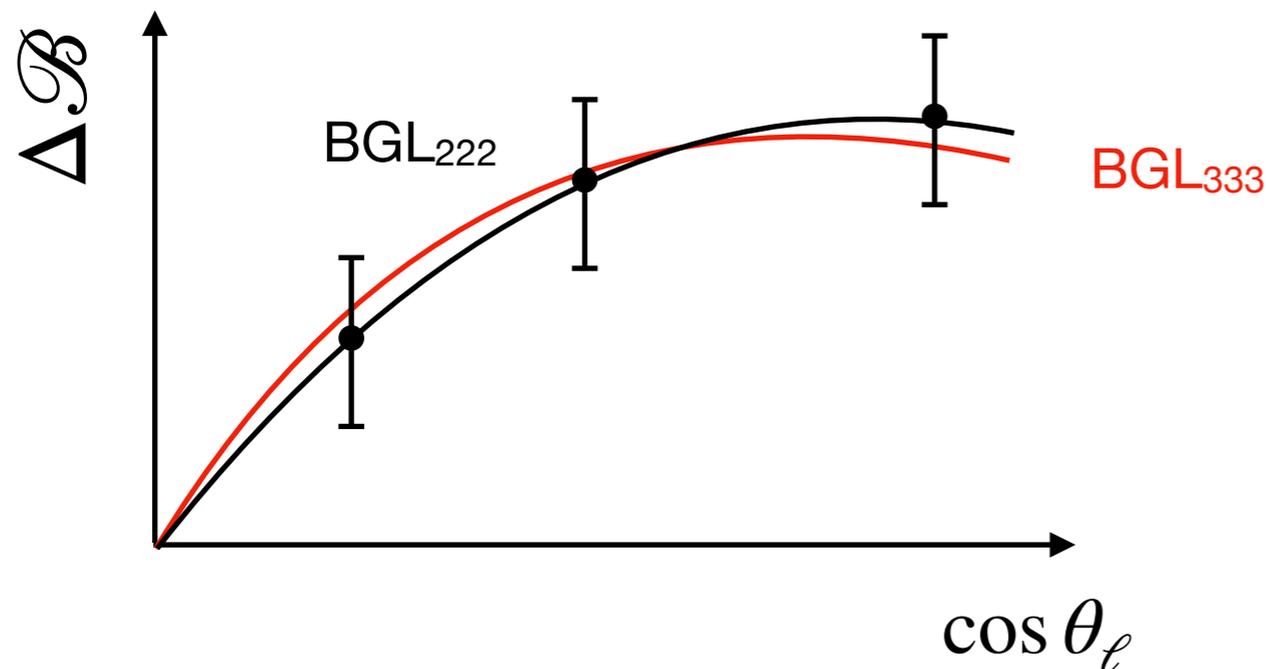
Use the central values of the **BGL₂₂₂** fit as a starting point to add **fine structure**

fit = fit to prel. 2017 Belle data

	'1-times'	'10-times'
Parameter	Value $\times 10^2$	Value $\times 10^2$
\tilde{a}_2	2.6954	26.954
\tilde{b}_2	-0.2040	-2.040
\tilde{c}_3	0.5350	5.350

Create a "true" higher order Hypothesis of order **BGL₃₃₃**

Has fine structure element the **current data cannot resolve**



Toy study to illustrate possible bias

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Toy Test

Produce **ensemble** of toy measurements using **meas. covariance** & **BGL₃₃₃** central values

Each toy is fitted to build the descendant tree and carry out a **NHT** to select its preferred **BGL_{n_an_bn_c}**

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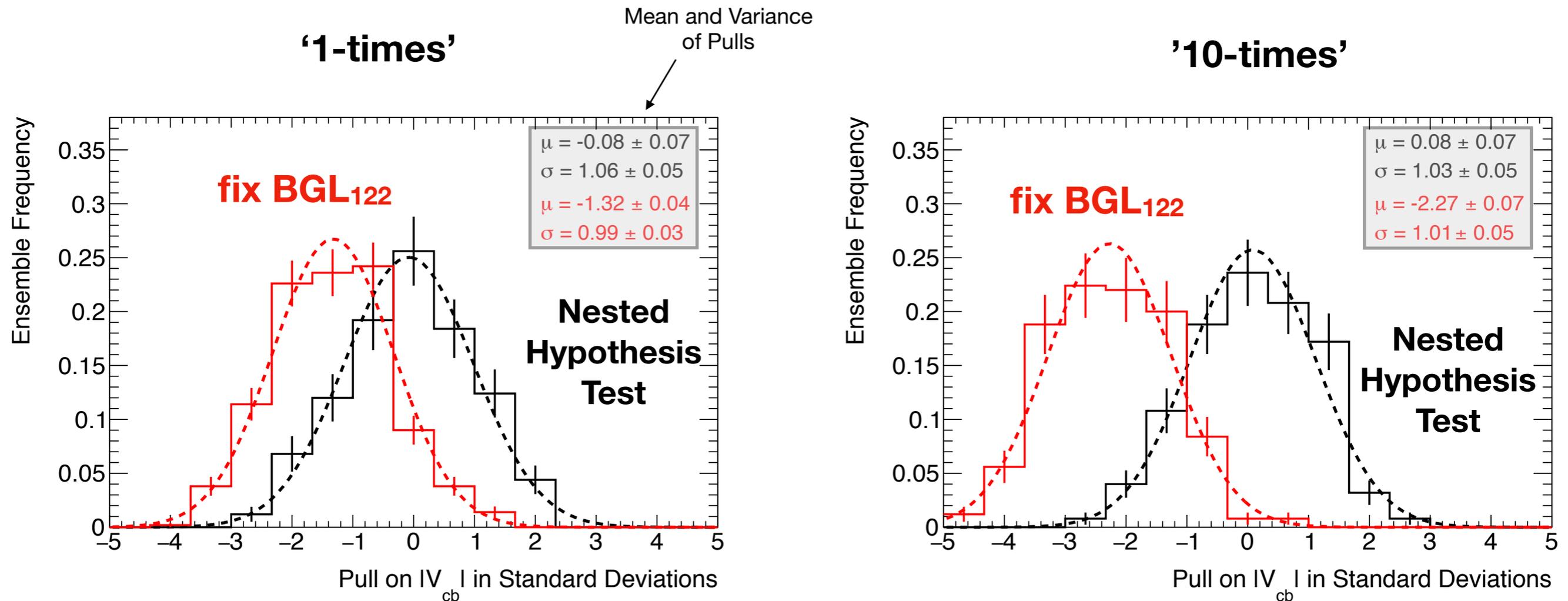
As calculated from selected BGL_{n_an_bn_c} fit of each toy

Construct Pulls

$$\text{Pull} = \frac{|V_{cb}|_{\text{true}} - |V_{cb}|_{\text{toy}}}{\Delta |V_{cb}|_{\text{toy}}}$$

If methodology unbiased, should follow a standard normal distribution (mean 0, width 1)

Bias



→ Procedure produces **unbiased** $|V_{cb}|$ values, **just picking a given hypothesis (BGL₁₂₂) does not**

Relative Frequency of selected Hypothesis:

	BGL ₁₂₂	BGL ₂₁₂	BGL ₂₂₁	BGL ₂₂₂	BGL ₂₂₃	BGL ₂₃₂	BGL ₃₂₂	BGL ₂₃₃	BGL ₃₂₃	BGL ₃₃₂	BGL ₃₃₃
1-times	6%	0%	37%	27%	6%	6%	11%	0%	2%	4%	0.4%
10-times	0%	0%	8%	38%	14%	8%	16%	3%	4%	8%	1%

Paradigm shift

My three take-away points:

1.

BGL removes the (theory)-model dependent assumptions of **CLN**; great step forward

BUT

still **choices** have to be made (**truncation**, **unitarity in or out**) that **influence** the **outcome** of the **interpretation** of the data

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What if someone comes along and wants to **fit something else** to the data with different assumptions?

BGL with updated pole masses?

DM from MNSL(*PoS LATTICE2022 (2023) 298*)

BGJD (*Eur.Phys.J.C* 80 (2020) 4, 347)

BLPR (*Phys. Rev. D* 95, 115008 (2017))

BLPRXP (*Phys. Rev. D* 106, 096015 (2022))

...

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...

3.

What if someone wants to **do something entirely else** with the **data** we have not thought of today?

E.g. look for a bump for a sterile neutrino at large MM^2 , search for RH currents in angular distributions

Possible Strategies

Publish either container that allows later reinterpretation

(includes final selected data, MC, etc.)



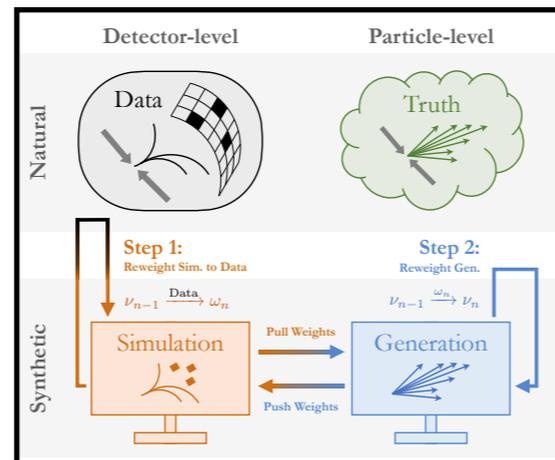
Very ambitious, but great goal!

- Not everybody agrees and not everybody agrees to what extent

Publish ND or unbinned unfolded measurements

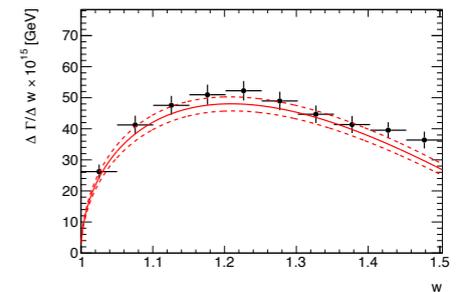
Very challenging, binned: curse of dimensionality (5D measurement essentially)

Unbinned unfolding cool new idea, beats high dimensionality



Omnifold: unbinned unfolding
Phys. Rev. Lett. 124, 182001 (2020)

Publish 1D Measurements of partial BF's



Belle started doing this in 2017

Followed up in 2018 and 2023

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opendata
CERN

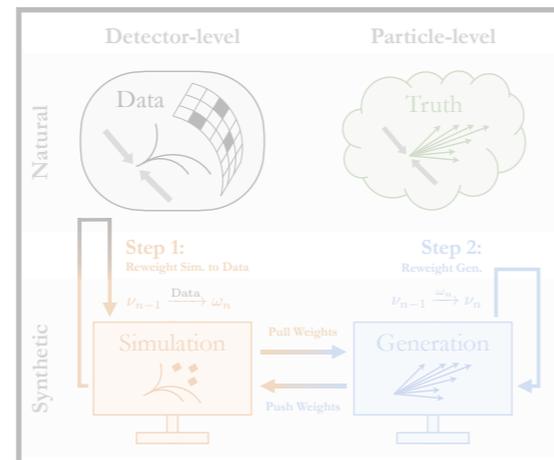
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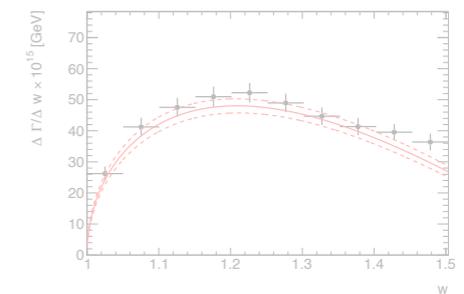


Omnifold: unbinned unfolding
Phys. Rev. Lett. 124, 182001 (2020)

Somewhere in between?

Without losing too much interesting information?

Publish 1D Measurements of partial BF's



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Full Angular Information **without** going to 4D

Full angular information can be encoded into **12 coefficients** :

$$\frac{d\Gamma}{dq^2 d\cos\theta_V d\cos\theta_\ell d\chi} = \frac{G_F^2 |V_{cb}|^2 m_B^3}{2\pi^4} \times \left\{ \begin{aligned} & J_{1s} \sin^2 \theta_V + J_{1c} \cos^2 \theta_V \\ & + (J_{2s} \sin^2 \theta_V + J_{2c} \cos^2 \theta_V) \cos 2\theta_\ell \\ & + J_3 \sin^2 \theta_V \sin^2 \theta_\ell \cos 2\chi \\ & + J_4 \sin 2\theta_V \sin 2\theta_\ell \cos \chi + J_5 \sin 2\theta_V \sin \theta_\ell \cos \chi \\ & + (J_{6s} \sin^2 \theta_V + J_{6c} \cos^2 \theta_V) \cos \theta_\ell \\ & + J_7 \sin 2\theta_V \sin \theta_\ell \sin \chi + J_8 \sin 2\theta_V \sin 2\theta_\ell \sin \chi \\ & + J_9 \sin^2 \theta_V \sin^2 \theta_\ell \sin 2\chi \end{aligned} \right\}.$$

Each of these coefficients is a function of $q^2 \sim w$



With some smart folding, one can “easily” determine them

Based on the ideas of:

JHEP 05 (2013) 043

JHEP 05 (2013) 137

Phys. Rev. D 90, 094003 (2014)

<http://cds.cern.ch/record/1605179>

8 Coefficients relevant in massless limit & SM

How can we measure these coefficients?

Step 1: bin up phase-space in $q^2 \sim w$ in however many bins you can afford

How can we measure these coefficients?

Step 1: bin up phase-space in $q^2 \sim w$ in however many bins you can afford

Step 2: Determine the # of signal events in specific phase-space regions

The coefficients are related to a weighted sum of events in a given q^2 bin

$$J_i = \frac{1}{N_i} \sum_{j=1}^8 \sum_{k,l=1}^4 \eta_{ij}^\chi \eta_{ik}^{\theta_\ell} \eta_{il}^{\theta_V} \left[\chi^i \otimes \theta_\ell^j \otimes \theta_V^k \right]$$

Normalization
Factor

Weights

Phase space region

J_i	η_i^χ	$\eta_i^{\theta_\ell}$	$\eta_i^{\theta_V}$	normalization N_i
J_{1s}	{+}	{+, a, a, +}	{-, c, c, -}	$2\pi(1)2$
J_{1c}	{+}	{+, a, a, +}	{+, d, d, +}	$2\pi(1)(2/5)$
J_{2s}	{+}	{-, b, b, -}	{-, c, c, -}	$2\pi(-2/3)2$
J_{2c}	{+}	{-, b, b, -}	{+, d, d, +}	$2\pi(-2/3)(2/5)$
J_3	{+, -, -, +, +, -, -, +}	{+}	{+}	$4(4/3)^2$
J_4	{+, +, -, -, -, -, +, +}	{+, +, -, -}	{+, +, -, -}	$4(4/3)^2$
J_5	{+, +, -, -, -, -, +, +}	{+}	{+, +, -, -}	$4(\pi/2)(4/3)$
J_{6s}	{+}	{+, +, -, -}	{-, c, c, -}	$2\pi(1)2$
J_{6c}	{+}	{+, +, -, -}	{+, d, d, +}	$2\pi(1)(2/5)$
J_7	{+, +, +, +, -, -, -, -}	{+}	{+, +, -, -}	$4(\pi/2)(4/3)$
J_8	{+, +, +, +, -, -, -, -}	{+, +, -, -}	{+, +, -, -}	$4(4/3)^2$
J_9	{+, +, -, -, +, +, -, -}	{+}	{+}	$4(4/3)^2$

$$a = 1 - 1/\sqrt{2}, b = a\sqrt{2}, c = 2\sqrt{2} - 1, d = 1 - 4\sqrt{2}/5$$

E.g. for J_3 : **Split** χ into **2 Regions**

$$'+' : \chi \in [0, \pi/4], [3/4\pi, 5/4\pi], [7/4\pi, 2\pi]$$

\tilde{N}_+

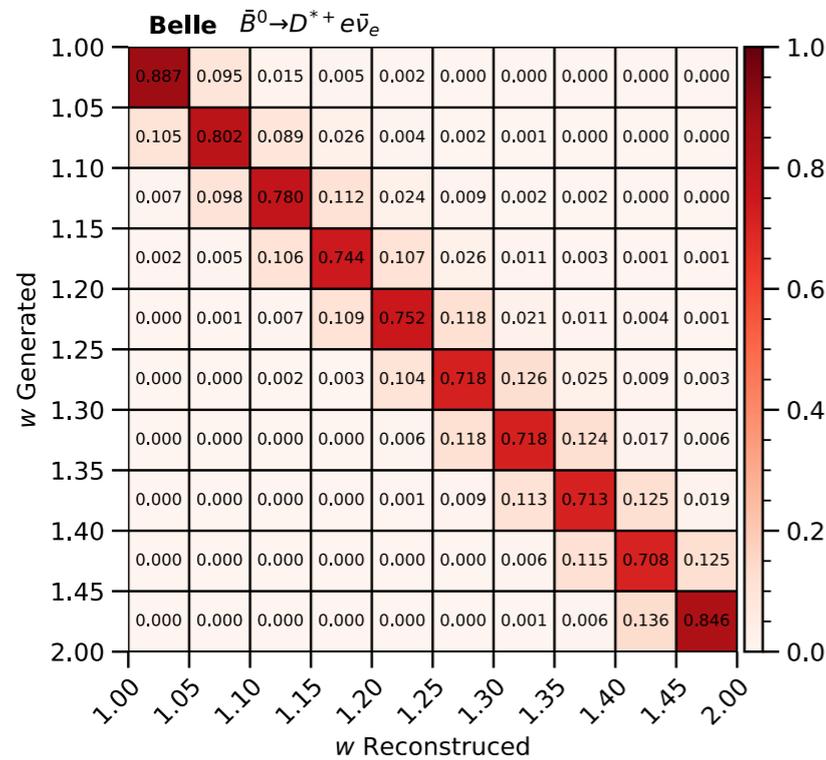
$$'-' : \chi \in [\pi/4, 3/4\pi], [5/4\pi, 7/4\pi]$$

\tilde{N}_-

Step 3: Reverse Migration and Acceptance Effects

Resolution effects: events with a given “true” value of $\{q^2, \cos \theta_\ell, \cos \theta_V, \chi\}$ can fall into different reconstructed bins

E.g. w migration matrix



[arXiv:2301.07529](https://arxiv.org/abs/2301.07529) [hep-ex]

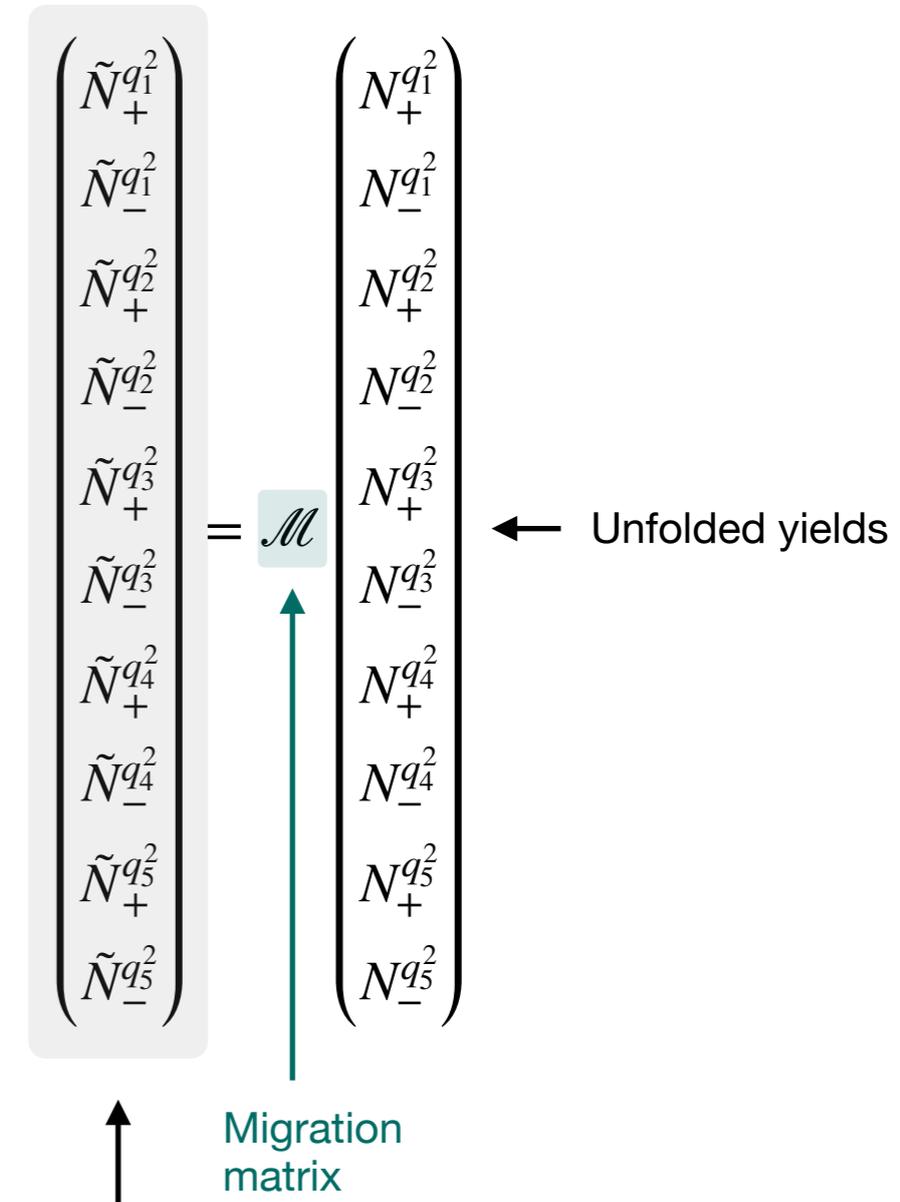
Acceptance x Efficiency Corrections:

$$N_+^{q_i^2} \cdot e_{\text{eff},+,q_i^2}^{-1} = n_+^{q_i^2}$$

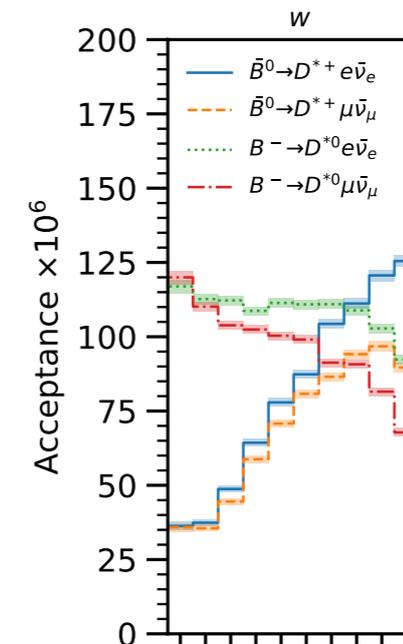
$$N_-^{q_i^2} \cdot e_{\text{eff},-,q_i^2}^{-1} = n_-^{q_i^2}$$

Unfolded yields

← Acceptance / Eff. corrected yields



Bkg subtracted yields



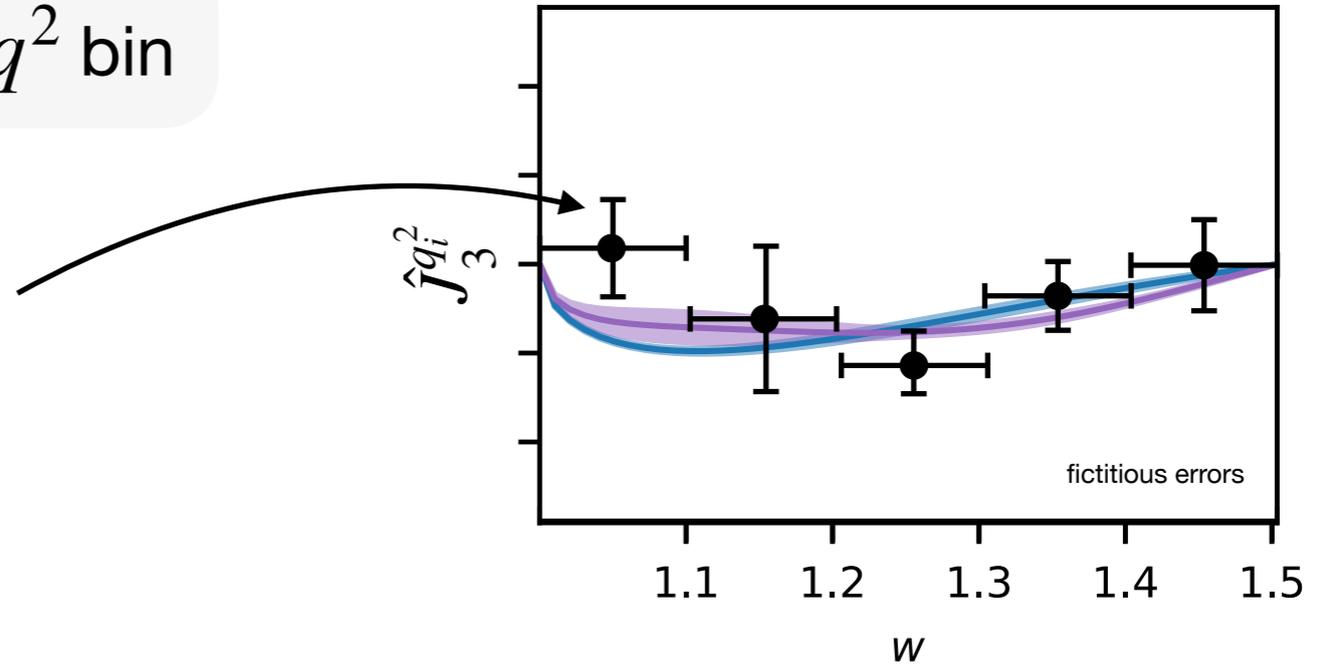
[arXiv:2301.07529](https://arxiv.org/abs/2301.07529) [hep-ex]

Step 4: Calculate J_i for a given w/q^2 bin

$$\frac{n_+^{q_i^2}}{n_-^{q_i^2}} \rightarrow \hat{J}_3^{q_i^2} = \frac{1}{\Gamma} \times \frac{n_+^{q_i^2} - n_-^{q_i^2}}{4(4/3)^2}$$

Normalization

$$\Gamma = \frac{8}{9}\pi \left(3 \sum_i J_{1c}^{q_i^2} + 6 \sum_i J_{1s}^{q_i^2} - \sum_i J_{2c}^{q_i^2} - 2 \sum_i J_{2s}^{q_i^2} \right)$$



More **involved** for the **other** coefficients: need full experimental covariance between all measured w/q^2 bins and coefficients (statistical overlap, systematics)

SM:

$$\{ J_{1s}^{q_i^2}, J_{1c}^{q_i^2}, J_{2s}^{q_i^2}, J_{2c}^{q_i^2}, J_3^{q_i^2}, J_4^{q_i^2}, J_5^{q_i^2}, J_{6s}^{q_i^2} \}$$

e.g. **5 x 8 = 40 coefficients**

or full thing (SM + NP)

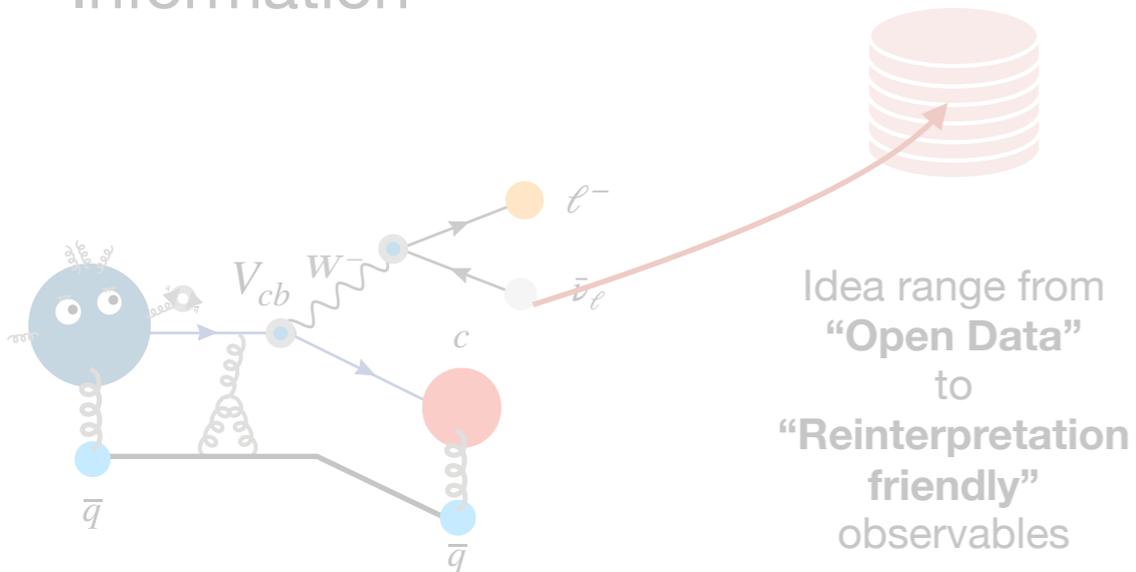
with **5 x 12 = 60 coefficients**

J_i	η_i^x	$\eta_i^{\theta_e}$	$\eta_i^{\theta_V}$	normalization N_i
J_{1s}	{+}	{+, a, a, +}	{-, c, c, -}	$2\pi(1)2$
J_{1c}	{+}	{+, a, a, +}	{+, d, d, +}	$2\pi(1)(2/5)$
J_{2s}	{+}	{-, b, b, -}	{-, c, c, -}	$2\pi(-2/3)2$
J_{2c}	{+}	{-, b, b, -}	{+, d, d, +}	$2\pi(-2/3)(2/5)$
J_3	{+, -, -, +, +, -, -, +}	{+}	{+}	$4(4/3)^2$
J_4	{+, +, -, -, -, -, +, +}	{+, +, -, -}	{+, +, -, -}	$4(4/3)^2$
J_5	{+, +, -, -, -, -, +, +}	{+}	{+, +, -, -}	$4(\pi/2)(4/3)$
J_{6s}	{+}	{+, +, -, -}	{-, c, c, -}	$2\pi(1)2$
J_{6c}	{+}	{+, +, -, -}	{+, d, d, +}	$2\pi(1)(2/5)$
J_7	{+, +, +, +, -, -, -, -}	{+}	{+, +, -, -}	$4(\pi/2)(4/3)$
J_8	{+, +, +, +, -, -, -, -}	{+, +, -, -}	{+, +, -, -}	$4(4/3)^2$
J_9	{+, +, -, -, +, +, -, -}	{+}	{+}	$4(4/3)^2$

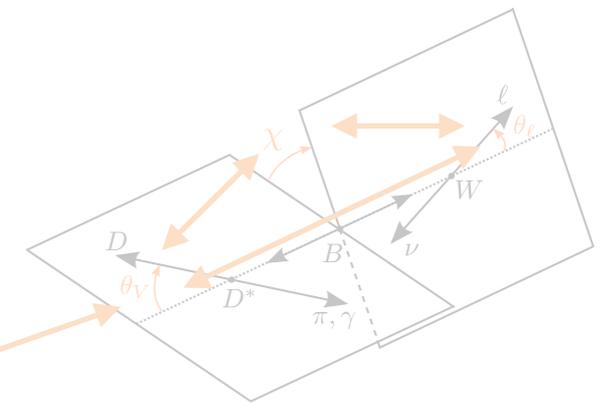
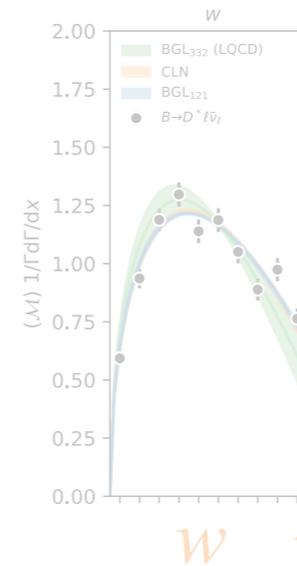
$$a = 1 - 1/\sqrt{2}, b = a\sqrt{2}, c = 2\sqrt{2} - 1, d = 1 - 4\sqrt{2}/5$$

Talk Overview

1. Why we need to do better to preserve our experimental Information

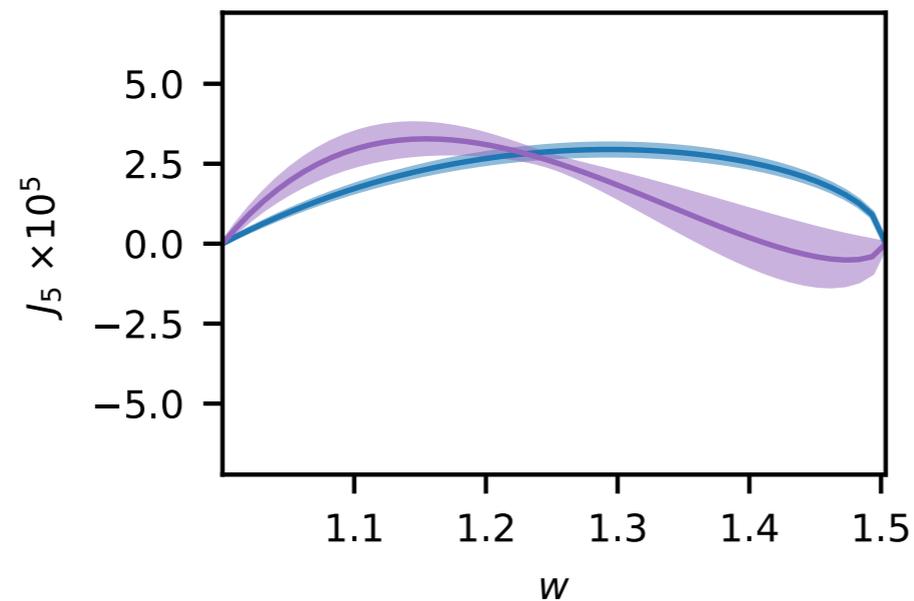


2. From 1D projections to full angular information



Working around the curse of dimensionality

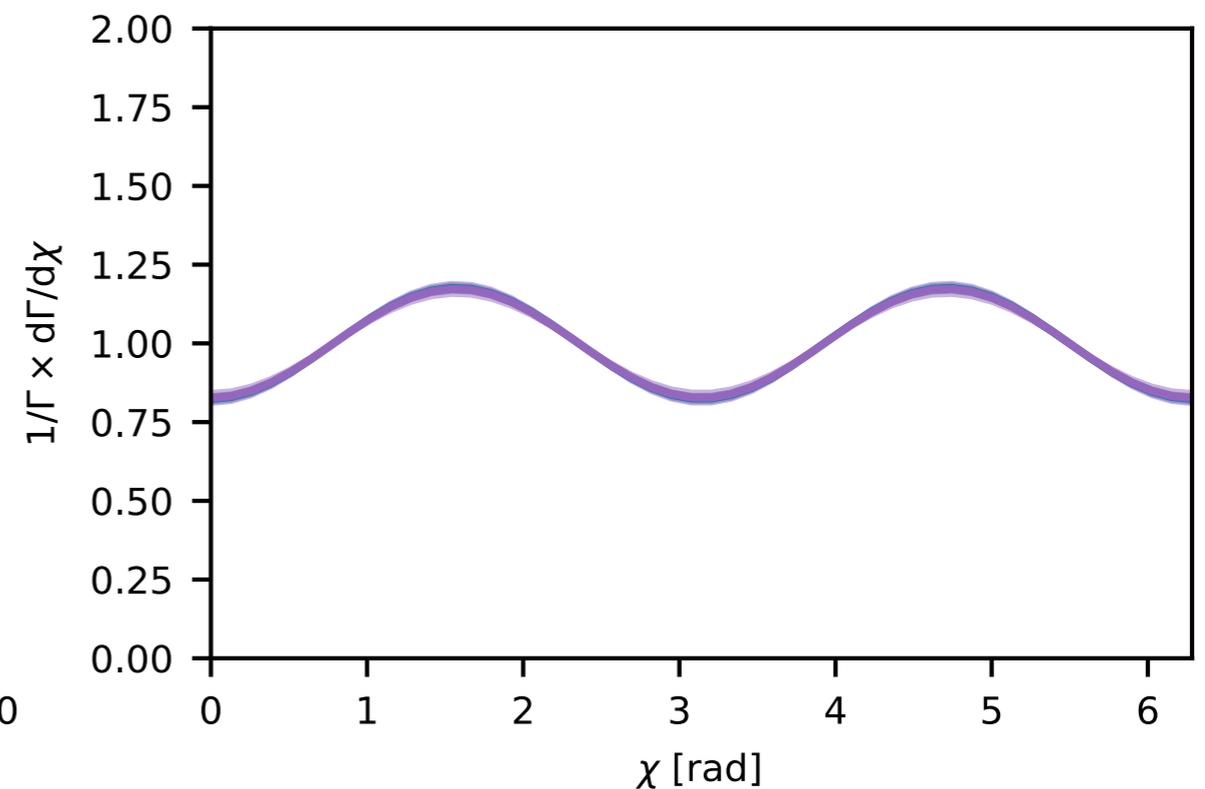
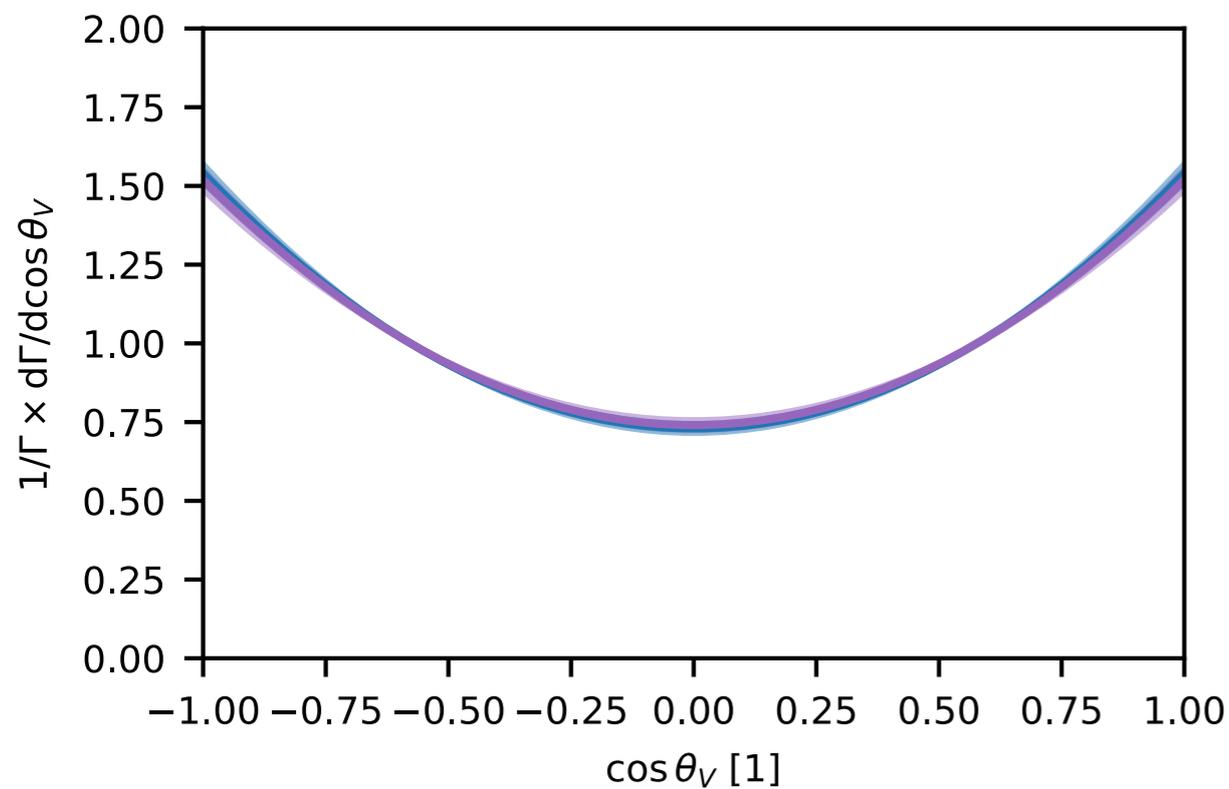
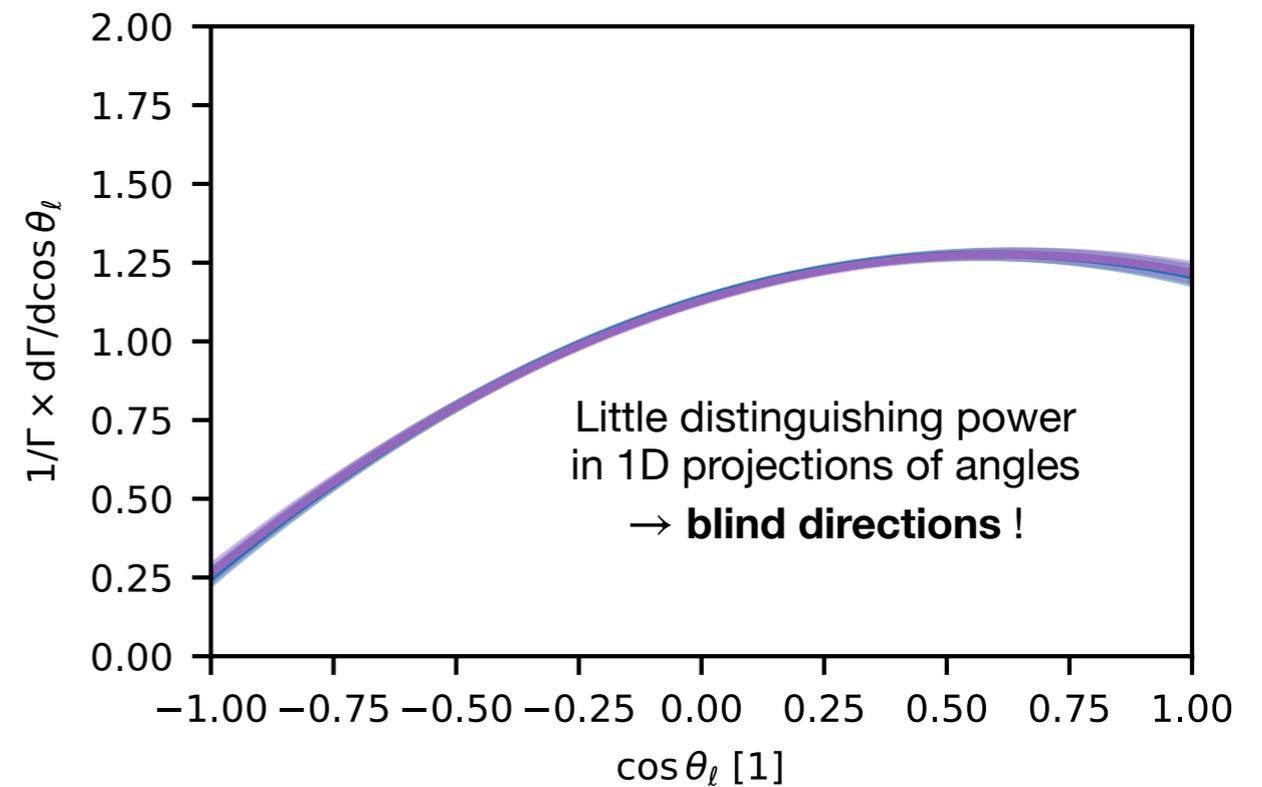
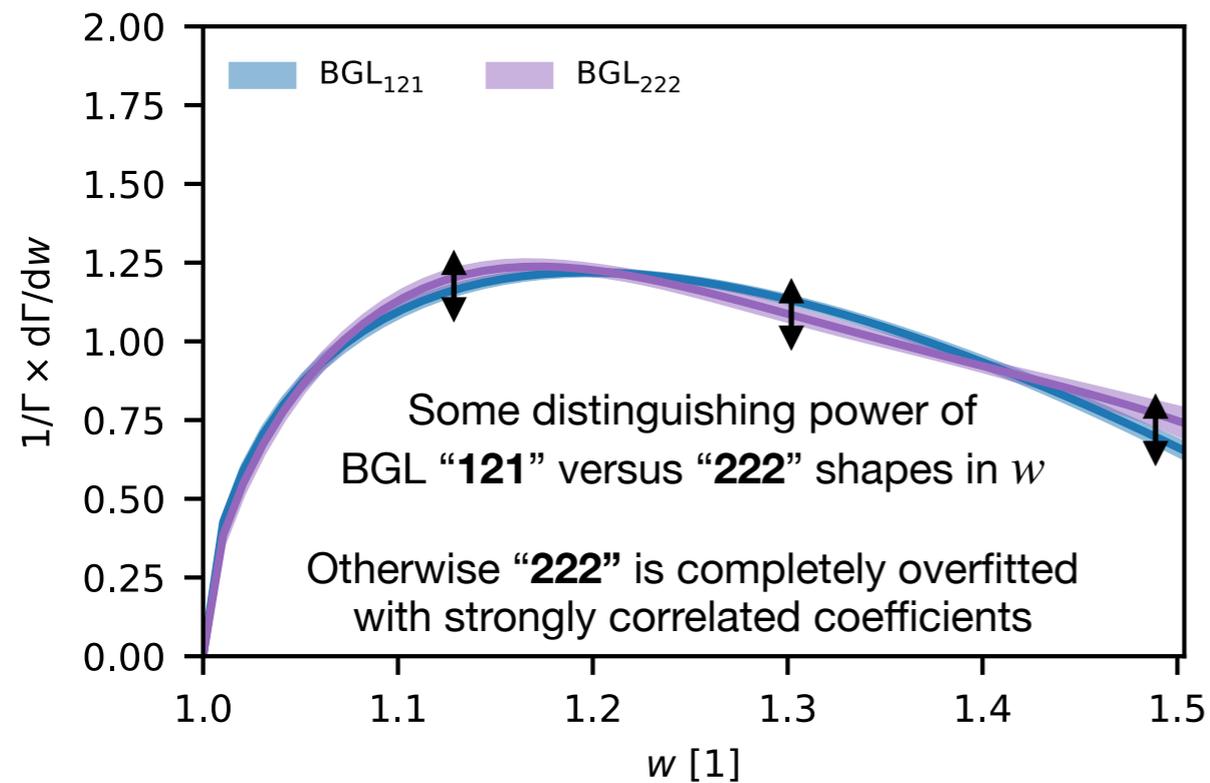
3. Potential of full angular fits



1D versus Full Angular Sensitivities

Errors and central values from
1D projection fits of
[arXiv:2301.07529](https://arxiv.org/abs/2301.07529) (Table XVI)

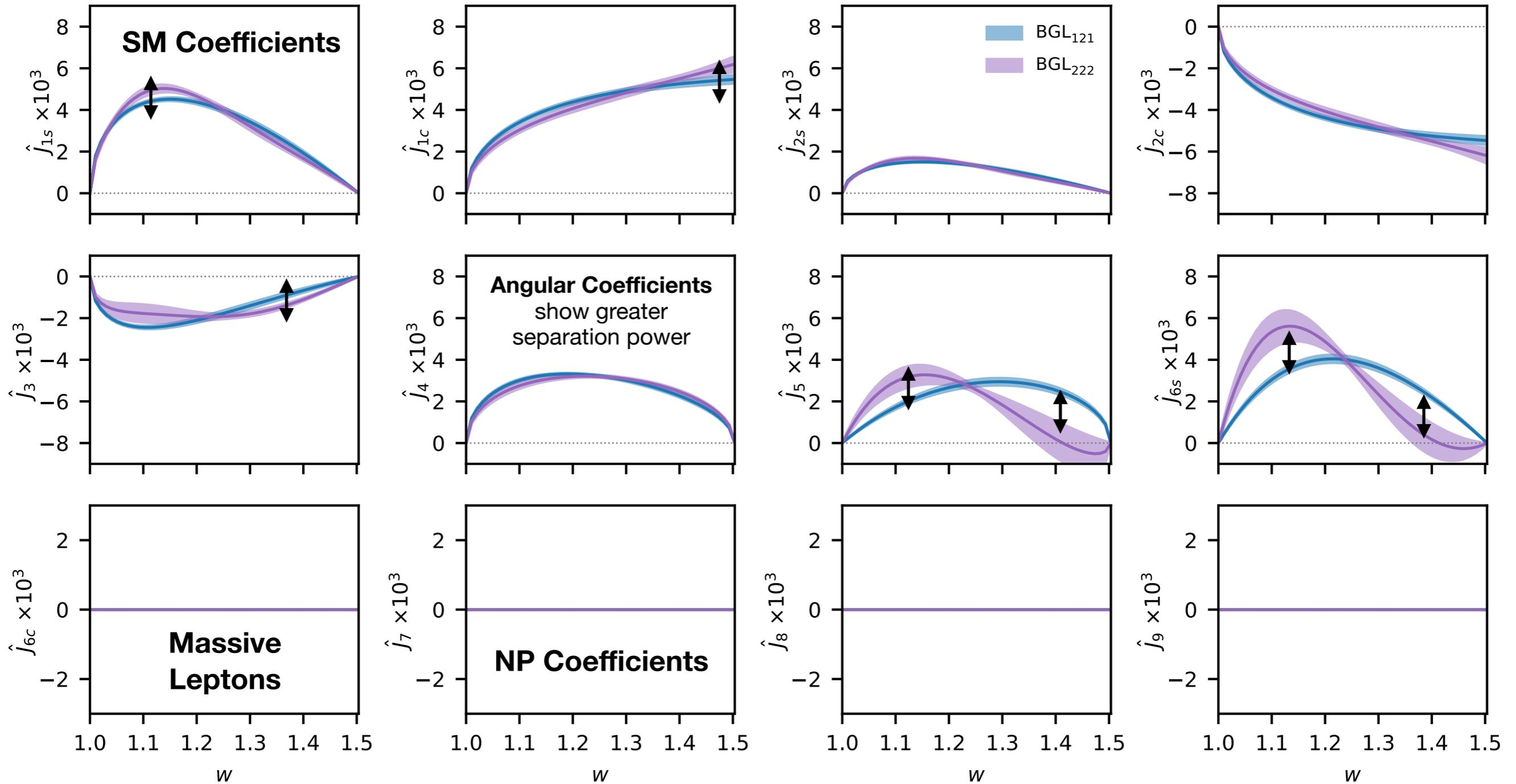
29



1D versus Full Angular Sensitivities

Errors and central values from
1D projection fit of
[arXiv:2301.07529](https://arxiv.org/abs/2301.07529) (Table XVI)

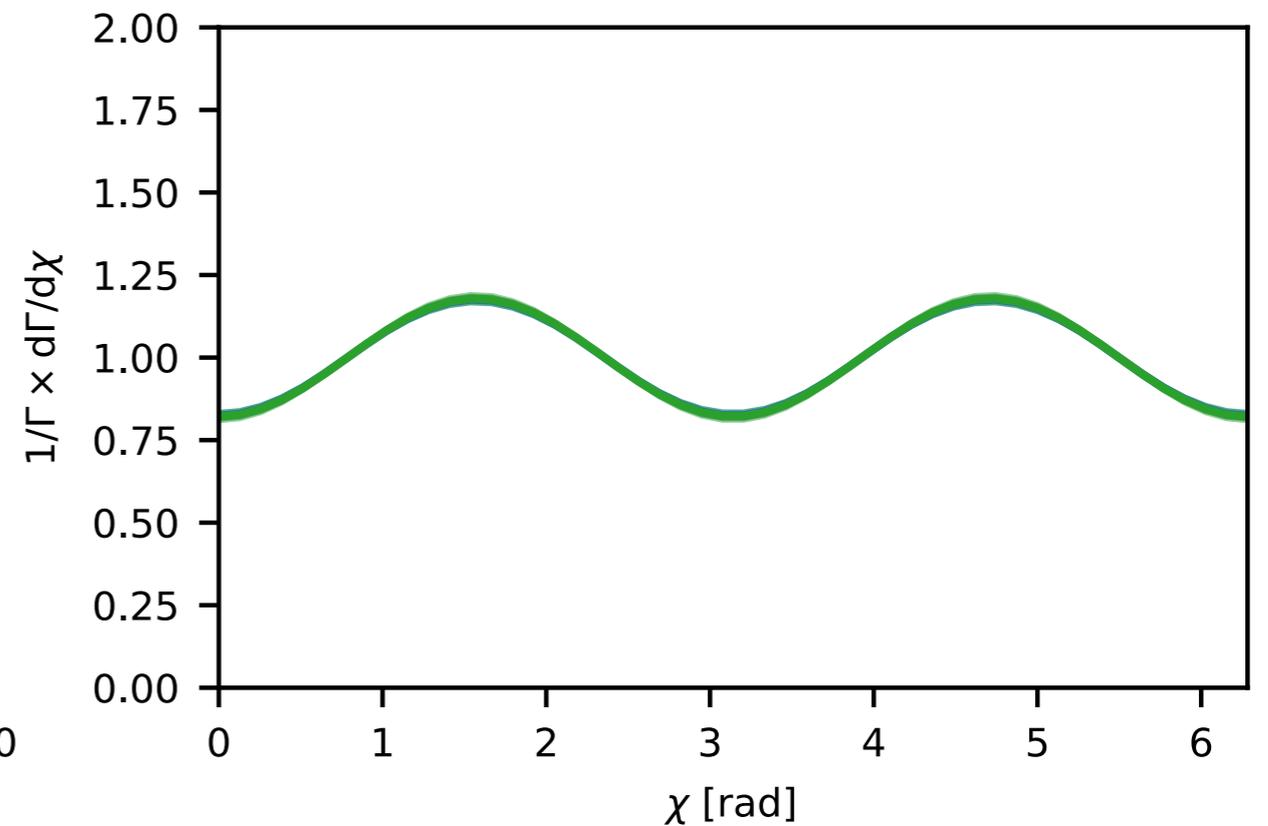
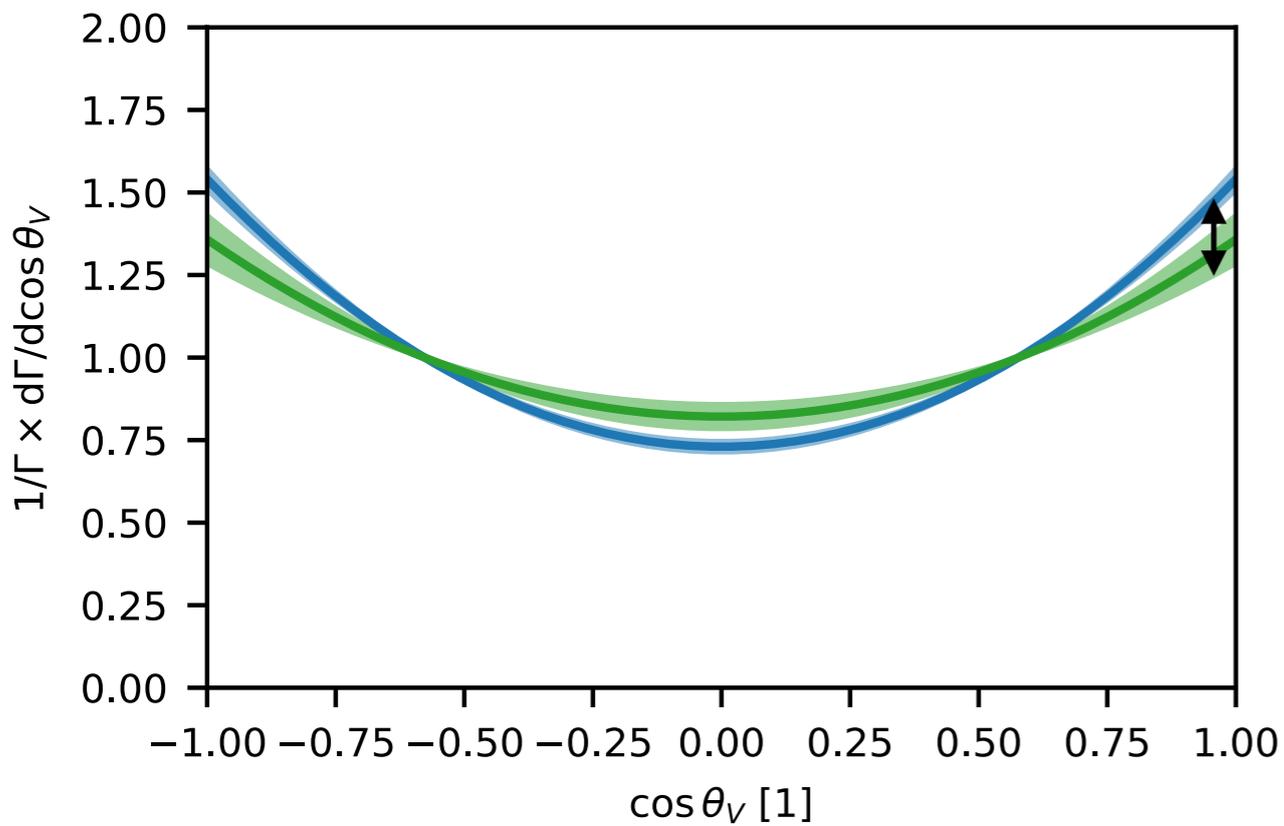
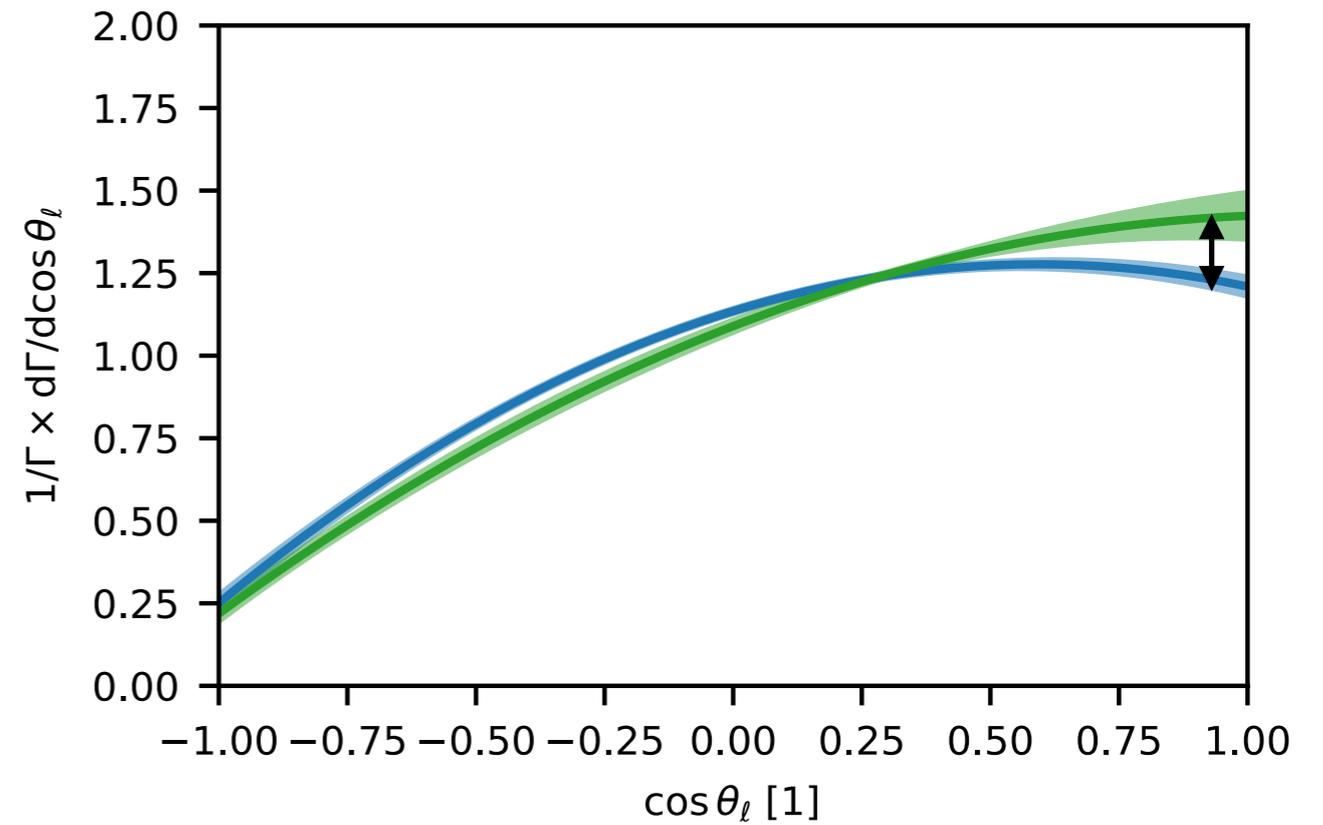
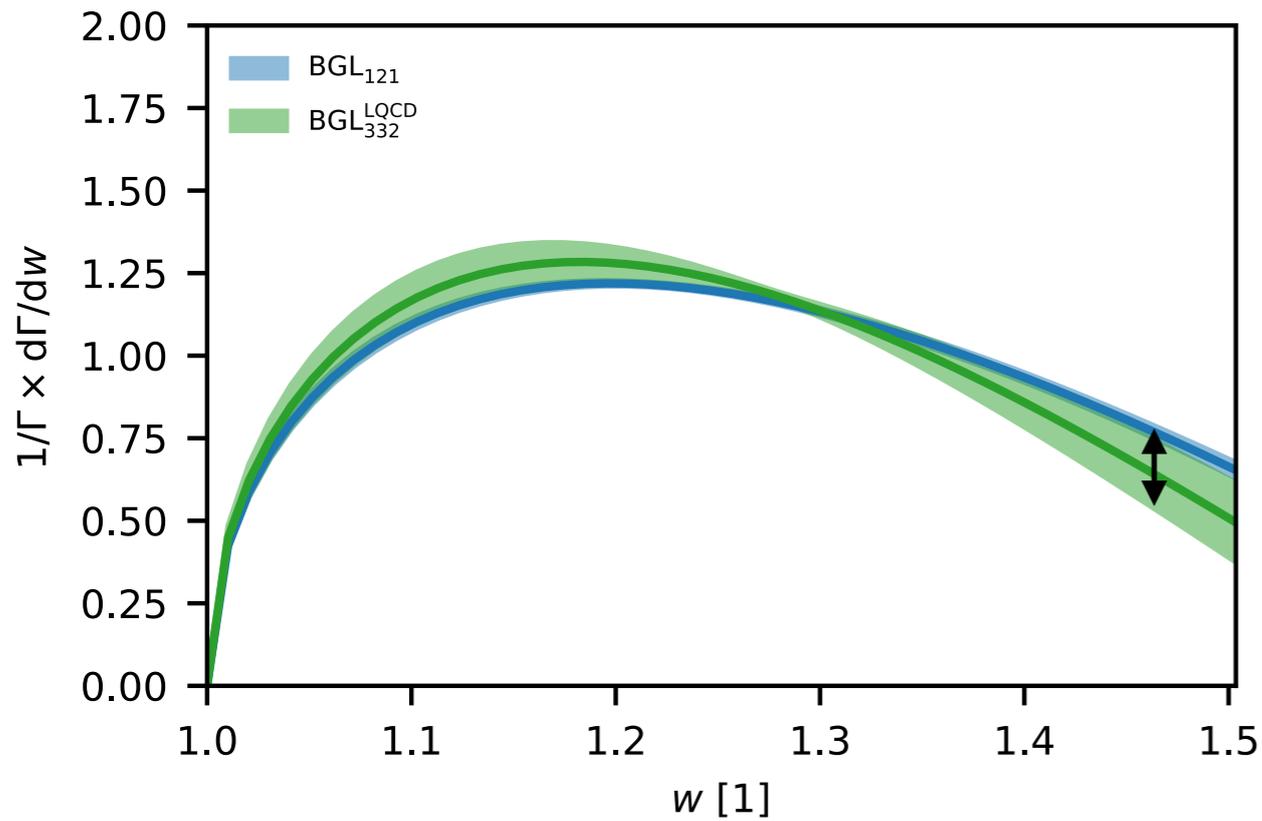
30



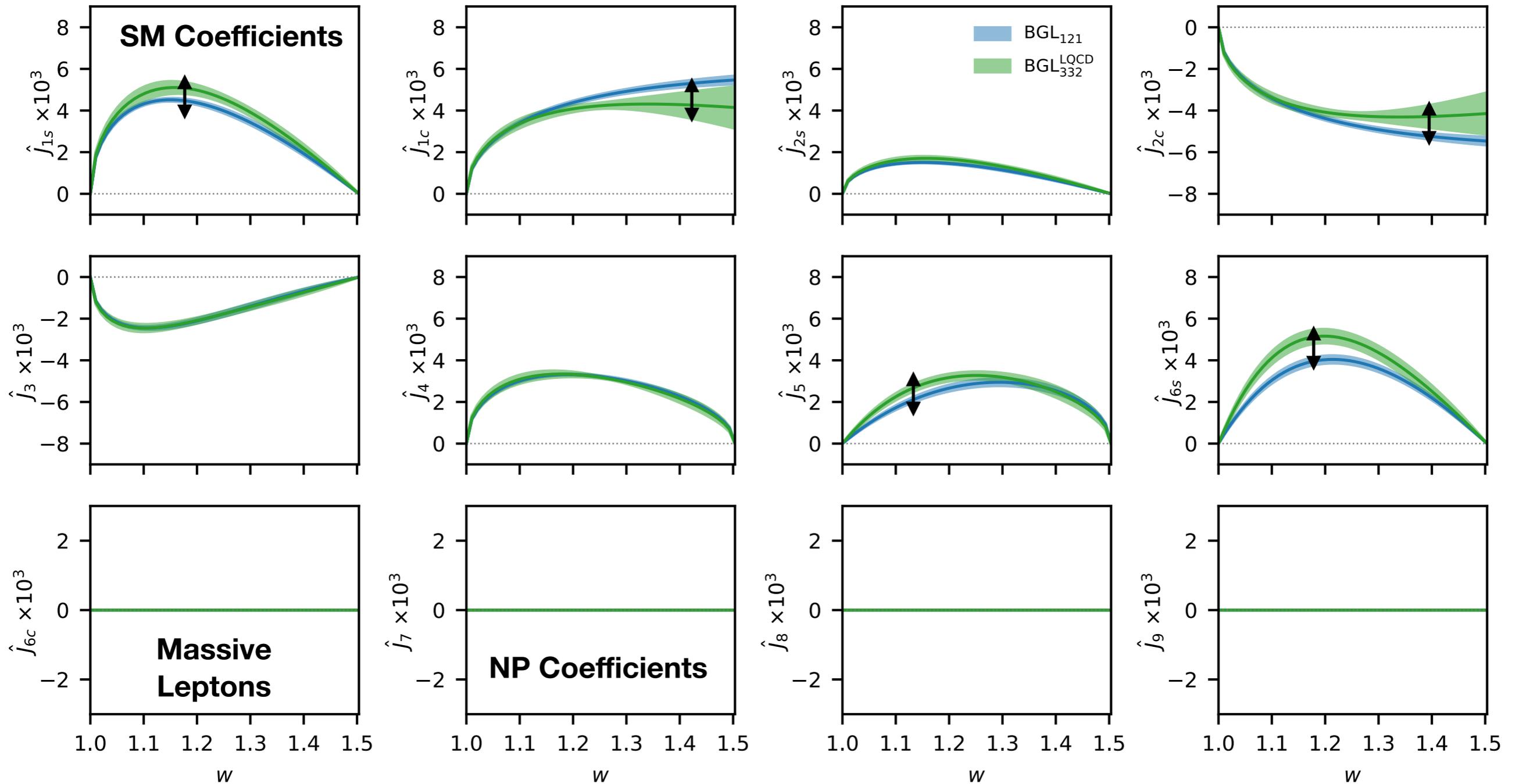
1D versus Full Angular Sensitivities

BGL121 1D projection fit of
arXiv:2301.07529 (Table XVI) or
FNAL/MILC prediction
[arXiv:2105.14019]

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1D versus Full Angular Sensitivities



Angular Coefficients also will allow us to better investigate what is going on with **lattice** versus **data tensions**..

Summary

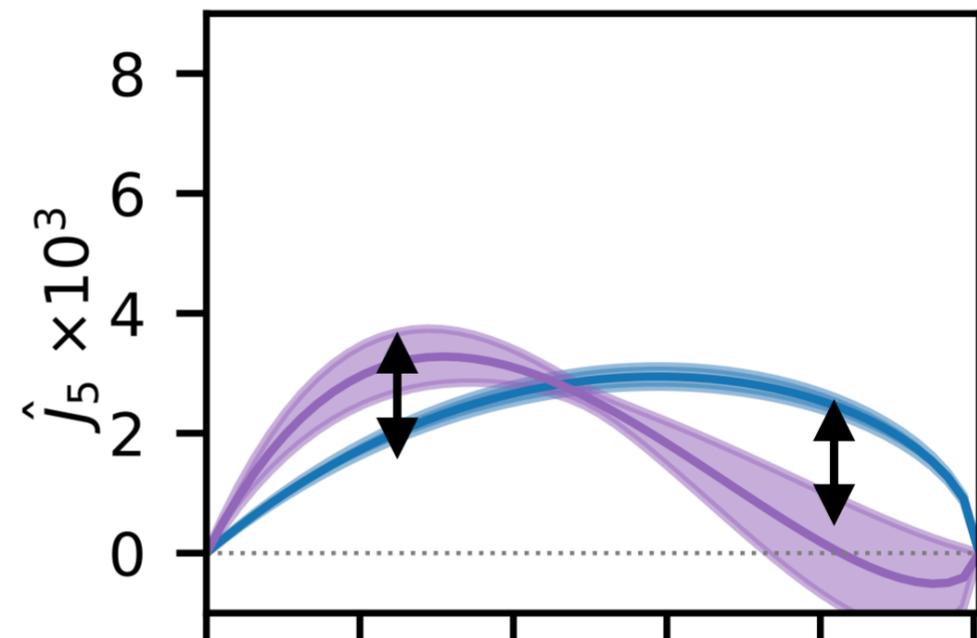
Angular information is crucial to better study $B \rightarrow D^* \ell \bar{\nu}_\ell$:

The **future** will hopefully go towards **open data** ... but part of a longer discussion.

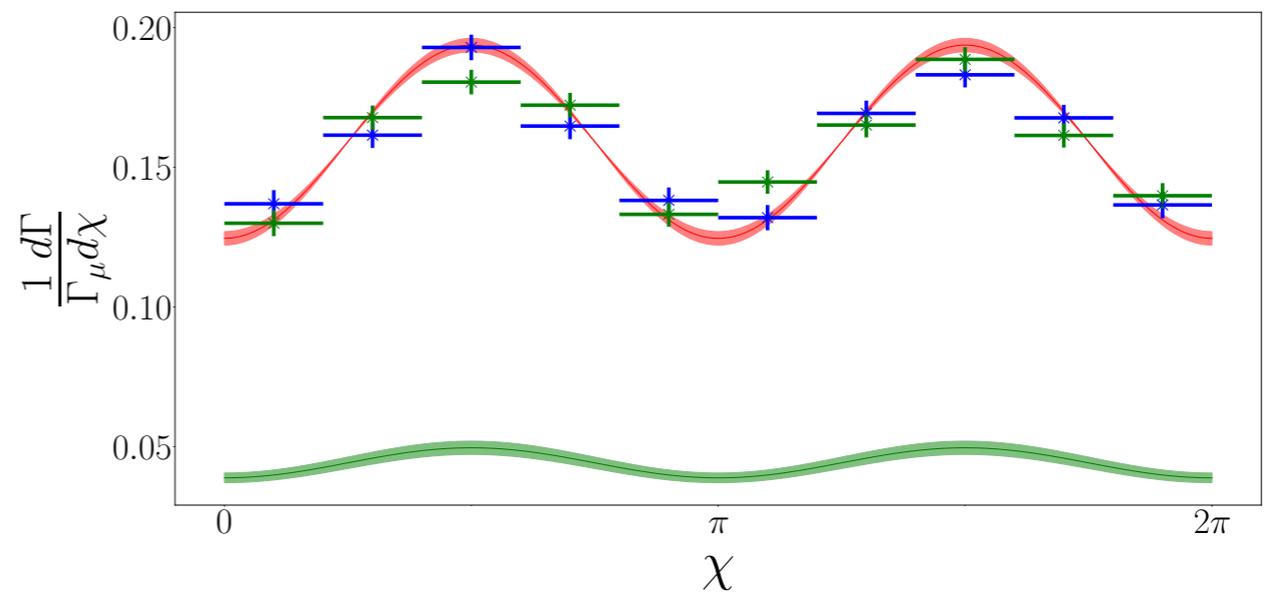
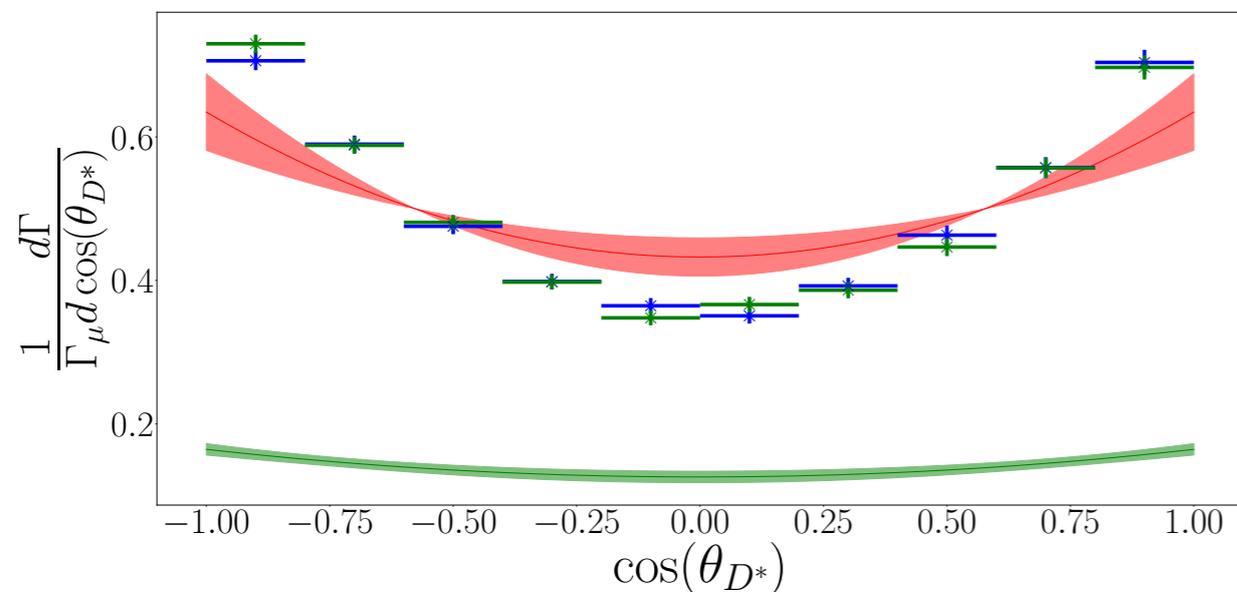
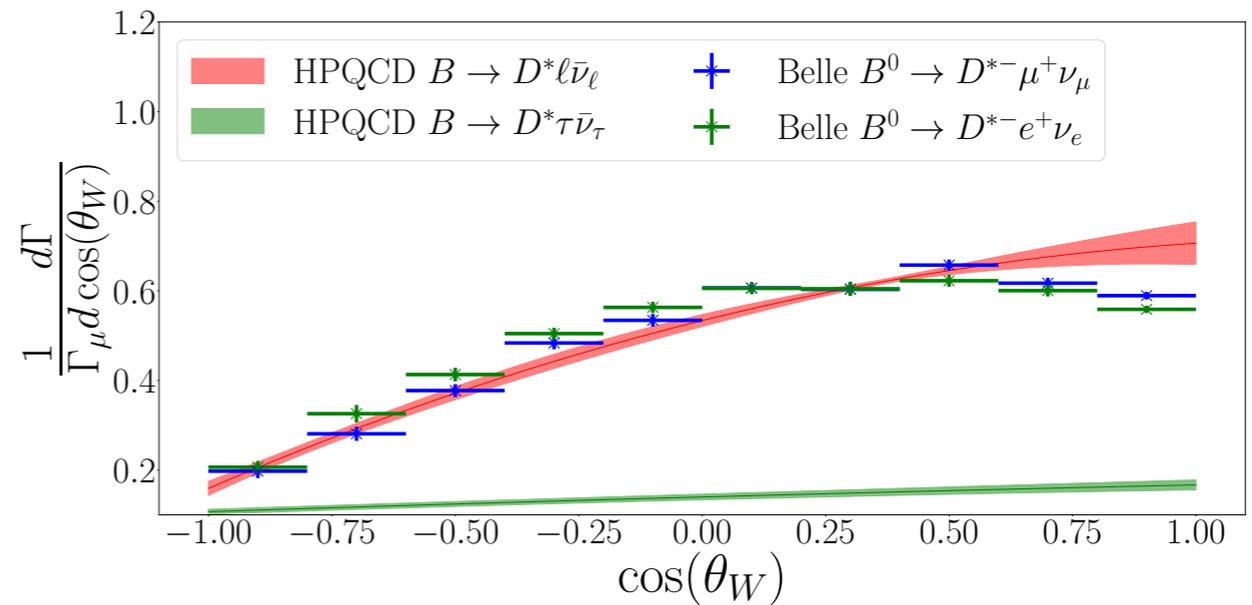
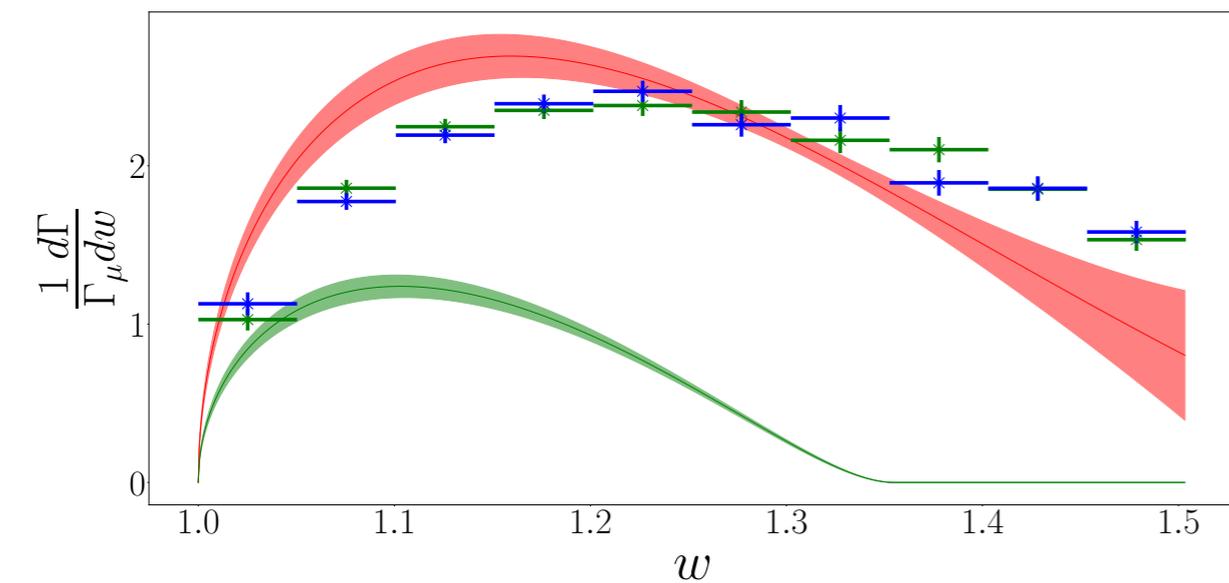
Unbinned unfolded data is a great new idea; best preserves the properties of the underlying data; but many details need to be ironed out

In the meantime: **Binned angular coefficients** seem a very promising strategy to preserve our measurements.

$$\frac{d\Gamma}{dq^2 d \cos \theta_V d \cos \theta_\ell d \chi} = \frac{G_F^2 |V_{cb}|^2 m_B^3}{2\pi^4} \times \left\{ \begin{aligned} & J_{1s} \sin^2 \theta_V + J_{1c} \cos^2 \theta_V \\ & + (J_{2s} \sin^2 \theta_V + J_{2c} \cos^2 \theta_V) \cos 2\theta_\ell \\ & + J_3 \sin^2 \theta_V \sin^2 \theta_\ell \cos 2\chi \\ & + J_4 \sin 2\theta_V \sin 2\theta_\ell \cos \chi + J_5 \sin 2\theta_V \sin \theta_\ell \cos \chi \\ & + (J_{6s} \sin^2 \theta_V + J_{6c} \cos^2 \theta_V) \cos \theta_\ell \\ & + J_7 \sin 2\theta_V \sin \theta_\ell \sin \chi + J_8 \sin 2\theta_V \sin 2\theta_\ell \sin \chi \\ & + J_9 \sin^2 \theta_V \sin^2 \theta_\ell \sin 2\chi \end{aligned} \right\}.$$

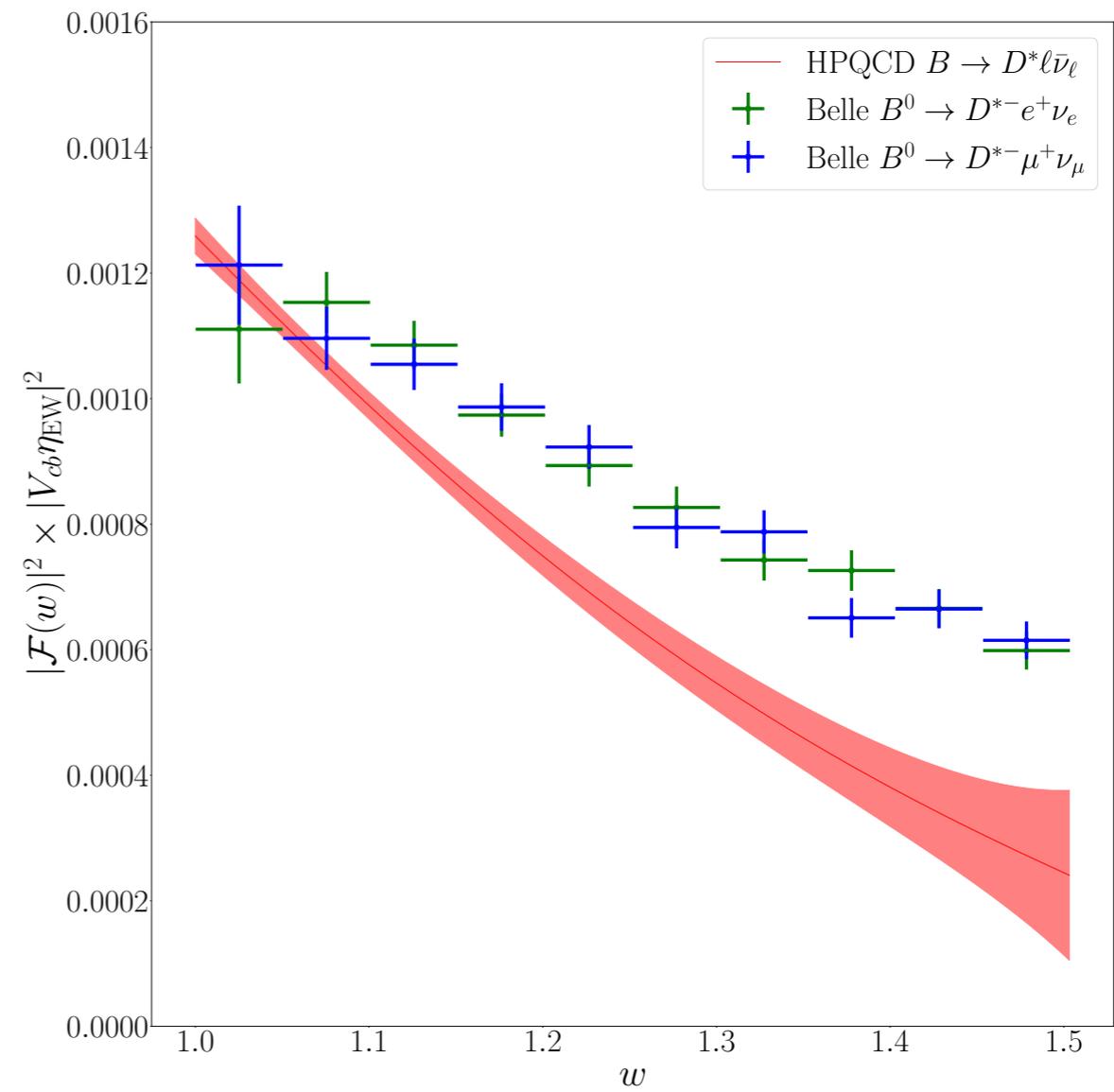
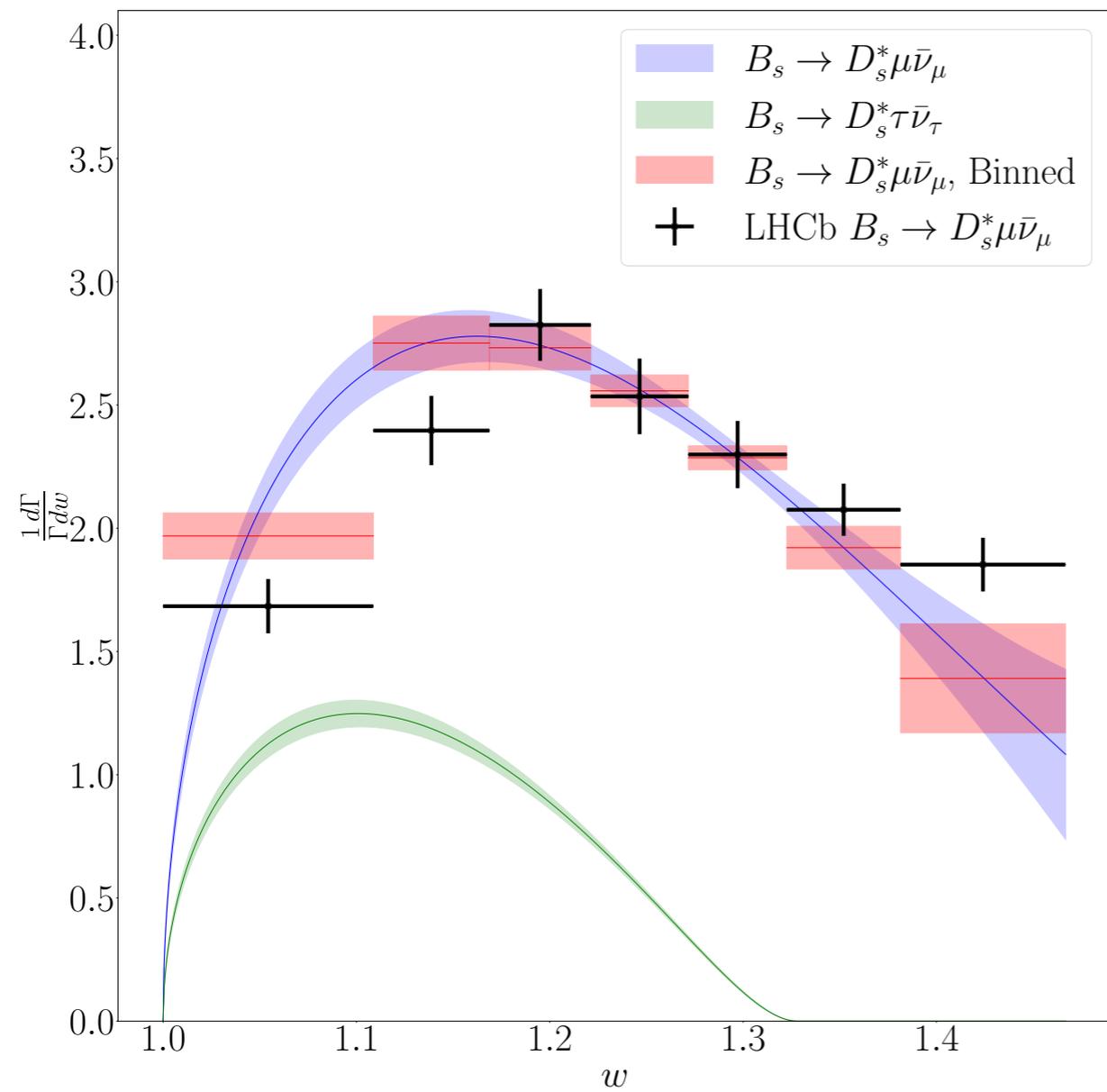


Backup

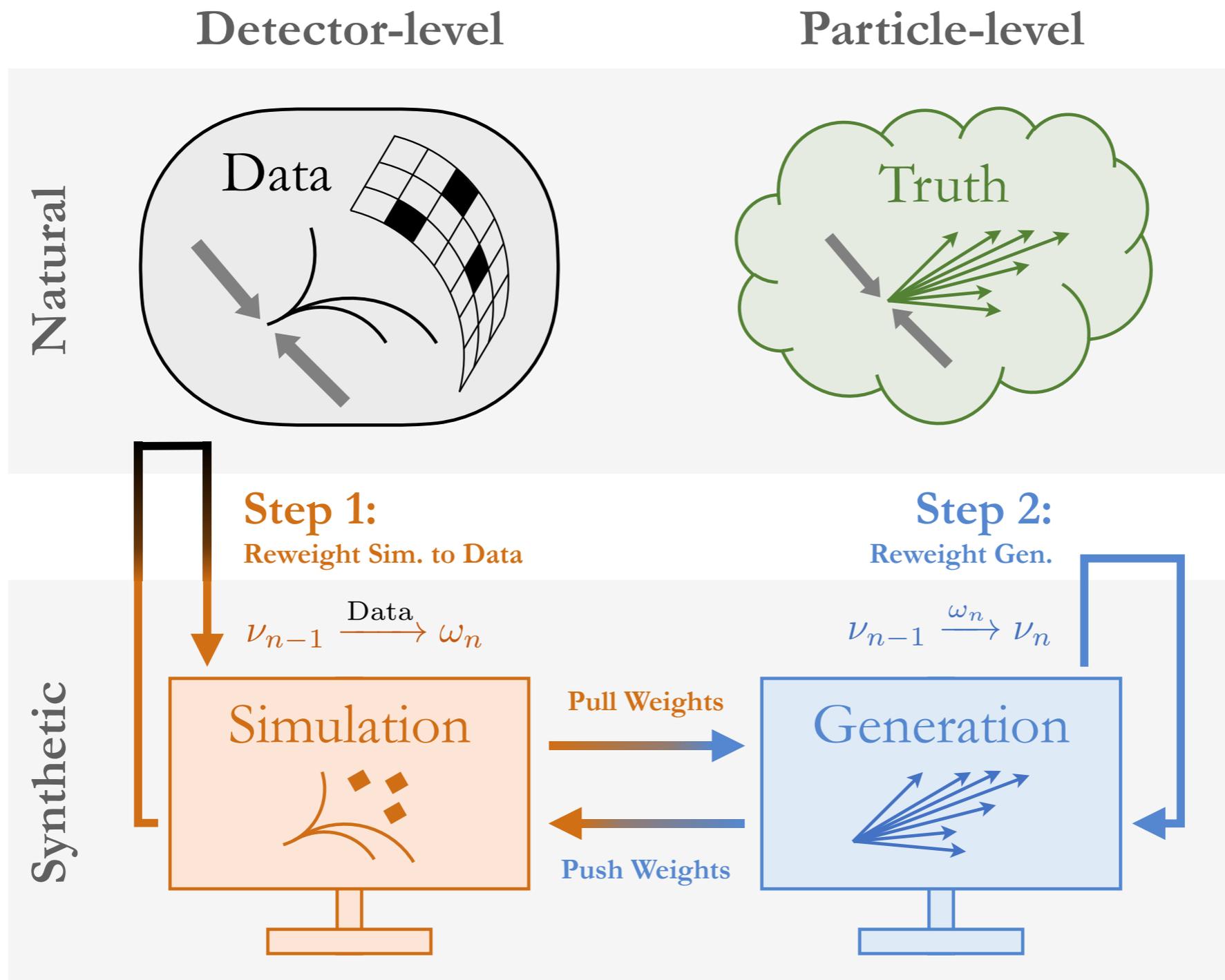


Is it meaningful to combine LQCD and data that do not agree in shape?

What does this mean for our $|V_{cb}|$ values? Can we trust $\mathcal{F}(1)$?



Same data / MC disagreement?



$$p_{\text{unfolded}}^{(n)}(t) = \nu_n(t) p_{\text{Gen.}}(t).$$

- UNIFOLD: A single observable as input. This is an unbinned version of IBU.
- MULTIFOLD: Many observables as input. Here, we use the six jet substructure observables in Fig. 2 to derive the detector response.
- OMNIFOLD: The full event (or jet) as input, using the full phase space information.

