

New physics search with angular distribution of $B \rightarrow D^* l \nu$ decay

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13 April 2023 @ Frascati

Outline

Introduction

- Angular distribution and new physics search
- Form factor dependence

The impact of the new lattice data

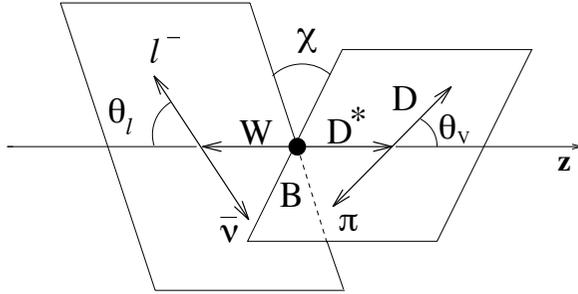
- Fermilab and JLQCD result on the form factors
- Impacts of the lattice results

New Physics fit of experimental data including lattice data

- Toy study of unbanned analysis
- Preliminary results

Introduction

Introduction

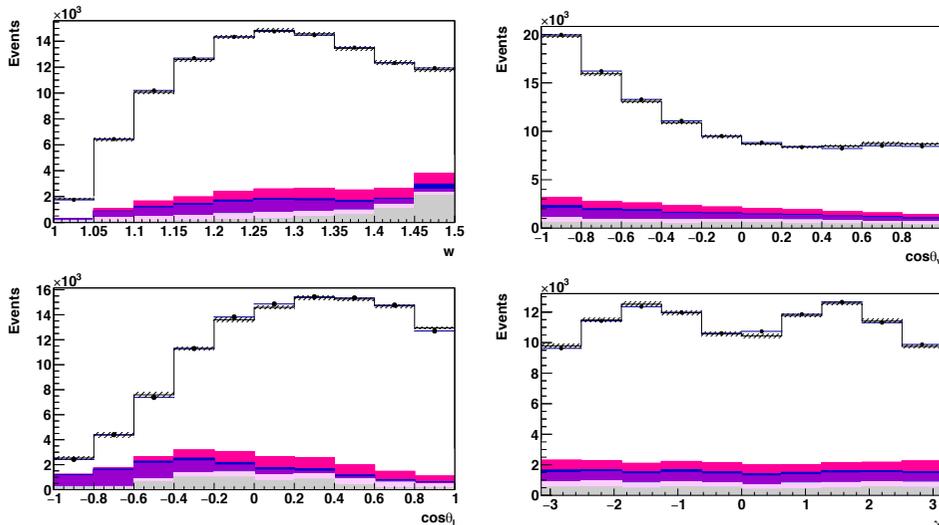


$$\frac{d\Gamma(B^0 \rightarrow D^{*-} \ell^+ \nu_\ell)}{dw d\cos\theta_\ell d\cos\theta_\nu d\chi} =$$

$$\frac{\eta_{EW}^2 3m_B m_{D^*}^2}{4(4\pi)^2} G_F^2 |V_{cb}|^2 \sqrt{w^2 - 1} (1 - 2wr + r^2)$$

$$\left\{ (1 - \cos\theta_\ell)^2 \sin^2\theta_\nu H_+^2(w) + (1 + \cos\theta_\ell)^2 \sin^2\theta_\nu H_-^2(w) \right. \\ \left. + 4 \sin^2\theta_\ell \cos^2\theta_\nu H_0^2 - 2 \sin^2\theta_\ell \sin^2\theta_\nu \cos 2\chi H_+(w) H_-(w) \right. \\ \left. - 4 \sin\theta_\ell (1 - \cos\theta_\ell) \sin\theta_\nu \cos\theta_\nu \cos\chi H_+(w) H_0(w) \right. \\ \left. + 4 \sin\theta_\ell (1 + \cos\theta_\ell) \sin\theta_\nu \cos\theta_\nu \cos\chi H_-(w) - H_0(w) \right\} \quad (5)$$

Belle angular analysis



- 4 dimensional binned analysis
- **SM is assumed**
- Simultaneous fit of form factors and V_{cb} (**with one lattice input**)

Introduction

• Helicity amplitudes in BGL

$$H_{\pm}(w) = f(w) \mp m_B |\mathbf{p}_{D^*}| g(w)$$

$$H_0(w) = \frac{\mathcal{F}_1(w)}{\sqrt{q^2}}$$

$$z \equiv \frac{\sqrt{w+1} - \sqrt{2}}{\sqrt{w+1} + \sqrt{2}}$$

$$r = m_{D^*}/m_B$$

$$g(z) = \frac{1}{P_g(z)\phi_g(z)} \sum_{n=0}^N a_n^g z^n, \quad f(z) = \frac{1}{P_f(z)\phi_f(z)} \sum_{n=0}^N a_n^f z^n, \quad \mathcal{F}_1(z) = \frac{1}{P_{\mathcal{F}_1}(z)\phi_{\mathcal{F}_1}(z)} \sum_{n=0}^N a_n^{\mathcal{F}_1} z^n$$

• Helicity amplitudes in CNL

$$H_{\pm}(w) = m_B \sqrt{r} (w+1) h_{A_1}(w) \left[1 \mp \sqrt{\frac{w-1}{w+1}} R_1(w) \right]$$

$$H_0(w) = m_B \sqrt{r} (w+1) \frac{1-r}{\sqrt{q^2}} h_{A_1}(w) \left[1 + \frac{w-1}{1-r} (1 - R_2(w)) \right]$$

$$h_{A_1}(w) = h_{A_1}(1) (1 - 8\rho_{D^*}^2 + (53\rho_{D^*}^2 - 15)z^2 - (231\rho_{D^*}^2 - 91)z^2)$$

$$R_1(w) = R_1(1) - 0.12(w-1) + 0.05(w-1)^2$$

$$R_2(w) = R_2(1) + 0.11(w-1) - 0.06(w-1)^2$$

New physics search with angular analysis

Is it possible to do new physics search like B->K* II?

$$\mathcal{H}_{eff} = \frac{4G_F}{\sqrt{2}} V_{cb} [C_{V_L}^l O_{V_L}^l + C_{V_R}^l O_{V_R}^l + C_S^l O_S^l + C_P^l O_P^l + C_T^l O_T^l]$$

$$O_{V_L}^l = (\bar{c}_L \gamma^\mu b_L) (\bar{\ell}_L \gamma_\mu \nu_{\ell L})$$

$$O_{V_R}^l = (\bar{c}_R \gamma^\mu b_R) (\bar{\ell}_L \gamma_\mu \nu_{\ell L})$$

$$O_S^l = (\bar{c}_L b_R) (\bar{\ell}_R \nu_{\ell L})$$

$$O_P^l = (\bar{c} \gamma^5 b) (\bar{\ell}_R \nu_{\ell L})$$

$$O_T^l = (\bar{c}_R \sigma^{\mu\nu} b_L) (\bar{\ell}_R \sigma_{\mu\nu} \nu_{\ell L})$$

B->K* II angular analysis:

- 4 dimensional **unbinned maximum likelihood analysis**
- Simultaneous fit of form factors (nuisance parameters) and **several Wilson coefficients**

Example of the Right-Handed (RH) model

- Differential decay rate with RH current

$$\begin{aligned} \frac{d\Gamma(\bar{B} \rightarrow D^*(\rightarrow D\pi) \ell^- \bar{\nu}_\ell)}{dw d\cos\theta_V d\cos\theta_\ell d\chi} &= \frac{6m_B m_{D^*}^2}{8(4\pi)^4} \sqrt{w^2 - 1} (1 - 2wr + r^2) G_F^2 |V_{cb}|^2 \times \mathcal{B}(D^* \rightarrow D\pi) \\ &\times \left\{ J_{1s} \sin^2 \theta_V + J_{1c} \cos^2 \theta_V + (J_{2s} \sin^2 \theta_V + J_{2c} \cos^2 \theta_V) \cos 2\theta_\ell \right. \\ &\quad + J_3 \sin^2 \theta_V \sin^2 \theta_\ell \cos 2\chi \\ &\quad + J_4 \sin 2\theta_V \sin 2\theta_\ell \cos \chi + J_5 \sin 2\theta_V \sin \theta_\ell \cos \chi \\ &\quad + (J_{6s} \sin^2 \theta_V + J_{6c} \cos^2 \theta_V) \cos \theta_\ell \\ &\quad + J_7 \sin 2\theta_V \sin \theta_\ell \sin \chi + J_8 \sin 2\theta_V \sin 2\theta_\ell \sin \chi \\ &\quad \left. + J_9 \sin^2 \theta_V \sin^2 \theta_\ell \sin 2\chi \right\} \end{aligned}$$

$$J_{1s} = \frac{3}{2}(H_+^2 + H_-^2)(|C_{V_L}|^2 + |C_{V_R}|^2) - 6H_+H_- \text{Re}[C_{V_L}C_{V_R}^*]$$

$$J_{1c} = 2H_0^2(|C_{V_L}|^2 + |C_{V_R}|^2 - 2\text{Re}[C_{V_L}C_{V_R}^*])$$

$$J_{2s} = \frac{1}{2}(H_+^2 + H_-^2)(|C_{V_L}|^2 + |C_{V_R}|^2) - 2H_+H_- \text{Re}[C_{V_L}C_{V_R}^*]$$

$$J_{2c} = -2H_0^2(|C_{V_L}|^2 + |C_{V_R}|^2 - 2\text{Re}[C_{V_L}C_{V_R}^*])$$

$$J_3 = -2H_+H_- (|C_{V_L}|^2 + |C_{V_R}|^2) + 2(H_+^2 + H_-^2) \text{Re}[C_{V_L}C_{V_R}^*]$$

$$J_4 = (H_+H_0 + H_-H_0)(|C_{V_L}|^2 + |C_{V_R}|^2 - 2\text{Re}[C_{V_L}C_{V_R}^*])$$

$$J_5 = -2(H_+H_0 - H_-H_0)(|C_{V_L}|^2 - |C_{V_R}|^2)$$

$$J_{6s} = -2(H_+^2 - H_-^2)(|C_{V_L}|^2 - |C_{V_R}|^2)$$

$$J_{6c} = 0$$

$$J_7 = 0$$

$$J_8 = 2(H_+H_0 - H_-H_0) \text{Im}[C_{V_L}C_{V_R}^*]$$

$$J_9 = 2(H_+^2 - H_-^2) \text{Im}[C_{V_L}C_{V_R}^*]$$

Example of the Right-Handed (RH) model

ZR. Huang, C.D. Lu, R.T. Tang, E.K.
Phys.Rev.D 105 (2022) 1, 013010

- Unbinned maximum likelihood analysis

$$\chi^2(\vec{v}) = \chi_{\text{angle}}^2(\vec{v}) + \chi_{w\text{-bin}}^2(\vec{v})$$

\vec{v} : Fit parameters
(FF, C_{VR} etc...)

- Angular part:

$$\chi_{\text{angle}}^2(\vec{v}) = \sum_{w\text{-bin}=1}^{10} \left[\sum_{ij} N_{\text{event}} \hat{V}_{ij}^{-1} (\langle g_i \rangle^{\text{exp}} - \langle g_i^{\text{th}}(\vec{v}) \rangle) (\langle g_j \rangle^{\text{exp}} - \langle g_j^{\text{th}}(\vec{v}) \rangle) \right]_{w\text{-bin}}$$

Normalised PDF

$$\begin{aligned} \hat{f}_{\langle g \rangle}(\cos \theta_V, \cos \theta_\ell, \chi) = & \frac{9\pi}{8} \times \left\{ \frac{1}{6} (1 - 3\langle g_{1c} \rangle + 2\langle g_{2s} \rangle + \langle g_{2c} \rangle) \sin^2 \theta_V + \langle g_{1c} \rangle \cos^2 \theta_V \right. \\ & + \langle g_{2s} \rangle \sin^2 \theta_V + \langle g_{2c} \rangle \cos^2 \theta_V \cos 2\theta_\ell \\ & + \langle g_3 \rangle \sin^2 \theta_V \sin^2 \theta_\ell \cos 2\chi \\ & + \langle g_4 \rangle \sin 2\theta_V \sin 2\theta_\ell \cos \chi + \langle g_5 \rangle \sin 2\theta_V \sin \theta_\ell \cos \chi \\ & + (\langle g_{6s} \rangle \sin^2 \theta_V + \langle g_{6c} \rangle \cos^2 \theta_V) \cos \theta_\ell \\ & + \langle g_7 \rangle \sin 2\theta_V \sin \theta_\ell \sin \chi + \langle g_8 \rangle \sin 2\theta_V \sin 2\theta_\ell \sin \chi \\ & \left. + \langle g_9 \rangle \sin^2 \theta_V \sin^2 \theta_\ell \sin 2\chi \right\} \end{aligned} \quad (46)$$

$$\langle J_i \rangle_{w\text{-bin}} \equiv \int_{w\text{-bin}} J_i dw.$$

$$\langle g_i \rangle \equiv \frac{\langle J_i \rangle}{6\langle J_{1s} \rangle + 3\langle J_{1c} \rangle - 2\langle J_{2s} \rangle - \langle J_{2c} \rangle}$$

Example of the Right-Handed (RH) model

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- Unbinned maximum likelihood analysis

$$\chi^2(\vec{v}) = \chi_{\text{angle}}^2(\vec{v}) + \chi_{w\text{-bin}}^2(\vec{v})$$

\vec{v} : Fit parameters
(FF, C_{VR} etc...)

- N_{event} part:

$$\chi_{w\text{-bin}}^2(\vec{v}) = \sum_{w\text{-bin}=1}^{10} \frac{([N]_{w\text{-bin}} - \alpha \langle \Gamma \rangle_{w\text{-bin}})^2}{[N]_{w\text{-bin}}}$$

$$\langle \Gamma \rangle_{w\text{-bin}} = \frac{6m_B m_{D^*}^2}{8(4\pi)^4} G_F^2 |V_{cb}|^2 \times \mathcal{B}(D^* \rightarrow D\pi) \frac{8}{9\pi} \left\{ 6\langle J'_{1s} \rangle_{w\text{-bin}} + 3\langle J'_{1c} \rangle_{w\text{-bin}} - 2\langle J'_{2s} \rangle_{w\text{-bin}} - \langle J'_{2c} \rangle_{w\text{-bin}} \right\}$$

$$\alpha \equiv \frac{4N_{B\bar{B}}}{1 + f_{+0}} \tau_{B^0} \times \epsilon \mathcal{B}(D^* \rightarrow D\pi) \mathcal{B}(D \rightarrow K\pi)$$

Necessity of form factor inputs to fit C_{VR}

- Branching ratio

$$C_w \equiv \sqrt{w^2 - 1}(1 - 2wr + r^2)$$

$$\text{BR}(B \rightarrow D^* \ell \nu)$$

$$\propto G_F |V_{cb}|^2 \int dw C_w \left[(2f(w)^2 + \frac{\mathcal{F}_1(w)^2}{q^2}) |C_{VL} - C_{VR}|^2 + 2g(w)^2 m_B^2 |\mathbf{p}_D|^2 |C_{VL} + C_{VR}|^2 \right]$$

V_{cb} and form factor parameters have to be fixed to get C_{VR}

- Forward-backward asymmetry

$$A_{\theta_\ell} \equiv \frac{\int_0^1 d\Gamma(\cos \theta_\ell) d \cos \theta_\ell - \int_{-1}^0 d\Gamma(\cos \theta_\ell) d \cos \theta_\ell}{\int_0^1 d\Gamma(\cos \theta_\ell) d \cos \theta_\ell + \int_{-1}^0 d\Gamma(\cos \theta_\ell) d \cos \theta_\ell}$$

$$\propto \frac{\int dw C_w \left[f(w)g(w)m_B |\mathbf{p}_D| (|C_{VL}|^2 - |C_{VR}|^2) \right]}{\int dw C_w \left[(2f(w)^2 + \frac{\mathcal{F}_1(w)^2}{q^2}) |C_{VL} - C_{VR}|^2 + 2g(w)^2 m_B^2 |\mathbf{p}_D|^2 |C_{VL} + C_{VR}|^2 \right]}$$

Form factor parameters have to be fixed to get C_{VR}

Impact of the new lattice data

Combining Belle & lattice results

*D. Ferlewicz, Ph. Urquijo, E. Waheed
arXiv:2008.09341*

- Assuming the SM, we fit the Belle data and lattice data simultaneously.

$$\chi_{\text{tot}}^2(\vec{v}) = \chi_{\text{exp}}^2(\vec{v}) + \chi_{\text{latt}}^2(\vec{v}_0)$$

$$\vec{v} = (a_i, b_i, c_i, d_i, V_{cb})$$

$$\vec{v}_0 = (f, g, \mathcal{F}_1, \mathcal{F}_2)$$

a_i, b_i, \dots : BGL parameters

Be careful with the Bc inputs!

The latest Lattice result on form factors

Fermilab-MILK [arXiv.2105.14019](#)
 JLQCD [arXiv.2304.xxxx](#)
 HPQCD [arXiv.2304.03137](#)

- Lattice QCD groups determined, all form factors (FF) necessary to describe the $B \rightarrow D^* l \nu$ decay, including their w dependence.
- In lattice QCD method, the w dependence of FF is obtained by chiral extrapolation. For example, the Fermilab group provided the FF at different point of w and correlation matrix.

$$\chi_{\text{latt}}^2(\vec{v}_0) = (\vec{v}_0 - \vec{v}_0^{\text{latt}}) V^{-1} (\vec{v}_0 - \vec{v}_0^{\text{latt}})^T$$

TABLE XX: Synthetic data of the chiral-continuum extrapolation for the form factors used in the z expansion with their correlation matrix.

	Value	Correlation Matrix											
		$f(1.03)$	$g(1.03)$	$\mathcal{F}_1(1.03)$	$\mathcal{F}_2(1.03)$	$f(1.10)$	$g(1.10)$	$\mathcal{F}_1(1.10)$	$\mathcal{F}_2(1.10)$	$f(1.17)$	$g(1.17)$	$\mathcal{F}_1(1.17)$	$\mathcal{F}_2(1.17)$
$f(1.03)$	5.77(11)	1.0000	0.1156	0.9885	0.6710	0.9247	0.1029	0.8596	0.6518	0.5743	0.0723	0.5229	0.4407
$g(1.03)$	0.371(14)	0.1156	1.0000	0.1304	0.3489	0.1245	0.9354	0.1564	0.3253	0.1057	0.6819	0.1373	0.2375
$\mathcal{F}_1(1.03)$	18.73(38)	0.9885	0.1304	1.0000	0.7304	0.9084	0.1153	0.9065	0.7204	0.5616	0.0816	0.5753	0.4991
$\mathcal{F}_2(1.03)$	2.175(70)	0.6710	0.3489	0.7304	1.0000	0.6024	0.3259	0.7364	0.9346	0.3584	0.2449	0.4923	0.6078
$f(1.10)$	5.49(12)	0.9247	0.1245	0.9084	0.6024	1.0000	0.1781	0.9031	0.6784	0.7973	0.2001	0.6881	0.5825
$g(1.10)$	0.330(14)	0.1029	0.9354	0.1153	0.3259	0.1781	1.0000	0.1954	0.3664	0.2416	0.8429	0.2442	0.3480
$\mathcal{F}_1(1.10)$	17.52(45)	0.8596	0.1564	0.9065	0.7364	0.9031	0.1954	1.0000	0.8430	0.6986	0.2002	0.8116	0.7270
$\mathcal{F}_2(1.10)$	1.912(69)	0.6518	0.3253	0.7204	0.9346	0.6784	0.3664	0.8430	1.0000	0.5238	0.3358	0.6981	0.7960
$f(1.17)$	5.23(17)	0.5743	0.1057	0.5616	0.3584	0.7973	0.2416	0.6986	0.5238	1.0000	0.3769	0.8364	0.7089
$g(1.17)$	0.290(17)	0.0723	0.6819	0.0816	0.2449	0.2001	0.8429	0.2002	0.3358	0.3769	1.0000	0.3571	0.4456
$\mathcal{F}_1(1.17)$	16.46(70)	0.5229	0.1373	0.5753	0.4923	0.6881	0.2442	0.8116	0.6981	0.8364	0.3571	1.0000	0.9218
$\mathcal{F}_2(1.17)$	1.692(91)	0.4407	0.2375	0.4991	0.6078	0.5825	0.3480	0.7270	0.7960	0.7089	0.4456	0.9218	1.0000

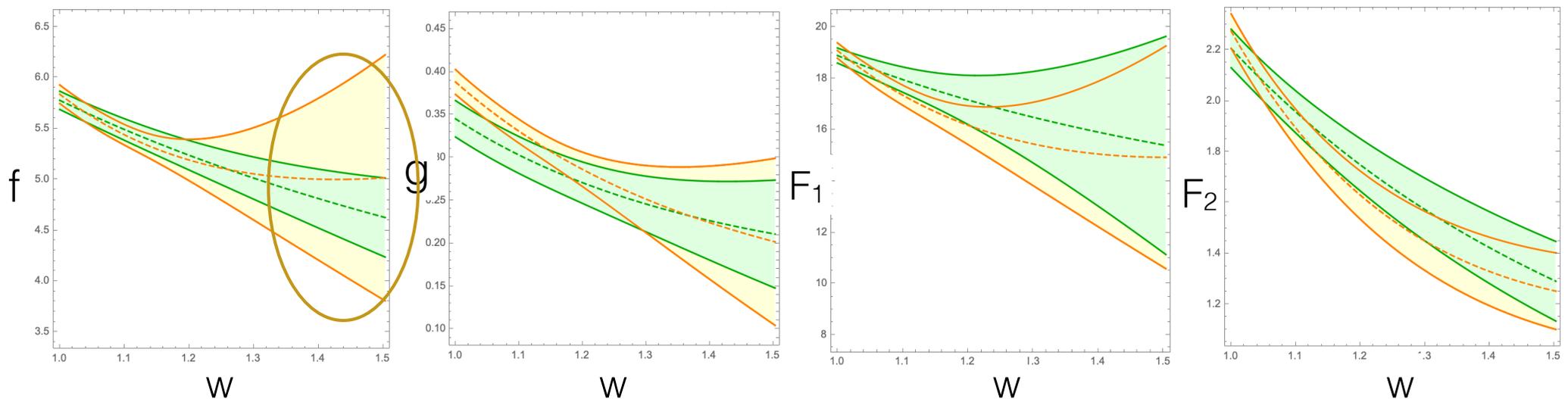
Comparison of two Lattice results

preliminary



- Using the lattice values for form factors at low w , we obtain the BGL parameters (e.g. up to quadratic term).
- As a result, the higher w regions are less constrained.

Without unitarity bound



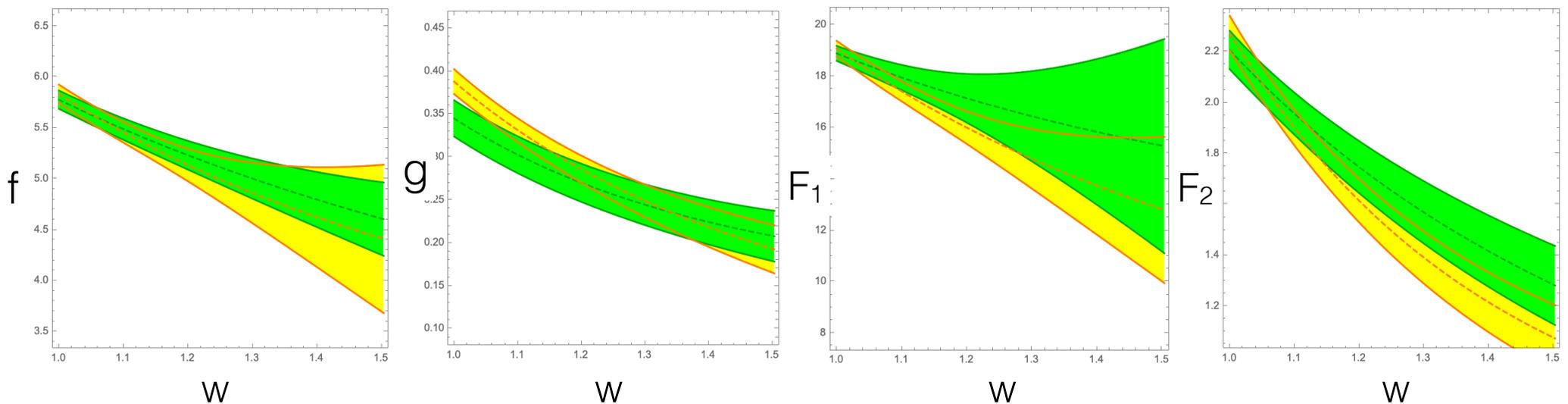
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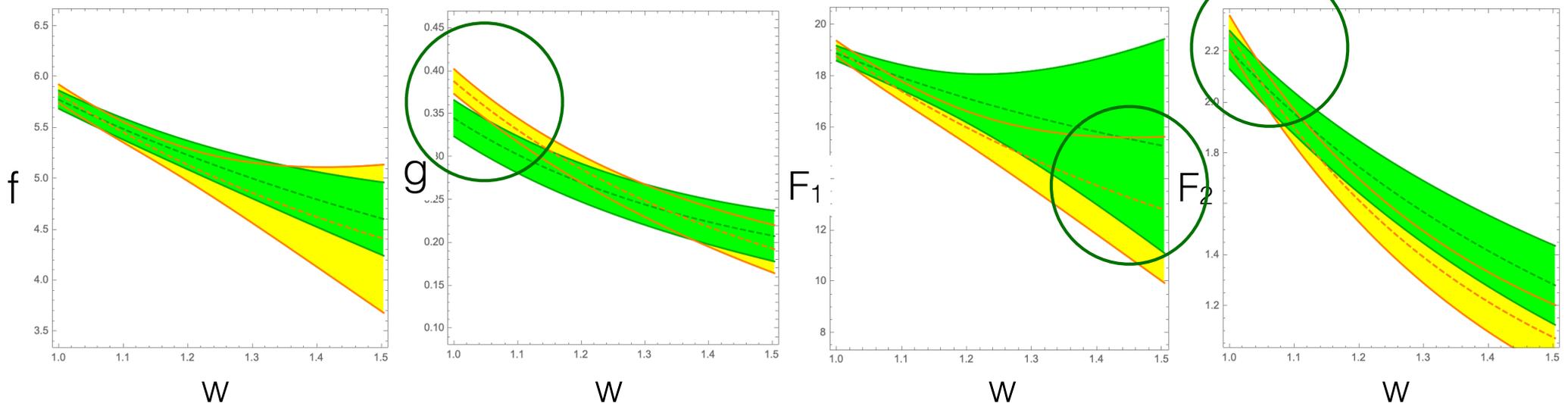
Comparison of two Lattice results

preliminary



- Using the lattice values for form factors at low w , we obtain the BGL parameters (e.g. up to quadratic term).
- As a result, the higher w regions are less constrained.

With unitarity bound



There are small differences between two results but within errors.

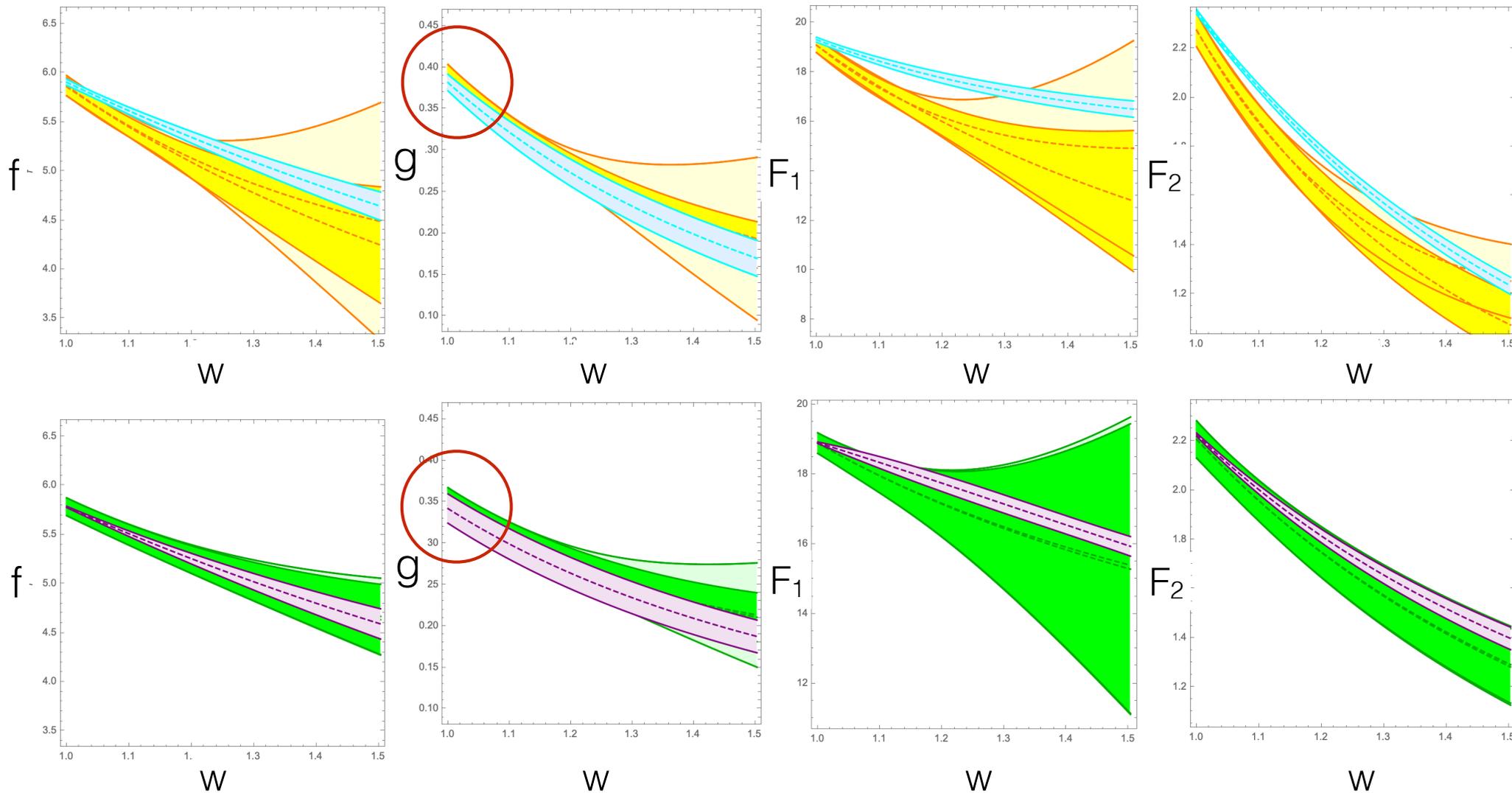
Lattice + Belle combined results

preliminary

Fermilab-MILK

JLQCD

+Belle '18 (arXiv:1809.03290)



JLQCD has slightly better agreement with Belle.

Toy study of the unbinned analysis

Toy study

Z.R. Huang, C.D. Lu, R.T.Tang, E.K.
Phys.Rev.D 105 (2022) 1, 013010
T. Kappor, Z.R. Huang, E.K.
arXiv:2304.xxxxx

1. Generate “fake-data” with the Belle '18 fitted parameters.
2. Fit the fake-data with the theory formula including **new physics parameters together with the lattice data**

$$\mathcal{H}_{eff} = \frac{4G_F}{\sqrt{2}} V_{cb} [C_{V_L}^l O_{V_L}^l + C_{V_R}^l O_{V_R}^l + C_S^l O_S^l + C_P^l O_P^l + C_T^l O_T^l]$$

$$O_{V_L}^l = (\bar{c}_L \gamma^\mu b_L)(\bar{\ell}_L \gamma_\mu \nu_{\ell L})$$

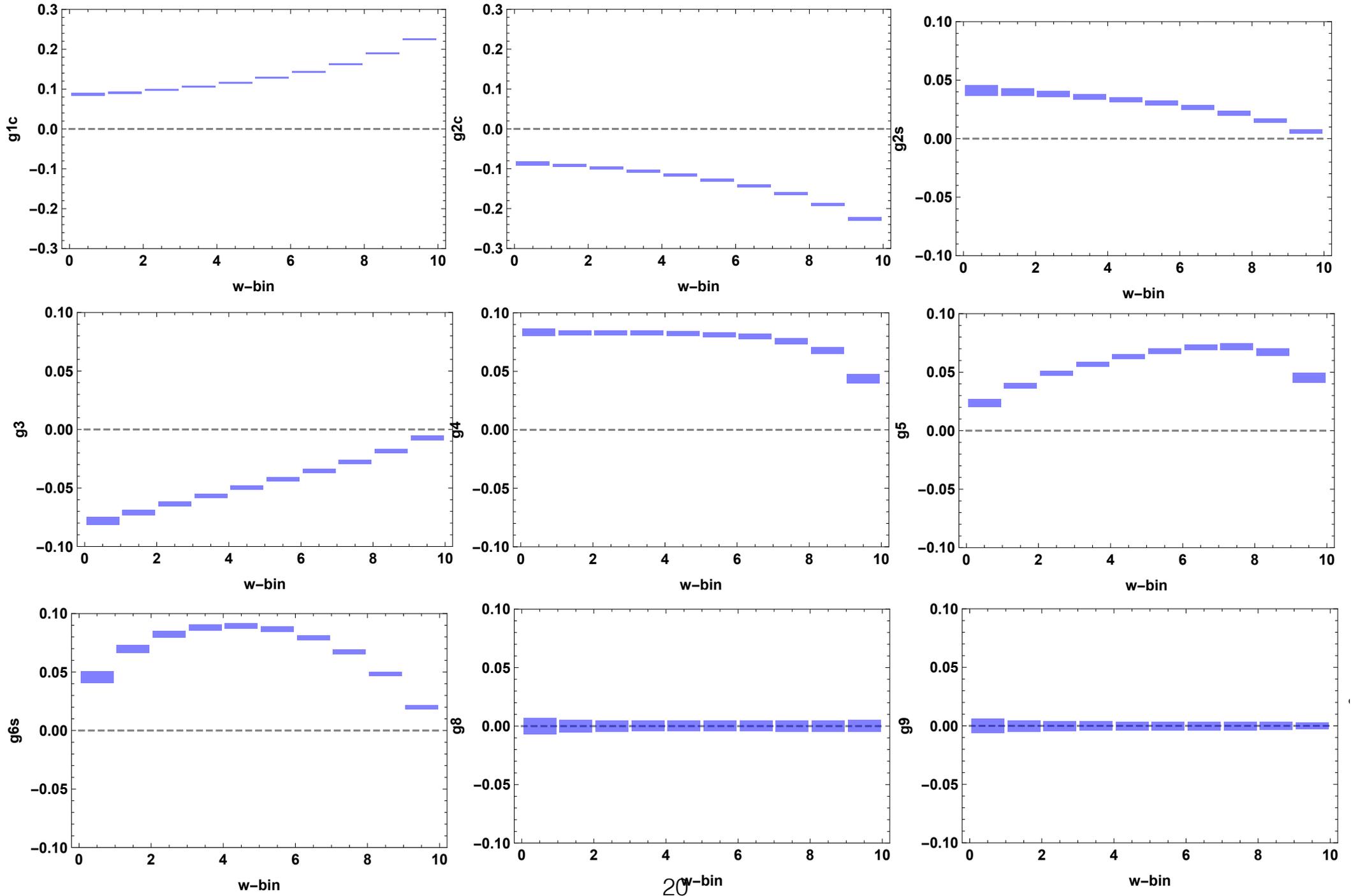
$$O_{V_R}^l = (\bar{c}_R \gamma^\mu b_R)(\bar{\ell}_L \gamma_\mu \nu_{\ell L})$$

$$O_S^l = (\bar{c}_L b_R)(\bar{\ell}_R \nu_{\ell L})$$

$$O_P^l = (\bar{c} \gamma^5 b)(\bar{\ell}_R \nu_{\ell L})$$

$$O_T^l = (\bar{c}_R \sigma^{\mu\nu} b_L)(\bar{\ell}_R \sigma_{\mu\nu} \nu_{\ell L})$$

Fake data



SM fit

Z.R. Huang, C.D. Lu, R.T. Tang, E.K.
Phys.Rev.D 105 (2022) 1, 013010

Reproducing Belle study

$$(\tilde{a}_g^{0,1,\dots}, \tilde{a}_f^{0,1,\dots}, \tilde{a}_{\mathcal{F}_1}) = \alpha V_{cb}(a_g^{0,1,\dots}, a_f^{0,1,\dots}, a_{\mathcal{F}_1}) \leftarrow \text{normalisation}$$

$$a_0^f = 2m_B \sqrt{r} P_f(0) \phi_f(0) h_{A_1}(1)$$

$$h_{A_1}(1) = 0.906 \pm 0.013 \leftarrow \text{lattice input}$$

$$\vec{v} = (\tilde{a}_f^0, \tilde{a}_f^1, \tilde{a}_{\mathcal{F}_1}, \tilde{a}_{\mathcal{F}_2}, \tilde{a}_g^0) = (0.051, 0.066, 0.027, -0.329, 0.093) \times 10^{-2}$$

$$\sigma_{\vec{v}} = (0.0004, 0.016, 0.006, 0.119, 0.001) \times 10^{-2}$$

$$\rho_{\vec{v}} = \begin{pmatrix} 1. & -0.816 & -0.73 & 0.586 & -0.002 \\ -0.816 & 1. & 0.525 & -0.415 & -0.046 \\ -0.73 & 0.525 & 1. & -0.969 & -0.004 \\ 0.586 & -0.415 & -0.969 & 1. & 0.003 \\ -0.002 & -0.046 & -0.004 & 0.003 & 1. \end{pmatrix}$$

$$V_{cb} = (38.7 \pm 0.6) \times 10^{-3}$$

Statistical error to the form factors are 20-80% better than the Belle study (only), probably due to the unbanned analysis.

Right-Handed fit

Z.R. Huang, C.D. Lu, R.T. Tang, E.K.
Phys.Rev.D 105 (2022) 1, 013010

Previous study (before lattice result)

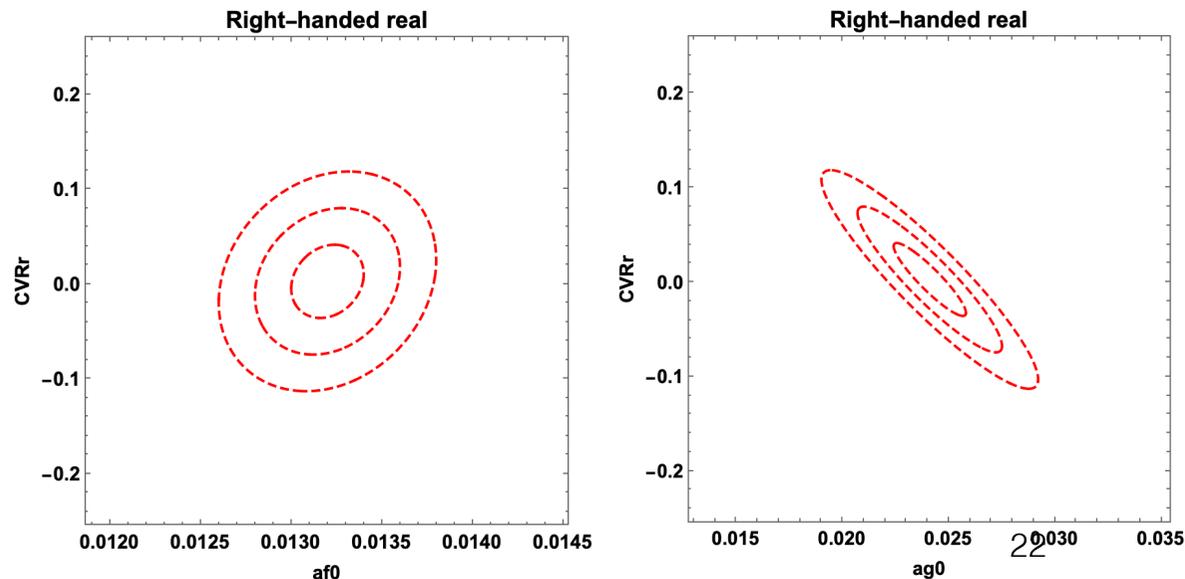
$$a_0^f = 0.0132 \pm 0.0002 \quad \leftarrow \text{lattice input}$$

$$a_g^0 = 0.0240 \pm 0.0007 \quad \leftarrow \text{Belle input}$$

We use only angular dependence and assume C_{VR} is real

$$\vec{v} = (a_f^0, a_f^1, a_{\mathcal{F}_1}, a_{\mathcal{F}_2}, a_g^0, C_{VR}) = (0.0132, 0.0169, 0.007, -0.0852, 0.0241, 0.0024)$$

$$\sigma_{\vec{v}} = (0.0002, 0.0173, 0.0041, 0.0556, 0.0005, 0.0234)$$

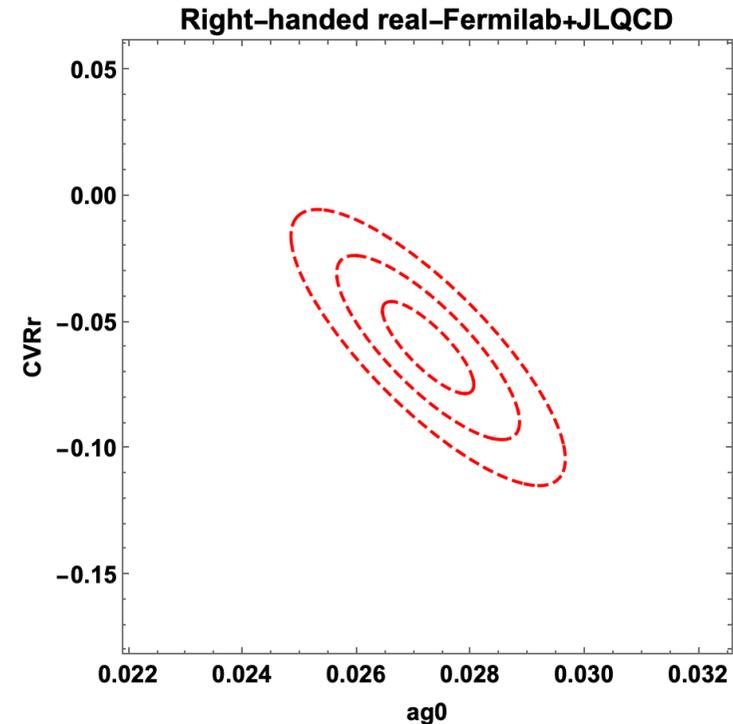
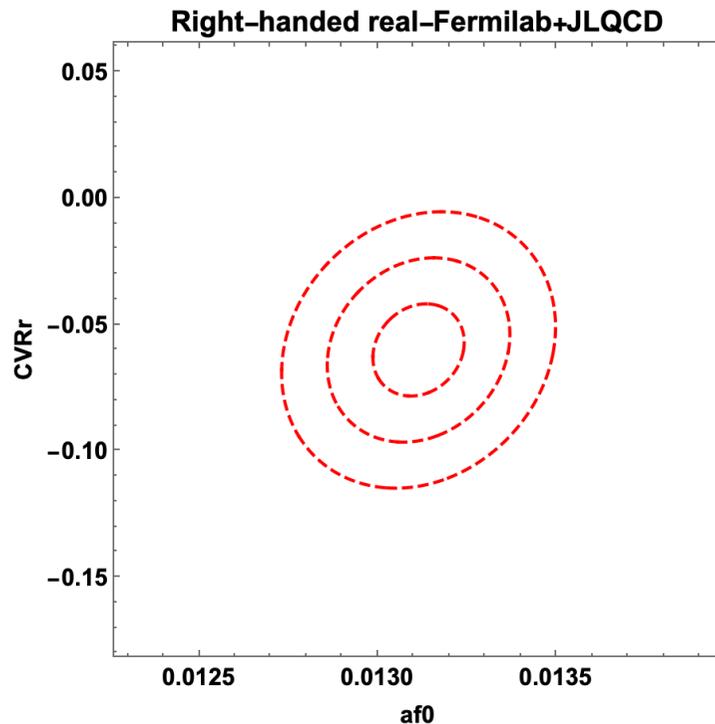


With the current statistic,
 C_{VR} can be measured at
 $\sim 3\%$ precision (stat only)!

Right-Handed fit including lattice data

T. Kappor, Z.R. Huang, E.K.
arXiv:2304.xxxxx

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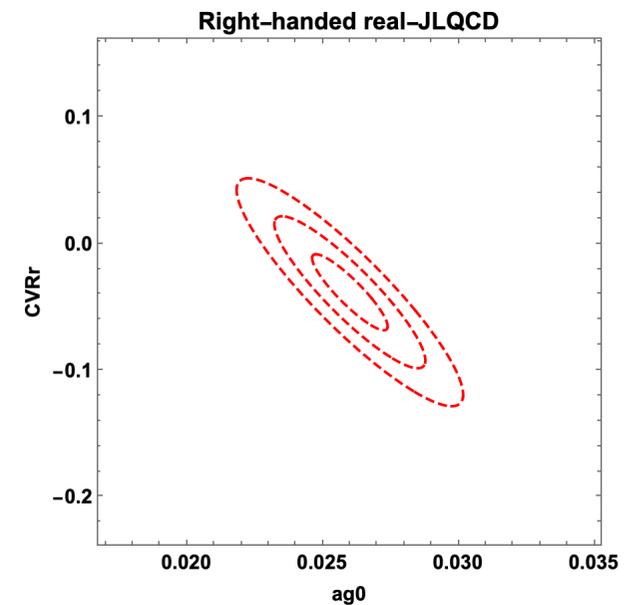
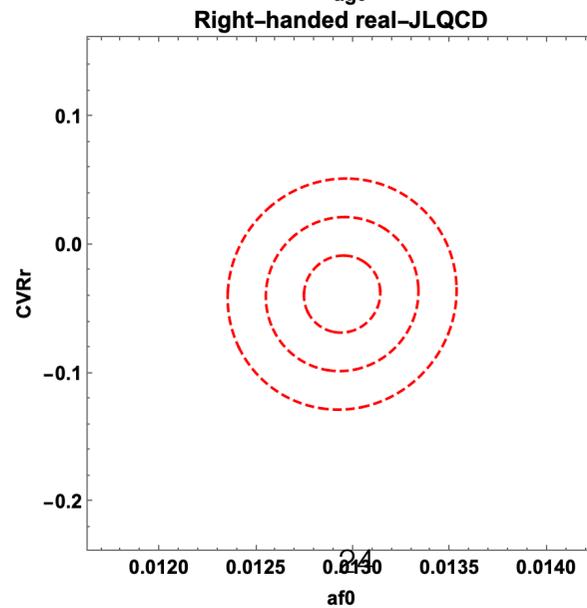
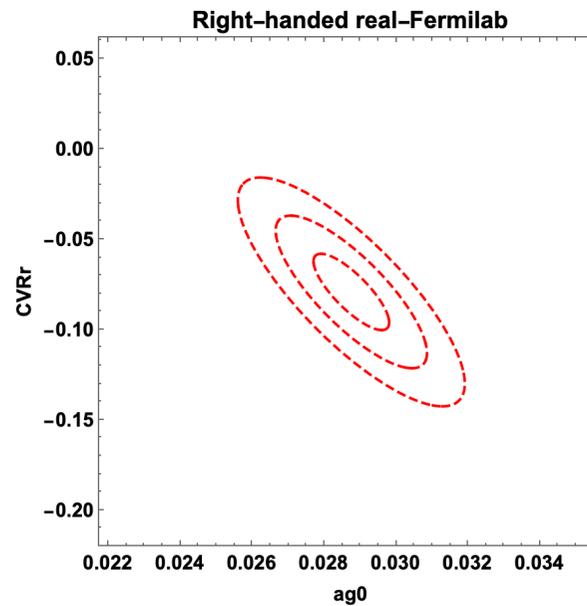
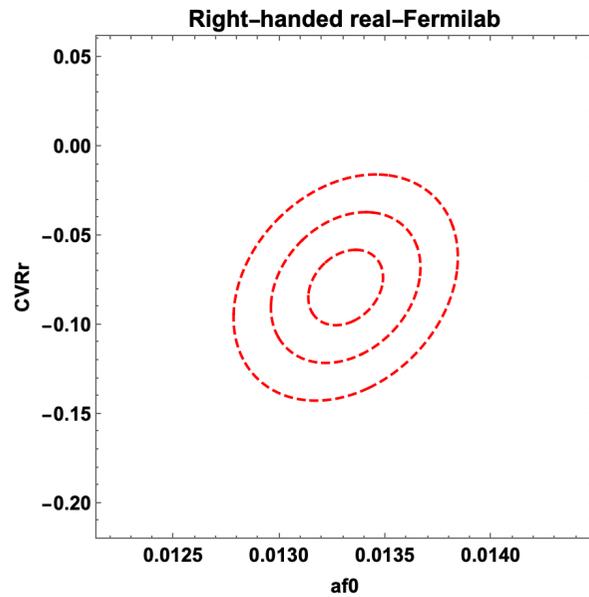


The larger a_{g0} value observed by Fermilab/MILK comparing to the Belle measurement could be compensated by the negative C_{VR} in RH model.

Right-Handed fit including lattice data

We use only angular dependence and assume C_{VR} is real

T. Kappor, Z.R. Huang, E.K.
arXiv:2304.xxxxx



Conclusions

- Belle has been studying the angular distribution to constrain the form factors within SM.
- There are now **three lattice QCD results** on the $B \rightarrow D^*$ Form Factors (HPQCD includes the tensor FF!).
- Thus, we are **ready to move to BSM fit!**
- We performed toy study of the unbanned maximum likelihood method of Belle data to RH model including the lattice data.
- We showed that the observed small tension on the vector FF could be compensated by the RH contribution.