SM predictions on LFU and implications

Nico Gubernari

Open LHCb Workshop on semileptonic exclusive $b \to c$ decays

INFN - Laboratori Nazionali di Frascati

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Introduction

Hadronic matrix elements

study $\boldsymbol{B}$-meson decays to test the SM and extract its parameters (e.g., $\boldsymbol{V_{cb}}$)
factorise decay amplitude (neglecting QED corrections)

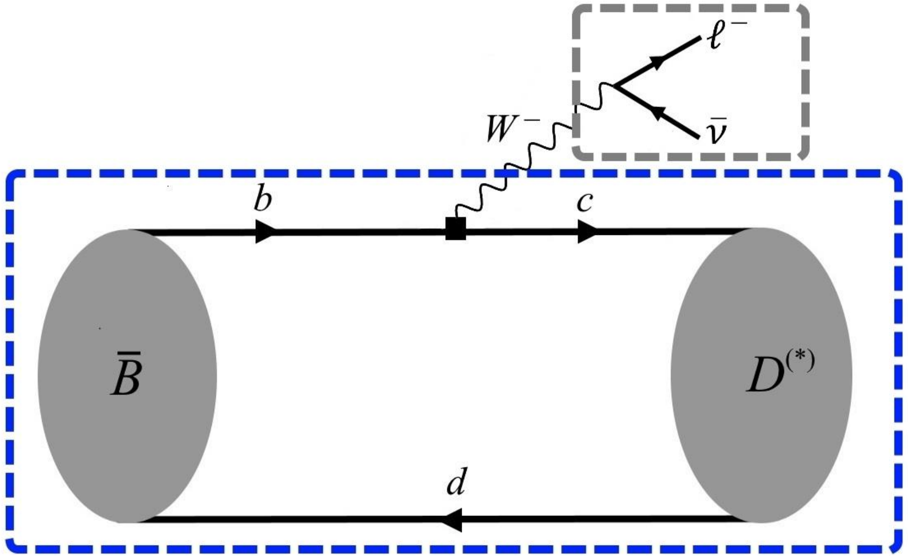
charged currents: $\langle \bar{D}^{(*)}\ell\nu_\ell|\mathcal{O}_{eff}|B\rangle = \langle \ell\nu_\ell|\mathcal{O}_{lep}|0\rangle\langle D^{(*)}|\mathcal{O}_{had}|B\rangle$

neutral currents: $\langle K^{(*)}\ell^+\ell^-|\mathcal{O}_{eff}|B\rangle = \langle \ell\ell|\mathcal{O}_{lep}|0\rangle\langle K^{(*)}|\mathcal{O}_{had}|B\rangle$ + non−fact.

leptonic matrix elements: perturbative objects, high accuracy
QED corrections mostly unknown but small (~1%)

hadronic matrix elements: non-perturbative QCD effects,
usually large uncertainties (~10%)

(local) hadronic matrix elements are crucial
to obtain precise predictions for $b \to c\ell\nu$ decays



Definition of the form factors

form factors (FFs) parametrize exclusive hadronic matrix elements

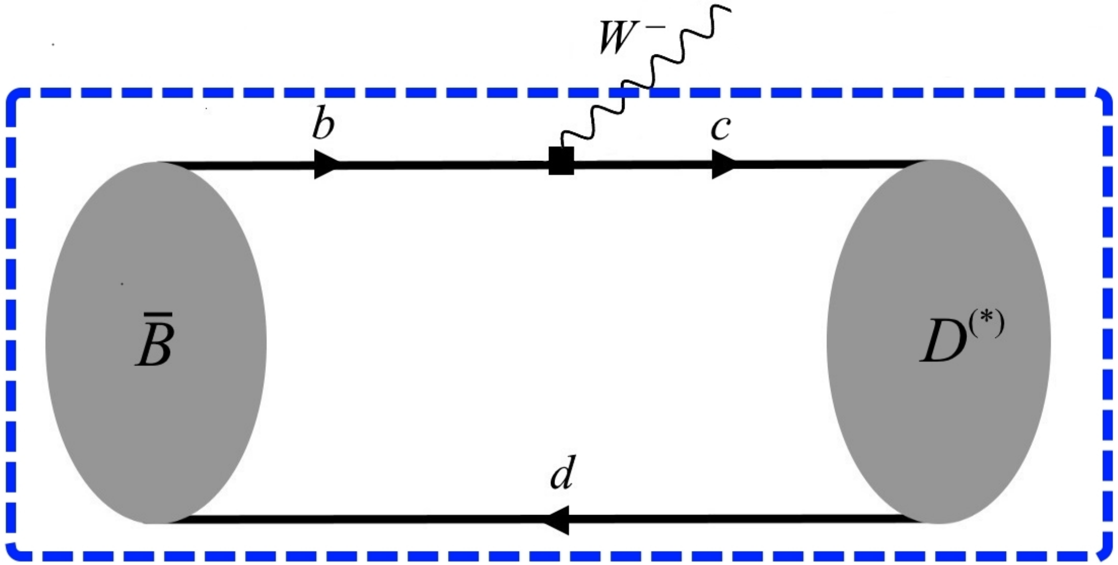
$$\langle D(k)|\bar{c}\,\gamma_\mu b|B(q+k)\rangle = 2\,k_\mu f_+(q^2) + q_\mu\big(f_+(q^2) + f_-(q^2)\big)$$

$$\langle D(k)|\bar{c}\,\sigma_{\mu\nu}q^\nu b|B(q+k)\rangle = \frac{i f_T(q^2)}{m_B + m_P}\big(q^2(2k+q)_\mu - (m_B^2 - m_P^2)q_\mu\big)$$

decomposition follows from Lorentz invariance

FFs are functions of the momentum transferred q^2
(q^2 is the dilepton mass squared)

2(+1) independent $B \to D$ FFs
4(+3) independent $B \to D^*$ FFs



Optimised observables and LFU

test the lepton flavour universality to test the SM

lepton flavour universality (LFU) = the 3 lepton generations have the same couplings to the gauge bosons

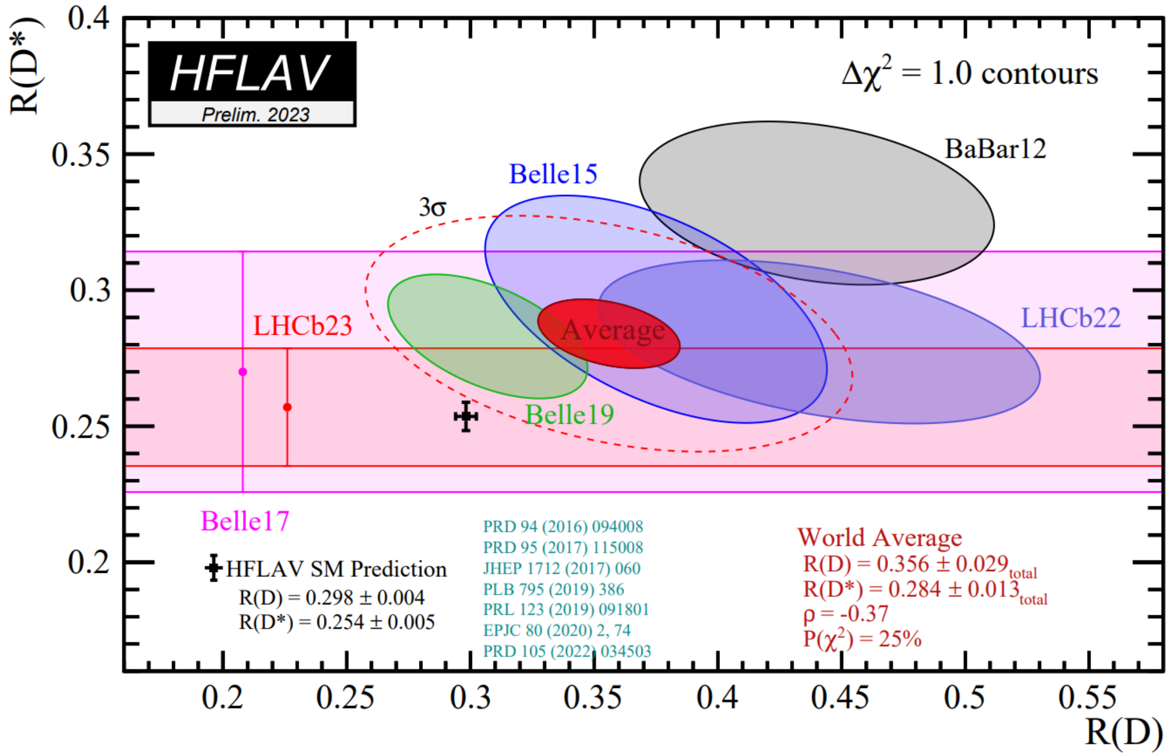
violations of LFU $\Longrightarrow$ new physics

define observables smartly to reduce FFs uncertainties and cancel V_{cb}

observables to test LFU

$$R(D^{(*)}) = \frac{\Gamma(B \to D^{(*)}\tau\,\nu)}{\Gamma(B \to D^{(*)}\ell\,\nu)}$$

3.2 σ tension between the SM and data



Form factors calculations

Methods to compute FFs

non-perturbative techniques are needed to compute FFs

1. Lattice QCD (LQCD)
   numerical evaluation of correlators in a finite and discrete space-time
   more efficient usually at **high q^2**
   reducible systematic uncertainties

2. Light-cone sum rules (LCSRs)
   based on unitarity, analyticity, and quark-hadron duality approximation
   need universal non-perturbative inputs (B-meson distribution amplitudes)
   only applicable at **low q^2**
   non-reducible systematic uncertainties

complementary approaches to calculate FFs
in the long run LQCD will dominate the theoretical predictions (smaller and reducible syst unc.)

State of the art

- $B \to D$
  LQCD calculations available at **high** q^2
  [FNAL/MILC 2015] [HPQCD 2015]

- $B \to D^*$
  LQCD calculations available at **high** q^2
  [FNAL/MILC 2021] [JLQCD w.i.p.]
  in the **whole** semileptonic region of q^2
  [HPQCD 2023]

- $B_s \to D_s$
  LQCD calculations available
  in the **whole** semileptonic region of q^2
  [HPQCD 2019]

- $B_s \to D_s^*$
  LQCD calculations available
  in the **whole** semileptonic region of q^2
  [HPQCD 2021] [HPQCD 2023]

LCSRs available for the four processes at **low** q^2

how to **combine** different calculations for the same channel?
how to obtain result in the **whole** semileptonic region if not available from LQCD?

FFs extrapolation and LFU results

Parametrization for FFs

when LQCD data are available only at high q^2
obtain FFs in the whole semileptonic region by either

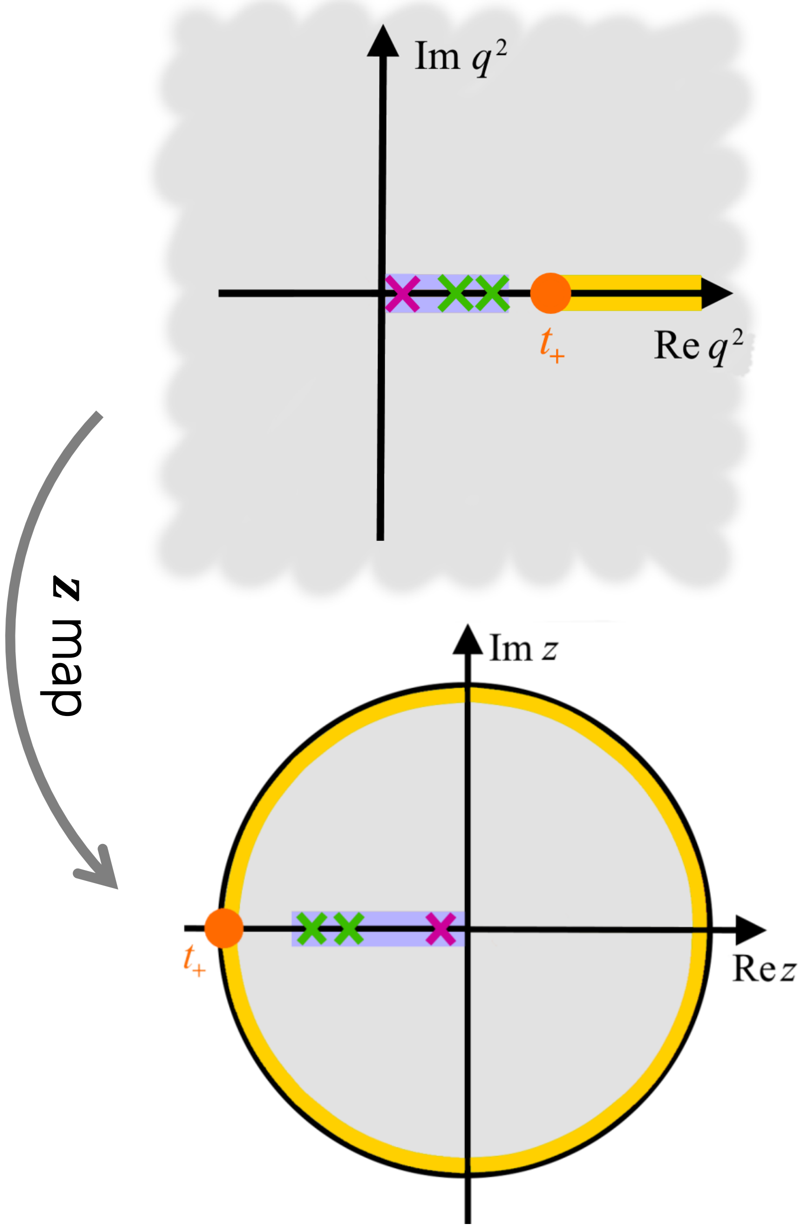
- extrapolating LQCD calculations to low q^2
- or combining LQCD and LCSRs

FFs are analytic functions of q^2 except for
branch cut for $q^2 > t_+ = \left(M_B + M_{D^{(*)}}\right)^2$

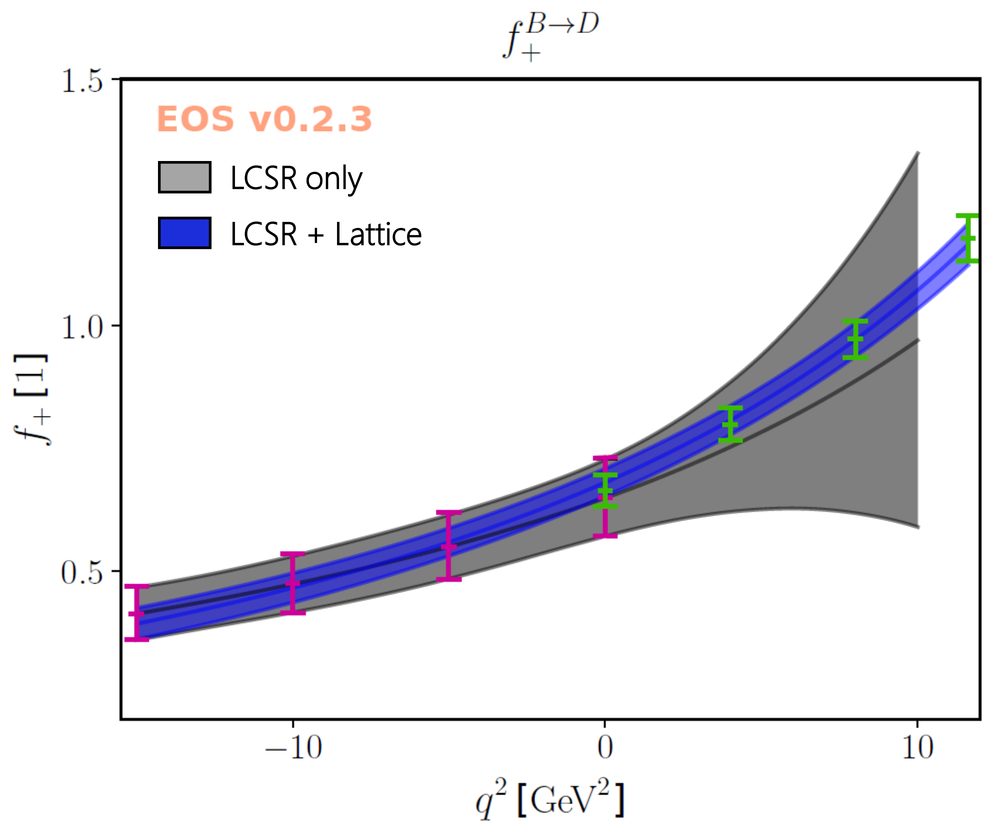
fit results to a z parametrization = Taylor series (standard approach)
[Boyd/Grinstein/Lebed 1997] [Bourrely/Caprini/Lellouch 2008]
[Bharucha/Straub/Zwicky 2015] [...]

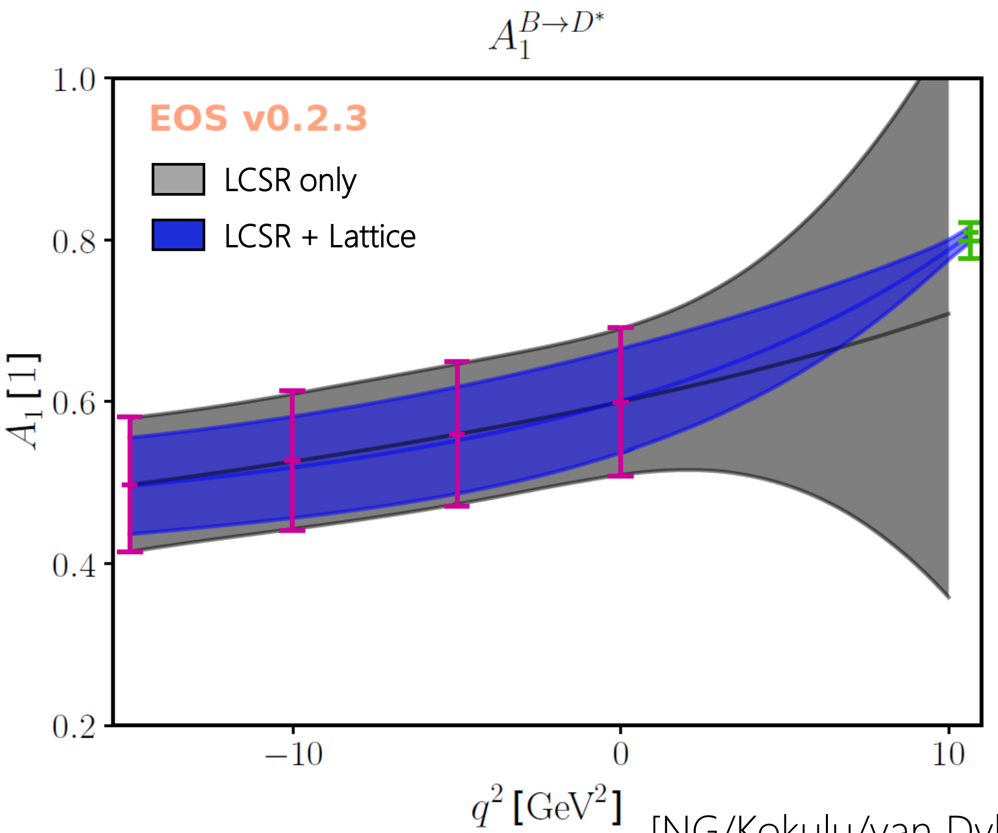
$$\mathrm{FF} \propto \sum_{n=0}^{\infty} \alpha_n^{\mathrm{FF}}\, z^k$$

$$z(q^2) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+}}{\sqrt{t_+ - q^2} + \sqrt{t_+}}$$



Combine LQCD and LCSRs with naïve z param.





[NG/Kokulu/van Dyk 2018]

combine LQCD and LCSRs to obtain the FF values to the whole semileptonic region

good agreement between lattice and LCSRs calculations

use only first 3 terms in the z parametrization $\Longrightarrow$ what is the truncation error?

Unitarity bounds

use analyticity, unitarity, and quark hadron duality to obtain constraints on the z (BGL) parametrization

unitarity bounds: [Boyd/Grinstein/Lebed 1994]

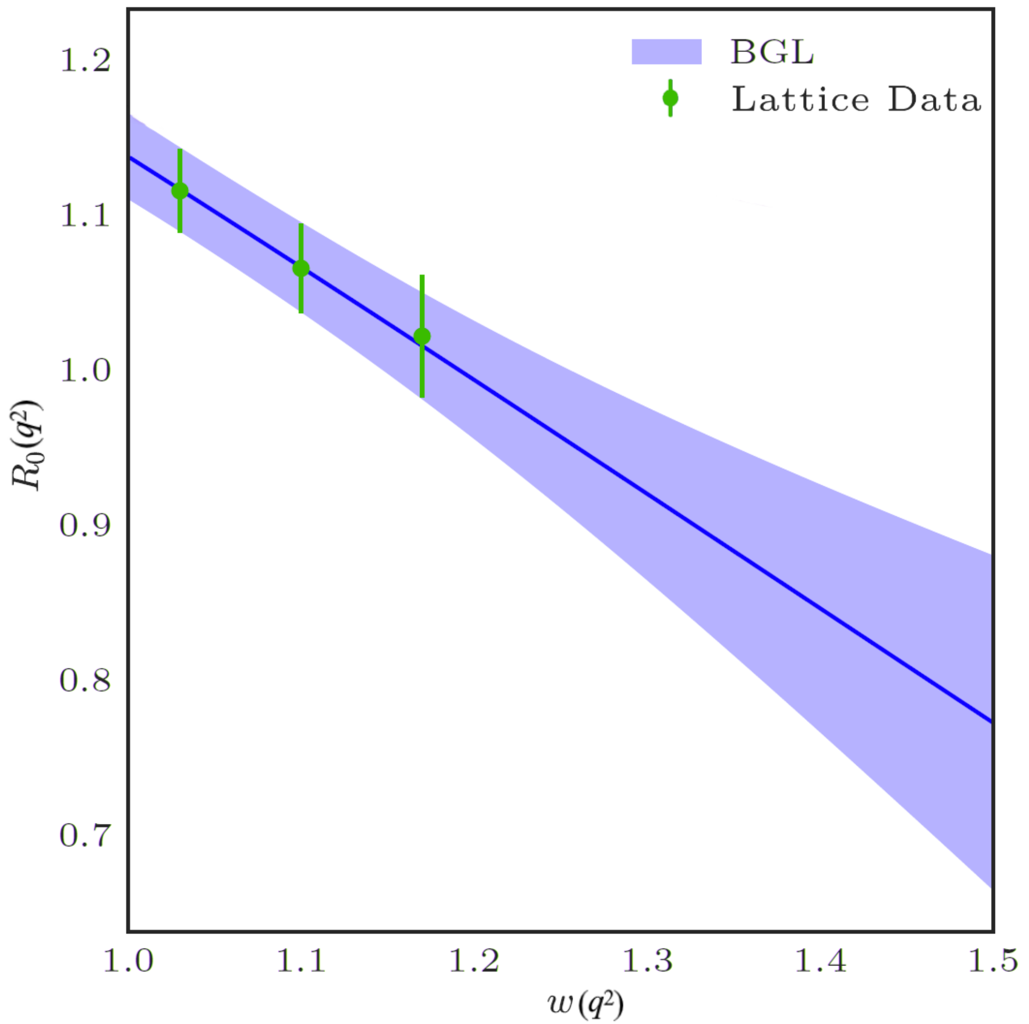
$$\mathrm{FF}(z) = \frac{1}{\mathcal{B}(z)\phi(z)} \sum_{n=0}^{\infty} a_n^{\mathrm{FF}} z^k \qquad 1 > \sum_{n=0}^{\infty} \left| a_n^{\mathrm{FF}} \right|^2$$

determine the truncation error

two different ways to apply use the bounds:

1. "standard" BGL fit
2. dispersive matrix method

two methods substantially equivalent



[FNAL/MILC 2021] [see Alejandro's talk]

HQE for the $B_{(s)} \to D_{(s)}^{(*)}$ FFs

use heavy-quark limit ($m_{b,c} \to \infty$) to relate $B_{(s)} \to D_{(s)}$ FFs to $B_{(s)} \to D_{(s)}^{*}$ FFs

heavy-quark expansion (HQE) for $B_{(s)} \to D_{(s)}^{(*)}$ FFs

$$FF^{B\to D^{(*)}}(q^2) = c_0 \xi(q^2) + c_1 \frac{\alpha_s}{\pi} C_i(q^2) + c_2 \frac{1}{m_b} L_i(q^2) + c_3 \frac{1}{m_c} L_i(q^2) + c_4 \frac{1}{m_c^2} l_i(q^2)$$

$$FF^{B_s\to D_s^{(*)}}(q^2) = c_0 \xi^s(q^2) + c_1 \frac{\alpha_s}{\pi} C_i(q^2) + c_2 \frac{1}{m_b} L_i^s(q^2) + c_3 \frac{1}{m_c} L_i^s(q^2) + c_4 \frac{1}{m_c^2} l_i(q^2)$$

include $1/m_c^2$ corrections [Bordone/Jung/van Dyk 2019]

all $B \to D^{(*)}$ and $B_s \to D_s^{(*)}$ FFs parametrized in terms of **14 Isgur-Wise functions**

alternative method to include $1/m_c^2$ corrections proposed in Bernlochner F. at al. (2022)
less parameters but model dependent

LQCD calculations must fulfil these relations (within errors)

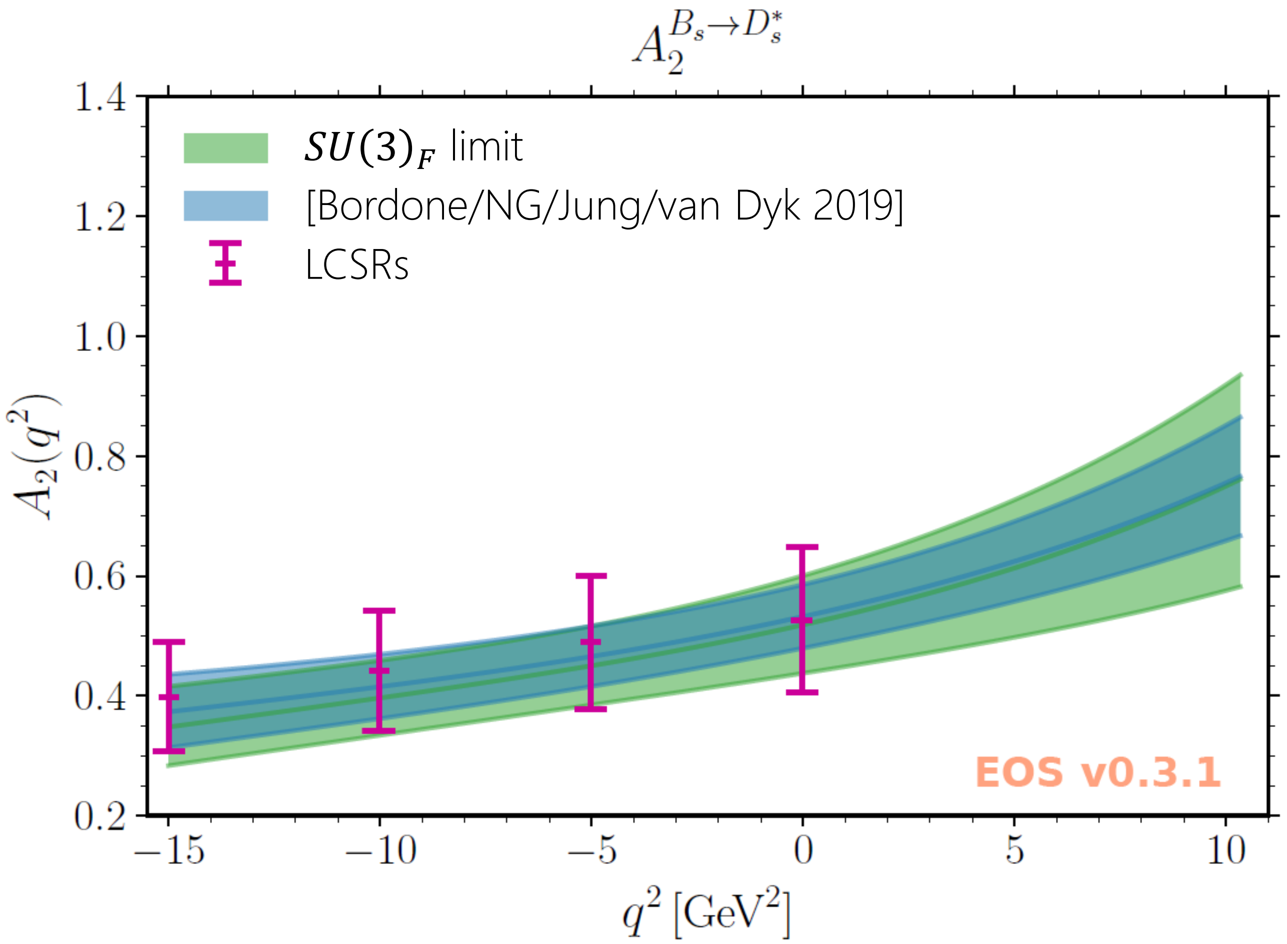
HQE FFs results

fit Isgur-Wise functions to

- LQCD
- LCSRs for the FFs
- SVZ sum rules for Isgur-Wise functions
- unitarity bounds
- with and w/o exp data

results for all $B \to D^{(*)}$ FFs and $B_s \to D_s^{(*)}$ FFs in the whole physical phase space

inclusion of $1/m_c^2$ corrections is necessary
CLN parametrization not sufficient anymore
(only includes $1/m_{b,c}$ corrections)



Some (concerning) comparison

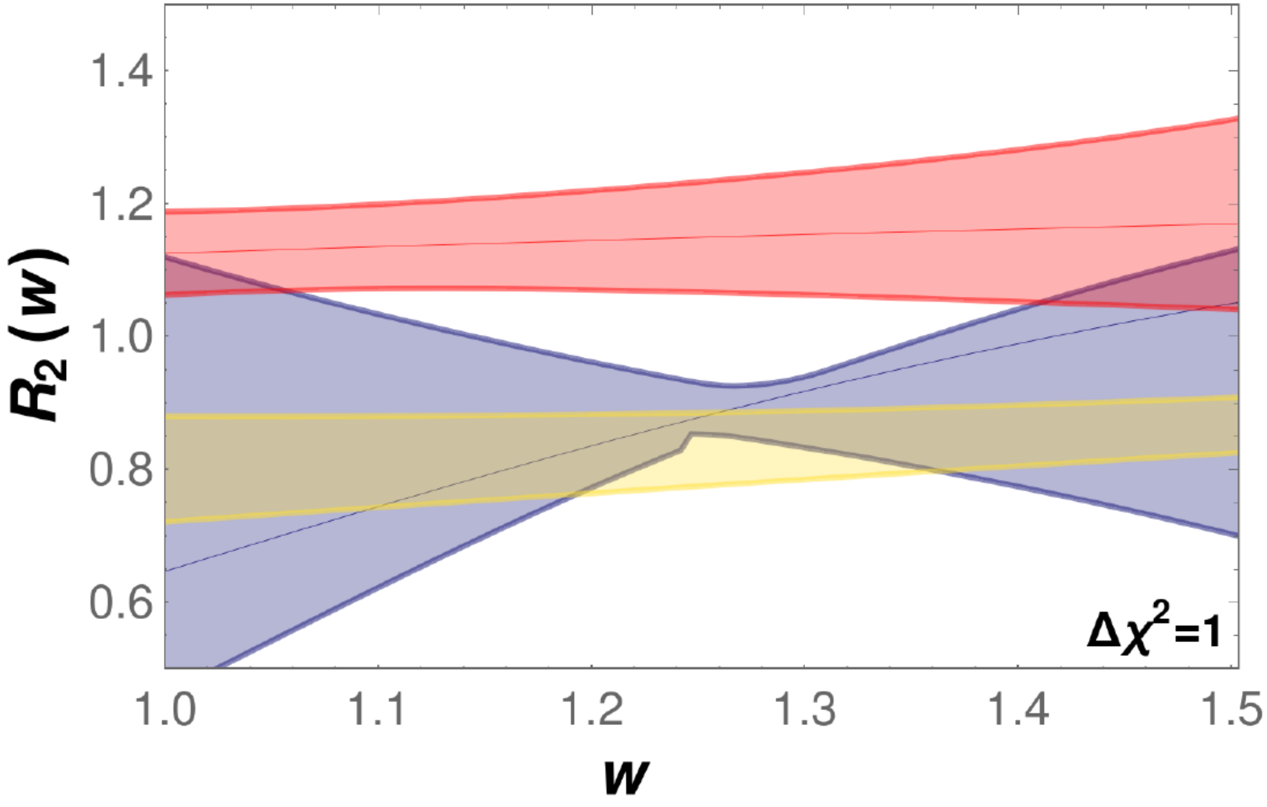
tension between experimental measurements (BGL)
and FNAL/MILC 2021 (HPQCD 2023)

tension between HQE ($1/m_c^2$)
and FNAL/MILC 2021 (HPQCD 2023)

solid pheno analyses need stable inputs

discussion about different approaches
(parametrizations) is useless if inputs are faulty

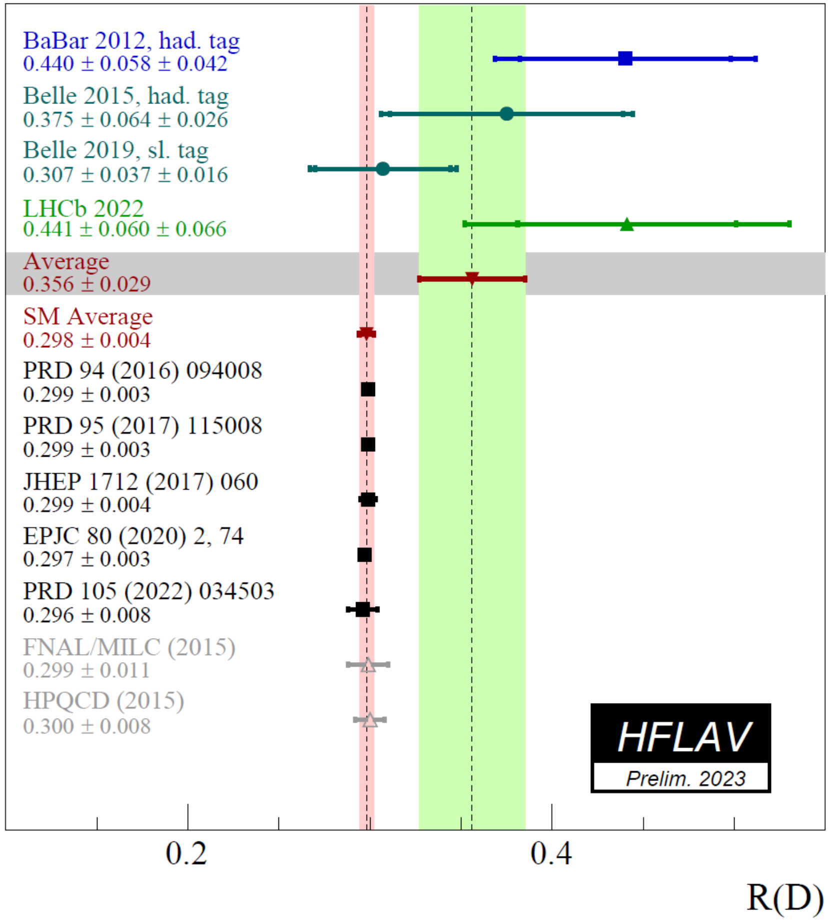
until LQCD results are well understood
theory predictions ($\boldsymbol{R(D^{(*)})}$) and $|\boldsymbol{V_{cb}}|$ extractions
cannot be trusted

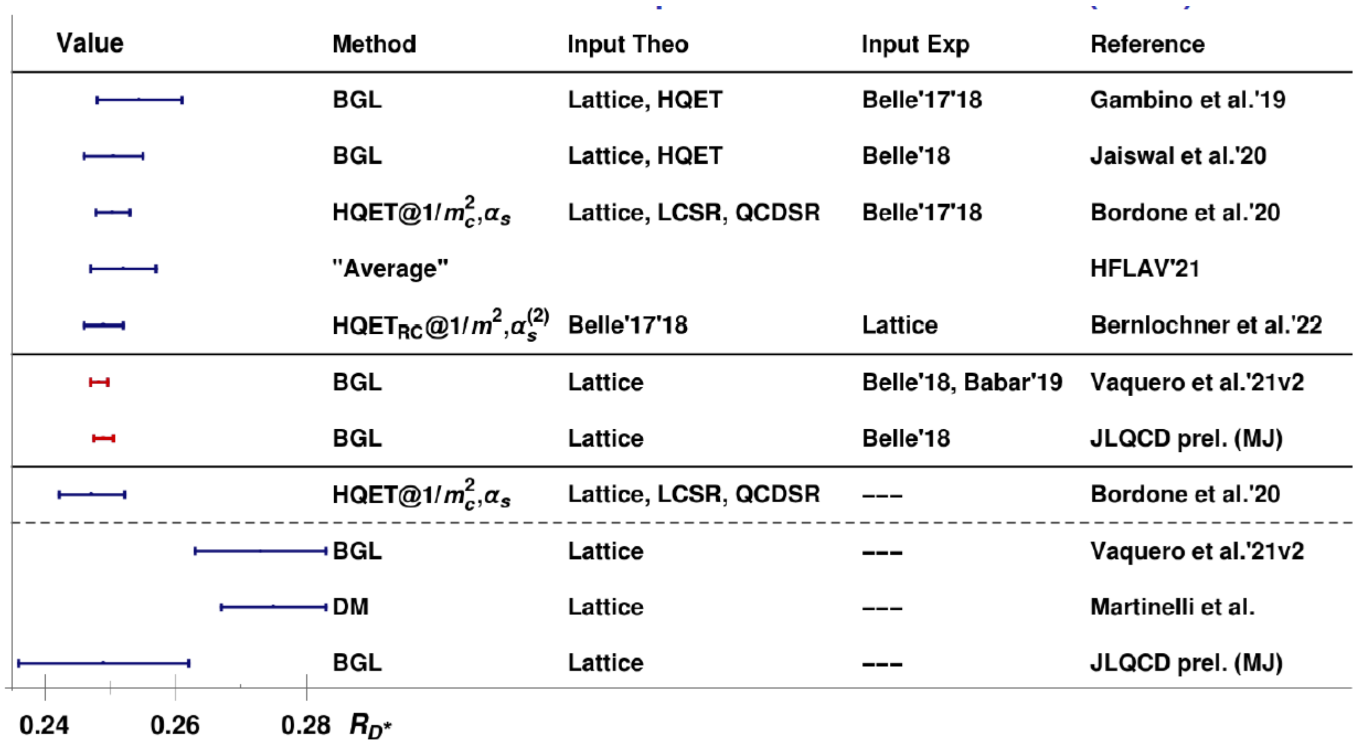


[credit: Martin Jung – LHCb impl. 2022]
[see also Alejandro's talk]

$R(D^{(*)})$ results

once the FFs are known it is trivial to predict the LFU ratios $R(D^{(*)})$ (and $R(D_s^{(*)})$)





[credit: Martin Jung – LHCb impl. 2022]

excellent agreement btw. SM predictions for $R(D)$, (worse) agreement btw. SM predictions for $R(D^*)$

solve $R(D^*) \Longrightarrow$ NP in $B \to D^*(e,\mu)\nu$

$B \to D^{**}$ form factors

D^{**} mesons

why study $B \to D^{**}\ell\nu$ decays?

- alternative way to study $b \to c\ell\nu$ transitions ($R(D^{**})$ ratios, $|V_{cb}|$ etc.)
- background in $B \to D^{*}\ell\nu$ measurements
- understand the gap inclusive vs. sum of exclusive $B \to X_c\ell\nu$

Meson	j	J^P	Mass [MeV]	Width [MeV]
$D_0^*(2300)$	$\frac{1}{2}$	0^+	2343 ± 10	229 ± 16
$D_1(2430) \equiv D_1'$	$\frac{1}{2}$	1^+	2412 ± 9	314 ± 29
$D_1(2420) \equiv D_1$	$\frac{3}{2}$	1^+	2422.1 ± 0.6	31.3 ± 1.9
$D_2^*(2460)$	$\frac{3}{2}$	2^+	2461.1 ± 0.8	47.3 ± 0.8

$\boldsymbol{B \to D_2^*}$ **FFs** already calculated with light-con sum rules (LCSRs) [Aliev et al 2019]

we calculated $\boldsymbol{B \to D_1}$ and $\boldsymbol{B \to D_1'}$ FFs for the first time [NG/Khodjamirian/Mandal/Mannel 2023]

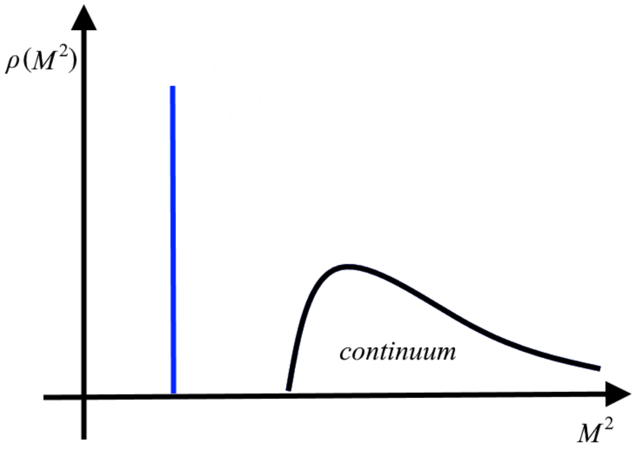
$\boldsymbol{B \to D_0^*}$ **FFs** w.i.p. [NG/Khodjamirian/Mandal/Mannel w.i.p.]

New LCSRs

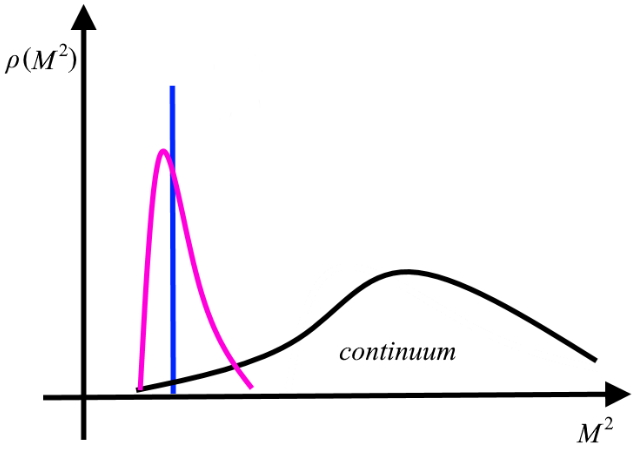
define a correlator and study spectral density

$$\Pi(k,q) = i\int \mathrm{d}^4x\, e^{ikx}\langle 0|T\{J_{int}(x), J_{weak}(0)\}|B(k+q)\rangle$$

usual LCSRs
(e.g. $B \to D$) one ground state



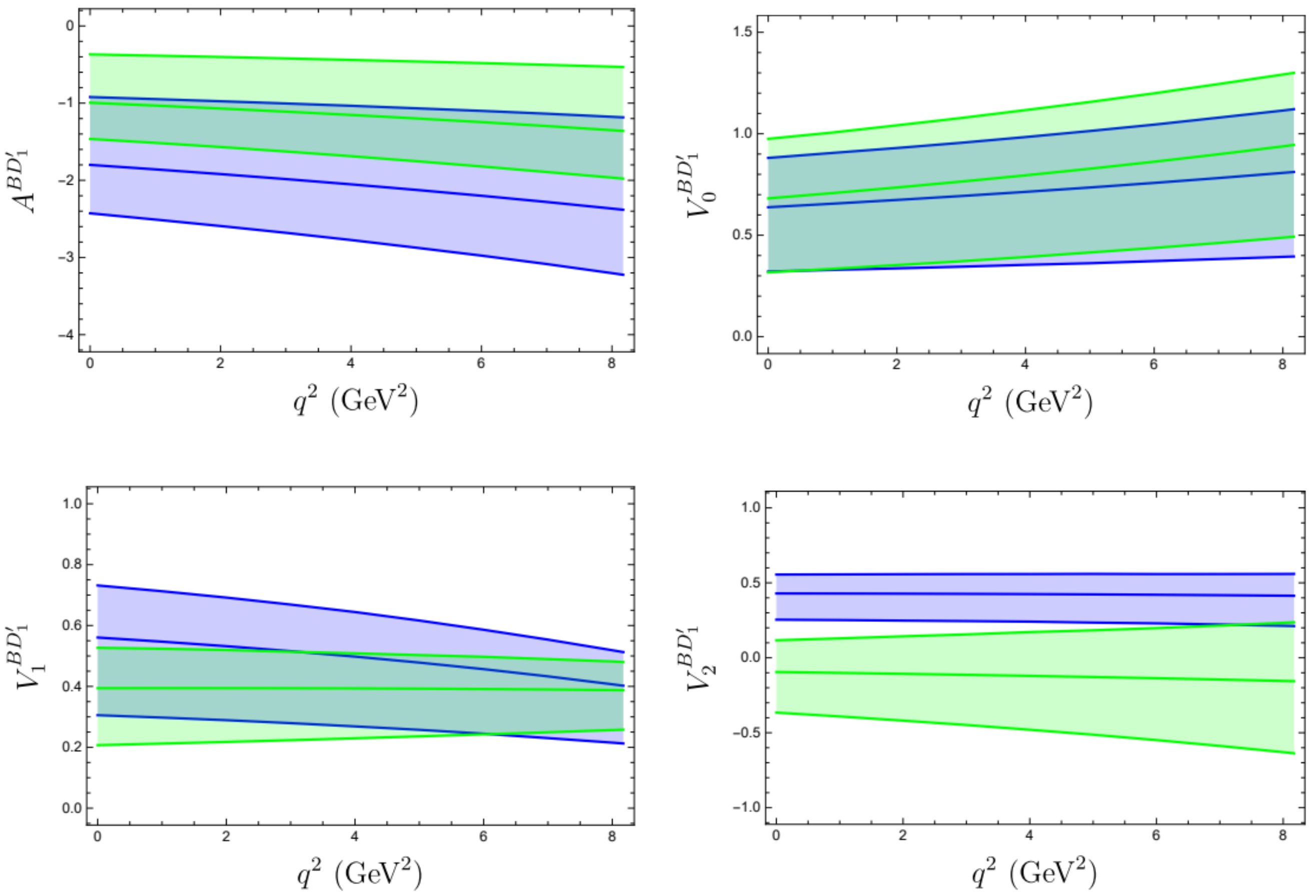
two states (D_1 and D_1') with similar masses and $J^P = 1^+$ (cannot be disentangled using a standard LCSRs)



define new type of LCSRs to deal with states with similar masses

Numerical results

new method yields a **twofold ambiguity** (could be resolved with more experimental data or LQCD results)



[NG/Khodjamirian/Mandal/Mannel 2023]

both solutions give

$R(D_1) = 0.10 \pm 0.02$

$R(D_1') = 0.10 \pm 0.03$

in agreement with Bernlochner, Ligeti et al.

Summary and conclusion

Summary and conclusion

- **combine** theory inputs using $\boldsymbol{z}$ **parametrization** (amazing progress by recent LQCD calculations)

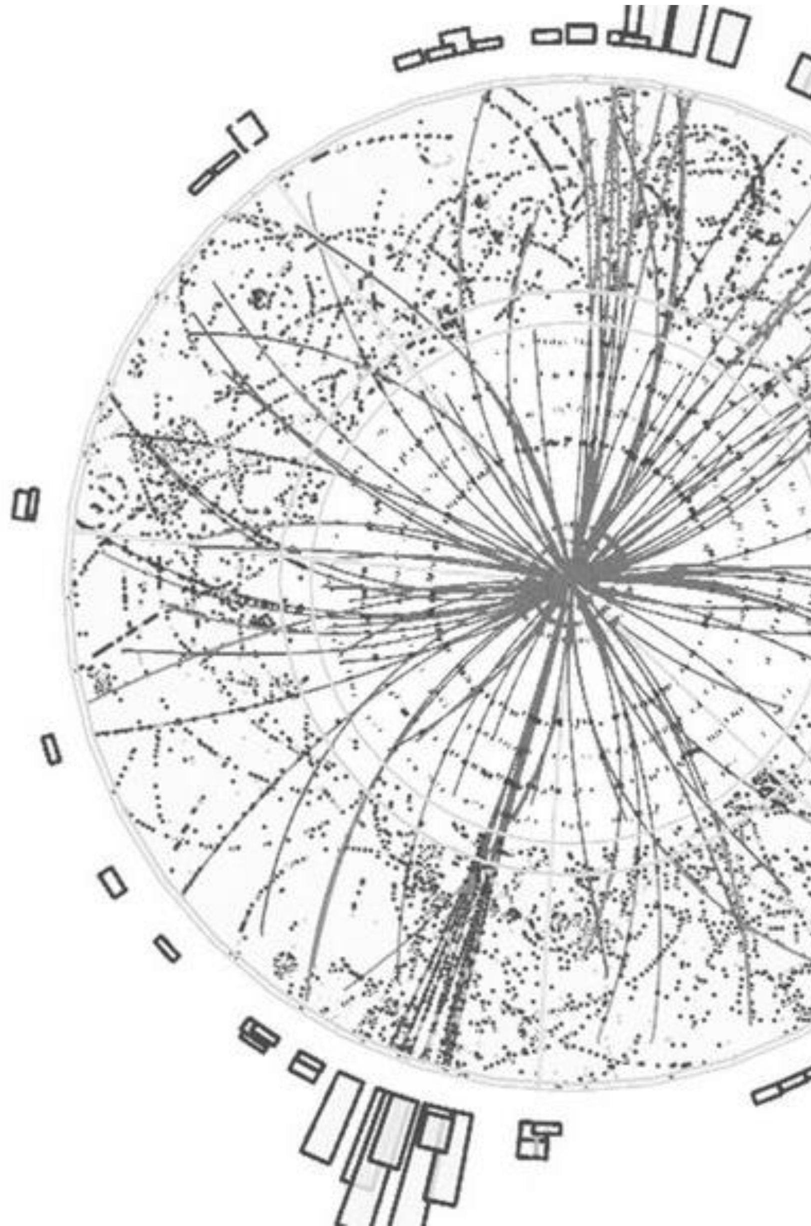
- z parametrization must be **truncated**
  $\Longrightarrow$ control the truncation error using **unitarity bounds**

- **HQET** gives additional and precious constraints but…

- **CLN** parametrization **not sufficient** anymore
  $\Longrightarrow$ include $1/m_c^2$ corrections

- use HQET and dispersive bounds for better precision

- puzzle in the non-zero recoil $B \to D^*$ FFs from LQCD ([FNAL/MILC 2021] [HPQCD 2023])
  $\Longrightarrow$ understand these results otherwise theory predictions ($\boldsymbol{R(D^{(*)})}$) and $|\boldsymbol{V_{cb}}|$ extractions cannot be trusted



Thank you!