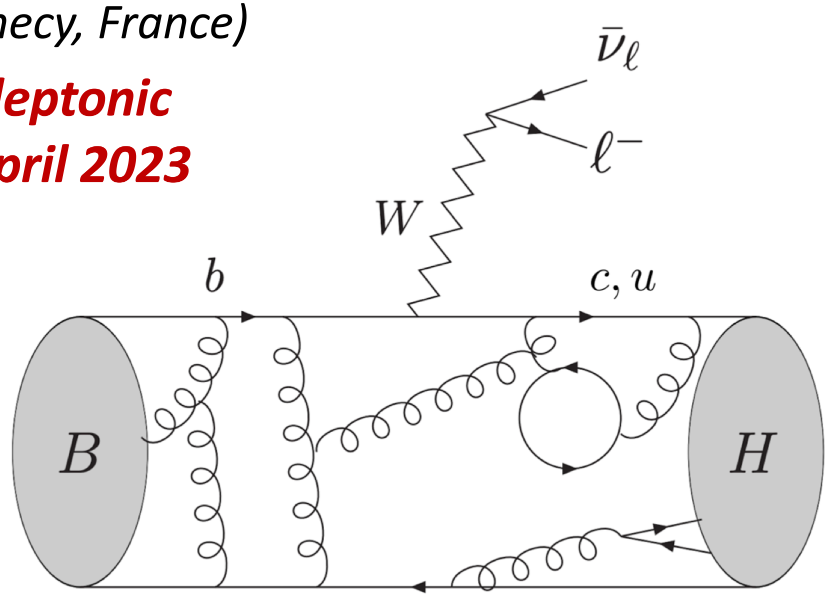
Results with unitarity based Dispersion Matrix approach

Work in collaboration with G. Martinelli, M. Naviglio and S. Simula

[PRD ’21 (2105.02497), PRD ’21 (2105.07851), PRD ’22 (2105.08674), EPJC ‘22 (2109.15248), PRD ‘22 (2204.05925)]

Ludovico Vittorio (LAPTh & CNRS, Annecy, France)

Open LHCb Workshop on semileptonic exclusive $b \rightarrow c$ decays – 12nd April 2023



(from J.Phys.G 46 (2019) 2, 023001)

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1. CKM matrix elements puzzles

A non-negligible tension exists between the inclusive and the exclusive determinations of |Vcb| and |Vub|, for instance in the latter case:

$$|V_{cb}| \times 10^3 = 39.36(68)$$ *EXCLUSIVE*

VS

$$|V_{cb}| \times 10^3 = 42.00(65)$$ *INCLUSIVE*

$$|V_{cb}|_{\rm incl} \times 10^3 = 42.16 \pm 0.50$$

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$$|V_{cb}|_{\rm incl} \times 10^3 = 41.69 \pm 0.63$$

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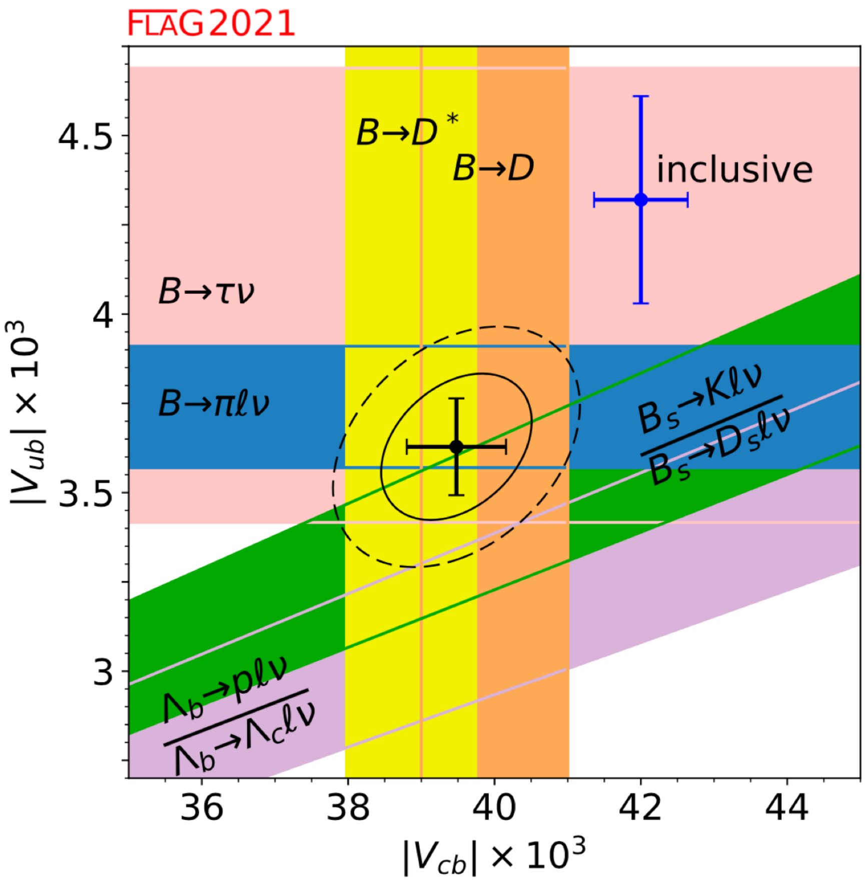

FLAG Review 2021 [EPJC '22 (2111.09849)]

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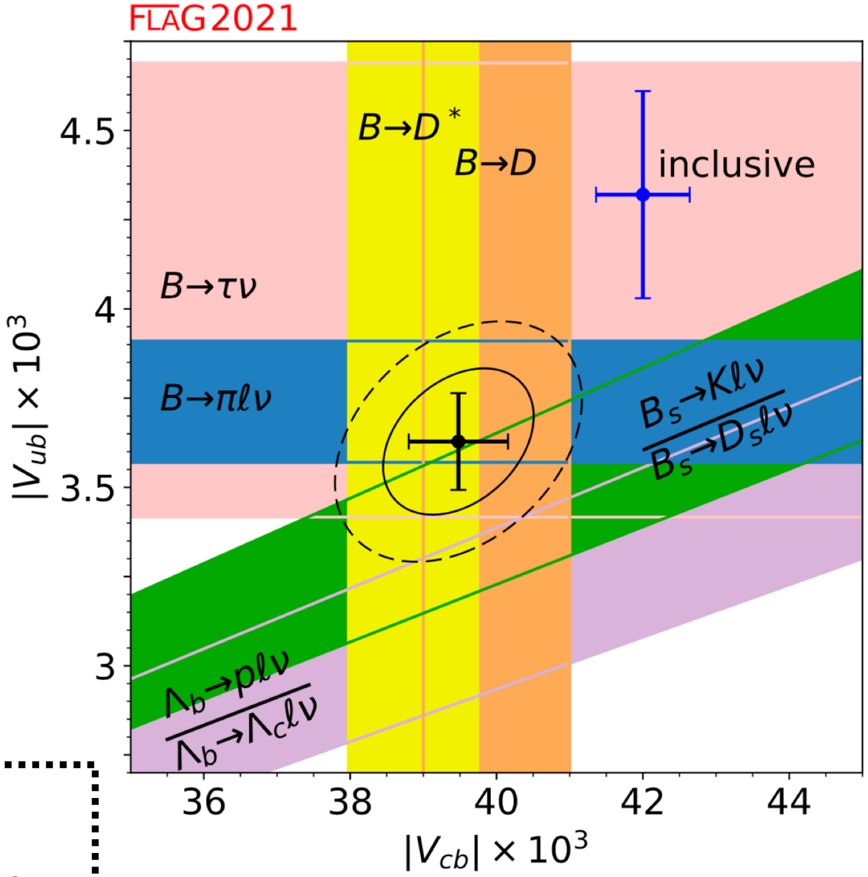
$\sim 3\sigma$ discrepancy



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2. Lepton Flavour Universality (Violation)

Lepton Flavour Universality (LFU) is one of the pillars of the SM. According to this principle, all the three types of charged lepton particles (namely the electrons, the muons and the taus) interact in the same way with the gauge bosons, independently of their generation. In other words, in the SM the gauge interactions are LFU.

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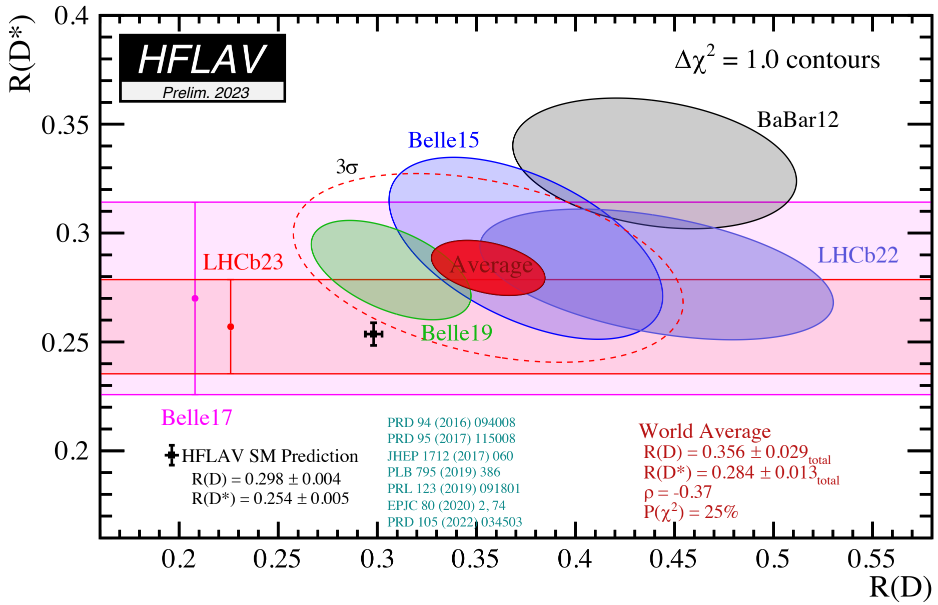
Two types: **i) Lepton Flavour Universality Violation in charged currents**

$$\mathcal{R}(D) = \frac{\mathcal{B}(B \to D\tau\nu_\tau)}{\mathcal{B}(B \to D\ell\nu_\ell)},$$

$$\mathcal{R}(D^*) = \frac{\mathcal{B}(B \to D^*\tau\nu_\tau)}{\mathcal{B}(B \to D^*\ell\nu_\ell)}$$

3.2σ
discrepancy



HFLAV Collaboration, PRD '23 [arXiv:2206.07501] (updated plot)

The central role of the Form Factors (FFs) in excl. semil. *B* decays

- Production of a pseudoscalar meson (*i.e. D,π*):

$$\frac{d\Gamma}{dw} = \frac{G_F^2 |V_{cb}|^2 \eta_{EW}^2}{48\pi^3} \frac{4r\, m_D^3\, (m_B + m_D)^2\, (w^2-1)^{3/2}}{(1+r)^2} \boxed{|f_+(w)|^2}$$

- Production of a vector meson (*i.e. D**):

$$\frac{d\Gamma(B \to D^*(\to D\pi)\ell\nu)}{dw\, d\cos\theta_\ell\, d\cos\theta_v\, d\chi} = \frac{G_F^2 |V_{cb}|^2 \eta_{EW}^2}{4(4\pi)^4} 3 m_B m_{D^*}^2 \sqrt{w^2-1}$$

$$\times B(D^* \to D\pi)\{(1-\cos\theta_\ell)^2 \sin^2\theta_v |H_+|^2$$

$$+ (1+\cos\theta_\ell)^2 \sin^2\theta_v |H_-|^2 + 4\sin^2\theta_\ell \cos^2\theta_v |H_0|^2$$

$$- 2\sin^2\theta_\ell \sin^2\theta_v \cos 2\chi H_+ H_-$$

$$- 4\sin\theta_\ell (1-\cos\theta_\ell) \sin\theta_v \cos\theta_v \cos\chi H_+ H_0$$

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$$H_\pm(w) = \boxed{f(w)} \mp m_B m_{D^*} \sqrt{w^2-1}\, \boxed{g(w)}$$

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relation between the momentum transfer and the recoil

$$\boxed{q^2 = m_B^2 + m_P^2 - 2 m_B m_P\, w}$$

If the lepton is **NOT** massless? Two other FFs!

$$\boxed{f_0(w)}\ (\text{pseudoscalar}),\ \boxed{P_1(w)}\ (\text{vector})$$

The Dispersive Matrix (DM) method

Our goal is to describe the FFs using a novel, non-perturbative and model independent approach: starting from the available LQCD computations of the FFs in the high-q^2 (or low-w) regime, we extract the FFs behaviour in the low-q^2 (or high-w) region!

- Pioneering works from S. Okubo [PRD, 3 (1971); PRD, 4 (1971)], C. Bourrely et al [NPB, 189 (1981)] and L. Lellouch [NPB, 479 (1996)]
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How does it work?

The DM method

Let us focus on a generic FF *f*: we can define

$$\mathbf{M} = \begin{pmatrix} \chi & \phi f & \phi_1 f_1 & \phi_2 f_2 & \dots & \phi_N f_N \\ \phi f & \frac{1}{1-z^2} & \frac{1}{1-z z_1} & \frac{1}{1-z z_2} & \dots & \frac{1}{1-z z_N} \\ \phi_1 f_1 & \frac{1}{1-z_1 z} & \frac{1}{1-z_1^2} & \frac{1}{1-z_1 z_2} & \dots & \frac{1}{1-z_1 z_N} \\ \phi_2 f_2 & \frac{1}{1-z_2 z} & \frac{1}{1-z_2 z_1} & \frac{1}{1-z_2^2} & \dots & \frac{1}{1-z_2 z_N} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \phi_N f_N & \frac{1}{1-z_N z} & \frac{1}{1-z_N z_1} & \frac{1}{1-z_N z_2} & \dots & \frac{1}{1-z_N^2} \end{pmatrix}$$

$$\phi_i f_i \equiv \phi(z_i) f(z_i) \;\; (\text{with } i = 1, 2, \dots N)$$

$$z(t) = \frac{\sqrt{\frac{t_+ - t}{t_+ - t_-}} - 1}{\sqrt{\frac{t_+ - t}{t_+ - t_-}} + 1}$$

$$t_\pm \equiv (m_B \pm m_D)^2$$

t: momentum transfer

The DM method

Non-perturbative values of the susceptibilities from the dispersion relations (see PRD '21 (2105.07851) and 2202.10285)

Estimates of the FFs, computed on the lattice

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One can show that

$$\det \mathbf{M} \geq 0$$

$$\Downarrow$$

$$f_{\rm lo}(z) \leq f(z) \leq f_{\rm up}(z)$$

Values of the momentum transfer @ which FFs are computed on the lattice

The DM method

The positivity of the original inner products guarantee that $\det \mathbf{M} \geq 0$: the solution of this inequality can be computed analitically, bringing to

LOWER bound

$$\boxed{\beta - \sqrt{\gamma}} \leq f(z) \leq \boxed{\beta + \sqrt{\gamma}}$$

UPPER bound

$$\beta = \frac{1}{d(z)\phi(z)} \sum_{j=1}^{N} f_j \phi_j d_j \frac{1 - z_j^2}{z - z_f} \qquad \gamma = \frac{1}{d^2(z)\phi^2(z)} \frac{1}{1 - z^2} \left[\chi - \sum_{i,j=1}^{N} f_i f_j \phi_i \phi_j d_i d_j \frac{(1 - z_i^2)(1 - z_j^2)}{1 - z_i z_j} \right]$$

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UNITARITY FILTER: unitarity is satisfied if γ is semipositive definite, namely if

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Unitarity is built-in!

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- Phenomenological applications:

i) **For the LFU observables**, we sum over all these integrals to cover the full q2-range

ii) **For the CKM matrix element**, we compare our theoretical determinations of the d.d.w with the corresponding experimental measurements, obtaining **bin-per-bin estimates of |Vcb|**

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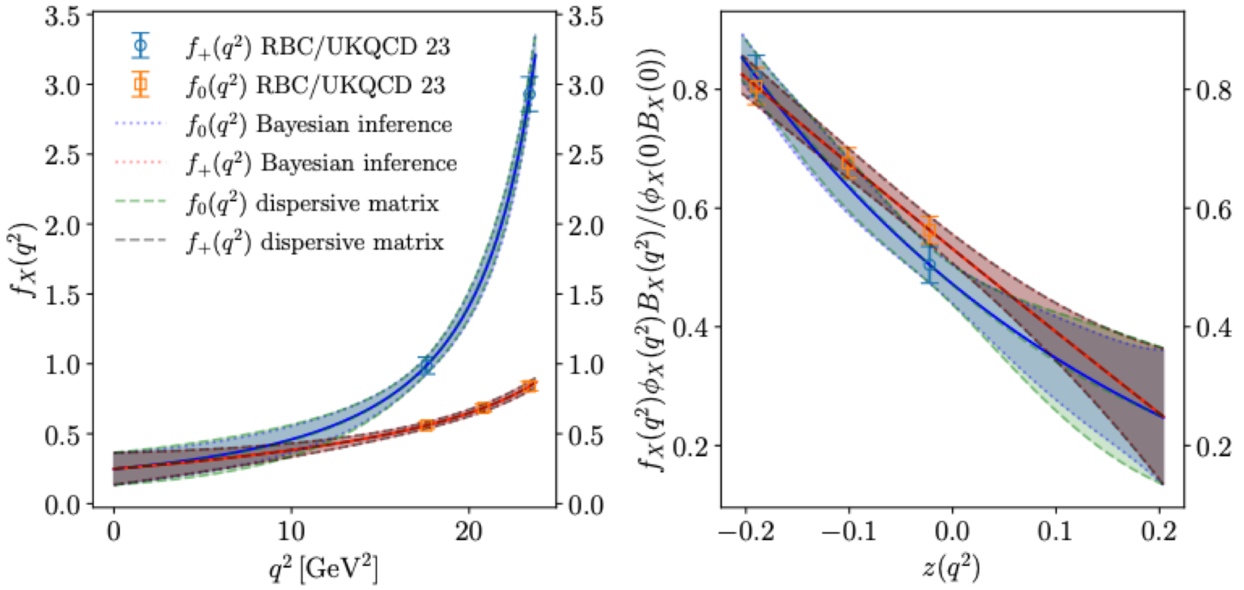
→ **Simple implementation!**

A recent counter-check of the DM method

Results III: Bayesian Inference vs Dispersive Matrix Method



Application to $B_s \to K$: **identical results!**

- BI and DM produce equivalent results
- BI simpler to implement + easily to combine with multiple data sets
- BI provides numerical value of the coefficients

Tsang's talk @ Moriond EW 2023

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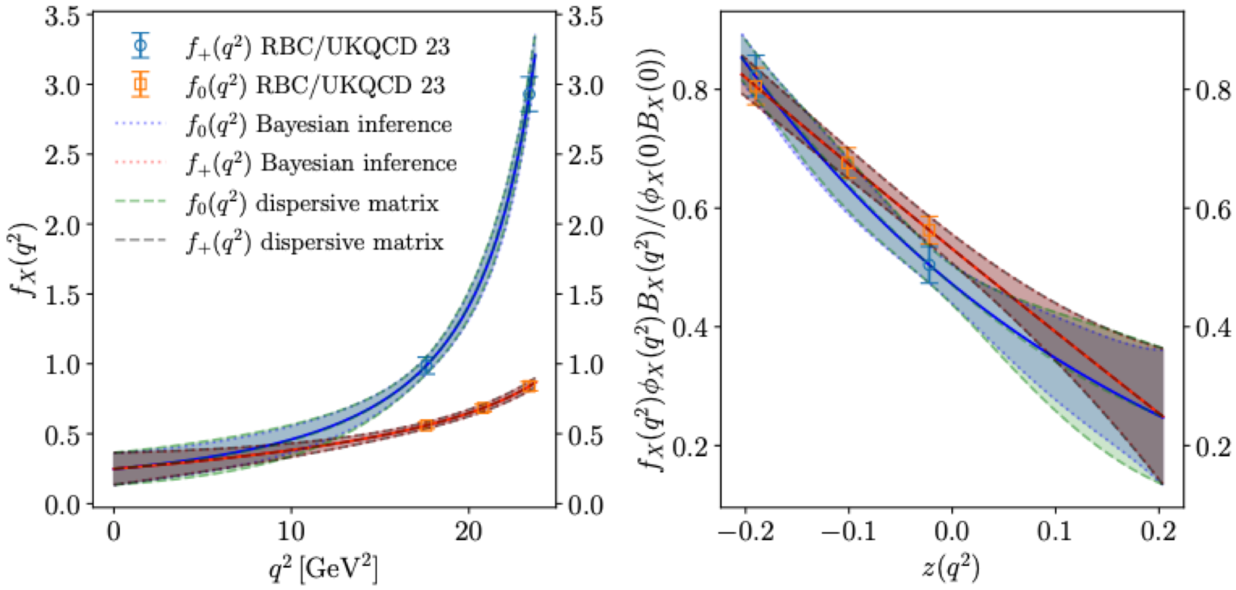

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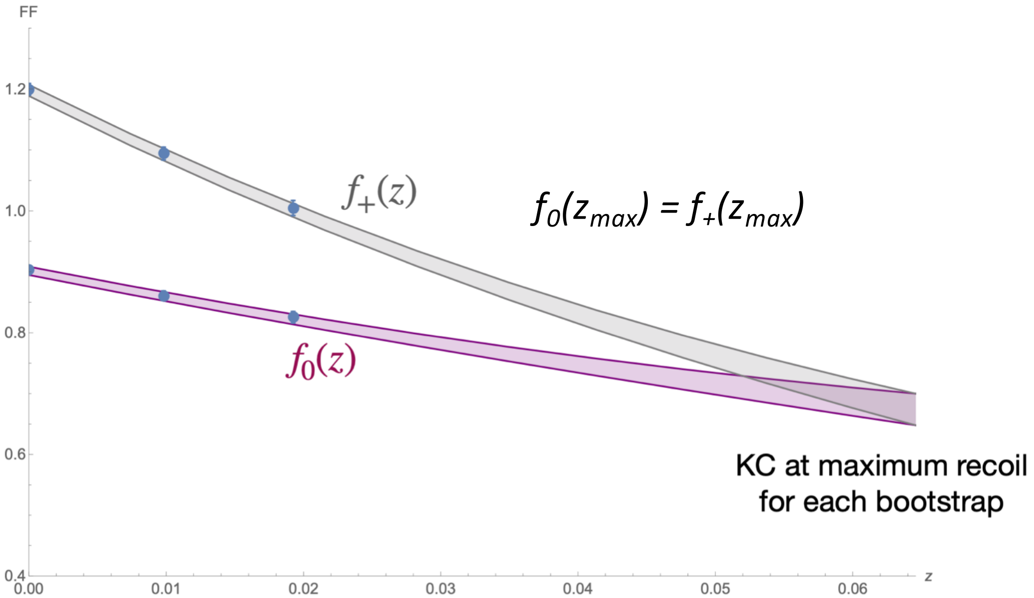
Application to $B_s \to K$: **identical results!**

All the details of the new ***Bayesian Inference (B.I.)*** *method can be found in:*
i) arXiv:2303.11285
ii) arXiv:2303.11280

The simplest example: semileptonic **B → D** decays

In **PRD ’21 (arXiv:2105.08674)**, our DM method has been applied to B → D decays:

- **3 FNAL/MILC data for each FF**: final results contained in **PRD ‘15 (arXiv:1503.07237)**



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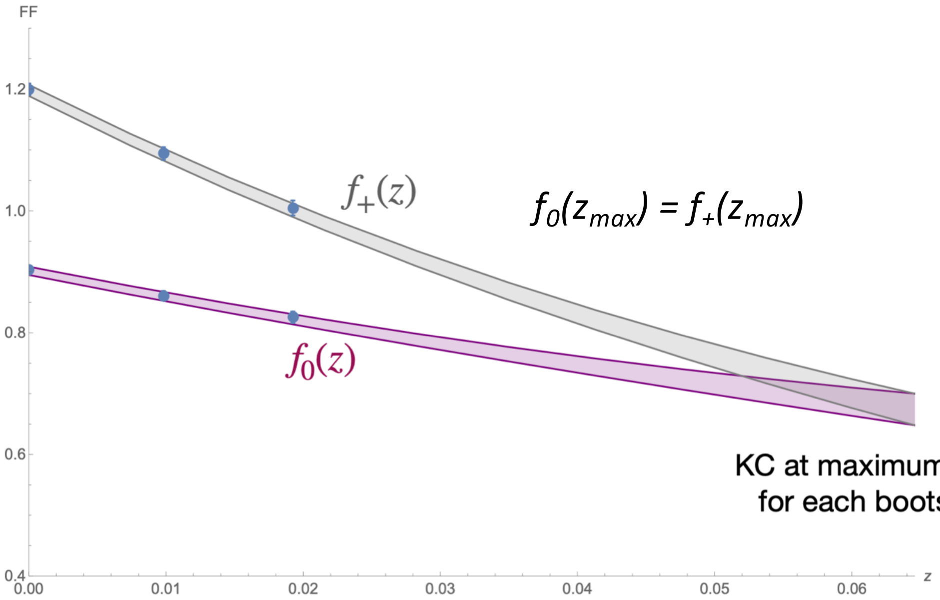

*Recalling that for production of a pseudoscalar meson (i.e. D) in case of **massive** lepton:*

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2 |V_{cb}|^2 \eta_{EW}^2}{24\pi^3} \left(1 - \frac{m_\ell^2}{q^2}\right)^2 \times \left[|\vec{p}_D|^3 \left(1 + \frac{m_\ell^2}{2q^2}\right) |f^+(q^2)|^2 + m_B^2 |\vec{p}_D| \left(1 - \frac{m_D^2}{m_B^2}\right)^2 \frac{3m_\ell^2}{8q^2} |f^0(q^2)|^2 \right]$$

FULLY-THEORETICAL ESTIMATE!

$$R(D) = 0.296 \pm 0.008$$

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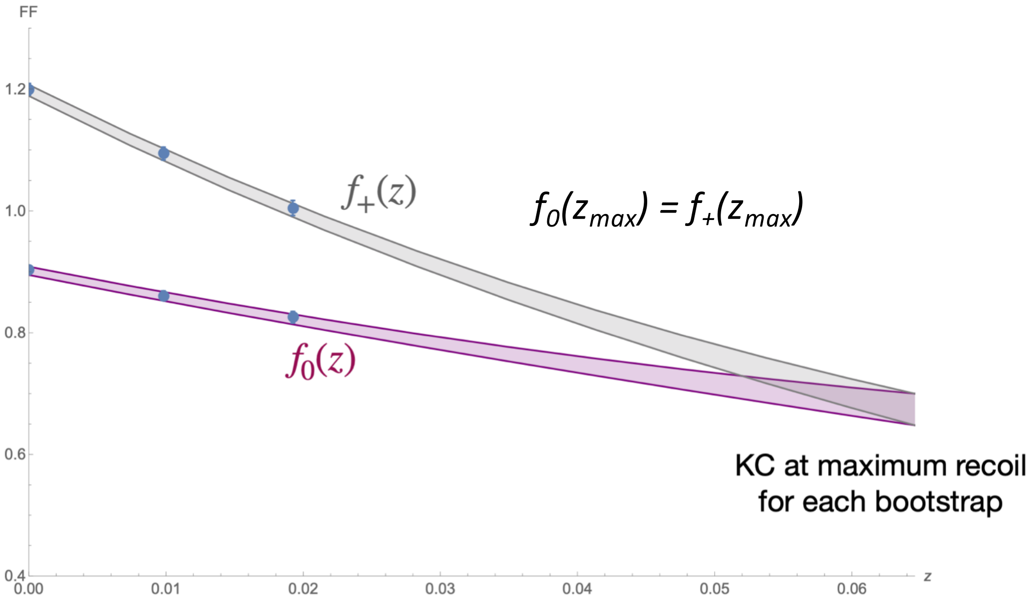
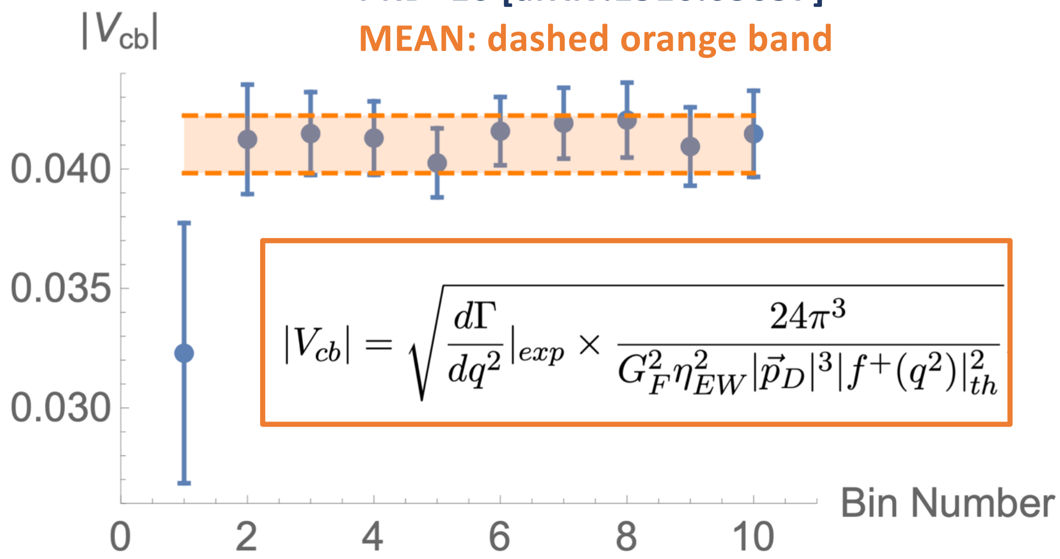


Blue points from Belle data:
PRD ’16 [arXiv:1510.03657]
MEAN: dashed orange band



$$R(D) = 0.296 \pm 0.008$$

$$|V_{cb}| \times 10^3 = 41.0 \pm 1.2$$

The “problematic” semileptonic **B → D*** channel

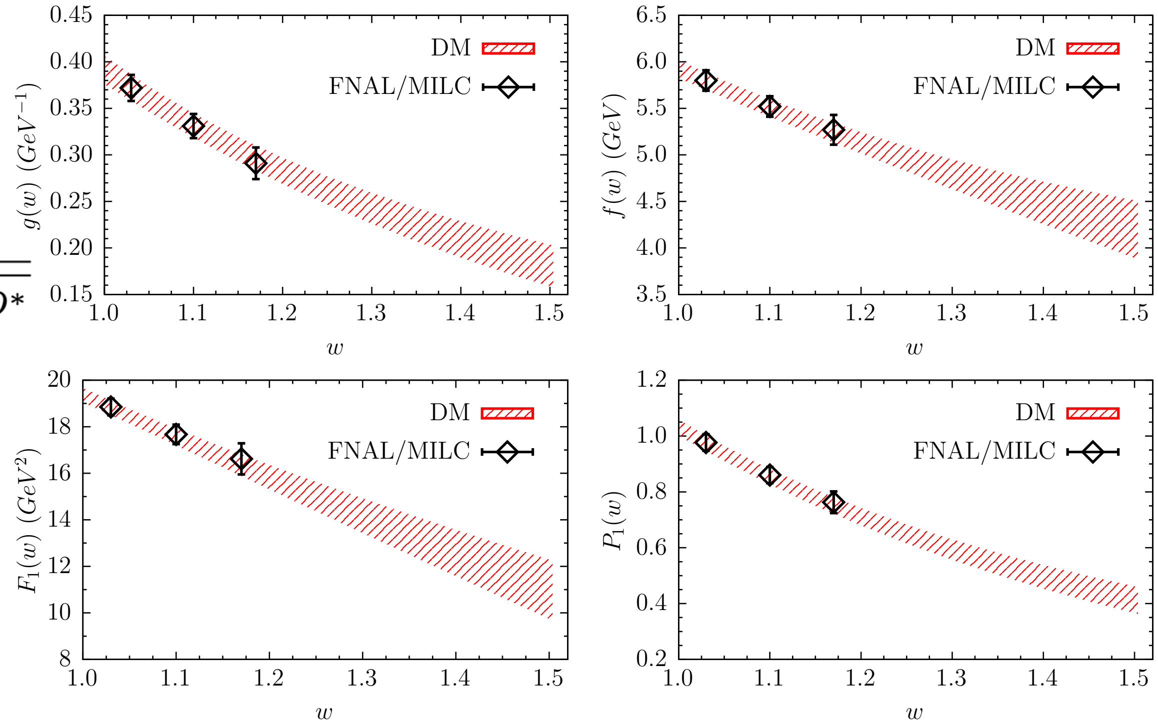
In **EPJC ‘22 (arXiv:2109.15248)**, we have studied the final results of the FNAL/MILC computations of the FFs

- **3 FNAL/MILC data (diamonds) for each FF**: final results contained in **EPJC ‘22 arXiv:2105.14019 [hep-lat]**

Two kinematical constraints (KCs):

$$\mathcal{F}_1(1) = (m_B - m_{D^*}) f(1)$$

$$P_1(w_{max}) = \frac{\mathcal{F}_1(w_{max})}{(1 + w_{max})(m_B - m_{D^*})\sqrt{m_B m_{D^*}}}$$



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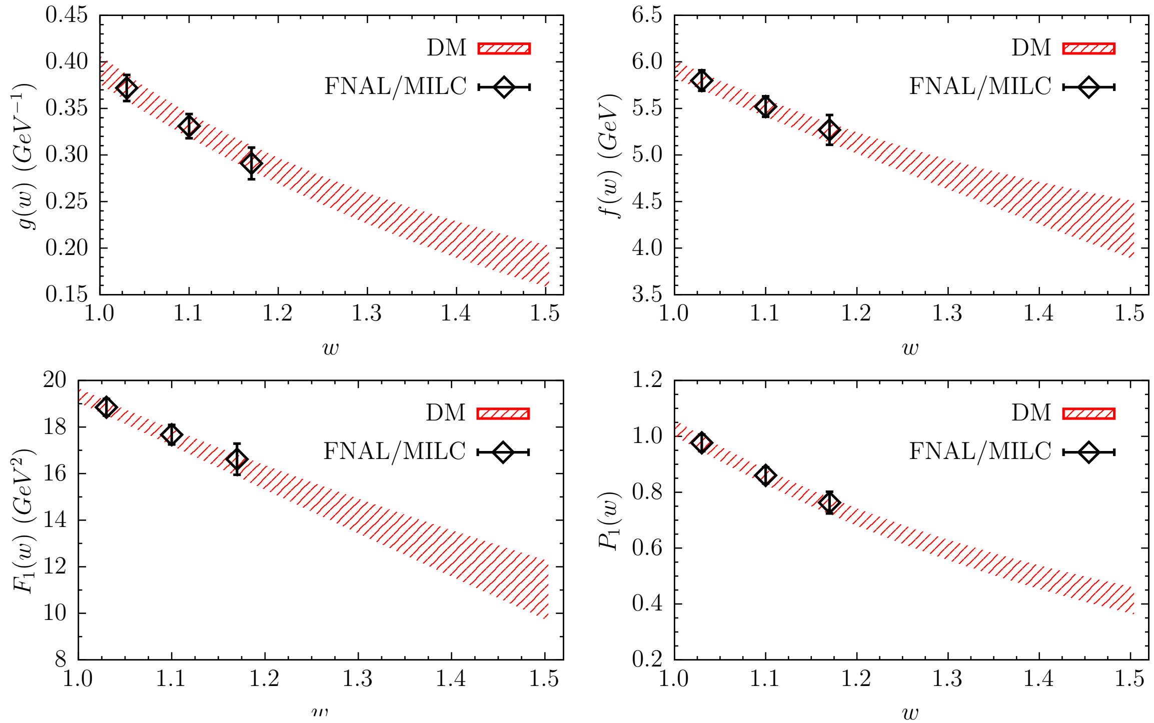
*Using the unitarity bands of the FFs, we can compute new fully-theoretical expectation values of the anomaly **R(D*)**:*

DM prediction

$$R(D^*) = 0.275 \pm 0.008$$

Less than 1σ compatibility!



$$R(D^*)|_{\rm exp} = 0.284 \pm 0.013$$

HFLAV Coll. (https://hflav-eos.web.cern.ch/hflav-eos/semi/winter23_prel/html/RDsDsstar/RDRDs.html)

Why not doing a global fit of lattice and exp. data

Note that one can use **also** experimental data (in addition to the LQCD ones) to constrain the shape of the FFs…

$$d\Gamma/dx, \;\; x = w, \cos\theta_l, \cos\theta_v, \chi$$

Belle Coll.: arXiv:1702.01521, PRD '19 [arXiv:1809.03290]

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Let us see this in detail: *let us consider the BGL fits performed by FNAL/MILC Collaborations in EPJC '22 arXiv:2105.14019*

Basics of BGL: the hadronic FFs corresponding to definite spin-parity can be represented as an expansion, originating from unitarity, analyticity and crossing symmetry, in terms of the conformal variable *z*, for instance

$$g(z) = \frac{1}{\sqrt{\chi_{1^-}(q_0^2)}} \frac{1}{\phi_g(z, q_0^2)\, P_{1^-}(z)} \sum_{n=0}^{\infty} a_n\, z^n$$

Unitarity:

$$\sum_{n=0}^{\infty} a_n^2 \leq 1$$

Boyd, Grinstein and Lebed, Phys. Lett. B353, 306 (1995)
Boyd, Grinstein and Lebed, Nucl. Phys. B461, 493 (1996)
Boyd, Grinstein and Lebed, Phys. Rev. D 56, 6895 (1997)

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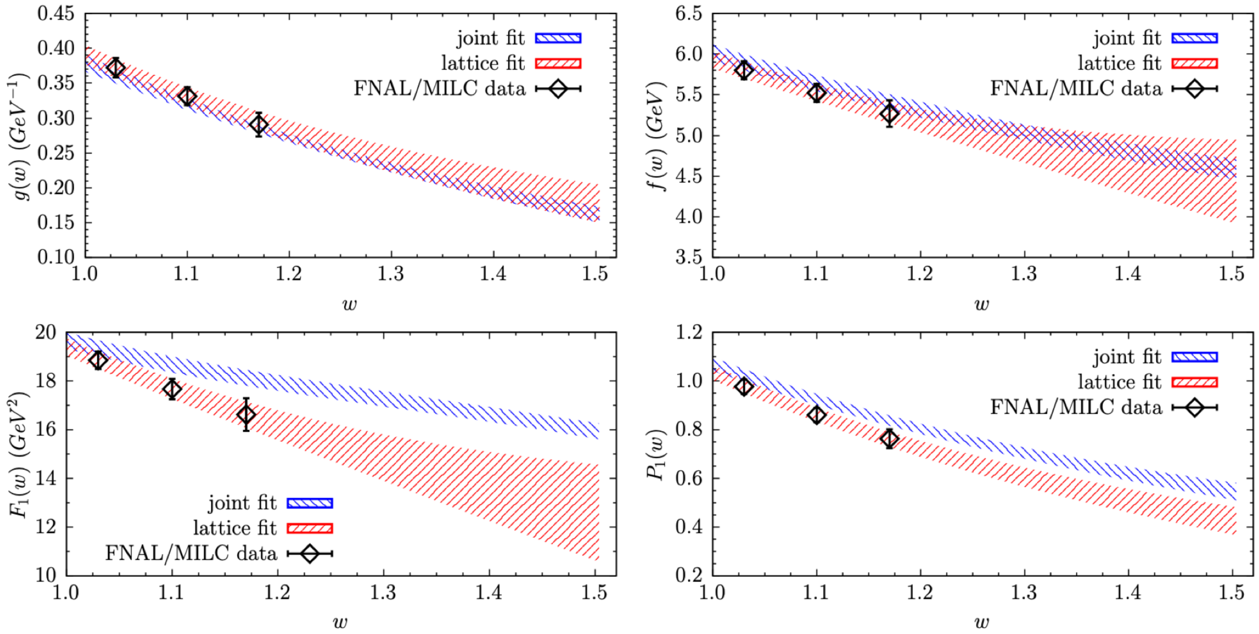


joint fit:
BGL fit of LQCD points + Belle + BaBar exp. data

$|V_{cb}| \cdot 10^3 = 38.40 \pm 0.74$

$R(D^*) = 0.2483 \pm 0.0013$

lattice fit:
quadratic BGL fit of LQCD points only

$|V_{cb}| > |V_{cb}|^{\text{joint fit}}$?

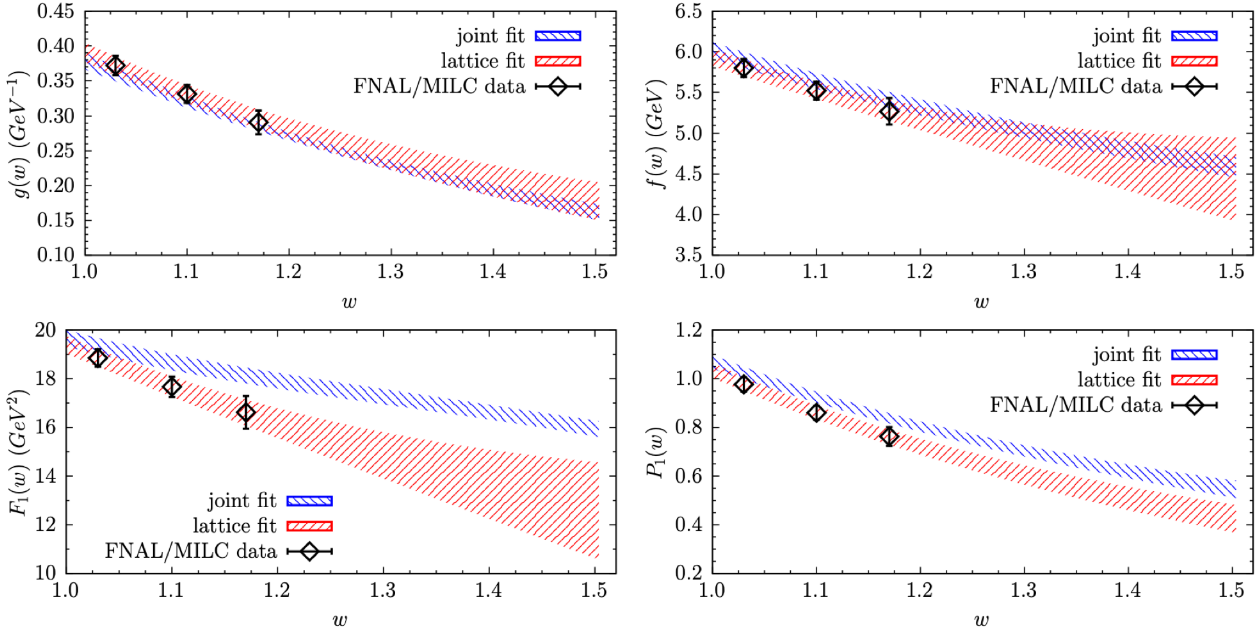
$R(D^*) = 0.265 \pm 0.013$

simultaneous fit of the lattice points and experimental data to determine the shape of the FFs and to extract $|V_{cb}|$

*** slope differences between exp's and theory $\rightarrow$ bias on $|V_{cb}|^{\text{joint fit}}$? ***

Why not doing a global fit of lattice and exp. data

OUR UNDERSTANDING: to avoid any bias in the description of the final shape of the FFs, we want to first analyse the lattice data and *then* compare the results with experiments!



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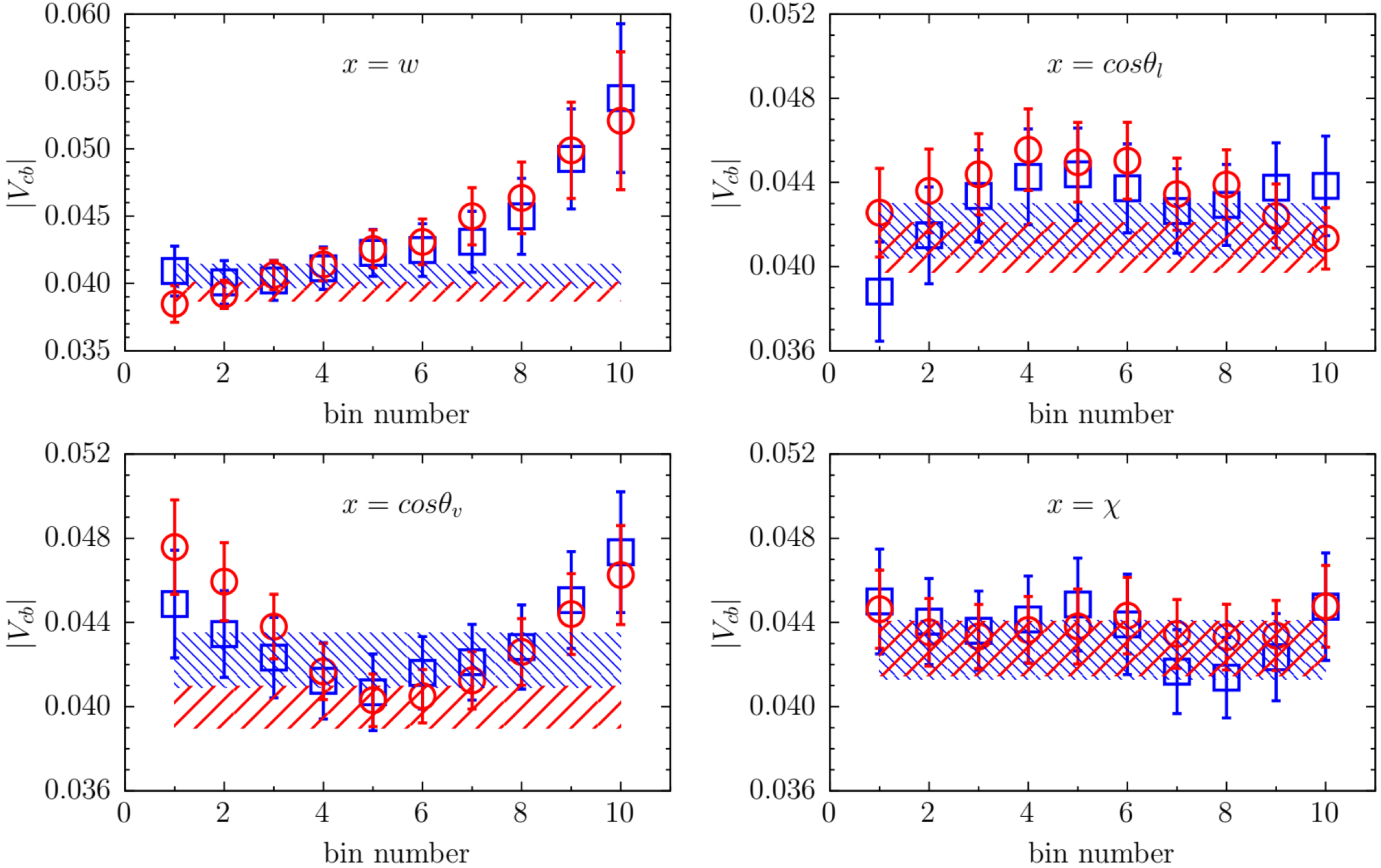
Our proposal: ***bin-per-bin exclusive Vcb*** determination through unitarity



Blue squares: arXiv:1702.01521

Red points: arXiv:1809.03290

$$|V_{cb}|_i \equiv \sqrt{\frac{(d\Gamma/dx)_i^{exp.}}{(d\Gamma/dx)_i^{th.}}}$$

Exclusive Vcb determination through unitarity

experiment	$\lvert V_{cb}\rvert(x=w)$	$\lvert V_{cb}\rvert(x=\cos\theta_l)$	$\lvert V_{cb}\rvert(x=\cos\theta_v)$	$\lvert V_{cb}\rvert(x=\chi)$
Ref. [14]	0.0405 (9)	0.0417 (13)	0.0422 (13)	0.0427 (14)
$\chi^2/(\text{d.o.f.})$	1.01	0.89	0.66	0.72
Ref. [15]	0.0394 (7)	0.0409 (12)	0.0400 (10)	0.0427 (13)
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Exclusive Vcb determination through unitarity

To compute the final average of these Vcb estimates:

$$\mu_x = \frac{1}{N}\sum_{k=1}^{N} x_k,$$

$$\sigma_x^2 = \frac{1}{N}\sum_{k=1}^{N}\sigma_k^2 + \frac{1}{N}\sum_{k=1}^{N}(x_k-\mu_x)^2$$

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Final DM estimate

$$|V_{cb}| \times 10^3 = 41.3 \pm 1.7$$

Compatible with the (most recent) **inclusive** values!

$$|V_{cb}|_{\rm incl} \times 10^3 = 42.16 \pm 0.50$$

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MESSAGE OF THE TALK:
treat experimental and LQCD data differently to determine |Vcb| and R(D*)

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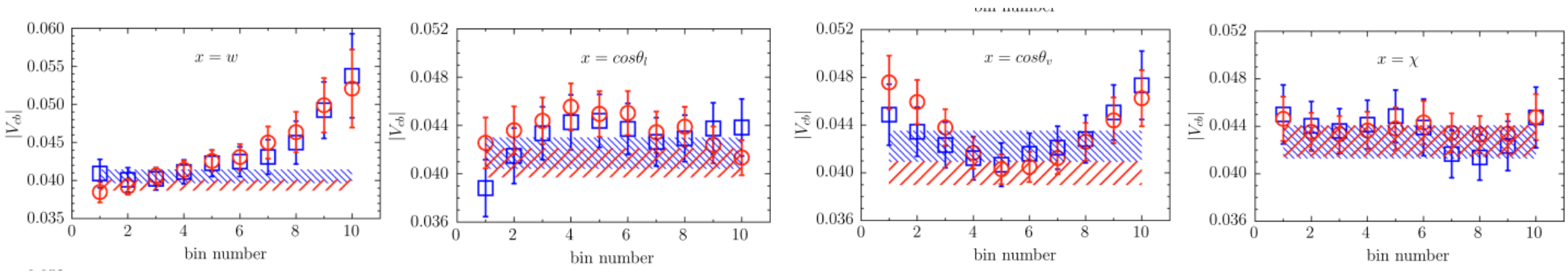
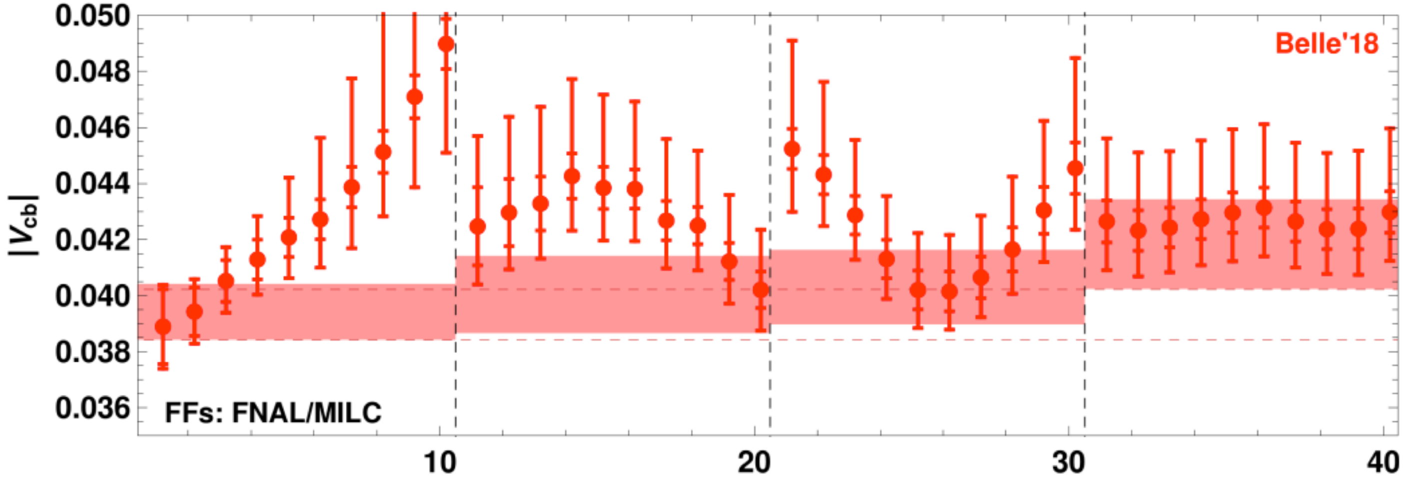
Critical understanding of the results obtained so far

1. Does the DM method modify the mean values/the correlations of the FFs?





Jung's talk @ LHCb Implications Workshop 2022 (CERN)

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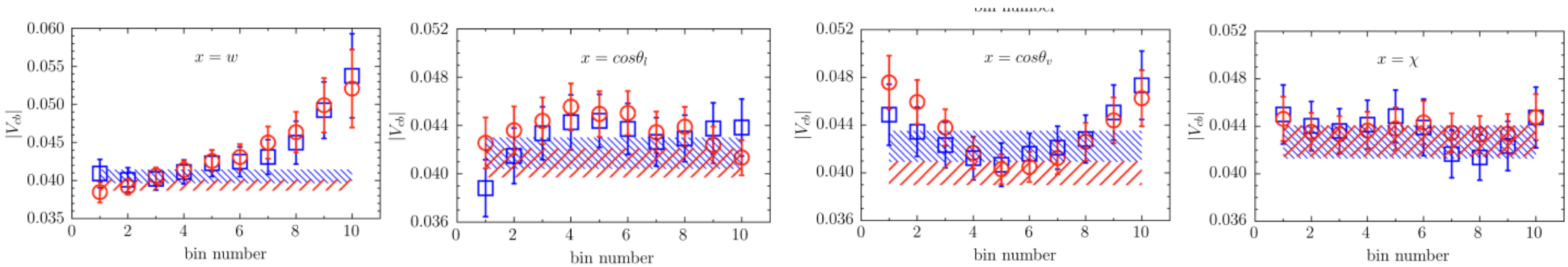
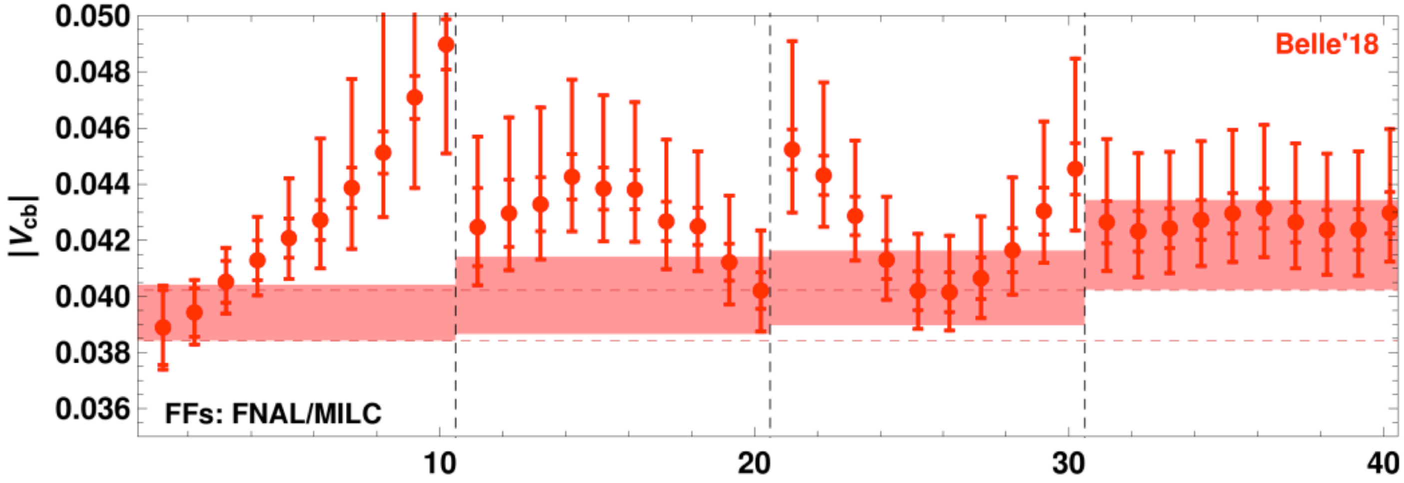


i) Same bin-per-bin values of Vcb
ii) Same mean values for each kinematical variable!

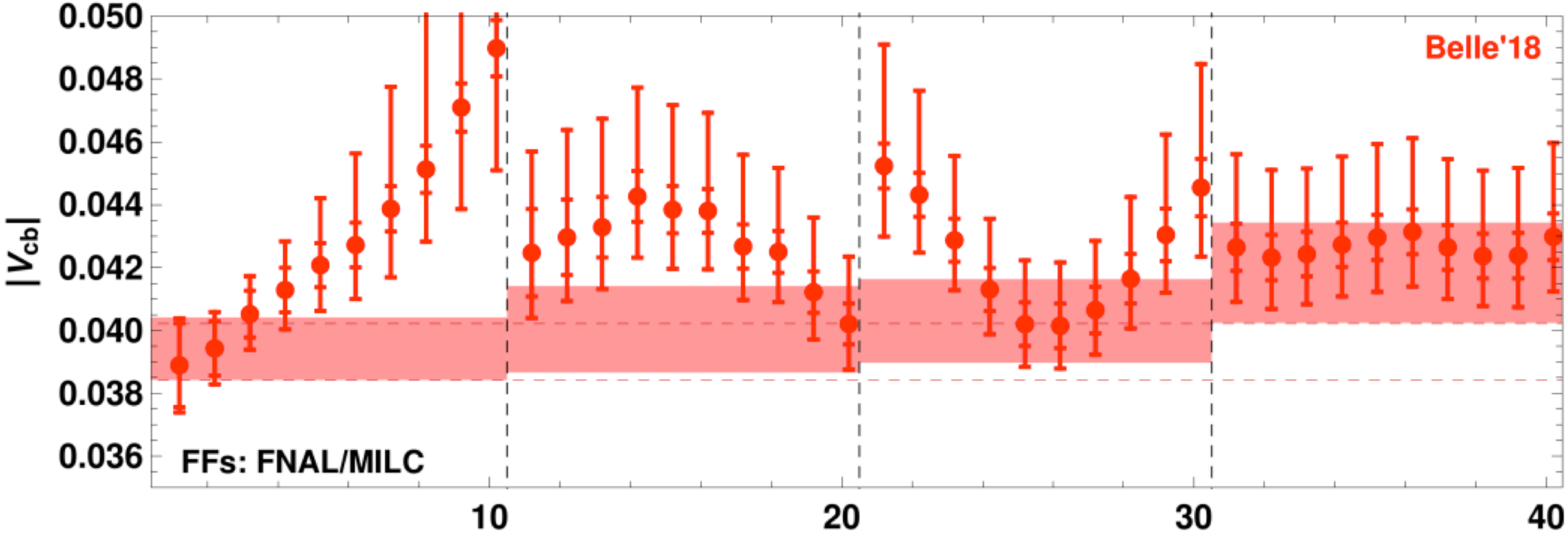


Jung's talk @ LHCb Implications Workshop 2022 (CERN)

Critical understanding of the results obtained so far

2. Why not a 40x40 correlated average?

However, Martin does use another (obviously legitimate) way to compute the average, *i.e.* a 40x40 correlated average:



Jung's talk @ LHCb Implications Workshop 2022 (CERN)

Unfilled band

$$V_{cb}^{\rm FM} = (39.3 \pm 0.9) \times 10^{-3}$$

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Important issue: other way to determine Vcb from Belle experiments, namely the study the total decay width!

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$$\mathcal{B}(B \to D^* \ell \nu) = (4.95 \pm 0.11 \pm 0.22) \times 10^{-2}$$

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through the unitary DM bands of the FFs

$$|V_{cb}| = (42.9 \pm 1.8) \cdot 10^{-3}$$

$$|V_{cb}| = (43.3 \pm 1.6) \cdot 10^{-3}$$

L. Vittorio, "The D(M)M perspective on Flavour Physics" , https://ricerca.sns.it/handle/11384/125744

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averages	Belle [2]	Belle [3]
Eq. (28) from bins	41.8 ± 1.5	40.8 ± 1.7
correlated average from bins	40.67 ± 0.88	39.43 ± 0.66
from total rate	42.9 ± 1.8	43.3 ± 1.6

FUNDAMENTAL QUESTION: how to justify this??

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This problem is absent here !

Summary of the DM study of LQCD and exps. data

According to our prescription, **the shape of the FFs have to be constrained by using *only* the results of the LQCD computations** on the lattice. In this way:

- the estimate of R(D*) is fully-theoretical
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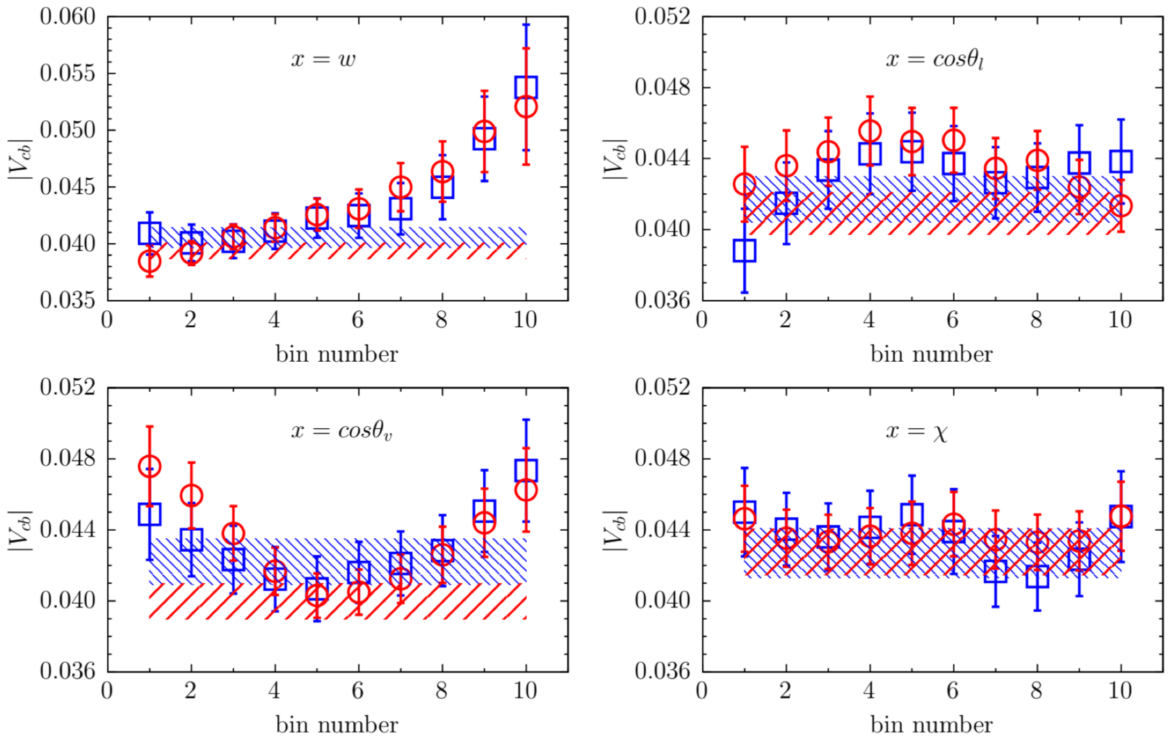

Remark 1

The value of $|V_{cb}|$ exhibits some dependence on the specific w-bin. The value obtained adopting a constant fit is dominated by the bins at small values of the recoil w, where direct lattice data are available and the lenght of the momentum extrapolation is limited.

Remark 2

The value of $|V_{cb}|$ deviates from a constant fit for $x = \cos(\theta_v)$. If we try a quadratic fit of the form

$$|V_{cb}|\left[1 + \delta B\, \cos^2(\theta_v)\right]$$

we get $\delta B \neq 0$ (2-3σ level) and $|V_{cb}|$ more consistent between the two sets of Belle data, but still in agreement with the value of $|V_{cb}|$ obtained with a constant fit

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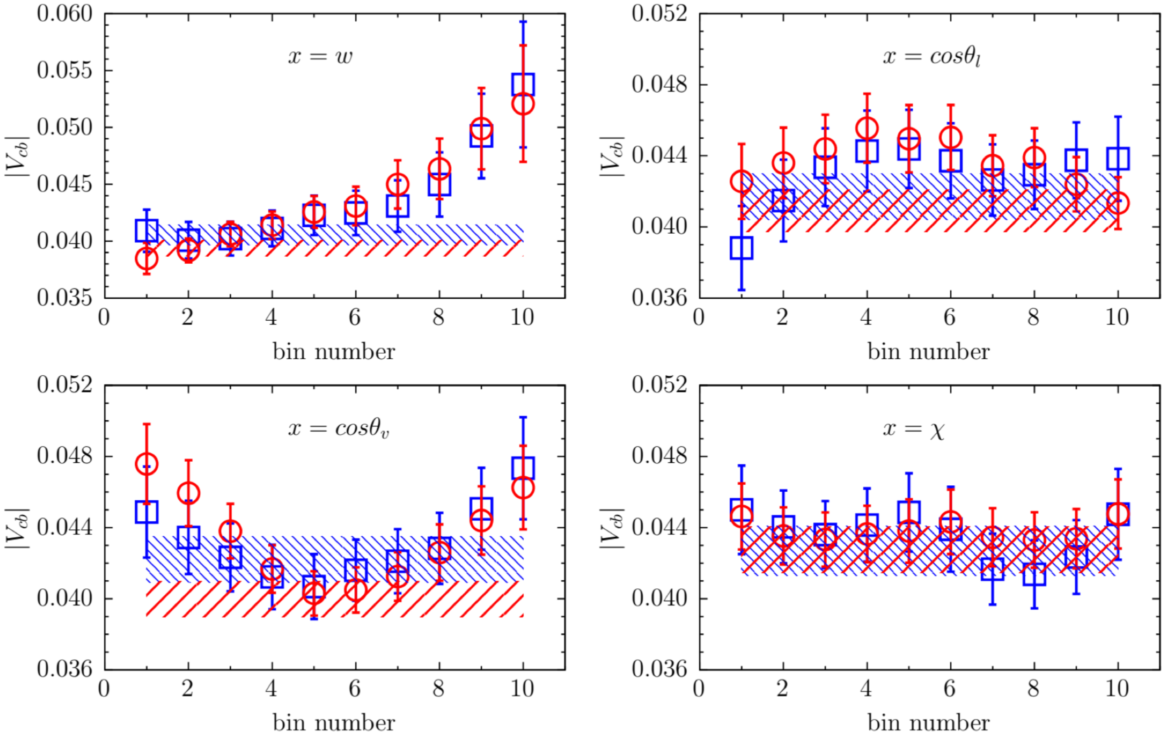

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OTHER DATA ARE FUNDAMENTAL!

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From the experimental point of view, a huge effort has been done in the recent past:
- New Belle data: arXiv:2301.07529 [hep-ex] (B \to D*)
- New Belle-II data (conf. paper): arXiv:2210.13143 [hep-ex] (B \to D) & arXiv:2301.04716 [hep-ex] (B \to D*)

See F. Bernlocher's talk

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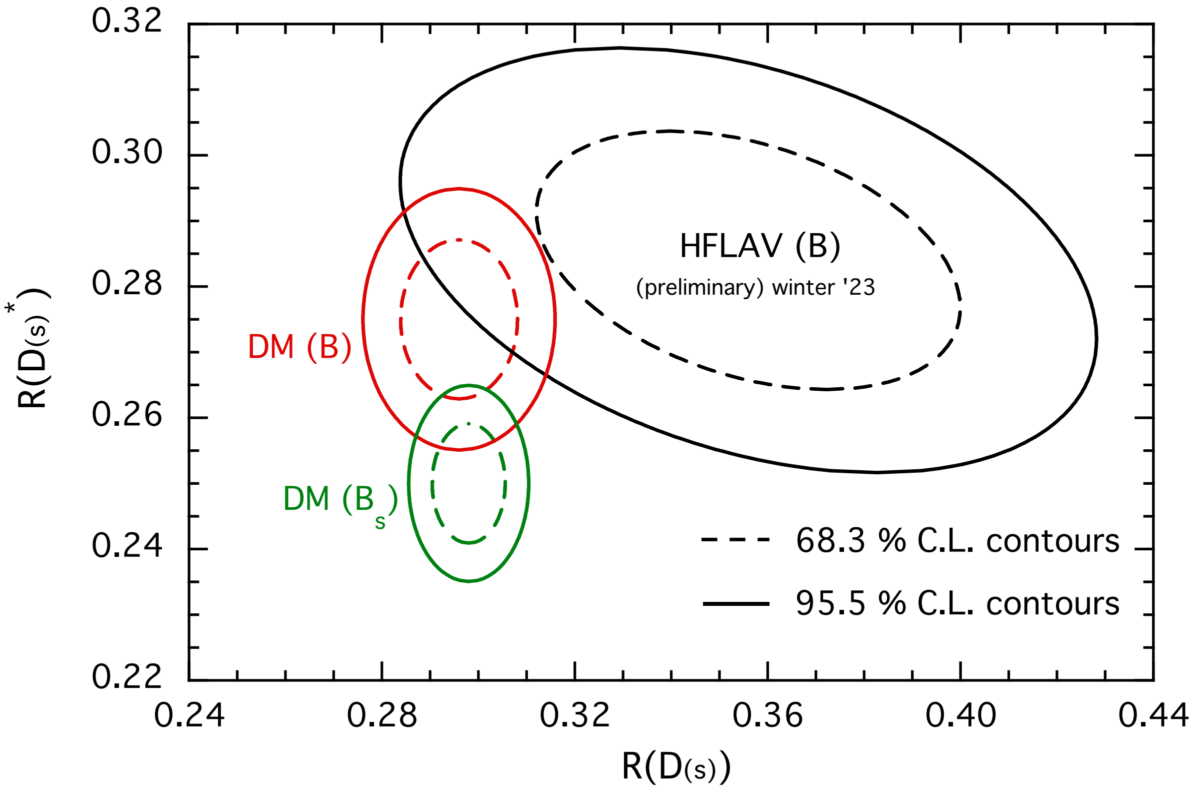
From the theoretical point of view, new LQCD data are awaited in the near future from JLQCD Collaboration. See arXiv:2304.03137 from HPQCD Collaboration for a very recent computation of the FFs

Conclusions

LFU observables



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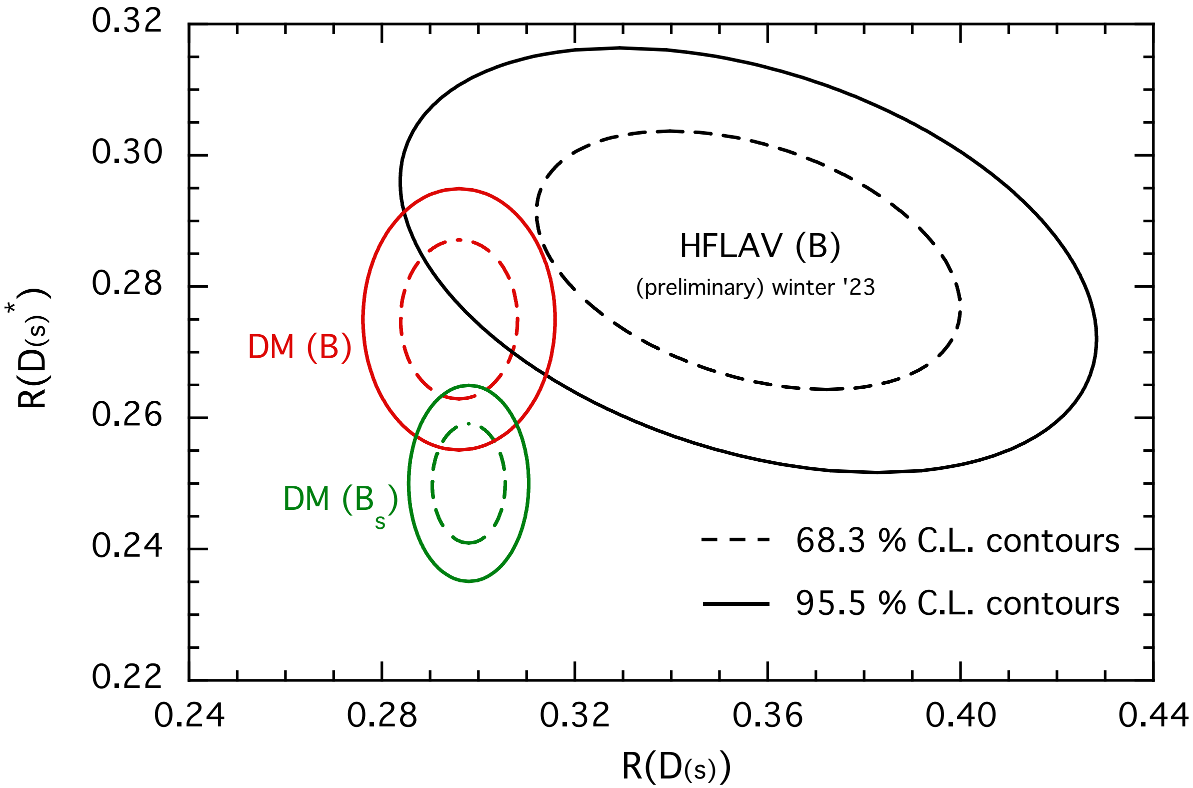

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If you are interested in semileptonic Bs → Ds decays, we can discuss about the DM application to these decays (back-up slides)

Summary of all the DM results

CKM parameters Vub and Vcb

	decays	DM	FLAG '21	inclusive
$\lvert V_{cb}\rvert \cdot 10^3$	$B_{(s)} \to D_{(s)}^{(*)}$	41.4 (8)	39.48 (68)	42.16 (50)
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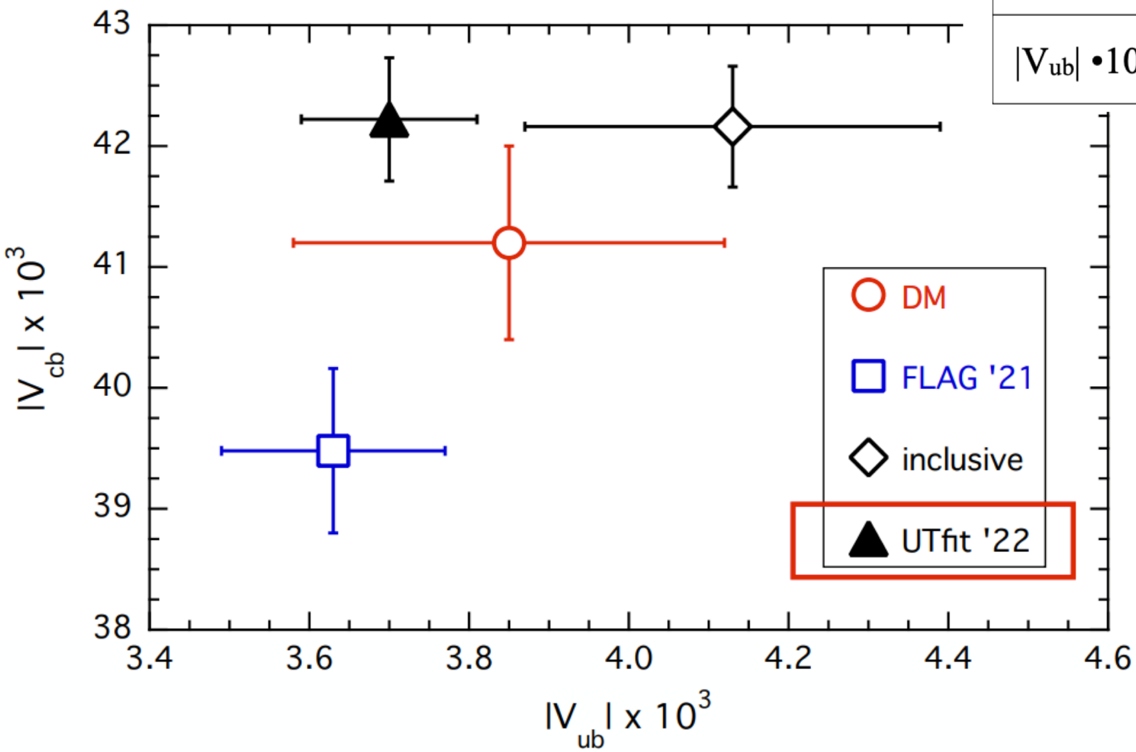

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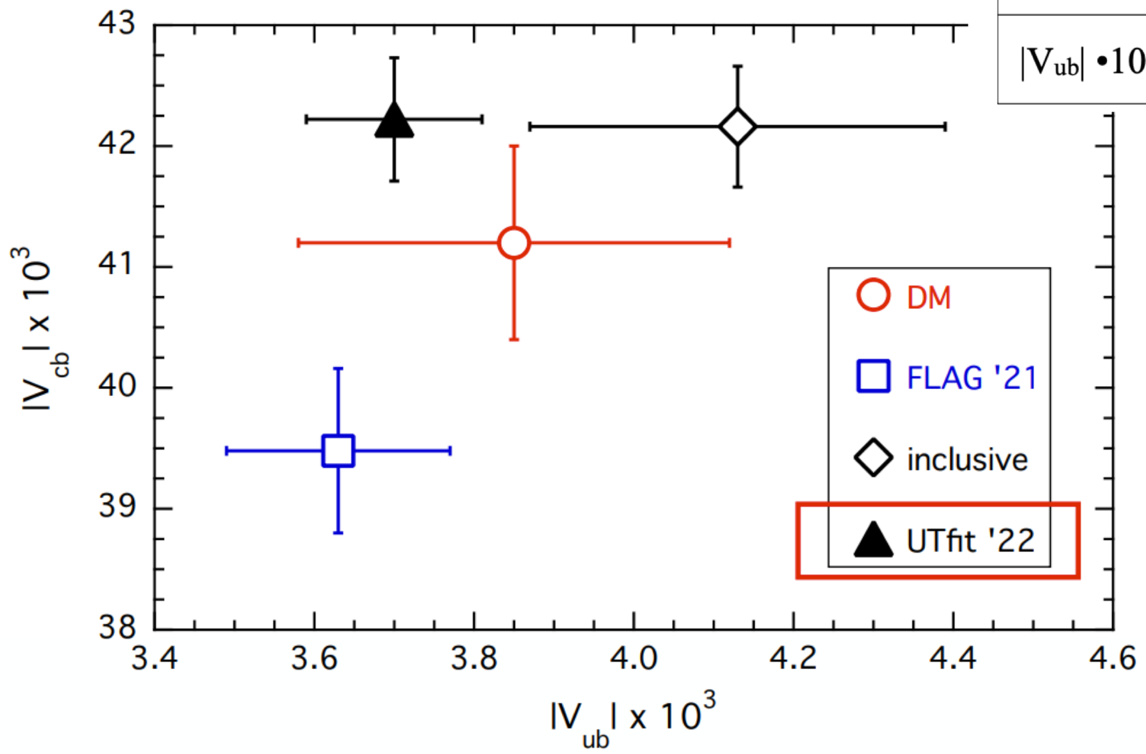

i) JHEP '22 [arXiv:2202.10285]
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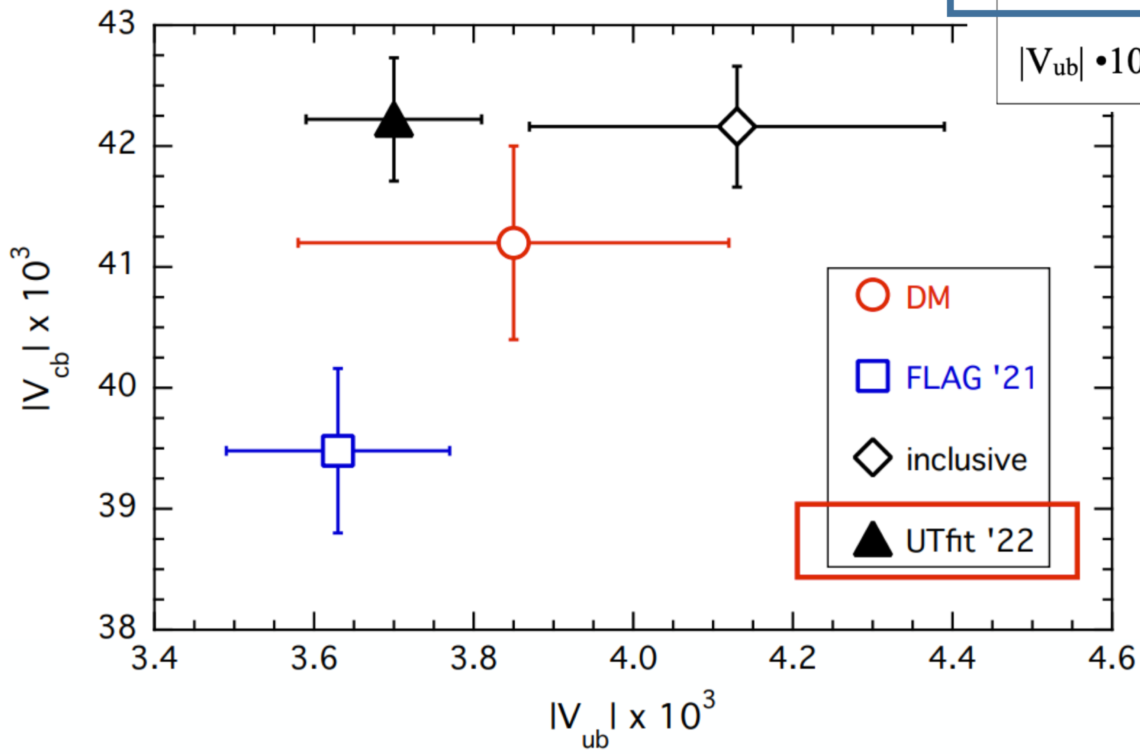

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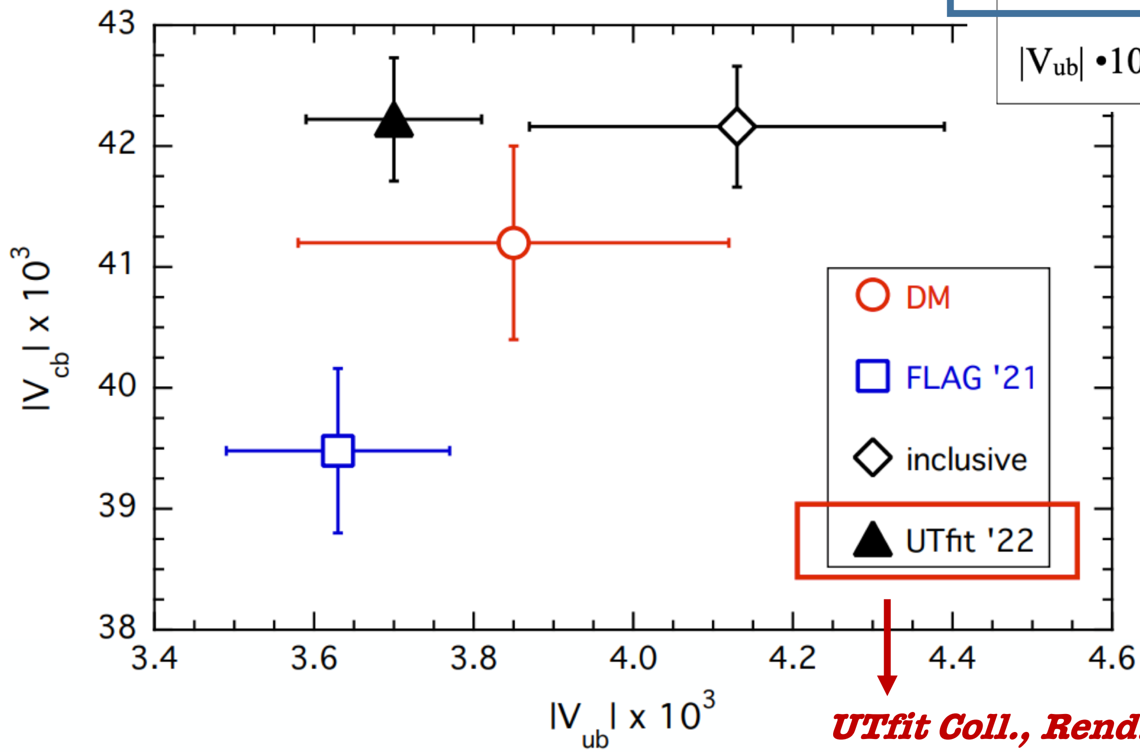

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UTfit Coll., Rend. Lincei Sci.Fis.Nat. 34 (2023) 37-57 [arXiv:2212.03894]

THANKS FOR

YOUR ATTENTION!

BACK-UP SLIDES

Statistical and systematic uncertainties

How can we finally combine all the N_U lower and upper bounds of both the FFs??

One bootstrap event case:

after a single extraction, we have one value of the lower bound f_L and one value of the upper one f_U for each FF. Assuming that the true value of each FF can be **everywhere inside the range (f_U - f_L) with equal probability**, we associate to the FFs a *flat* distribution

$$P(f_{0(+)}) = \frac{1}{f_{U,0(+)} - f_{L,0(+)}}\Theta(f_{0(+)} - f_{L,0(+)})\Theta(f_{U,0(+)} - f_{0(+)})$$

Many bootstrap events case:

how to mediate over the whole set of bootstrap events? Since the lower and the upper bounds of a generic FF are deeply correlated, we will assume a multivariate Gaussian distribution:

$$P(f_L, f_U) = \frac{\sqrt{\det \rho}}{2\pi} \exp\left[-\frac{\rho_{up,up}(f_U - \langle f_U\rangle)^2 + \rho_{lo,lo}(f_L - \langle f_L\rangle)^2 + 2\rho_{lo,up}(f_U - \langle f_U\rangle)(f_L - \langle f_L\rangle)}{2}\right]$$

In conclusion, we can combine the bounds of each FF in a final mean value and a final standard deviation, defined as

$$\langle f\rangle = \frac{\langle f_L\rangle + \langle f_U\rangle}{2},$$

$$\sigma_f = \frac{1}{12}(\langle f_U\rangle - \langle f_L\rangle)^2 + \frac{1}{3}(\sigma^2_{f_{lo}} + \sigma^2_{f_{up}} + \rho_{lo,up}\sigma_{f_{lo}}\sigma_{f_{up}})$$

NO PARAMETRIZATION ADOPTED!!!

Kinematical Constraints (KCs)

REMINDER: after the unitarity filter we were left with $N_U < N$ *survived events!!!*

Let us focus on the pseudoscalar case. Since by construction the following *kinematical constraint* holds

$$f_0(0) = f_+(0)$$

we will filter only the $N_{KC} < N_U$ events for which the two bands of the FFs intersect each other @ $t = 0$.
Namely, for each of these events we also define

$$\phi_{lo} = \max[F_{+,lo}(t=0), F_{0,lo}(t=0)]$$

$$\phi_{up} = \min[F_{+,up}(t=0), F_{0,up}(t=0)]$$

From WE theorem

$$\langle D(p_D)| V^\mu |B(p_B)\rangle \quad = \quad f_+(p_B + p_D)^\mu + f_-(p_B - p_D)^\mu$$

One then defines

$$f_0(q^2) = f_+(q^2) + \frac{q^2}{m_B^2 - m_D^2} f_-(q^2)$$

$$\langle D(p_D)\,|\,V^\mu\,|\,B(p_B)\rangle = f^+(q^2)\left(p_B^\mu + p_D^\mu - \frac{m_B^2 - m_D^2}{q^2}q^\mu\right) + f^0(q^2)\frac{m_B^2 - m_D^2}{q^2}q^\mu$$

Kinematical Constraints (KCs)

We then consider a **modified matrix**

$$\mathbf{M_C} = \begin{pmatrix} \phi f|\phi f\rangle & \langle \phi f|g_t\rangle & \langle \phi f|g_{t_1}\rangle & \cdots & \langle \phi f|g_{t_n}\rangle & \langle \phi f|g_{t_{n+1}}\rangle \\ \langle g_t|\phi f\rangle & \langle g_t|g_t\rangle & \langle g_t|g_{t_1}\rangle & \cdots & \langle g_t|g_{t_n}\rangle & \langle g_t|g_{t_{n+1}}\rangle \\ \langle g_{t_1}|\phi f\rangle & \langle g_{t_1}|g_t\rangle & \langle g_{t_1}|g_{t_1}\rangle & \cdots & \langle g_{t_1}|g_{t_n}\rangle & \langle g_{t_1}|g_{t_{n+1}}\rangle \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \langle g_{t_n}|\phi f\rangle & \langle g_{t_n}|g_t\rangle & \langle g_{t_n}|g_{t_1}\rangle & \cdots & \langle g_{t_n}|g_{t_n}\rangle & \langle g_{t_n}|g_{t_{n+1}}\rangle \\ \langle g_{t_{n+1}}|\phi f\rangle & \langle g_{t_{n+1}}|g_t\rangle & \langle g_{t_{n+1}}|g_{t_1}\rangle & \cdots & \langle g_{t_{n+1}}|g_{t_n}\rangle & \langle g_{t_{n+1}}|g_{t_{n+1}}\rangle \end{pmatrix}$$

with $t_{n+1} = 0$. Hence, we compute the new lower and upper bounds of the FFs in this way. For each of the N_{KC} events, we extract $N_{KC,2}$ values of $f_0(0) = f_+(0) \equiv f(0)$ with uniform distribution defined in the range $[\phi_{lo}, \phi_{up}]$. Thus, for both the FFs and for each of the N_{KC} events we define

$$\begin{aligned} F_{lo}(t) &= \min[F_{lo}^1(t), F_{lo}^2(t), \cdots, F_{lo}^{N_{KC,2}}(t)], \\ F_{up}(t) &= \max[F_{up}^1(t), F_{up}^2(t), \cdots, F_{up}^{N_{KC,2}}(t)] \end{aligned}$$

Non-perturbative computation of the susceptibilities

In **PRD '21 [arXiv:2105.07851]**, we have presented the results of the first computation on the lattice of the susceptibilities for the $b \to c$ quark transition, using the N_f=2+1+1 gauge ensembles generated by ETM Collaboration.

How are they defined? The starting point is the HVP tensor:

$$\Pi^V_{\mu\nu}(Q) = \int d^4x \; e^{-iQ\cdot x} \langle 0 | T \left[\bar{b}(x) \gamma^E_\mu c(x) \; \bar{c}(0) \gamma^E_\nu b(0) \right] | 0 \rangle$$
$$= -Q_\mu Q_\nu \Pi_{0^+}(Q^2) + (\delta_{\mu\nu} Q^2 - Q_\mu Q_\nu) \Pi_{1^-}(Q^2)$$

To compute the susceptibilities on the lattice, we start from the Euclidean correlators:

$$\chi_{0^+}(Q^2) \equiv \frac{\partial}{\partial Q^2} \left[Q^2 \Pi_{0^+}(Q^2) \right] = \int_0^\infty dt \; t^2 j_0(Qt) \; C_{0^+}(t) \; , \quad \xrightarrow{W.\,I.} \quad \frac{1}{4} \int_0^\infty dt' \, t'^4 \, \frac{j_1(Qt')}{Qt'} \left[(m_b - m_c)^2 C_S(t') + Q^2 C_{0^+}(t') \right]$$

$$\chi_{1^-}(Q^2) \equiv -\frac{1}{2} \frac{\partial^2}{\partial^2 Q^2} \left[Q^2 \Pi_{1^-}(Q^2) \right] = \frac{1}{4} \int_0^\infty dt \; t^4 \frac{j_1(Qt)}{Qt} \; C_{1^-}(t)$$

$$\chi_{0^-}(Q^2) \equiv \frac{\partial}{\partial Q^2} \left[Q^2 \Pi_{0^-}(Q^2) \right] = \int_0^\infty dt \; t^2 j_0(Qt) \; C_{0^-}(t) \; , \quad \xrightarrow{W.\,I.} \quad \frac{1}{4} \int_0^\infty dt' \, t'^4 \, \frac{j_1(Qt')}{Qt'} \left[(m_b + m_c)^2 C_P(t') + Q^2 C_{0^-}(t') \right]$$

$$\chi_{1^+}(Q^2) \equiv -\frac{1}{2} \frac{\partial^2}{\partial^2 Q^2} \left[Q^2 \Pi_{1^+}(Q^2) \right] = \frac{1}{4} \int_0^\infty dt \; t^4 \frac{j_1(Qt)}{Qt} \; C_{1^+}(t)$$

Non-perturbative computation of the susceptibilities

Let us choose for the moment zero Q^2:

$$\chi_{0^+}(Q^2=0) = \int_0^\infty dt\ t^2\ C_{0^+}(t)\ ,$$

$$\chi_{1^-}(Q^2=0) = \frac{1}{12}\int_0^\infty dt\ t^4\ C_{1^-}(t)\ ,$$

$$\chi_{0^-}(Q^2=0) = \int_0^\infty dt\ t^2\ C_{0^-}(t)\ ,$$

$$\chi_{1^+}(Q^2=0) = \frac{1}{12}\int_0^\infty dt\ t^4\ C_{1^+}(t)\ .$$

$$\chi_{0^+}(Q^2=0) = \frac{1}{12}(m_b-m_c)^2\int_0^\infty dt\ t^4\ C_S(t)$$

$$\chi_{0^-}(Q^2=0) = \frac{1}{12}(m_b+m_c)^2\int_0^\infty dt\ t^4\ C_P(t)$$

$$C_{0^+}(t) = \boxed{\widetilde{Z}_V^2}\int d^3x \langle 0|T\left[\bar{q}_1(x)\gamma_0 q_2(x)\ \bar{q}_2(0)\gamma_0 q_1(0)\right]|0\rangle\ ,$$

$$C_{1^-}(t) = \boxed{\widetilde{Z}_V^2}\frac{1}{3}\sum_{j=1}^{3}\int d^3x \langle 0|T\left[\bar{q}_1(x)\gamma_j q_2(x)\ \bar{q}_2(0)\gamma_j q_1(0)\right]|0\rangle\ ,$$

$$C_{0^-}(t) = \boxed{\widetilde{Z}_A^2}\int d^3x \langle 0|T\left[\bar{q}_1(x)\gamma_0\gamma_5 q_2(x)\ \bar{q}_2(0)\gamma_0\gamma_5 q_1(0)\right]|0\rangle\ ,$$

$$C_{1^+}(t) = \boxed{\widetilde{Z}_A^2}\frac{1}{3}\sum_{j=1}^{3}\int d^3x \langle 0|T\left[\bar{q}_1(x)\gamma_j\gamma_5 q_2(x)\ \bar{q}_2(0)\gamma_j\gamma_5 q_1(0)\right]|0\rangle\ ,$$

$$C_S(t) = \boxed{\widetilde{Z}_S^2}\int d^3x \langle 0|T\left[\bar{q}_1(x) q_2(x)\ \bar{q}_2(0) q_1(0)\right]|0\rangle\ ,$$

$$C_P(t) = \boxed{\widetilde{Z}_P^2}\int d^3x \langle 0|T\left[\bar{q}_1(x)\gamma_5 q_2(x)\ \bar{q}_2(0)\gamma_5 q_1(0)\right]|0\rangle\ ,$$

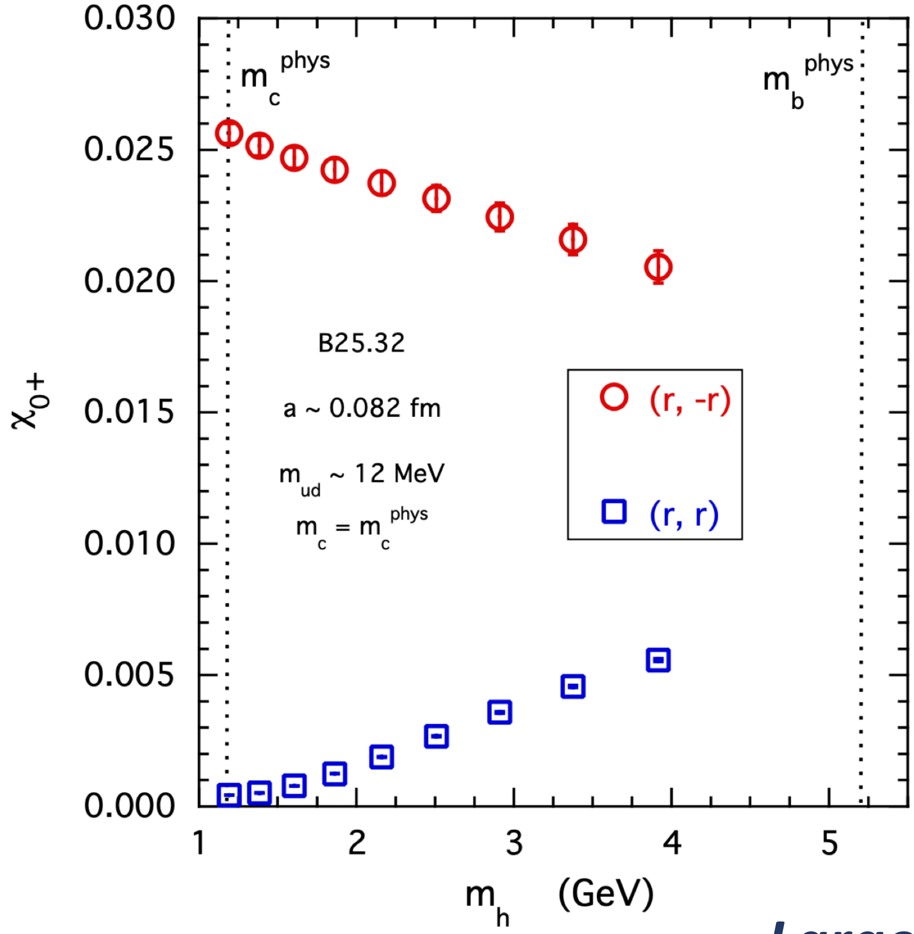
We are working in twisted mass LQCD: the Wilson parameter *r* can be equal or opposite for the two quarks in the currents

→ Two possible **independent** combinations of *(r_1,r_2)!*

Z: appropriate renormalization constants

N. Carrasco et al. [ETM Coll.], NPB 887 (2014) [arXiv:1403.4504]

Non-perturbative computation of the susceptibilities



Following set of masses:

$$m_h(n) = \lambda^{n-1}\ m_c^{phys} \qquad \text{for } n = 1, 2, \ldots$$

$$m_h = a\mu_h/(Z_P a)$$

$$\lambda \equiv [m_b^{phys}/m_c^{phys}]^{1/10} = [5.198/1.176]^{1/10} \simeq 1.1602$$

Nine masses values!

$$m_h(1) = m_c^{phys}$$

$$m_h(9) \simeq 3.9\ \text{GeV} \simeq 0.75\ m_b^{phys}$$

***r*: Wilson parameter**

Large discretisation effects and contact terms

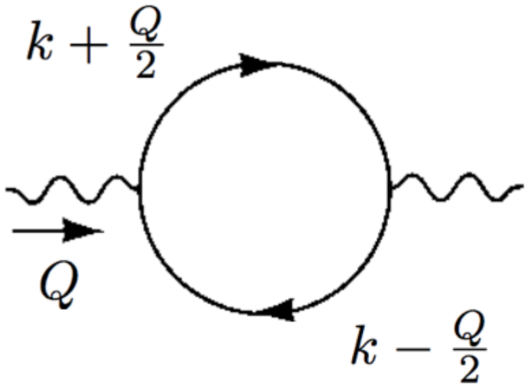
Contact terms & perturbative subtraction

In **twisted mass LQCD:**

$$\Pi_V^{\alpha\beta} = \int_{-\pi/a}^{+\pi/a} \frac{d^4k}{(2\pi)^4}\,\mathrm{Tr}\Big[\gamma^\alpha G_1(k+\frac{Q}{2})\gamma^\beta G_2(k-\frac{Q}{2})\Big],$$

$$G_i(p) = \frac{-i\gamma_\mu \mathring{p}_\mu + \mathcal{M}_i(p) - i r_i \mu_{q,i}\gamma_5}{\mathring{p}_\mu^2 + \mathcal{M}_i^2(p) + \mu_{q,i}^2}$$

$$\mathring{p}_\mu \equiv \frac{1}{a}\sin(ap_\mu), \qquad \mathcal{M}_i(p) \equiv m_i + \frac{r_i}{2} a\hat{p}_\mu^2, \qquad \hat{p} \equiv \frac{2}{a}\sin\Big(\frac{ap_\mu}{2}\Big).$$



$$\begin{aligned}\Pi_V^{\alpha\beta} &= a^{-2}(Z_1^I + (r_1^2 - r_2^2)Z_2^I + (r_1^2 - r_2^2)(r_1^2 + r_2^2)Z_3^I)g^{\alpha\beta} \\ &+ (\mu_1^2 Z^{\mu_1^2} + \mu_2^2 Z^{\mu_2^2} + \mu_1\mu_2 Z^{\mu_1\mu_2})g^{\alpha\beta} + (Z_1^{Q^2} + (r_1^2 - r_2^2)Z_2^{Q^2})Q\cdot Q g^{\alpha\beta} \\ &+ (Z_1^{Q^\alpha Q^\beta} + (r_1^2 - r_2^2)Z_2^{Q^\alpha Q^\beta})Q^\alpha Q^\beta + r_1 r_2(a^{-2}Z_1^{r_1r_2}g^{\alpha\beta} + (Z_2^{r_1r_2} + (r_1^2 + r_2^2)Z_3^{r_1r_2} \\ &+ (r_1^4 + r_2^4)Z_4^{r_1r_2})Q\cdot Q g^{\alpha\beta} + (\mu_1^2 Z_5^{r_1r_2} + \mu_2^2 Z_6^{r_1r_2})g^{\alpha\beta}) + O(a^2),\end{aligned}$$

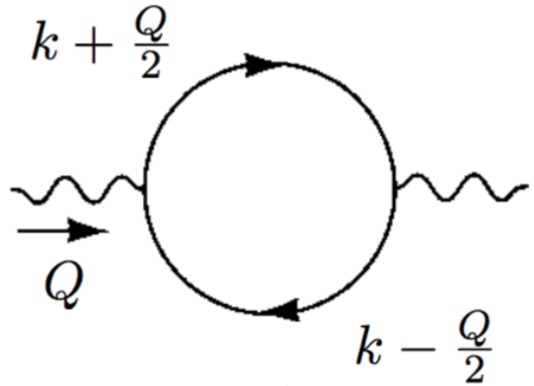
CONTACT TERMS!!!

F. Burger et al., ETM Coll., JHEP '15 [arXiv:1412.0546]

Contact terms & perturbative subtraction

In **twisted mass LQCD**:

$$\Pi_V^{\alpha\beta} = \int_{-\pi/a}^{+\pi/a} \frac{d^4k}{(2\pi)^4}\, \mathrm{Tr}\left[\gamma^\alpha G_1(k+\frac{Q}{2})\gamma^\beta G_2(k-\frac{Q}{2})\right],$$



Thus, by separating the *longitudinal* and the *transverse* contributions, we can compute the susceptibilities for all the spin-parity quantum numbers **in the free theory on the lattice**, *i.e.* at order $\mathcal{O}(\alpha_s^0)$ using twisted-mass fermions!

$$\chi_j^{free} = \boxed{\chi_j^{LO}} + \boxed{\chi_j^{discr}}$$

LO term of PT @ $\mathcal{O}(\alpha_s^0)$ contact terms and discretization effects @ $\mathcal{O}(\alpha_s^0 a^m)$ with $m \geq 0$

Perturbative subtraction:

$$\chi_j \to \chi_j - \left[\chi_j^{free} - \chi_j^{LO}\right]$$

ETMC ratio method & final results

For the extrapolation to the physical *b*-quark point we have used the ETMC ratio method:

$$R_j(n; a^2, m_{ud}) \equiv \frac{\chi_j[m_h(n); a^2, m_{ud}]}{\chi_j[m_h(n-1); a^2, m_{ud}]} \boxed{\frac{\rho_j[m_h(n)]}{\rho_j[m_h(n-1)]}}$$

to ensure that $\lim_{n\to\infty} R_j(n) = 1$

$$\rho_{0^+}(m_h) = \rho_{0^-}(m_h) = 1 \ ,$$
$$\rho_{1^-}(m_h) = \rho_{1^+}(m_h) = (m_h^{pole})^2$$

All the details are deeply discussed in **PRD '21 [2105.07851]**. In this way, we have obtained the first lattice QCD determination of susceptibilities of heavy-to-heavy (and heavy-to-light, see **JHEP '22 [2202.10285]**) transition current densities:

b → c

	Perturbative	With subtraction	Non-perturbative	With subtraction
$\chi_{V_L}\,[10^{-3}]$	6.204(81)	—	7.58(59)	—
$\chi_{A_L}\,[10^{-3}]$	24.1	19.4	25.8(1.7)	21.9(1.9)
$\chi_{V_T}\,[10^{-4}\ \text{GeV}^{-2}]$	6.486(48)	5.131(48)	6.72(41)	5.88(44)
$\chi_{A_T}\,[10^{-4}\ \text{GeV}^{-2}]$	3.894	—	4.69(30)	—

Differences with PT? **~4% for 1^-, ~7% for 0^-, ~20 % for 0^+ and 1^+**

Critical understanding of the results obtained so far

2. Does the DM method modify the mean values/the correlations of the FFs?

Unitarity constraint

Substituting BGL into the unitarity constraint

$$\frac{1}{2\pi i}\oint_C \frac{dz}{z}\left|B_X(q^2)\phi_X(q^2,t_0)f_X(q^2)\right|^2 \leq 1\,,$$

gives

$$|\boldsymbol{a}_X|^2 \leq 1$$

(see Paper 2 for details of a modified version of this constraint)

- **MUST** be satisfied for any believable fit!
- Any z-expansion fit must necessarily be truncated.
- Strong constraint on allowed size of coefficients $a_{X,i}$.
- How can we best make use of this?

Tsang's talk @ Moriond EW 2023

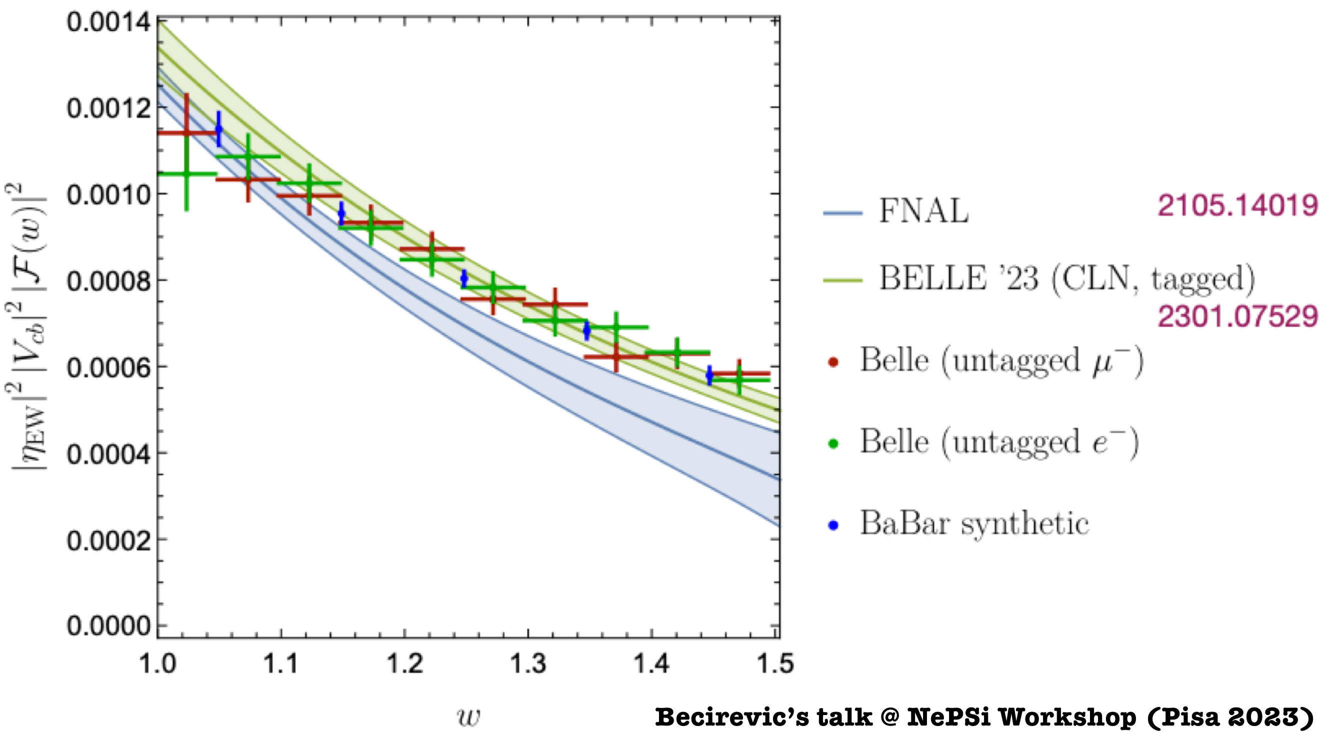
One performs a Bayesian modified BGL fit, in which unitarity is built-in through a prior on the modified BGL coefficients

All the details of the new Bayesian Inference (B.I.) method can be found in:
i) arXiv:2303.11285
ii) arXiv:2303.11280

The importance of new data for semileptonic B \to D* decays

We still do not have a control over hadronic uncertainties with

$$\frac{d\mathcal{B}(B \to D^* \ell \bar{\nu})}{dq^2} = \frac{1}{2 m_B m_{D^*}} \frac{d\mathcal{B}(B \to D^* \ell \bar{\nu})}{dw} \propto |V_{cb}|^2 |\mathcal{F}(w)|^2$$



Becirevic's talk @ NePSi Workshop (Pisa 2023)

It is evident that FNAL/MILC and Belle '23 have practically identical slopes, they differ only for the normalization, *i.e.* Vcb…

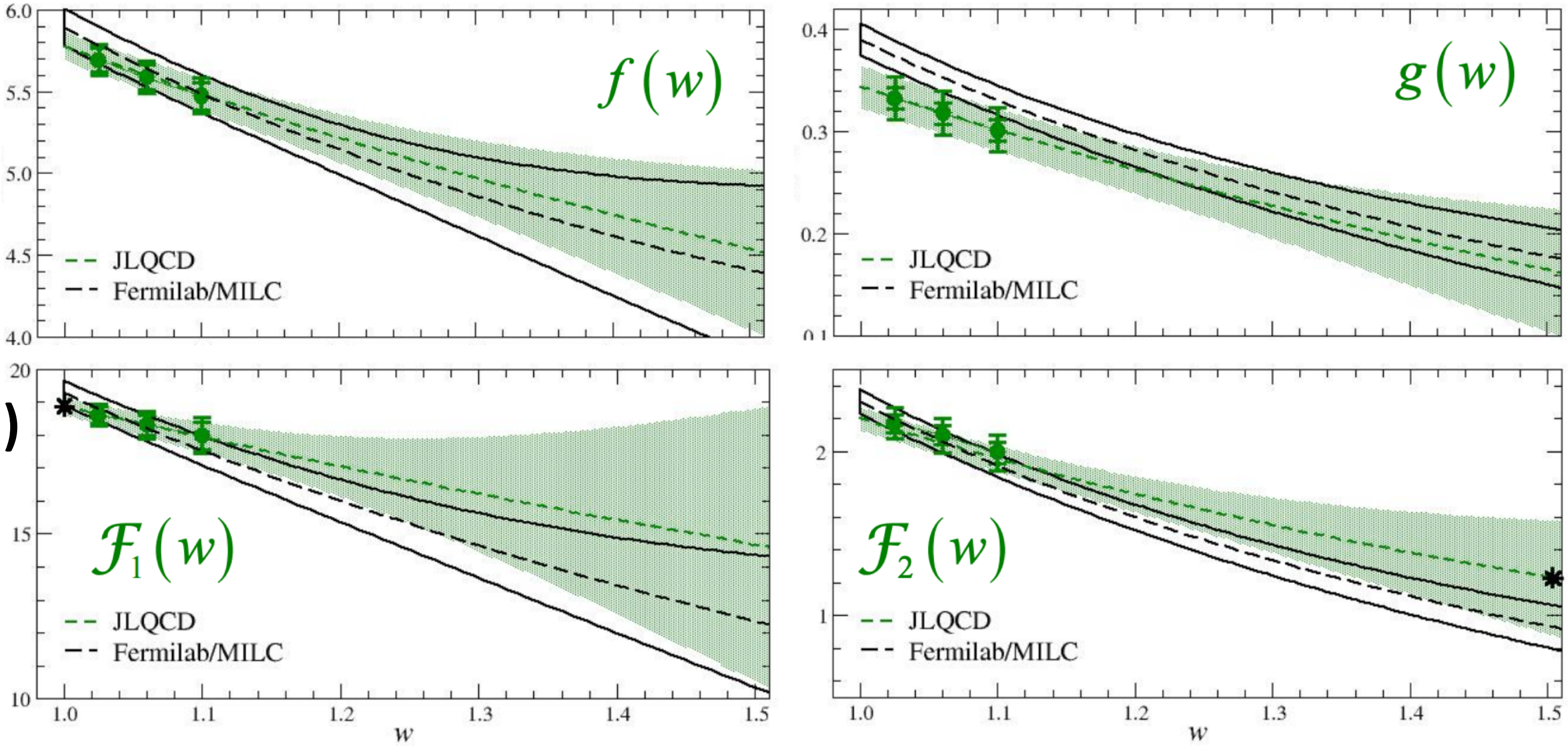
Belle '23 data prefer a higher Vcb, similar to the DM one!
With a BGL parametr.:

$$|V_{cb}| = (40.6 \pm 0.9) \times 10^{-3}$$

Belle Coll., arXiv:2301.04716 hep-ex]

Future perspectives for LQCD data

JLQCD (prelim.)

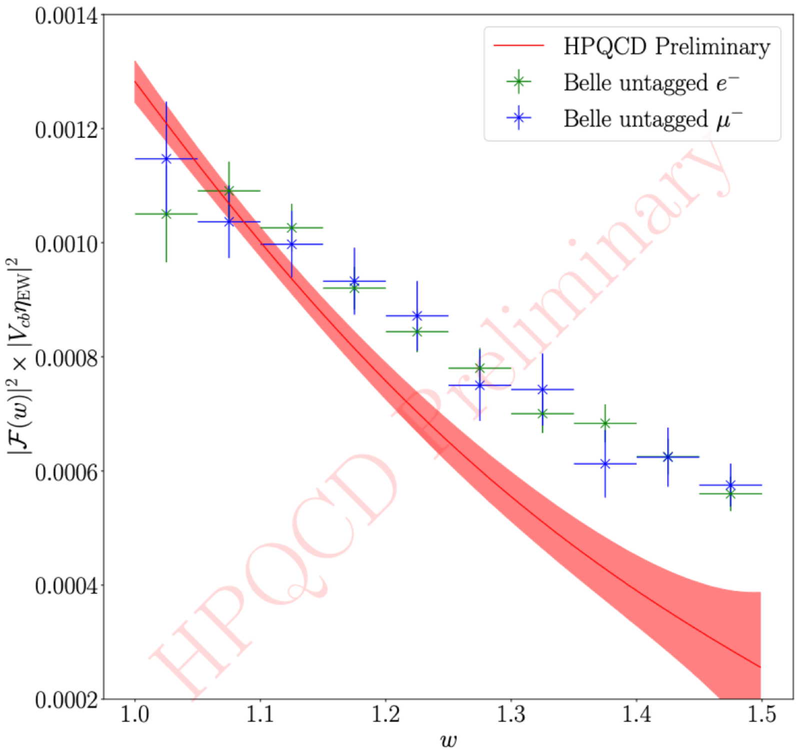


Kaneko's talk @ "Challenges in Semileptonic B decays 2022" Workshop

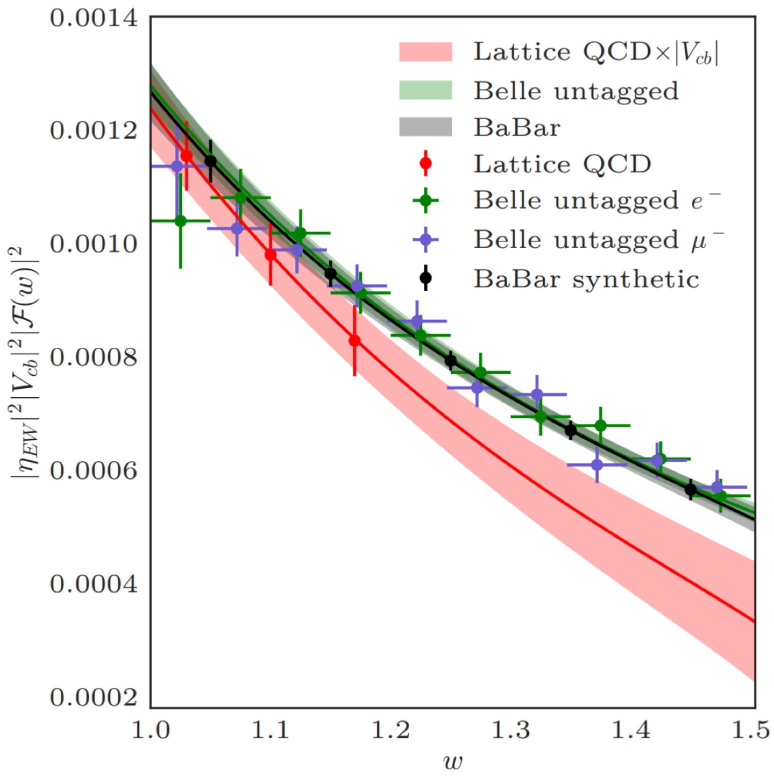
Future perspectives for LQCD data

Harrison's talk @ "Challenges in Semileptonic B decays 2022" Workshop

HPQCD (prelim.)



FNAL/MILC, arXiv:arXiv:2105.14019 [hep-lat]



FNAL/MILC

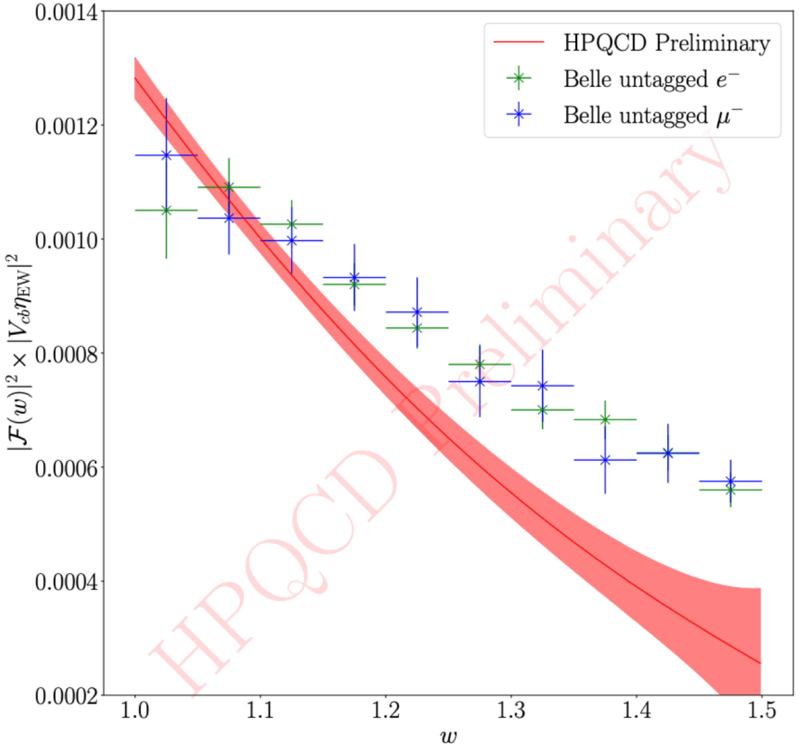
$$\frac{\mathrm{d}\Gamma(\bar{B}\to D^*\ell\,\bar{\nu})}{\mathrm{d}w} = \frac{G_F^2|V_{cb}|^2}{48\pi^3}(m_B-m_{D^*})^2\,m_{D^*}^3\sqrt{w^2-1}\,(w+1)^2$$
$$\times\left[1+\frac{4w}{w+1}\,\frac{m_B^2-2w\,m_Bm_{D^*}+m_{D^*}^2}{(m_B-m_{D^*})^2}\right]|\mathcal{F}(w)|^2$$

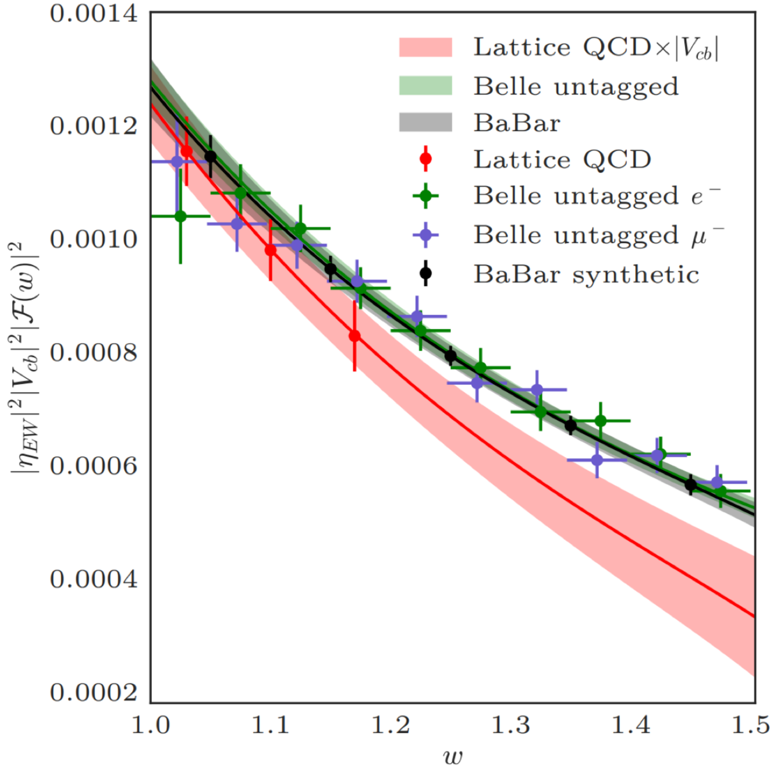
Future perspectives for LQCD data

Harrison's talk @ "Challenges in Semileptonic B decays 2022" Workshop

FNAL/MILC, arXiv:arXiv:2105.14019 [hep-lat]

HPQCD (prelim.)





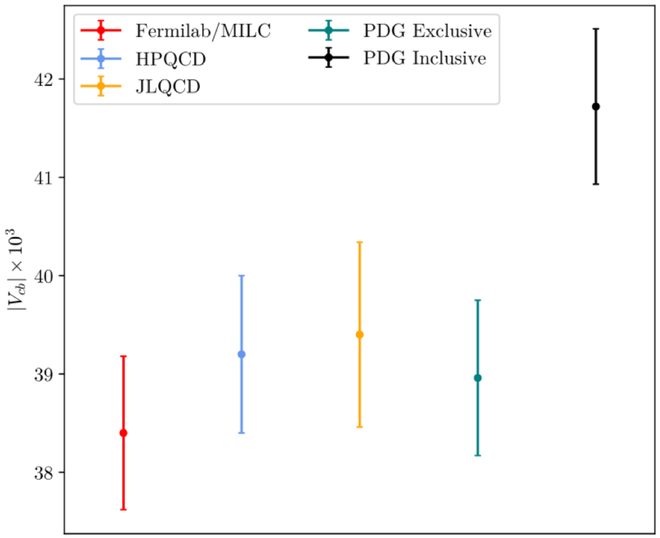
FNAL/MILC

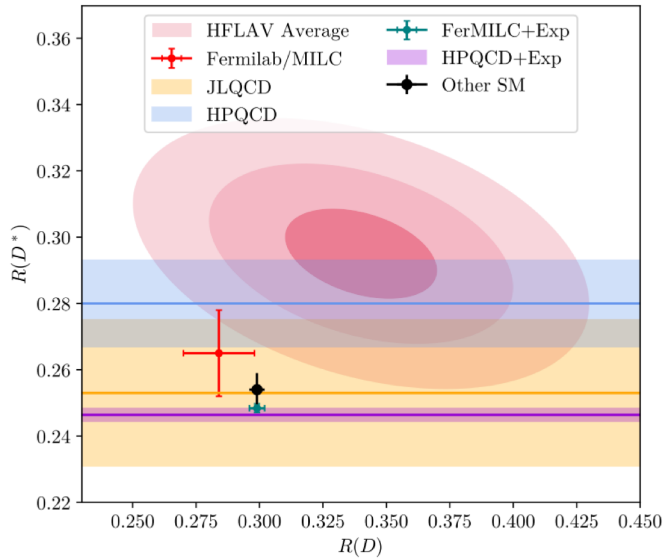
$$\frac{\mathrm{d}\Gamma(\bar{B} \to D^* \ell\, \bar{\nu})}{\mathrm{d}w} = \frac{G_F^2 |V_{cb}|^2}{48\pi^3} (m_B - m_{D^*})^2\, m_{D^*}^3 \sqrt{w^2 - 1}\, (w+1)^2 \times \left[1 + \frac{4w}{w+1} \frac{m_B^2 - 2w\, m_B m_{D^*} + m_{D^*}^2}{(m_B - m_{D^*})^2}\right] |\mathcal{F}(w)|^2$$

CONCLUSION: FNAL/MILC and HPQCD have similar shape, which is different from Belle & different from JLQCD (which is affected by higher uncertainties…)

Future perspectives for LQCD data

Comparison of results





$$|V_{cb}|^{\text{JLQCD}} = 39.40(94) \times 10^{-3}$$

$$|V_{cb}|^{\text{HPQCD}} = 39.2(8) \times 10^{-3}$$

$$|V_{cb}|^{\text{FerMILC}} = 38.17(85) \times 10^{-3}$$

$$|V_{cb}|^{\text{Excl}} = 38.96(79) \times 10^{-3}$$

$$|V_{cb}|^{\text{Incl}} = 41.72(79) \times 10^{-3}$$

$$R(D^*)^{\text{JLQCD}} = 0.253(22)$$

$$R(D^*)^{\text{HPQCD}} = 0.280(13)$$

$$R(D^*)^{\text{FerMILC}} = 0.265(13)$$

$$R(D^*)^{\text{HFLAV}} = 0.295(14)$$

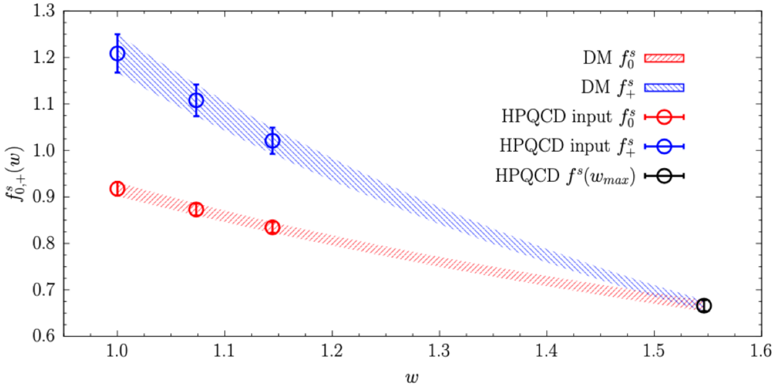
$$R(D^*)^{\text{SM}} = 0.254(5)$$

Vaquero's talk @ First LatticeNET Workshop (Benasque 2022)

Quick parenthesis: semileptonic $B_s \to D_s^{(*)}$ decays

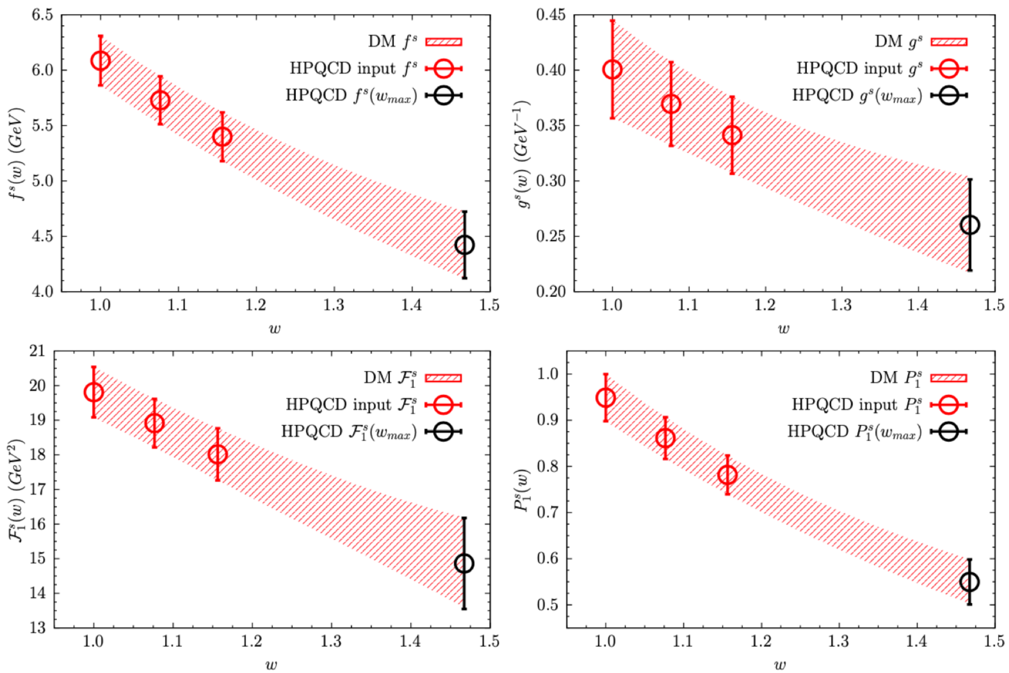
In **PRD '22 [arXiv:2204.05925]**, our DM method has been applied to **semileptonic $B_s \to D_s^{(*)}$ decays**. LQCD form factors taken from the results of the fits preformed by the HPQCD Collaboration in **PRD '20 [arXiv:1906.00701]** ($B_s \to D_s$) and **PRD '22 [arXiv:2105.11433]** ($B_s \to D_s^*$): we extract 3 data points for the FFs at small values of the recoil and apply the DM approach.

$B_s \to D_s \ell \nu_\ell$



* nice agreement in the whole kinematical range

$B_s \to D_s^* \ell \nu_\ell$



Vcb from semileptonic $B_s \to D_s^{(*)}$ decays

* two sets of experimental data from LHCb collaboration: arXiv:2001.03225, 2003.08453, 2103.06810

two different runs at LHC

* first analysis: ratios of branching ratios [2103.06810]

$$\frac{\mathscr{B}(B_s \to D_s \mu \nu_\mu)}{\mathscr{B}(B \to D \mu \nu_\mu)} = 1.09 \pm 0.05_{stat} \pm 0.06_{syst} \pm 0.05_{inputs} = 1.09 \pm 0.09$$

$$\frac{\mathscr{B}(B_s \to D_s^* \mu \nu_\mu)}{\mathscr{B}(B \to D^* \mu \nu_\mu)} = 1.06 \pm 0.05_{stat} \pm 0.07_{syst} \pm 0.05_{inputs} = 1.06 \pm 0.10$$

\- using the PDG values for $\mathscr{B}(B \to D^{(*)} \mu \nu_\mu)$ and the B_s-meson lifetime one gets

$$\Gamma^{\text{LHCb}}(B_s \to D_s \mu \nu_\mu) = (1.04 \pm 0.10) \cdot 10^{-14} \text{ GeV}$$
$$\Gamma^{\text{LHCb}}(B_s \to D_s^* \mu \nu_\mu) = (2.26 \pm 0.24) \cdot 10^{-14} \text{ GeV}$$

to be compared with

$$\Gamma^{\text{DM}}(B_s \to D_s \mu \nu_\mu)/|V_{cb}|^2 = (6.04 \pm 0.23) \cdot 10^{-12} \text{ GeV}$$
$$\Gamma^{\text{DM}}(B_s \to D_s^* \mu \nu_\mu)/|V_{cb}|^2 = (1.39 \pm 0.11) \cdot 10^{-11} \text{ GeV}$$

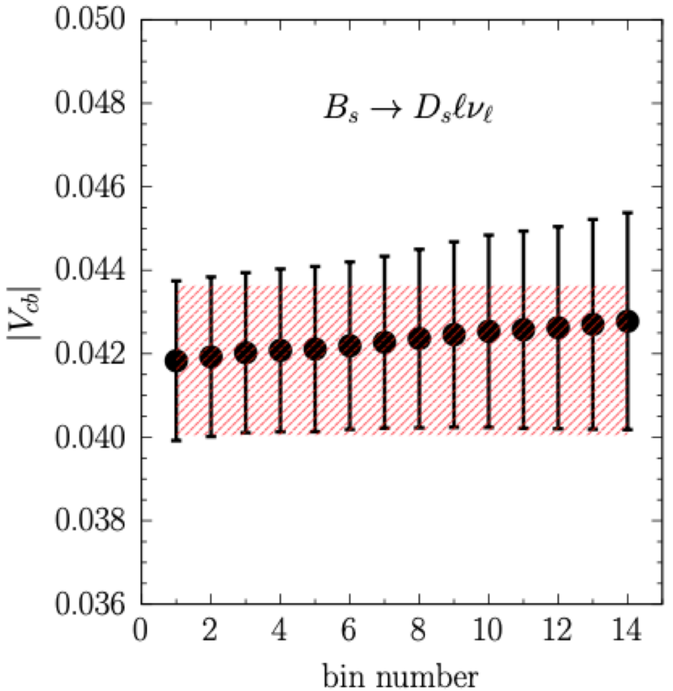
decays	$\lvert V_{cb}\rvert^{\text{DM}} \cdot 10^3$
$B_s \to D_s \ell \nu_\ell$	41.5 ± 2.1
$B_s \to D_s^* \ell \nu_\ell$	40.3 ± 2.7

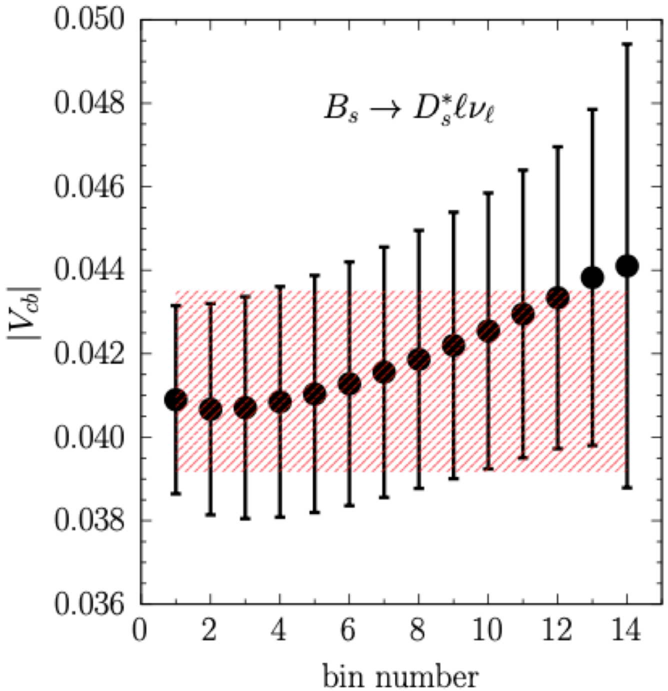
Many thanks to Silvano Simula!

Vcb from semileptonic $B_s \to D_s^{(*)}$ decays

* second analysis: differential decay rates reconstructed from the LHCb fits of $p_\perp$ distributions (BGL/CLN parameterizations for the FFs) carried out in arXiv:2001.03225 (see also arXiv:2103.06810)

bin-per-bin analysis: $|V_{cb}|_j \equiv \sqrt{\frac{d\Gamma^{\rm LHCb}/dw_j}{d\Gamma^{\rm DM}/dw_j}}$ $\quad j = 1, \ldots, N_{bins}$ $\quad$ we adopted $N_{bins} = 14$ w-bins





correlated weighted averages

$$|V_{cb}| = \frac{\sum_{i,j=1}^{N_{\rm bins}} (\mathbf{C}^{-1})_{ij} |V_{cb}|_j}{\sum_{i,j=1}^{N_{\rm bins}} (\mathbf{C}^{-1})_{ij}}$$

$$\sigma^2_{|V_{cb}|} = \frac{1}{\sum_{i,j=1}^{N_{\rm bins}} (\mathbf{C}^{-1})_{ij}}$$

decays	$\|V_{cb}\|^{\rm DM} \cdot 10^3$
$B_s \to D_s \ell \nu_\ell$	41.8 ± 1.8
$B_s \to D_s^* \ell \nu_\ell$	41.3 ± 2.2

$|V_{cb}|^{\rm LHCb} \cdot 10^3 = 41.7 \pm 1.6$

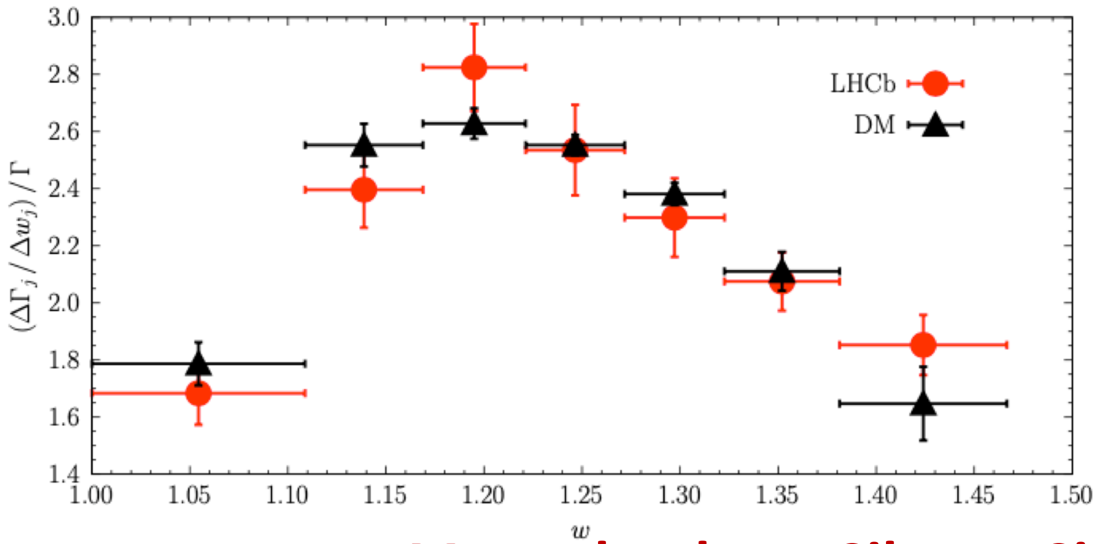
Many thanks to Silvano Simula!

Vcb from semileptonic $B_s \to D_s^{(*)}$ decays

* third analysis: LHCb ratios from arXiv:2003.08453

$$\Delta r_j = \frac{\Delta\Gamma_j(B_s \to D_s^* \mu\nu_\mu)}{\Gamma(B_s \to D_s^* \mu\nu_\mu)} \qquad j = 1,\ldots,7$$

j	1	2	3	4	5	6	7
w-bin	1.000 - 1.1087	1.1087 - 1.1688	1.1688 - 1.2212	1.2212 - 1.2717	1.2717 - 1.3226	1.3226 - 1.3814	1.3814 - 1.4667
Δw_j	0.1087	0.0601	0.0524	0.0505	0.0509	0.0588	0.0853
$\Delta r_j^{\mathrm{LHCb}}$	0.183(12)	0.144(8)	0.148(8)	0.128(8)	0.117(7)	0.122(6)	0.158(9)
Δr_j^{DM}	0.1942(82)	0.1534(45)	0.1377(28)	0.1289(18)	0.1212(20)	0.1241(40)	0.1405(110)



consistency within $\sim 1\sigma$

⇩

shape of theoretical FFs is consistent with the one of the experimental data

Many thanks to Silvano Simula!

Vcb from semileptonic $B_s \to D_s^{(*)}$ decays

* to determine $|V_{cb}|$ we evaluate the integrated differential decay rates for each bin

$$\Delta\Gamma_j^{exp} = \Delta r_j^{LHCb} \cdot \Gamma^{LHCb}(B_s \to D_s^* \mu \nu_\mu) \qquad j = 1,\ldots,7$$

and the covariance matrix: $\Gamma_{ij}^{exp} = R_{ij}^{LHCb}\left[\overline{\Gamma}^2 + \sigma_{\overline{\Gamma}}^2\right] + \Delta r_i^{LHCb} \Delta r_j^{LHCb} \sigma_{\overline{\Gamma}}^2$

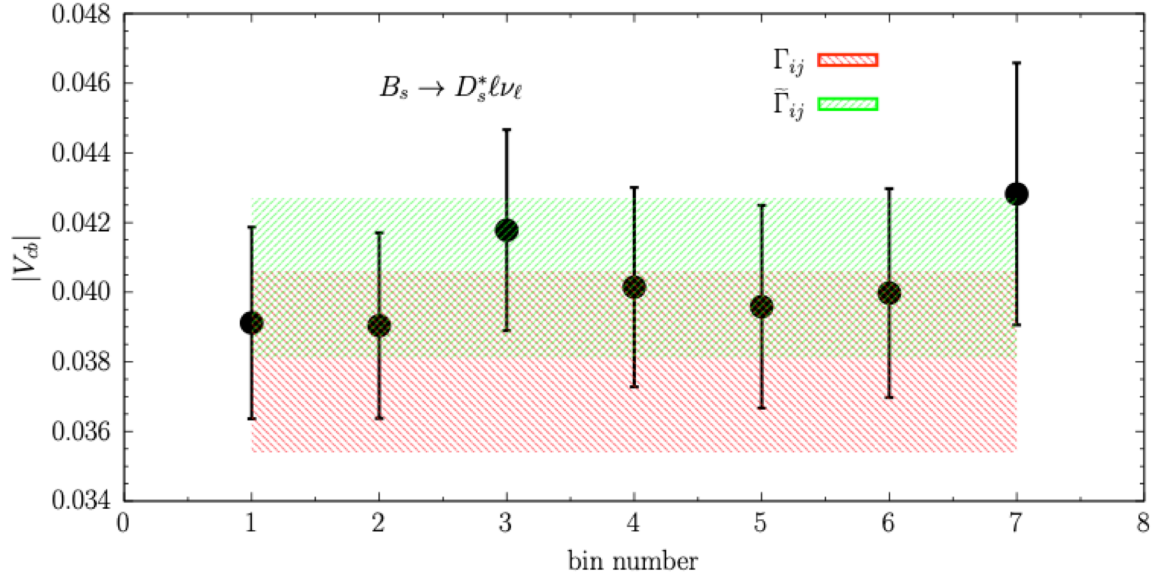
general property: $\sum_{i,j=1}^{N_{bins}} \Gamma_{ij}^{exp} = \sigma_{\overline{\Gamma}}^2 \iff \sum_{i=1}^{N_{bins}} \Delta r_i^{LHCb} = 1$ and $\sum_{i,j=1}^{N_{bins}} R_{ij}^{LHCb} = 0$

$\Gamma^{LHCb}(B_s \to D_s^* \mu \nu_\mu)$ from arXiv:2103.06810

$$\overline{\Gamma} \pm \sigma_{\overline{\Gamma}} = (2.26 \pm 0.24) \cdot 10^{-14}\ \text{GeV}$$

*** uncorrelated with Δr_j^{LHCb} ***

- D'Agostini effect (NIMA '94): negative bias on constant fits to data affected by an overall normalization uncertainty; it depends upon $\sigma_{\overline{\Gamma}}$ and $\Delta r_i^{LHCb} \neq \Delta r_j^{LHCb}$



<u>modified covariance matrix</u>

$$\widetilde{\Gamma}_{ij}^{exp} = R_{ij}^{LHCb}\left[\overline{\Gamma}^2 + \sigma_{\overline{\Gamma}}^2\right] + \sigma_{\overline{\Gamma}}^2 / N_{bins}^2$$

$$\sum_{i,j=1}^{N_{bins}} \widetilde{\Gamma}_{ij}^{exp} = \sum_{i,j=1}^{N_{bins}} \Gamma_{ij}^{exp} = \sigma_{\overline{\Gamma}}^2$$

correlated weighted averages

$|V_{cb}| \cdot 10^3 = 38.0 \pm 2.6$

$|V_{cb}| \cdot 10^3 = 40.4 \pm 2.3$

Many thanks to Silvano Simula!

Quick parenthesis: semileptonic $B_s \to D_s^{(*)}$ decays

Without entering in the details of this analysis, phenomenological applications give the results

$$|V_{cb}|^{\mathrm{DM}} \cdot 10^3 = 41.7 \pm 1.9 \qquad \text{from } B_s \to D_s \ell \nu_\ell \text{ decays}$$

$$= 40.7 \pm 2.4 \qquad \text{from } B_s \to D_s^* \ell \nu_\ell \text{ decays}$$

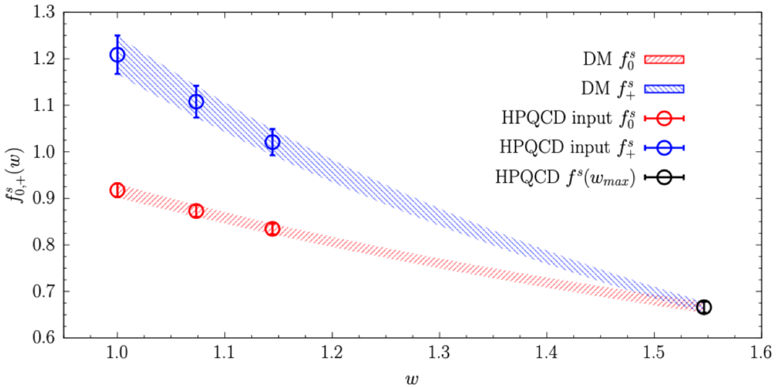
through available exps. data by LHCb Collaboration
(PRD '20 [2001.03225], JHEP '20 [2003.08453])

$$R(D_s) = 0.298\ (5)$$

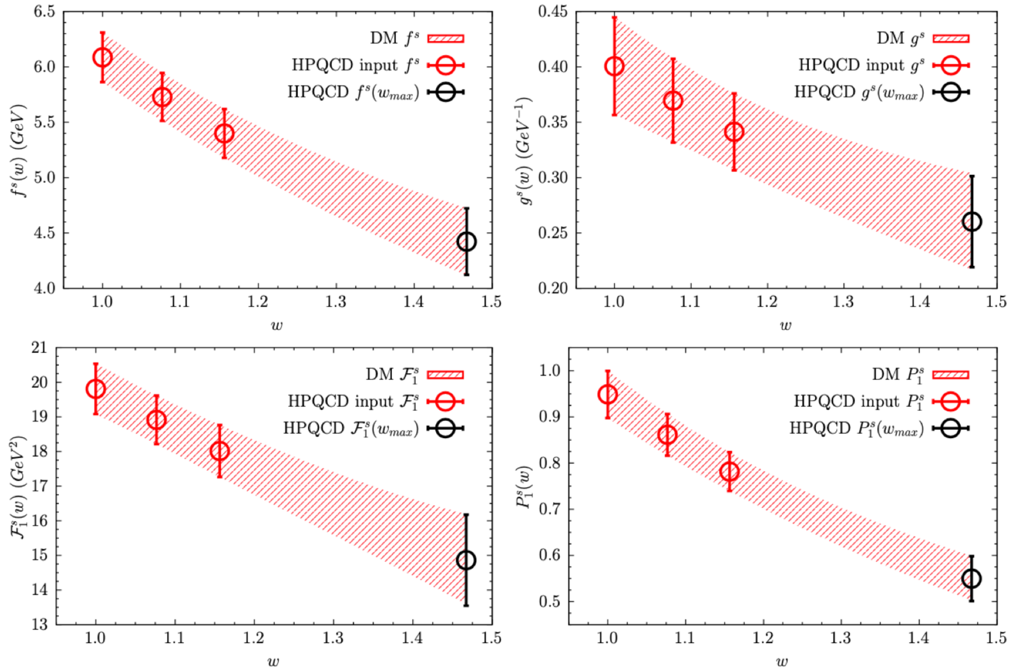
$$R(D_s^*) = 0.250\ (6)$$

fully-theoretical

$B_s \to D_s \ell \nu_\ell$



$B_s \to D_s^* \ell \nu_\ell$



* nice agreement in the whole kinematical range

Results of the last UTfit analysis within the SM

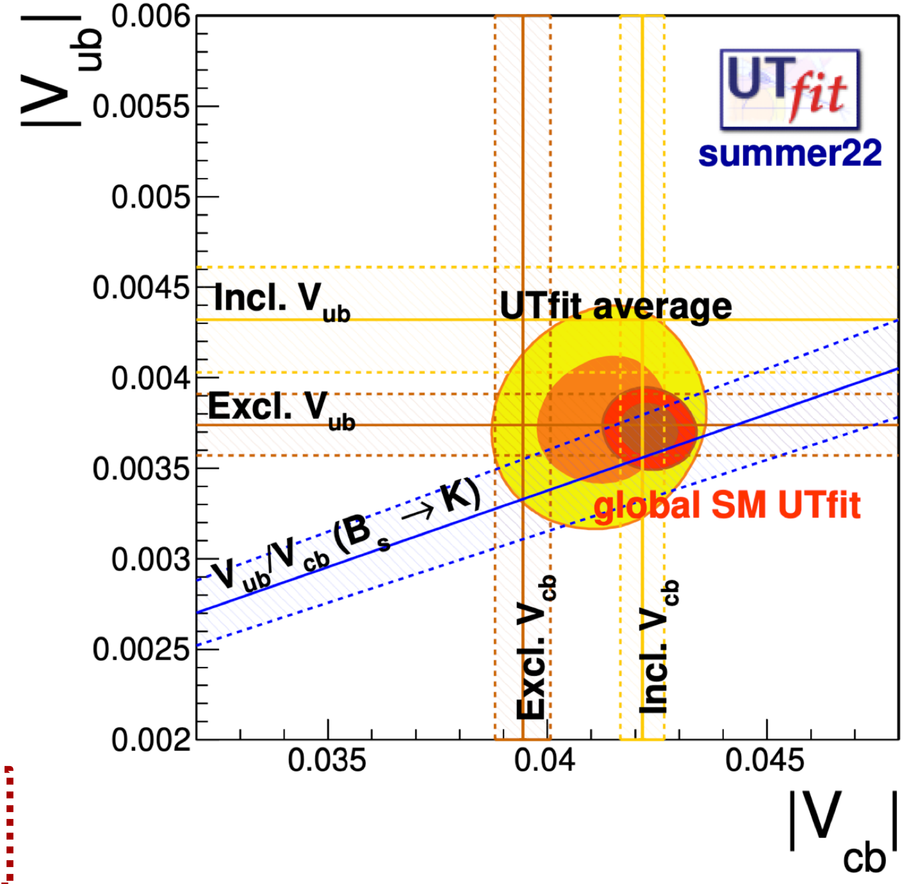
UTfit

www.utfit.org

RELOADED

M.Bona, M. Ciuchini, D. Derkach, F. Ferrari, E. Franco, V. Lubicz, G. Martinelli, D. Morgante, M. Pierini, L. Silvestrini, S. Simula, A. Stocchi, C. Tarantino, V. Vagnoni, M. Valli and L. Vittorio

NB: the UTA favours a large value of |Vcb| (close to the inclusive value) and a small value of |Vub| (close to the exclusive one)



UTfit Collaboration, arXiv:2212.03894