# $b \to c$ semileptonic decays at non-zero recoil

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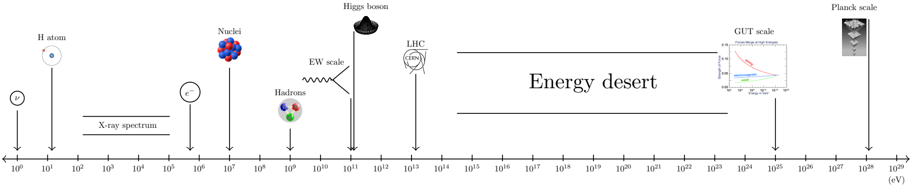
AGENCIA ESTATAL DE INVESTIGACIÓN

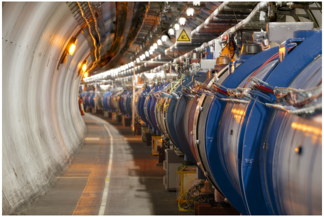
# Motivation: Searches for new physics

- The Standard Model (SM) describes phenomena in a wide range of scales
- Yet, we expect it to fail at some point
    - Hierarchy problem, too many parameters, absence of gravity, dark matter/energy, neutrino mixing...
    - SM regarded as an Effective Field Theory (EFT)
- New physics searches more important than ever



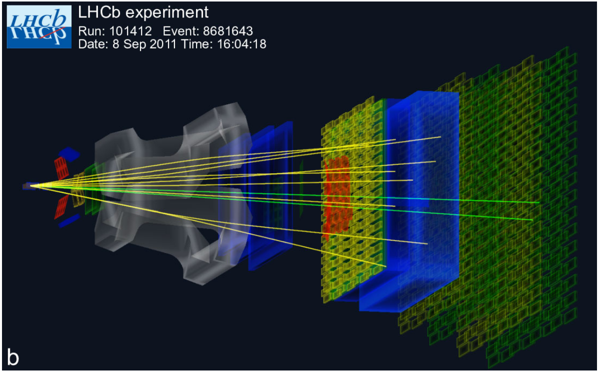
# Motivation: Searches for new physics

Energy frontier



Cosmology frontier

Intensity frontier



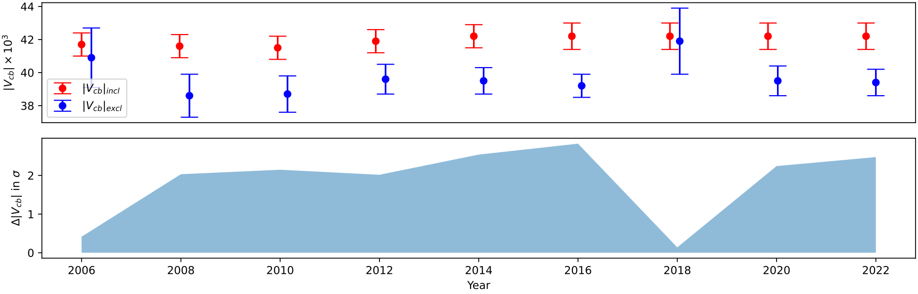

- High expectations with the LHC
- Intensity frontier becoming increasingly important

# Motivation: New physics in the flavor sector of the SM

## The CKM matrix

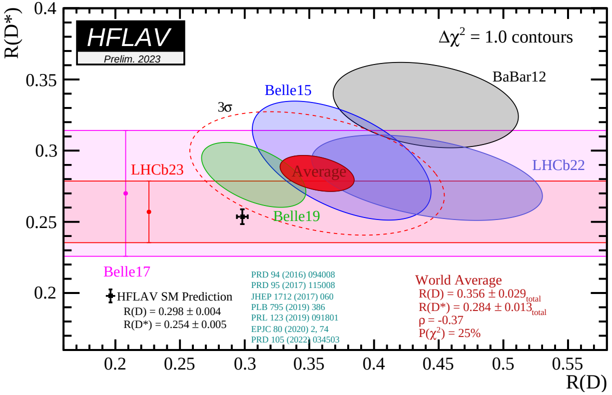
$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & \boldsymbol{V_{cb}} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

- Matrix must be unitary (preserve the norm)
- Tensions have been there for a long time
- Evolution of the tensions according to PDG



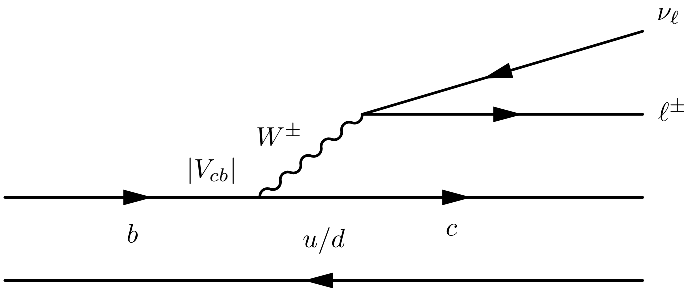
# Motivation: Tensions in lepton universality ratios

$$R\left(D^{(*)}\right) = \frac{\mathcal{B}\left(B \to D^{(*)}\tau\nu_\tau\right)}{\mathcal{B}\left(B \to D^{(*)}\ell\nu_\ell\right)}$$




- Current $\approx 3.3\sigma$ tension with the SM (HFLAV)

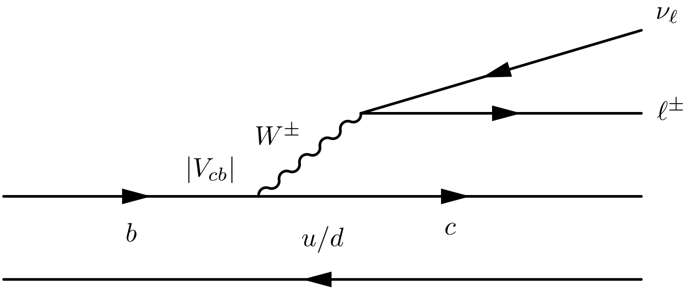
# Semileptonic $B$ decays on the lattice: Exclusive $|V_{cb}|$




$$\underbrace{\frac{d\Gamma}{dw}\left(\bar{B} \to D^{(*)}\ell\bar{\nu}_\ell\right)}_{\text{Experiment}} = \underbrace{K_{D^{(*)}}(w, m_\ell)}_{\text{Known factors}} \underbrace{|F(w)|^2}_{\text{Theory}} \times |V_{cb}|^2 , \qquad w = v_{D^{(*)}} \cdot v_B$$

- The amplitude $F = \mathcal{F}, \mathcal{G}$ must be calculated in LQCD
  - Data more precise at $w$ close to 1
- $K_{D^{(*)}}(w, m_\ell) \propto (w^2 - 1)^{\frac{n}{2}}, n = 1, 3$ require extrapolation of experimental data

# Semileptonic $B$ decays on the lattice: Universality ratios




$$R(D^{(*)}) = \frac{\int_1^{w_{\mathrm{Max},\tau}} dw\, K_{D^{(*)}}(w, m_\tau)\, |F(w)|^2 \times \cancel{|V_{cb}|^2}}{\int_1^{w_{\mathrm{Max}}} dw\, K_{D^{(*)}}(w, 0)\, |F(w)|^2 \times \cancel{|V_{cb}|^2}}$$

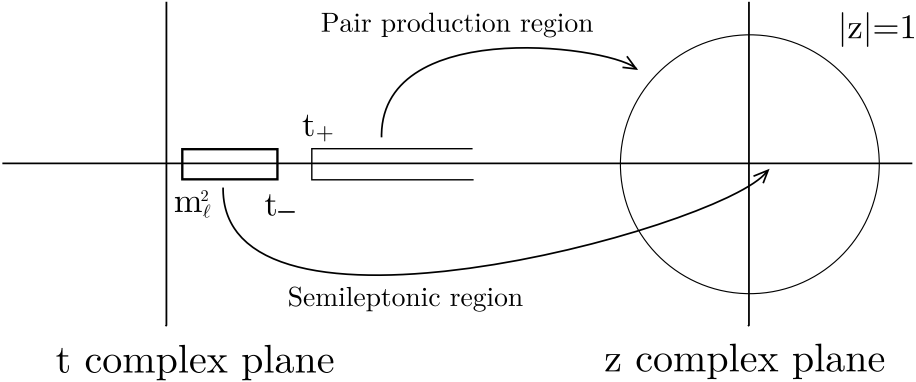
- The universality ratio depends only on the form factors
- It is possible to extract $R(D^{(*)})$ without experimental data!

# Semileptonic $B$ decays on the lattice: Parametrizations

Most parametrizations perform an expansion in the $z$ parameter

$$\frac{1+z}{1-z} = \sqrt{\frac{t_+ - t}{t_+ - t_-}}, \qquad z = \frac{\sqrt{w+1} - \sqrt{2N}}{\sqrt{w+1} + \sqrt{2N}}$$

with $t_\pm = (m_B \pm m_{D^*})^2\,, \quad t = (p_B - p_{D^*})^2\,, \quad w = v_B \cdot v_{D^*}$



# Semileptonic $B$ decays on the lattice: Parametrizations

- Boyd-Grinstein-Lebed (BGL) — *Phys. Rev. Lett. 74 (1995) 4603-4606*; *Phys.Rev. D56 (1997) 6895-6911*; *Nucl.Phys. B461 (1996) 493-511*

$$f_X(w) = \frac{1}{B_{f_X}(z)\phi_{f_X}(z)} \sum_{n=0}^{\infty} a_n z^n$$

  - $B_{f_X}$ Blaschke factors, includes contributions from the poles
  - $\phi_{f_X}$ is called *outer function* and must be computed for each form factor
  - Weak unitarity constraints $\sum_n |a_n|^2 \leq 1$

- Caprini-Lellouch-Neubert (CLN) — *Nucl. Phys. B530 (1998) 153-181*

$$F(w) \propto 1 - \rho^2 z + cz^2 - dz^3, \quad \text{with } c = f_c(\rho),\, d = f_d(\rho)$$

  - Relies strongly on HQET, spin symmetry and (old) inputs
  - Tightly constrains $F(w)$: four independent parameters, one relevant at $w = 1$

- Current consensus: abandon CLN
  - Spiritual sucessors of CLN — Bernlochner *et al. Phys.Rev.D 95 (2017) 115008, Phys.Rev.D 97 (2018) 059902*

Bordone, Gubernari, Jung, Straub, Van Dyk... *Eur.Phys.J.C 80 (2020) 74, Eur.Phys.J.C 80 (2020) 347, JHEP 01 (2019) 009*

# Semileptonic $B$ decays on the lattice: Parametrizations

- Dispersive approach

Bourrely *et al.* *Nucl.Phys.B 189 (1981) 157*, Lellouch *Nucl.Phys.B 479 (1996) 353*

Di Carlo *et al.* *Phys.Rev.D 104 (2021) 054502*

- Express unitarity bounds as a norm, define an inner product

$$\langle \phi f | \phi f \rangle = \frac{1}{2\pi i} \int_{|z|=1} \frac{dz}{z} \left| \phi(z, q_0^2) f(z) \right|^2 \leq \chi(q_0^2), \quad \langle g | h \rangle \equiv \frac{1}{2\pi i} \int_{|z|=1} \frac{dz}{z} \bar{g}(z) h(z)$$

- Use Cauchy integral theorem to test unitarity in synthetic data at $z = z_{t_1}, z_{t_2} \ldots$

$$g_t(z) \equiv \frac{1}{1 - \bar{z}_t z}$$

$$\langle g_t | \phi f \rangle = \phi(z_t, q_0^2) f(z_t)$$

$$\det\mathcal{M} = \begin{vmatrix} \langle \phi f | \phi f \rangle & \langle \phi f | g_{t_1} \rangle & \langle \phi f | g_{t_2} \rangle & \cdots \\ \langle g_{t_1} | \phi f \rangle & \langle g_{t_1} | g_{t_1} \rangle & \langle g_{t_1} | g_{t_2} \rangle & \cdots \\ \langle g_{t_2} | \phi f \rangle & \langle g_{t_2} | g_{t_1} \rangle & \langle g_{t_2} | g_{t_2} \rangle & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{vmatrix} \geq 0$$

**Matrix $\mathcal{M}$ positive semidefinite**

**L. Vittorio next talk**

# Semileptonic $B$ decays on the lattice: Heavy quarks

- Heavy quark treatment in Lattice QCD
  - For light quarks ($m_l \lesssim \Lambda_{QCD}$), leading discretization errors $\sim \alpha_s^k (a\Lambda_{QCD})^n$
  - For heavy quarks ($m_Q > \Lambda_{QCD}$), discretization errors grow as $\sim \alpha_s^k (am_Q)^n$
- Need special actions to describe the bottom quark, difficult renormalization
  - Relativistic HQ actions (f.i. FermiLab)
  - Non-Relativistic QCD (NRQCD)
- If the action is improved enough, one can treat the bottom as a light quark
  - Highly improved action AND small lattice spacing
  - Use unphysical values for $m_b$ and extrapolate

The discretization errors needn't disappear **as long as we keep them under control**

# Semileptonic $B$ decays on the lattice: Formalism

- $P \to P$ Form factors

$$\frac{\langle D(p_D)|\,\mathcal{V}^\mu\,|\bar{B}(p_B)\rangle}{2\sqrt{m_B\,m_{D^*}}} = \boldsymbol{h_+}(w)\,(v_B^\mu + v_D^\mu) + \boldsymbol{h_-}(w)\,(v_B^\mu - v_D^\mu)$$

- $P \to V$ Form factors

$$\frac{\langle D^*(p_{D^*},\epsilon^\nu)|\,\mathcal{V}^\mu\,|\bar{B}(p_B)\rangle}{2\sqrt{m_B\,m_{D^*}}} = \frac{1}{2}\epsilon^{\nu*}\varepsilon^{\mu\nu}_{\ \ \rho\sigma}v_B^\rho v_{D^*}^\sigma \boldsymbol{h_V}(w)$$

$$\frac{\langle D^*(p_{D^*},\epsilon^\nu)|\,\mathcal{A}^\mu\,|\bar{B}(p_B)\rangle}{2\sqrt{m_B\,m_{D^*}}} =$$

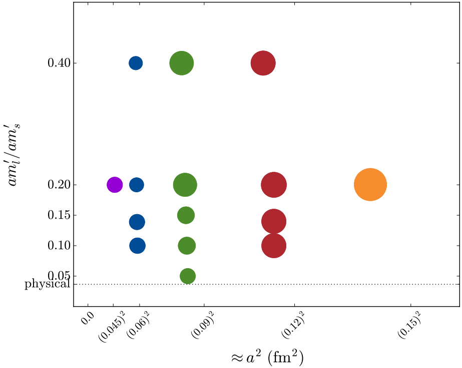
$$\frac{i}{2}\epsilon^{\nu*}\left[g^{\mu\nu}\,(1+w)\,\boldsymbol{h_{A_1}}(w) - v_B^\nu\,(v_B^\mu \boldsymbol{h_{A_2}}(w) + v_{D^*}^\mu \boldsymbol{h_{A_3}}(w))\right]$$

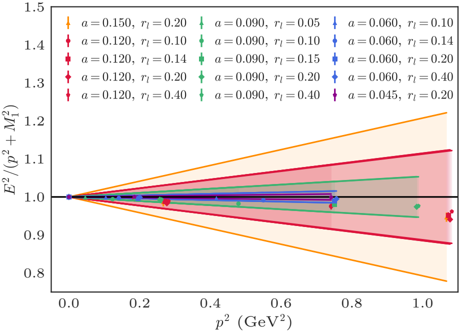
- $\mathcal{V}$ and $\mathcal{A}$ are the vector/axial currents in the continuum
- The $h_X$ enter in the definition of the decay amplitudes
- We can calculate $h_X$ directly from the lattice

# Semileptonic $B$ decays on the lattice: Fermilab/MILC

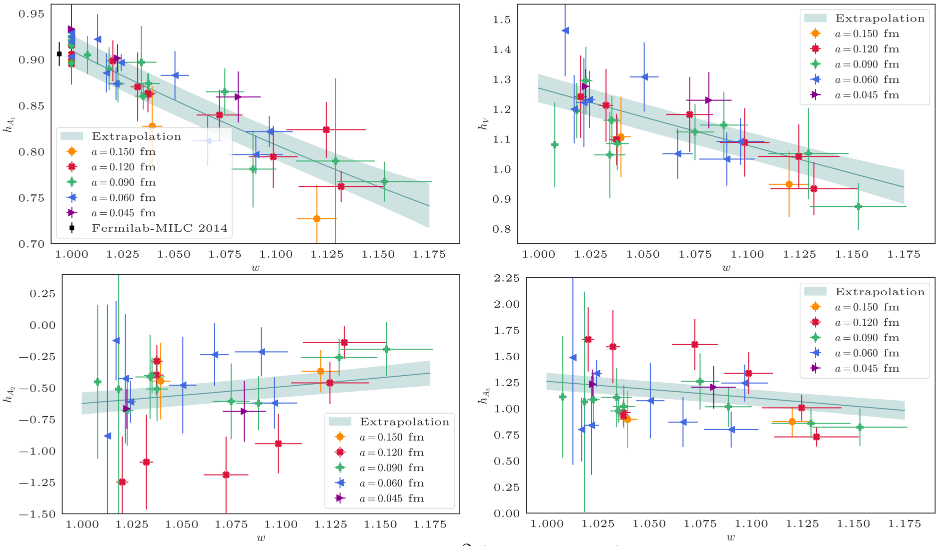
- Using 15 $N_f = 2 + 1$ MILC ensembles of sea asqtad quarks
- The heavy quarks are treated using the Fermilab action
- Lightest $m_\pi \approx 180$ MeV

Eur. Phys. J. C 82 (2022) 1141

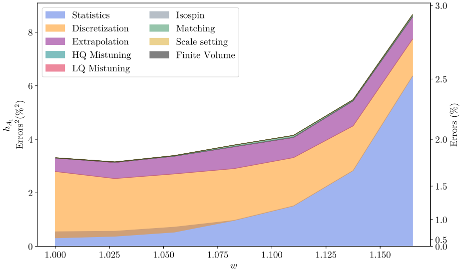


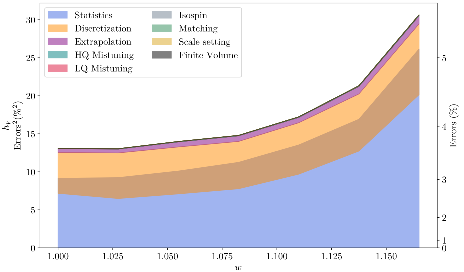



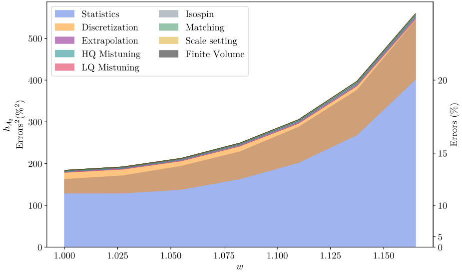
# Semileptonic $B$ decays on the lattice: Fermilab/MILC

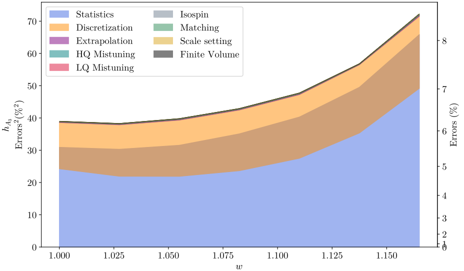



Combined fit $\chi^2/\text{dof} = 85.2/95$



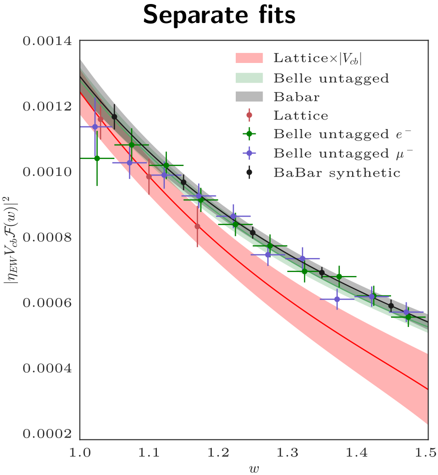



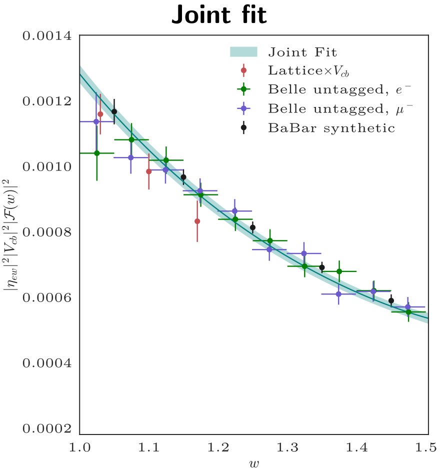




Largest systematic errors come from discretization

# Semileptonic $B$ decays on the lattice: Fermilab/MILC



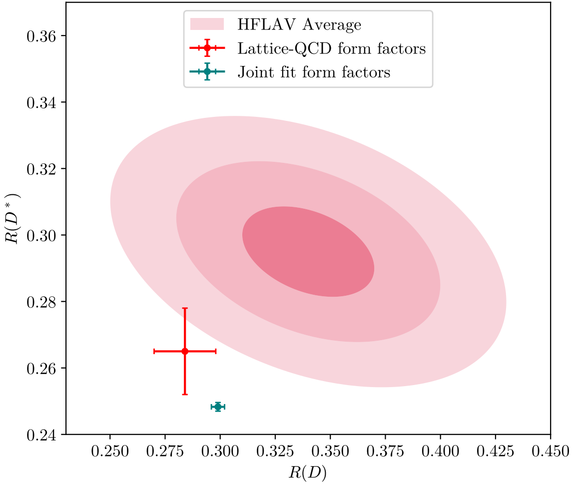

| Fit | Lattice | Exp | Lat + Belle | Lat + BaBar | Lat + Exp |
|---|---|---|---|---|---|
| $\chi^2/\mathrm{dof}$ | $0.63/1$ | $104/76$ | $111/79$ | $8.50/4$ | $126/84$ |

**Unblinded, final result** $|V_{cb}| = 38.40(78) \times 10^{-3}$

# Semileptonic $B$ decays on the lattice: Fermilab/MILC

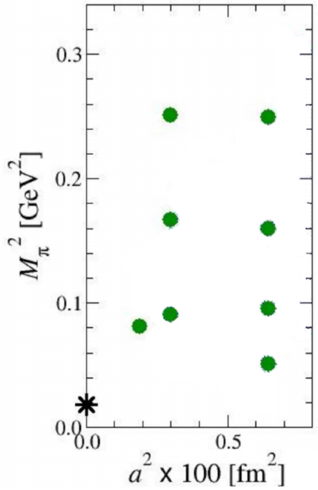
$$R(D^*)_{\mathbf{Lat}} = 0.265(13) \quad R(D^*)_{\mathbf{Lat+Exp}} = 0.2483(13)$$

Phys.Rev.D**92** (2015), 034506; Phys.Rev.D**100** (2019), 052007; Phys.Rev.D**103** (2021), 079901; Phys.Rev.Lett. **123** (2019), 091801

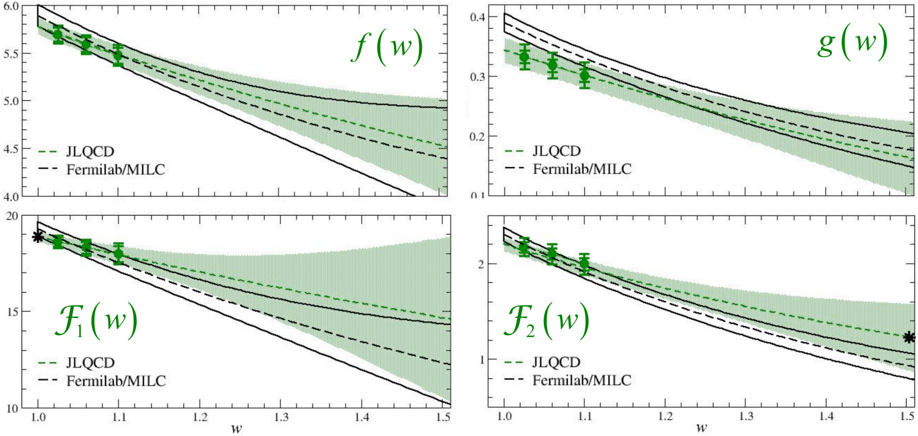


# Semileptonic $B$ decays on the lattice: JLQCD

- Using 8 $N_f = 2 + 1$ ensembles of sea DW quarks
- The heavy quarks use the same DW action
  - Simulations at unphysical $b$ masses
  - Requires extrapolation
  - Easier and more precise renormalization
- $m_\pi$ is as small as $230$ MeV
  - Stable $D^*$

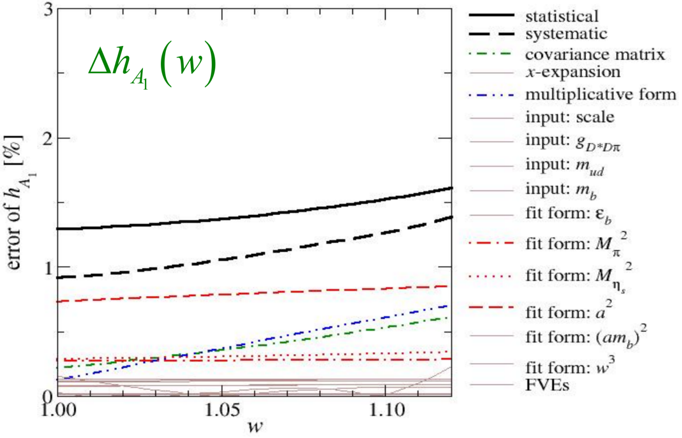


# Semileptonic $B$ decays on the lattice: JLQCD



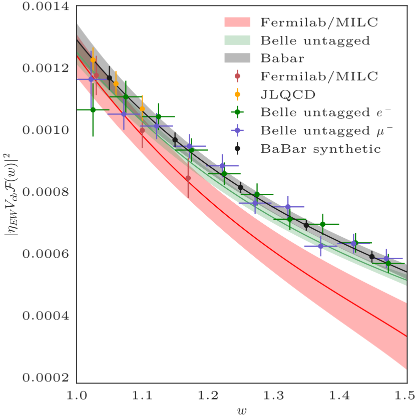

- Milder slope than Fermilab/MILC, but reasonable agreement

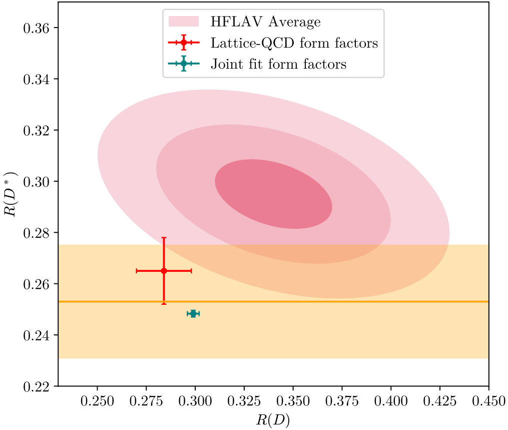
# Semileptonic $B$ decays on the lattice: JLQCD




- Discretization errors dominate the systematic contributions
- Statistical errors are the largest contribution in most ff

# Semileptonic $B$ decays on the lattice: JLQCD




$$|V_{cb}|^{\text{JLQCD}} = 39.85(95) \times 10^{-3}$$

$$|V_{cb}|^{\text{FerMILC}} = 38.60(86) \times 10^{-3}$$

$$R(D^*)^{\text{JLQCD}} = 0.253(22)$$

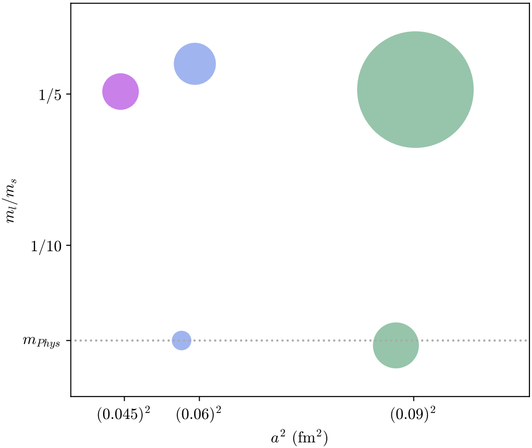
$$R(D^*)^{\text{FerMILC}} = 0.265(13)$$

- Fit to Belle dataset, no Coulomb factor
- Combined fit $\chi^2/\text{dof} = 0.94$

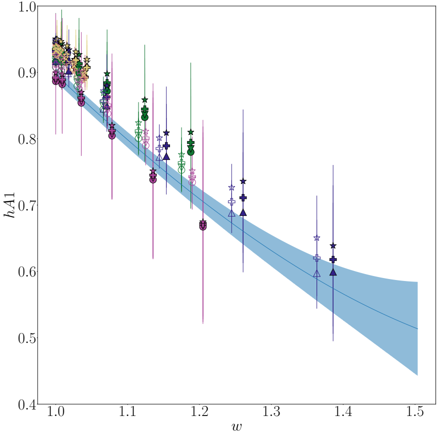
# Semileptonic $B$ decays on the lattice: HPQCD

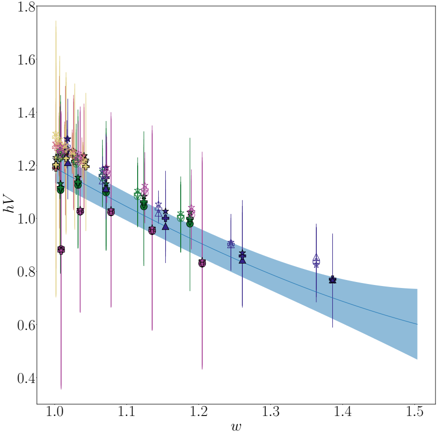
- Using 5 $N_f = 2 + 1 + 1$ MILC ensembles of sea HISQ quarks
- The $b$ quark uses the HISQ action and unphysical masses
- $m_\pi$ ranges from $330$ MeV to $129$ MeV

arXiv:2304.03137



# Semileptonic $B$ decays on the lattice: HPQCD



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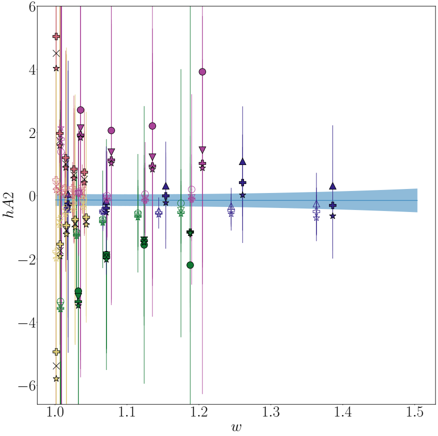


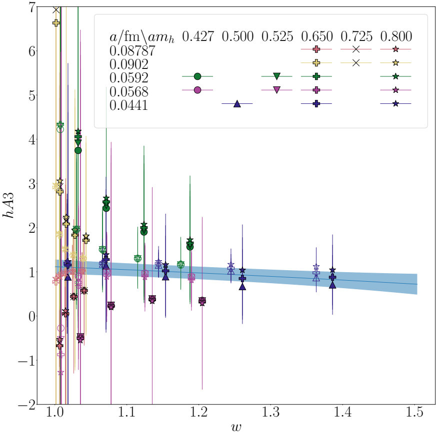

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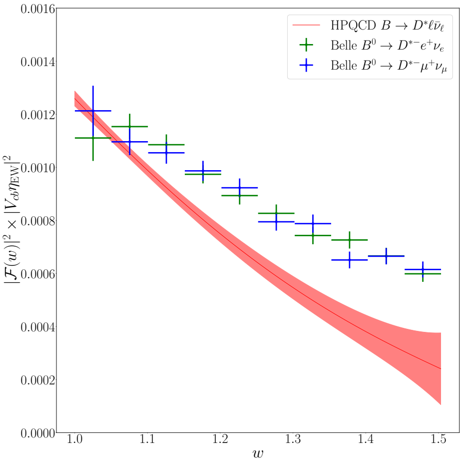
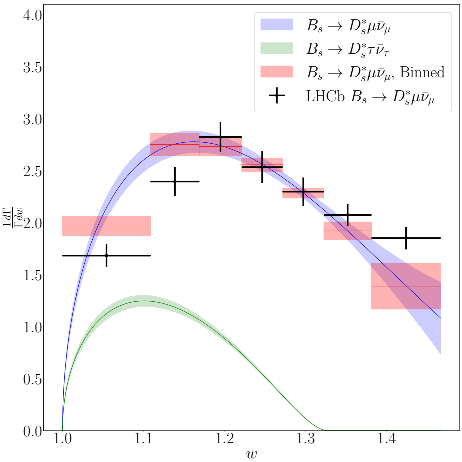
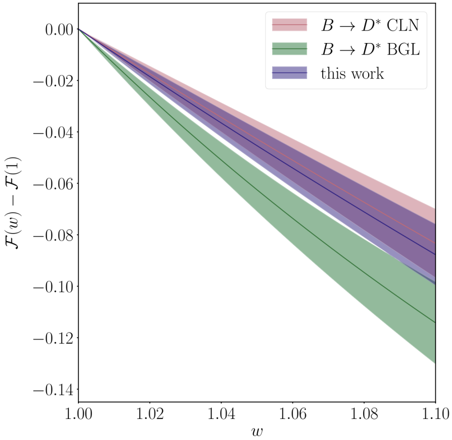


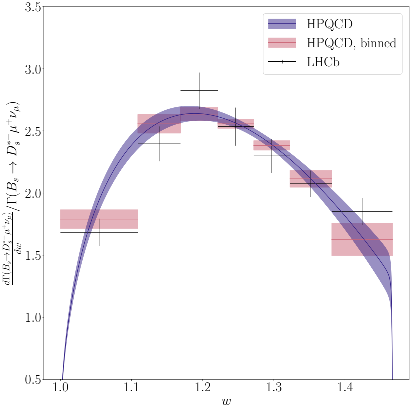




$$\chi^2_{\rm aug}/{\rm dof} = 1.3 \qquad |V_{cb}| = 39.31(74)$$

Belle + HPQCD data combined fit, Coulomb factor included

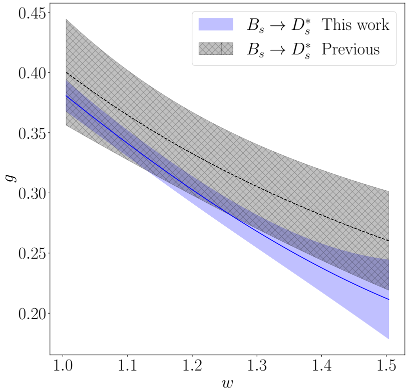
# Semileptonic $B$ decays on the lattice: HPQCD

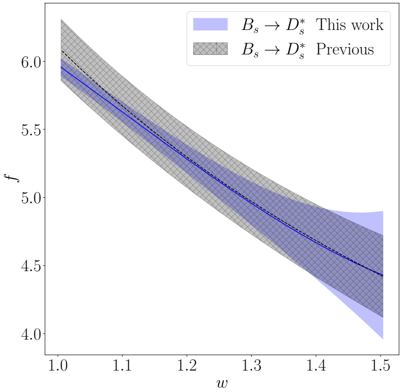


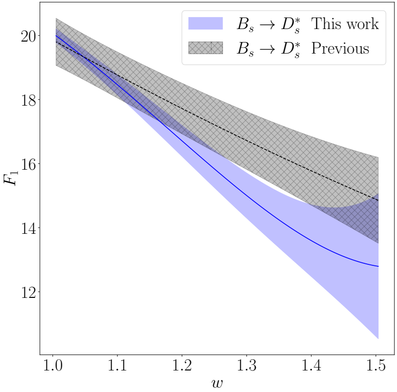

Previous analysis closer to experiment

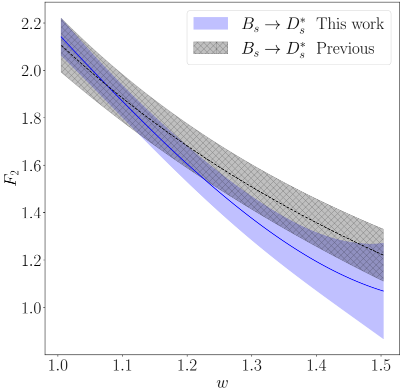
# Semileptonic $B$ decays on the lattice: HPQCD



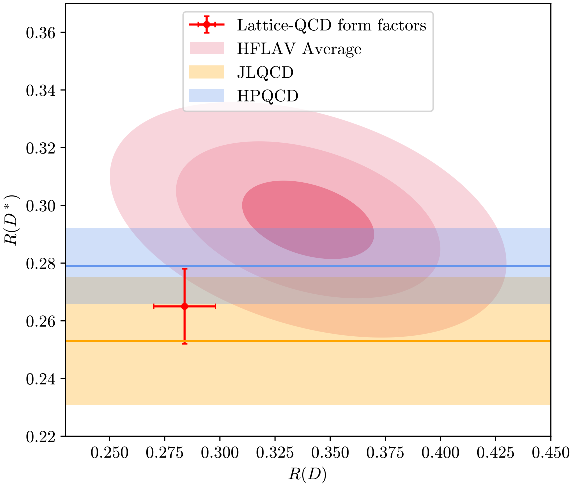
# Semileptonic $B$ decays on the lattice: HPQCD



# Semileptonic $B$ decays on the lattice: HPQCD

$$R(D^*) = 0.279(13) \quad R(D_s^*) = 0.265(9)$$



# Upcoming analyses and summary

- Fermilab lattice and MILC $B_{(s)} \to D_{(s)}^{(*)} \ell\nu$ with HISQ + Fermilab and all HISQ
  - Allows to test systematics associated to heavy-quark discretization
  - The analyses are done at the same time as $B \to \pi/K$ and will offer correlated $V_{ub}$ vs $V_{cb}$ plots

- Exciting times in flavor physics
  - Good progress, both in theoretical and experimental fronts
    - Consolidation of existing channels
    - New channels emerging
- Current results are not conclusive:
  - Lattice results have not converged yet
    - Another iteration is needed ($\sim 5$ years)
  - The inclusive-exclusive tension in $|V_{cb}|$ **remains unsolved**
  - Results show $R(D^*)$ closing the gap with experiment, **but still not conclusive**
- As we reduce our errors, new problems arise
  - Stability of the $D^*$ meson (narrow resonance)
  - QED effects (Coulomb factor and beyond)

- Expect interesting results from the flavor sector in the next years

# THANK YOU

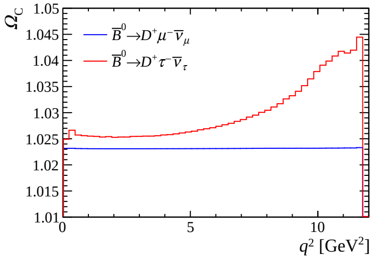
# BACKUP SLIDE

# Semileptonic $B$ decays on the lattice: QED effects

- Most important correction: Coulomb factor $(1+\alpha\pi) = 1.023$

  D. Atwood, W. Marciano, Phys.Rev.D**41** (1990), 1736

  - **Not** included in PHOTOS
  - Applies to decays with a charged $D^{(*)}$
  - Experiments should distinguish between both decays
  - Structure-dependent corrections $\approx (1+\alpha/\pi)$
- Velocity-dependent correction, but $\approx$ constant for light leptons
- Current consensus (Barolo) is to include it as much as possible



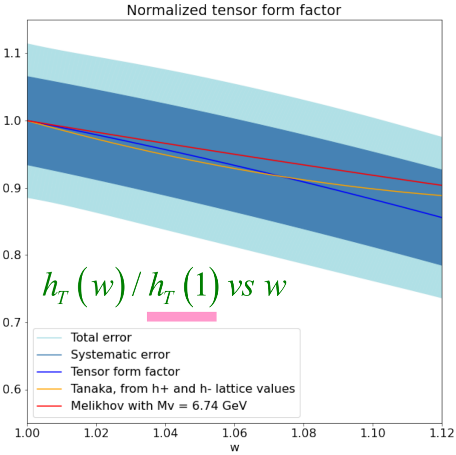

S. Cali, S. Klaver, M. Rotondo, B. Sciascia, Eur.Phys.J.C**79** (2019), 744

# Tensor form factors: JLQCD

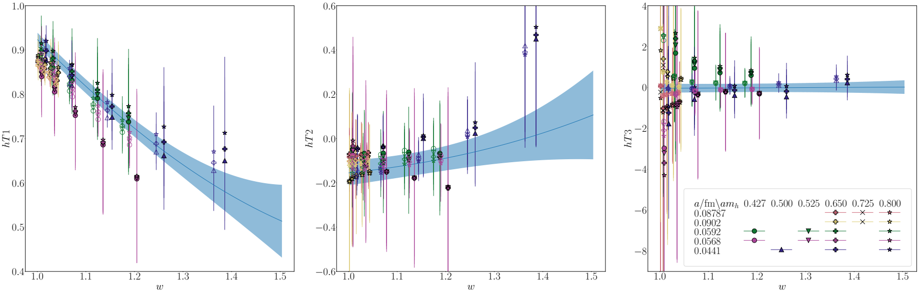
- JLQCD calculation of the tensor ff $h_T$ for $B \to D\ell\nu$

$$\langle D(p')|T_{\mu\nu}|B(p)\rangle = i\,(v'^{\mu}v^{\nu} - v'^{\nu}v^{\mu})$$

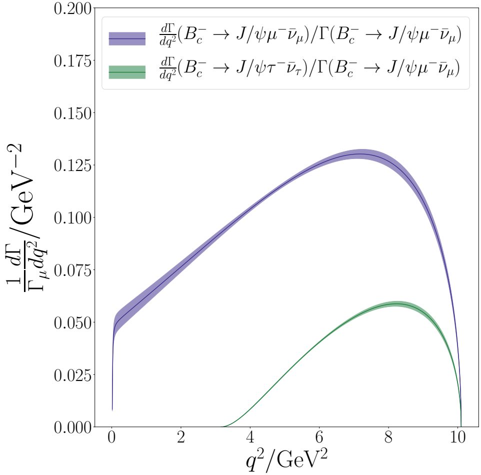
  - Renormalization WIP
  - Consistent with phenomenology



# Tensor form factors: HPQCD



# Other channels: $B_c \to J/\psi \ell \nu$ by HPQCD




$$R(J/\psi) = 0.2582(38)$$

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