# Nuclear muon capture: A perfect probe of the neutrinoless double beta decay

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Contents:

- Intro: Rare weak decays and $g_A$
- Double beta decays and the $g_A$ problem
- Effective value of $g_A$ from different perspectives
- Introducing OMC
- OMC: Recent results

# On the effective value of the weak axial coupling $g_A$

The PCVC hypothesis $\Rightarrow g_A = 1.27$

$\Downarrow$ Non-nucleonic degrees of freedom (delta resonances)

$\Downarrow$ Nuclear many-body effects (two-body currents)

$\Downarrow$ Nuclear-model effects (valence-space and configuration-space truncations, neglect of three-nucleon forces,...)

Effective value: $g_A^{\rm eff}$

# Effective value of $g_A$ affects everything

Motivation:

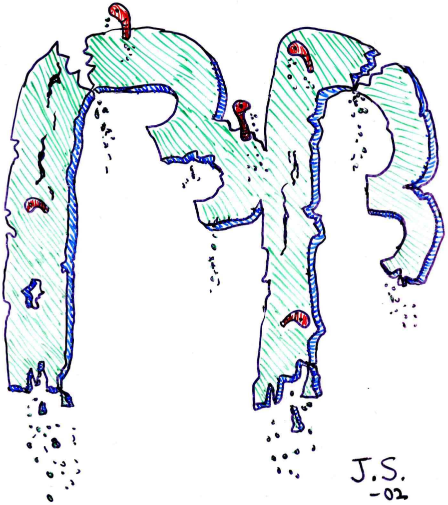
Effective value of the weak coupling $g_A$ is involved in all weak processes, and thus have impact on

- studies of rare $\beta$ decays
- processes in neutrino physics ($\beta\beta$ decay, low-energy (anti)neutrino-nucleus scattering, nuclear muon capture, ...)
- processes in astrophysics (allowed and forbidden $\beta$ decays, (anti)neutrino-nucleus scattering cross sections,...)

$\Downarrow$

Affects (strongly) the determination of neutrino properties!

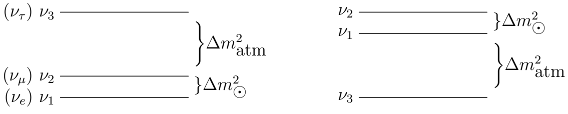
# Motivation for the Work: Double Beta Decay



# Neutrinoless double $\beta^-$ decay

## $0\nu\beta\beta$ Decay is Able to:

- Reveal if the neutrino is a Majorana particle
- Probe the neutrino effective mass
  $\langle m_\nu \rangle = \sum_{j=\mathrm{light}} \lambda_j^{\mathrm{CP}} |U_{ej}|^2 m_j$
- Probe the degenerate or inverted mass hierarchies (next-generation experiments!)
- Probe possibly the CP phases (nuclear matrix elements are critical!)



Normal hierarchy

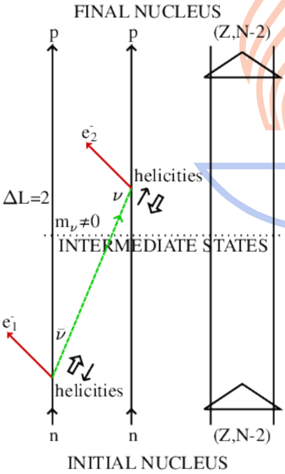
Inverted hierarchy

$$\Delta m^2_\odot = 7.67^{+0.16}_{-0.19} \times 10^{-5}\,\mathrm{eV}^2$$

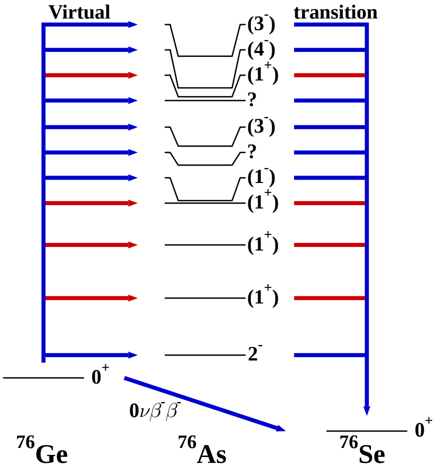
$$\Delta m^2_{\mathrm{atm}} = 2.39^{+0.11}_{-0.08} \times 10^{-3}\,\mathrm{eV}^2$$

[Global $3\nu$ oscillation analysis (2008)]

MASS MODE:
$T_{1/2}^{-1} \propto \langle m_\nu \rangle^2$



# $0\nu\beta\beta$ decay from nuclear-structure point of view



Decay rate:

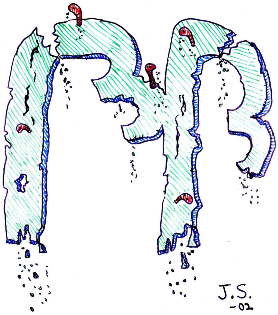
$$\frac{\ln 2}{T_{1/2}} = (g_{\rm A})^4 g^{(0\nu)}(Q)[M^{(0\nu)}]^2 \langle m_\nu \rangle^2$$

- $g^{(0\nu)}(Q) \propto Q^5$ is the phase-space factor
- $M^{(0\nu)}$ = NUCLEAR MATRIX ELEMENT
- Effective neutrino mass:

$$\langle m_\nu \rangle = \sum_{j=\text{light}} \lambda_j^{\rm CP} |U_{ej}|^2 m_j$$

- Light and heavy Majorana-neutrino exchange: J. Hyvärinen and J.S., Phys. Rev. C 91 (2015) 024613

# Motivation for the studies of $g_{\rm A}^{\rm eff}$



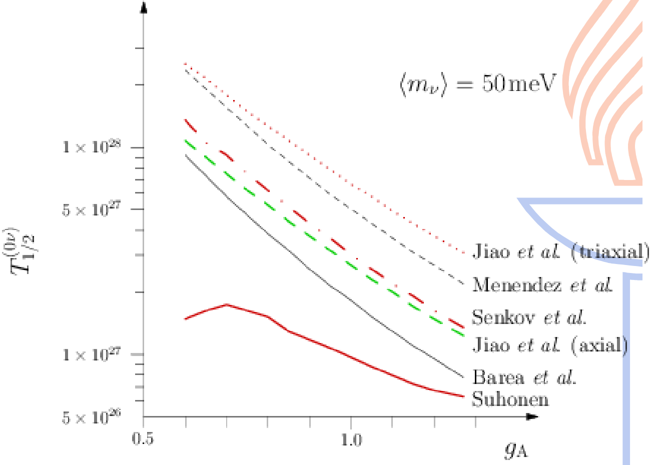
- DECAY:

$$\boxed{2\nu\beta\beta - \text{rate} \sim \left|M^{(2\nu)}_{\rm GTGT}\right|^2 = ({\color{red}g_{\rm A}})^4 \left|\sum_{m,n} \frac{M_{\rm L}(1^+_m)M_{\rm R}(1^+_n)}{D_m}\right|^2}$$

$$\boxed{0\nu\beta\beta - \text{rate} \sim \left|M^{(0\nu)}_{\rm GTGT}\right|^2 = ({\color{red}g_{\rm A,0\nu}})^4 \left|\sum_{J^\pi} (0^+_f||\mathcal{O}^{(0\nu)}_{\rm GTGT}(J^\pi)||0^+_i)\right|^2}$$

# Example: $0\nu\beta\beta$ NMEs of $^{76}$Ge, effect on the half-life

- Jiao *et al.*: Phys. Rev. C 96 (2017) 054310 (GCM+ISM)
- Menendez *et al.*: Nucl. Phys. A 818 (2009) 139 (ISM)
- Senkov *et al.*: Phys. Rev. C 93 (2016) 044334 (ISM)
- Barea *et al.*: Phys. Rev. C 91 (2015) 034304 (IBM-2)
- Suhonen: Phys. Rev. C 96 (2017) 055501 (pnQRPA + $g_{\mathrm{pp}}$ + isospin restoration + data on $2\nu\beta\beta$)

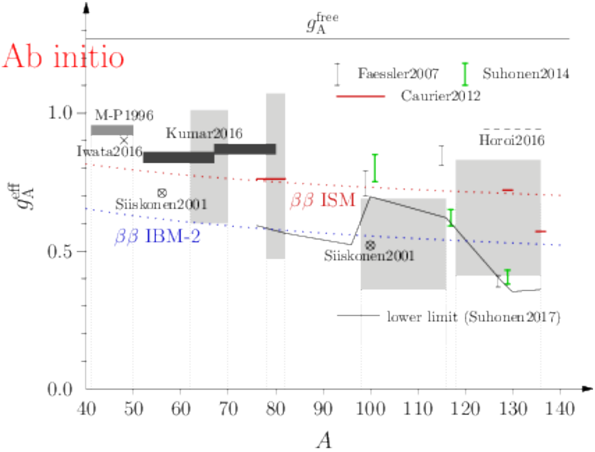


# What do we know about the effective value of $g_A$?

Till now we have info from the following processes:

- studies of Gamow-Teller $\beta$ decays
- studies of first-forbidden unique $\beta$ decays
- studies of two-neutrino $\beta\beta$ decays
- studies of higher-forbidden non-unique $\beta$ spectral shapes

# Collection of results extracted from the GT $\beta^{\pm}$/EC and $2\nu\beta\beta$ calculations

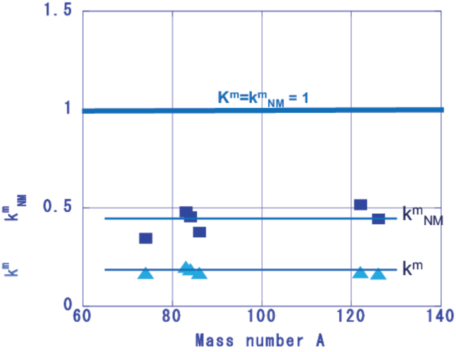


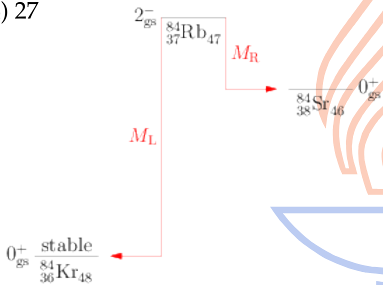
Ab initio: P. Gysbers *et al.*, Nature Physics 15 (2019) 428

- Faessler2007: pnQRPA A. Faessler *et al.*, arXiv 0711.3996v1 [Nucl-th]
- Suhonen2014: pnQRPA J. Suhonen *et al.*, Nucl. Phys. A 924 (2014) 1
- Suhonen2017: pnQRPA J. Suhonen, Phys. Rev. C 96 (2017) 055501
- Caurier2012: ISM E. Caurier *et al.*, Phys. Lett. B 711 (2012) 62
- Horoi2016: ISM M. Horoi *et al.*, Phys. Rev. C 93 (2016) 024308
- M-P1996: ISM G. Martínez-Pinedo *et al.*, Phys. Rev. C 53 (1996) R2602
- Iwata2016: ISM Y. Iwata *et al.*, Phys. Rev. Lett. 116 (2016) 112502
- Kumar2016: ISM V. Kumar *et al.*, J. Phys. G 43 (2016) 105104 Phys. Lett. B 711 (2012) 62
- Siiskonen2001: ISM T. Siiskonen *et al.*, Phys. Rev. C 63 (2001) 055501
- $\beta\beta$ ISM and IBM-2: J. Barea *et al.*, Phys. Rev. C 87 (2013) 014315
- Light hatched regions: pnQRPA H. Ejiri *et al.*, J. Phys. G 42 (2015) 055201 ; P. Pirinen *et al.*, Phys. Rev. C 91 (2015) 054309 ; F. Deppisch *et al.*, Phys. Rev. C 94 (2016) 055501

# Global study for the first-forbidden ($K = 1$) spin-dipole $2^-_{gs} \to 0^+_{gs}$ decays

H. Ejiri, N. Soukouti and J. S., Spin-dipole nuclear matrix elements for double beta decays and astro-neutrinos, Phys. Lett. B 729 (2014) 27





$$\bar{M}(\mathrm{SD}2^-) = \sqrt{M_L M_R}$$

$$\langle k \rangle = \left\langle \frac{\bar{M}_{\mathrm{exp}}(\mathrm{SD}2^-)}{\bar{M}_{\mathrm{qp}}(\mathrm{SD}2^-)} \right\rangle \approx 0.18$$

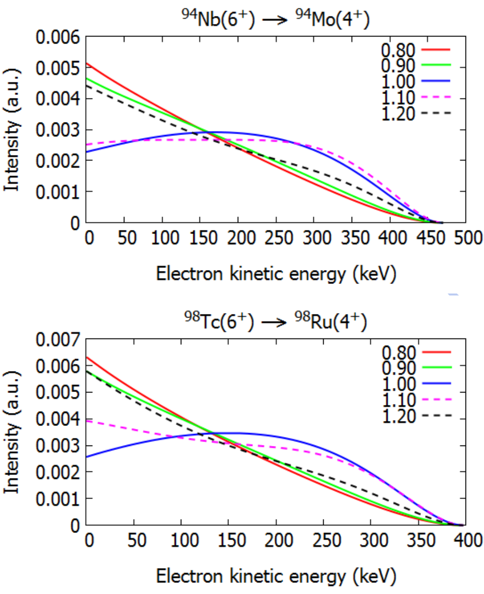
$$\langle k_{\mathrm{NM}} \rangle = \left\langle \frac{\bar{M}_{\mathrm{exp}}(\mathrm{SD}2^-)}{\bar{M}_{\mathrm{pnQRPA}}(\mathrm{SD}2^-)} \right\rangle \approx 0.45$$

$$\Rightarrow \bar{g}_A^{\mathrm{eff}} \approx 0.57$$

# $\beta$-electron spectra can depend on the value of $g_A$

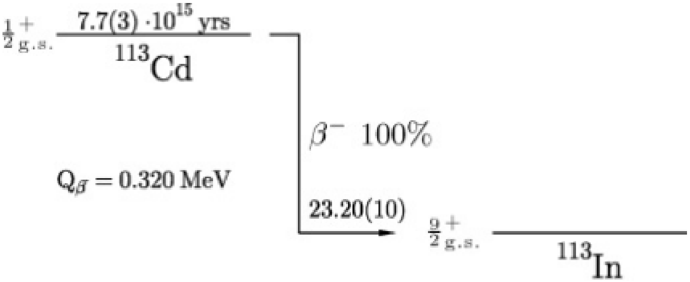
Normalized ISM-computed electron spectra for the 2*nd*-forbidden nonunique $\beta^-$ decays of $^{94}$Nb and $^{98}$Tc ($g_V = 1.0$).

From: J. Kostensalo and J. S., $g_A$-driven shapes of electron spectra of forbidden $\beta$ decays in the nuclear shell model, Phys. Rev. C 96 (2017) 024317



# EXAMPLE: 4*th*-forbidden nonunique decay of $^{113}$Cd

4*th*-forbidden nonunique $\beta^-$ transition $^{113}\mathrm{Cd}(1/2^+) \rightarrow {}^{113}\mathrm{In}(9/2^+)$



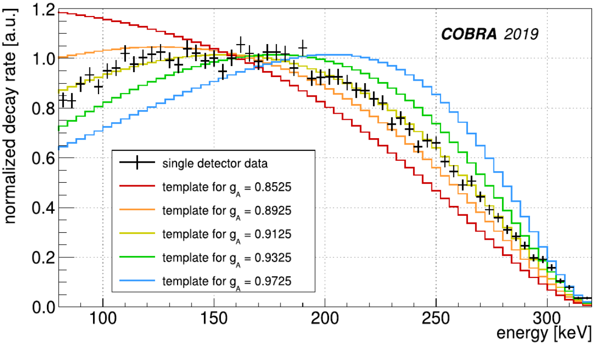
Calculated by using the Interacting Shell Model (ISM), the Microscopic Quasiparticle-Phonon Model (MQPM) and the microscopic Interacting Boson-Fermion Model (IBFM-2).

# Decay of $^{113}$Cd – Comparison with data

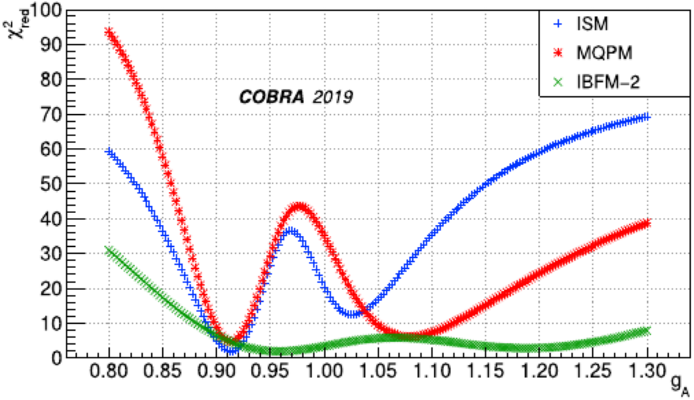
Normalized electron spectra for the 4*th*-forbidden nonunique $\beta^-$ transition $^{113}\mathrm{Cd}(1/2^+) \rightarrow {}^{113}\mathrm{In}(9/2^+)$ ($g_\mathrm{V} = 1.0$).

Experimental data from The COBRA collaboration: PLB2020: L. Bodenstein-Dresler *et al.*, Phys. Lett. B 800 (2020) 135092.

Measured spectrum by detector no. 54:



# Decay of $^{113}$Cd – Comparison with data



$$\text{PLB2020} : \bar{g}_{\rm A}(\text{ISM}) = 0.914 \pm 0.008; \text{PLB2021} := 0.907 \pm 0.064$$
$$\text{PLB2020} : \bar{g}_{\rm A}(\text{MQPM}) = 0.910 \pm 0.013; \text{PLB2021} := 0.993 \pm 0.063$$
$$\text{PLB2020} : \bar{g}_{\rm A}(\text{IBFM-2}) = 0.955 \pm 0.035; \text{PLB2021} := 0.828 \pm 0.140$$

# Decay of $^{113}$Cd – $g_A^{\text{eff}}$ using spectral moments

SMM = Spectral Moments Method

$$\mu_n = \int_{w_{\text{thr}}}^{w_0} S(w_e) w_e^n \mathrm{d}w_e \,,$$

$n = 0$ ↔ area under the spectral curve ↔ $T_{1/2}$

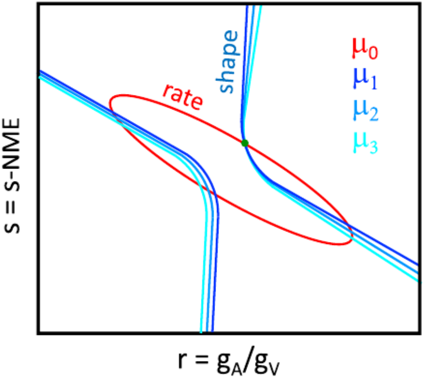
$n = 1$ ↔ mean energy

$n = 2$ ↔ variance

Usually only first few moments $\mu$ are enough!

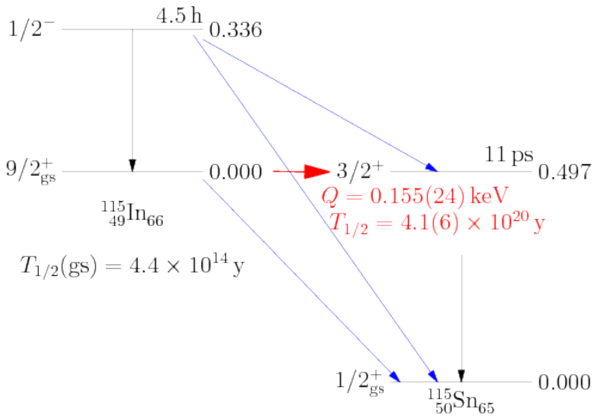
Result from

J. Kostensalo, E. Lisi, A. Marrone and J. S., $^{113}$Cd $\beta$-decay spectrum and $g_A$ quenching using spectral moments, Phys. Rev. C 107 (2023) 055502.



$$\bar{g}_A(\text{ISM}) = 0.96 - 0.99$$
$$\bar{g}_A(\text{IBFM-2}) = 1.03 - 1.13$$
$$\bar{g}_A(\text{MQPM}) = 1.02 - 1.07$$

# EXAMPLE: 4*th*-forbidden nonunique transition $^{115}\mathrm{In}(9/2^+) \rightarrow {}^{115}\mathrm{Sn}(1/2^+)$

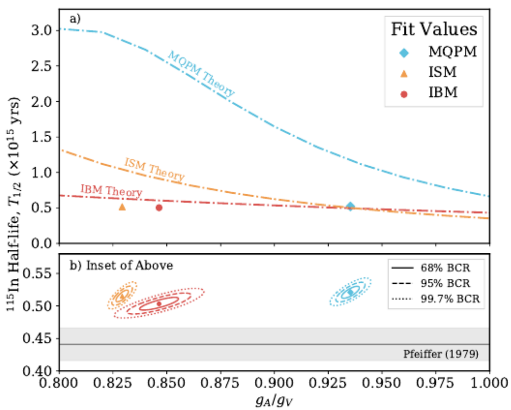


Interesting ultra-low $Q$-value transition: The 2*nd*-forbidden unique transition $^{115}\mathrm{In}(9/2^+) \rightarrow {}^{115}\mathrm{Sn}(3/2^+)$ has the smallest known $Q$ value of a nuclear transition: J. S. E. Wieslander *et al.*, Phys. Rev. Lett. 103 (2009) 122501; B. J. Mount *et al.*, Phys. Rev. Lett. 103 (2009) 122502.

# Decay of $^{115}$In – Comparison with data

Normalized electron spectra for the 4*th*-forbidden nonunique $\beta^-$ decay $^{115}\mathrm{In}(9/2^+) \rightarrow {}^{115}\mathrm{Sn}(1/2^+)$ ($g_\mathrm{V} = 1.0$).

Result from The CEA-CNRS-CSNSM-INR-JYFL-MIT-LUKE-UCB collaboration: A. F. Leder *et al.*, Phys. Rev. Lett. 129 (2022) 232502.



$$\begin{aligned} \bar{g}_\mathrm{A}(\mathrm{ISM}) &= 0.830 \pm 0.002 \\ \bar{g}_\mathrm{A}(\mathrm{IBFM\text{-}2}) &= 0.845 \pm 0.006 \\ \bar{g}_\mathrm{A}(\mathrm{MQPM}) &= 0.936 \pm 0.003 \end{aligned}$$

# Conclusions about the effective $g_A$

Conclusion 1:

The long chain of ISM calculations and the recent pnQRPA and IBM-2 calculations of Gamow-Teller $\beta$ decays and $2\nu\beta\beta$ decays are (surprisingly!) consistent with each other and clearly point to a $A$-dependent quenched $g_A$

Conclusion 2:

The spectrum-shape method (SSM) and the spectral moments method (SMM) for forbidden non-unique $\beta$ decays seem robust tools (largely independent of the nuclear model, the assumed Hamiltonian and mean field) to search for the effective value of $g_A$ and to try to solve other problems, like those related to the reactor-$\bar{\nu}_e$ spectra

## Still more information on the value of the weak couplings

These methods are now available:

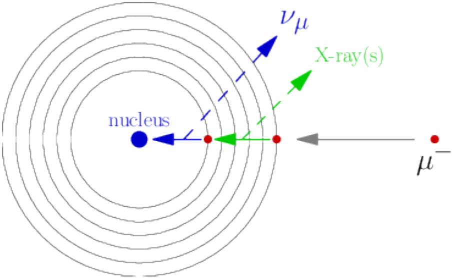
**For low momentum exchanges ($g_{\mathrm{A}}$):**

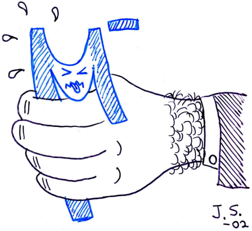
- Study half-lives of $\beta$ decays ($1^+$ and $2^-$ states)
- Study half-lives of $2\nu\beta\beta$ decays ($1^+$ states)
- Study electron spectral shapes of $\beta$ decays ($J^\pi$ states)
- Study charge-exchange reactions
- Study double charge-exchange reactions

**For high momentum exchanges like $0\nu\beta\beta$ decay ($g_{\mathrm{A},0\nu}$):**

- Study charge-exchange reactions
- Study double charge-exchange reactions
- Study nuclear muon capture ($J^\pi$ states)

# How about the ordinary Muon Capture (OMC)?





Nuclear muon capture:

$$\text{OMC: } {}^A_Z\text{X} + \mu^- \rightarrow {}_{Z-1}{}^{A}\text{Y} + \nu_\mu$$

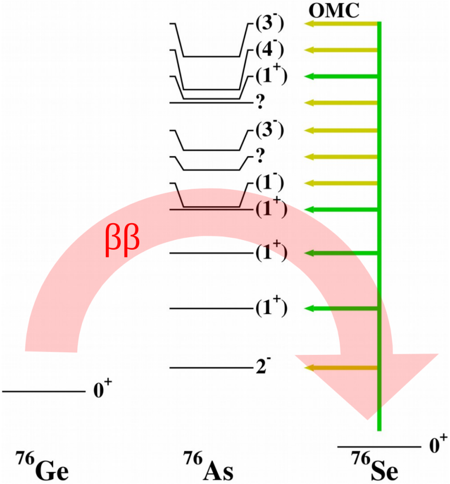
Also:

Muon decay: $\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu \ (\tau = 2.2\mu\text{s})$

OMC probability $\sim Z^4$
(in Fe 91% are captured,
breakeven at $Z \sim 11$)

# Ordinary muon capture (OMC) on $^{76}$Se

$$^{76}\text{Se} + \mu^- \rightarrow {}^{76}\text{As} + \nu_\mu$$



$$\boxed{m_\mu c^2 \approx 105\,\text{MeV}}$$

- OMC and $0\nu\beta\beta$ operate in the $q \approx 100\,\text{MeV}$ momentum-exchange region $\Rightarrow g_{\text{A},0\nu}(J^\pi)$
- Induced currents ($g_\text{P}$!) are activated

Experiments:

RCNP, Osaka ; J-PARC MLF, Japan ; PSI, Villigen, Switzerland

# The capture rate of OMC

The muon-capture rate (in units of 1/s) can be written as:

$$W = 2P\frac{2J_f+1}{2J_i+1}\left(1-\frac{q}{m_\mu + AM_N}\right)q^2 \, ,$$

where $q$ is OMC $Q$-value (essentially the magnitude of the muon-neutrino momentum) and $M_N$ the (average) nucleon mass. Here

$$P = \frac{1}{2}\sum_{\kappa u}\Big| g_{\mathrm{V}}(q^2)P^{(1)}_{\kappa u} + g_{\mathrm{A}}(q^2)P^{(2)}_{\kappa u} - \frac{g_{\mathrm{V}}(q^2)}{M_N}P^{(3)}_{\kappa u} + \sqrt{3}\frac{q}{2M_N}g_{\mathrm{V}}(q^2)P^{(4)}_{\kappa u}$$
$$+ \sqrt{6}\frac{q}{2M_N}\left(g_{\mathrm{V}}(q^2) - g_{\mathrm{M}}(q^2)\right)P^{(5)}_{\kappa u} - \frac{g_{\mathrm{A}}(q^2)}{M_N}P^{(6)}_{\kappa u} + \sqrt{\frac{1}{3}}\frac{q}{2M_N}\left(g_{\mathrm{P}}(q^2) - g_{\mathrm{A}}(q^2)\right)P^{(7)}_{\kappa u}\Big|^2$$

Compare with the inverse half-life of the $0\nu\beta\beta$ decay:

$$\left(T^{(0\nu)}_{1/2}\right)^{-1} = G_{0\nu}\left(\frac{\langle m_\nu\rangle}{m_e}\right)^2\Big|[g_{\mathrm{V}}(q^2)]^2M^{(0\nu)}_{\mathrm{F}} + [g_{\mathrm{A}}(q^2)]^2M^{(\mathrm{AA})}_{\mathrm{GT}} - \frac{q^2}{3M_N}g_{\mathrm{A}}(q^2)g_{\mathrm{P}}(q^2)M^{(\mathrm{AP})}_{\mathrm{GT}}$$
$$+ \frac{q^4}{12M_N^2}[g_{\mathrm{P}}(q^2)]^2M^{(\mathrm{PP})}_{\mathrm{GT}} + \frac{q^2}{6M_N^2}[g_{\mathrm{M}}(q^2)]^2M^{(\mathrm{MM})}_{\mathrm{GT}} - [g_{\mathrm{A}}(q^2)]^2M^{(0\nu)}_{\mathrm{T}}\Big|^2$$

# OMC first suggested as an experimental probe for $0\nu\beta\beta$ matrix elements in:

Pioneering works:

M. Kortelainen and J. S., Ordinary muon capture as a probe of virtual transitions of $\beta\beta$ decay, *Europhysics Letters* **58** (2002) 666-672

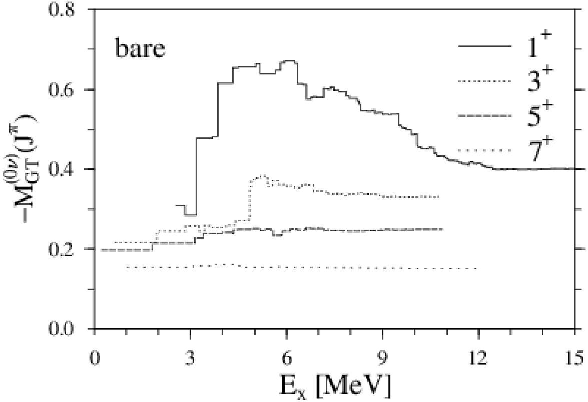
M. Kortelainen and J. S., Microscopic study of muon-capture transitions in nuclei involved in double-beta-decay processes, *Nuclear Physics* **A 713** (2003) 501-521

M. Kortelainen and J. S., Nuclear muon capture as a powerful probe of double-beta decays in light nuclei, *Journal of Physics G: Nucl. Part. Phys.* **30** (2004) 2003-2018

Original theory from: M. Morita and A. Fujii, Theory of allowed and forbidden transitions in muon capture reactions, Phys. Rev. 118 (1960) 606.

# Cumulative sums for the $0\nu\beta\beta$ matrix elements

NME calculated for the $0\nu\beta\beta$ decay of $^{48}$Ca using the nuclear shell model

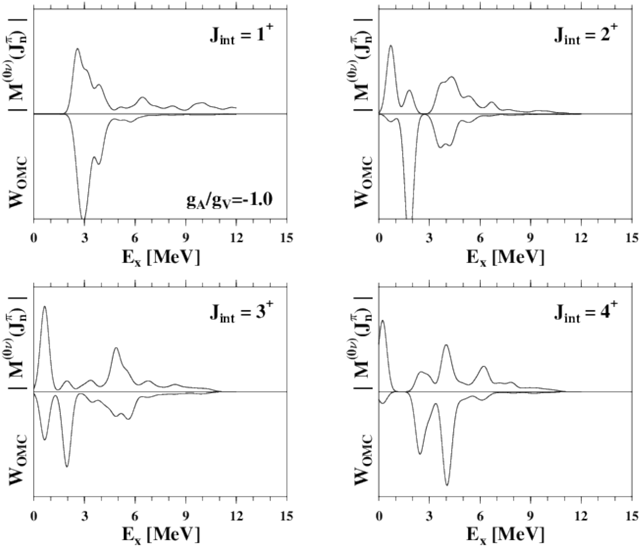


$$M^{(0\nu)} = \sum_{J^\pi} M^{(0\nu)}(J^\pi) \, ,$$

FPBP interaction
in the pf shell:

$M(1^+) = -0.402$
$M(2^+) = -0.304$
$M(3^+) = -0.332$
$M(4^+) = -0.183$
$M(5^+) = -0.249$
$M(6^+) = -0.102$
$M(7^+) = -0.151$
TOTAL $= -1.723$

# Comparison of OMC rates and $0\nu\beta\beta$ NMEs



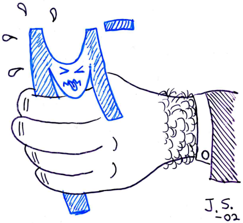
# Recently: OMC in medium-heavy and heavy nuclei

There are and will be more data on:

PARTIAL CAPTURE RATES of OMC

and in particular:

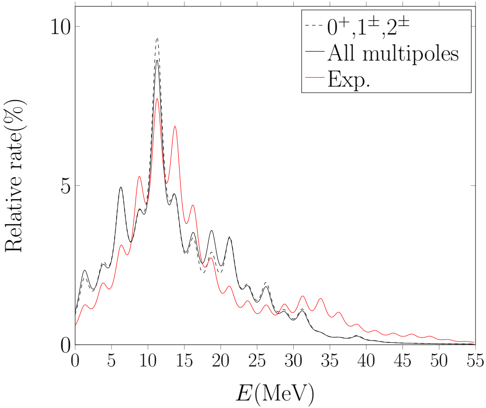
OMC STRENGTH DISTRIBUTIONS



Now we need:

Large-basis (with Wood-Saxon single-particle energies) no-core pnQRPA calculations with realistic effective two-nucleon interactions.

# RECENT WORK on OMC strength distributions: OMC on $^{100}$Mo



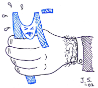
First evidence on OMC giant resonance:

L. Jokiniemi, J. S., H. Ejiri, I.H. Hashim, Pinning down the strength function for ordinary muon capture on $^{100}$Mo, Phys. Lett. B 794 (2019) 143.

Experiments: MuSIC beam channel at RCNP (Research Center for Nuclear Physics), Osaka, Japan

D2 beam channel in J-PARC (Japan Proton Accelerator Research Complex) MLF, Ibaraki, Japan

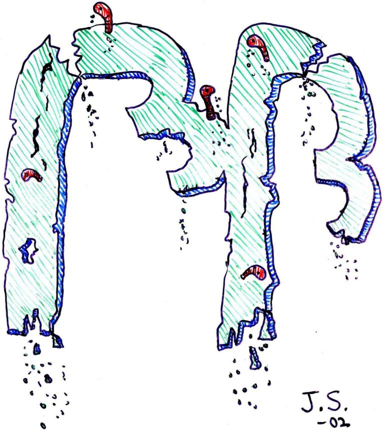
Ongoing work: experiments at the $\mu$E4 beamline at PSI by The MONUMENT Collaboration

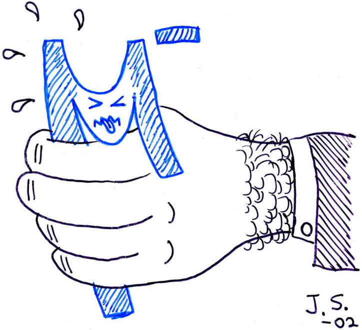


# Recent work: OMC vs. $0\nu\beta\beta$ decay

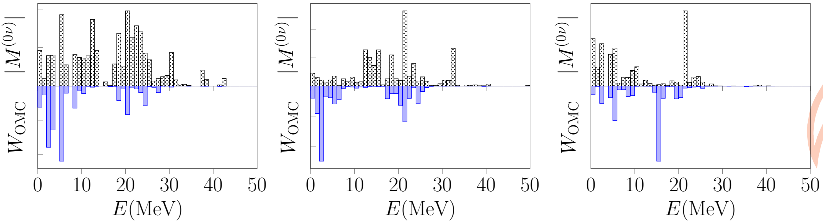
Studied in: L. Jokiniemi and J. S., Comparative analysis of muon-capture and $0\nu\beta\beta$-decay matrix elements, Phys. Rev. C 102 (2020) 024303

VS.

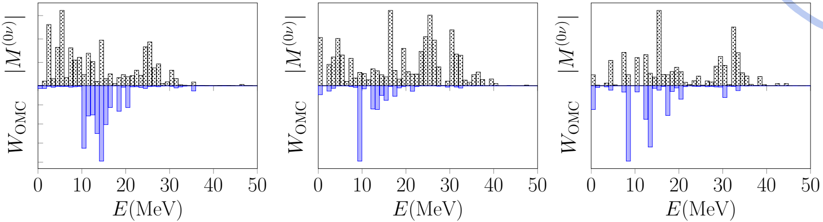




# Comparative analysis between OMC rates and $0\nu\beta\beta$ NME for $^{76}$Ge

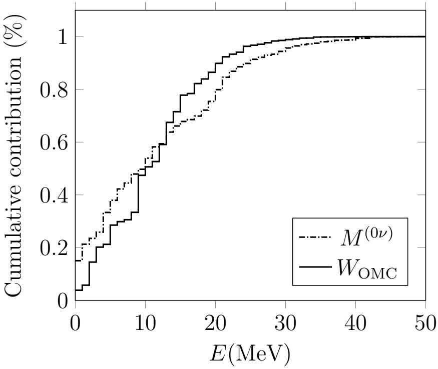


(a) $J^\pi = 1^+$ (b) $J^\pi = 2^+$ (c) $J^\pi = 3^+$

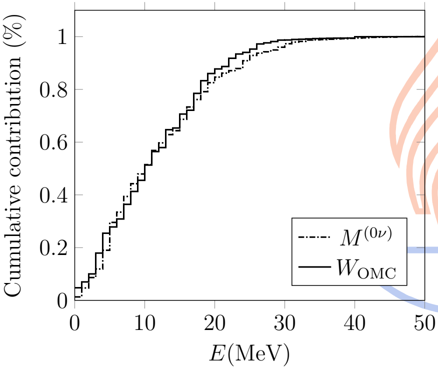


(d) $J^\pi = 1^-$ (e) $J^\pi = 2^-$ (f) $J^\pi = 3^-$

# Cumulative relative (%) OMC rates and $0\nu\beta\beta$ NMEs



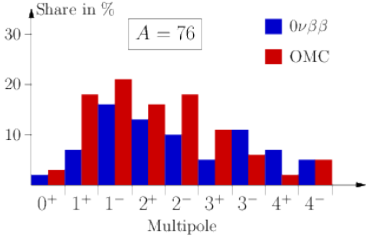
$A = 76$



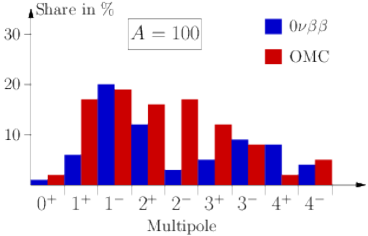
$A = 136$

From: L. Jokiniemi and J. S., Comparative analysis of muon-capture and $0\nu\beta\beta$-decay matrix elements, Phys. Rev. C 102 (2020) 024303

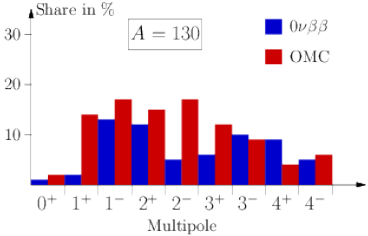
# Comparison of the OMC and $0\nu\beta\beta$ multipole decompositions



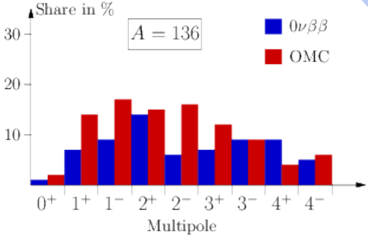
(a) OMC: 100%, $0\nu\beta\beta$: 76%



(b) OMC: 98%, $0\nu\beta\beta$: 68%



(c) OMC: 96%, $0\nu\beta\beta$: 63%



(d) OMC: 95%, $0\nu\beta\beta$: 67%

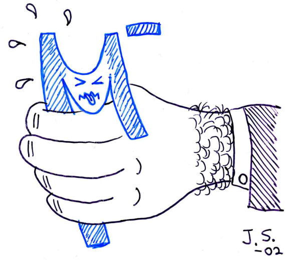
# Recent and very recent work: OMC partial capture rates to individual final states

There are and will be more data on:

OMC CAPTURE RATES

to

INDIVIDUAL FINAL STATES

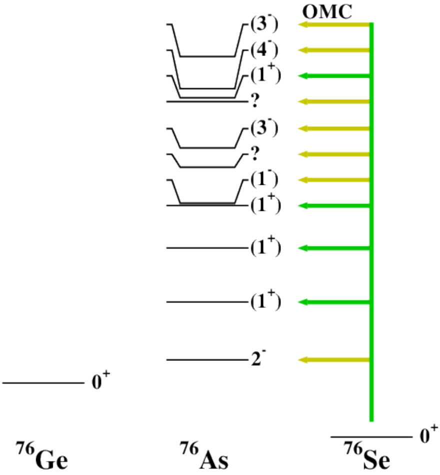


Now we can use:

pnQRPA theory, the nuclear shell model and ab initio methods

# OMC to individual $J^\pi$ states

OMC on $^{76}$Se:



OMC on $^{76}$Se: Rates to states $J^\pi$ in $^{76}$As below some 1 MeV: no-core large-basis pnQRPA calculation

$(g_V(0) = 1.0, g_A(0) = 0.8, g_P(0) = 7.0)$

| $J^\pi$ | Exp. (1/s) | Th. (1/s) |
|---|---|---|
| $0^+$ | 5120 | 414 |
| $1^+$ | 218 240 | 236 595 |
| $1^-$ | 31 360 | 28 991 |
| $2^+$ | 120 960 | 114 016 |
| $2^-$ | 145 920 + g.s. | 177 802 |
| $3^+$ | 60 160 | 55 355 |
| $3^-$ | 53 120 | 34 836 |
| $4^+$ | - | 2797 |
| $4^-$ | 30 080 | 23 897 |

Data from: D. Zinatulina *et al.*, Phys. Rev. C 99 (2019) 024327

Calculation from: L. Jokiniemi and J.S., Phys. Rev. C 100 (2019) 014619

# NEW: Add two-body meson-exchange currents

## Quenching of the weak couplings by 2BC:

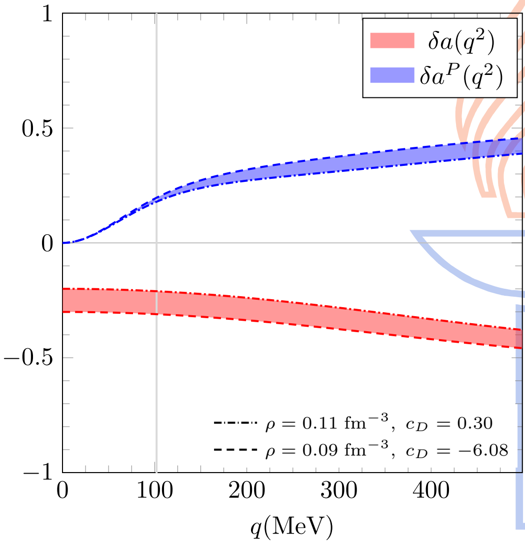
In addition to the one-body (weak nucleon) current (1BC) one can take into account the *meson exchanges* by adding (normal-ordered one-body part of) the two-body current (2BC) through the replacements:

$$g_A(q^2) \to (1 + \delta_a(q^2))g_A(q^2)\,,$$

$$g_P(q^2) \to \left(1 - \frac{q^2 + m_\pi^2}{q^2}\delta_a^P(q^2)\right)g_P(q^2)$$

See: M. Hoferichter *et al.*, Phys. Rev. D 102 (2020) 074018

NOTE: Does not account for deficiencies in the many-body framework!



# OMC on $^{12}$C to individual $J^{\pi}$ states in $^{12}$B: 2BCs added

Shell-model calculated capture rates in units of $10^3$ 1/s, with $g_{\rm V}(0) = 1.0$, $g_{\rm A}(0) = 1.27$, $g_{\rm P}(0)/g_{\rm A}(0) = 6.8$ (PCAC, Goldberger-Treiman):

| $J^{\pi}$ | Exp. | Th.: 1BC | Th.: 1BC+2BC* |
|---|---|---|---|
| $1^+_{\rm gs}$ | $5.68^{+0.14}_{-0.23}$ | 6.48 | 3.98 – 4.45 |
| $2^+_1$ | $0.31^{+0.09}_{-0.07}$ | 0.42 | 0.30 – 0.32 |
| $2^+_2$ | $0.026^{+0.015}_{-0.011}$ | 0.011 | 0.008 – 0.009 |

* The spread comes from the spread in the assumed values of the EFT low-energy constants

Data from: Y. Abe *et al.*, Phys. Rev. C 93 (2016) 054608

Nuclear shell-model calculation from: L. Jokiniemi, T. Miyagi, S. R. Stroberg, J. D. Holt, J. Kotila and J.S., Phys. Rev. C 107 (2023) 014327

# Recent work continues: OMC and double positron decays

NEW:

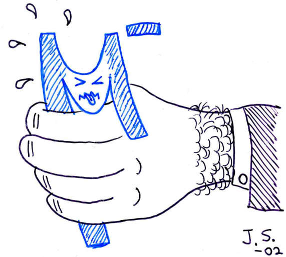
OMC

probing the

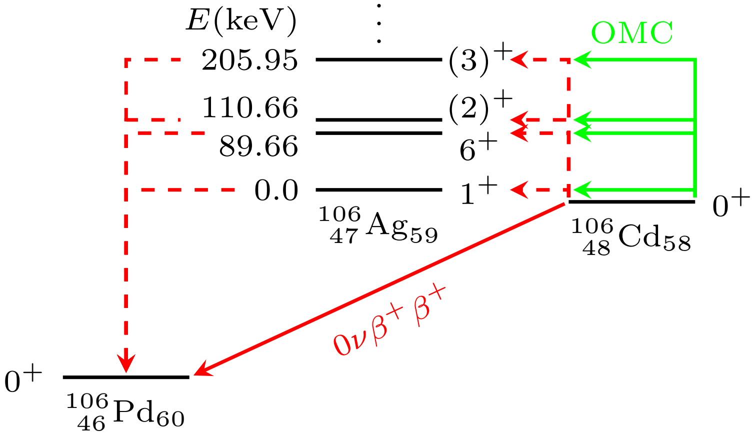
INITIAL BRANCH

of

DOUBLE BETA DECAY

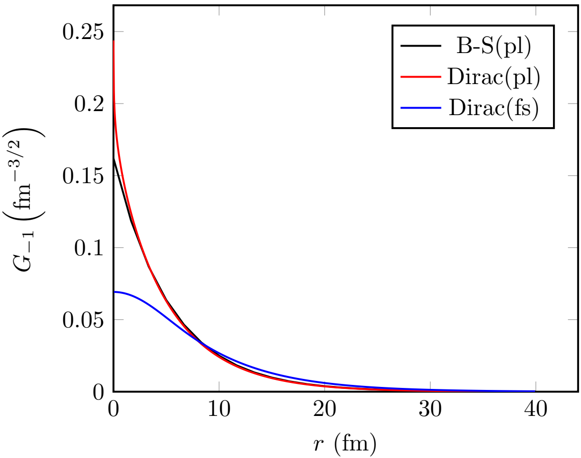


# OMC and $0\nu\beta^+\beta^+$ decay of $^{106}$Cd



L. Jokiniemi, J.S. and J. Kotila, Comparative Analysis of nuclear matrix elements of $0\nu\beta^+\beta^+$ decay and muon capture in $^{106}$Cd, Front. Phys. 9 (2021) 652536

# OMC on $^{106}$Cd: Muon orbital wave function



B-S: Bethe-Salpeter point-like nucleus approximation;

Dirac: Numerical solution of the Dirac equation;

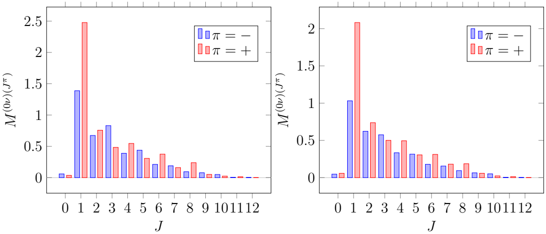
pl: point-like nucleus;

fs: finite-size nucleus.

$$\mathrm{B-S}: \psi_\mu(\mathbf{r}) = \begin{pmatrix} -\mathrm{i}F_{-1}\chi_\mu \\ G_{-1}\chi_\mu \end{pmatrix} ; \quad F_{-1} = -\sqrt{\frac{1-\gamma}{1+\gamma}}G_{-1} ; \quad G_{-1} = \left(\frac{2Z}{a_0}\right)^{3/2}\sqrt{\frac{1+\gamma}{2\Gamma(2\gamma+1)}}\left(\frac{2Zr}{a_0}\right)^{\gamma-1} e^{-Zr/a_0}$$

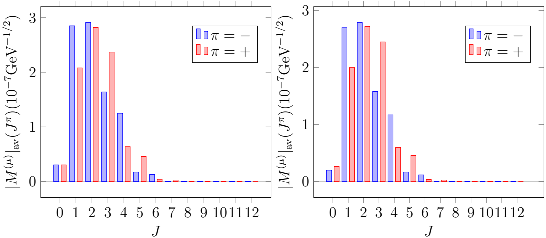
# $^{106}$Cd: OMC and $0\nu\beta^+\beta^+$ multipole decompositions

## $0\nu\beta\beta$ decay



**(a) Basis: 'Adj.'** **(b) Basis: 'WS'**

## OMC



**(a) Basis: 'Adj.'** **(b) Basis: 'WS'**

# And finally: OMC on $^{136}$Ba to states in $^{136}$Cs

NEW:

OMC

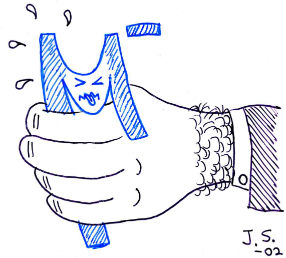
on

$^{136}$Ba,

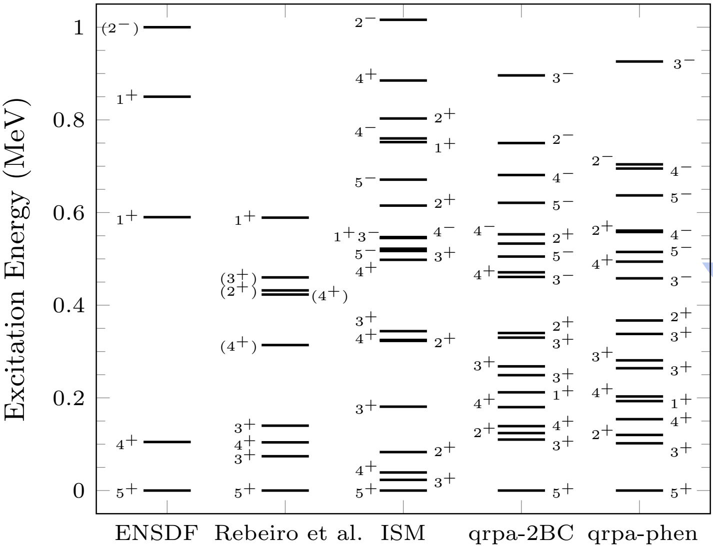
the $0\nu\beta\beta$ daughter of

$^{136}$Xe

Two-body currents (2BC) included and results compared with a phenomenological approach for both the ISM and the pnQRPA



# Comparison of the ISM- and pnQRPA-computed energy spectra in $^{136}$Cs



The ISM and pnQRPA are surprisingly compatible below the considered 1 MeV of excitation

# Comparison of OMC rates to different multipole states in $^{136}$Cs

| $J^\pi$ | ISM-2BC | ISM-phen | QRPA-2BC | QRPA-phen |
|---|---|---|---|---|
| $1^+$ | 4.8 (6.2) | 5.2 | 243.0 (303.4) | 206.9 |
| $2^+$ | 32.7 (38.1) | 34.9 | 211.0 (239.5) | 201.1 |
| $2^-$ | 41.5 (53.3) | 44.4 | 26.6 (26.0) | 14.2 |
| $3^+$ | 28.3 (36.2) | 29.9 | 104.8 (132.9) | 87.8 |
| $3^-$ | 8.9 (11.0) | 9.7 | 41.6 (49.5) | 38.7 |
| $4^+$ | 9.3 (10.5) | 9.8 | 20.8 (22.6) | 20.4 |
| $4^-$ | 14.3 (18.4) | 15.0 | 25.1 (31.8) | 21.2 |
| $5^+$ | 0.1 (0.1) | 0.0 | 0.5 (0.6) | 0.5 |
| $5^-$ | 0.6 (0.7) | 0.6 | 0.8 (0.9) | 0.8 |

All OMC rates in units of $10^3$ 1/s.
Phenomenological $g_{\rm A}^{\rm eff} = 0.93$ for the ISM (M. Horoi and B. A. Brown, Phys. Rev. Lett. 110 (2013) 222502) and $g_{\rm A}^{\rm eff} = 0.83$ ($g_{\rm ph} = 1.18$, $g_{\rm pp} = 0.7$) for the pnQRPA (P. Pirinen and J. S., Phys. Rev. C 91 (2015) 054309). $g_{\rm P}/g_{\rm A} = 6.8$ (Goldberger-Treiman relation).

FROM: P. Gimeno, L. Jokiniemi, J. Kotila, M. Ramalho and J. S., Ordinary muon capture on $^{136}$Ba: Comparative study using the shell model and pnQRPA, Universe 2023, 9, 270 (2023)

# Conclusions and outlook

## Conclusions:

- The effective value of $g_{\rm A}$ is involved in all weak processes, and thus has impact on studies of rare $\beta$ decays, neutrino physics and astrophysics
- The OMC can test the weak axial couplings at the momentum-exchange region relevant for the $0\nu\beta\beta$ decay; it is also a sensitive probe of nuclear wave functions in this context
- OMC to individual states can be a probe of the effective value of the induced pseudoscalar coupling $g_{\rm P}$ (see, e.g., T Siiskonen, J.S. and M. Hjorth-Jensen, Phys. Rev. C 59 (1999) R1839)

## Outlook:

- Measurements of the OMC rates for the $0\nu\beta^-\beta^-$-decay daughters ($0\nu\beta^+\beta^+$-decay mothers ) will yield important information on the (induced) axial couplings relevant for $0\nu\beta\beta$ decay
- Ongoing experiments, like the MONUMENT, will produce interesting new information on OMC strength functions and rates for transitions to individual states

# The (hopefully happy) end

THANKS FOR PATIENCE!