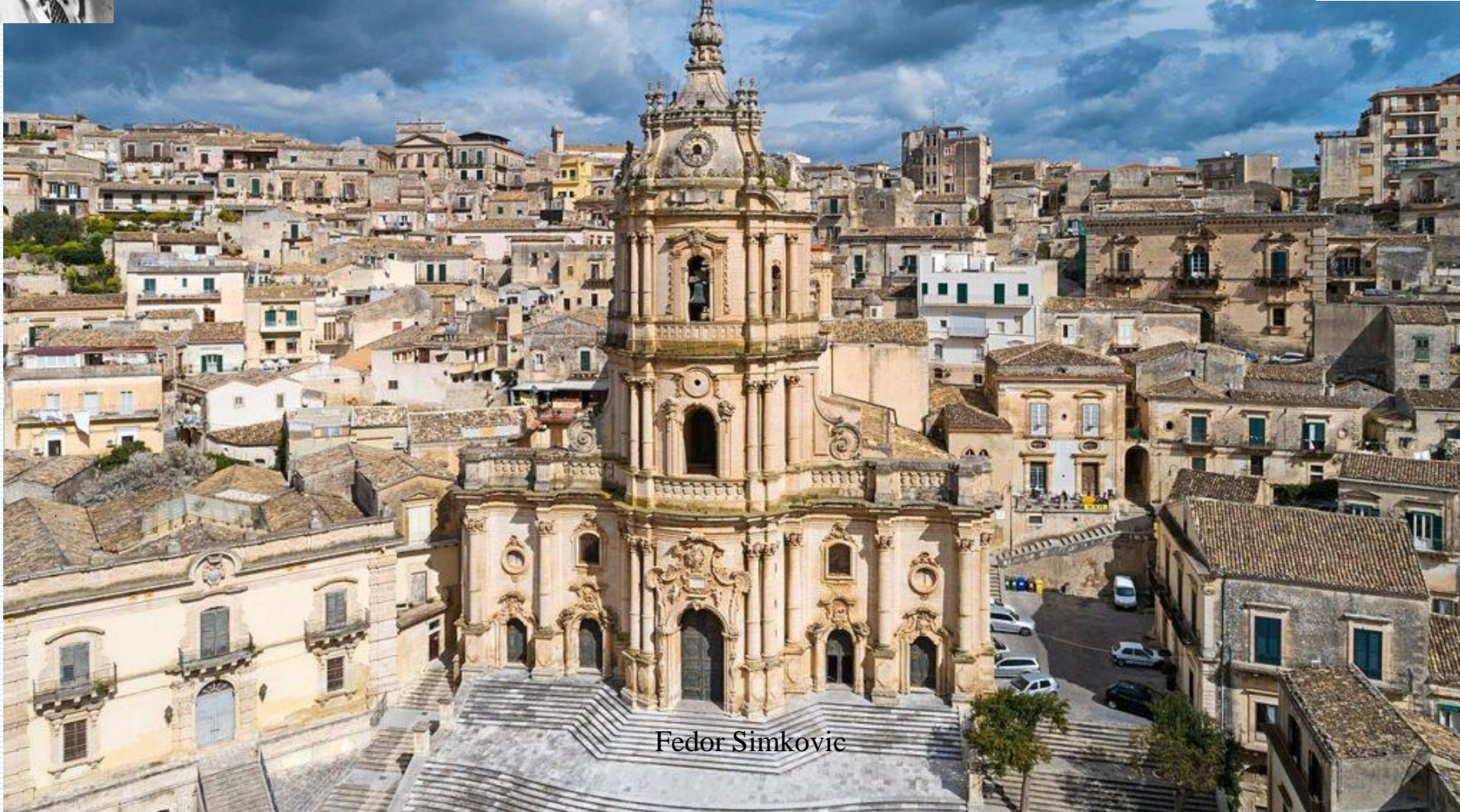
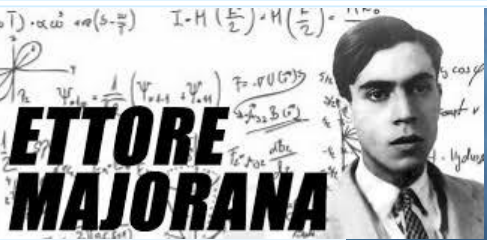
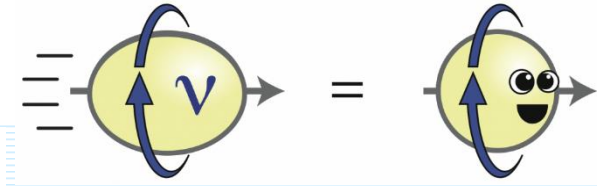
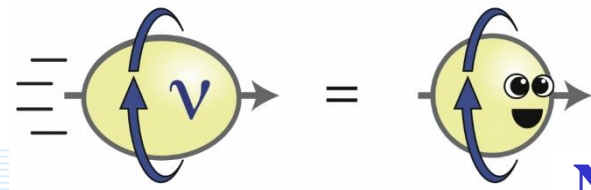


MAYORANA workshop, Modica, Sicily

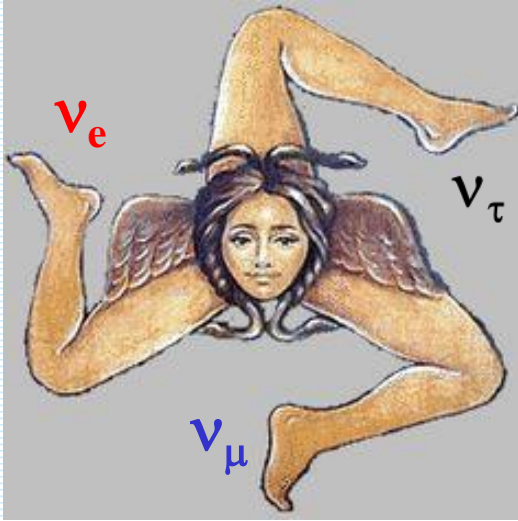
July 12-14, 2023

Neutrino masses, oscillations, and $0\nu\beta\beta$ -decay

Fedor Šimkovic



OUTLINE



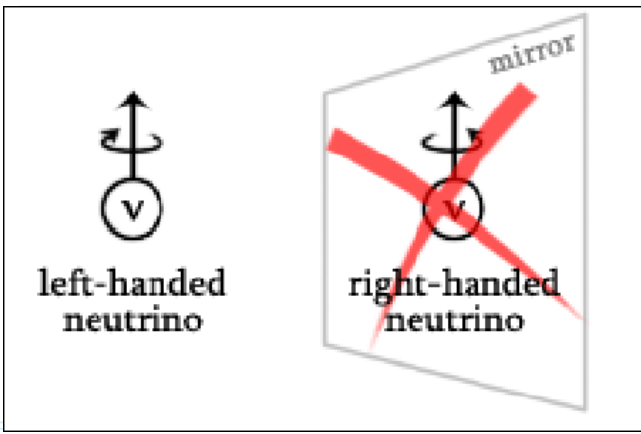
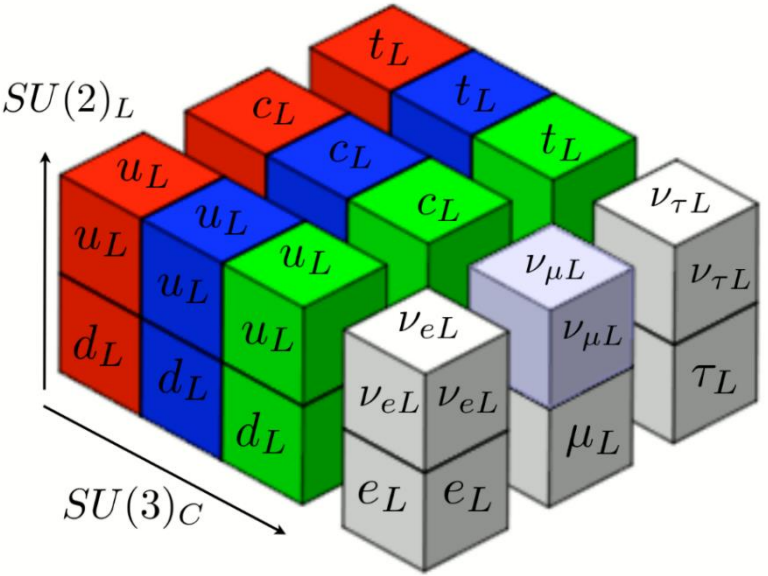
- I. Introduction*
- II. Neutrino oscillations as a single Feynman diagram
(QFT formalism, nu-antineu oscil.)*
- III. $0\nu\beta\beta$ -decay mechanisms
(QCSS scenario, Quasi-Dirac ν)*
- IV. The $0\nu\beta\beta$ -decay NME, effective g_A , and supporting
nuclear physics experiments
(muon capture, DCE)*
- V. The $2\nu\beta\beta$ -decay and new physics
(sterile heavy ν , right-handed ν)*
- V. Outlook*

Acknowledgements: F. Deppisch, R. Dvornický, L. Graf, A. Khatun, S. Kovalenko, M. Krivoruchenko, A. Smetana, P. Vogel, and other colleagues and friends.

Standard Model
 (an astonishing successful theory,
 based on few principles)

ν is a **special particle** in SM:

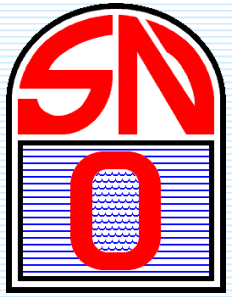
- It is the only fermion that **does not carry electric charge** (like γ , g , H^0)
- There are only **left-handed ν 's** (ν_{eL} , $\nu_{\mu L}$, $\nu_{\tau L}$)
- **ν -mass** can not be generated with any renormalizable coupling with the Higgs fields through SSB



ν 's oscillations experiments
 \Rightarrow tiny neutrino masses (!)
 \Rightarrow Beyond SM physics (!)



7/13/2023



, etc

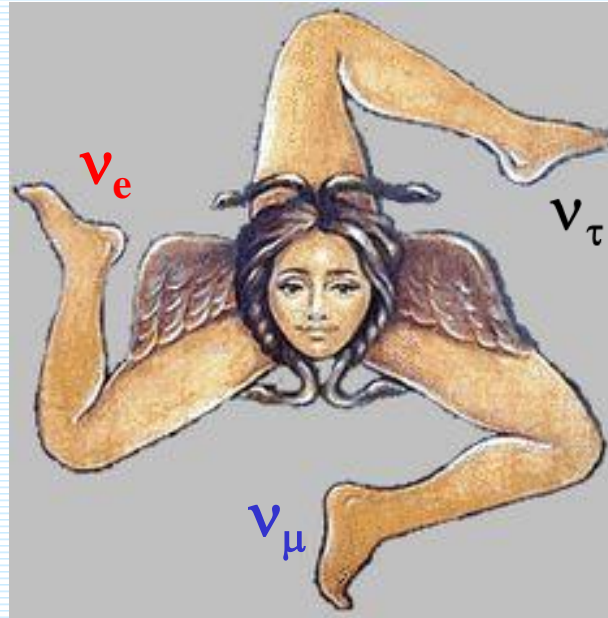


After 93/67 years we know

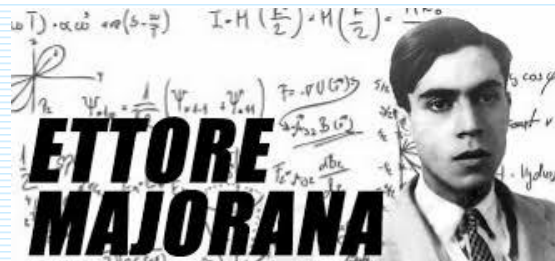
Fundamental ν properties

No answer yet

- 3 families of light (V-A) neutrinos:
 ν_e, ν_μ, ν_τ
- ν are massive:
we know mass squared differences
- relation between flavor states and mass states (neutrino mixing)

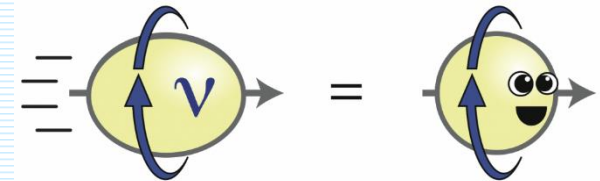


- Are ν Dirac or Majorana?
- Is there a CP violation in ν sector?
- Are neutrinos stable?
- What is the magnetic moment of ν ?
- **Sterile neutrinos?**
- Statistical properties of ν ? Fermionic or partly bosonic?



Currently main issue

Nature, Mass hierarchy, CP-properties, sterile ν



The observation of neutrino oscillations has opened a new excited era in neutrino physics and represents a big step forward in our knowledge of neutrino properties



Majorana fermions

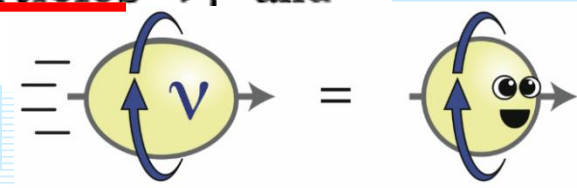
Ettore Majorana

Teoria simmetrica dell'elettrone e del positrone
(A symmetric theory of electrons and positrons).
Il Nuovo Cimento, 14: 171–184, 1937.) 171

ν is its own antiparticle

It follows from the above assumptions that in vacuum a neutrino can be transformed into an antineutrino and vice versa. This means that the neutrino and antineutrino are "mixed" particles, i.e., a symmetric and antisymmetric combination of two truly neutral Majorana particles ν_1 and ν_2 of different combined parity.⁵

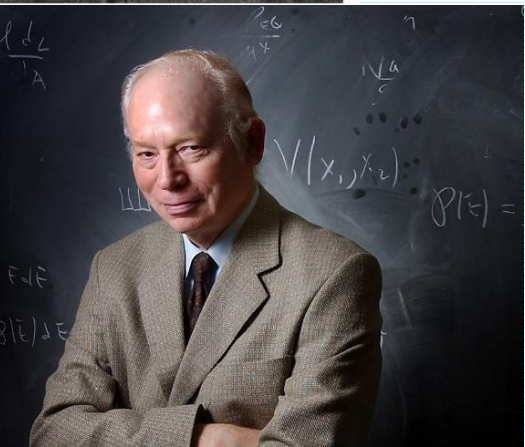
$\nu \leftrightarrow$ anti- ν oscillation



Bruno Pontecorvo
Inverse beta processes and nonconservation of lepton charge
Zhur. Eksptl'. i Teoret. Fiz. 34, 247 (1958)



Steve Weinberg
 ν -mass generation via d=5 eff. oper. related to unknown high energy scale (GUT?)



thought massless back in 1979. Weinberg does not take credit for predicting neutrino masses, but he thinks it's the right interpretation. What's more, he says, the non-renormalisable interaction that produces the neutrino masses is probably also accompanied with non-renormalisable interactions that produce proton decay and other things that haven't been observed, such as violation of baryon-number conservations. "We don't know anything about the details of those terms, but I'll swear they are there."

$$R_{12} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad R_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{12} & s_{12} \\ 0 & -s_{12} & c_{12} \end{pmatrix}$$

$$\tilde{R}_{13} = \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta} & 0 & c_{13} \end{pmatrix}$$

3 neutrino masses, 2 mass squared differences

$$\delta m^2 = m_2^2 - m_1^2, \quad \Delta m^2 = m_3^2 - (m_1^2 + m_2^2)/2$$

$$U = R_{23} \tilde{R}_{13} R_{12}$$

3 mixing angles
CP-phase

$$|\nu_\alpha\rangle = \sum_{j=1}^3 U_{\alpha j}^* |\nu_j\rangle$$

($\alpha = e, \mu, \tau$)

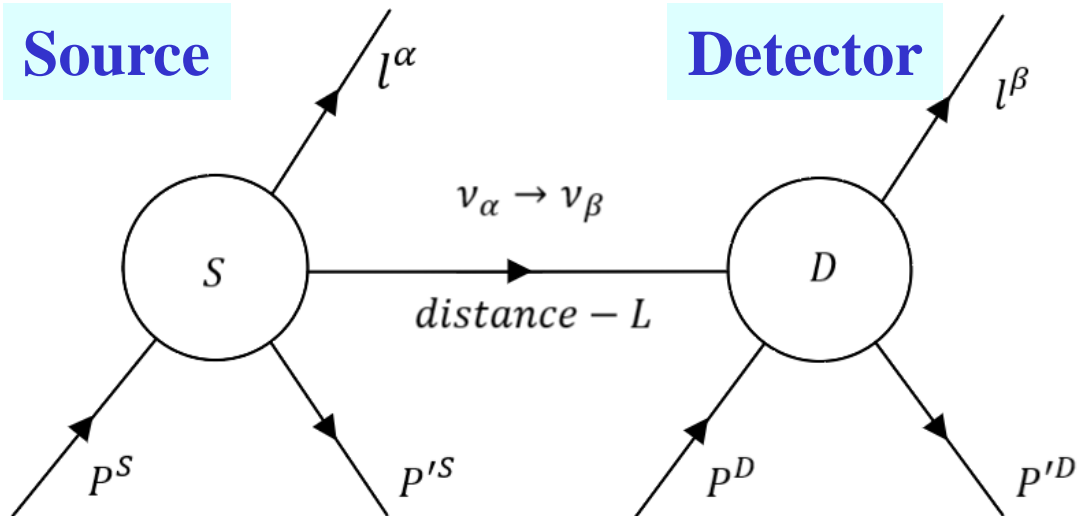
**Global neutrino
oscillations analysis
(PRD 101, 116013 (2020))**

	best - fit	1σ	2σ	3σ
Normal hierarchy (NH)				
$\delta m^2 / 10^{-5} \text{ eV}^2$	7.34	7.20-7.51	7.05-7.69	6.92-7.90
$\Delta m^2 / 10^{-3} \text{ eV}^2$	2.485	2.453-2.514	2.419-2.547	2.2389-2.578
$\sin^2 \theta_{12} / 10^{-1}$	3.05	2.92-3.19	2.78-3.32	2.65-3.47
$\sin^2 \theta_{13} / 10^{-2}$	2.22	2.14-2.28	2.07-2.34	2.01-2.41
$\sin^2 \theta_{23} / 10^{-1}$	5.45	4.98-5.65	4.54-5.81	4.36-5.95
δ / π	1.28	1.10-1.66	0.95-1.90	0-0.07 \oplus 0.81-2.00
Inverted hierarchy (IH)				
$\delta m^2 / 10^{-5} \text{ eV}^2$	7.34	7.20-7.51	7.05-7.69	6.92-7.91
$-\Delta m^2 / 10^{-3} \text{ eV}^2$	2.465	2.434-2.495	2.404-2.526	2.374-2.556
$\sin^2 \theta_{12} / 10^{-1}$	3.03	2.90-3.17	2.77-3.31	2.64-3.45
$\sin^2 \theta_{13} / 10^{-2}$	2.23	2.17-2.30	2.10-2.37	2.03-2.43
$\sin^2 \theta_{23} / 10^{-1}$	5.51	5.17-5.67	4.60-5.82	4.39-5.96
		\oplus 5.31-6.10		
δ / π	1.52	1.37-1.65	1.23-1.78	1.09-1.90

Neutrino oscillations (Quantum Mechanics Approach)

Source

Detector



Massive neutrinos and neutrino oscillations

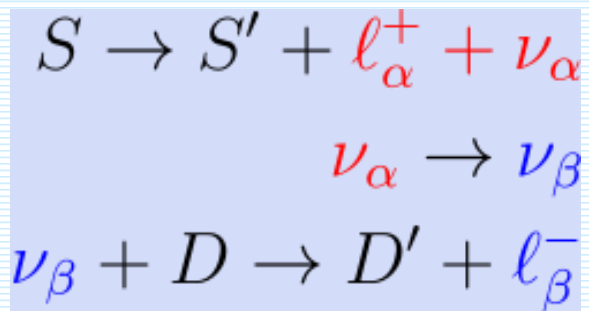
S. M. Bilenky

Joint Institute of Nuclear Research, Dubna, Union of Soviet Socialist Republics

S. T. Petcov

Institute of Nuclear Research and Nuclear Energy, Bulgarian Academy of Sciences, 1784 Sofia, People's Republic of Bulgaria

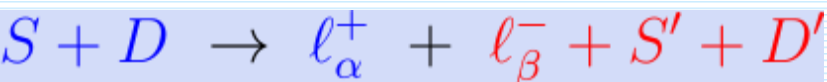
The theory of neutrino mixing and neutrino oscillations, as well as the properties of massive neutrinos (Dirac and Majorana), are reviewed. More specifically, the following topics are discussed in detail: (i) the possible types of neutrino mass terms; (ii) oscillations of neutrinos (iii) the implications of *CP* invariance for the mixing and oscillations of neutrinos in vacuum; (iv) possible varieties of massive neutrinos (Dirac, Majorana, pseudo-Dirac); (v) the physical differences between massive Dirac and massive Majorana neutrinos and the possibilities of distinguishing experimentally between them; (vi) the electromagnetic properties of massive neutrinos. Some of the proposed mechanisms of neutrino mass generation in gauge theories of the electroweak interaction and in grand unified theories are also discussed. The lepton number nonconserving processes $\mu \rightarrow e\gamma$ and $\mu \rightarrow 3e$ in theories with massive neutrinos are considered. The basic elements of the theory of neutrinoless double- β decay are discussed as well. Finally, the existing data on neutrino masses, oscillations of neutrinos, and neutrinoless double- β decay are briefly reviewed. The main emphasis in the review is on the general model-independent results of the theory. Detailed derivations of these are presented.



$$\Gamma_{osc} = \int \frac{d\Phi_{\nu}(E_{\nu})}{dE_{\nu}} \frac{\mathcal{P}_{\alpha\beta}(E_{\nu}, L)}{4\pi L^2} \sigma(E_{\nu}) dE_{\nu}$$

Rev. Mod. Phys.
59, 671 (1987)
961 citations
(inspire hep)

Process is governed by
the oscillation probability



Fedor Simkovic

$$\mathcal{P}_{\alpha\beta}(E_{\nu}, L) = \left| \sum_{j=1}^3 U_{\alpha j}^{*} U_{\beta j} e^{-i m_j^2 L / (2E_{\nu})} \right|^2$$

$$\langle f|S^{(2)}|i\rangle = -i \int d^4x_1 J_S^\mu(P'_S, P_S) e^{i(P_\alpha + P'_S - P_S) \cdot x_1} \times$$

$$\int d^4x_2 J_D^\mu(P'_D, P_D) e^{i(P_\beta + P'_D - P_D) \cdot x_2} \sum_{k=1}^3 U_{\alpha k}^* U_{\beta k} \times$$

$$\bar{v}(P_\alpha; \lambda_\alpha) \gamma_\mu (1 - \gamma_5) D(x_2 - x_1, m_k) (1 - \gamma_5) \gamma_\nu u(P_\beta; \lambda_\beta)$$

Neutrino oscillations as a single Feynman diagram
 (within QFT, Walter Grimus approach revisited)
 e-Print: [2212.13635](https://arxiv.org/abs/2212.13635) [hep-ph]

The neutrino emission and detection are identified with the charged-current vertices of a single second-order **Feynman diagram** for the underlying process, enclosing neutrino propagation between these two points.

~~$$D(x; m) = \theta(x_0) D^-(x; m) + \theta(-x_0) D^+(x; m),$$~~

$$D^\pm(x; m) = \int \frac{d\mathbf{q}}{(2\pi)^3} \frac{\mp(-\mathbf{q} \cdot \vec{\gamma} + \omega\gamma^0) + m}{2\omega} e^{\pm i(-\mathbf{q} \cdot \mathbf{x} + \omega x_0)}$$

Integration over time variables results in **energy conservation** and **energy denominator**

$$2\pi i \frac{\delta(E_\beta + E'_D - E_D + E_\alpha + E'_S - E_S)}{\omega + E_\alpha + E'_S - E_S + i\varepsilon}$$

Neutrino propagation

$$\int \frac{d\mathbf{q}}{(2\pi)^3} \frac{\not{q} + m_k}{2\omega(\omega + E_\alpha + E'_S - E_S + i\varepsilon)} e^{i\mathbf{q}\cdot(\mathbf{x}_2 - \mathbf{x}_1)}$$

$$\simeq \frac{1}{4\pi} \frac{e^{ip_k|\mathbf{x}_2 - \mathbf{x}_1|}}{|\mathbf{x}_2 - \mathbf{x}_1|} (Q_k + m_k) \simeq e^{i\mathbf{p}_k \cdot \mathbf{x}_D} e^{-i\mathbf{p}_k \cdot \mathbf{x}_S} \frac{e^{ip_k L}}{L} (Q_k + m_k)$$

$$Q_k \equiv (E_\nu, \mathbf{p}_k), \quad \mathbf{p}_k = p_k (\mathbf{x}_2 - \mathbf{x}_1) / |\mathbf{x}_2 - \mathbf{x}_1|, \quad p_k = \sqrt{E_\nu^2 - m_k^2}$$

$$E_\nu = E_S - E'_S - E_\alpha \text{ (source)} = E_\beta + E'_D - E_D \text{ (detector)}$$

Energy conservation

Amplitude (there is no factorization of source and detector!)

Energy conservation

Momentum conservation
at source

Momentum conservation
at detector

$$\langle f | S^{(2)} | i \rangle = (2\pi)^7 \delta(E_f - E_i) \sum_k U_{\alpha k} U_{\beta k}^* \frac{e^{i\mathbf{p}_k L}}{4\pi L} \times$$

$$T_k^{\alpha\beta} \delta_{V_S}^3(\mathbf{p}_k + \mathbf{p}_\alpha + \mathbf{p}'_S - \mathbf{p}_S) \delta_{V_D}^3(\mathbf{p}_\beta + \mathbf{p}'_D - \mathbf{p}_D - \mathbf{p}_k)$$

with

$$E_f - E_i = E_\beta + E'_D - E_D + E_\alpha + E'_S - E_S$$

$$T_k^{\alpha\beta} = J_S^\mu(P'_S, P_S) J_D^\nu(P'_D, P_D) \bar{v}(P_\alpha; \lambda_\alpha) \gamma_\mu (1 - \gamma_5) \not{Q}_k \gamma_\nu u(P_\beta; \lambda_\beta)$$

Master formula

$$d\Gamma^{\alpha\beta}(L) = \sum_{km} U_{\alpha k} U_{\beta k}^* U_{\alpha m} U_{\beta m}^* \frac{e^{i(p_k - p_m)L}}{4\pi L^2} \times \mathcal{F}_{km}^{\alpha\beta}$$

$$\delta(\mathbf{p}_k + \mathbf{p}_\alpha + \mathbf{p}'_S - \mathbf{p}_S) \delta(\mathbf{p}_\beta + \mathbf{p}'_D - \mathbf{p}_D - \mathbf{p}_m)$$

$$\frac{(2\pi)^7}{4E_S E_D} \delta(E_\beta + E'_D - E_D + E_\alpha + E'_S - E_S) \times$$

$$\frac{1}{\hat{J}_S \hat{J}_D} \frac{d\mathbf{p}_\alpha}{2E_\alpha (2\pi)^3} \frac{d\mathbf{p}_\beta}{2E_\beta (2\pi)^3} \frac{d\mathbf{p}'_S}{2E'_S (2\pi)^3} \frac{d\mathbf{p}'_D}{2E'_D (2\pi)^3}$$

with

$$\mathcal{F}_{km}^{\alpha\beta} = 4\pi \sum_{\text{spin}} \frac{1}{2} \left(T_k^{\alpha\beta} (T_m^{\alpha\beta})^* + T_m^{\alpha\beta} (T_k^{\alpha\beta})^* \right)$$

$$\langle \Phi^{S,D}(\mathbf{P}_i) | \Phi^{S,D}(\mathbf{P}_k) \rangle = (2\pi)^3 2E_k \delta_{V_{S,D}}^3(\mathbf{P}_i - \mathbf{P}_k)$$

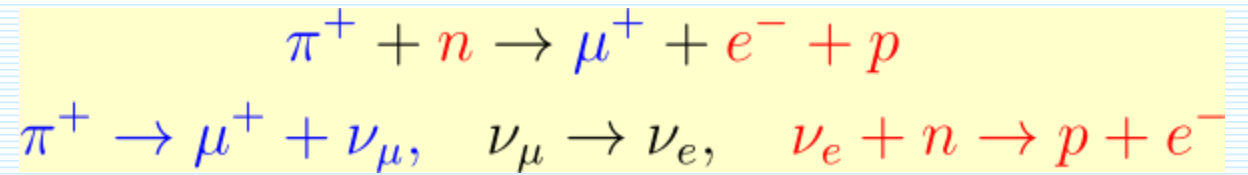
Two normalization volumes:

- i) source;
- ii) Detector.

$$\delta_V^3(\mathbf{Q}_n - \mathbf{P}) \delta_V^3(\mathbf{Q}_m - \mathbf{P}) \simeq$$

$$\frac{V}{(2\pi)^3} \frac{1}{2} \left(\delta_V^3(\mathbf{Q}_n - \mathbf{P}) + \delta_V^3(\mathbf{Q}_m - \mathbf{P}) \right)$$

An example:
 e-Print: [2212.13635](https://arxiv.org/abs/2212.13635) [hep-ph]



$$\Gamma_{osc}^{\pi^+ n} = \int \frac{d\Phi_\nu(E'_\nu)}{dE'_\nu} \frac{\mathcal{P}_{\nu_\mu \nu_e}(E'_\nu)}{4\pi L^2} \sigma(E'_\nu) dE'_\nu$$

$$= \frac{1}{2\pi^2} G_\beta^2 \left(\frac{f_\pi}{\sqrt{2}} \right)^2 \frac{m_\mu^2}{m_\pi} E_\nu^2 \frac{P_{\nu_\mu \nu_e}(E_\nu)}{4\pi L^2} (g_V^2 + 3g_A^2) p_e E_e$$

with

$$\mathcal{P}_{\alpha\beta}(E_\nu, L) = \left| \sum_{j=1}^3 U_{\alpha j}^* U_{\beta j} e^{-im_j^2 L / (2E_\nu)} \right|^2$$

**Standard QM
 approach**

**New QFT
 approach**

**(no decoherence,
 no factorization of
 two processes)**

$$\Gamma_{QFT}^{\pi^+ n} = \frac{1}{2\pi^2} G_\beta^2 \left(\frac{f_\pi}{\sqrt{2}} \right)^2 \frac{m_\mu^2}{m_\pi} E_\nu^2 \frac{\mathcal{P}_{\mu e}^{QFT}(E_\nu)}{4\pi L^2} (g_V^2 + 3g_A^2) p_e E_e$$

with

$$\mathcal{P}_{\alpha\beta}^{QFT}(E_\nu) = \frac{1}{2} \sum_{km} U_{\beta k} U_{\alpha k}^* U_{\beta k}^* U_{\alpha k} e^{i(p_m - p_k)L} \left(1 + \frac{p_k p_m}{E_\nu^2} \right)$$

Nuovo Cim. 14,
322 (1937)



neutrino ↔ antineutrinos oscillations

Second order process
with real intermediate neutrinos

$$S + D \rightarrow \ell_{\alpha}^{+} + \ell_{\beta}^{+} + S' + D'$$

$$S \rightarrow S' + \ell_{\alpha}^{+} + \nu_{\alpha}, \quad \nu_{\alpha} \rightarrow \bar{\nu}_{\beta}, \quad \bar{\nu}_{\beta} + D \rightarrow D' + \ell_{\beta}^{+}$$

Amplitude proportional to **v-mass**

$$T_k^{\alpha\beta} = J_S^{\mu}(P'_S, P_S) J_D^{\nu}(P'_D, P_D) \times \bar{v}(P_{\alpha}; \lambda_{\alpha}) \gamma_{\mu} (1 - \gamma_5) m_k \gamma_{\nu} u(P_{\beta}; \lambda_{\beta})$$

Replacement:

$$U_{\alpha k} \rightarrow U_{\alpha k}^{*}$$
$$U_{\beta m}^{*} \rightarrow U_{\beta m}$$

Particular process:

$$\pi^{+} + p \rightarrow \mu^{+} + e^{+} + n$$

Production rate

$$\Gamma_{QFT}^{\pi^+ p} = \frac{1}{2\pi^2} G_{\beta}^2 \left(\frac{f_{\pi}}{\sqrt{2}} \right)^2 \frac{m_{\mu}^2}{m_{\pi}} E_{\nu}^2 \frac{P_{\nu\mu\bar{\nu}e}^{QFT}(E_{\nu}, L)}{4\pi L^2} (g_V^2 + 3g_A^2) p_e E_e$$

Oscillation probability

$$\mathcal{P}_{\alpha\bar{\beta}}^{QFT}(E_{\nu}, L) \equiv |\langle \nu_{\beta} | \bar{\nu}_{\alpha} \rangle|^2$$
$$= \left| \sum_{j=1}^3 U_{\alpha j}^{*} U_{\beta j} \frac{m_j}{E_{\nu}} e^{-im_j^2 L / (2E_{\nu})} \right|^2$$

**Effective Majorana mass $m_{\beta\beta}$
can be strongly suppressed (!) ...**

For $L=0$

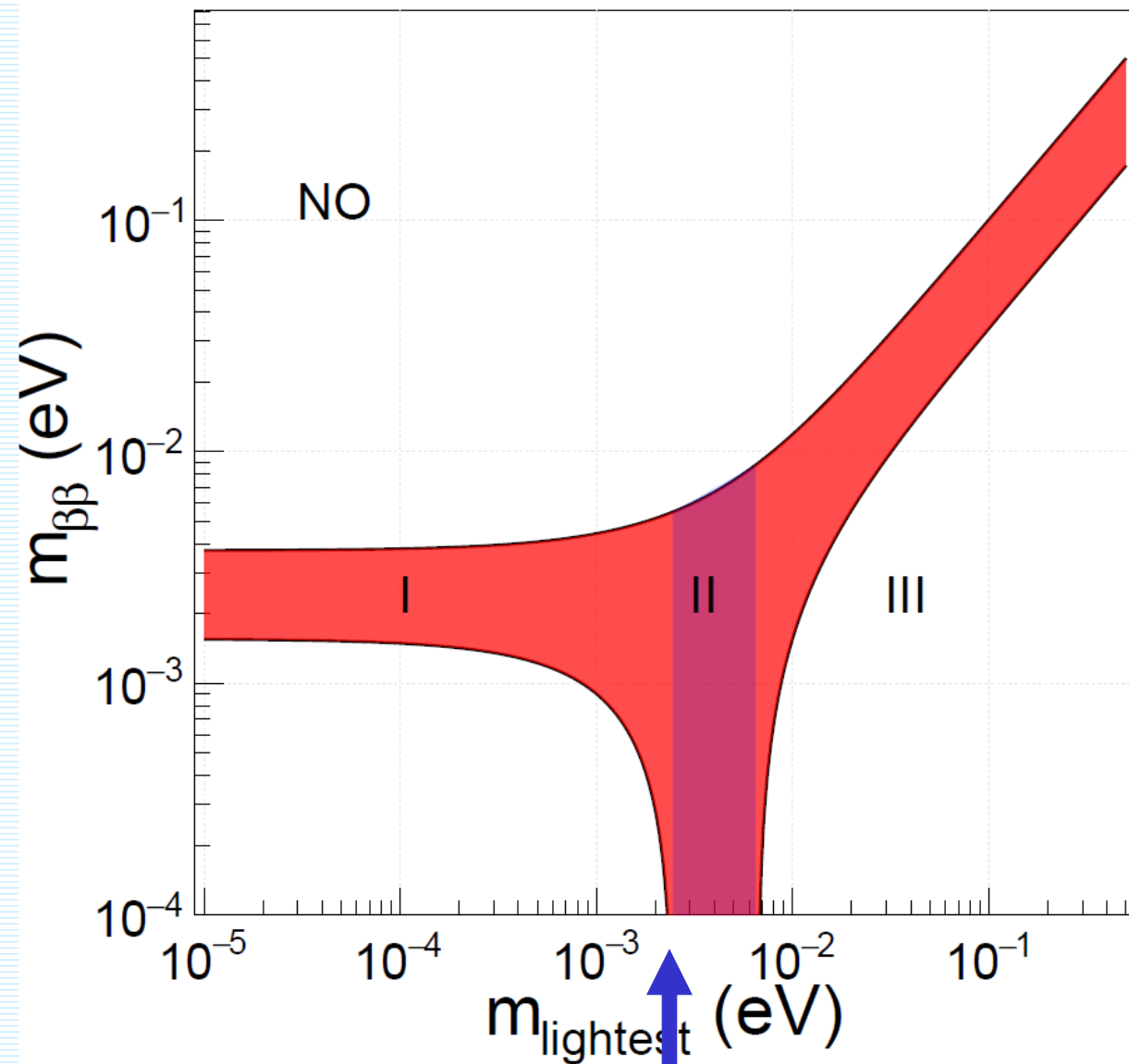
$$\mathcal{P}_{\alpha\bar{\beta}}(E_\nu, L=0) = \frac{m_{\beta\beta}^2}{E_\nu^2}$$

$$= \frac{1}{E_\nu^2} \left| \sum_{j=1}^3 U_{\alpha j} U_{\beta j} m_j \right|^2$$

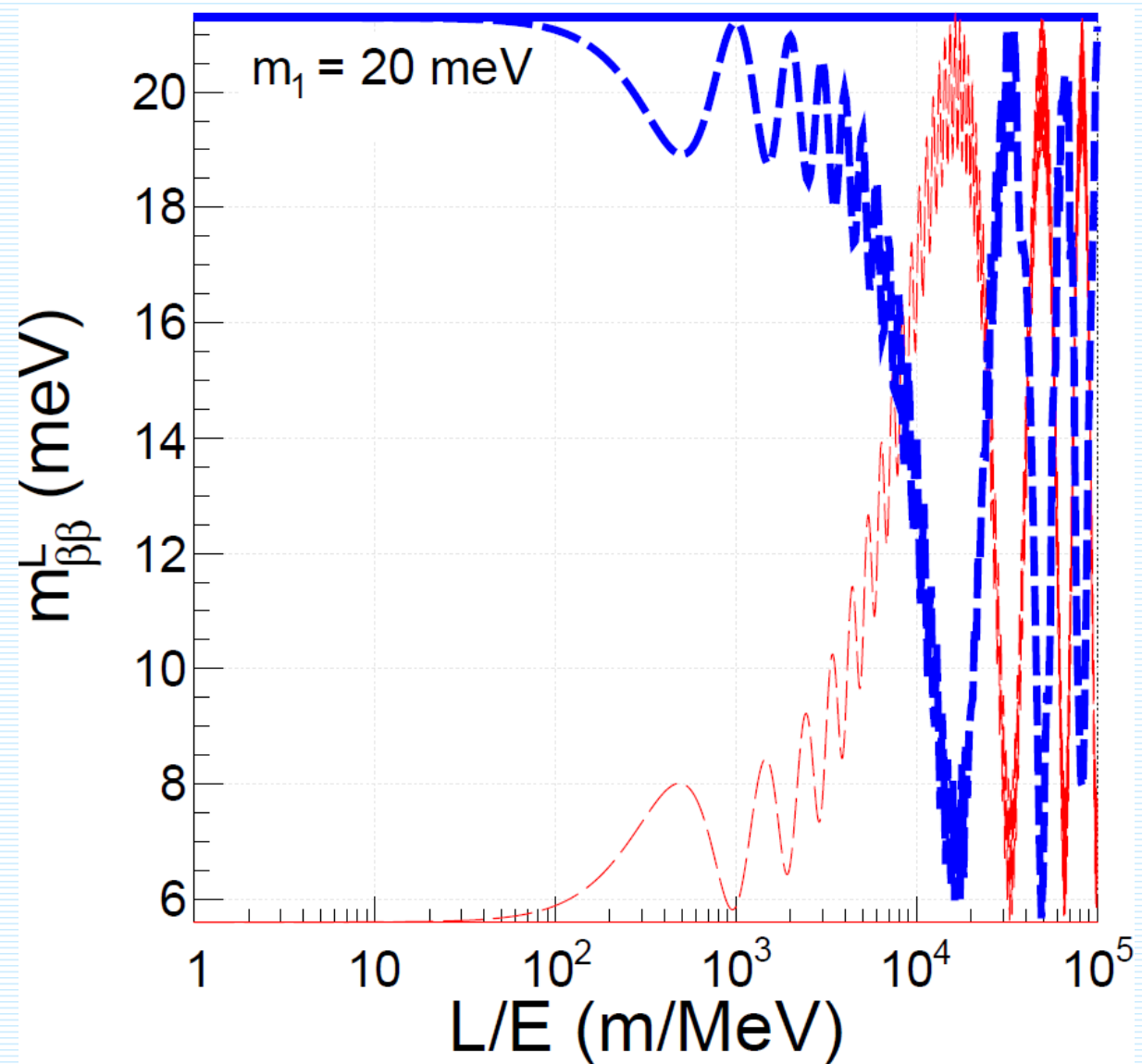
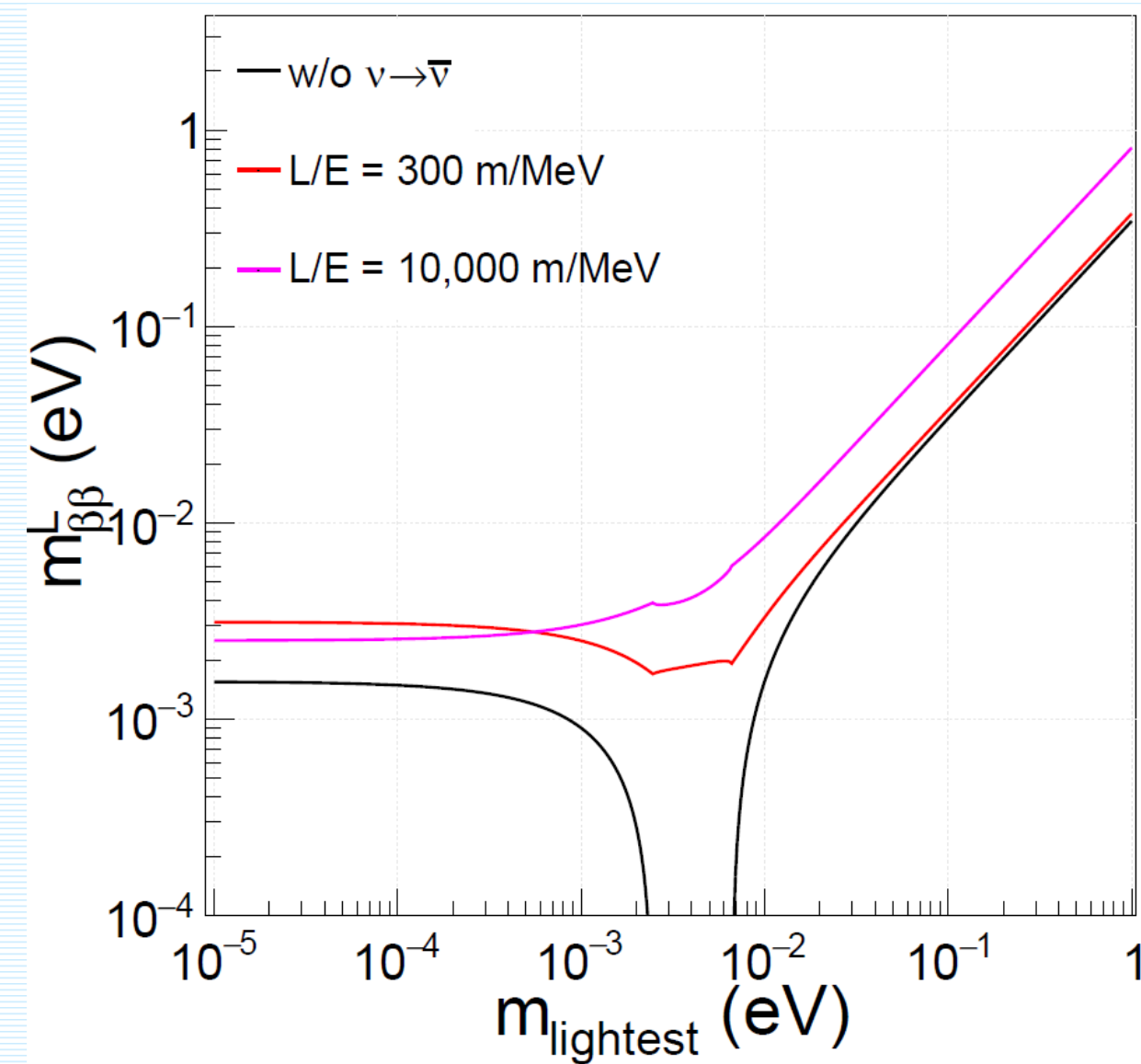
$$m_{\beta\beta} = |\rho_1 e^{2i\phi_1} + \rho_2 e^{2i\phi_2} + \rho_3|$$

$$\rho_1 = c_{12}^2 c_{13}^2 m_1, \quad \rho_2 = s_{12}^2 c_{13}^2 m_2, \quad \rho_3 = s_{13}^2 m_3$$

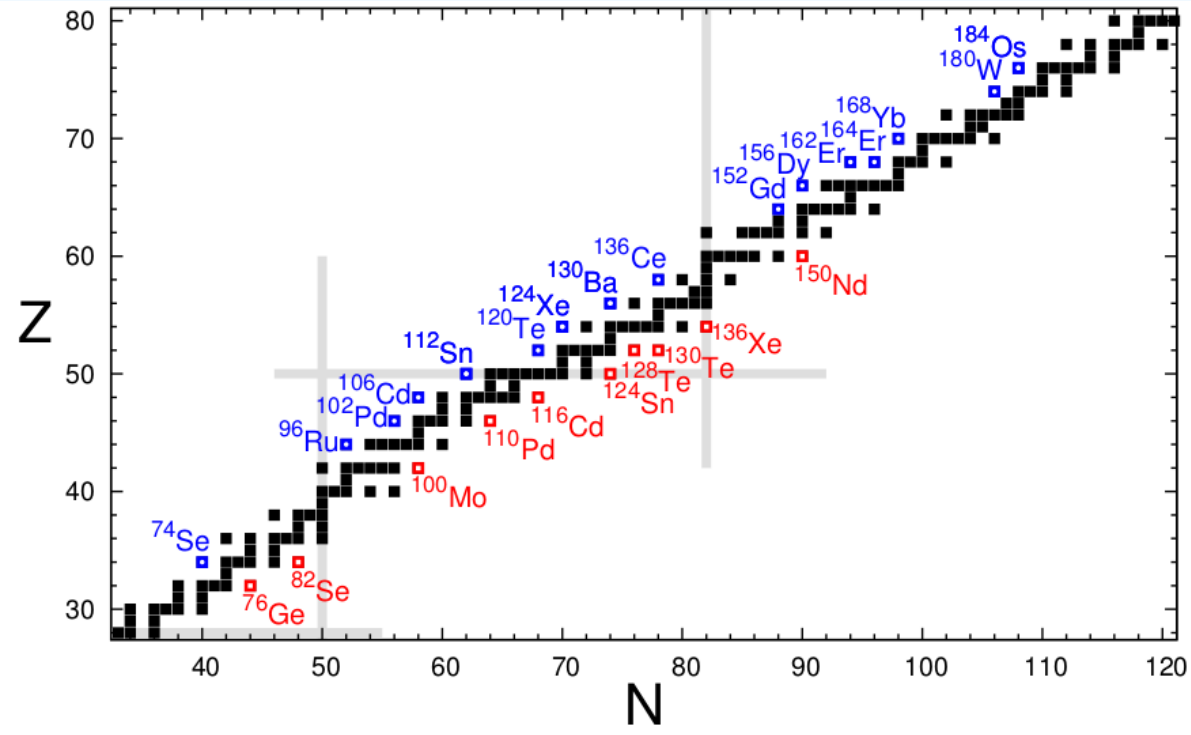
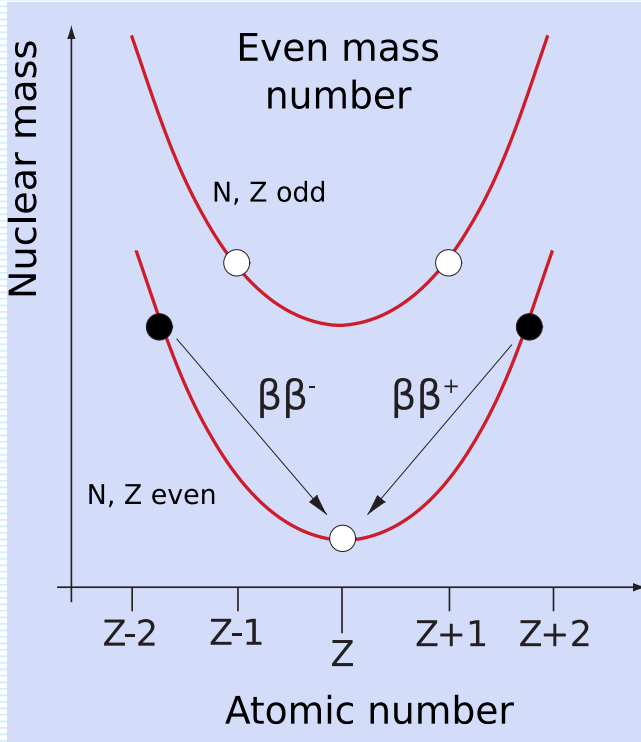
$$\min_{\phi_1, \phi_2} m_{\beta\beta} = \begin{cases} |\rho_2 - \rho_3| - \rho_1, & \text{if } \rho_1 < |\rho_2 - \rho_3| & \text{: region I,} \\ 0, & \text{if } |\rho_2 - \rho_3| \leq \rho_1 \leq \rho_2 + \rho_3 & \text{: region II,} \\ \rho_1 - (\rho_2 + \rho_3), & \text{if } \rho_2 + \rho_3 < \rho_1 & \text{: region III.} \end{cases}$$



Dependence of m_{ee}^L on m_{lightest} and L/E



Nuclear double- β decay
(even-even nuclei, pairing int.)

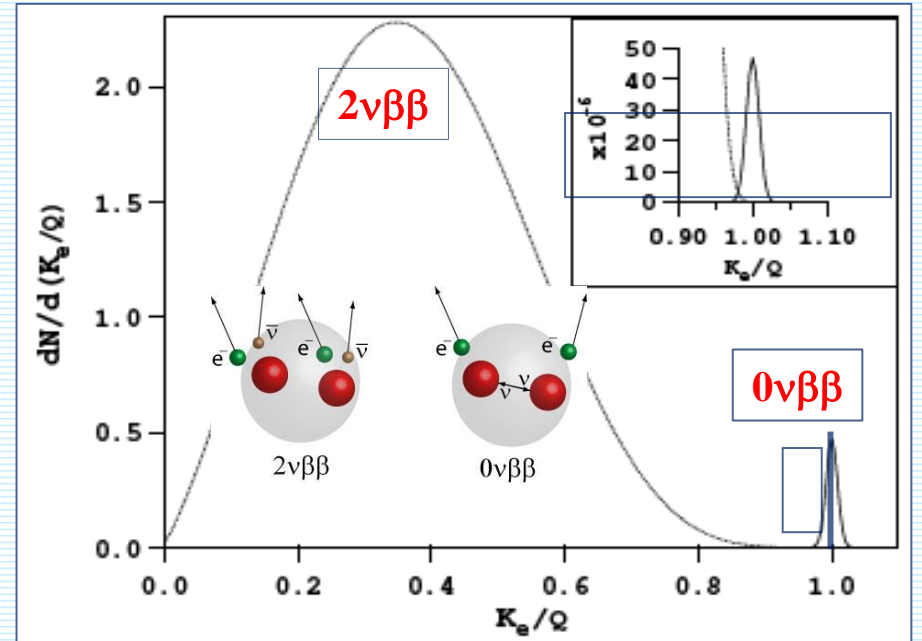


Phys. Rev. 48, 512 (1935)

Two-neutrino double- β decay – LN conserved
 $(A,Z) \rightarrow (A,Z+2) + e^- + e^- + \bar{\nu}_e + \nu_e$
 Goepert-Mayer – 1935. 1st observation in 1987

Phys. Rev. 56, 1184 (1939)

Neutrinoless double- β decay – LN violated
 $(A,Z) \rightarrow (A,Z+2) + e^- + e^-$ (Furry 1937)
 Not observed yet. Requires massive Majorana ν 's



$$(A,Z) \rightarrow (A,Z+2) + e^- + e^-$$

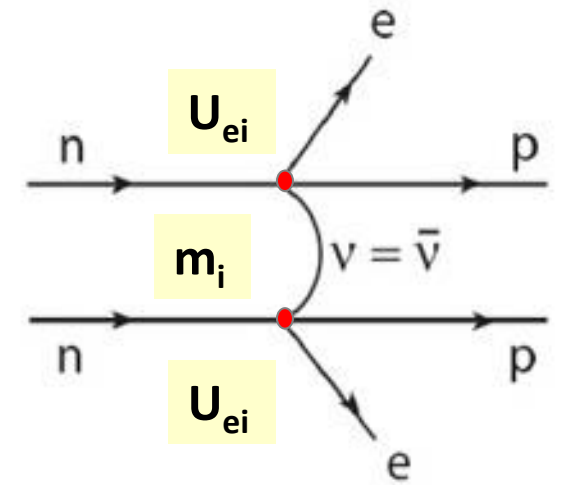
$0\nu\beta\beta$ -decay

(LNV at \approx GUT scale, exchange of three light ν)

$$\left(T_{1/2}^{0\nu}\right)^{-1} = \left|\frac{m_{\beta\beta}}{m_e}\right|^2 g_A^4 \left|M_\nu^{0\nu}\right|^2 G^{0\nu}$$

Phase space factor
well understood

*NME must be evaluated
using tools of nuclear theory*

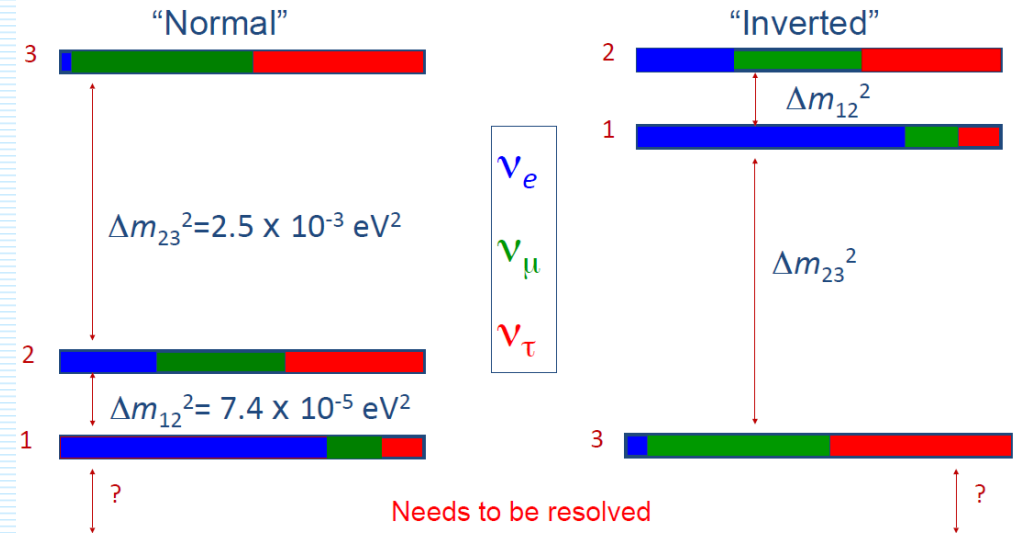


*Effective Majorana mass
can be evaluated. It depends on*

$m_1, m_2, m_3, \theta_{12}, \theta_{13}, \alpha_1, \alpha_2$

(3 unknown parameters: $m_1/m_3, \alpha_1, \alpha_2$
and ν -mass hierarchy)

$$m_{\beta\beta} = \left| c_{13}^2 c_{12}^2 e^{i\alpha_1} m_1 + c_{13}^2 s_{12}^2 e^{i\alpha_2} m_2 + s_{13}^2 m_3 \right|$$



$$U^{PMNS} = \begin{pmatrix} c_{12}c_{13} & c_{13}s_{12} & e^{-i\delta}s_{13} \\ -c_{23}s_{12} - e^{i\delta}c_{12}s_{13}s_{23} & c_{12}c_{23} - e^{i\delta}s_{12}s_{13}s_{23} & c_{13}s_{23} \\ s_{12}s_{23} - e^{i\delta}c_{12}c_{23}s_{13} & -e^{i\delta}c_{23}s_{12}s_{13} - c_{12}s_{23} & c_{13}c_{23} \end{pmatrix} \begin{pmatrix} e^{i\alpha_1} & 0 & 0 \\ 0 & e^{i\alpha_2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Effective Majorana

ν -mass $m_{\beta\beta}$

(prediction
due ν -oscillations)

Constraint from cosmology

$$\Sigma = m_1 + m_2 + m_3$$

$$< 0.90 \text{ eV}$$

$$< 0.26 \text{ eV (Planck coll.)}$$

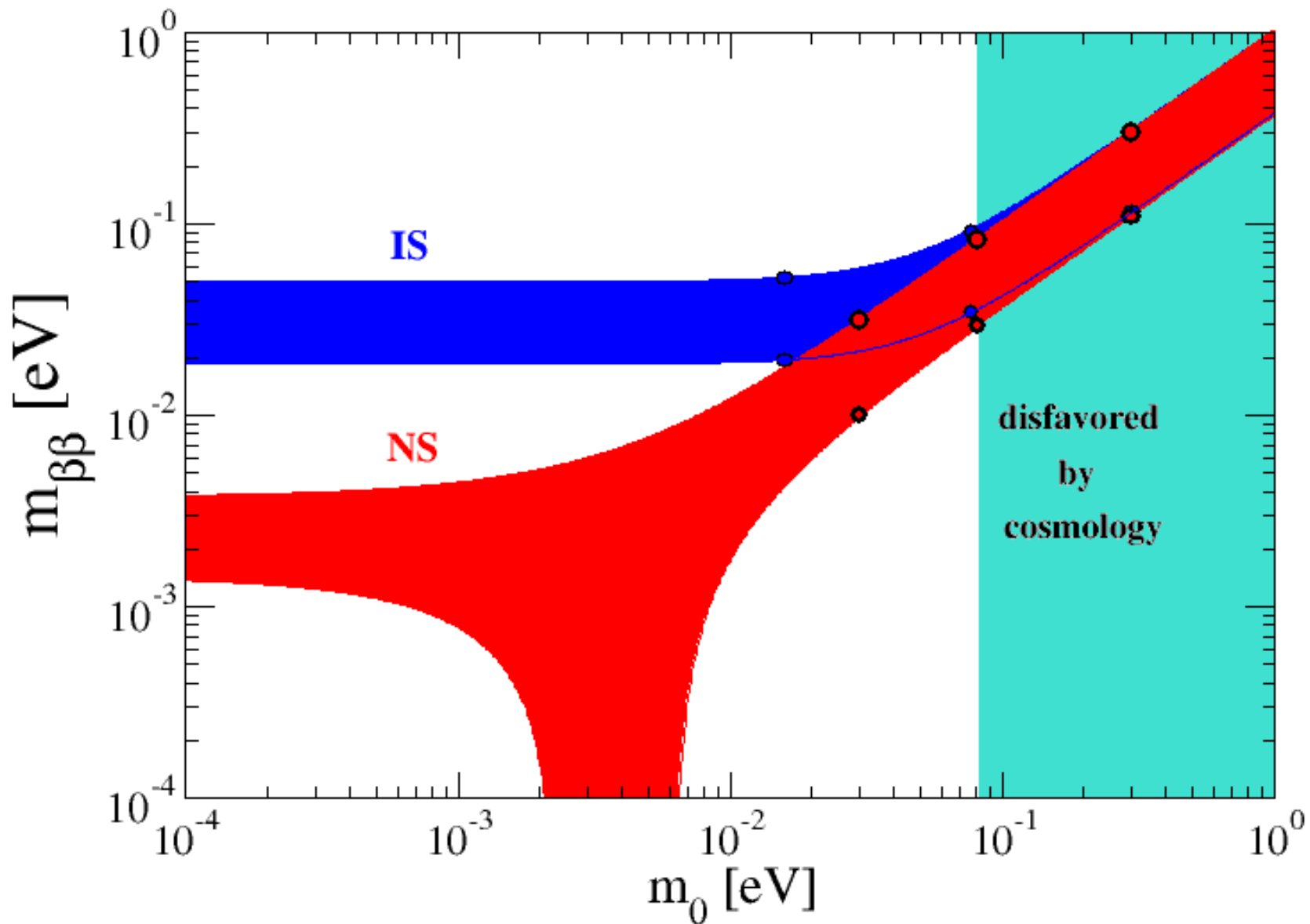
$$< 0.12 \text{ eV}$$

Contrary, the constraint from
 $0\nu\beta\beta$ -decay (KLZ)

$$m_{\beta\beta} < 0.036\text{-}0.156 \text{ eV}$$

implies

$$\Sigma < 0.12 \text{ eV}$$



$0\nu\beta\beta$ decay isotopes and experiments

[Current CANDLES detector]



CANDLE
CaF
scintillating
crystal



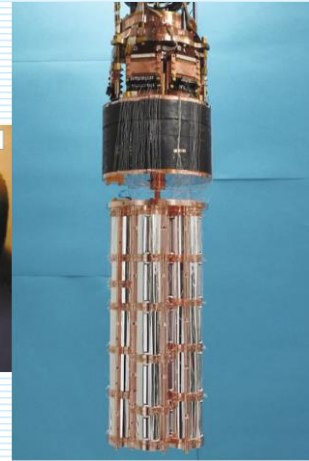
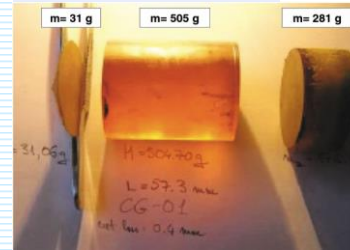
SuperNEMO
Se source foil

GERDA, MAJORANA
Ge crystal

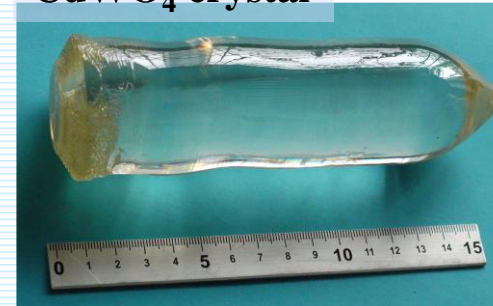


Candidates	$Q_{\beta\beta}$ (MeV)	N.A. (%)
$^{48}\text{Ca} \rightarrow ^{48}\text{Ti}$	4.268	0.187
$^{76}\text{Ge} \rightarrow ^{76}\text{Se}$	2.039	7.8
$^{82}\text{Se} \rightarrow ^{82}\text{Kr}$	2.998	8.8
$^{96}\text{Zr} \rightarrow ^{96}\text{Mo}$	3.356	2.8
$^{100}\text{Mo} \rightarrow ^{100}\text{Ru}$	3.034	9.7
$^{110}\text{Pd} \rightarrow ^{110}\text{Cd}$	2.017	11.7
$^{116}\text{Cd} \rightarrow ^{116}\text{Sn}$	2.813	7.5
$^{124}\text{Sn} \rightarrow ^{124}\text{Te}$	2.293	5.8
$^{130}\text{Te} \rightarrow ^{130}\text{Xe}$	2.528	34.1
$^{136}\text{Xe} \rightarrow ^{136}\text{Ba}$	2.458	8.9
$^{150}\text{Nd} \rightarrow ^{150}\text{Sm}$	3.371	5.6

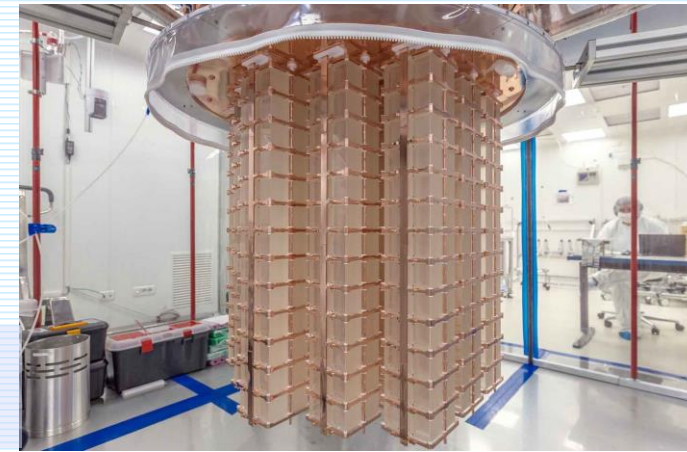
CUPID-0
ZnSe
scintillating
crystal



Aurora
 CdWO_4 crystal



CUORE
 TeO_2 crystal



Amore
 CaMoO_4 crystal



EXO, KamLAND-Zen
Liquid Xe



Leading limits in each $0\nu\beta\beta$ isotope (unquenched g_A)

A monoenergetic peak at the Q-value is searched for.
Need a large amount of decay isotope and low radioactive environment

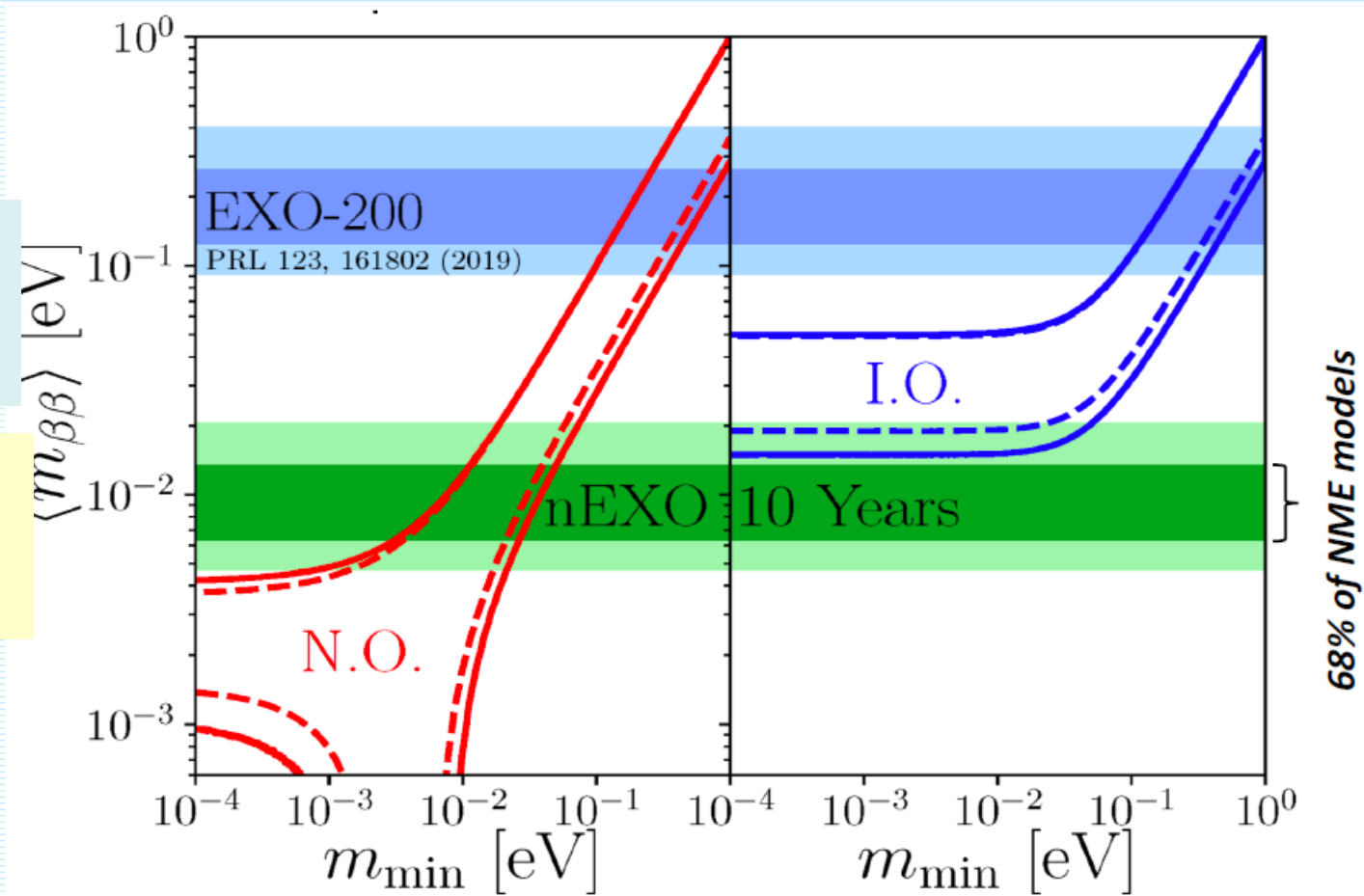
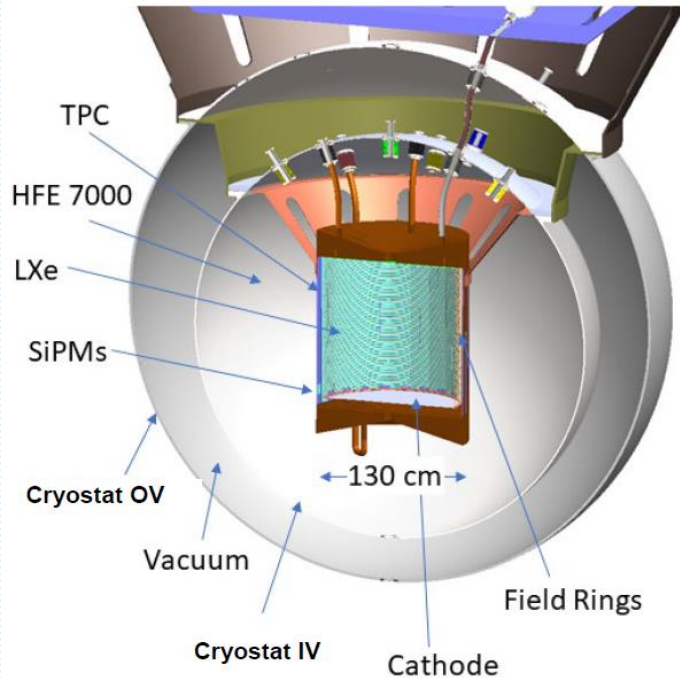
Experiment	Isotope	Exposure [kg yr]	$T_{1/2}^{0\nu}$ [10^{25} yr]	$m_{\beta\beta}$ [meV]
Gerda	^{76}Ge	127.2	18	79-180
Majorana	^{76}Ge	26	2.7	200-433
CUPID-0	^{82}Se	5.29	0.47	276-570
NEMO3	^{100}Mo	34.3	0.15	620-1000
CUPID-Mo	^{100}Mo	2.71	0.18	280-490
Amore	^{100}Mo	111	0.095	1200-2100
CUORE	^{130}Te	1038.4	2.2	90-305
EXO-200	^{136}Xe	234.1	3.5	93-286
KamLAND-Zen	^{136}Xe	970	23	36-156

nEXO

5 ton-class ^{136}Xe $0\nu\beta\beta$ experiment

EXO-200, 1st 100 kg-class $0\nu\beta\beta$ -experiment, excellent background-essential for nEXO design, Sensitivity increased linearly with exposure.

nEXO, discovery $0\nu\beta\beta$ experiment, reaches sensitivity of 10^{28} yr in 6.5 yr data taking, probes $m_{\beta\beta}$ down to 15 meV, scalable experiment.

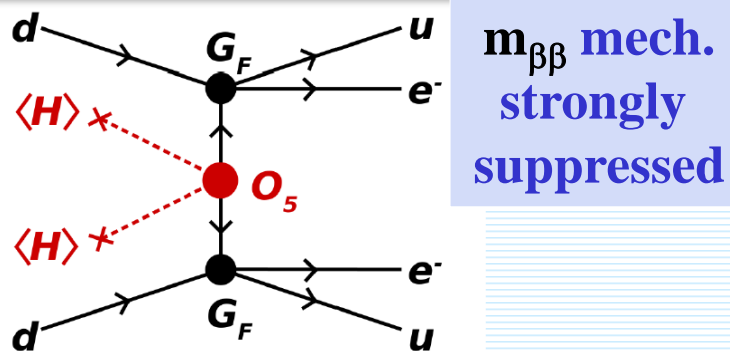
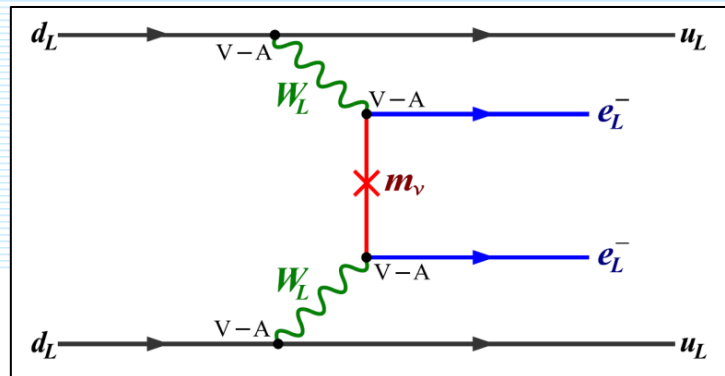


	isotope	$m_{\beta\beta}$ [meV] 90% excl. sensitivity	$m_{\beta\beta}$ [meV] 3 σ discovery potential
Legend	^{76}Se	8.2	11.1
CUPID	^{100}Mo	11.1	12.0
nEXO	^{136}Xe	12.9	15.0

$0\nu\beta\beta$ governed by exotic mechanisms

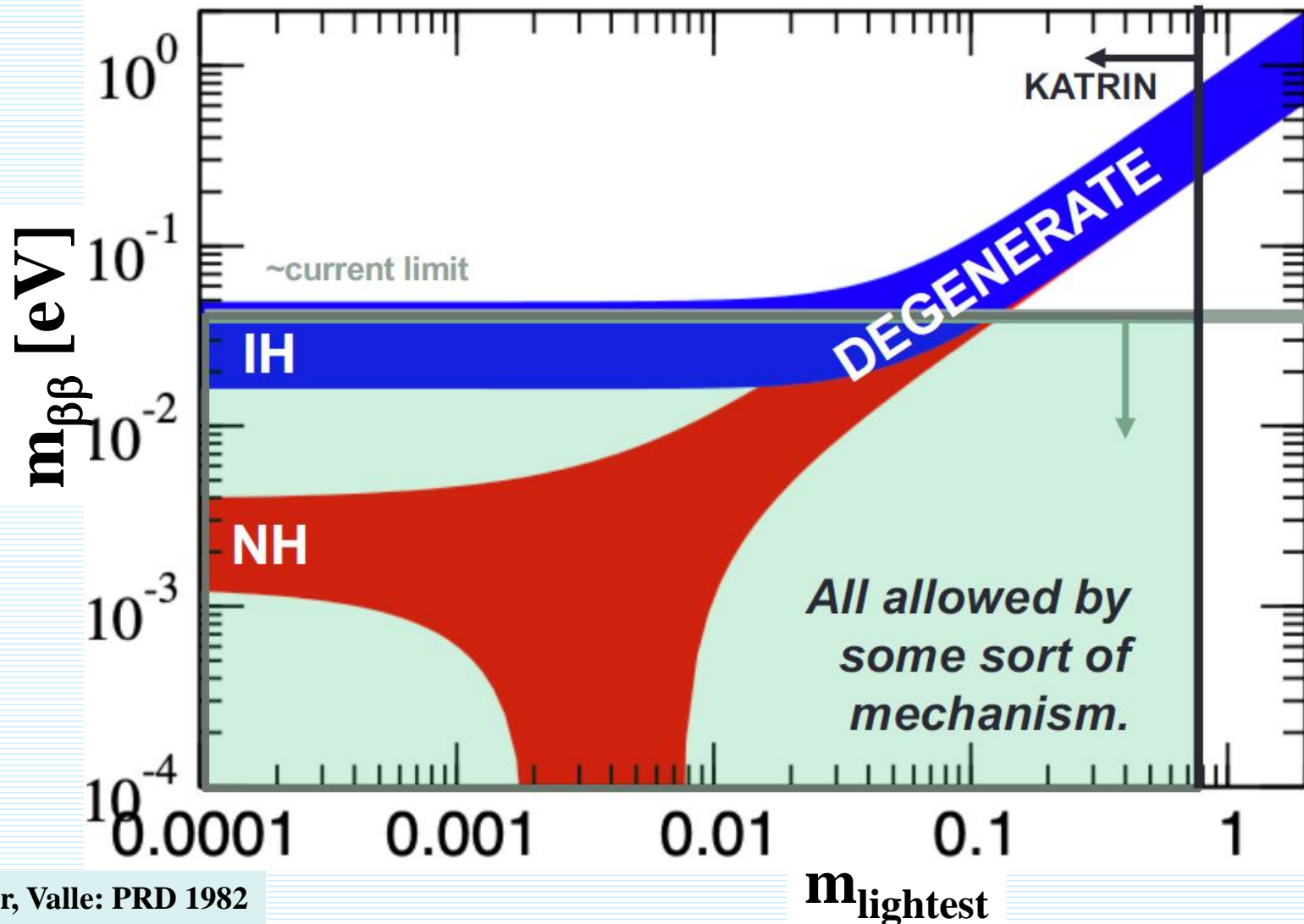
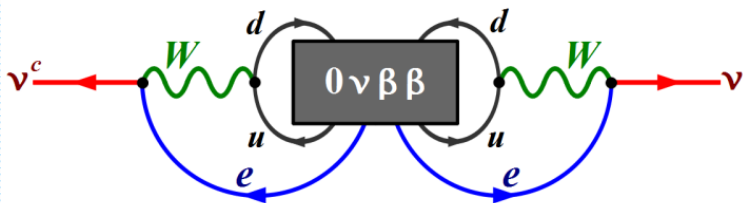
Light ν -mass mechanism can be strongly suppressed: $m_{\beta\beta} < 1$ meV

- It is not possible to discover $0\nu\beta\beta$ with 10-100 ton-class experiment
- It should be a **subject of theory** to justify it
- There might be a dominance of other $0\nu\beta\beta$ mechanisms



$m_{\beta\beta}$ mech. strongly suppressed

Any $0\nu\beta\beta$ mech. generates a small correction to ν -mass



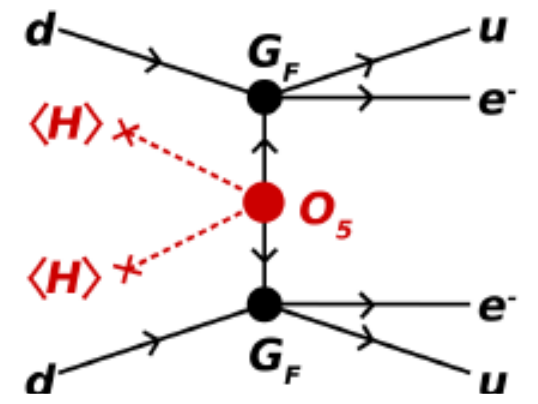
Beyond the SM physics

$$\mathcal{L} = \mathcal{L}_{SM}^{(4)} + \frac{1}{\Lambda} \sum_i c_i^{(5)} \mathcal{O}_i^{(5)} + \frac{1}{\Lambda^2} \sum_i c_i^{(6)} \mathcal{O}_i^{(6)} + O\left(\frac{1}{\Lambda^3}\right)$$

Amplitude for $(A,Z) \rightarrow (A,Z+2) + 2e^-$ can be divided into:

long range: $d=7$

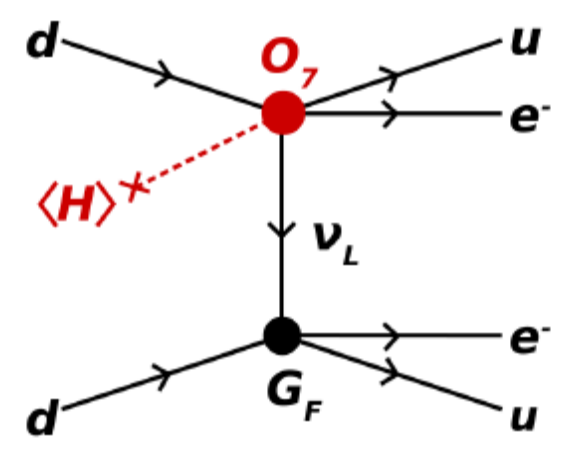
mass mechanism: $d=5$



$$\mathcal{O}_W \propto \frac{c_{ij}}{\Lambda} (L_i H)(L_j H)$$

Weinberg, 1979

+

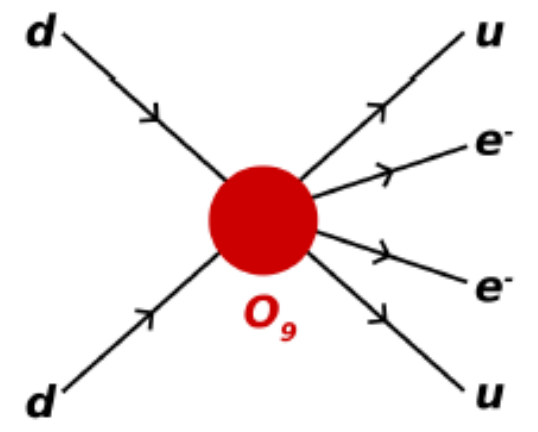


- $\mathcal{O}_2 \propto LLLe^c H$
- $\mathcal{O}_3 \propto LLQd^c H$
- $\mathcal{O}_4 \propto LL\bar{Q}\bar{u}^c H$
- $\mathcal{O}_8 \propto L\bar{e}^c \bar{u}^c d^c H$

Babu, Leung: 2001
de Gouvea, Jenkins: 2007

short range: $d=9$ ($d=11$)

+



- $\mathcal{O}_5 \propto LLQd^c H H H^\dagger$
- $\mathcal{O}_6 \propto LL\bar{Q}\bar{u}^c H H^\dagger H$
- $\mathcal{O}_7 \propto LQ\bar{e}^c \bar{Q} H H H^\dagger$
- $\mathcal{O}_9 \propto LLLe^c Le^c$
- $\mathcal{O}_{10} \propto LLLe^c Qd^c$
- $\mathcal{O}_{11} \propto LLQd^c Qd^c$
-

Valle

Quark Condensate Seesaw Mechanism for Neutrino Mass

PRD 103, 015007 (2021).

This operator contributes to the **Majorana-neutrino mass matrix** due to chiral symmetry breaking via the **light-quark condensate**.

The SM gauge-invariant effective operators

$$\mathcal{O}_7^{u,d} = \frac{\tilde{g}_{\alpha\beta}^{u,d}}{\Lambda^3} \overline{L}_\alpha^C L_\beta H \left\{ (\overline{Q} u_R), (\overline{d}_R Q) \right\}$$

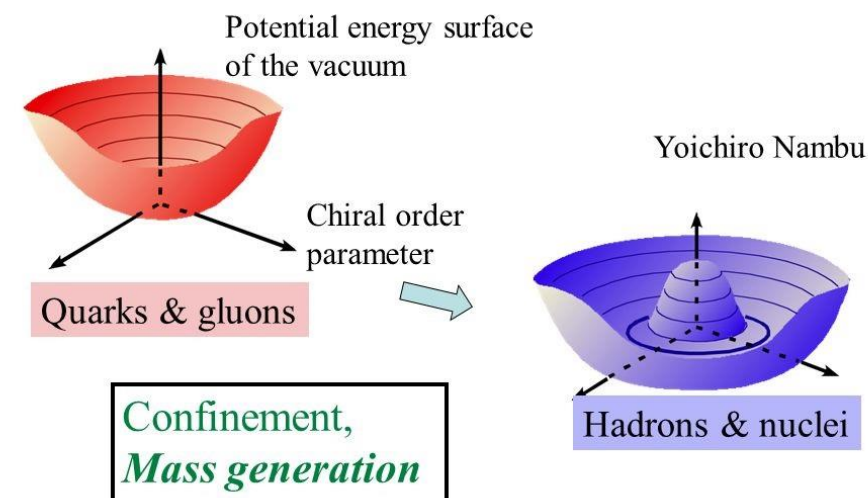
After the **EWSB** and **ChSB** one arrives at the Majorana mass matrix of active neutrinos

$$\begin{aligned} m_{\alpha\beta}^\nu &= g_{\alpha\beta} v \frac{\langle \overline{q}q \rangle}{\Lambda^3} \\ &= g_{\alpha\beta} v \left(\frac{\omega}{\Lambda} \right)^3 \end{aligned}$$

$$g_{\alpha\beta} = g_{\alpha\beta}^u + g_{\alpha\beta}^d, \quad v/\sqrt{2} = \langle H^0 \rangle$$

$$\omega = -\langle \overline{q}q \rangle^{1/3}, \quad \langle \overline{q}q \rangle^{1/3} \approx -283 \text{ MeV}_{\text{vic}}$$

Spontaneous breaking of *chiral* (χ) symmetry

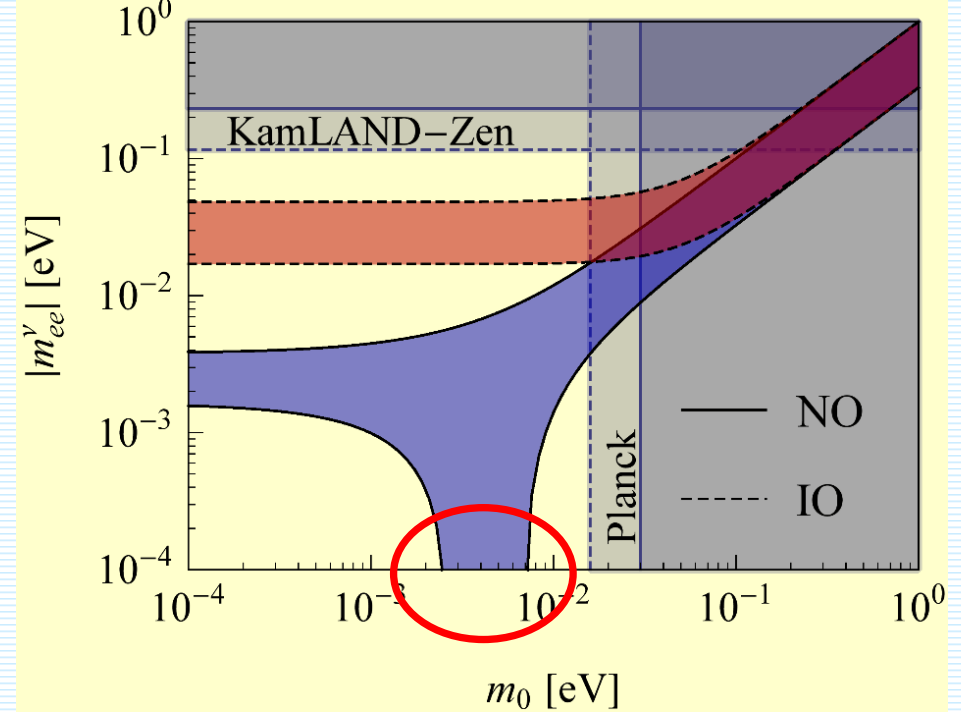


$\Lambda \sim$ **a few TeV**
we get the neutrino mass in the **sub-eV ballpark**

The genuine QCSS scenario
(predicts NH and ν -mass spectrum)

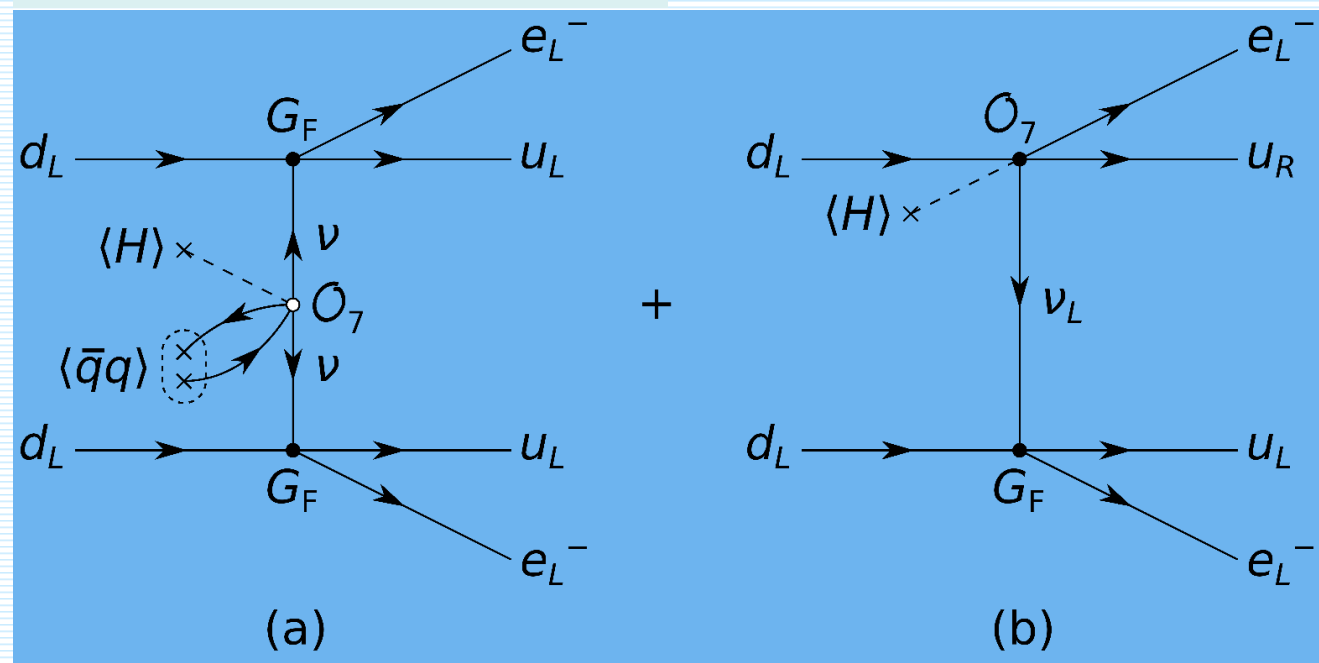
$$\mathcal{L}_7 = \frac{1}{\sqrt{2}} \sum_{\alpha\beta} \frac{v}{\Lambda^3} \overline{\nu_{\alpha L}^C} \nu_{\beta L} (g_{\alpha\beta}^u \overline{u}_L u_R + g_{\alpha\beta}^d \overline{d}_R d_L) + \text{H.c.}$$

$$m_{\alpha\beta}^\nu = -\frac{g_{\alpha\beta}}{\sqrt{2}} v \frac{\langle \bar{q}q \rangle}{\Lambda^3} = \frac{g_{\alpha\beta}}{\sqrt{2}} v \left(\frac{\omega}{\Lambda} \right)^3$$



(a) PRL 112, 142503 (2014).

(b) PLB 453, 194 (1999).



Neutrino spectrum (NH) !!!

- $2 \text{ meV} < m_1 < 7 \text{ meV}$
- $9 \text{ meV} < m_2 < 11 \text{ meV}$
- $50 \text{ meV} < m_3 < 51 \text{ meV}$

Prediction for m_β
 $9 \text{ meV} < m_\beta < 12 \text{ meV}$

Prediction for cosmology (Σ)
 $62 \text{ meV} < m_1 + m_2 + m_3 < 69 \text{ meV}$

Six Quasi-Dirac neutrinos and $0\nu\beta\beta$ -decay

Symmetry 12, 1310 (2020).

M_D - 3x3 complex matrix (18 real numb.)

$M_{L,R}$ - 3x3 symmetric matrix (12 real numb.)
(42 parameters)

Dirac-Majorana mass term

$$\mathcal{L}_m = \frac{1}{2} \begin{pmatrix} \overline{\nu}_L & \overline{\nu}_R^c \end{pmatrix} \mathcal{M} \begin{pmatrix} \nu_L^c \\ \nu_R \end{pmatrix} + h.c.$$

$$U^T \tilde{M} U = \mathcal{M}$$

Diagonalization: 6x6 unitary mixing matrix
(15 mixing angles plus 15 phases)

$$U = \mathcal{X} \cdot A \cdot S$$

Product of 3 unitary matrices.
A and **S** mix exclusively active
and sterile neutrino flavors, each
given by 3 angles and 3 phases.

$$A \equiv \begin{pmatrix} U^T & 0 \\ 0 & 1 \end{pmatrix}$$
$$S \equiv \begin{pmatrix} 1 & 0 \\ 0 & V^\dagger \end{pmatrix}$$

$$\mathcal{M} = \begin{pmatrix} M_L & M_D \\ M_D^T & M_R \end{pmatrix}$$

$$|M_{L,R}| \ll |M_D|$$

6 eigenvalues:

3 Dirac masses $m_{1,2,3}$, 3 mass splitting $\epsilon_{1,2,3}$

$$m_i^\pm = \pm m_i + \epsilon_i$$

$$\mathcal{X} = \begin{pmatrix} 1 & X^\dagger \\ -X & 1 \end{pmatrix} + O(X^2)$$

X given by 9 angles and 9 phases,
small parameters.

Simplified Quasi-Dirac neutrino mixing scheme (6x6 generalization of the PMNS matrix)

$$\mathcal{U}_{\text{QD}} = \frac{1}{\sqrt{2}} \begin{pmatrix} U & U \\ -V^* & V^* \end{pmatrix}$$

$$m_i^\pm = \pm m_i + \epsilon \quad (\epsilon > 0)$$

Oscillation probabilities among
active neutrinos

3 Dirac masses and 1 universal Majorana mass
splitting ϵ

$$\begin{aligned}
 P_{\alpha\beta} = & \delta_{\alpha\beta} - \sum_{i=1}^3 |U_{\alpha i}|^2 |U_{\beta i}|^2 \sin^2 \frac{m_i \epsilon}{E} L - \sum_{i>j=1}^3 \Re(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*) \left(\sin^2 \frac{\Delta m_{ij}^2 + 2\epsilon \Delta m_{ij}}{4E} L \right. \\
 & \left. + \sin^2 \frac{\Delta m_{ij}^2 - 2\epsilon \Sigma m_{ij}}{4E} L + \sin^2 \frac{\Delta m_{ij}^2 + 2\epsilon \Sigma m_{ij}}{4E} L + \sin^2 \frac{\Delta m_{ij}^2 - 2\epsilon \Delta m_{ij}}{4E} L \right) \\
 & + \frac{1}{2} \sum_{i>j=1}^3 \Im(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*) \left(\sin \frac{\Delta m_{ij}^2 + 2\epsilon \Delta m_{ij}}{2E} L + \sin \frac{\Delta m_{ij}^2 - 2\epsilon \Sigma m_{ij}}{2E} L \right. \\
 & \left. + \sin \frac{\Delta m_{ij}^2 + 2\epsilon \Sigma m_{ij}}{2E} L + \sin \frac{\Delta m_{ij}^2 - 2\epsilon \Delta m_{ij}}{2E} L \right)
 \end{aligned}$$

The survival probability of electron antineutrinos

Quasi-Dirac neutrinos and constraints on neutrino masses

$$\begin{aligned}
 P_{\bar{\nu}_e \rightarrow \bar{\nu}_e}(\epsilon \neq 0) &= P_{\bar{\nu}_e \rightarrow \bar{\nu}_e}(\epsilon = 0) - \frac{\epsilon^2 L^2}{E^2} \left[c_{13}^4 c_{12}^4 m_1^2 + c_{13}^4 s_{12}^4 m_2^2 + s_{13}^4 m_3^2 \right] \\
 &\quad - \frac{\epsilon^2 L^2}{4E^2} \left[4 c_{13}^4 s_{12}^2 c_{12}^2 \Sigma m_{21}^2 \cos \frac{\Delta m_{21}^2 L}{2E} + 4 s_{13}^2 c_{13}^2 c_{12}^2 \Sigma m_{31}^2 \cos \frac{\Delta m_{31}^2 L}{2E} \right. \\
 &\quad \left. + 4 s_{13}^2 c_{13}^2 s_{12}^2 \Sigma m_{32}^2 \cos \frac{\Delta m_{32}^2 L}{2E} \right] + \mathcal{O}(\epsilon^4),
 \end{aligned}$$

Tritium β -decay

$$\begin{aligned}
 m_\beta &= \sqrt{m_1^2 c_{12}^2 c_{13}^2 + m_2^2 c_{13}^2 s_{12}^2 + m_3^2 s_{13}^2 + \epsilon^2} \\
 &= m_\beta^{(0)} \left(1 + \frac{1}{2} \left(\frac{\epsilon}{m_\beta^{(0)}} \right)^2 + \dots \right)
 \end{aligned}$$

Cosmology

$$\frac{1}{2} \sum_{i=1}^3 \left| \tilde{\mathcal{M}}_{ii} \right| = \sum_{i=1}^3 m_i$$

Restriction from **Daya-Bay data** (3σ):

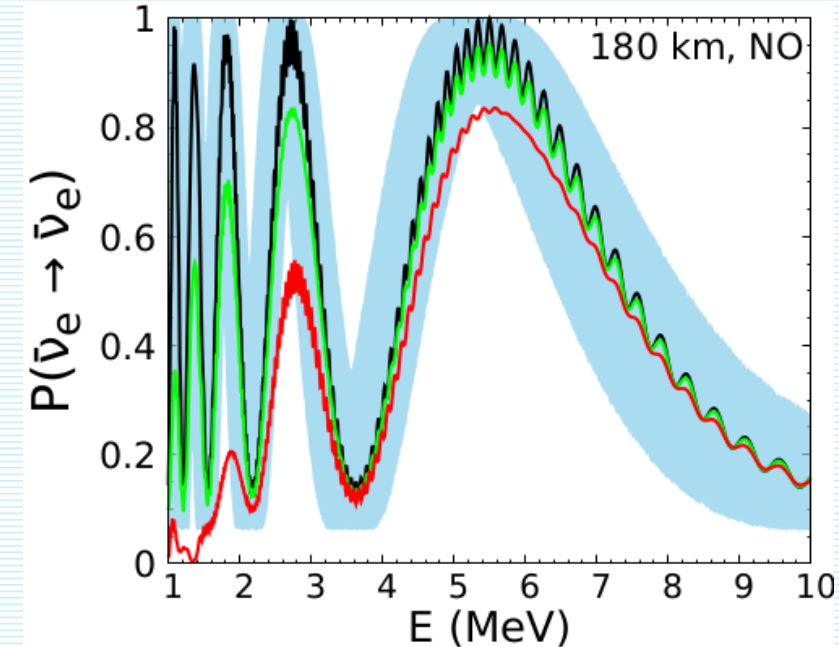
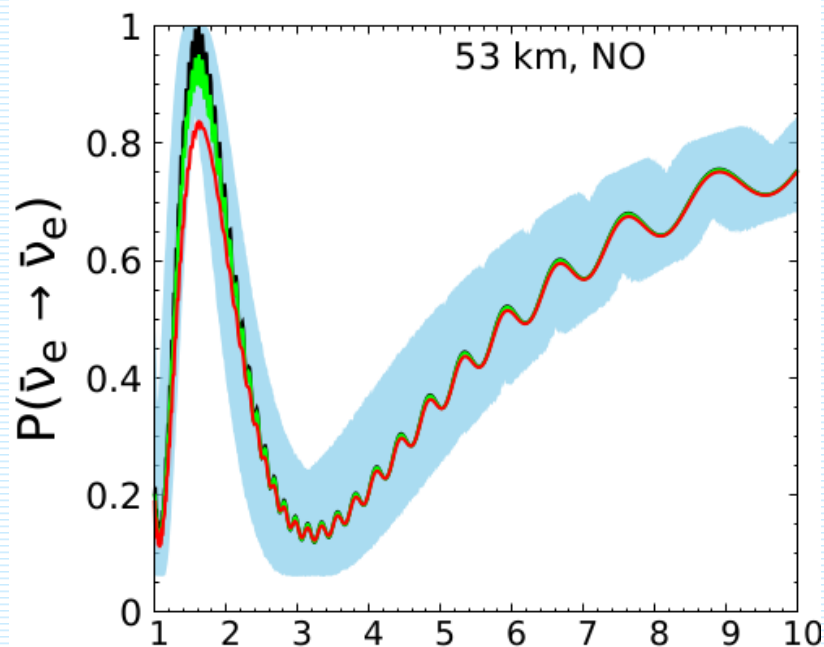
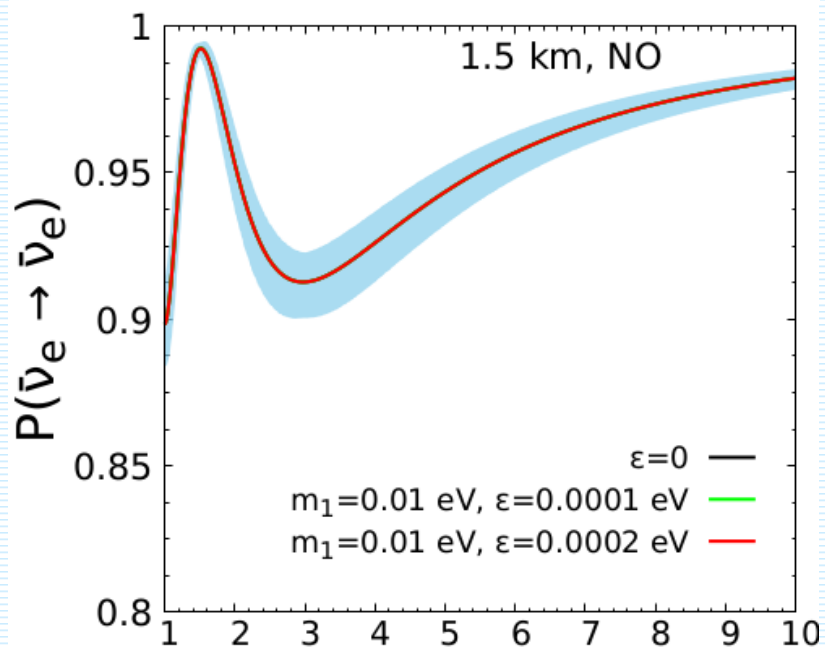
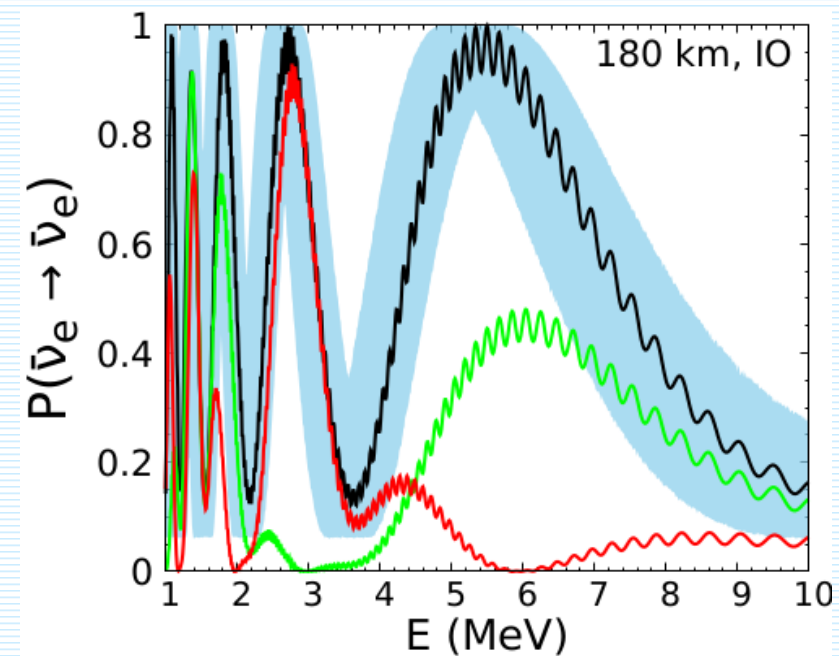
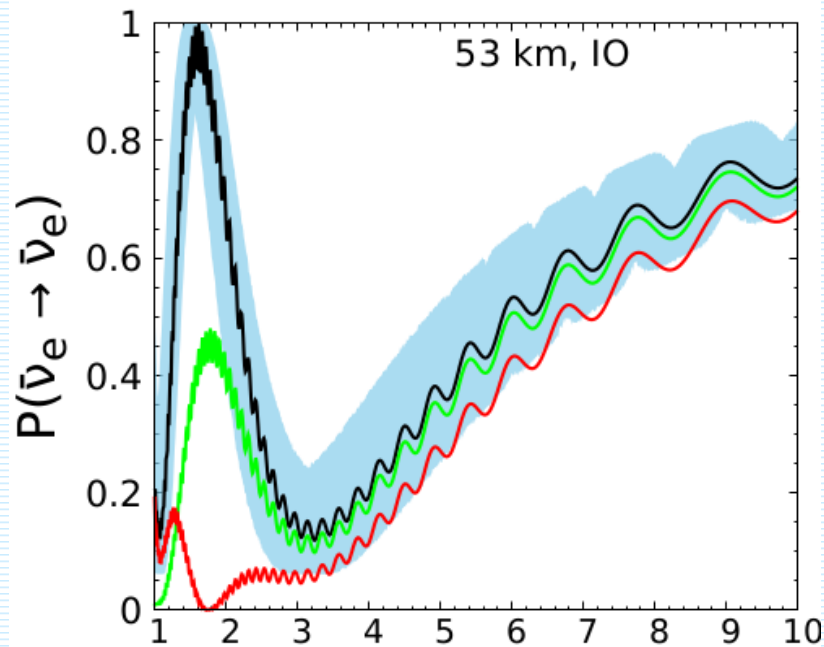
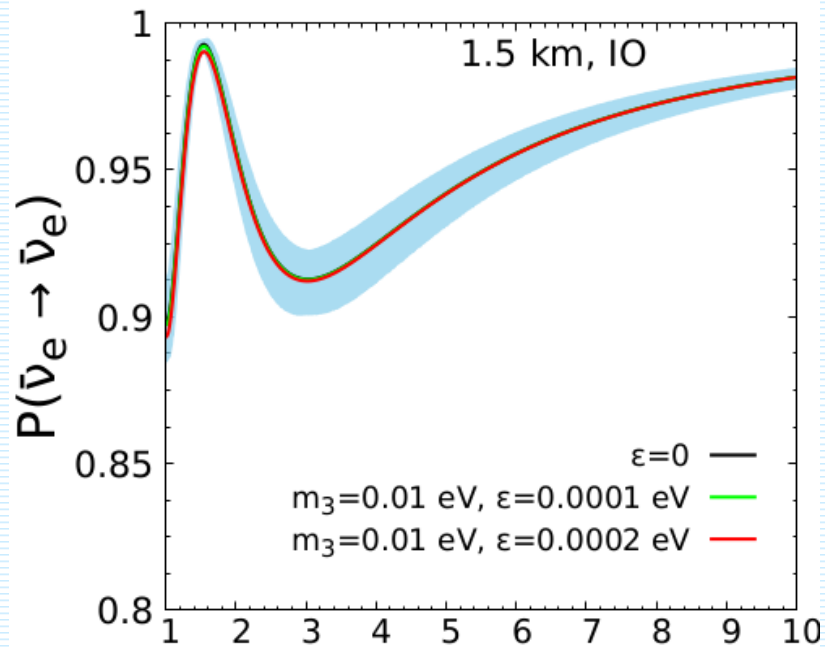
Survival probabilities with non-zero ϵ are the same **3 ν** cases.

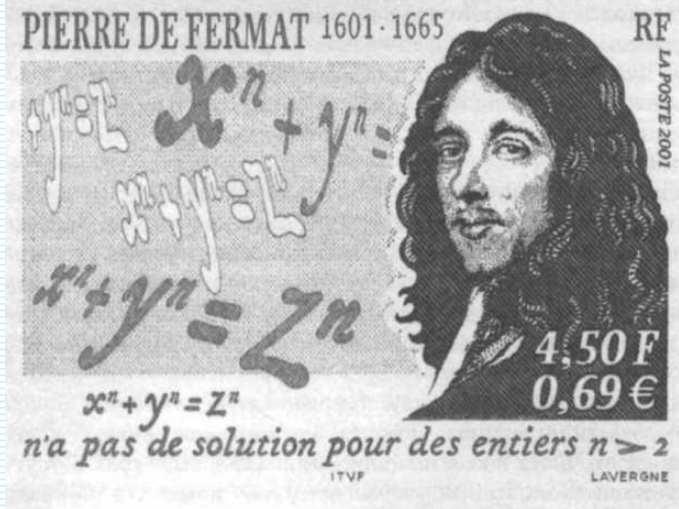
$0\nu\beta\beta$ -decay

$$\begin{aligned}
 m_{\beta\beta} &= [M_L]_{ee} \\
 &= \epsilon \left| c_{12}^2 c_{13}^2 + e^{2i\alpha_{21}} c_{13}^2 s_{12}^2 + e^{2i\alpha_{31}} s_{13}^2 \right| \text{ for Simkovic}
 \end{aligned}$$

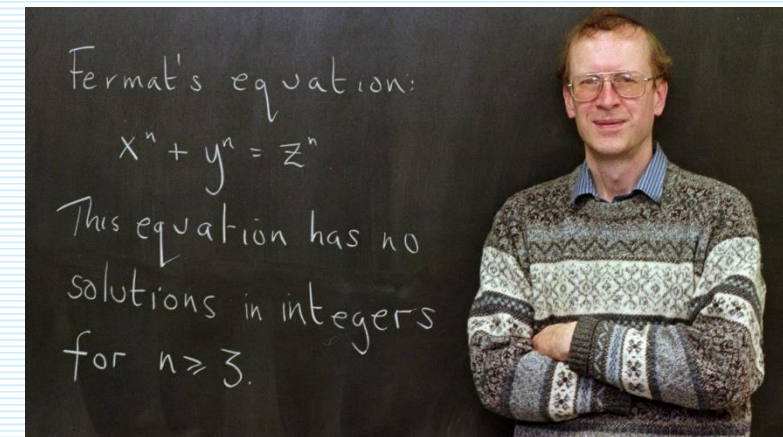
$$\begin{aligned}
 m_{\beta\beta} &\lesssim 30 \text{ meV for NO} \\
 &\lesssim 1 \text{ meV for IO}
 \end{aligned}$$

Quasi-Dirac neutrino oscillations at different distances





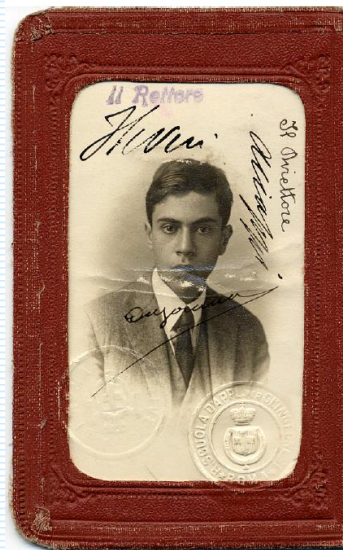
Around 1637, Pierre de Fermat wrote in the margin of a book that the more general equation $a^n + b^n = c^n$ had no solutions in positive integers if n is an integer greater than 2.



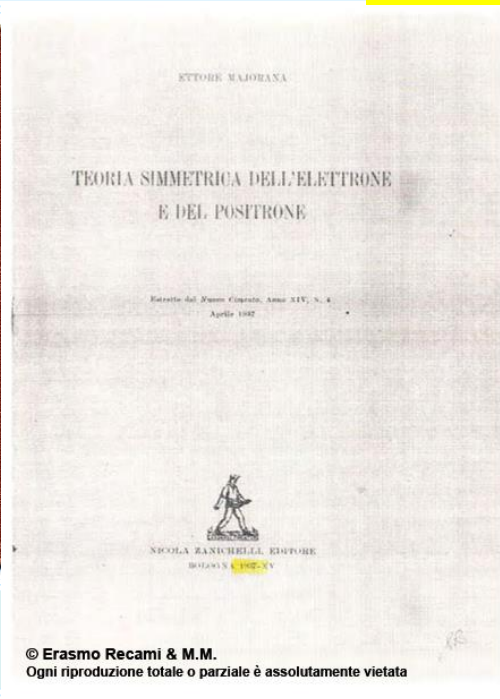
The proof was published by Andrew Wiles in 1995.

After 358 years

Some long-standing tasks of humanity ...



1937



After 85 years

n-ton-class $0\nu\beta\beta$ exp. with discovery potential
KamLAND-Zen 800
SNO+
LEGEND
nEXO
NEXT
CUPID
 etc

After ? years

If $m_{\beta\beta} < 1$ meV, what technology is needed for observation of $0\nu\beta\beta$?

Evaluation of the $0\nu\beta\beta$ -decay NMEs calculation - Approximations needed

Nuclear Shell Model (Madrid-Strasbourg, Michigan, Tokyo): Relatively small model space (1 shell), all correlations included, solved by direct diagonalization

QRPA (Tuebingen-Bratislava-Calltech, Jyvaskyla, Chapel Hill, Lanzhou): Several major shells, only simple correlations included

Interacting Boson Method (Yale-Concepcion): Small space, important proton-neutron Pairing correlations missing

Projected Hartree-Fock-Bogoliubov Method (Lucknow): Several major shells, missing GT proton-neutron residual interaction.

Energy Density Functional theory (Madrid, Beijing): >10 shells, important proton-neutron pairing missing

An initio approaches:

The nuclear w. f. of
(A,Z), (A,Z+1)*, (A,Z+2)
Many-body methods
of choice:

The $0\nu\beta\beta$ nuclear transition operators
(F, GT, and tensor type):

- + Transition operators involving complete set of states of intermediate nucleus
- + Transition operators in closure approximation – just two-body operators
- + Chiral effective field theory two-body transition operators with contact term

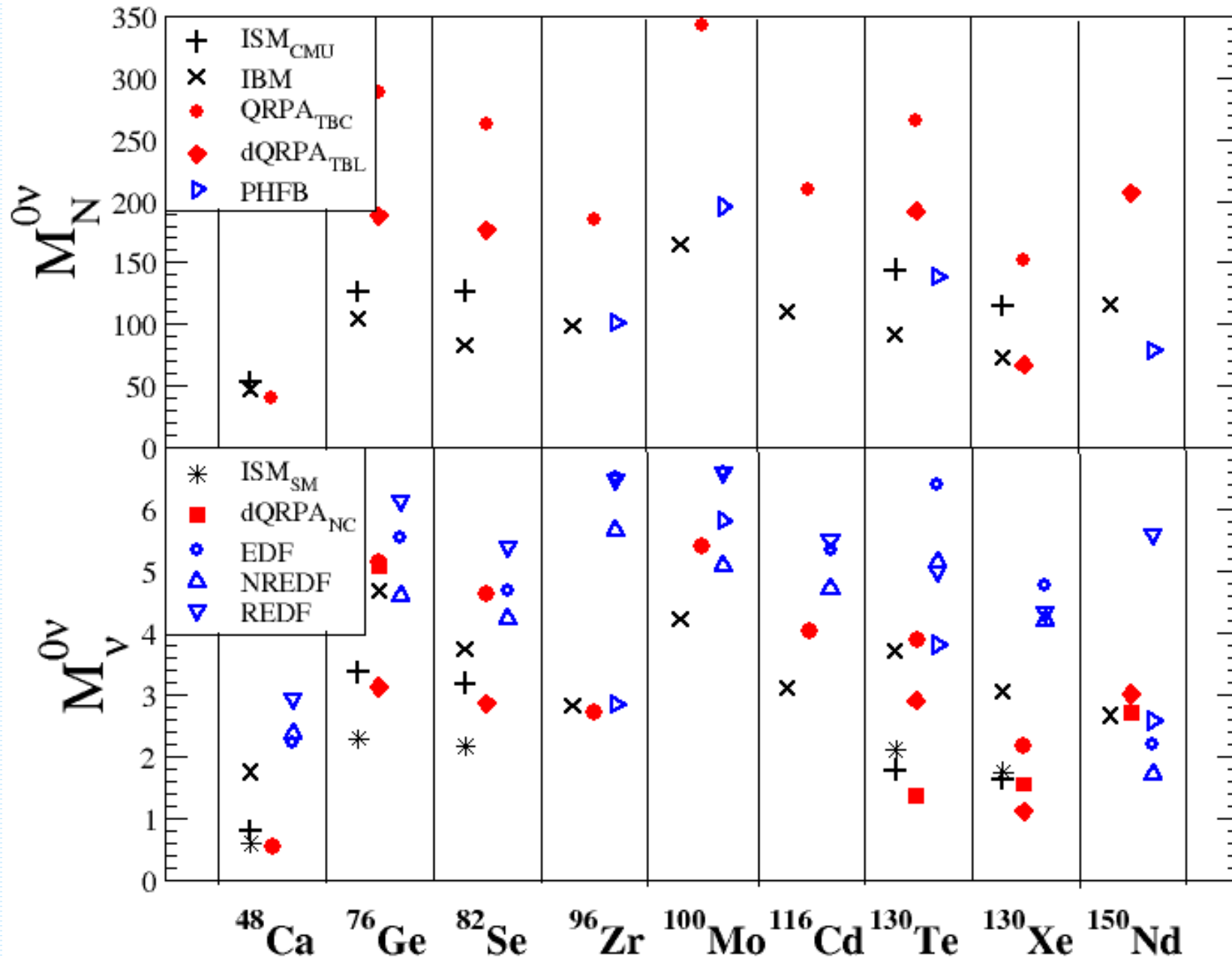
! Isospin, and spin-isospin symmetries
($M_{Fcl} \approx 0$, M_{GTcl} strongly suppressed):

Initial 0^+ g.s.: (T, T)

Final 0^+ g.s.: (T-2, T-2) $\Rightarrow \Delta T = 2$ (!)

Till now, this issue addressed only within the QRPA
PRC 98, 064325 (2018)

**$0\nu\beta\beta$ -decay
NME
status 2022**



All
models
missing
essential
physics

Impossible
to assign
rigorous
uncertainties

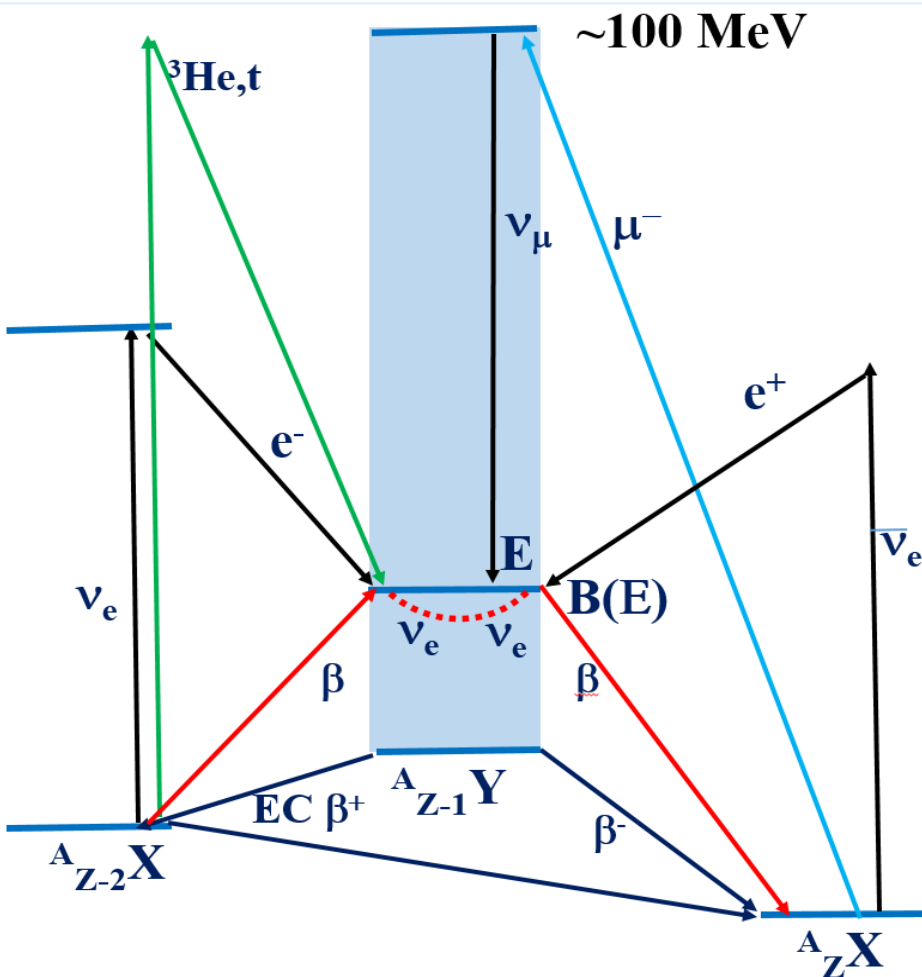
Differences:

- Many-body approxim.
- Size of the m.s.
- Residual interactions

unquenched g_A

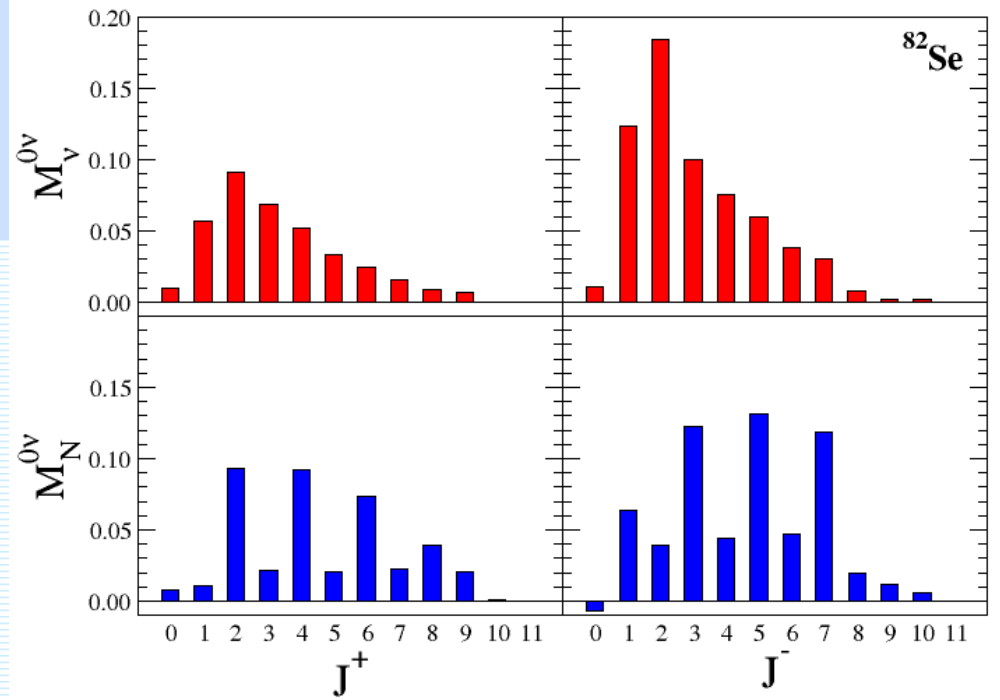
Supporting nuclear physics experiments (Measurements still not conclusive for $0\nu\beta\beta$ NME)

- ✓ β -decay, EC and $2\nu\beta\beta$ decay
- ✓ μ -capture
- ✓ (π^+, π^-) , single charge exchange
- ✓ $({}^3\text{He}, t)$, $(d, {}^2\text{He})$, transfer reactions
- ✓ γ -ray spectroscopy, $\gamma\gamma$ -decay
- ✓ A promising experimental tool:
Heavy-Ion Double Charge-Exchange



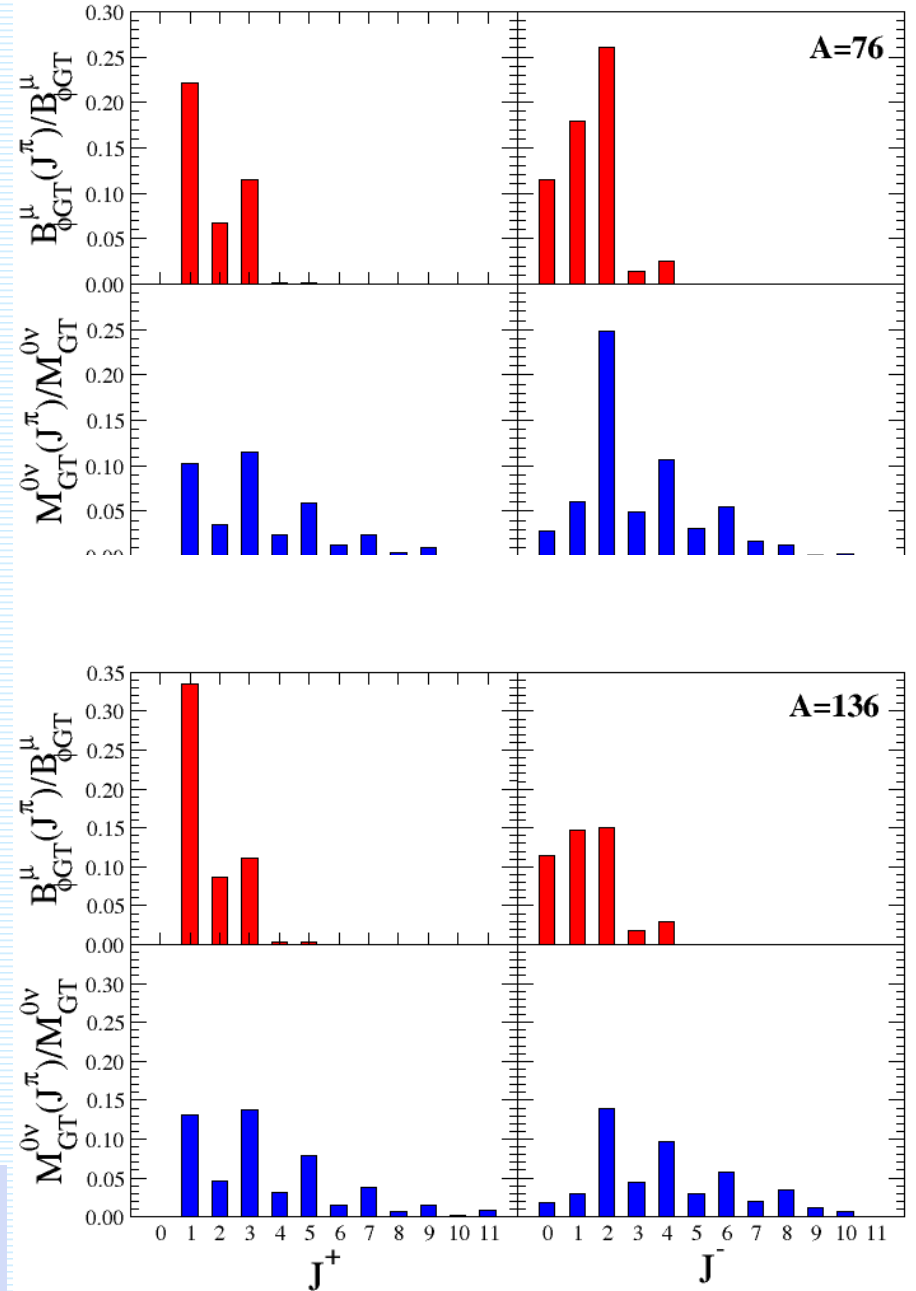
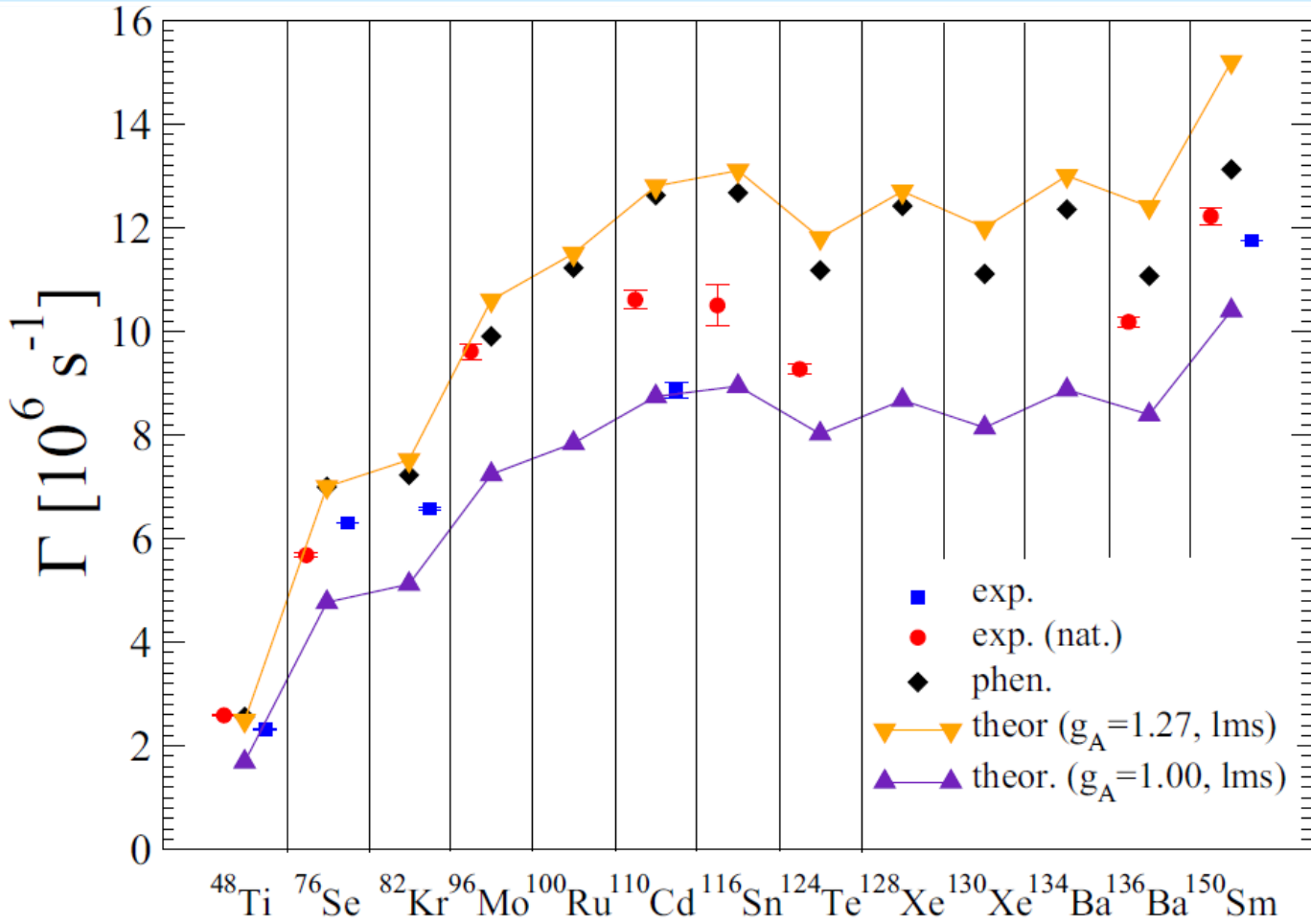
Multipole decomposition of light and heavy $0\nu\beta\beta$ -decay NMEs normalized to unity

Higher multipoles are populated mostly due large ν -momenta transfer



Muon capture rates evaluated within QRPA

In agreement with soft quenching ($g_A \approx 1.1$)

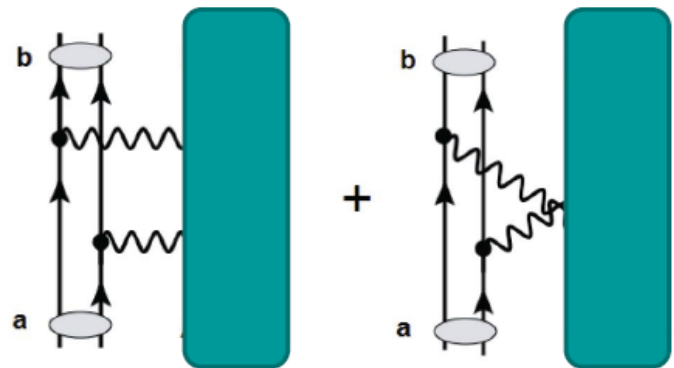


Multipole decomposition of B_{GT}^{μ} and $M_{GT}^{0\nu}$

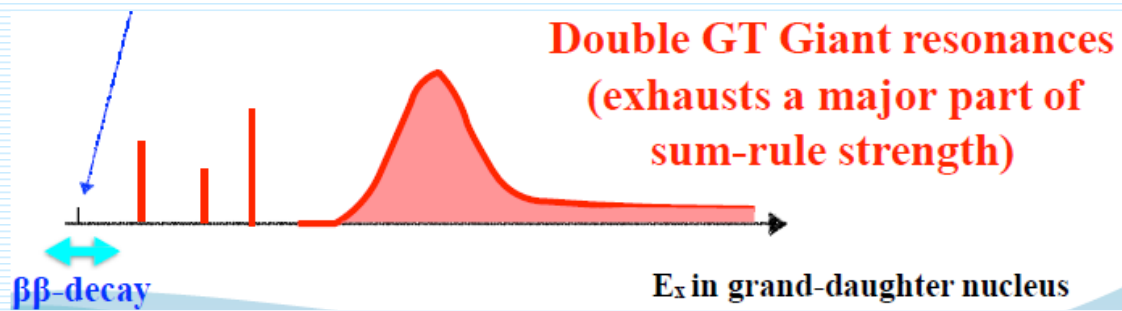
PRC 102, 034301 (2020).

7/13/2023

Experiment Monument at PSI
will study contributions from all multipoles

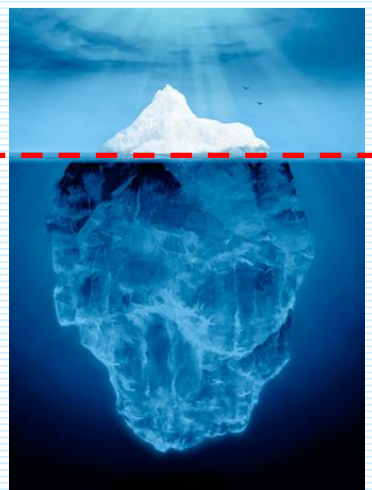


**Heavy-ion DCE
as surrogate processes
of $\beta\beta$ -decay**



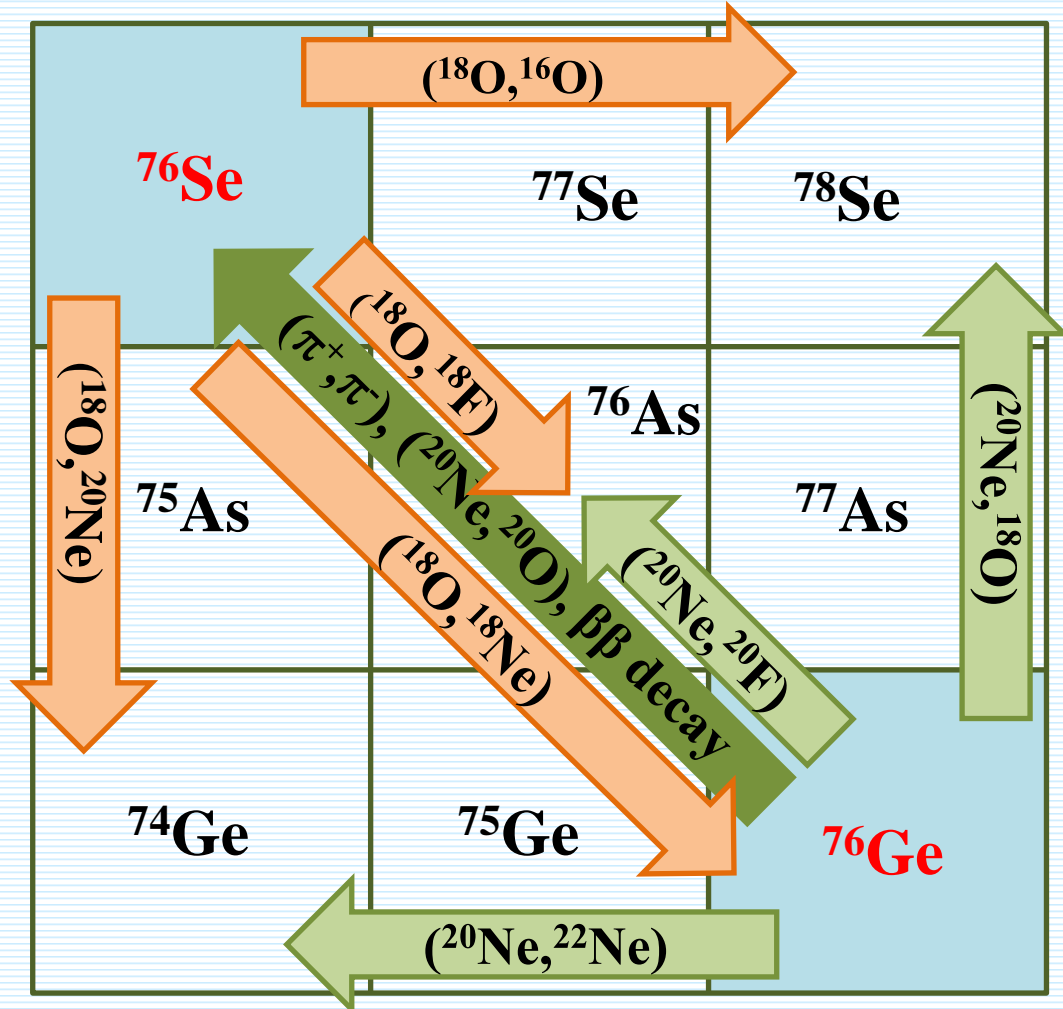
- ✓ Induced by strong interaction
- ✓ Sequential nucleon transfer mechanism 4th order: Kinematical matching
- ✓ Meson exchange mechanism 1st or 2nd order
- ✓ Possibility to go in both directions
- ✓ Low cross section

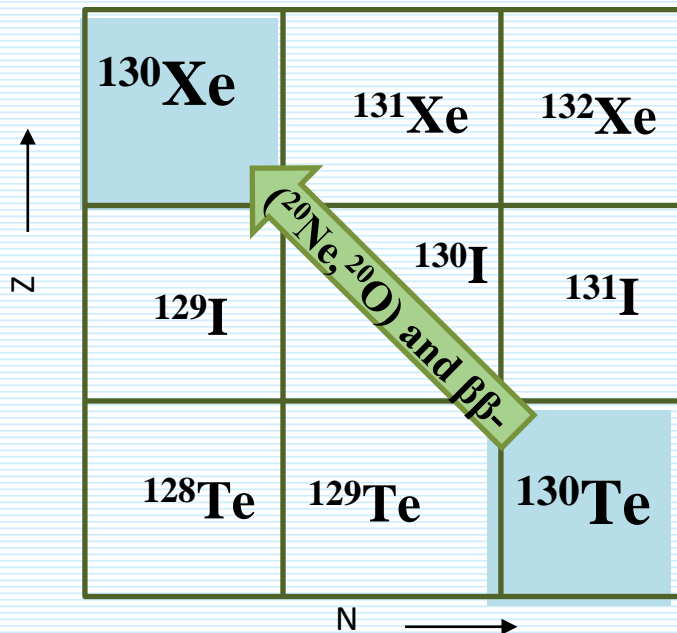
Tiny amount of DGT strength for low lying states



Sum rule almost exhausted by DGT Giant Mode, still not observed

RIKEN
RCNP
Future:
INFN-LNS



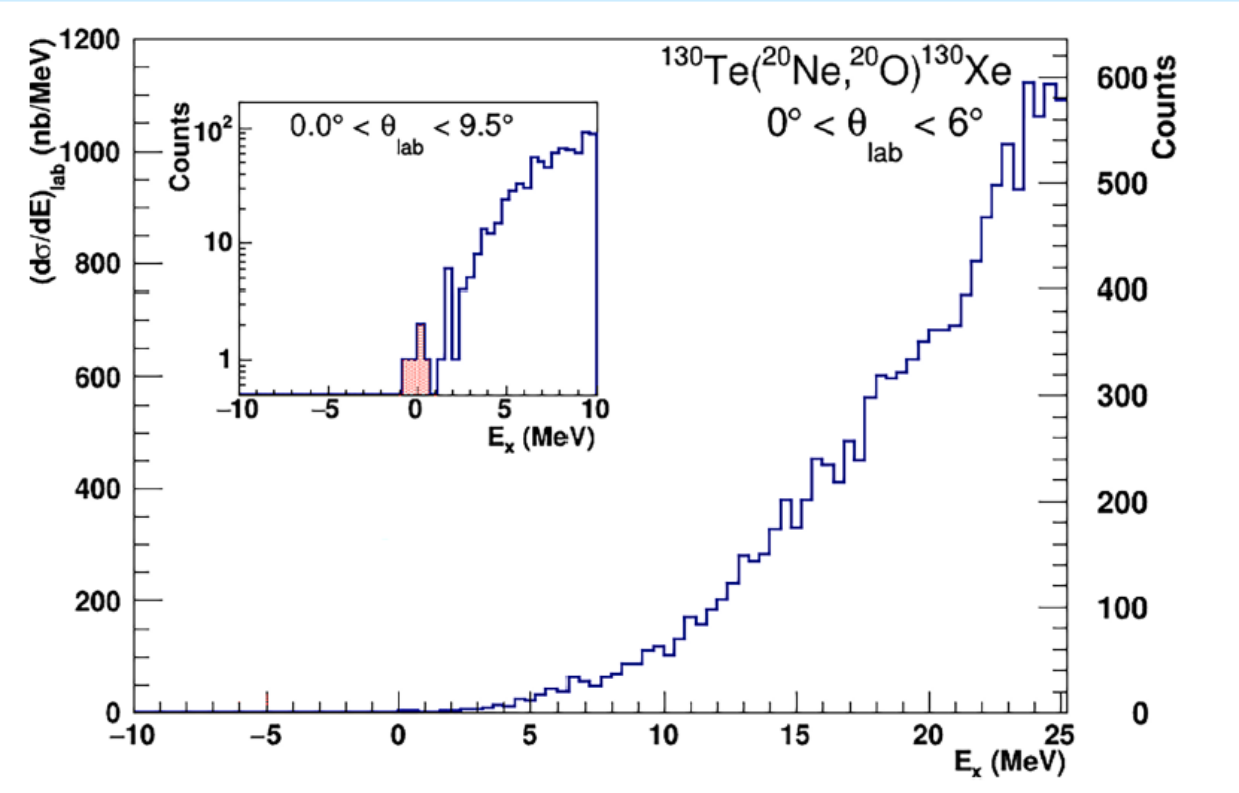


- **g.s. → g.s. transition maybe isolated**
- **Absolute cross section measured**

Resolution ~ 500 keV FWHM

No spurious counts at $-10 < E_x < -2$ MeV

The $^{130}\text{Te}(^{20}\text{Ne}, ^{20}\text{O})^{130}\text{Xe}$ DCE reaction

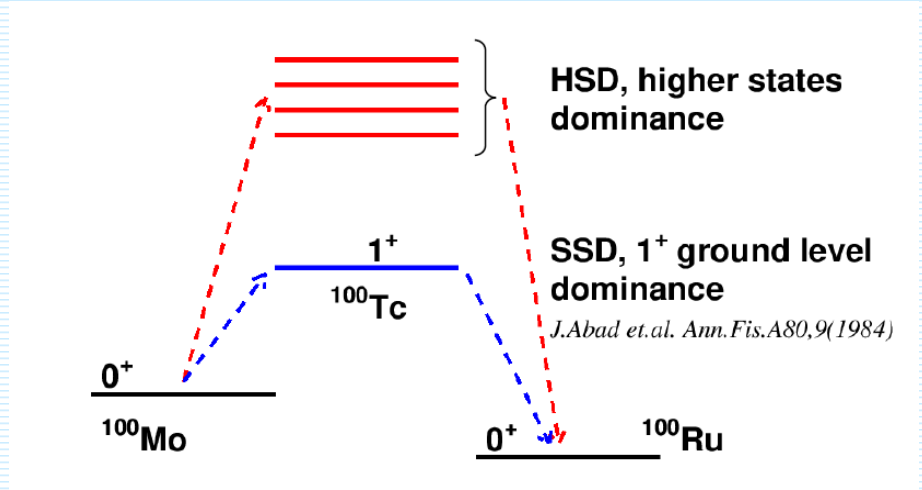


State (MeV)	Counts	Absolute cross section (nb)	Cross section 95% limit (nb)
g.s. (0 ⁺) + 2 ⁺ (536 keV)	5	13	[3--18]

Analysis of cross-section sensitivity < 0.1 nb in the Region Of Interest

Understanding of the $2\nu\beta\beta$ -decay NMEs is of crucial importance for correct evaluation of the $0\nu\beta\beta$ -decay NMEs

There is no reliable calculation of the $2\nu\beta\beta$ -decay NMEs yet

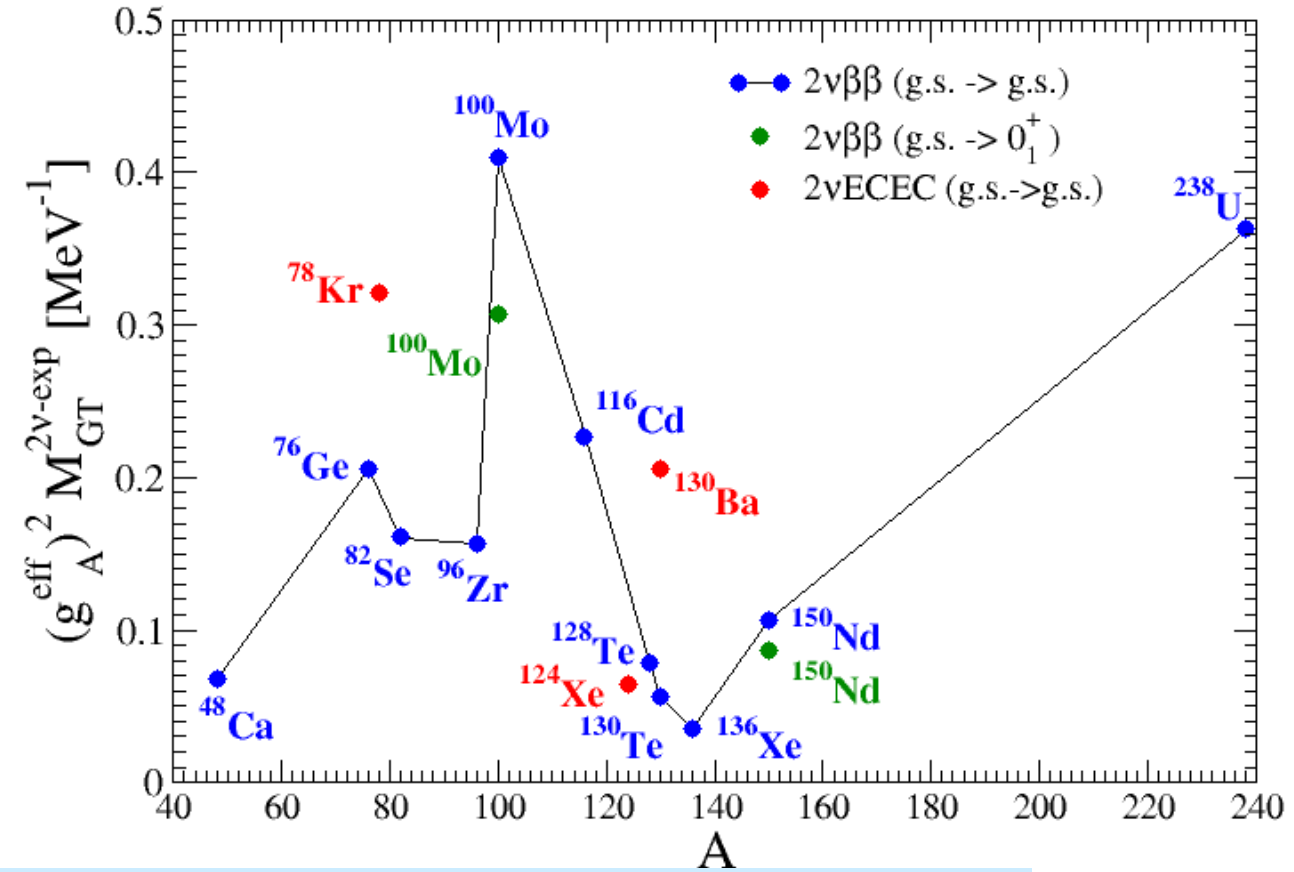


$$M_{GT}^{2\nu} = \sum_m \frac{\langle 0_f^+ || \tau^+ \sigma || 1_m^+ \rangle \langle 1_m^+ || \tau^+ \sigma || 0_i^+ \rangle}{E_m - E_i + \Delta}$$

The **spread** of $2\nu\beta\beta$ and $0\nu\beta\beta$ NMEs is large and small, respectively.

Reasons:

- $2\nu\beta\beta$ NMEs governed by contribution from single (1^+) multipole unlike $0\nu\beta\beta$ NMEs by contributions from all **multipoles** (J^π) of int. nucl.
- Neutrino **potential** vs $2\nu\beta\beta$ potential



7/13/2 Both $2\nu\beta\beta$ and $0\nu\beta\beta$ operators connect the same states. Both change two neutrons into two protons. Explaining $2\nu\beta\beta$ -decay is necessary but not sufficient

Improved description of the $0\nu\beta\beta$ -decay rate (a way to fix g_A^{eff})

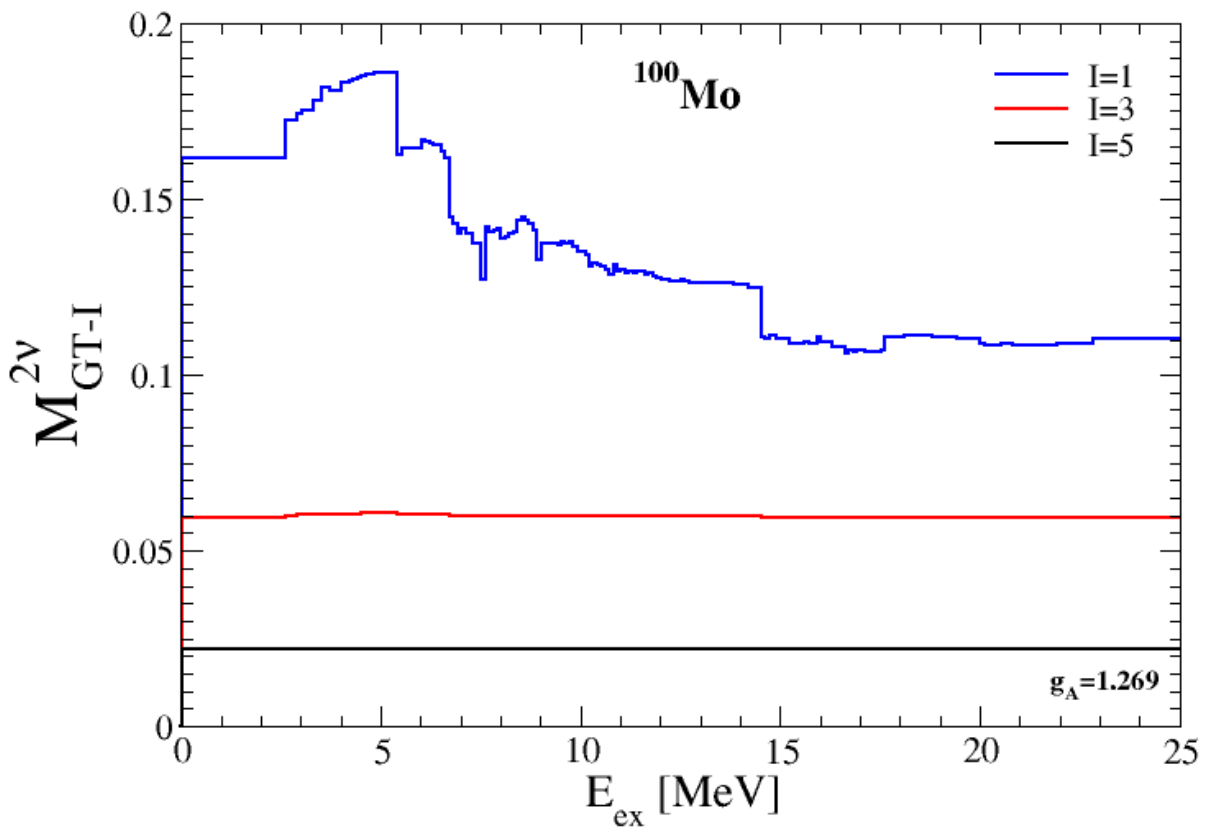
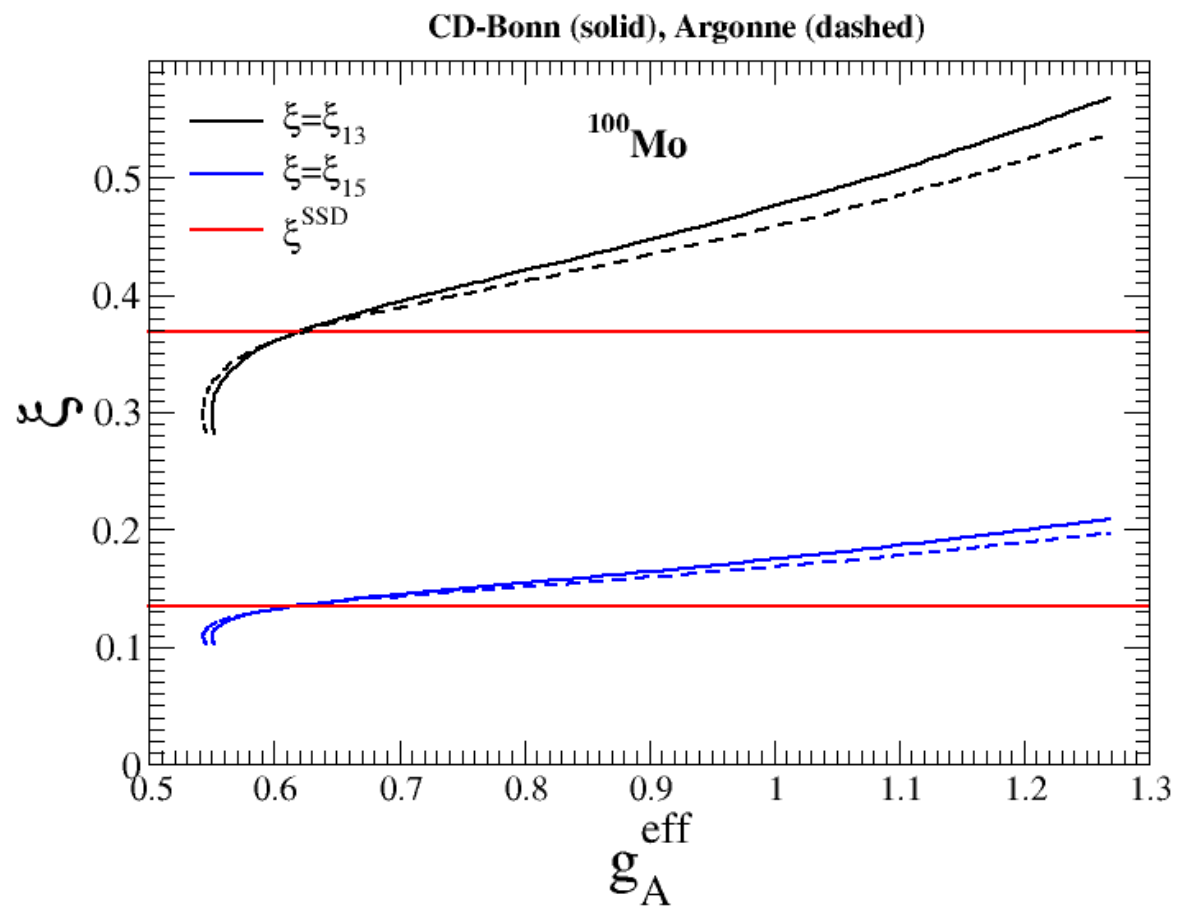
PRC 97, 034315 (2018)

$$[T_{1/2}^{2\nu\beta\beta}]^{-1} \simeq (g_A^{\text{eff}})^4 \frac{|M_{GT-3}^{2\nu}|^2}{|\xi_{13}^{2\nu}|^2} (G_0^{2\nu} + \xi_{13}^{2\nu} G_2^{2\nu})$$

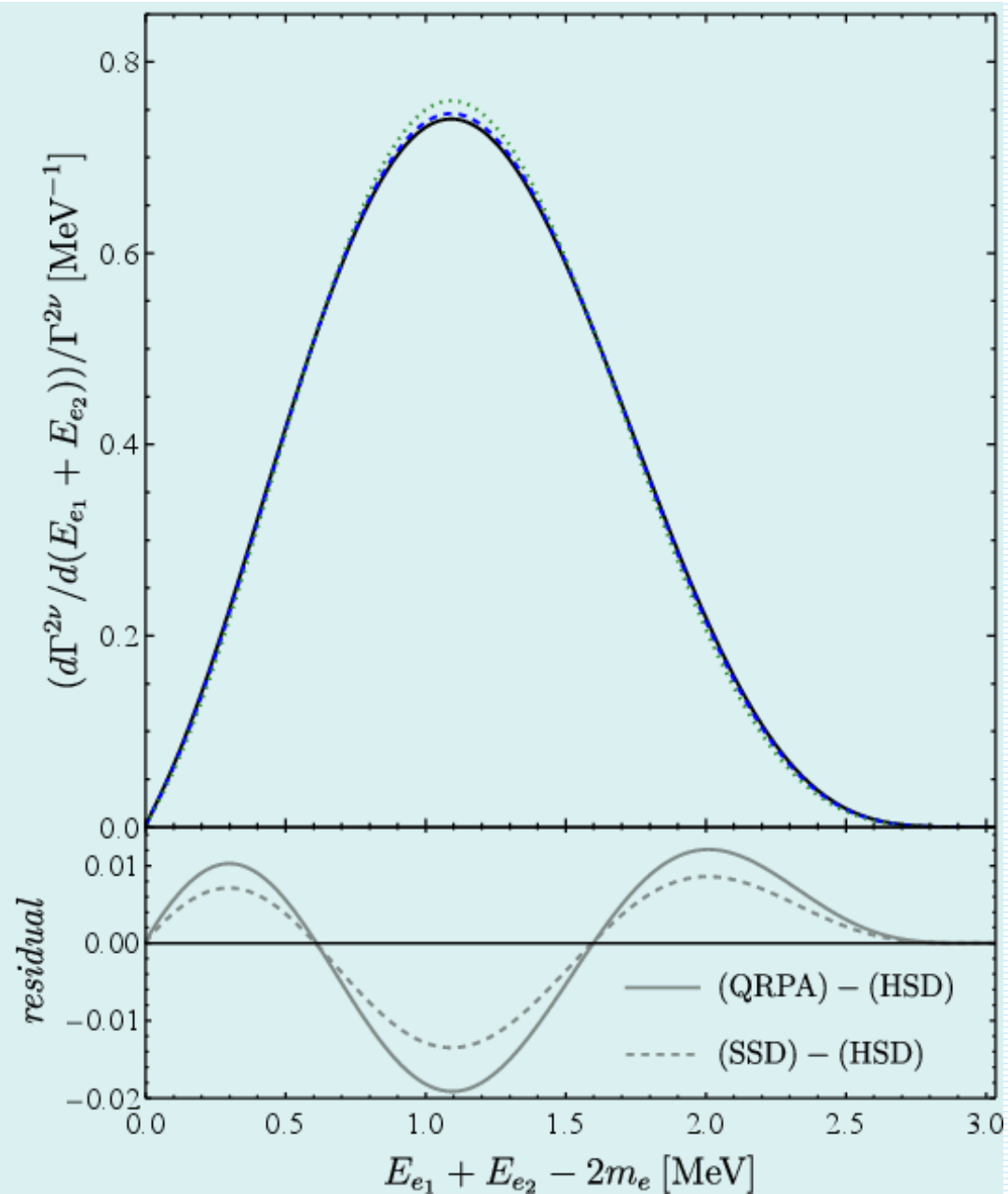
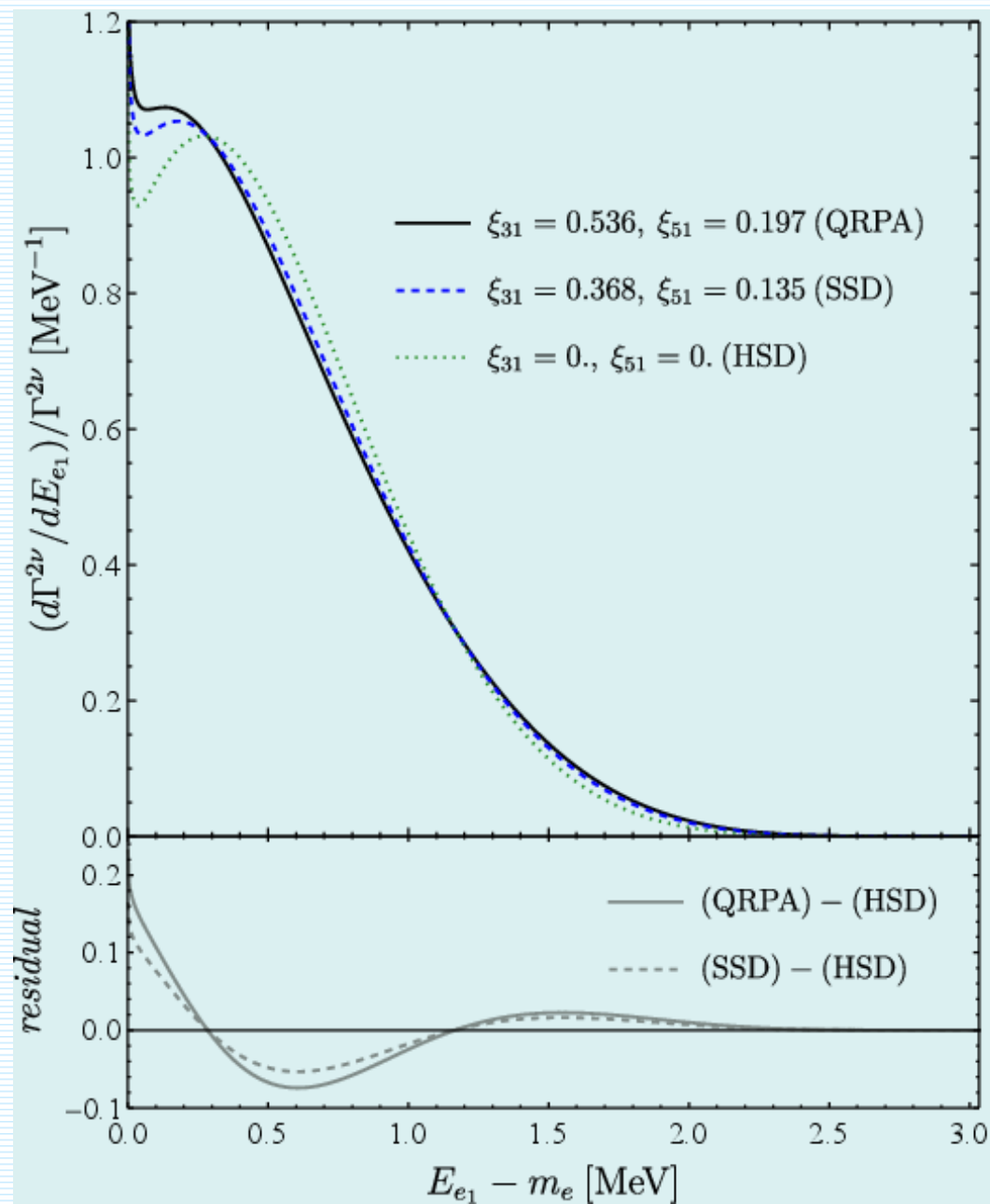
$$M_{GT}^{K,L} = m_e \sum_n M_n \frac{E_n - (E_i + E_f)/2}{[E_n - (E_i + E_f)/2]^2 - \varepsilon_{K,L}^2}$$

$$M_{GT-1}^{2\nu} = \sum_n M_n \frac{1}{(E_n - (E_i + E_f)/2)} \quad \xi_{13}^{2\nu} = \frac{M_{GT-3}^{2\nu}}{M_{GT-1}^{2\nu}}$$

$$M_{GT-3}^{2\nu} = \sum_n M_n \frac{4 m_e^3}{(E_n - (E_i + E_f)/2)^3} \quad \xi_{15}^{2\nu} = \frac{M_{GT-5}^{2\nu}}{M_{GT-1}^{2\nu}}$$



Determining g_A from the $2\nu\beta\beta$ differential characteristics



$2\nu\beta\beta$ is sensitive to New Physics as well

All 100 kg- and ton-class $0\nu\beta\beta$ experiments can also study a diverse range of **exotic phenomena**, e.g. through **spectral distortion** in $2\nu\beta\beta$. Future searches will probe the $2\nu\beta\beta$ with **high statistics** about 10^5 - 10^6 events.

Common subjects:

Majoron(s) emission
(partly)bosonic neutrinos,
Lorentz invariance violation

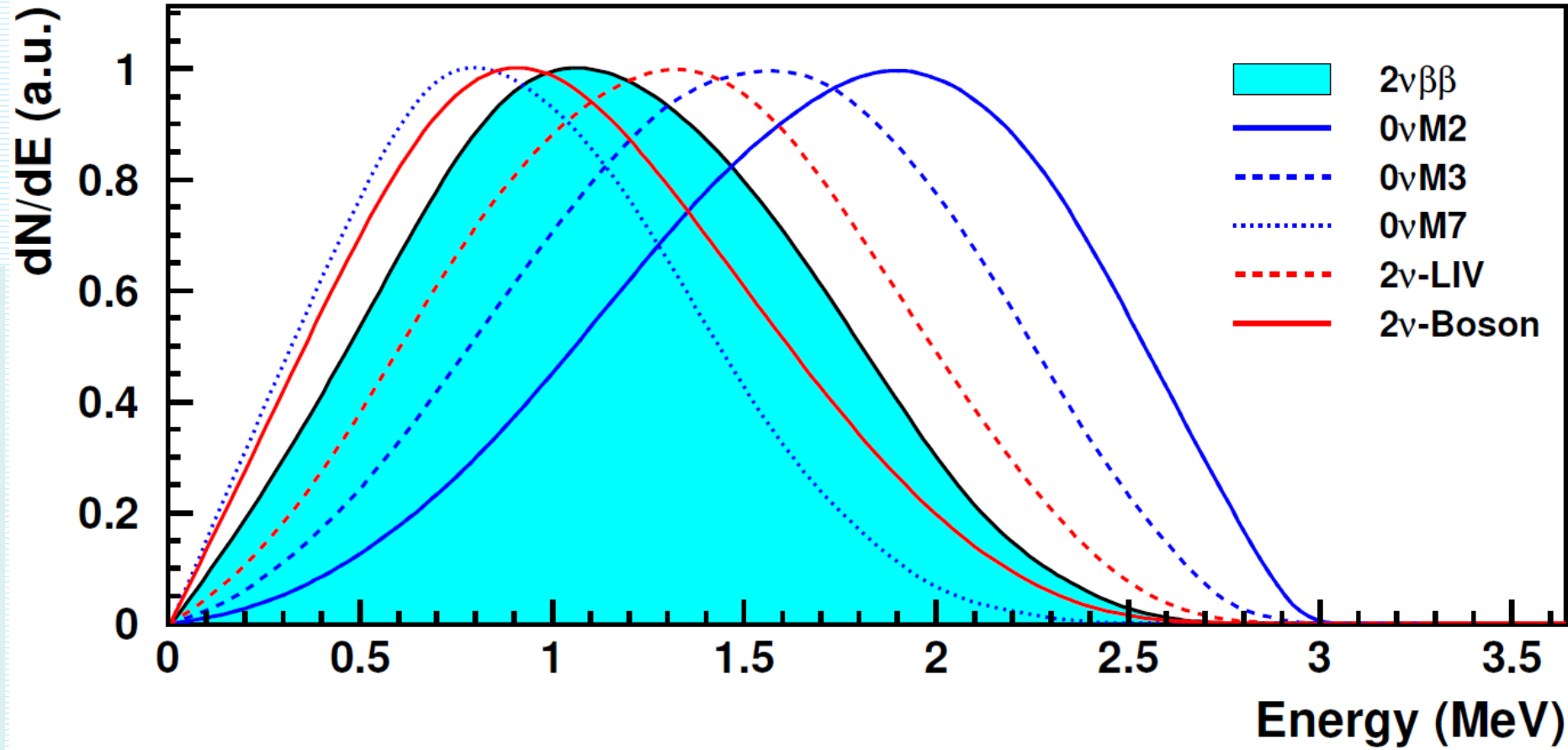
Recent subjects:

Lepton-number conserving right-handed currents
(PRL 125 (2020) 17, 171801)

Neutrino self-interactions
(PRD 102 (2020) 5, 051701)

Sterile neutrino and light fermion searches through energy end point
(PRD 103 (2021) 5, 055019;

PLB 815 (2021) 136127)



$$\frac{d\Gamma}{d\varepsilon_1 d\varepsilon_2} = C(Q - \varepsilon_1 - \varepsilon_2)^n [p_1 \varepsilon_1 F(\varepsilon_1)] [p_2 \varepsilon_2 F(\varepsilon_2)]$$

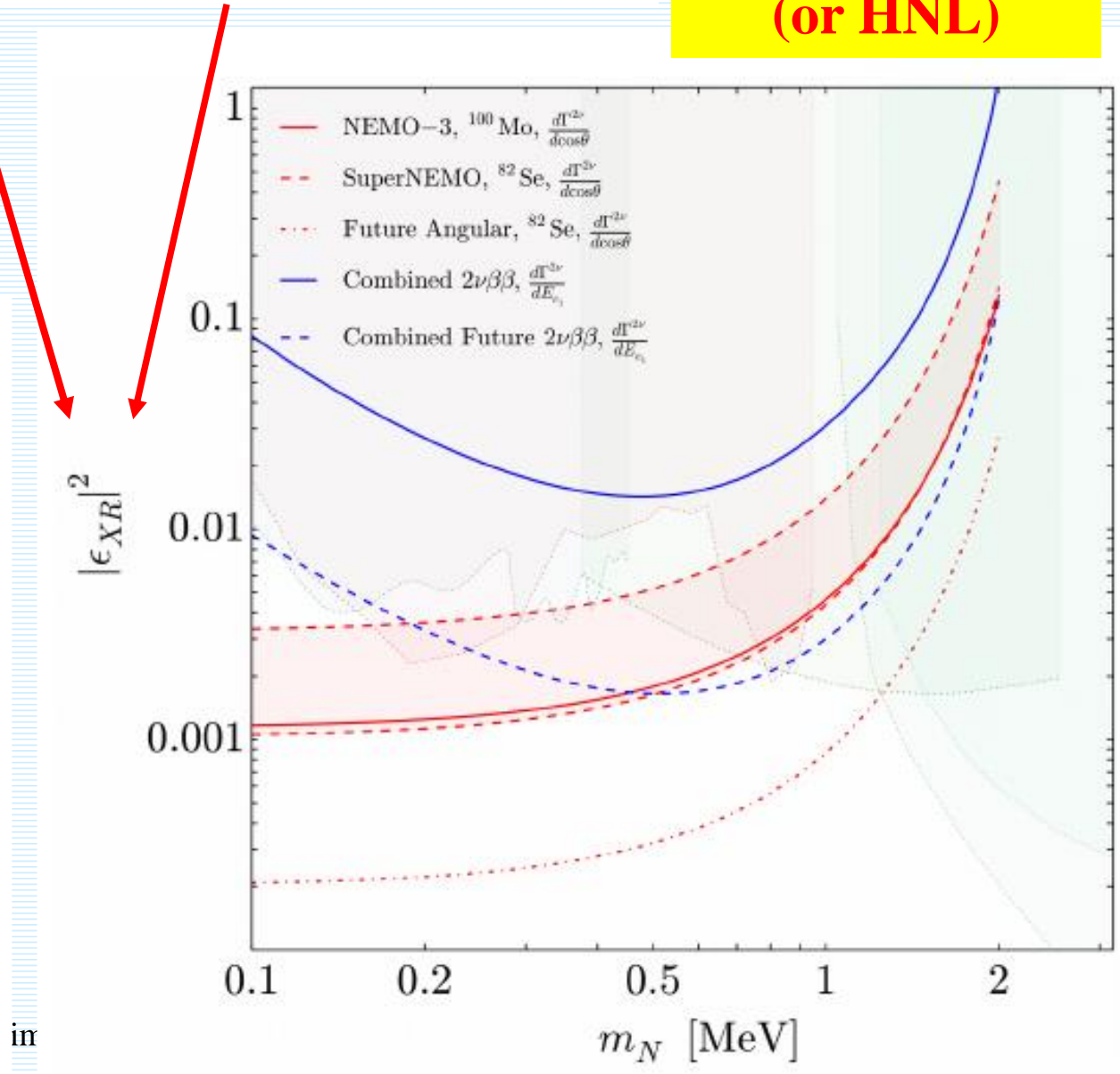
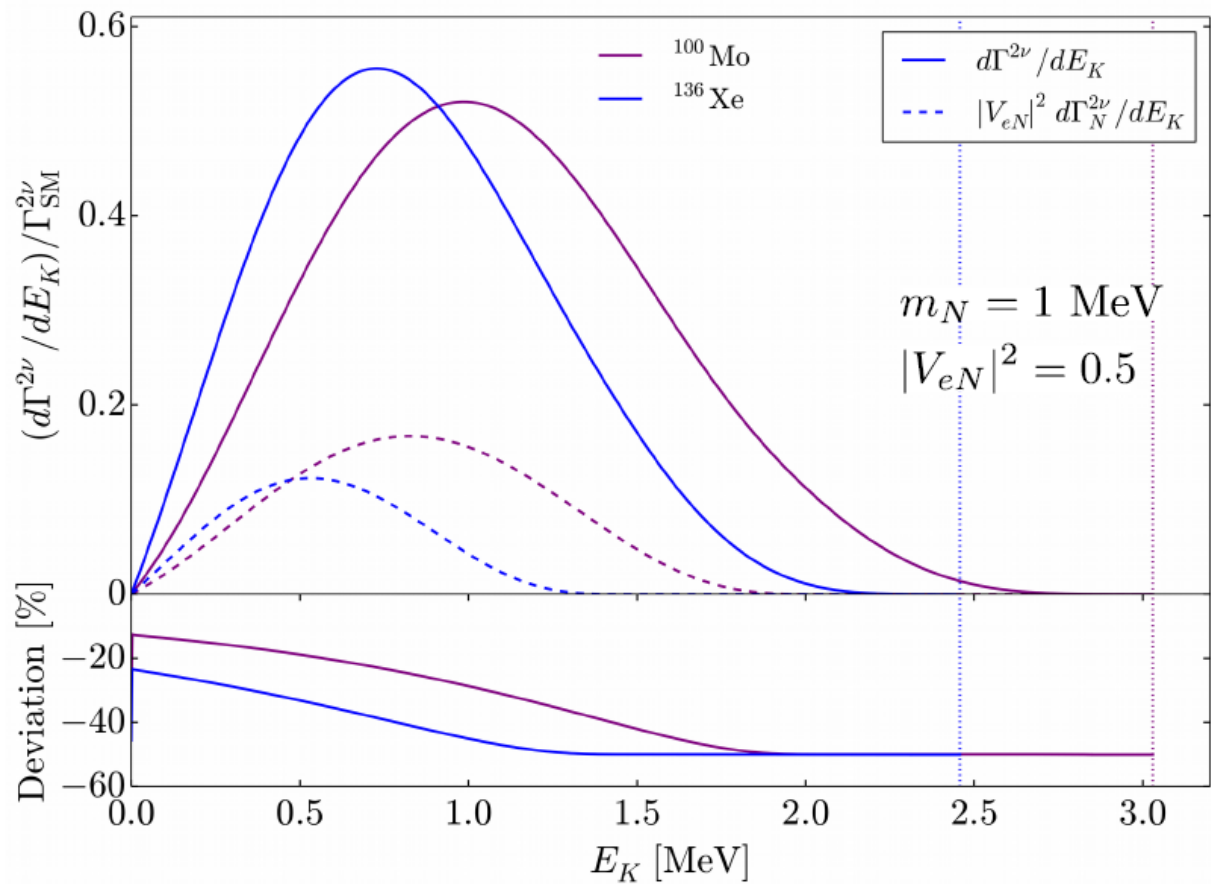
Spectral index n

$$\mathcal{L} = \frac{G_F \cos \theta_C}{\sqrt{2}} \left[(1 + \delta_{\text{SM}}) j_L^\mu J_{L\mu} + V_{eN} j_L^{N\mu} J_{L\mu} + \epsilon_{LR} j_R^{N\mu} J_{L\mu} + \epsilon_{RR} j_R^{N\mu} J_{R\mu} \right] + \text{h.c.}$$

**Exotic $2\nu\beta\beta$
Sterile Neutrino
(or HNL)**

Consider either mixing of **sterile ν** with **active ν** ,
or **right-handed currents**

Phys.Rev.D 103 (2021) 055019



SuperNEMO Double-Beta Decay Experiment

Full kinematics and precision measurements of $2\nu\beta\beta$

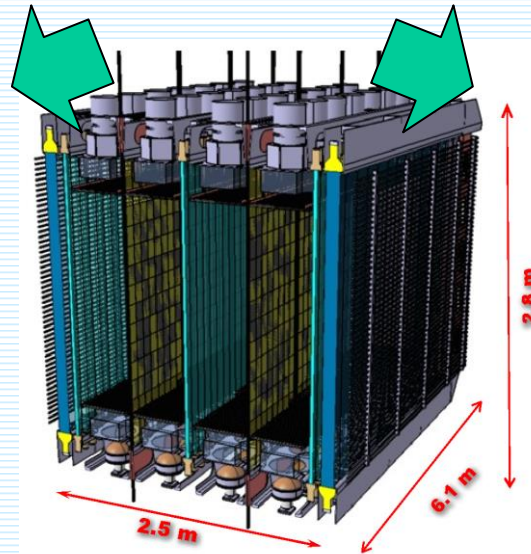
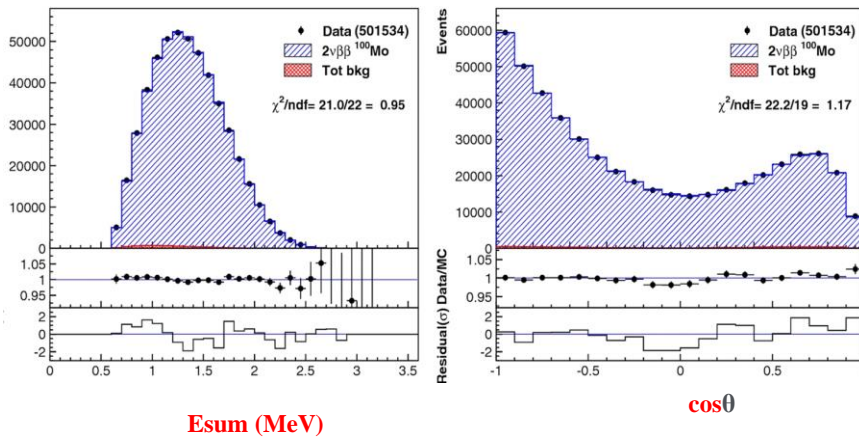
- Nuclear model constraints
- g_A quenching constraints
- Sterile neutrinos
- Right-handed currents
- $2\nu\beta\beta$ with emission of single e^- , etc (NEMO-3 analysis in preparation)



Understanding the Ultimate Reach of the Tracker-Calorimeter Technique

- Can the technique be used to confirm & probe a signal found in the next generation of $0\nu\beta\beta$ experiments?
- Explore alternative tracker-calorimeter technologies & different isotopes

NEMO3 100Mo total data



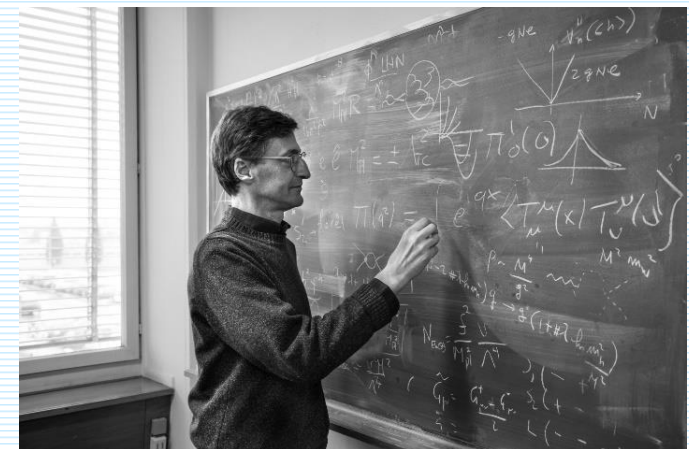


AI can not solve **v-physics** problems!

v-physics problems can be solved only by **v-experiments** (and **v-theory**), i.e., by **skilled experimentalists** and **theorists**



Fedor Simkovic



Thank you for your attention!