RECENT RESULTS IN THE THEORY OF LEPTON NUMBER VIOLATING PROCESSES

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INTRODUCTION

Neutrino-less double beta decay (DBD) has not been observed so far (2023).

Mechanism for this decay mostly considered:

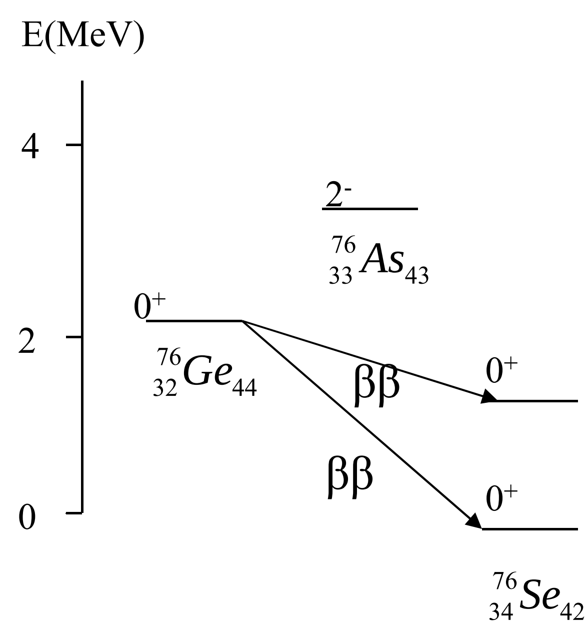
Mass mechanism.

Because of the non-observation so far of the mass mechanism it is of interest to consider other possible mechanisms of lepton number violating processes.

An exhaustive study of all possible other mechanisms has been recently done. An outline of the results will be presented in this talk.

DOUBLE BETA DECAY

$$^{A}_{Z}X_{N} \rightarrow {}^{A}_{Z\pm2}Y_{N\mp2} + 2e^{\mp} + anything$$



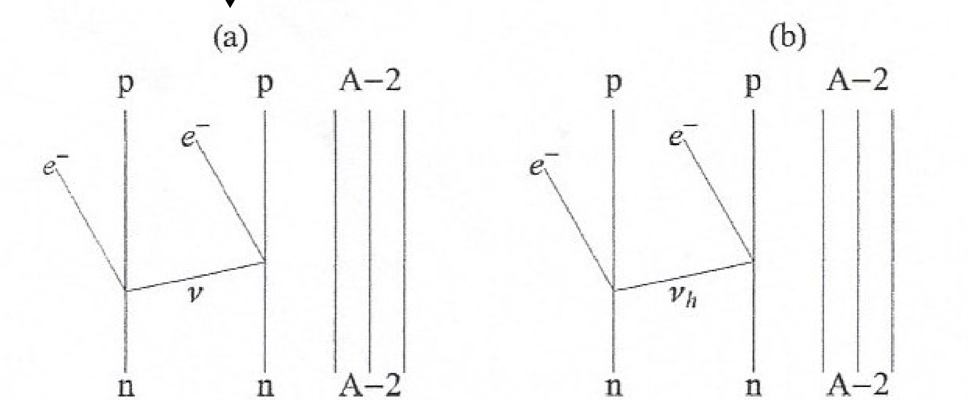
A. MASS MECHANISM

Standard mechanism of neutrino-less DBD

↓

Majorana particle:

$$\nu \equiv \overline{\nu}$$



Half-life for neutrino-less DBD

$$\left[\tau_{1/2}^{0\nu\beta\beta}(0^+ \rightarrow 0^+)\right]^{-1} = G_{0\nu}\left|M_{0\nu}\right|^2 \left|f(m_i, U_{ei})\right|^2$$

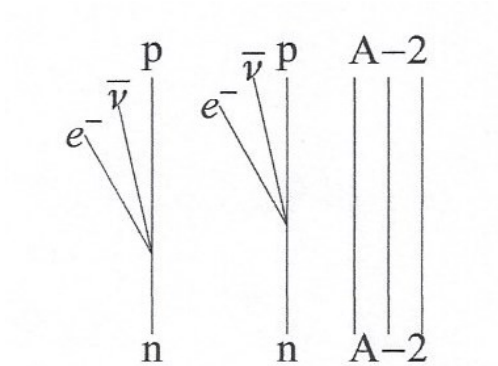
Phase-space factor (Atomic physics) — **PSF**

Matrix elements (Nuclear physics) — **NME**

Beyond the standard model (Particle physics)

Concomitant with neutrino-less DBD, there is DBD with the emission of two neutrinos. This process is allowed by the Standard Model.

$${}^{A}_{Z}X \rightarrow {}^{A}_{Z\pm2}Y_{N\mp2} + 2e^{\mp} + 2\overline{\nu}_e$$



The half-life for this process can be, to a good approximation, factorized in the form

$$\left[\tau_{1/2}^{2\nu}\right]^{-1} = G_{2\nu}\left|M_{2\nu}\right|^2$$

Phase-space factor
(Atomic Physics) PSF

Matrix elements
(Nuclear Physics) NME

To calculate the half-life, one needs phase space factors (PSF) and nuclear matrix elements (NME).

PHASE SPACE FACTORS (PSF)

All recent calculations make use of PSF given in J. Kotila and F. Iachello, Phys. Rev. C 85, 034316 (2012).

NUCLEAR MATRIX ELEMENTS (NME)

NME can be written as:

$$M_{0\nu} = g_A^2 M^{(0\nu)}$$

$$M^{(0\nu)} \equiv M_{GT}^{(0\nu)} - \left(\frac{g_V}{g_A}\right)^2 M_F^{(0\nu)} + M_T^{(0\nu)}$$

Several methods have been used to evaluate $M_{0\nu}$:

QRPA (Quasiparticle Random Phase Approximation)

ISM (Shell Model)

IBM-2 (Interacting Boson Model)

DFT (Density Functional Theory)

…

For 0ν processes two “mass” scenarios have been considered:

(1) Emission and re-absorption of a light ($m_{light} \ll 1$keV) neutrino.

(2) Emission and re-absorption of a heavy ($m_{heavy} \gg 1$GeV) neutrino.

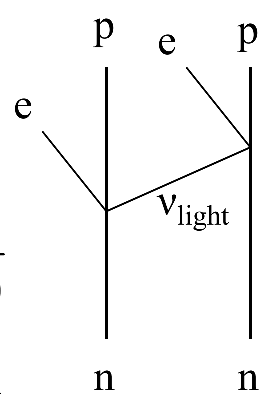
$$f = \frac{\langle m_\nu \rangle}{m_e}$$

$$f_h = m_p \left\langle \frac{1}{m_{\nu_h}} \right\rangle$$

$$\langle m_\nu \rangle = \sum_{k=light} (U_{ek})^2 m_k$$

$$\langle m_\nu^{-1} \rangle = \sum_{k=heavy} (U_{ek_h})^2 \frac{1}{m_{k_h}}$$

1



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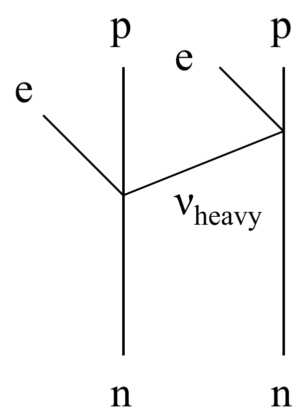


$$v(p) = \frac{2}{\pi} \frac{1}{p\left(p + \tilde{A}\right)}$$

$$\tilde{A} = 1.12 A^{1/2} (MeV)$$

Closure energy

$m_{\nu ligth} \ll 1$keV

Long range

$m_{\nu heavy} \gg 1$GeV

Short range

$$v(p) = \frac{2}{\pi} \frac{1}{m_p m_e}$$

QUENCHING OF g_A

A problem of all calculations is the quenching of g_A. Most results are presented with $g_A=1.269$.

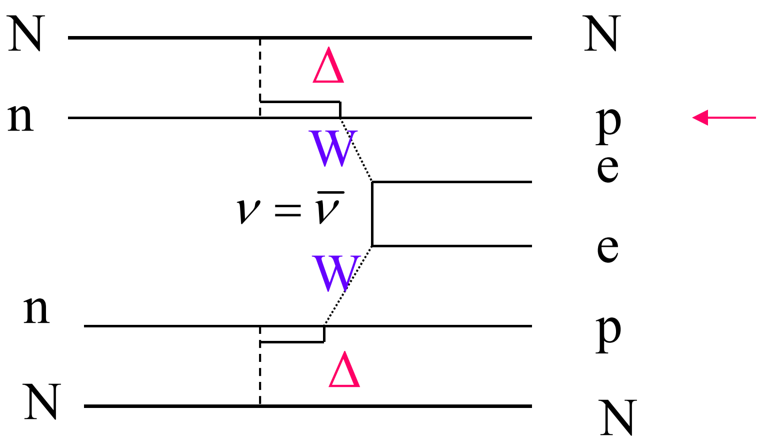
It is well-known from single β-decay/EC [¶] and from 2νββ that g_A is quenched in models of nuclei. Two reasons:

(i) Limited model space

(ii) Omission of non-nucleonic degrees of freedom (Δ,…)

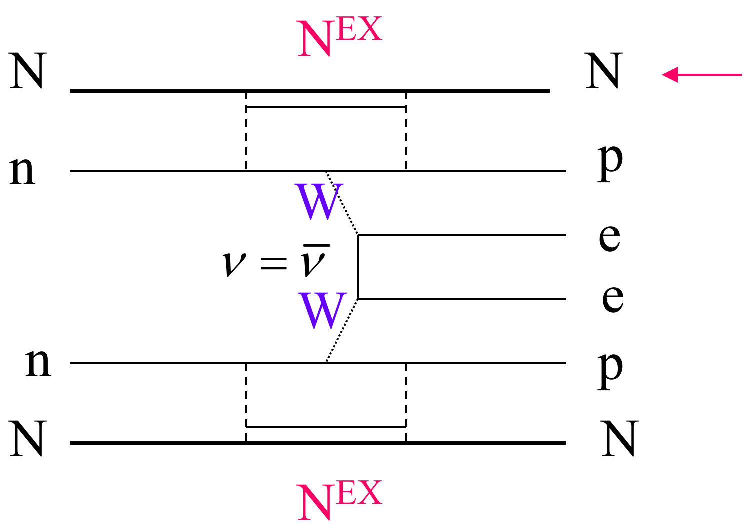
[¶] J. Fujita and K. Ikeda, Nucl. Phys. 67, 145 (1965).
D.H. Wilkinson, Nucl. Phys. A225, 365 (1974).

ORIGIN OF QUENCHING OF g_A



Quenching factor $q_\Delta \cong 0.7$

(Δ means excited states of the nucleon)



Quenching factor $q_{N^{EX}} \cong 0.7$
(nuclear model dependent)

(N^{EX} means excited states of the nucleus not included explicitly)

Maximal quenching:

$$Q = q_\Delta q_{N^{EX}} \cong 0.5$$

The axial vector coupling constant, g_A, appears to the second power in the NME

$$M_{2\nu} = g_A^2 M^{(2\nu)}$$

$$M_{0\nu} = g_A^2 M^{(0\nu)}$$

$$M^{(0\nu)} = M_{GT}^{(0\nu)} - \left(\frac{g_V}{g_A}\right)^2 M_F^{(0\nu)} + M_T^{(0\nu)}$$

and hence to the fourth power in the half-life!

Therefore, results with g_A=1.269 should be multiplied by 6-34 to have realistic estimates of expected half-lives. [See also, H. Robertson ¶, and S. Dell'Oro, S. Marcocci, F. Vissani#.]

¶ R.G.H. Robertson, Modern Phys. Lett. A 28, 1350021 (2013).

S. Dell'Oro, S. Marcocci, and F. Vissani, Phys. Rev. D 90, 033005 (2014).

The question of whether or not g_A in $0\nu\beta\beta$ is quenched as much as in $2\nu\beta\beta$ is of much debate. The two processes differ by the momentum transferred to the leptons. In $2\nu\beta\beta$ this is of the order of few MeV, while in $0\nu\beta\beta$ it is of the order of 100 MeV. The current view is that both factors, q_Δ and q_{Nex}, contribute to $2\nu\beta\beta$, while only q_Δ contributes to $0\nu\beta\beta$.

$$[m_\Delta - m_p = 294 \text{ MeV}, \qquad \langle m_{Nex}\rangle - m_N \sim 10 \text{ MeV}]$$

This problem is currently being addressed from various sides. Experimentally by measuring the matrix elements to and from the intermediate odd-odd nucleus in $2\nu\beta\beta$ decay by means of single charge exchange reactions (^{3}He,t)[§]. Theoretically, by using effective field theory (EFT) to estimate the effect of non-nucleonic degrees of freedom (two-body currents) [¶].

§ P. Puppe *et al.*, Phys. Rev. C 86, 044603 (2012).

¶ J. Menendez, D. Gazit, and A. Schwenk, Phys. Rev. Lett. 107, 062501 (2011).

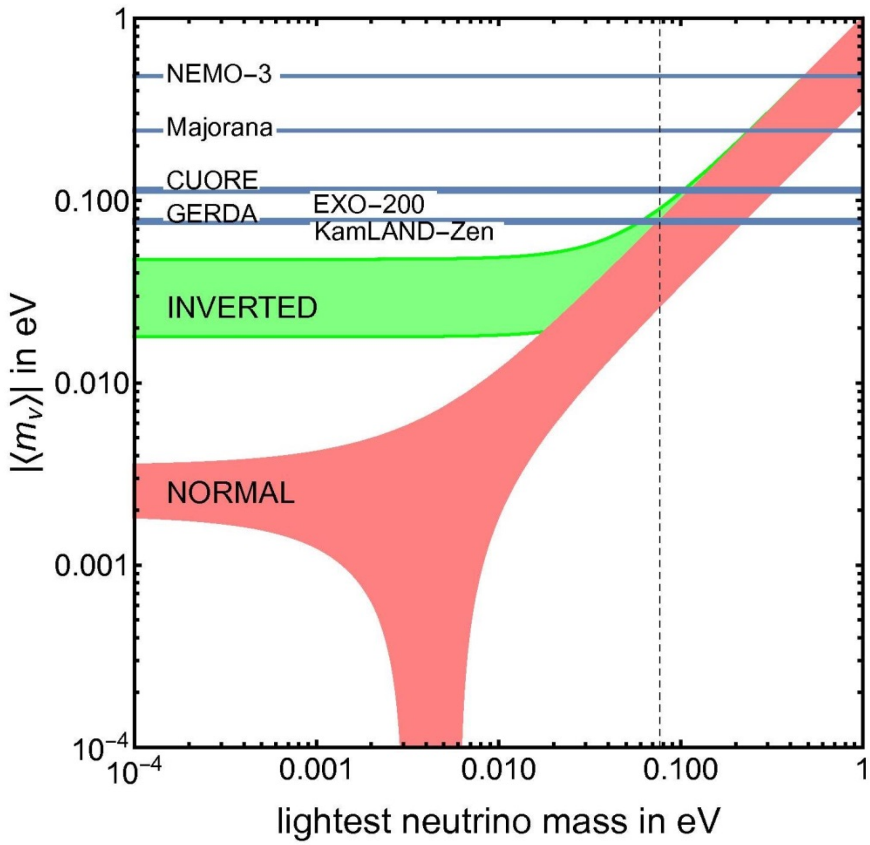
- Very recently, an experimental program (NUMEN) has been set up at LNS in Catania ¶ to measure both single and double charge exchange reaction intensities with heavy ions.

  This program will provide useful information on the Fermi and Gamow-Teller matrix elements of interest in 0νββ and 2νββ decay.

¶ F. Cappuzzello, C. Agodi *et al.*

MASS MECHANISM CONCLUSIONS

Current limits on the neutrino mass from $0\nu\beta^-\beta^-$ (light neutrino exchange) with g_A=1.269, IBM-2 NME, and KI PSF:



CUORE: K. Alfonso *et al.*, PRL 115, 102502 (2015); PRL 120,132501 (2018); D. Adams *et al.* PRL 124, 102501 (2020).
EXO-200: M. Auger *et al.*, PRL 109,032505 (2012); J.B. Albert *et* al. Nature 510, 229 (2014); G. Anton *et al*. PRL 123, 161802 (2019).
GERDA: M. Agostini *et al.*, PRL 111, 122503 (2013); Nature 544, 47 (2017); PRL 125, 252502 (2020).
KamLAND-Zen: A. Gando *et al.*, PRL 110, 062502 (2013); PRL 117, 082503 (2016). Addendum: PRL 117, 109903 (2016).
NEMO-3: R. Arnold *et al.*, PRD 92, 072011 (2015).
Majorana: C.E. Aalseth *et al.*, PRL 120, 132502 (2018).

The major remaining question is the value of g_A. Three scenarios are[¶,§] :

$$g_A = 1.269$$ ← Free value

$$g_A = 1$$ ← Quark value *

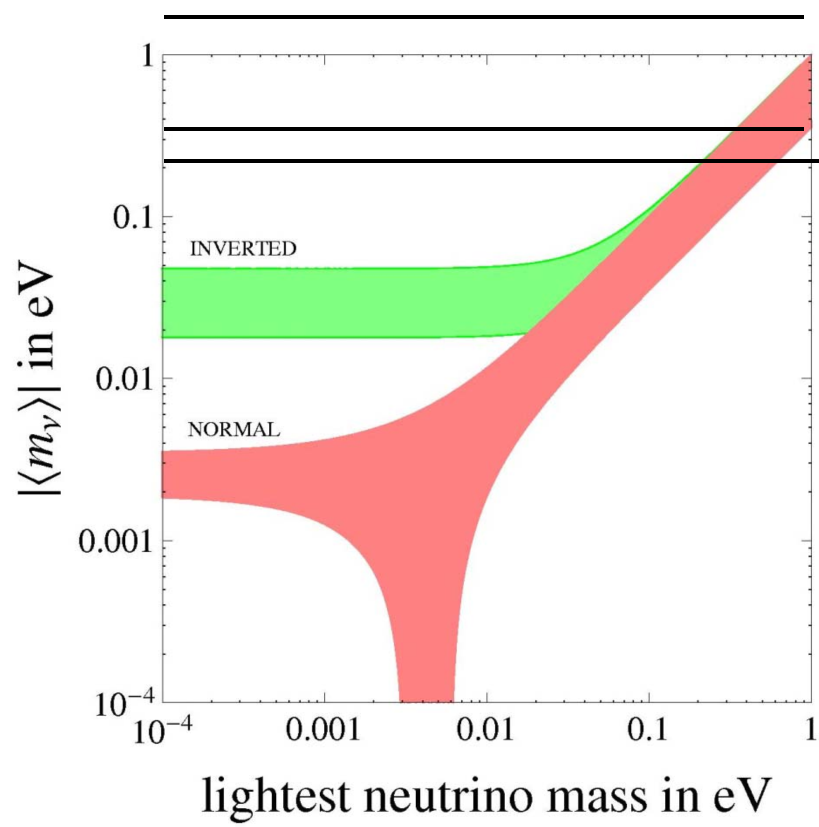
$$g_A = 1.269 A^{-0.18}$$ ← Maximal quenching

* Most likely value

¶ J. Barea, J. Kotila, and F. Iachello, Phys. Rev. C 87, 014315 (2013).

§ S. Dell'Oro, S. Marcocci, and F. Vissani, Phys. Rev. D 90, 033005 (2014).

If g_A is renormalized to ~ 1-0.5, all estimates for half-lives should be increased by a factor of ~ 4-34 and limits on the average neutrino mass should be increased by a factor ~ 1.6-6, making it very difficult to reach in the foreseeable future even the inverted region.



$g_A = 0.5$ $\langle m_\nu \rangle < 1.3 eV$

$g_A = 1$ $\langle m_\nu \rangle < 0.3 eV$

$g_A = 1.269$ $\langle m_\nu \rangle < 0.2 eV$

Limits from EXO in ^{136}Xe decay

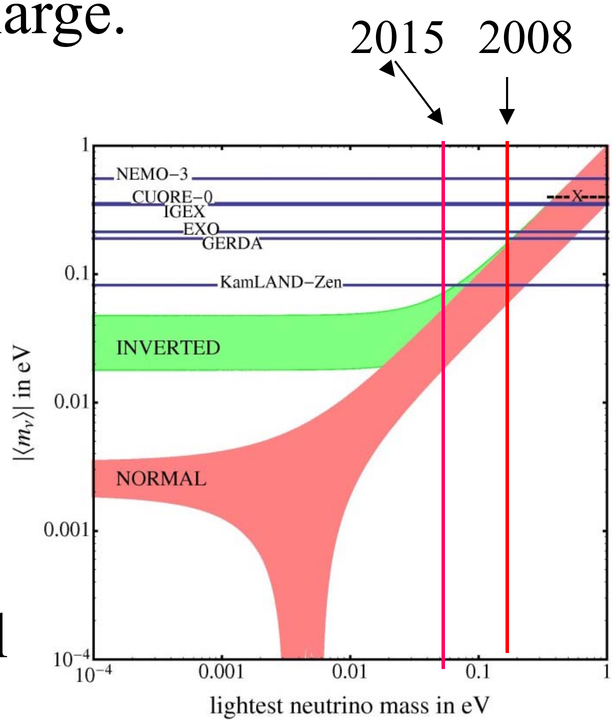
Possibilities to escape this negative conclusion are:
(1) Neutrino masses are degenerate and large.

This possibility will be in tension with the cosmological bound on the sum of the neutrino masses

$$\sum_i m_i \leq 0.6eV \quad (2008)$$

$$\sum_i m_i \leq 0.230eV \quad (2015)\ \text{Planck}\ ¶$$

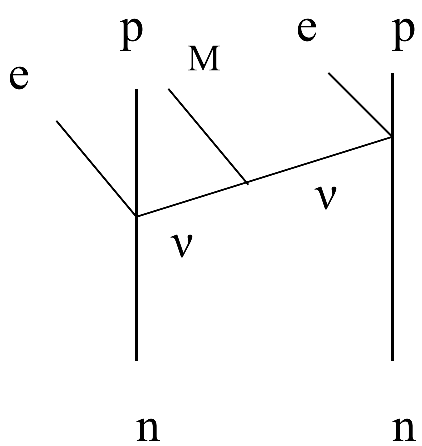
68% confidence level



¶ P.A.R. Ade *et al.* [Planck Collaboration], Astron. Astrophys. 594, A13 (2016).

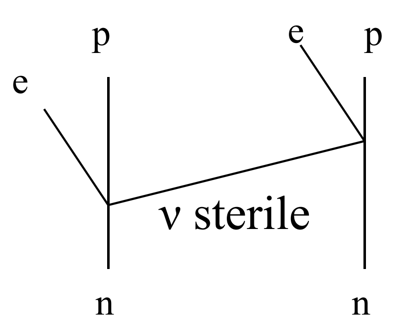
(2) Other scenarios (Majoron emission, sterile neutrinos, …) must be considered.

3



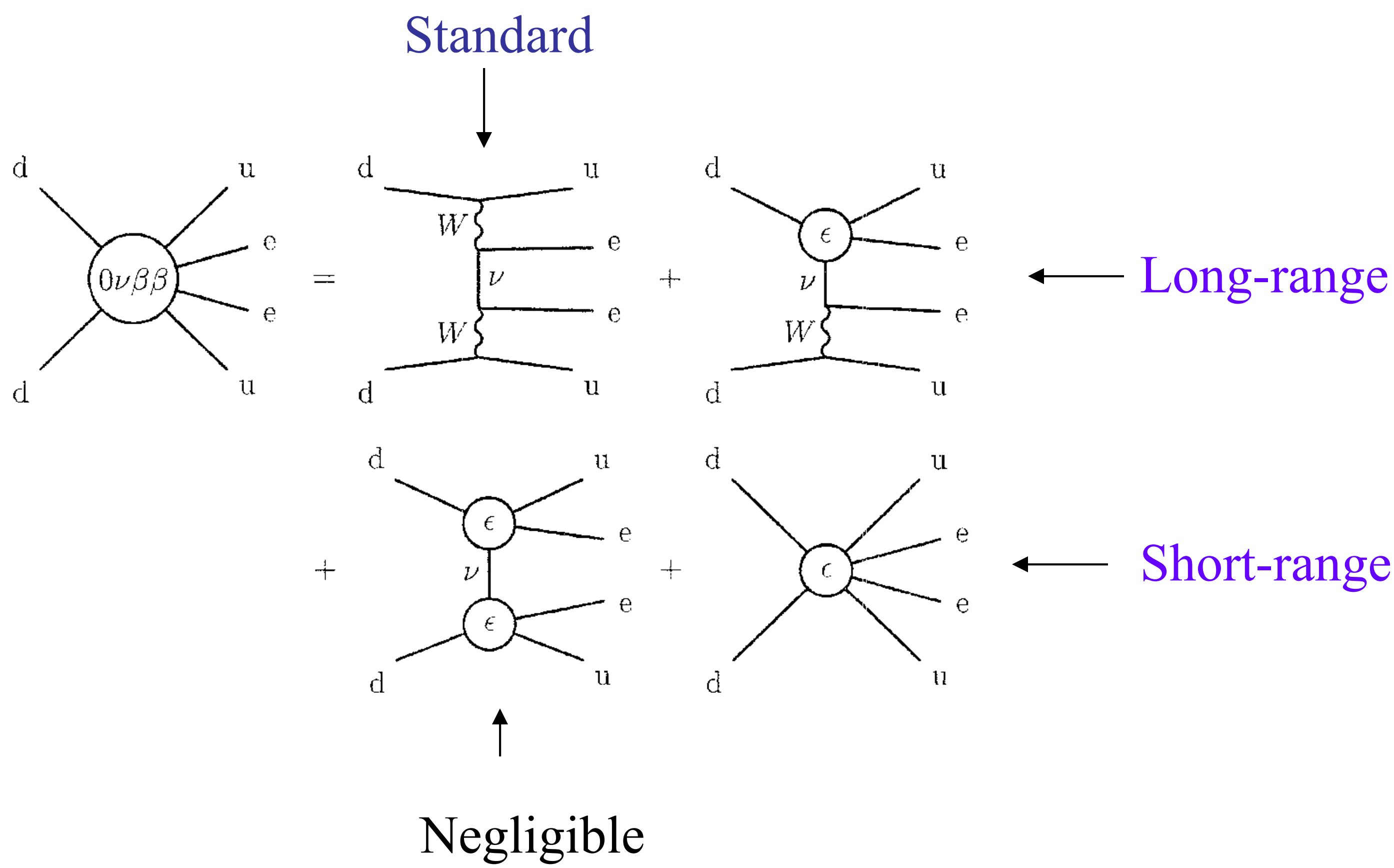
Majoron means a massless neutral boson

4



Sterile means no standard model interactions

(3) Other non-standard mechanisms contribute

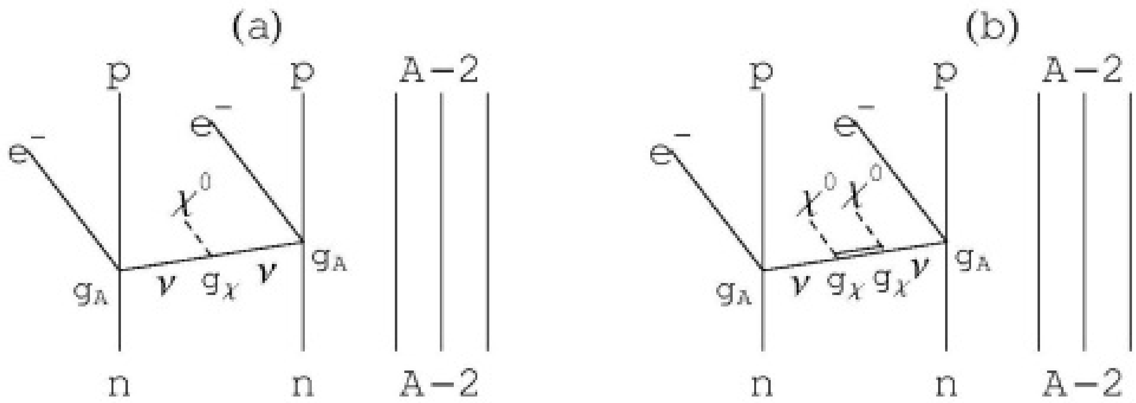


Scenario 2.1: MAJORON EMISSION ¶

0νββM decay:

$$(A,Z) \to (A,Z+2) + 2e^{-} + \chi_0$$

$$(A,Z) \to (A,Z+2) + 2e^{-} + 2\chi_0$$



Different models have been suggested:

TABLE I. Different Majoron-emitting models [14–17]. The third, fourth, and fifth columns indicate whether the Majoron is a Nambu-Goldstone boson, its leptonic charge L, and the model's spectral index, n. The sixth column indicates the nuclear matrix elements of Sec. II appropriate for each model.

Model	Decay mode	NG boson	L	n	NME
IB	$0\nu\beta\beta\chi_0$	No	0	1	M_1
IC	$0\nu\beta\beta\chi_0$	Yes	0	1	M_1
ID	$0\nu\beta\beta\chi_0\chi_0$	No	0	3	M_3
IE	$0\nu\beta\beta\chi_0\chi_0$	Yes	0	3	M_3
IIB	$0\nu\beta\beta\chi_0$	No	−2	1	M_1
IIC	$0\nu\beta\beta\chi_0$	Yes	−2	3	M_2
IID	$0\nu\beta\beta\chi_0\chi_0$	No	−1	3	M_3
IIE	$0\nu\beta\beta\chi_0\chi_0$	Yes	−1	7	M_3
IIF	$0\nu\beta\beta\chi_0$	Gauge boson	−2	3	M_2
"Bulk"	$0\nu\beta\beta\chi_0$	Bulk field	0	2	

¶ H. M. Georgi, S.L. Glashow, and S. Nussinov, Nucl. Phys. B 193, 297 (1981).

Half-life: $\left[\tau_{1/2}^{0\nu M}\right]^{-1} = G_{m\chi_0 n}^{(0)} \left|\left\langle g_{\chi_{ee}^M} \right\rangle\right|^{2m} g_A^4 \left|M_{0\nu M}^{(m,n)}\right|^2$

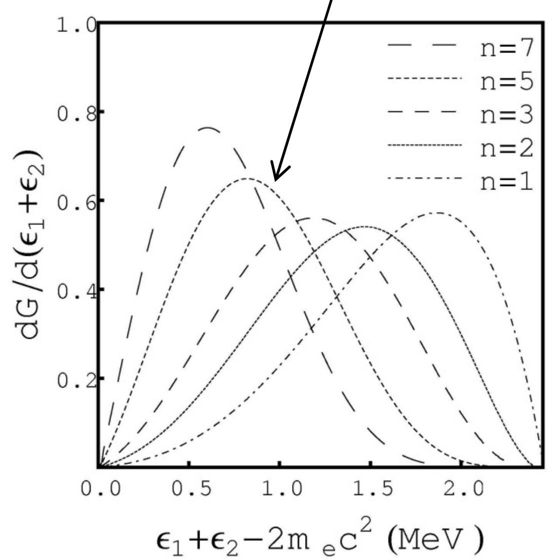
Phase Space Factor Coupling constant Nuclear matrix elements

n spectral index

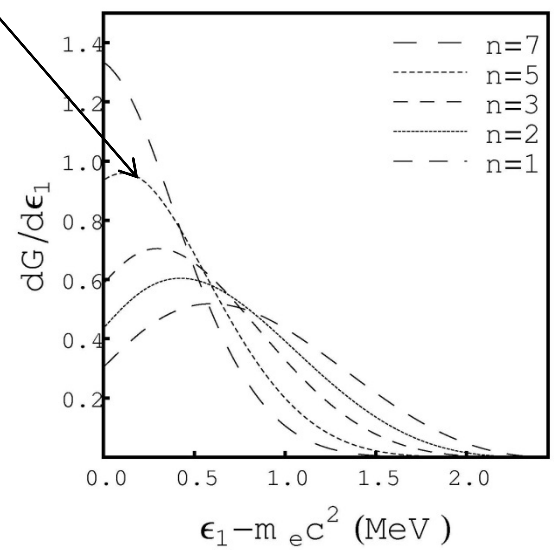
m=1,2 number of Majorons

2νββ (n=5)

PSF §



Summed electron spectra



Single electron spectra

§ J. Kotila, J. Barea and F. Iachello, Phys. Rev. C 91, 064310 (2015).

Nuclear matrix elements (NME) [¶,§]

$$M_1 = g_A^2\left[-\left(\frac{g_V^2}{g_A^2}\right)M_F + M_{GT} - M_T\right]$$

$$M_2 = g_A^2\left[\left(\frac{g_V}{g_A}\right)\frac{f_W}{3}M_{GTR} - \left(\frac{g_V}{g_A}\right)\frac{f_W}{6}M_{TR}\right]$$

$$M_3 = g_A^2\left[-\left(\frac{g_V^2}{g_A^2}\right)M_{F\omega^2} + M_{GT\omega^2} - M_{T\omega^2}\right]$$

Neutrino potentials

$$v_m = \frac{2}{\pi}\frac{1}{q(q+\tilde{A})}$$

$$v_R = \frac{2}{\pi}\frac{1}{Rm_p}\frac{q+\frac{\tilde{A}}{2}}{q(q+\tilde{A})^2}$$

$$v_{\omega^2} = \frac{2}{\pi}m_e^2\frac{q^2+\frac{9}{8}q\tilde{A}+\frac{3}{8}\tilde{A}^2\frac{q}{q+\tilde{A}}}{q^3(q+\tilde{A})^3}$$

¶ M. Hirsch, H.V. Klapdor-Kleingrothaus, S.G. Kovalenko and H. Pas, Phys. Lett. B 372, 8 (1996).
§ J. Kotila and F. Iachello, Phys. Rev. C 103, 044302 (2021).

Limits on half-lives can be set by high-statistics measurements of 2νββ decay, from which one can extract limits on coupling constants.

TABLE III. Limits on the Majoron-neutrino coupling constants $\langle g_{\chi_{ee}^M} \rangle$ for $g_A = 1$. PSF from Ref. [9]. NME from this paper.

Decay mode	Spectral index	Model type	$\mathcal{M}$	$G^{(0)}_{m\chi_0 n}$ [10^{-18} yr]	$\tau_{1/2}$ [yr]	$\lvert\langle g_{\chi_{ee}^M} \rangle\rvert$
^{76}Ge [32]						
$0\nu\beta\beta\chi_0$	1	IB,IC,IIB	6.64	44.2	$>4.2 \times 10^{23}$	$<3.5 \times 10^{-5}$
$0\nu\beta\beta\chi_0\chi_0$	3	ID,IE,IID	0.0026	0.22	$>0.8 \times 10^{23}$	<1.7
$0\nu\beta\beta\chi_0$	3	IIC,IIF	0.381	0.073	$>0.8 \times 10^{23}$	$<0.34 \times 10^{-1}$
$0\nu\beta\beta\chi_0\chi_0$	7	IIE	0.0026	0.420	$>0.3 \times 10^{23}$	<1.9
$0\nu\beta\beta\chi_0$	2	Bulk			$>1.8 \times 10^{23}$	
^{130}Te [29]						
$0\nu\beta\beta\chi_0$	1	IB,IC,IIB	4.40	413	$>2.2 \times 10^{21}$	$<2.4 \times 10^{-4}$
$0\nu\beta\beta\chi_0\chi_0$	3	ID,IE,IID	0.0013	3.21	$>0.9 \times 10^{21}$	<3.8
$0\nu\beta\beta\chi_0$	3	IIC,IIF	0.199	1.51	$>2.2 \times 10^{21}$	$<0.87 \times 10^{-1}$
$0\nu\beta\beta\chi_0\chi_0$	7	IIE	0.0013	14.4	$>0.9 \times 10^{21}$	<2.6
$0\nu\beta\beta\chi_0$	2	Bulk			$>2.2 \times 10^{21}$	
^{130}Te [23]						
$0\nu\beta\beta\chi_0$	1	IB,IC,IIB	4.40	413	$>1.6 \times 10^{22}$	$<8.8 \times 10^{-5}$
^{136}Xe [31]						
$0\nu\beta\beta\chi_0$	1	IB,IC,IIB	3.60	409	$>1.2 \times 10^{24}$	$<1.3 \times 10^{-5}$
$0\nu\beta\chi_0\chi_0$	3	ID,IE,IID	0.0011	3.05	$>2.7 \times 10^{22}$	<1.8
$0\nu\beta\beta\chi_0$	3	IIC,IIF	0.160	1.47	$>2.7 \times 10^{22}$	$<0.31 \times 10^{-1}$
$0\nu\beta\beta\chi_0\chi_0$	7	IIE	0.0011	12.5	$>6.1 \times 10^{21}$	<1.8
$0\nu\beta\beta\chi_0$	2	Bulk			$>2.5 \times 10^{23}$	
^{136}Xe [30]						
$0\nu\beta\beta\chi_0$	1	IB,IC,IIB	3.60	409	$>2.6 \times 10^{24}$	$<8.5 \times 10^{-6}$
$0\nu\beta\beta\chi_0\chi_0$	3	ID,IE,IID	0.0011	3.05	$>4.5 \times 10^{24}$	<0.49
$0\nu\beta\beta\chi_0$	3	IIC,IIF	0.160	1.47	$>4.5 \times 10^{24}$	$<0.24 \times 10^{-2}$
$0\nu\beta\beta\chi_0\chi_0$	7	IIE	0.0011	12.5	$>1.1 \times 10^{22}$	<1.6
$0\nu\beta\beta\chi_0$	2	Bulk			$>1.0 \times 10^{24}$	

R. Arnold *et al.* (NEMO3 Collaboration), Phys. Rev. Lett. 107, 062504 (2011).
C. Arnaboldi *et al.* (CUORE Collaboration), Phys. Lett. B 557,167 (2003).
A. Gando *et al.* (KamLAND-Zen Collaboration), Phys. Rev. C 86, 021601 (R) (2012).
J.B. Albert *et al.* (EXO-200 Collaboration), Phys. Rev. D 90, 092004 (2014).
S. Hemmer *et al.* (GERDA Collaboration), Eur. Phys. J. Plus 130, 139 (2015).

Scenario 2.2: STERILE NEUTRINOS §

§ B. Pontecorvo, Sov. Phys. JEPT 26, 984 (1968)

A scenario currently being extensively discussed is the mixing of additional "sterile" neutrinos.

[The question on whether or not "sterile" neutrinos exist is an active areas of research at the present time with experiments planned at FERMILAB and CERN-LHC.]

NME for sterile neutrinos of arbitrary mass can be calculated by using a transition operator as in scenario 1 and 2 but with

$$f = \frac{m_{\nu I}}{m_e}$$

$$v(p) = \frac{2}{\pi} \frac{1}{\sqrt{p^2 + m_{\nu I}^2}\left(\sqrt{p^2 + m_{\nu I}^2} + \tilde{A}\right)}$$

↑ Effective mass of the sterile neutrinos

IBM-2 NME for this scenario have been calculated ¶.

PSF are the same as in the standard mass scenario.

¶ J. Barea, J. Kotila and F. Iachello, Phys. Rev. D 92, 093001 (2015).

Half-life

$$\left[\tau_{1/2}^{0\nu_h}\right]^{-1} = G_{0\nu} g_A^4 \left|M^{(0\nu_h)}\right|^2 \left| m_p \sum_N (U_{eN})^2 \frac{m_N}{\langle p^2 \rangle + m_N^2} \right|^2$$

$$\langle p^2 \rangle = \frac{M^{(0\nu_h)}}{M^{(0\nu)}} m_p m_e$$

Limit of sterile neutrino contribution for a single neutrino of mass m_N and coupling U_{eN}.

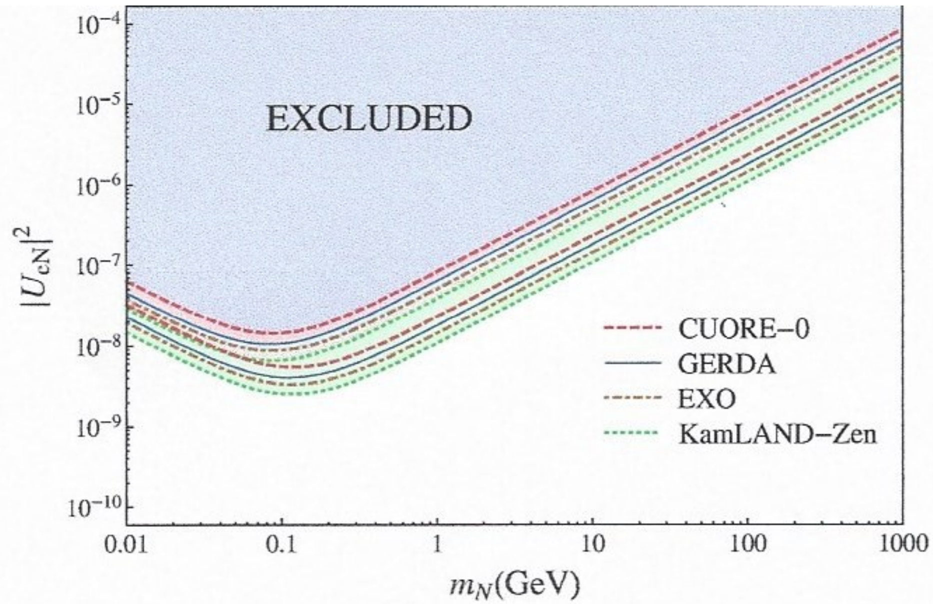


FIG. 4 (color online). Excluded values of $|U_{eN}|^2$ and m_N in the m_N-$|U_{eN}|^2$ plane, for $g_A = 1.269$. For each experiment, GERDA [17], CUORE-0 [18], KamLAND-Zen [19], and EXO [20], a band of values is given, corresponding to our error estimate.

Several types of sterile neutrinos have been suggested.

Scenario a: HEAVY STERILE NEUTRINOS

Sterile neutrinos with masses $m_{\nu I} \gg 1eV$

Possible values of the sterile neutrino, 4a,5a, 6a,…, masses in the keV-GeV range have been suggested by T. Asaka and M. Shaposhnikov, Phys. Lett. B620, 17 (2005) and T. Asaka, S. Blanchet, and M. Shaposhnikov, Phys. Lett. B631, 151 (2005).

Scenario b: LIGHT STERILE NEUTRINOS

Sterile neutrinos with masses $m_{\nu I} \sim 1eV$

Very recently C. Giunti and M. Laveder have suggested sterile neutrinos, 4b,…, with masses in the eV range to account for the reactor anomaly in oscillation experiments, G. Giunti, XVI International Workshop on Neutrino Telescopes, Venice, Italy, March 4, 2015.

CONTRIBUTIONS OF HYPOTHETICAL NEUTRINOS ALL ¶

Known neutrinos

Unknown light sterile

eV

keV

$$\left[\tau_{1/2}^{0\nu}\right]^{-1} = G_{0\nu}\left|\left[\frac{1}{m_e}\sum_{k=1}^{3}U_{ek}^2 m_k + \frac{1}{m_e}\sum_i U_{ei}^2 m_i + \frac{1}{m_e}\sum_j U_{ej}^2 m_j\right]M^{(0\nu)} + \left[m_p\sum_N U_{eN}^2 \frac{m_N}{\langle p^2\rangle + m_N^2} + m_p\sum_{k_h=1}^{3}U_{ek_h}^2\frac{1}{m_{k_h}}\right]M^{(0\nu_h)}\right|$$

Unknown heavy sterile

Unknown heavy neutrinos

¶ J. Barea, J. Kotila and F. Iachello, Phys. Rev. D 92, 093001 (2015).

Light sterile neutrinos $\langle m_{N,light} \rangle = \sum_{k=1}^{3} U_{ek}^2 m_k + \sum_i U_{ei}^2 m_i$

Half-life

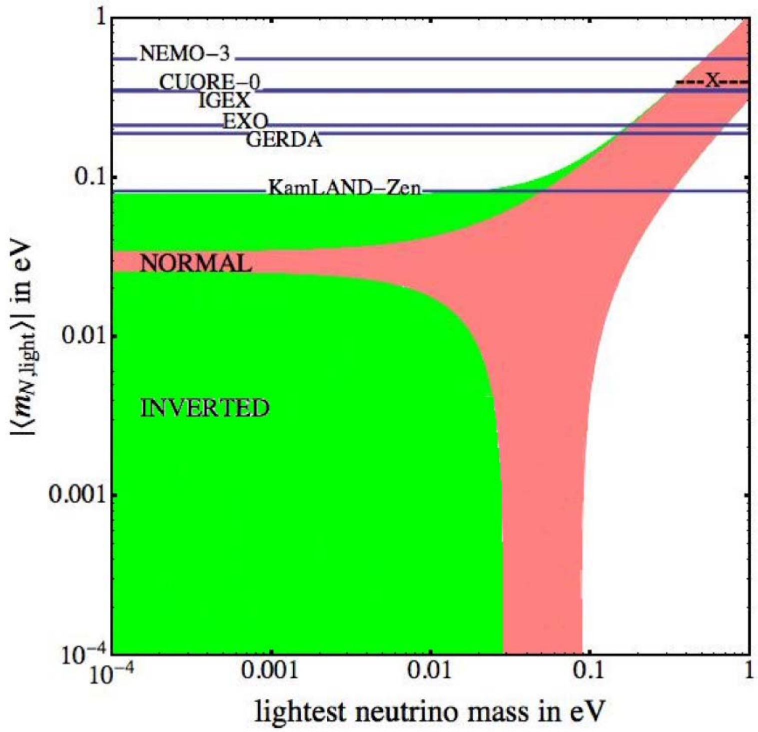
$$\left[\tau_{1/2}^{0\nu}\right]^{-1} = G_{0\nu} g_A^4 \left(\frac{\langle m_{N,light} \rangle}{m_e} \right)^2 \left| M^{(0\nu)} \right|^2$$

Simple case: a 4th neutrino with mass m_4=1eV and coupling $|U_{e4}|^2$ =0.03 ¶

$$\langle m_{N,light} \rangle = \sum_{k=1}^{3} U_{ek}^2 m_k + U_{e4}^2 e^{i\alpha_4} m_4$$

¶ C. Giunti and M. Laveder, *loc.cit*. (2015).

The presence of sterile neutrinos changes completely the picture



g_A=1.269

Figure courtesy of Jenni Kotila, adapted from J. Barea, J. Kotila and F. Iachello, *loc.cit*. (2015).

With sterile neutrinos (with properties of scenario 4b ¶) and g_A=1.269, the inverted hierarchy is reachable by GERDA-PHASE II and CUORE.

¶ C. Giunti and M. Laveder, *loc.cit.* (2015).

Scenario 3: NON-STANDARD MECHANISMS

3.1. Short-range mechanism §

$$L_{Short} = \frac{G_F^2 \cos^2\theta_c}{2m_p}\left[\varepsilon_1 JJj + \varepsilon_2 J^{\mu\nu}J_{\mu\nu}j + \varepsilon_3 J^{\mu}J_{\mu}j + \varepsilon_4 J^{\mu}J_{\mu\nu}j^{\nu} + \varepsilon_5 J^{\mu}Jj_{\mu}\right]$$

Half-life

$$\left[\tau_{1/2}^{0\nu}\right]^{-1} = G_{11+}^{(0)}\left|\sum_{I=1}^{3}\varepsilon_I^L M_I + \varepsilon_\nu M_\nu\right|^2 + G_{11+}^{(0)}\left|\sum_{I=1}^{3}\varepsilon_I^R M_I\right|^2 + G_{66}^{(0)}\left|\sum_{I=4}^{5}\varepsilon_I M_I\right|^2$$

$$+G_{11-}^{(0)} \times 2\,\mathrm{Re}\left[\left(\sum_{I=1}^{3}\varepsilon_I^L M_I + \varepsilon_\nu M_\nu\right)\left(\sum_{I=1}^{3}\varepsilon_I^R M_I\right)^*\right]$$

$$+G_{16}^{(0)} \times 2\,\mathrm{Re}\left[\left(\sum_{I=1}^{3}\varepsilon_I^L M_I - \sum_{I=1}^{3}\varepsilon_I^R + \varepsilon_\nu M_\nu\right)\left(\sum_{I=4}^{5}\varepsilon_I M_I\right)^*\right]$$

§ F.F. Deppisch, L. Graf, F. Iachello and J. Kotila, Phys. Rev. D 102, 095016 (2020)

Upper limits on effective mass and on short range couplings

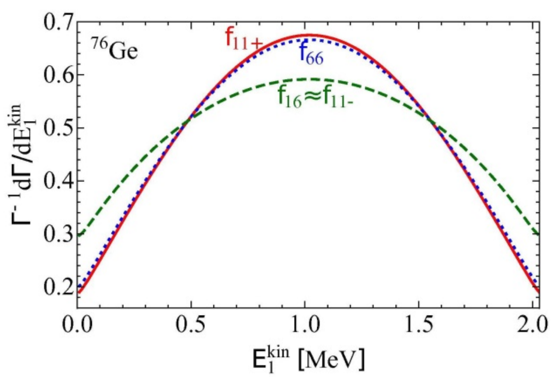
TABLE VIII. Upper limits on the effective $0\nu\beta\beta$ mass $|m_{\beta\beta}|$ and the short-range ϵ_I couplings in units of 10^{-10} from current experimental bounds $T_{1/2}^{\rm exp}$ at 90% C.L., assuming a single contribution at a time and $g_A = 1.0$. The chiralities of the involved quark currents are specified: the label XX stands for the case when both chiralities are the same, $XX = RR, LL$, and XY applies if the chiralities are different, $XY = RL, LR$. The limit on ϵ_4 applies for all chirality combinations.

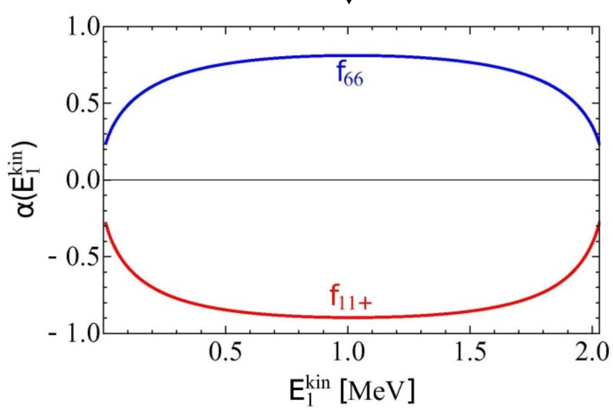
Isotope	$T_{1/2}^{\rm exp}$ [yr]		$\lvert m_{\beta\beta}\rvert$ [meV]	$\lvert\epsilon_1^{XX}\rvert$	$\lvert\epsilon_1^{XY}\rvert$	$\lvert\epsilon_2^{XX}\rvert$	$\lvert\epsilon_3^{XX}\rvert$ [10^{-10}]	$\lvert\epsilon_3^{XY}\rvert$	$\lvert\epsilon_4\rvert$	$\lvert\epsilon_5^{XX}\rvert$	$\lvert\epsilon_5^{XY}\rvert$
^{76}Ge	1.8×10^{26}	[9]	118	2.90	2.84	88.4	77.1	154	130	102	68.1
^{82}Se	2.4×10^{24}	[77]	599	15.9	15.5	445	375	768	654	764	440
^{96}Zr	9.2×10^{21}	[78]	9130	85.5	84.8	5640	8510	12600	11300	1200	1110
^{100}Mo	1.1×10^{24}	[79]	733	6.10	6.04	401	608	901	774	84.1	77.5
^{116}Cd	2.2×10^{23}	[80]	2720	22.3	22.1	1430	2090	3170	2800	321	294
^{128}Te	1.1×10^{23}	[81]	13300	283	277	9300	8080	17300	12100	7630	5390
^{130}Te	3.2×10^{25}	[82]	252	5.38	5.27	178	153	336	270	158	112
^{136}Xe	1.1×10^{26}	[83]	114	2.50	2.45	83.4	72.5	157	127	74	52.4
^{150}Nd	2.0×10^{22}	[84]	3830	45.5	45.1	2730	3590	6190	5240	659	596

How to distinguish the mass mechanism from mechanisms 4-5:

Angular distribution

↓





3.2. Long-range mechanism [¶] [§]

$$L_{Long} = \frac{G_F \cos\theta_c}{\sqrt{2}} \left[J^{\dagger}_{V-A,\mu} j^{\mu}_{V-A} + \sum_{\alpha,\beta}{}' \varepsilon_{\alpha\beta} J^{\dagger}_{\alpha} j_{\beta} \right] \qquad \alpha, \beta = S \pm P, V \pm A, T \pm T_5$$

↑ Mass mechanism

Each model of long-range ββ decay is defined by 12 coefficients

$$\varepsilon^{V\mp A}_{V\mp A}, \varepsilon^{S\mp P}_{S\mp P}, \varepsilon^{T\mp T_5}_{T\mp T_5}$$

Non-zero coefficients of some suggested models

Model	Non-zero ε	
L-R	$\varepsilon^{V-A}_{V+A}, \varepsilon^{V+A}_{V\mp A}$	3 parameters
SUSY	$\varepsilon^{S\mp P}_{S+P}, \varepsilon^{V-A}_{V-A}, \varepsilon^{T+T_5}_{T+T_5}$	4 parameters

[¶] A. Ali, A.V. Borisov and D.V. Zhuridov, arXiv:0706.4165v3[hep-ph]

[§] J. Kotila, J. Ferretti and F. Iachello, arXiv:2110.09141v1[hep-ph] 18 Oct 2021

L-R MODELS

L-R models were investigated by Doi *et al.* ¶ and Tomoda §

We have recalculated them recently # with IBM2 NME and KI PSF

$$\varepsilon^{V-A}_{V+A,i} = \kappa U_{ei}$$
$$\varepsilon^{V+A}_{V+A,i} = \lambda V_{ei}$$
$$\varepsilon^{V+A}_{V-A,i} = \eta U_{ei}$$

3 parameters

$$\langle m_\nu \rangle = \sum_i m_i U_{ei}^2$$

Standard mass mechanism

$$\langle \lambda \rangle = \lambda \sum_i U_{ei} V_{ei} \equiv \bar{\varepsilon}^{V+A}_{V+A}$$
$$\langle \eta \rangle = \eta \sum_i U_{ei} V_{ei} \equiv \bar{\varepsilon}^{V+A}_{V-A}$$
$$\langle \kappa \rangle = \kappa \sum_i U_{ei}^2 \equiv \bar{\varepsilon}^{V-A}_{V+A} = 0$$

2 parameters

¶ M. Doi, T. Kotani, H. Nishina, K. Okuda, and E. Takesugi, Prog. Theor. Phys. 66, 1739 (1981).

§ T. Tomoda, Rept. Prog. Phys. 54, 53 (1991).

J. Kotila, J. Ferretti and F. Iachello, *loc. cit.*

Half-life

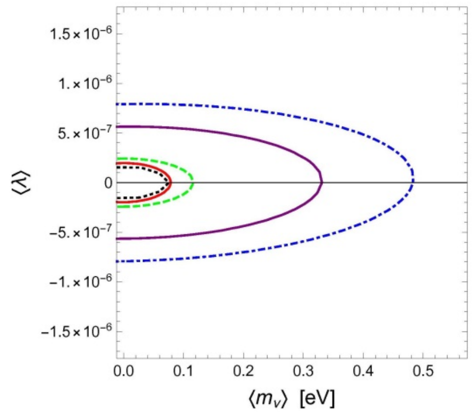
$$\left[\tau_{1/2}^{0\nu}\left(0^+ \rightarrow 0^+\right)\right]^{-1} = A^{(0)}$$

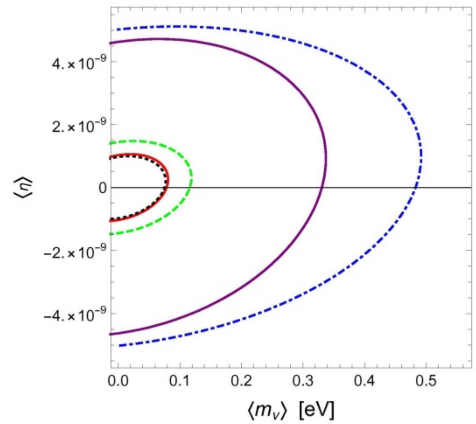
Angular coefficient

$$K = \frac{A^{(1)}}{A^{(0)}}$$

$$A^{(i)} = C_{mm}^{(i)}\left(\frac{\langle m_\nu \rangle}{m_e}\right)^2 + C_{\lambda\lambda}^{(i)}\langle \lambda \rangle^2 + C_{\eta\eta}^{(i)}\langle \eta \rangle^2 + 2C_{m\lambda}^{(i)}\frac{\langle m_\nu \rangle}{m_e}\langle \lambda \rangle$$
$$+2C_{m\eta}^{(i)}\frac{\langle m_\nu \rangle}{m_e}\langle \eta \rangle + 2C_{\lambda\eta}^{(i)}\langle \lambda \rangle\langle \eta \rangle$$

Limits on
L-R models

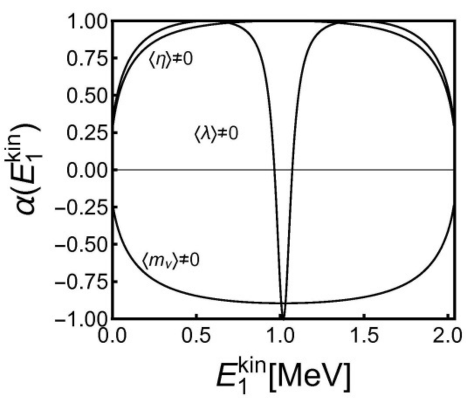


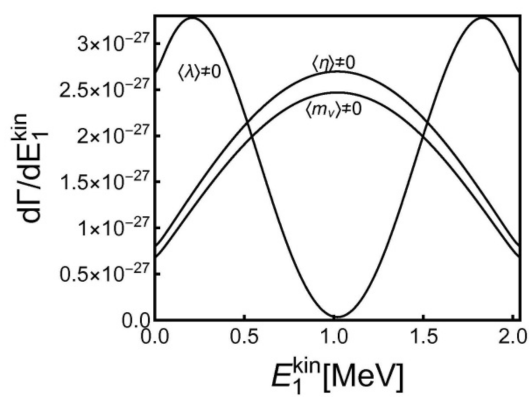


$T_{1/2}^{exp}$ [y]	$\frac{\langle m_\nu\rangle}{m_e}$	$\langle\lambda\rangle$	$\langle\eta\rangle$
^{76}Ge 1.8×10^{26} [2]	1.5×10^{-7}	2.0×10^{-7}	1.0×10^{-9}
^{82}Se 3.5×10^{24} [7]	6.5×10^{-7}	5.7×10^{-7}	4.6×10^{-9}
^{100}Mo 1.1×10^{24} [5]	9.4×10^{-7}	7.9×10^{-7}	5.0×10^{-9}
^{130}Te 3.2×10^{25} [1]	2.3×10^{-7}	2.4×10^{-7}	1.4×10^{-9}
^{136}Xe 1.1×10^{26} [4]	1.5×10^{-7}	1.6×10^{-7}	1.0×10^{-9}

How to distinguish L-R models from mass mechanism: Angular correlation and single electron energy distribution

^{76}Ge





SUSY MODELS

SUSY models were investigated by Pas *et al.* ¶

We have recalculated them with IBM2 NME and KI PSF §

$$\bar{\varepsilon}_{V-A}^{V-A} = \gamma \sum_i U_{ei}^2 \equiv \langle \gamma \rangle = 0$$

Non-zero coefficients

$$\bar{\varepsilon}_{S+P}^{S+P} = \theta \sum_i U_{ei} V'_{ei} \equiv \langle \theta \rangle$$

$$\bar{\varepsilon}_{S+P}^{S-P} = \tau \sum_i U_{ei} V'_{ei} \equiv \langle \tau \rangle$$

$$\bar{\varepsilon}_{T+T_5}^{T-T_5} = \varphi \sum_i U_{ei} V''_i \equiv \langle \varphi \rangle$$

3 parameters

$$A^{(i)} = C_{mm}^{(i)} \left(\frac{\langle m_\nu \rangle}{m_e} \right)^2 + C_{\theta\theta}^{(i)} \langle \theta \rangle^2 + C_{\tau\tau}^{(i)} \langle \tau \rangle^2 + C_{\varphi\varphi}^{(i)} \langle \varphi \rangle^2$$
$$+ 2C_{\theta\tau}^{(i)} \langle \theta \rangle \langle \tau \rangle + 2C_{\theta\varphi}^{(i)} \langle \theta \rangle \langle \varphi \rangle + 2C_{\tau\varphi}^{(i)} \langle \tau \rangle \langle \varphi \rangle$$
$$+ 2C_{m\theta}^{(i)} \frac{\langle m_\nu \rangle}{m_e} \langle \theta \rangle + 2C_{m\tau}^{(i)} \frac{\langle m_\nu \rangle}{m_e} \langle \tau \rangle + 2C_{m\varphi}^{(i)} \frac{\langle m_\nu \rangle}{m_e} \langle \varphi \rangle$$

$$\left[\tau_{1/2}^{0\nu} \left(0^+ \to 0^+ \right) \right]^{-1} = A^{(0)}$$

$$K = \frac{A^{(1)}}{A^{(0)}}$$

¶ H. Pas, M. Hirsch, H.V. Klapdor-Kleingrothaus and S.G. Kovalenko, Phys. Lett. B 453, 194 (1999).

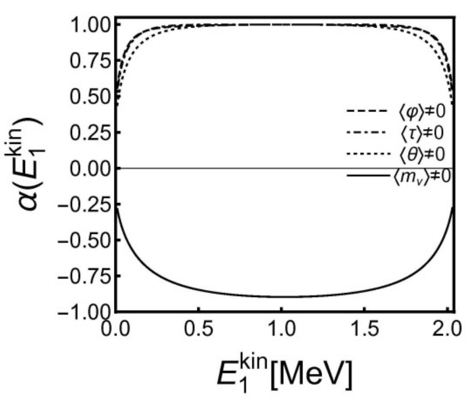
§ J. Kotila, J. Ferretti and F. Iachello, *loc.cit.* (2021).

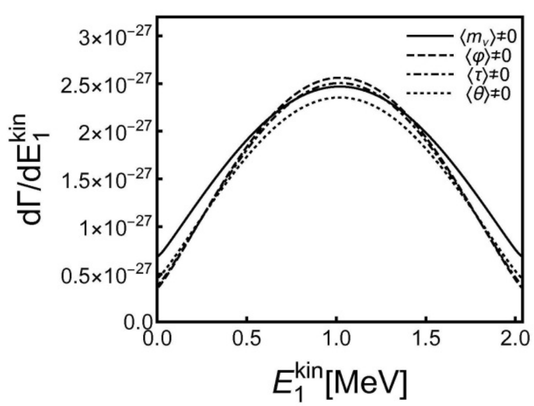
Limits on SUSY models

$T_{1/2}^{\exp}$ [y]	$\frac{\langle m_\nu \rangle}{m_e}$	$\langle \theta \rangle$	$\langle \tau \rangle$	$\langle \varphi \rangle$
^{76}Ge 1.8×10^{26} [2]	1.5×10^{-7}	2.9×10^{-7}	1.2×10^{-7}	3.3×10^{-8}
^{82}Se 3.5×10^{24} [7]	6.5×10^{-7}	1.0×10^{-6}	4.4×10^{-7}	1.2×10^{-7}
^{100}Mo 1.1×10^{24} [5]	9.4×10^{-7}	2.2×10^{-6}	4.1×10^{-7}	2.0×10^{-7}
^{130}Te 3.2×10^{25} [1]	2.3×10^{-7}	2.2×10^{-7}	1.0×10^{-7}	3.5×10^{-8}
^{136}Xe 1.1×10^{26} [4]	1.5×10^{-7}	1.4×10^{-7}	6.6×10^{-8}	2.3×10^{-8}

How to distinguish SUSY models from mass mechanism: Angular correlation

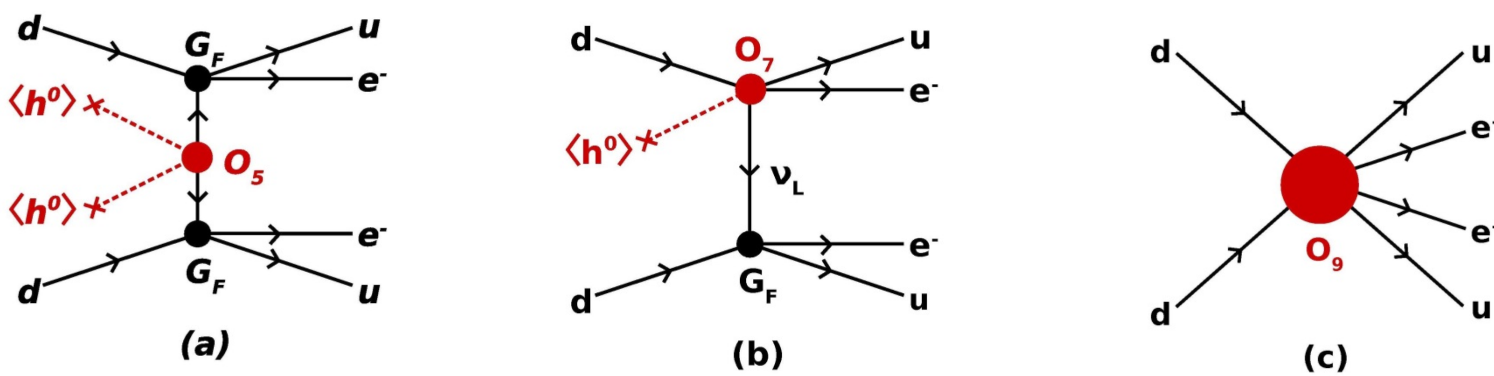
^{76}Ge





CONCLUSIONS

Complete study of all possible mechanisms of neutrinoless DBD up to dim-9 is now available



Dim5-Standard mass mechanism	Dim7-Non-standard Long-range mechanism	Dim9-Non-standard Short-range mechanism

Figure from F.F. Deppisch, L. Graf, F. Iachello and J. Kotila, Phys. Rev. D 102, 095016 (2020).

APPENDIX A: REFERENCES

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