The short-time approximation of nuclear responses to weak probes

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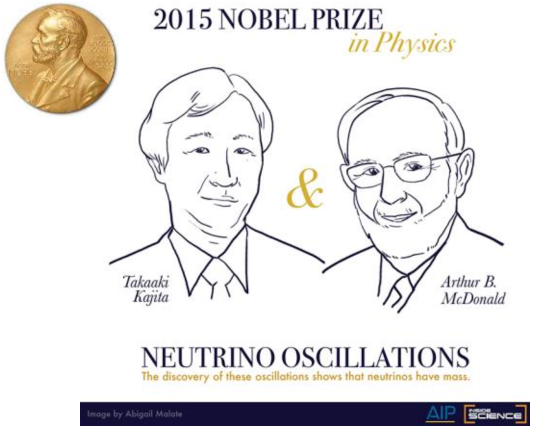
Motivation

Neutrinos produced in flavor eigenstates, travel as mass eigenstates, and due to **BSM non-zero mass** can oscillate between flavor states

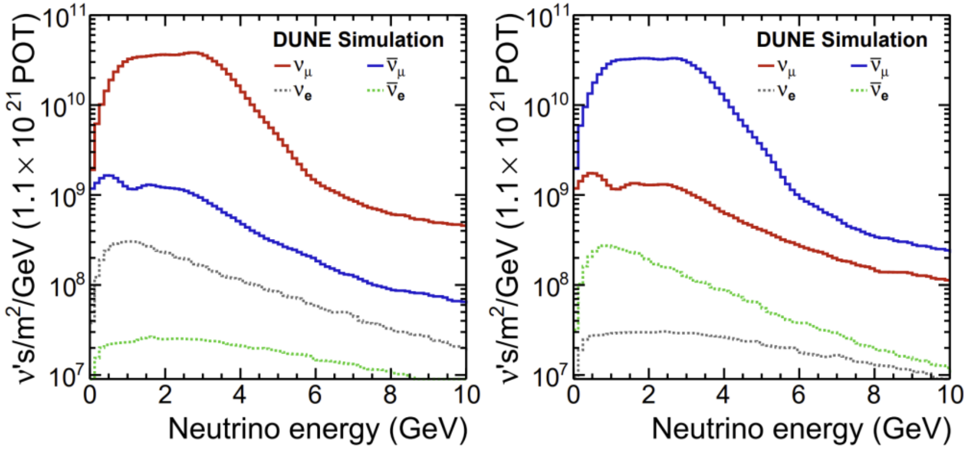
Experiments seek to measure oscillation parameters

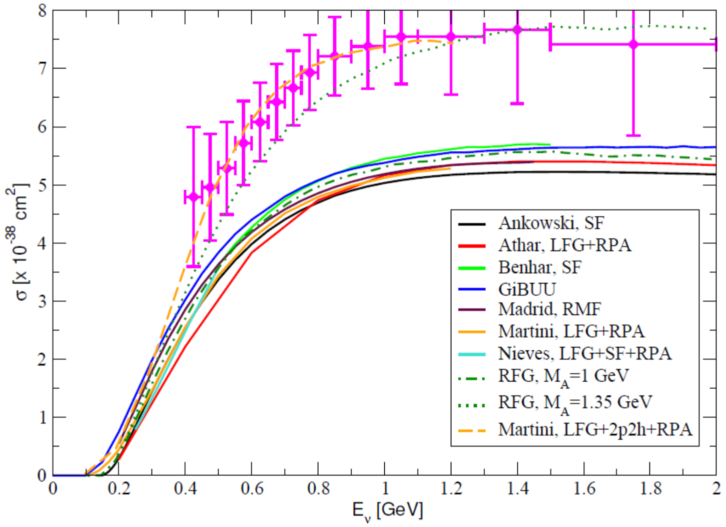
Length can be controlled in a long-baseline accelerator experiment

Accurate modeling of neutrino-nucleus cross-sections needed to infer beam energy



L. Alvarez-Ruso arXiv:1012.3871





DUNE, Eur. Phys. J. C 80, 978 (2020)

$$P_{\nu_\mu \to \nu_e} = \sin^2(2\theta)\sin^2\left(\frac{\Delta m_{21}^2 L}{2E_\nu}\right)$$

$$\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \end{pmatrix}$$

Real time response function

Generic form of the electroweak cross section: $d\sigma \propto L_{\mu\nu} R^{\mu\nu}$

$\mu, \nu = 0, x, y, z$

Standard definition of a response to an external probe with energy and momentum transfer ω and **q**:

$$R(\mathbf{q},\omega) = \sum_f \langle 0|\mathcal{O}^\dagger(\mathbf{q})|f\rangle\langle f|\mathcal{O}(\mathbf{q})|0\rangle\delta(E_f - E_0 - \omega)$$

Can be recast in terms of a real-time propagator:

$$R(\mathbf{q},\omega) = \int \frac{dt}{2\pi} e^{i(E_0+\omega)t}\langle 0|\mathcal{O}^\dagger(\mathbf{q})e^{-iHt}\mathcal{O}(\mathbf{q})|0\rangle$$

Pastore et al. PRC 101, 044612 (2020)
Andreoli et al. PRC 105, 014002 (2022)

Standard QMC approach: Euclidean Response

By wick rotating to imaginary time *τ = it*

$$R(\mathbf{q},\omega) = \int \frac{dt}{2\pi} e^{\omega\tau} \langle 0|\mathcal{O}^\dagger(\mathbf{q}) e^{-(H-E_0)\tau} \mathcal{O}(\mathbf{q})|0\rangle$$

Laplace Transform more easily evaluated with quantum Monte Carlo and is inverted using Maximum Entropy techniques **[Lovato et al PRC 91, 062501 (2015)]**:

$$\widetilde{R}(\mathbf{q},\tau) = \langle 0|\mathcal{O}^\dagger(\mathbf{q}) e^{-(H-E_i)\tau} \mathcal{O}(\mathbf{q})|0\rangle$$

One-body physics: Plane Wave Impulse Approximation

Factorize into a struck particle and A-1 spectator system

$$R(\mathbf{q},\omega) = \int d\mathbf{k} n(\mathbf{k}) r(\mathbf{k},\mathbf{q}) \delta\left(\omega - \frac{(\mathbf{k}+\mathbf{q})^2 - \mathbf{k}^2}{2m}\right)$$

Neglects two-body physics in the electroweak current operators

Missing Pauli blocking makes this a high-energy approximation

Valid when the momentum of the removed particle >> the typical momentum of particles in the system and final state interactions are small

Beyond PWIA: The short-time approximation

Want a method that reduces computational costs while retaining important two-body physics

Sum rules are determined by responses at $\tau=0$, high energy physics corresponds to short imaginary time propagations

Such an approximation is obtained retaining at most two active nucleons, first developed in **Pastore et al. PRC 101, 044612 (2020)**

Propagating at most two active nucleons is computationally less expensive and thus amenable to studying heavier nuclei of experimental relevance

Sum over two-body intermediate states allows investigation of exclusive processes and could allow one to study meson production

Two-body physics: Current-current correlator

For short imaginary times, one may expand the propagator as

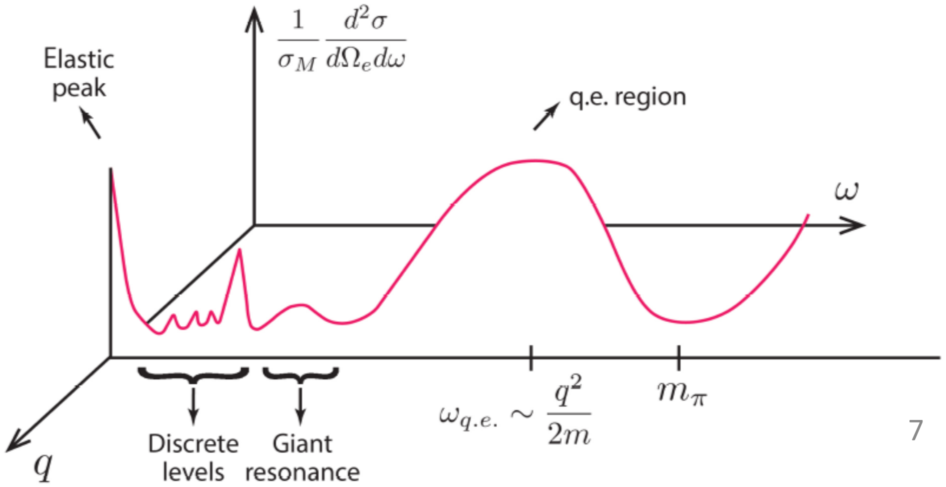
$$e^{-iHt} \approx 1 - iHt + \mathcal{O}(t^2) \approx 1 - i\left(\sum_i T_i + \sum_{i<j} v_{ij} + \ldots\right) t + \ldots$$

Making the above approximation, one only correlates two active nucleons at a time

Errors on the order $\mathcal{O}\left(\frac{\omega_{qe}^2}{B_{\text{pair}}^2}\right)$ in the region of the quasi-elastic peak

$$\omega_{qe} = \sqrt{q^2 + m^2} - m \qquad B_{\text{pair}} \approx \frac{2B}{A(A-1)} \lesssim 20 \text{ MeV}$$

Limits *q* where STA is valid

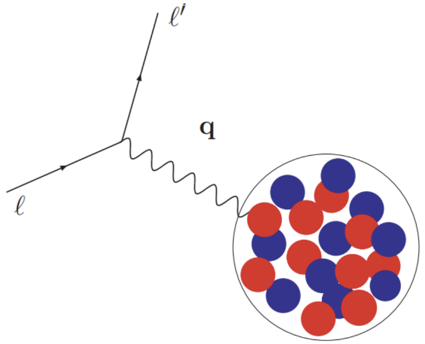


Pastore et al. PRC 101, 044612 (2020)
Andreoli et al. PRC 105, 014002 (2022)

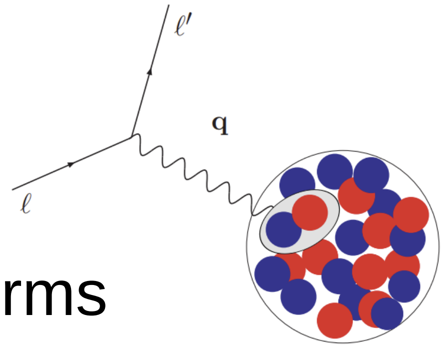
Two-body physics: Current-current correlator

Schematic EM and weak nuclear current : $\rho = \sum_i \rho_i + \sum_{ij} \rho_{ij}; \; \mathbf{j} = \sum_i \mathbf{j}_i + \sum_{ij} \mathbf{j}_{ij}$

Making the approximation of two active particles *i* and *j*



$$\begin{aligned}\mathcal{O}^\dagger(\mathbf{q})e^{-iHt}\mathcal{O}(\mathbf{q}) &= \left(\sum_i \mathcal{O}_i^\dagger(\mathbf{q}) + \sum_{i<j} \mathcal{O}_{ij}^\dagger(\mathbf{q})\right) e^{-iHt} \left(\sum_{i'} \mathcal{O}_{i'}(\mathbf{q}) + \sum_{i'<j'} \mathcal{O}_{i'j'}(\mathbf{q})\right) \\ &= \sum_i \mathcal{O}_i^\dagger(\mathbf{q})e^{-iHt}\mathcal{O}_i(\mathbf{q}) + \sum_{i\neq j} \mathcal{O}_i^\dagger(\mathbf{q})e^{-iHt}\mathcal{O}_j(\mathbf{q}) \\ &\quad + \sum_{i\neq j}\left(\mathcal{O}_i^\dagger(\mathbf{q})e^{-iHt}\mathcal{O}_{ij}(\mathbf{q}) + \mathcal{O}_{ij}^\dagger(\mathbf{q})e^{-iHt}\mathcal{O}_i(\mathbf{q}) + \mathcal{O}_{ij}^\dagger(\mathbf{q})e^{-iHt}\mathcal{O}_{ij}(\mathbf{q})\right)\end{aligned}$$



Retains important contributions coming from 1b*2b interference terms

Pastore et al. PRC 101, 044612 (2020)
Andreoli et al. PRC 105, 014002 (2022)

Generic expectation value

Using a complete set of two body-final states:

$$\langle \mathcal{O}_L^\dagger \mathcal{O}_R \rangle = \frac{N(N-1)}{2} \sum_{\alpha_1''\alpha_2''\alpha_1'\alpha_2'} \sum_{\alpha_{N-2}} \int d\mathbf{R}'' d\mathbf{r}'' d\mathbf{R}' d\mathbf{r}' d\mathbf{R}_{N-2}$$
$$\times \langle 0 | \mathcal{O}_L^\dagger | \mathbf{R}'' \mathbf{r}'' \alpha_1'' \alpha_2'' \mathbf{R}_{N-2} \alpha_{N-2} \rangle \langle \mathbf{R}'' | e^{-iH_{12}^{\mathrm{CM}} t} | \mathbf{R}' \rangle$$
$$\times \langle \mathbf{r}'' \alpha_1'' \alpha_2'' | e^{-iH_{12}^{\mathrm{rel}} t} | \mathbf{r}' \alpha_1' \alpha_2' \rangle \times \langle \mathbf{R}' \mathbf{r}' \alpha_1' \alpha_2' \mathbf{R}_{N-2} \alpha_{N-2} | \mathcal{O}_R | 0 \rangle$$

Integrations over coordinates may be performed with some numerical integration scheme (Gauss-Legendre, Monte Carlo, …)

Pastore et al. PRC 101, 044612 (2020)
Andreoli et al. PRC 105, 014002 (2022)

Variational Monte Carlo

Stochastic method to solve the Schrödinger Equation $H|\Psi\rangle = E|\Psi\rangle$ for some many-body Hamiltonian (Argonne v_{18}, χEFT, …)

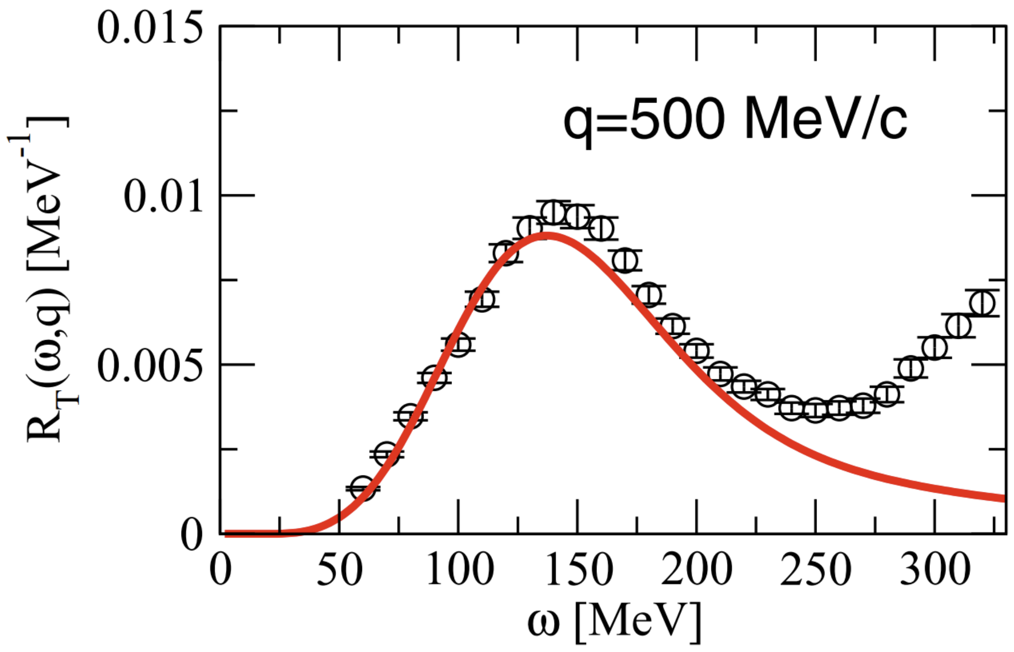
Generic fermion trial wave function may be written in terms of an anti-symmetric long-range term and a correlation operator

$$|\Psi_T\rangle = \hat{F}|\Phi\rangle$$

Embedded in the correlation operator are variational parameters that are optimized by minimizing the energy expectation value obtained by Monte Carlo integration:

$$E_V = \frac{\langle\Psi_V|H|\Psi_V\rangle}{\langle\Psi_V|\Psi_V\rangle} \geq E_0$$

Electromagnetic responses



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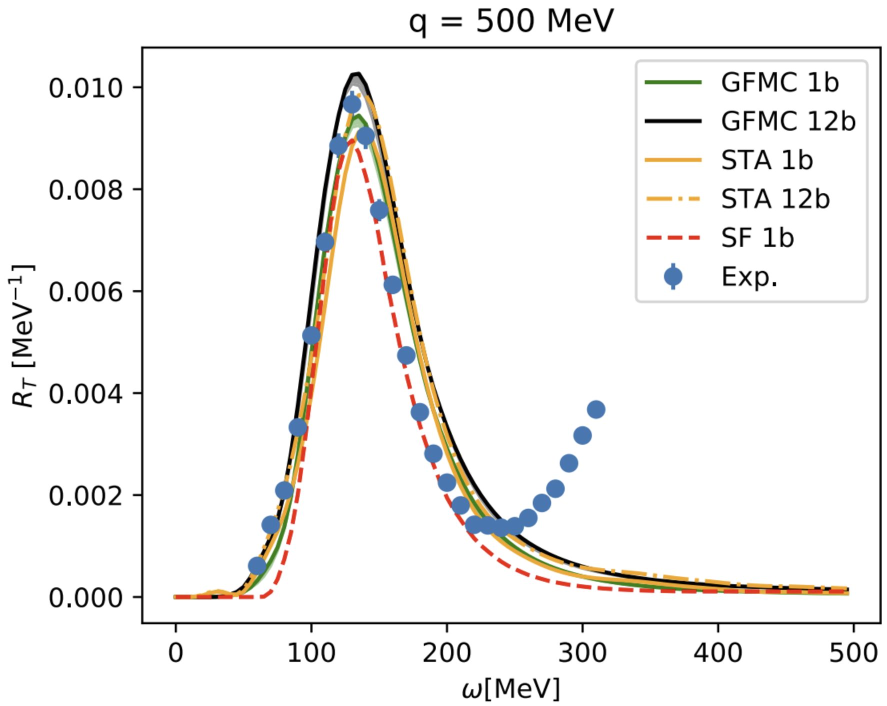
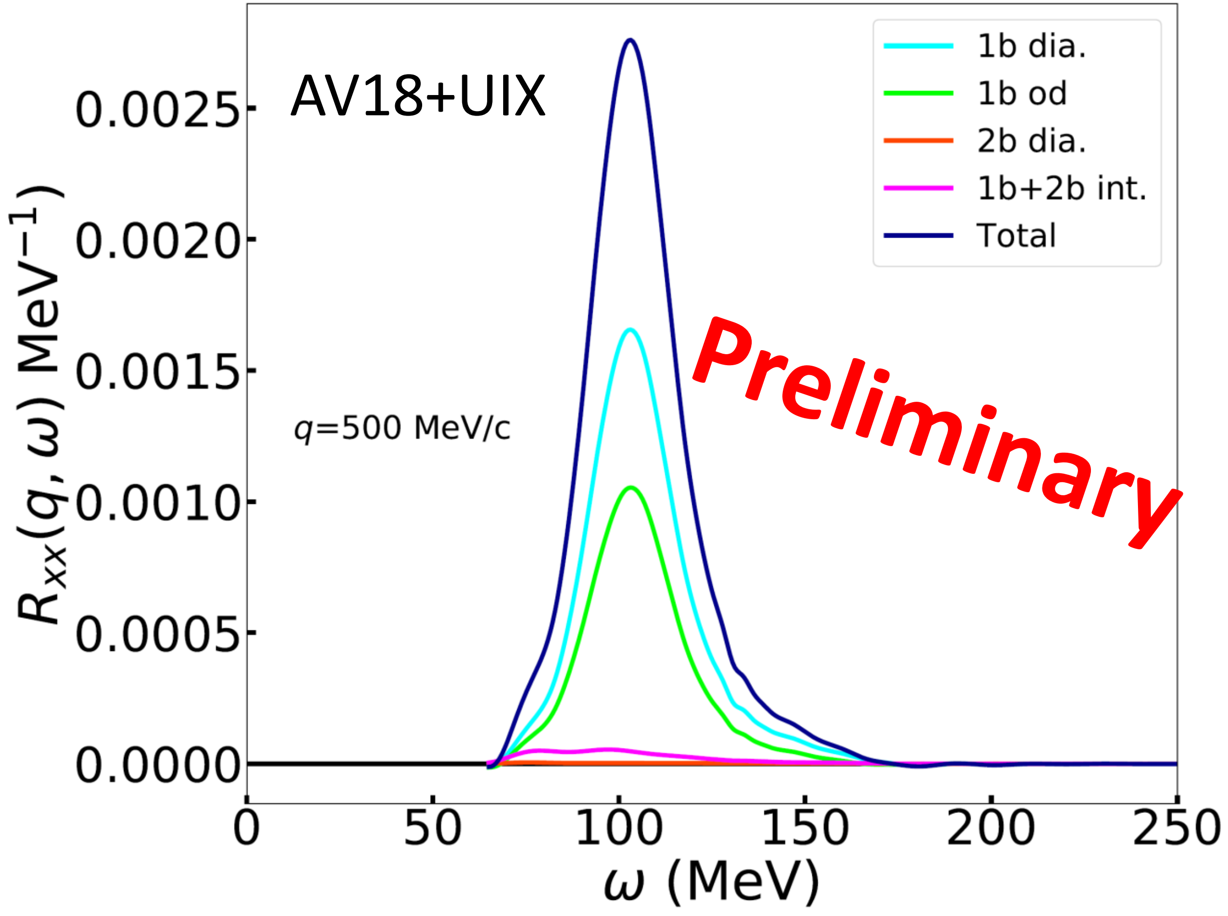


Andreoli et al. PRC 105, 014002 (2022)

Electromagnetic responses validated for A ≤ 4 nuclei with AV18+UIX wave functions

Weak neutral current response of ^{2}H



First step toward cross section benchmark with hyperspherical harmonics calculation of the cross section in **Shen et al. PRC 86, 035503 (2012)**

Development underway to study A=4 and A=12 neutral current responses

Exporting to other many-body methods will make A=16 and A=40 accessible to the STA

Outlook: charge-changing weak currents, relativistic kinematics, and radiative corrections to beta-decay

Overview and Outlook

STA is a factorization scheme that preserves sum rules, the physics of the PWIA, and two-particle correlations at short-times/high-energies

Good for high energy responses, but does not have information about low-lying excitations or collective behavior

Applied to the electromagnetic and NC response of the deuteron

Outlook: Computing ^{2}H NC cross sections, benchmarking NC response for other light nuclei, radiative corrections in beta decay

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