Multi-Aspect Young-ORiented Advanced Neutrino Academy (MAYORANA) - International School&Workshop

Nuclear Reactions for Weak Interactions

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Agenda

Lecture 1: Basic Concepts of Quantum-Mechanical Scattering Theory
Lecture 2: Nuclear Reaction Theory in a Nutshell
Lecture 3: Theory of Nuclear Direct Reactions
Lec ture 4: Optical Potentials and Elastic Scattering
Lecture 5: Perturbative Approach to Non-Elastic Reactions
Lecture 6: Single Charge Exchange (SCE) Reactions
Lecture 7: Light Ion SCE Reactions and beta-Decay
Lecture 8: The Gamow-Teller Quenching Mystery
Lecture 9: Nuclear Matrix Elements for Astrophysics
Lecture 10: Heavy Ion SCE Reactions
Lecture 11: Heavy Ion SCE Reactions at Relativistic Energies
Lecture 12: Double Charge Exchange (DCE) Reactions

Readings

- **Textbooks:**
  - M. L. Goldberger and K. M. Watson, ***Collision Theory*** (Wiley, New York, 1964); Dover reprint (2004).
  - N. Austern, ***Direct Nuclear Reaction Theories*** (Wiley, New York, 1970).
  - G. R. Satchler, ***Direct Nuclear Reactions*** (Oxford University Press, Oxford, 1983).
  - H. Feshbach, ***Theoretical Nuclear Physics: Nuclear Reactions*** (Wiley, New York, 1992).

- **Research Articles:**
  - H. Lenske, ***Theory and applications of nuclear direct reactions*,** Int.Jour.Mod.Phys. E30 (2021) 2130010
  - F. Cappuzzello, H. Lenske et al., **Shedding light on nuclear aspects of neutrinoless double beta decay by heavy-ion double charge exchange reactions**, *Prog.Part.Nucl.Phys.* 128 (2023) 103999
  - H. Lenske, F. Cappuzzello et al., **Heavy ion charge exchange reactions as probes for nuclear β-decay,** *Prog.Part.Nucl.Phys.* 109 (2019) 103716
  - H. Lenske, M. Dhar et al., ***Baryons and Baryon Resonances in Nuclear Matter***, *Prog.Part.Nucl.Phys.* 98 (2018) 119;

Lecture 1:
Basic Concepts and Characteristics of Nuclear Reaction

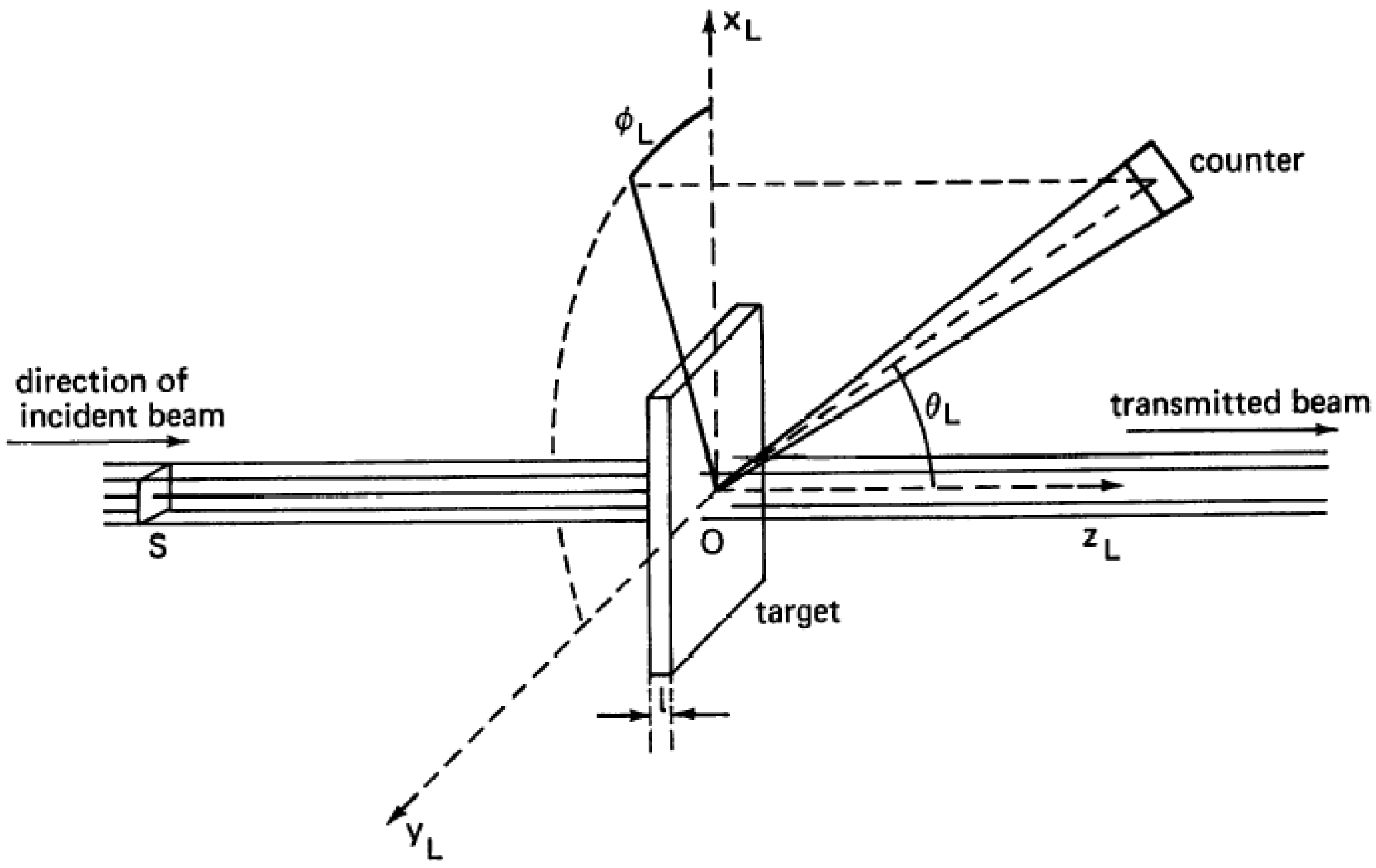


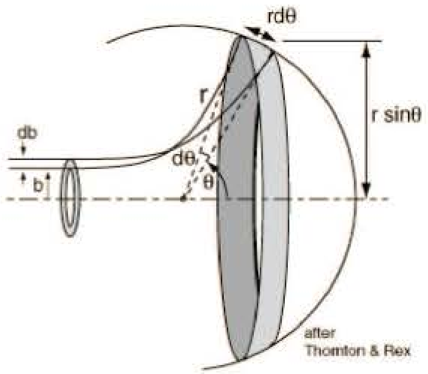
Fig. 1.2. Illustration of various quantities used in the definition of cross sections.

Recap of Classical Scattering Theory:

- In classical mechanics, for a central potential, $V(r)$, the angle of scattering is determined by **impact parameter** $b(\theta)$.
- The number of particles scattered per unit time between θ and $\theta + d\theta$ is equal to the number incident particles per unit time between b and $b + db$.
- Therefore, for incident flux j_I, the number of particles scattered into the solid angle $d\Omega = 2\pi \sin\theta \, d\theta$ per unit time is given by

$$N \, d\Omega = 2\pi \sin\theta \, d\theta \, N = 2\pi b \, db \, j_I$$

$$\text{i.e.} \quad \frac{d\sigma(\theta)}{d\Omega} \equiv \frac{N}{j_I} = \frac{b}{\sin\theta} \left| \frac{db}{d\theta} \right|$$



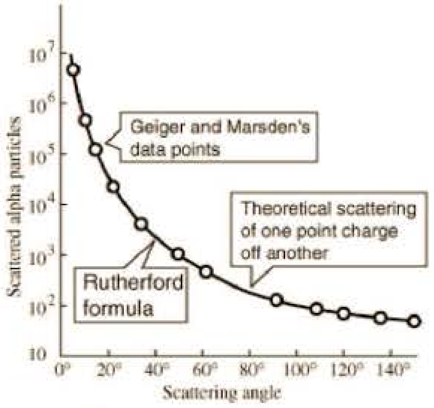
- For **classical Coulomb scattering**,

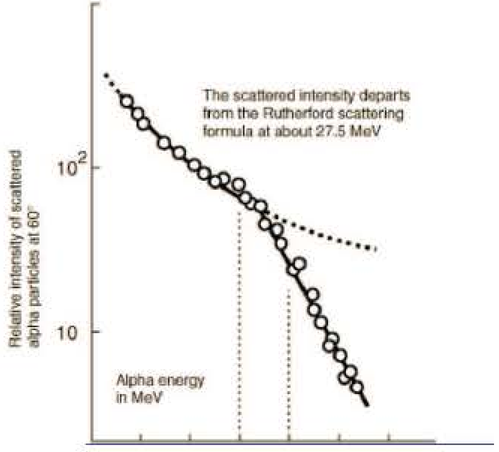
$$V(r) = \frac{\kappa}{r}$$

particle follows hyperbolic trajectory.

- In this case, a straightforward calculation obtains the **Rutherford formula:**

$$\frac{d\sigma}{d\Omega} = \frac{b}{\sin\theta} \left| \frac{db}{d\theta} \right| = \frac{\kappa^2}{16E^2} \frac{1}{\sin^4 \theta/2}$$





Quantum Mechanical Scattering

Scattering is a time-dependent process as expressed by the Schrödinger equation

$$i\hbar\frac{\partial}{\partial t}\Psi(\mathbf{r},t) = H(\mathbf{r},\mathbf{p})\Psi(\mathbf{r},t)$$

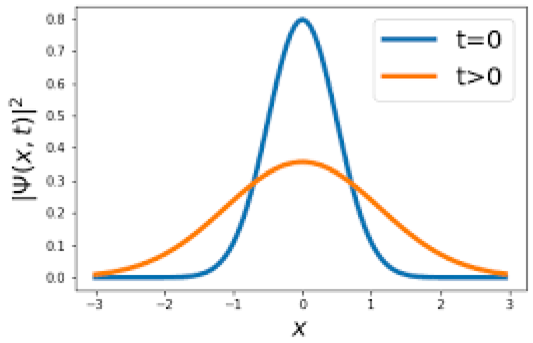
The formal solution is

$$\Psi(\mathbf{r},t) = e^{-iH(\mathbf{r},\mathbf{p})(t-t_0)}\Psi(\mathbf{r},t_0)$$

…propagating the initial state $\Psi(\mathbf{r},t_0)$ from $t_0 \rightarrow t$.

Free motion of a point particle ($t_0=0$):

$$\Psi_0(\vec{\mathbf{r}}) = \mathbf{A}(\gamma)\int\frac{\mathbf{d}^3\mathbf{k}}{(2\pi)^{3/2}}\mathbf{e}^{i\vec{\mathbf{k}}\bullet\vec{\mathbf{r}}-\frac{1}{2}\gamma^2\mathbf{k}^2} \quad ; \quad \langle\Psi_0|\Psi_0\rangle = 1$$



Nucleus-Nucleus Scattering

$$H(\vec{r},\vec{p}) \to H_{cm}(\vec{q},\xi) + \frac{\vec{P}^2}{2M}$$

Relative motion in the center-of-mass system:

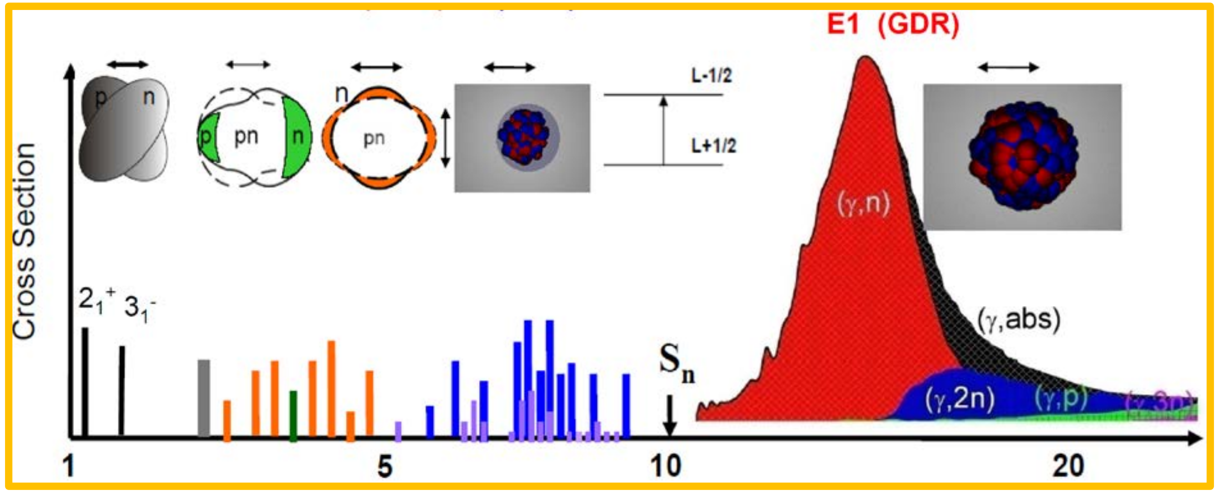
$$H_{cm}(\vec{q},\xi) = K_{PT}(\vec{q}) + V_{PT}(\vec{r},\xi_P,\xi_T) + H_P(\xi_P) + H_T(\xi_T)$$

$$Eigenstates: \Phi(t,\vec{x},\xi_P,\xi_T) = e^{-i\omega_c t}\psi_c(\vec{r},\vec{k})|c\rangle \quad ; \quad |c\rangle = [P_n \otimes T_m]$$

Nuclear Eigenstates

$$(H_A - E_{A_n})|A_n\rangle = 0 \quad ; \quad \text{A=P,T} \; ; \; n = 0\ldots\infty$$

In nuclear reactions two objects of a highly complex intrinsic structure are interacting!



A Real Physics Example:
Inelastic Scattering of a proton beam on a ^{54}Fe target at T_{lab}=61.7 MeV

Rule of Thumb:
Complexity of spectra increases with
ω, q
and for small impact parameters
$b \approx \ell/k$

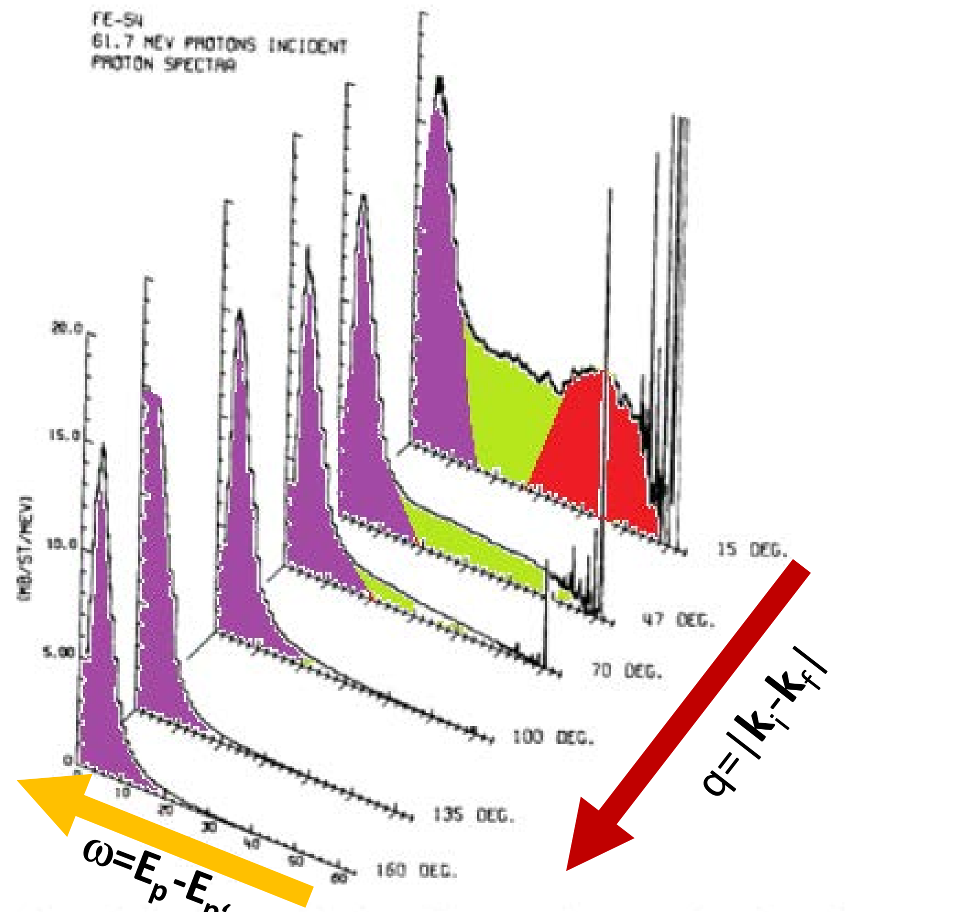


Fig. 2. Spectral angular distributions of inelastically scattered protons observed in the reaction ^{54}Fe(p,p') at the incident energy $T_{\rm Lab}=61.7$ MeV. The data were taken by Bertrand and Peelle.[4] The forward peaked direct reaction spectra are clearly distinguished from the compound emission spectra with an almost isotropic angular distribution.

Nuclear Cross sections
are depending on
energy (ω) transfer
and
momentum (q) transfer
covering
Discret Spectra
Giant resonances
Pre-Equilibirum States
Thermalized Compound States

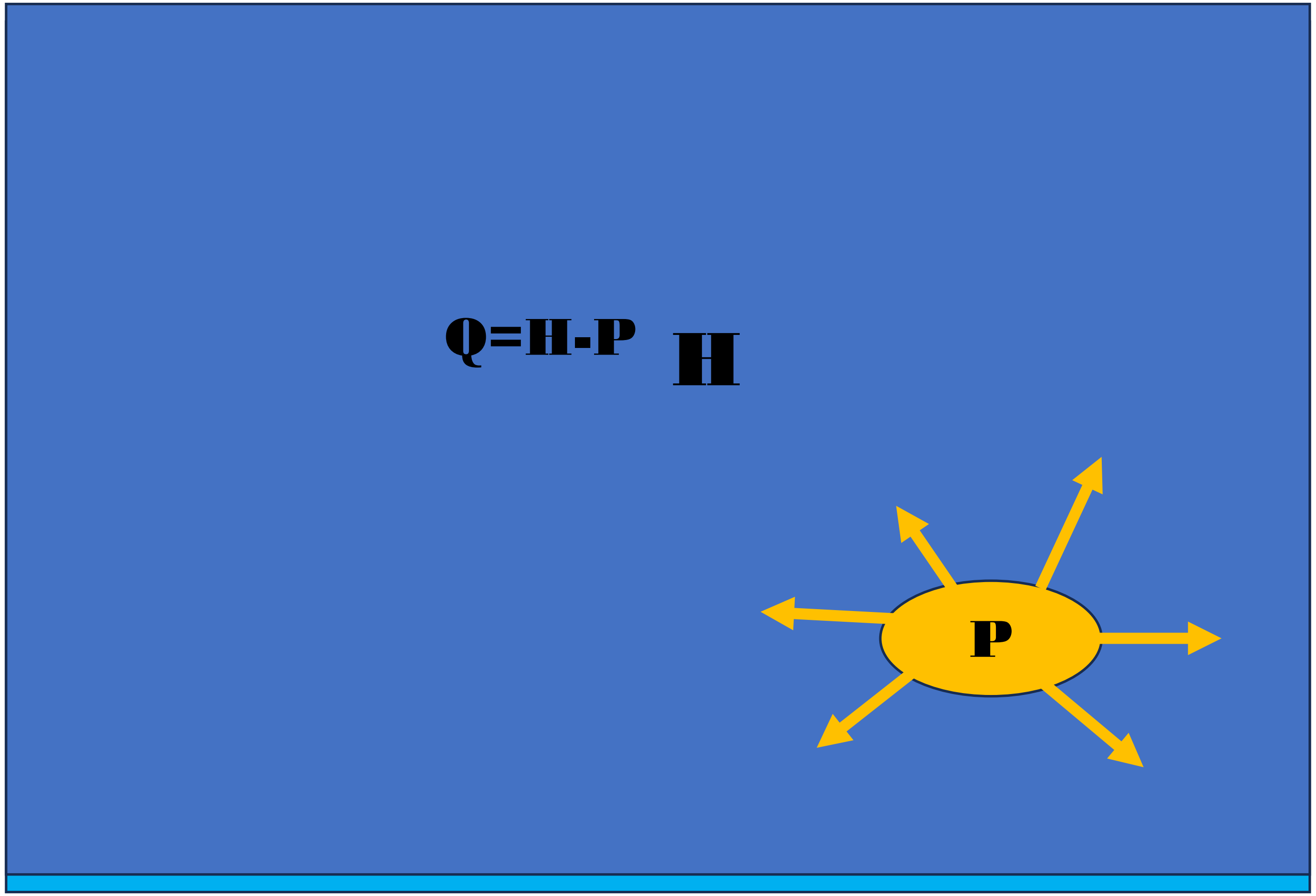
F. E. Bertrand and R. W. Peelle, *Phys. Rev. C* **8** (1973) 1045.

Lecture 2:
Nuclear Reaction Theory in a Nutshell

…for Stationary Scattering States:

$$\Phi(\vec{r},t,\xi) = e^{-iEt}\Psi(\vec{r},\xi):$$
$$\left(H(\vec{r},\vec{q},\xi) - E\right)\Psi(\vec{r},\xi) = 0$$

Division of the Hilbert-Space $\mathbb{H}=\mathbb{P}+\mathbb{Q}$ into a Model Space $\mathbb{P}$ and the Complementary Space $\mathbb{Q}$

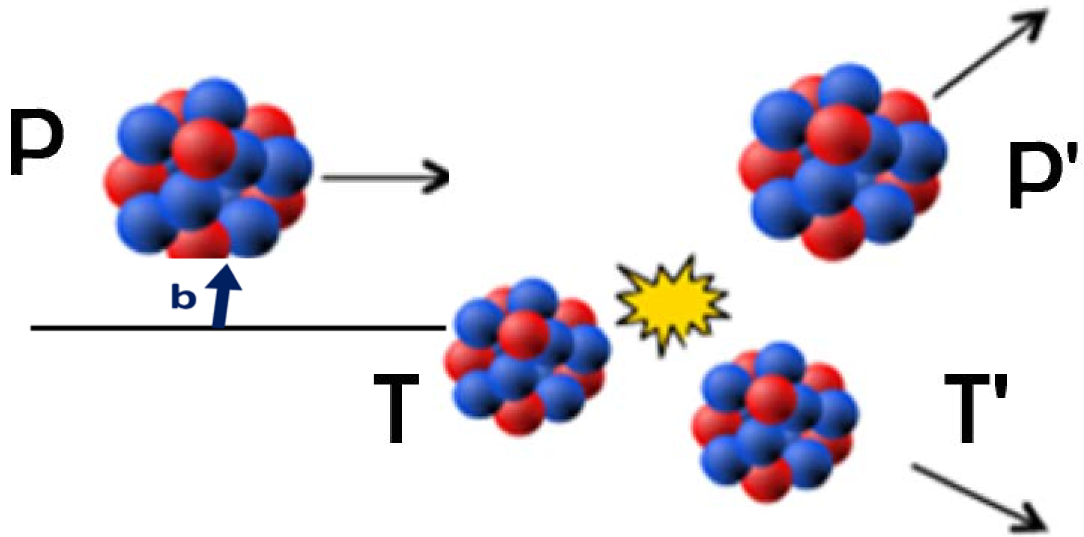


$\mathbb{P}$ and $\mathbb{Q}$ are coupled → exchange of probability flux $\mathbb{P} \leftrightarrow \mathbb{Q}$

Nuclear Reactions as Spectroscopic Tools:

Peripheral Reactions ↔ Direct Reactions

Elastic & Inelastic scattering, transfer reactions, charge exchange reactions…



Our P-Space Choice

Direct Reactions ↔ Grazing Reactions with Impact Parameters

$$b \sim R_T \sim r_0 A^{1/3}$$

$(r_0 \approx 1.12\ \mathrm{fm})$

Feshbach Projection

$$\boxed{(H-E)\Psi=0}$$

- Orthogonal Projectors P and Q with P+Q=1 and PQ=QP=0
- Hamitonian H = (P+Q)H(P+Q)=H_{PP}+H_{QQ}+H_{QP} +H_{PQ}
- Wavefunction Ψ=(P+Q)Ψ=Ψ_P+Ψ_Q

$$\boxed{\begin{aligned}&(H_{PP}-E)\Psi_P+H_{PQ}\Psi_Q=0\\&(H_{QQ}-E)\Psi_Q+H_{QP}\Psi_P=0\;\rightarrow\;\Psi_Q=\frac{1}{E-H_{QQ}+i\eta}H_{QP}\Psi_P\end{aligned}}$$

- Effective Wave Equation in P-space (H=K+V):

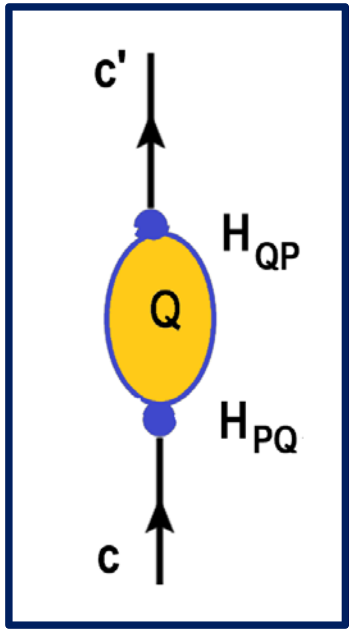
$$\boxed{\left(K_{PP}+V_{PP}+\Sigma_{PP}(E)-E\right)\Psi_P=0}$$

- Self-energy in P-space → non-Hermitian, energy dependent effective interaction:

$$\boxed{\Sigma_{PP}(E)=H_{PQ}\frac{1}{E-H_{QQ}+i\varepsilon}H_{QP}}$$

Structure of the Self-Energy Operator

$$\Sigma_{PP}(E) = \underbrace{H_{PQ}\frac{P}{E-H_{QQ}}H_{QP}}_{\text{„Off-shell“}} \underbrace{-i\pi H_{PQ}\delta\left(E-H_{QQ}\right)H_{QP}}_{\text{„On-shell“}}$$



$$\boxed{c, c' \in P}$$

Lecture 3:
Nuclear Direct Reaction Theory

Channel States, Channel Configurations, and Relatiive Motion

Expand the channel states Ψ_c into nuclear configurations and wave functions of relative motion:

$$\boxed{\Psi_c(\vec{r},\vec{k}_c)=\psi_c(\vec{r},\vec{k}_c)\left|c\right\rangle \;\; ; \;\; \left|c\right\rangle=\left[\Phi_P^{(c)}\otimes\Phi_T^{(c)}\right]_{L_cS_cJ_c\ldots} \;\; ; \;\; \left\langle c\middle|c'\right\rangle=\delta_{cc'}}$$

- **Projection onto the nuclear states c**
- **Integration over the intrinsic nuclear coordinates ξ={r,σ,τ…}**

Non-hermitian, non-local Self-Energy connecting P-space channel states c and c‘:

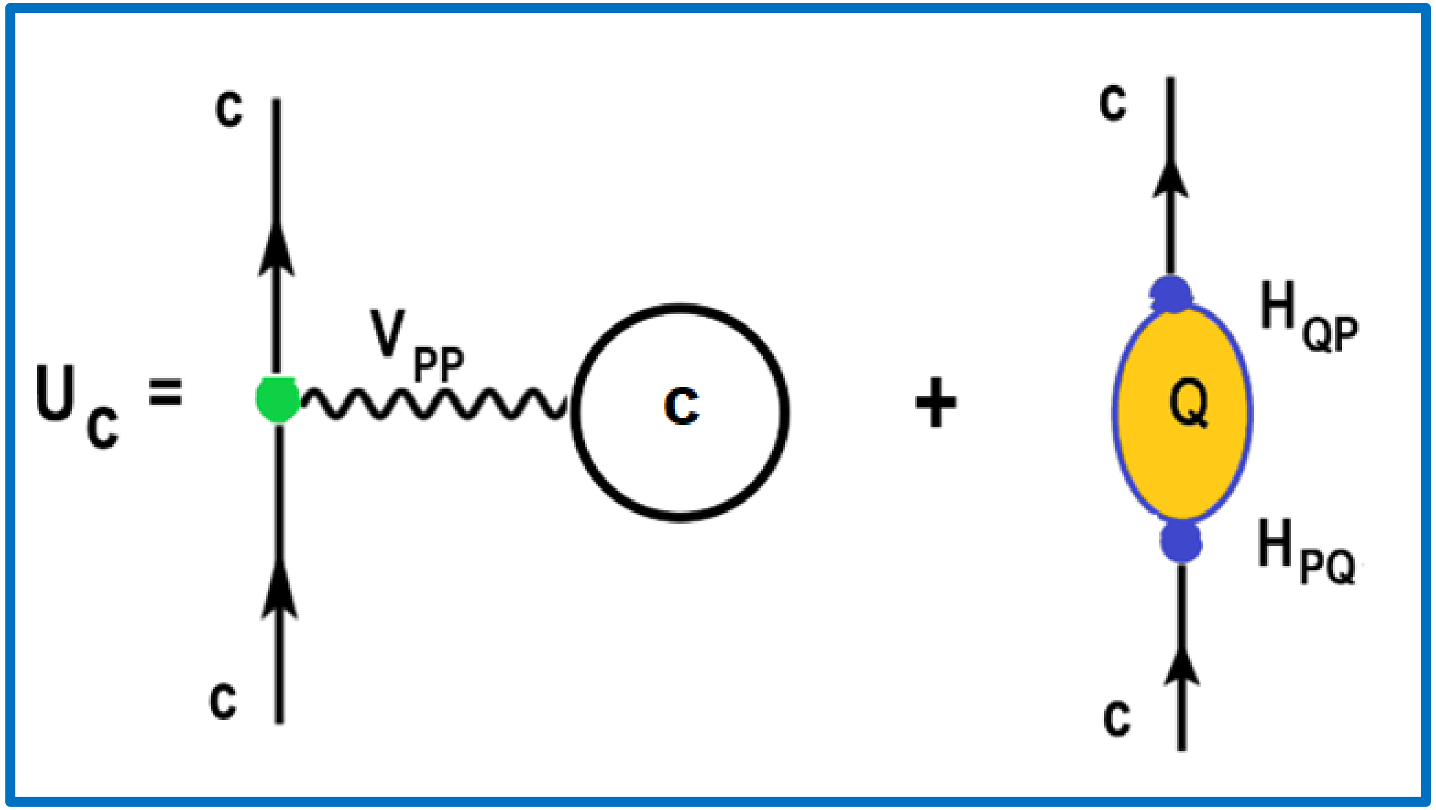
$$\boxed{\Sigma_{cc'}(\vec{r},\vec{r}\,'|E)=\sum_{n\in Q}\int\frac{d^3k_n}{(2\pi)^3}\left\langle c\right|H_{PQ}\left|\Psi_n\right\rangle(\vec{r})\left[\frac{P}{E-\omega_n(k_n)}-i\pi\delta\left(E-\omega_n(k_n)\right)\right]\left\langle\Psi_n\right|H_{QP}\left|c'\right\rangle(\vec{r}\,')}$$

Effective Channel Potentials in P-space for c=c‘

$$\boxed{U_c(\vec{r}\,|E)\approx\left\langle c\right|V_{PP}\left|c\right\rangle(\vec{r})+\Sigma_{cc}(\vec{r}\,|E)}$$

The Channel Potential

$$\boxed{U_c(\vec{r}\,|\,E) \approx \langle c|V_{PP}|c\rangle(\vec{r}) + \Sigma_{cc}(\vec{r}\,|\,E)}$$



Non-local (momentum dependent) dispersive self-energy

The Reaction Network in P-Space

Coupled Channels (CC) Problem in P-space:

$$\left(K_c + U_c(\vec{r}\,|\,E) - \varepsilon_c\right)\psi_c(\vec{r},\vec{k}_c) + \sum_{c'\neq c} F_{cc'}(\vec{r}\,|\,E)\psi_{c'}(\vec{r},\vec{k}_{c'}) = 0$$

$$F_{cc'}(\vec{r}\,|\,E) = \left\langle c\middle|V_{PP}\middle|c'\right\rangle(\vec{r}) + \Sigma_{cc'}(\vec{r}\,|\,E)$$

…to be solved under the asymptotic boundary condition:

$$\Psi_c \to e^{i\vec{k}_i \bullet \vec{r}_i}\left|a_i A_i\right\rangle \delta_{ic} + f_{ic}(\Omega_c)\frac{e^{ik_c r_c}}{r_c}\left|a_c A_c\right\rangle$$

…by expansion into (several hundred) partial waves and solved for a given incident channel c=i with plane and outgoing waves while for c≠i only outgoing spherical waves occur

Example: Elastic Scattering as 1-Channel Problem

Partial Wave expansion and wave equation

$$\boxed{\psi_c^{(+)}(\vec{r},\vec{k}_c)=\frac{4\pi}{k_c r}\sum_{\ell m}Y^*_{\ell m}(\hat{k})Y_{\ell m}(\hat{r})u_\ell(r,k_c)}\quad\longrightarrow\quad\boxed{\left(\frac{d^2}{dr^2}-\frac{\ell(\ell+1)}{r^2}-\frac{2m_c}{\hbar^2}U_c(r\,|\,E)+k_c^2(E)\right)u_\ell(r,k_c)=0}$$

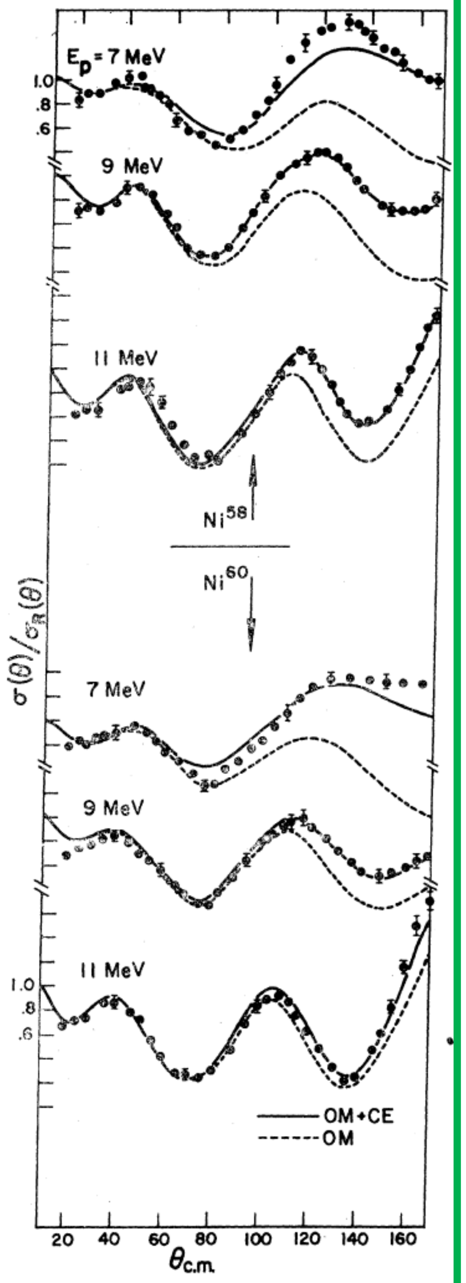
...matching at r >> R(U_c):

$$\boxed{A_\ell u_\ell(r,k_c)\to F_\ell(r,k_c)+C_\ell(k_c\,|\,U_c)H_\ell^{(+)}(r,k_c)}$$

Partial wave and total scattering amplitudes

$$\boxed{C_\ell\left(k_c\,|\,U_c\right)=\frac{1}{2i}\left(\eta_\ell e^{2i\delta_\ell}-1\right)\quad;\quad 0<\eta_\ell\le 1}$$

$$\boxed{f_c^{(elas)}(\vartheta)=\frac{1}{k_c}\sum_\ell(2\ell+1)\;P_\ell(\cos\vartheta)\;C_\ell(k_c\,|\,U_c)}\boxed{d\sigma_c^{(elas)}\sim\left|f_c^{(elas)}\left(\vartheta\right)\right|^2 d\Omega}$$



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Nucleon-Nucleus Optical-Model Parameters, $A > 40$, $E < 50$ MeV*

F. D. Becchetti, Jr., and G. W. Greenlees

- **Pioneering work of Becchetti and Greenless in 1969**
- **Systematic Study on Low-Energy proton-nucleus scattering**
- **U_c parametrized in terms of optical potentials of simple functioal form**
- **Wood-Saxon form factors**

Lecture 4
Optical Model Potentials and Elastic Scattering

Channel Potential and Optical Model Potential

$$\boxed{U_c\left(\vec{r}\,|\,E\right) \approx \langle c|V_{PP}|c\rangle + \Sigma_{cc}(\vec{r}\,|\,E)}$$

Introduce OMP as an AUXILLARY Potential

$$\mathrm{U}_c^{opt}(\vec{r}\,|\,E) \equiv V_c(\vec{r}\,|\,E) - iW_c(\vec{r}\,|\,E)$$

$$V_C(\vec{r}\,|\,E) \leftrightarrow \mathrm{Re}\left(\langle c|V_{PP}|c\rangle + \Sigma_{cc}(\vec{r}\,|\,E)\right) \;;\; W_c(\vec{r}\,|\,E) \leftrightarrow -\mathrm{Im}\left(\langle c|V_{PP}|c\rangle + \Sigma_c(\vec{r}\,|\,E)\right)$$

...imposed to reproduce as good as possible the scattering amplitude:

$$\boxed{f_c(U_c,\vartheta) = \langle \vec{k}_c|U_c|\psi_c^{(+)}\rangle = \langle \vec{k}_c|U_c \pm U_c^{opt}|\psi_c^{(+)}\rangle \approx f_c^{opt}(U_c^{opt},k_c) + \Delta(U_c,U_c^{opt})} \qquad \left(|\vec{k}_c\rangle \mapsto e^{i\vec{k}_c \bullet \vec{r}}\right)$$

$$\boxed{f_c^{opt}(U_c^{opt},k_c) = \langle \vec{k}_c|U_c^{opt}|\chi_c^{(+)}\rangle \;;\; \Delta(U_c,U_c^{opt}) = \langle \vec{k}_c|U_c - U_c^{opt}|\chi_c^{(+)}\rangle}$$

U^{opt} and the related distorted waves $\chi^{(+)}$ describe $\psi^{(+)}$ strictly spoken only in the asymptotic region but accounts globally for strong absorption at short distances.

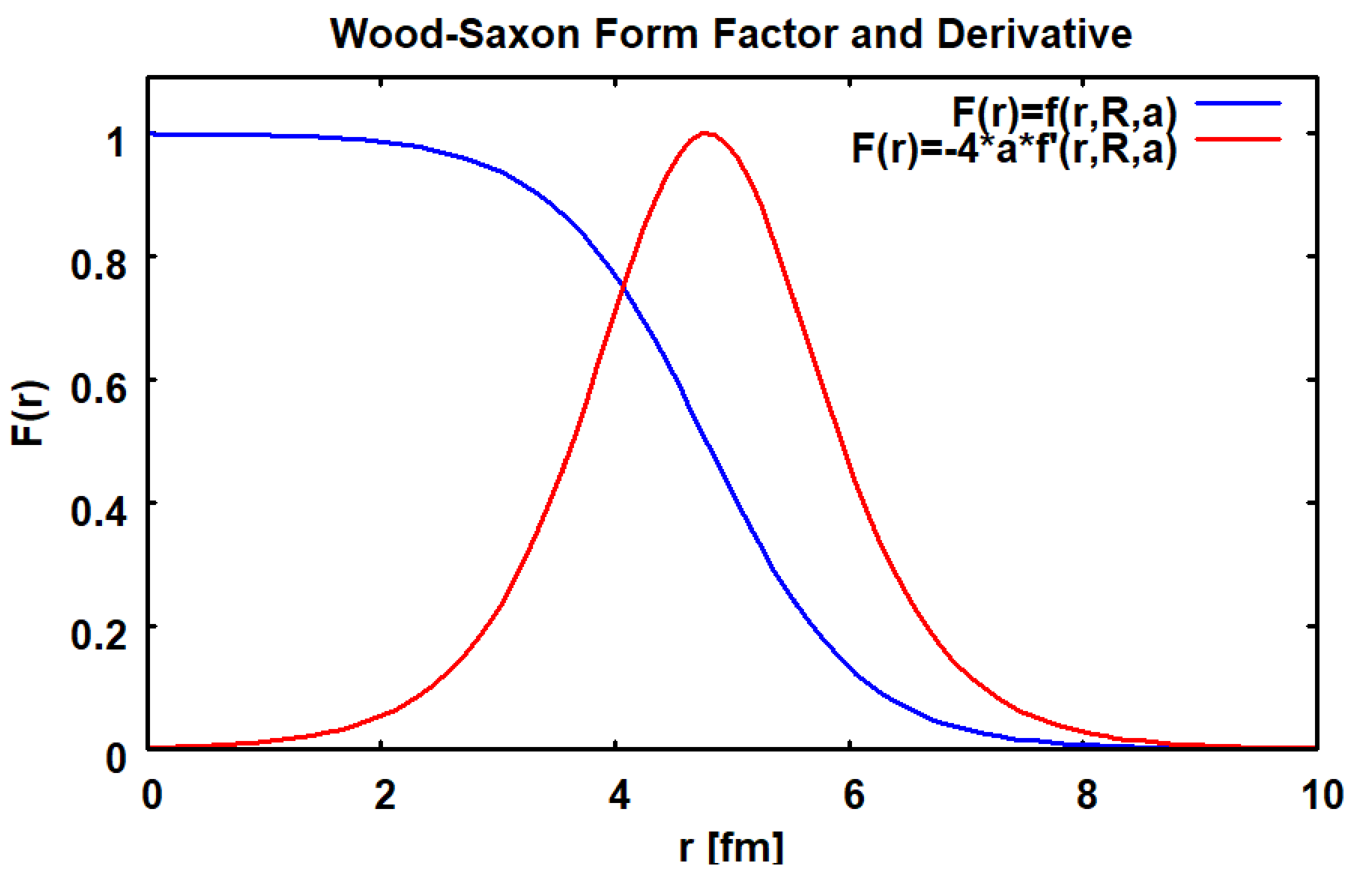
Phenomenological Optical Potentials

The general form for light (and heavy) ions is:

$$U(r) = V_c(r) - Vf_v(r, R_v, a_v) - i\left[W_{\mathrm{vol}} f_w(r, R_w, a_w) - 4a_w W_{\mathrm{surf}} \frac{d}{dr} f_w(r, R_w, a_w)\right]$$
$$+ V_{LS}\left[\frac{\hbar}{m_\pi c}\right]^2 (\boldsymbol{l} \cdot \mathbf{s}) \frac{1}{r} \frac{d}{dr} f_{LS}(r, R_{LS}, a_{LS})$$

Form factors of Wood-Saxon (or Fermi) shape:

$$f(r, R, a) = \frac{1}{1 + e^{\frac{r-R}{a}}} \quad ; \quad \frac{d}{dr} f(r, R, a) = -\frac{1}{a} e^{\frac{r-R}{a}} f^2(r, R, a)$$



A=56

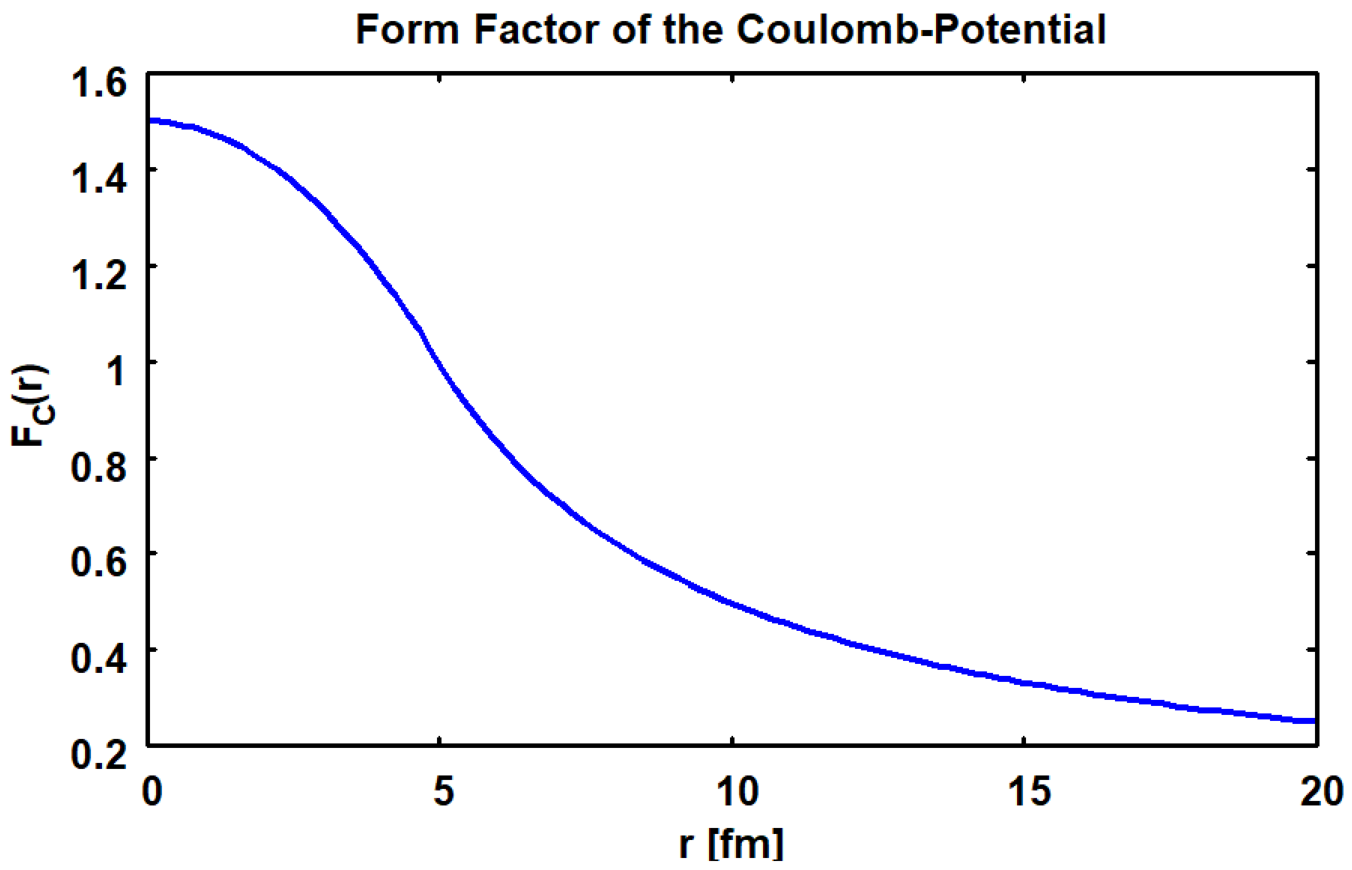
r_0=1.25 fm ; a=0.65 fm

R(A)=4.78 fm

Light Ions : $R(A)=r_0A^{1/3}$; $r_0 \sim 1.25$ fm ; a~0.6…0.7 fm

Heavy Ions A>10: $R(A_P,A_T)= r_0(A_P^{1/3}+A_T^{1/3})$; $r_0 \sim 1$ fm

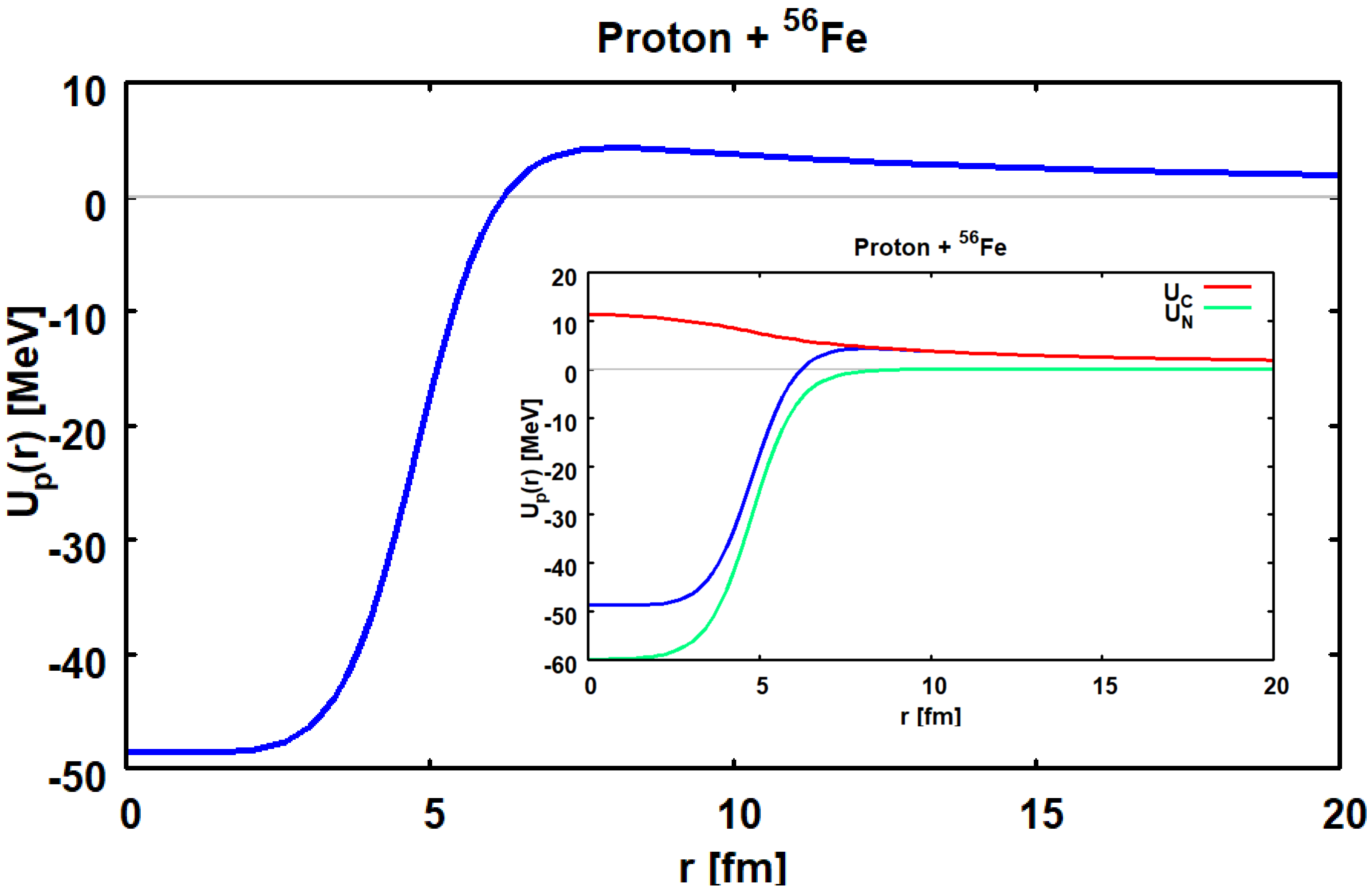
…plus Coulomb-Potential $U_C(r,R_C)$!



A=56, Z=26 (^{56}Fe)
r_{0C}=1.3 fm
$R_C(A)$=4.97 fm

$$U_C(r,R_C,Z)=\frac{Z}{R_C}\alpha_f\hbar c F_C(r,R_C) \quad (\alpha_f\hbar\approx 1.44 MeVfm)$$

$$F_C(r,R_C)=\frac{1}{2}\left(3-\left(\frac{r}{R_C}\right)^2\right)\Theta(R_C-r)+\frac{R_C}{r}\Theta(r-R_C) \quad \left(R_C(A)=r_{0C}A^{\frac{1}{3}}\right)$$



Proton + 56Fe
10
0
-10
-20
-30
-40
-50
Up(r) [MeV]
0
5
10
15
20
r [fm]
Proton + 56Fe
20
10
0
-10
-20
-30
-40
-50
-60
Up(r) [MeV]
UC
UN
0
5
10
15
20
r [fm]

OMP, Distorted Waves, and Elastic Scattering

$$\left(K_c + U_c^{opt}(r_c \mid E) - E\right)\chi_c^{(\pm)}(\vec{r}_c, \vec{k}_c) = 0$$

$$\chi_c^{(+)}(\vec{r}_c, \vec{k}_c) \to e^{i\vec{k}_c \bullet \vec{r}_c} + f_c(U_c^{opt}, \vartheta)\frac{e^{ik_c r_c}}{r} \quad ; \quad \chi_c^{(-)\dagger}(\vec{r}_c, \vec{k}_c) = \chi_c^{(+)}(\vec{r}_c, -\vec{k}_c)$$

Scattering amplitude and elastic angular distribution

$$f_c(U_c^{opt}, \vartheta) = \left\langle \vec{k}_c \middle| U_c^{opt} \middle| \chi_c^{(+)} \right\rangle \quad ; \quad d\sigma_c^{elas}(U_c^{opt}, \vartheta) \simeq \left| f_c(U_c^{opt}, \vartheta) \right|^2 d\Omega$$

Empirical approach:

parametrize U by a convenient functional form and fix the model parameters by χ^2-fit to data

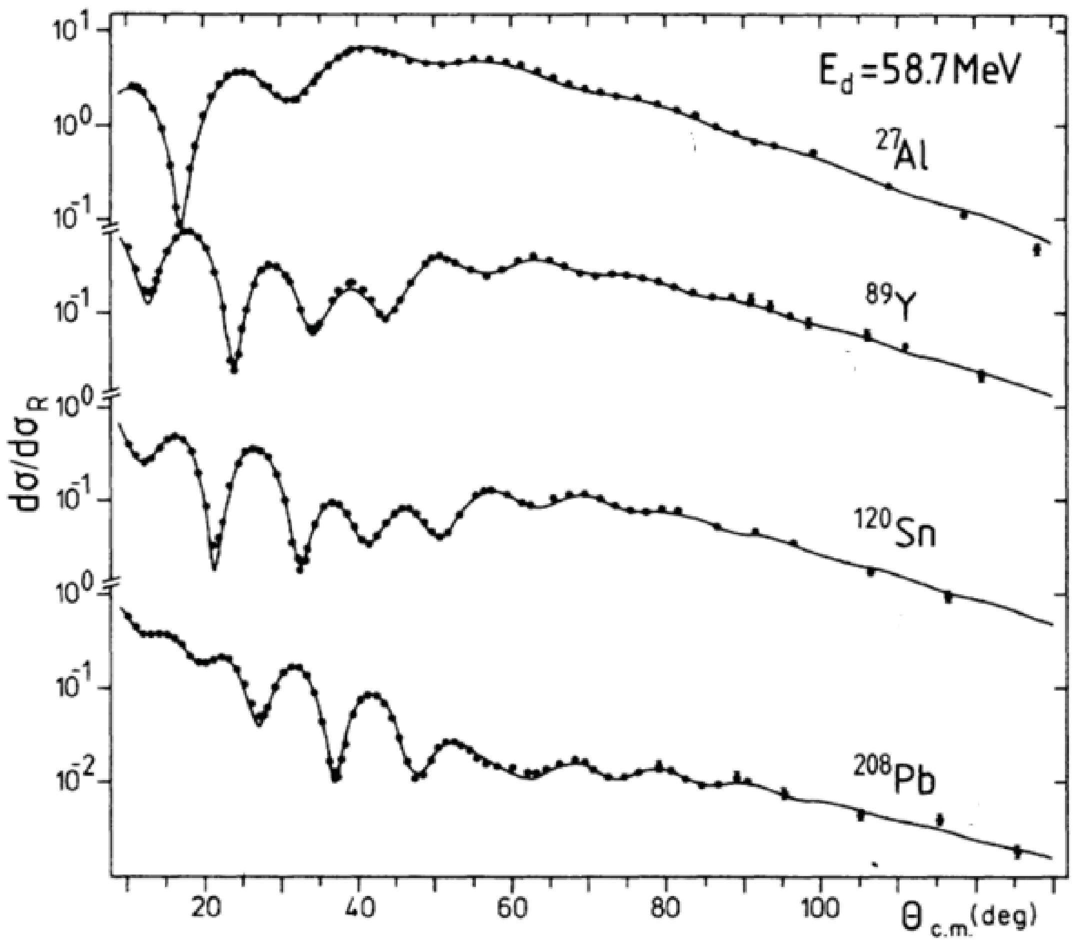
$$\chi^2(U_c) = \sum_n \frac{1}{w_n}\left(d\sigma_c^{elast}(U_c^{opt}, \vartheta_n) - d\sigma_c^{\exp}(\vartheta_n)\right)^2$$

$$d\sigma_c^{\exp}\left(\vartheta_n\right) \triangleq d\sigma_c^{\exp}\left(U_c, \vartheta_n\right)$$

High-Precision Phenomenological OMP-Fit to Elastic d+A Data

(Hinterberger et al., Phys. Rev. C38:1153 (1988))

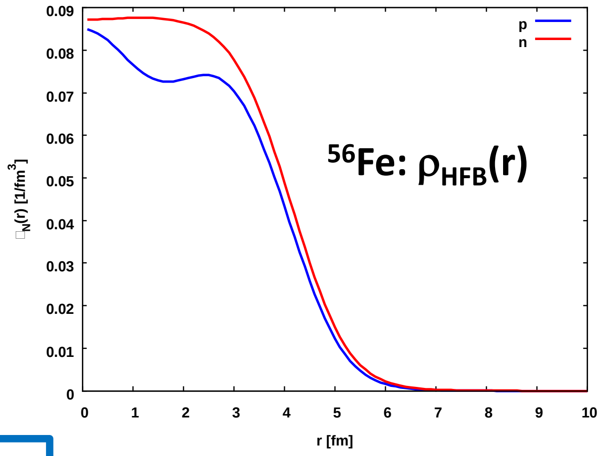
$$d\sigma_c^{(elas)} \sim \left| f_c^{(elas)}(\vartheta) \right|^2 d\Omega$$



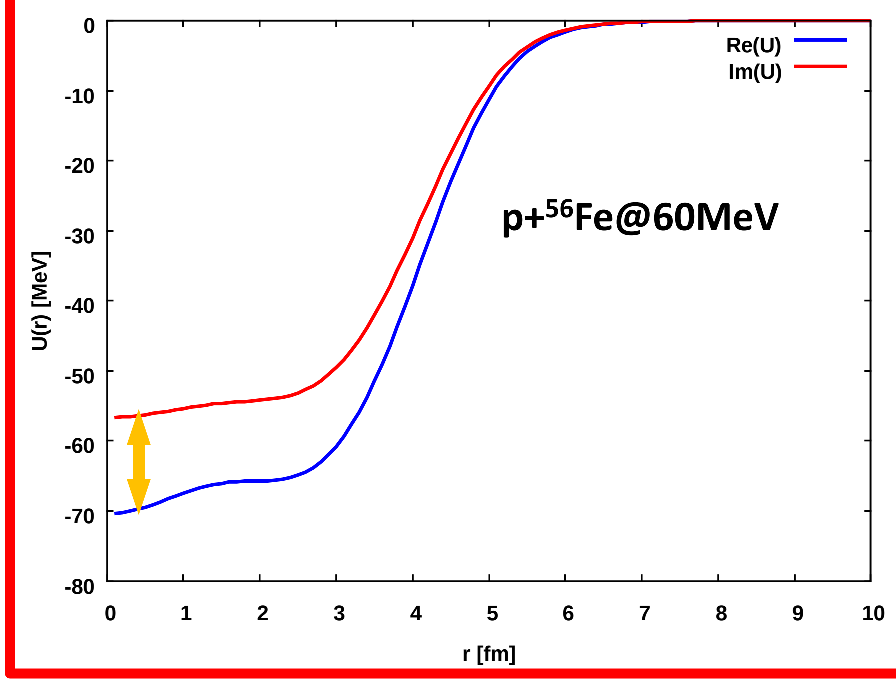
See also: H. Lenske, Int. Journ. Mod. Phys.E Vol. 30 (2021) 2130010

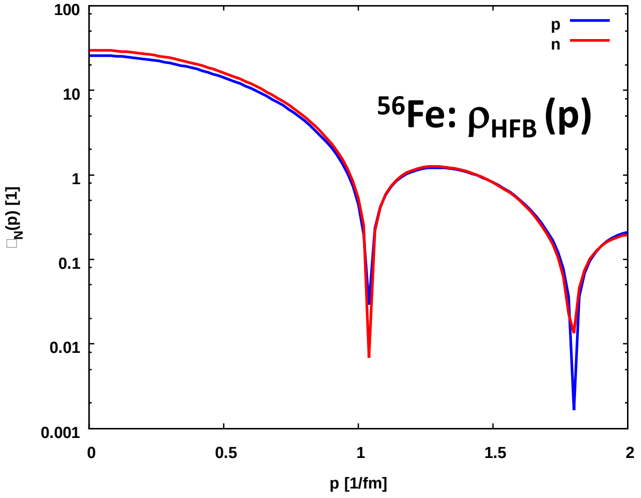
Microscopic Folding Optical Potentials

- **Proton and neutron ground state densities of target (and projectile) - HFB**
- **Projectile-target interaction → NN T-Matrix in momentum representation**
- **Folding of densities and interactions → Potential depending on r**

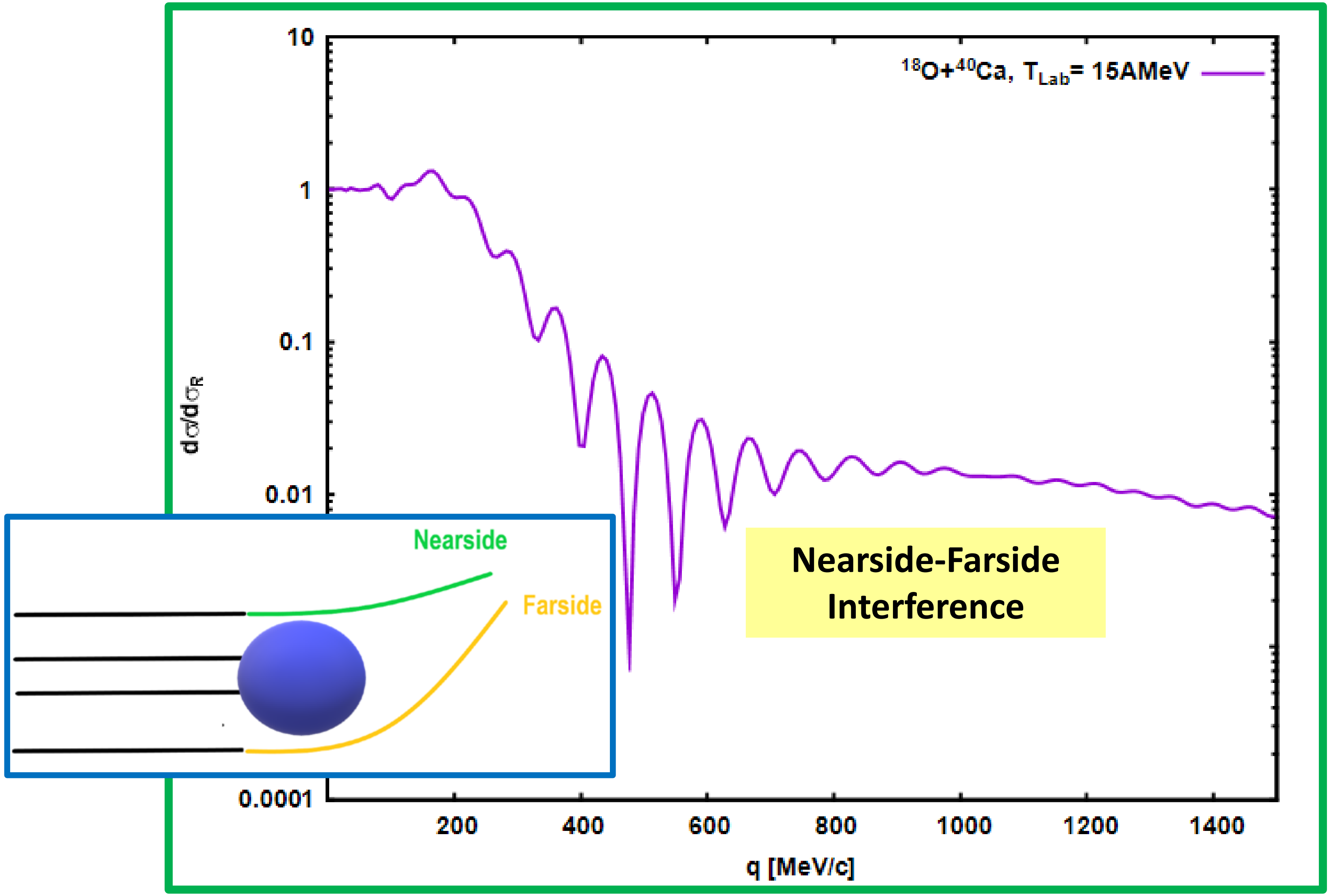


$$U_{fold}(r,E)=\int\frac{d^3p}{(2\pi)^3}e^{i\vec{p}\bullet\vec{r}}\left(T_{00}(p\,|\,E)\left(\rho_n(p)+\rho_p(p)\right)\pm T_{01}(p\,|\,E)\left(\rho_n(p)-\rho_p(p)\right)+V_C(p)\rho_C(p)\right)$$





$^{18}O + ^{40}Ca$ Elastic Scattering
Microscopic Double-Folding OMP



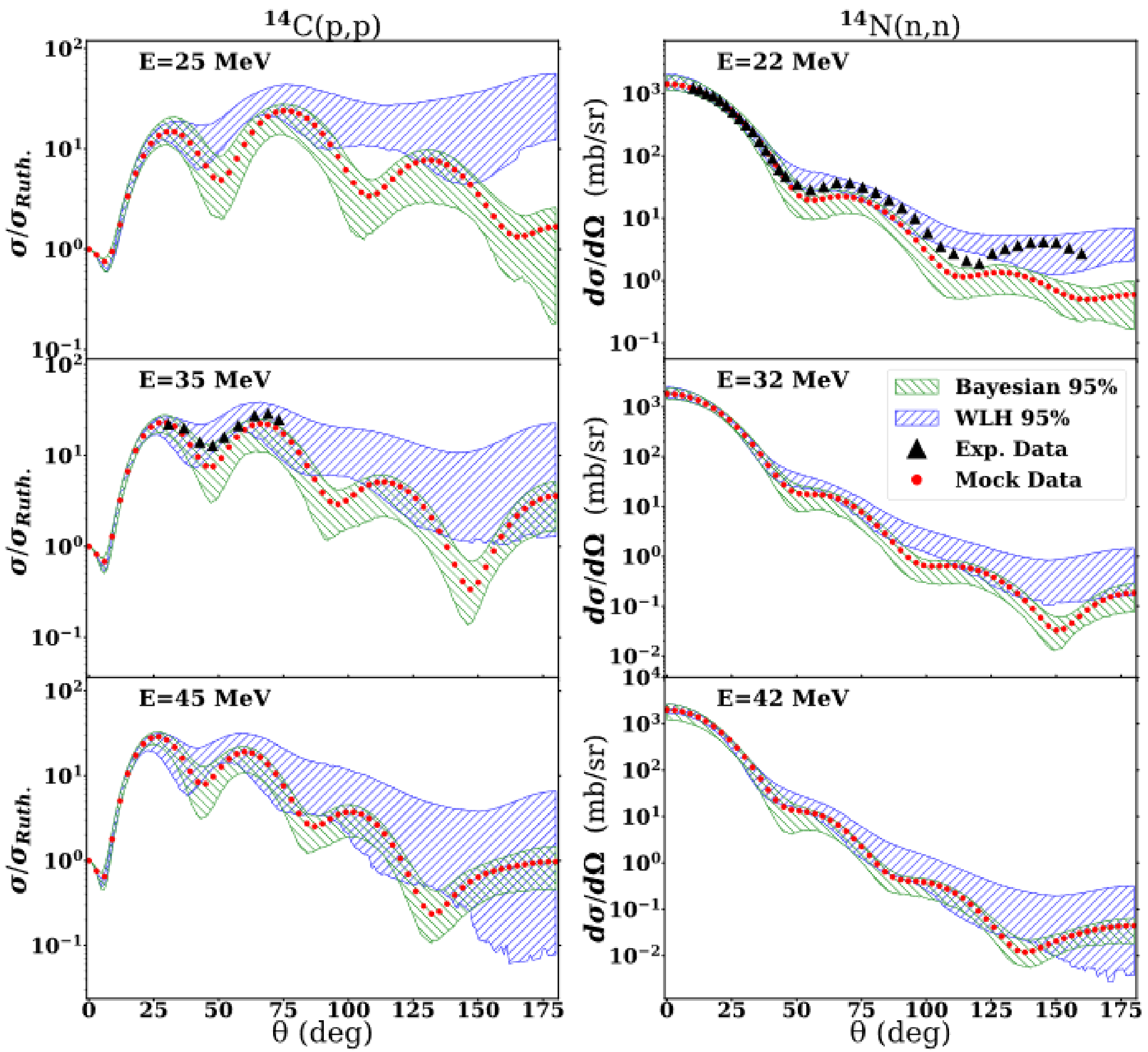
Optical Potentials from Effective Field Theory and Bayesian Optimization

The Whitehead-Lee-Holt (WLH) Method:

- **Use χEFT and many-body perturbation theory**
- **Derive $\Sigma_{cc}(\rho|E)$ in infinite matter**
- **Determine $U_{cA}(r|E)$ in LDA: $\Sigma_{cc}(r|E) \equiv \Sigma_{cc}(\rho_A(r)|E)$**
- **For numerical work, reparametrize $U_{cA}(r|E)$ in Wood-Saxon form:**

$$U(r,E) = -\mathcal{U}_V f(r; r_V, a_V) - i\mathcal{U}_W f(r; r_W, a_W) + i4a_S\mathcal{U}_S \frac{d}{dr} f(r; r_S, a_S) + \mathcal{U}_{SO}\frac{1}{m_\pi^2}\frac{1}{r}\frac{d}{dr} f(r; r_{SO}, a_{SO})\vec{\ell}\cdot\vec{\sigma},$$

- ***A posteriori* Bayesian optimization by fit to data**
- **Extend the fit to (p,n) charge exchange data and elastic scattering in the incident and exit channel**



T.R. Whitehead et al., PHYS. REV. C 105, 054611 (2022)

Multi-Aspect Young-ORiented Advanced Neutrino Academy (MAYORANA) - International School&Workshop

Nuclear Reactions for Weak Interactions
Part 2

Horst Lenske

Institut für Theoretische Physik, JLU Giessen

and

NUMEN Collaboration

JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN

Institut für Theoretische Physik

NUMEN

Agenda

Lecture 1: Basic Concepts of Quantum-Mechanical Scattering Theory
Lecture 2: Nuclear Reaction Theory in a Nutshell
Lecture 3: Theory of Nuclear Direct Reactions
Lecture 4: Optical Potentials and Elastic Scattering
We are here:
Lecture 5: Perturbative Approach to Non-Elastic Reactions
Lecture 6: Single Charge Exchange (SCE) Reactions
Lecture 7: Light Ion SCE Reactions and beta-Decay
Lecture 8: The Gamow-Teller Quenching Mystery
Lecture 9: Nuclear Matrix Elements for Astrophysics
Lecture 10: Heavy Ion SCE Reactions
Lecture 11: Heavy Ion SCE Reactions at Relativistic Energies
Lecture 12: Double Charge Exchange (DCE) Reactions

Lecture 5:
Perturbative Approach to Non-Elastic Reactions
The Distorted Wave Born Approxiamtion
(DWBA)

The Weak Coupling Limit:
Distorted Wave Born Approximation (DWBA)

$$\left(K_c + U_c^{opt}(\vec{r}\,|\,E) - \varepsilon_c\right)\chi_{ci}^{(+)}(\vec{r},\vec{k}_c) + F_{ci}(\vec{r}\,|\,E)\chi_i^{(+)}(\vec{r},\vec{k}_i) = 0$$

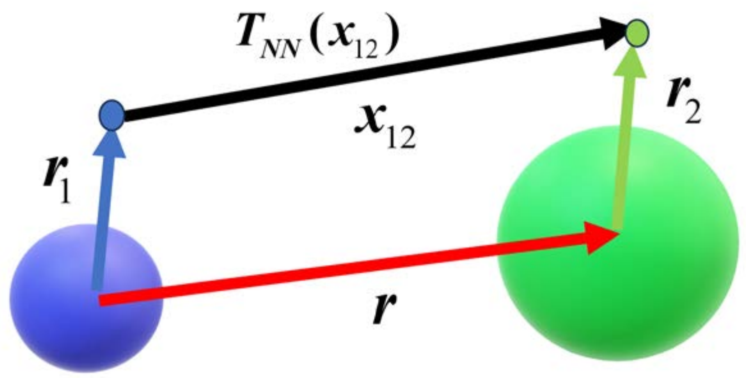
$$\chi_{ci}^{(+)}(\vec{r},\vec{k}_c) \approx \int d^3r' G_c^{(+)}(\vec{r},\vec{r}\,'\,|\,\varepsilon_c)F_{ci}(\vec{r}\,'\,|\,E)\chi_i^{(+)}(\vec{r}\,',\vec{k}_i)$$

$$\mathrm{f}_{c'i}(\vec{k}_c,\vec{k}_i) \approx \mathrm{f}_{c'i}^{(DWBA)}(\vec{k}_{c'},\vec{k}_i) = \left\langle \chi_c^{(-)} \middle| F_{ci} \middle| \chi_i^{(+)} \right\rangle$$

Decomposition of the reaction form factor $F_{cc'}$ into multipole components $F_{(LS)JM}$

$$\frac{d\sigma_{ci}}{d\Omega} \simeq \sum_{LS,JM} \left| \left\langle \chi_c^{(-)} \middle| F_{(LS)JM}(\vec{r}) \middle| \chi_i^{(+)} \right\rangle \right|^2 = \sum_{\ell=0,1,2...} A_\ell^{(cc')}(k_c,k_i) P_\ell(\cos\vartheta)$$

The Reaction Form Factor



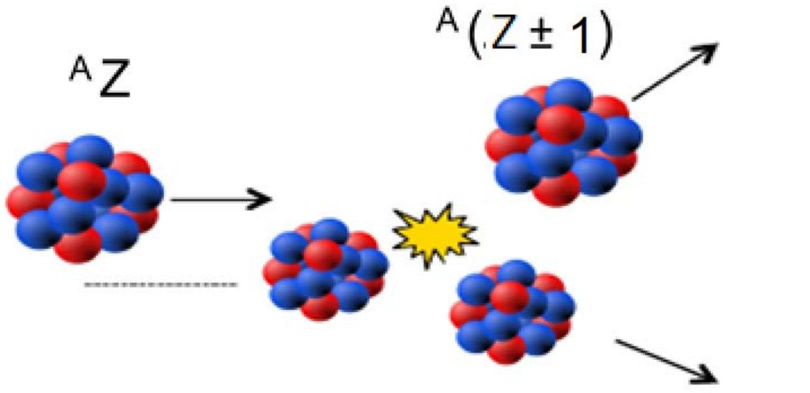
$$\vec{x}_{12} = \vec{r}_2 - \vec{r}_1 + \vec{r}$$

$$T_{NN}(\vec{x}_{12}) = \int \frac{d^3p}{(2\pi)^3} e^{i\vec{p}\bullet\vec{x}_{12}} T_{NN}(\vec{p}) = \int \frac{d^3p}{(2\pi)^3} e^{i\vec{p}\bullet(\vec{r}_2-\vec{r}_1+\vec{r})} \sum_{\mu=S,T,Tn} V_\mu(p^2)\left[O_\mu(1)\otimes O_\mu(2)\right]$$

Reaction Form Factor describing the nuclear transitions in α=a+A → β=b+B

$$F_{\alpha\beta}(\vec{r}) = \langle aA|T_{NN}(\vec{x}_{12})|bB\rangle = \int \frac{d^3p}{(2\pi)^3} e^{i\vec{p}\bullet\vec{r}} \sum_{\mu=S,T,Tn} V_\mu(p^2)\Big[\underbrace{\langle a|e^{-i\vec{p}\bullet\vec{r}_1}O_\mu(1)|b\rangle}_{\text{OBTD}} \otimes \underbrace{\langle A|e^{i\vec{p}\bullet\vec{r}_2}O_\mu(2)|B\rangle}_{\text{OBTD}}\Big]$$

Lecture 6:
Single Charge Exchange Reactions



Nuclear Single Charge Exchange Reaction by Isovector Meson Exchange

$$ {}^{a}_{z}a + {}^{A}_{Z}A \rightarrow {}^{a}_{z\pm1}b + {}^{A}_{Z\mp1}B, $$

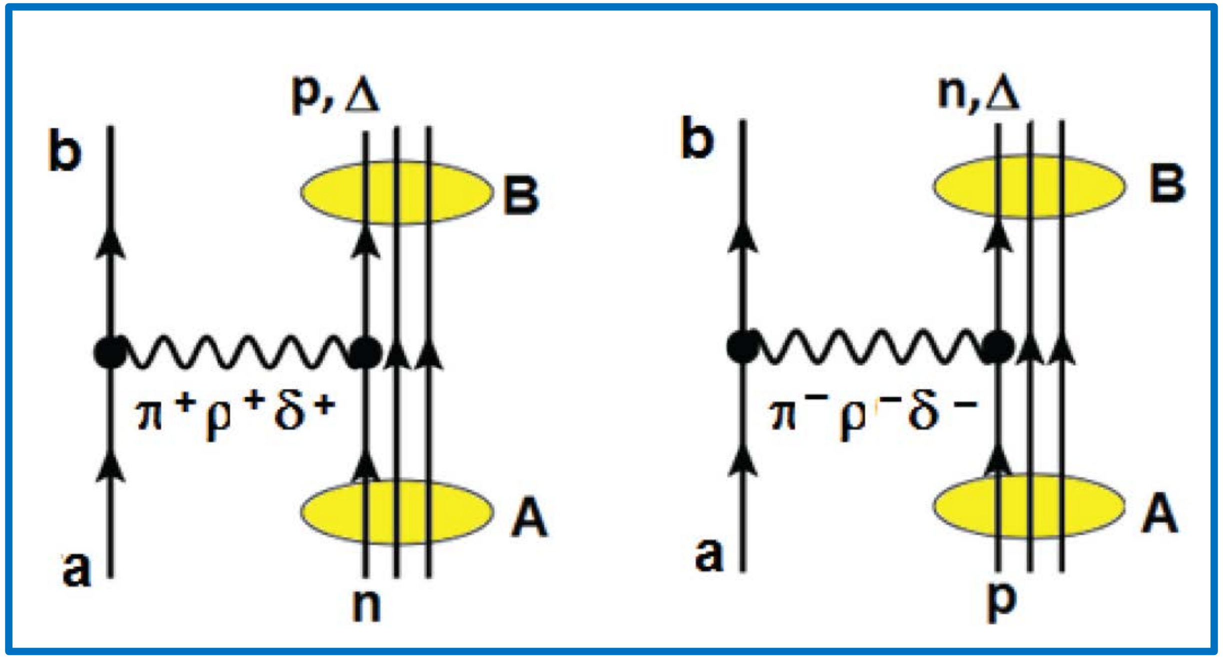
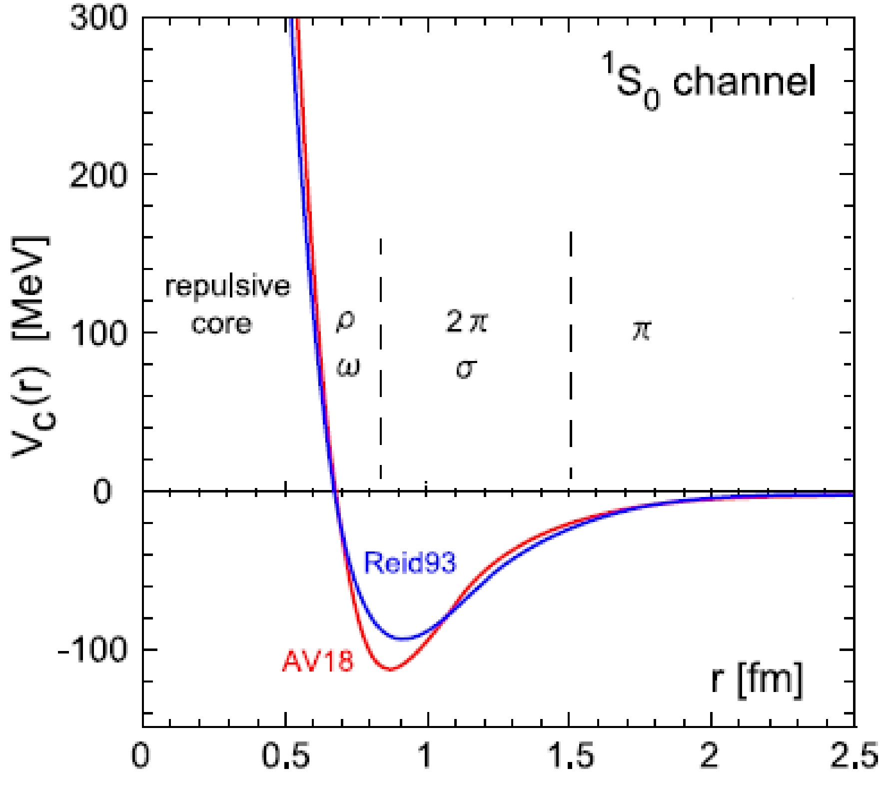


FIG. 1. Graphical representation of a single-charge exchange heavy-ion reaction by hadronic interactions corresponding to $\nu\beta$ processes. Both (n, p)-type (left) and (p, n)-type (right) reactions, as seen in the $A \rightarrow B$ transition in target system, are displayed, indicating also the exchanged meson.

NN OBE-Interaction at Tree-Level



Singularity requires complete summation of the Scattering Series:

$$T_{NN} = V_{NN} + \int V_{NN} G_{NN} T_{NN} = \frac{1}{1 + \int V_{NN} G_{NN}} V_{NN}$$

Operator Structure of the Fierz-Transformed (Anti-Symmetrized) NN T-Matrix

$$T_{NN} = V_{NN} + \int V_{NN} G_{NN} T_{NN}$$

$$T_{NN}(\mathbf{p}) = \sum_{S=0,1,T=0,1} \left\{ V_{ST}^{(C)}(p^2)[\boldsymbol{\sigma}_a \cdot \boldsymbol{\sigma}_A]^S + \delta_{S1} V_T^{(Tn)}(p^2) S_{12}(\mathbf{p})[\boldsymbol{\tau}_a \cdot \boldsymbol{\tau}_A]^T \right\}.$$

Rank-2 Spin-Tensor Interaction

$$S_{12}(\mathbf{p}) = \frac{1}{p^2}(3\boldsymbol{\sigma}_a \cdot \mathbf{p}\, \boldsymbol{\sigma}_A \cdot \mathbf{p} - \boldsymbol{\sigma}_a \cdot \boldsymbol{\sigma}_A p^2) = \sqrt{\frac{24\pi}{5}} \sum_M Y_{2M}^*(\hat{\mathbf{p}}) S_{2M} \; ; \; S_{2M} = [\boldsymbol{\sigma}_1 \otimes \boldsymbol{\sigma}_2]_{2M}$$

$$T_{NN}(\mathbf{p}) = \sum_{S,T} \left\{ V_{ST}^{(C)}(p^2) O_{ST}(1) O_{ST}(2) + \delta_{S1} V_T^{(Tn)}(p^2) \sqrt{\frac{24\pi}{5}} Y_2^*(\hat{\mathbf{p}}) \cdot [O_{ST}(1) \otimes O_{ST}(2)]_2 \right\}$$

$$O_{ST}(i) = (\boldsymbol{\sigma}_i)^S (\boldsymbol{\tau}_i)^T$$

H.L. et al., Phys. Rev. C 98 044620 (2018)

SCE Form Factors, Reaction Kernel and Amplitudes, Cross Section

$$\mathcal{R}_{\mathrm{ST}}(\mathbf{p},\mathbf{r}) = \frac{1}{4\pi} e^{i\mathbf{p}\cdot\mathbf{r}} O_{\mathrm{ST}}, \quad F^{(XY)}_{\mathrm{ST}}(\mathbf{p}) = \langle J_Y M_Y | \mathcal{R}_{\mathrm{ST}}(\mathbf{p},\mathbf{r}_X) | J_X M_X \rangle.$$

$$K^{(ST)}_{\alpha\beta}(\mathbf{p}) = (4\pi)^2 \left\{ V^{(C)}_{\mathrm{ST}}(p^2) F^{(ab)\dagger}_{\mathrm{ST}}(\mathbf{p}) \cdot F^{(AB)}_{\mathrm{ST}}(\mathbf{p}) + \delta_{S1} \sqrt{\frac{24\pi}{5}} V^{(Tn)}_{\mathrm{ST}}(p^2) Y^*_2(\hat{\mathbf{p}}) \cdot \left[F^{(ab)\dagger}_{\mathrm{ST}}(\mathbf{p}) \otimes F^{(AB)}_{\mathrm{ST}}(\mathbf{p}) \right]_2 \right\},$$

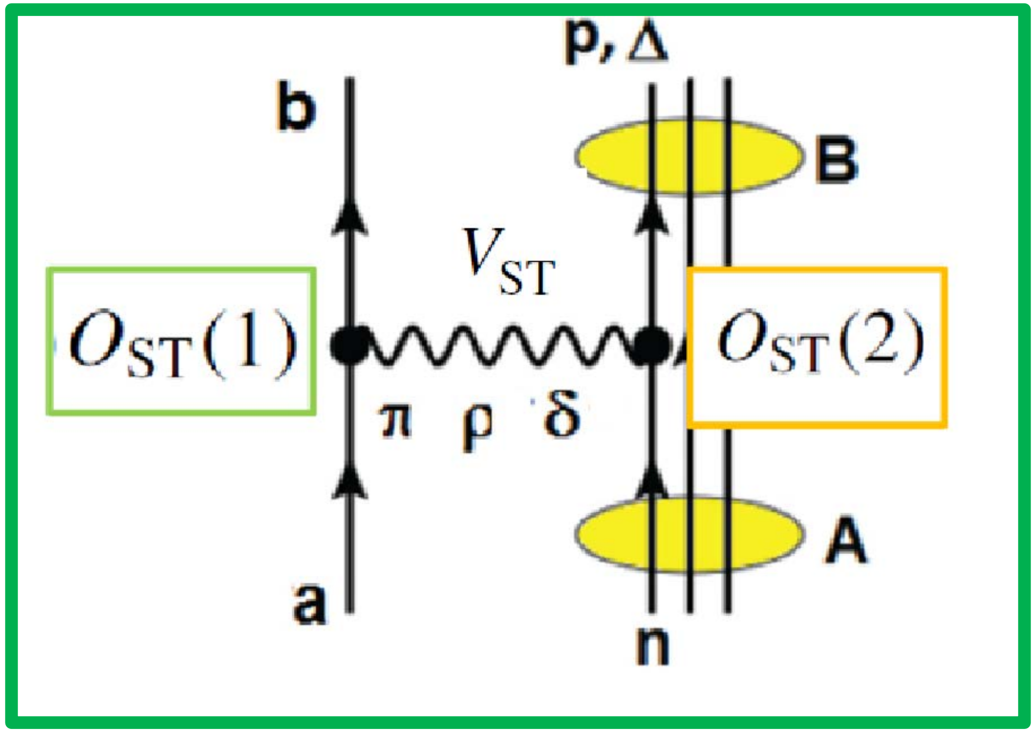
$$M_{\alpha\beta}(\mathbf{k}_\alpha,\mathbf{k}_\beta) = \int \frac{d^3p}{(2\pi)^3} \langle \chi^{(-)}_\beta | e^{-i\mathbf{p}\cdot\mathbf{r}} \sum_{ST} K^{(ST)}_{\alpha\beta}(\mathbf{p}) | \chi^{(+)}_\alpha \rangle \qquad N_{\alpha\beta}(\mathbf{k}_\alpha,\mathbf{k}_\beta,\mathbf{p}) = \frac{1}{(2\pi)^3} \langle \chi^{(-)}_\beta | e^{-i\mathbf{p}\cdot\mathbf{r}} | \chi^{(+)}_\alpha \rangle,$$

$$M_{\alpha\beta}(\mathbf{k}_\alpha,\mathbf{k}_\beta) = \int d^3p\, \mathcal{U}_{\alpha\beta}(\mathbf{p}) N_{\alpha\beta}(\mathbf{k}_\alpha,\mathbf{k}_\beta,\mathbf{p}),$$

$$d^2\sigma_{\alpha\beta} = \frac{m_\alpha m_\beta}{(2\pi\hbar^2)^2} \frac{k_\beta}{k_\alpha} \frac{1}{(2J_a+1)(2J_A+1)} \sum_{M_a,M_A\in\alpha;M_b,M_B\in\beta} |M_{\alpha\beta}(\mathbf{k}_\alpha,\mathbf{k}_\beta)|^2 d\Omega$$

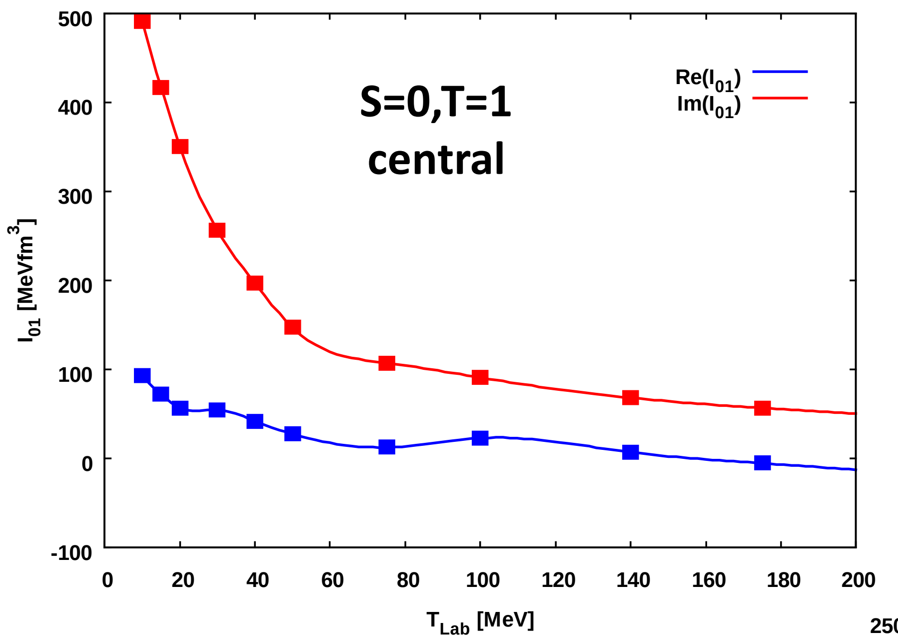
Vertex Structure of Mesonic Single Charge Exchange

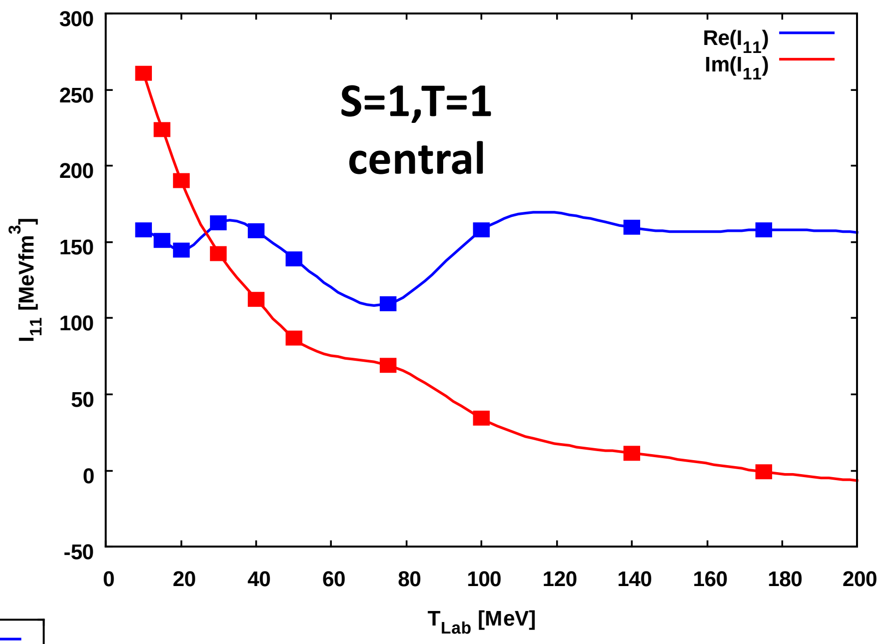
…for (p,n), (^{3}He,^{3}H)…(^{12}C,^{12}N)…(^{48}Ti,^{48}Sc)…(^{124}Sn,^{124}Sb) reactions on any target and at any energy!



$$O_{ST}(i) = (\boldsymbol{\sigma}_i)^S (\boldsymbol{\tau}_i)^T$$

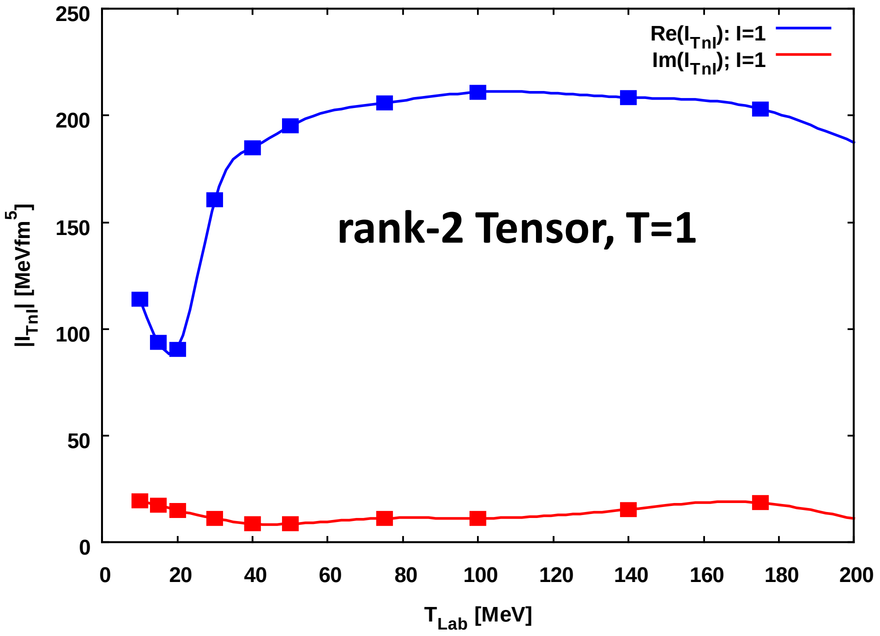
Energy Dependence of Isovector NN-Interaction Strengths





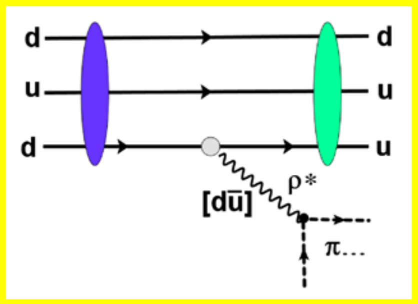
$$I_{ST}(T_{lab}) = V_{ST}(p=0|T_{lab})$$

$$I_{ST}(T_{lab}) \approx g^2_{eff}(T_{lab})/m^2_{eff}$$

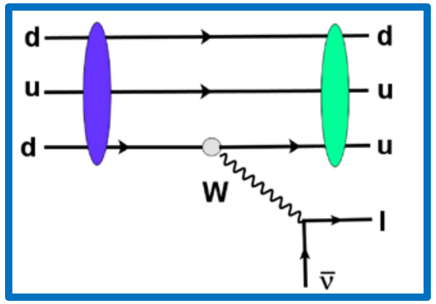


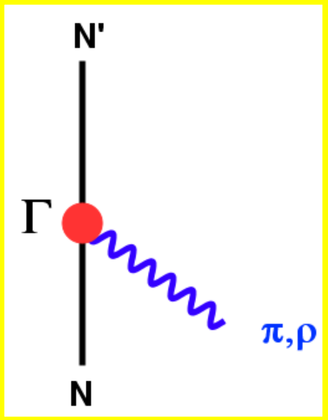
NN Strong Interaction and Weak Interaction Vertices

Strong Interaction



Weak Interaction



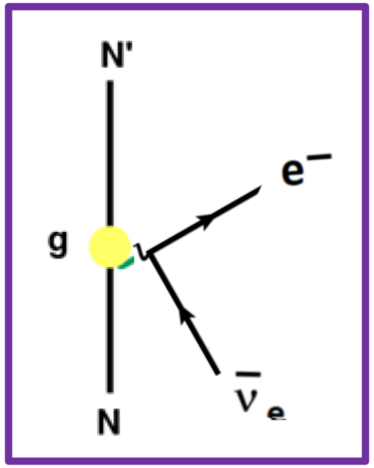


$$\pi\,\&\,\delta : \Gamma_{01}\left(q^2\right)\boldsymbol{\tau}_\pm \quad \leftrightarrow \quad g_F\left(q^2\right)\boldsymbol{\tau}_\pm \quad \text{"Fermi"}$$

$$\pi\,\&\,\rho : \Gamma_{11}\left(q^2\right)\boldsymbol{\sigma}_1\boldsymbol{\tau}_\pm \quad \leftrightarrow \quad g_A\left(q^2\right)\boldsymbol{\sigma}\ \boldsymbol{\tau}_\pm \quad \text{"Gamow-Teller"}$$

$$\rho : \Gamma_{T1}\left(q^2\right)\boldsymbol{\sigma}\times \mathrm{q}\ \boldsymbol{\tau}_\pm \quad \leftrightarrow \quad g_M\left(q^2\right)\boldsymbol{\sigma}\times \mathrm{q}\ \boldsymbol{\tau}_\pm \quad \text{"weak magnetic"}$$

$$+\ldots$$



pion-exchange: $\sigma_1\cdot\vec{q}\sigma_2\cdot\vec{q} = \frac{q^2}{3}\left(S_{12}+\sigma_1\cdot\sigma_2\right)$

rho-exchange : $\sigma_1\times\vec{q}\sigma_2\times\vec{q} = \sigma_1\cdot\sigma_2 q^2 - \sigma_1\cdot\vec{q}\sigma_2\cdot\vec{q}$

$$S_{12} = \frac{1}{q^2}\left[3\sigma_1\cdot\vec{q}\sigma_2\cdot\vec{q} - \sigma_1\cdot\sigma_2 q^2\right]$$

Lecture 7:
Light Ion SCE Reactions and Nuclear beta-Decay

The Game Changer:
Simplicity of (p,n) and (3He,t) SCE Reactions above T_{lab}> 100 AMeV

$$\boxed{\begin{aligned}\frac{d\sigma_{GT}}{d\Omega}(q,\omega) &\simeq K(\omega)N_{\sigma\tau}|J_{\sigma\tau}(q)|^2 B(GT)\\ &= \hat{\sigma}_{GT}(q,\omega)B(GT),\end{aligned}}$$

SCE Cross Sections factorize into a unit cross section $\hat{\sigma}_{GT}$ and a nuclear matrix element B(GT)!

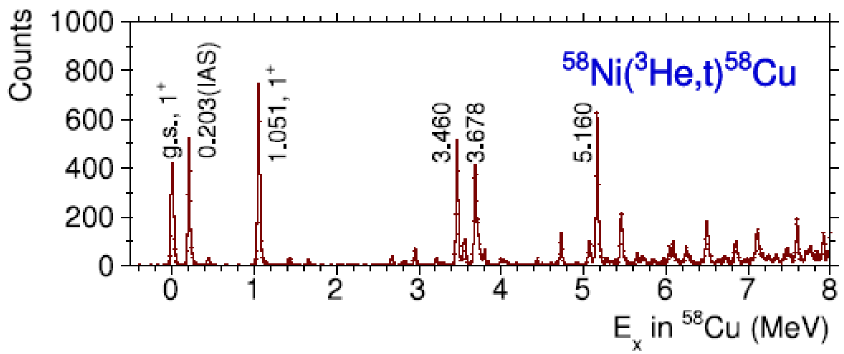
(late 1970ties at IUCF)

Spoiler:

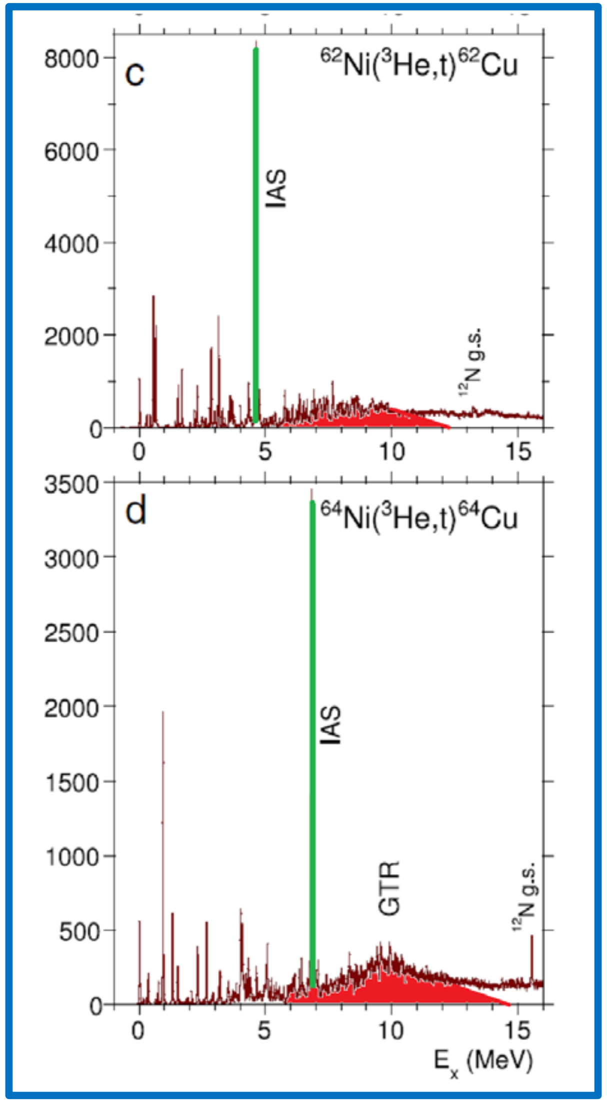
This is not always the case but depends esp. on the multipolarity!

Characteristics of Light Ion Single Charge Exchange Reactions on a Medium-Mass Nucleus

ΔE=45KeV (!)



Y. Fujita et al. / Progress in Particle and Nuclear Physics 66 (2011) 549

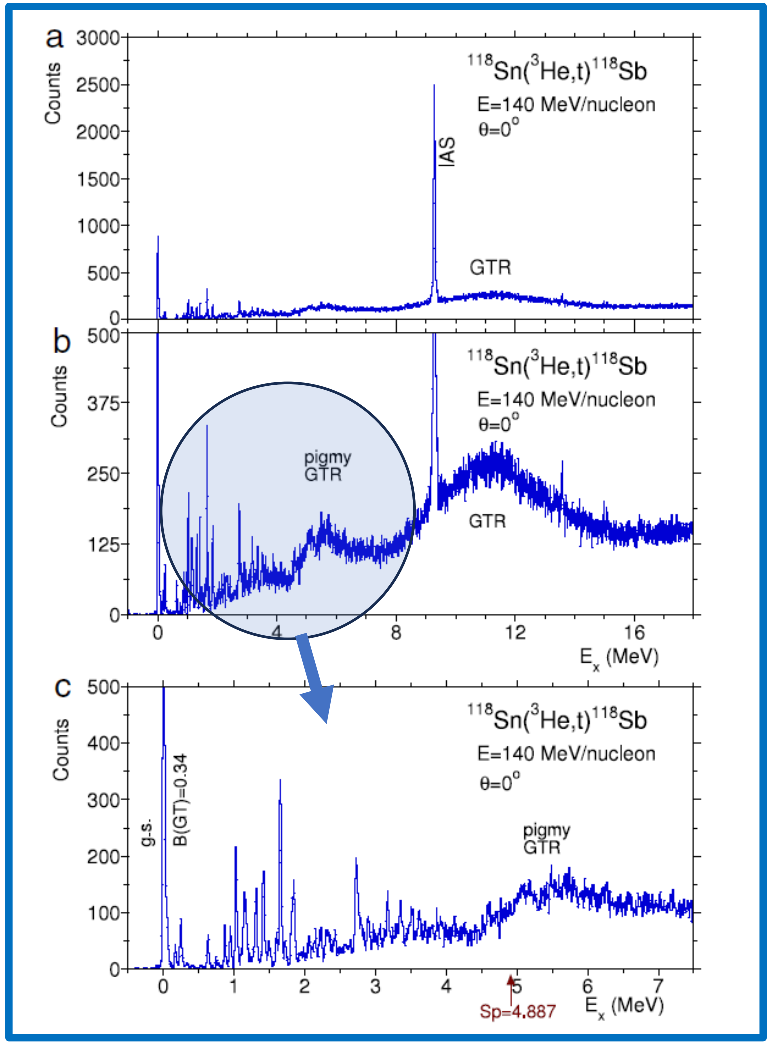


The (^{3}He, t) spectra on nickel target nuclei.

T_{lab}(^{3}He)=140 AMeV=420 MeV
Research Center for Nuclear Physics (RCNP), Osaka University

Characteristics of Light Ion Single Charge Exchange Reactions on a Heavy Nucleus

$T_{lab}(^3He)$=140 AMeV=420 MeV
RCNP@Osaka



$^{118}Sn(^3He, t)\ ^{118}Sb$ spectrum. The discrete g.s, low-lying states, and the IAS are prominent

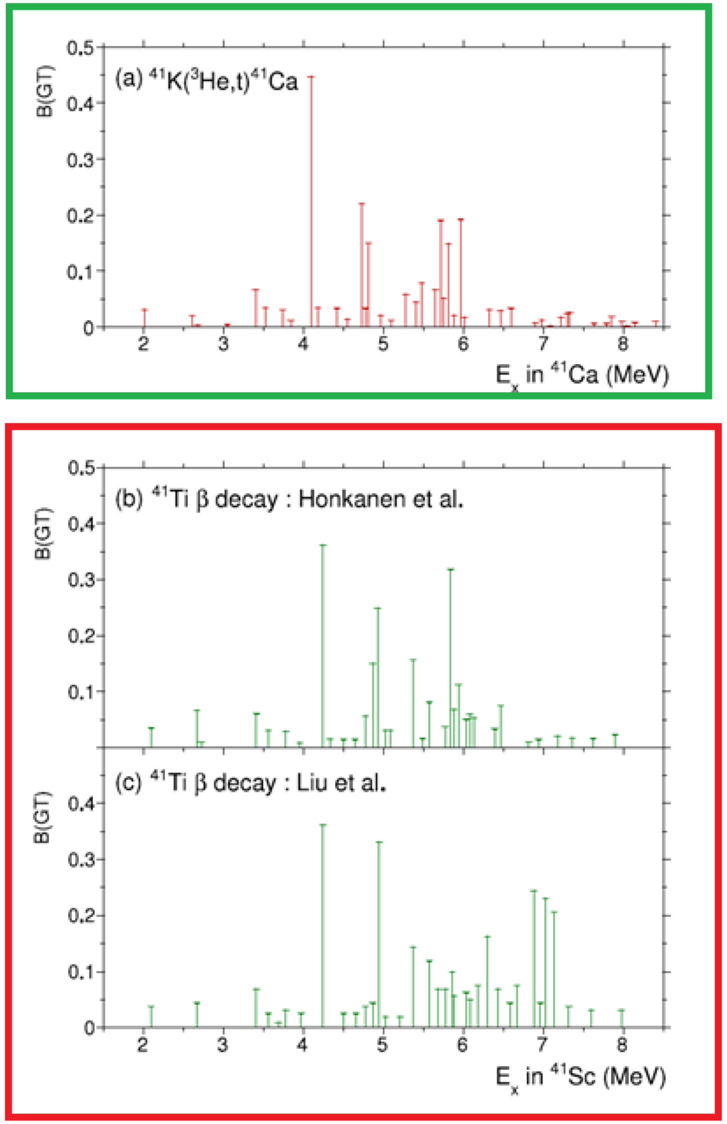
$^{118}Sn(^3He, t)\ ^{118}Sb$ spectrum with an expanded vertical scale.

$^{118}Sn(^3He, t)\ ^{118}Sb$ spectrum with expanded vertical and energy scales.

ΔE=45KeV (!)

Y. Fujita et al. / Progress in Particle and Nuclear Physics 66 (2011) 549

NME from (^{3}He,t) and β-Decay

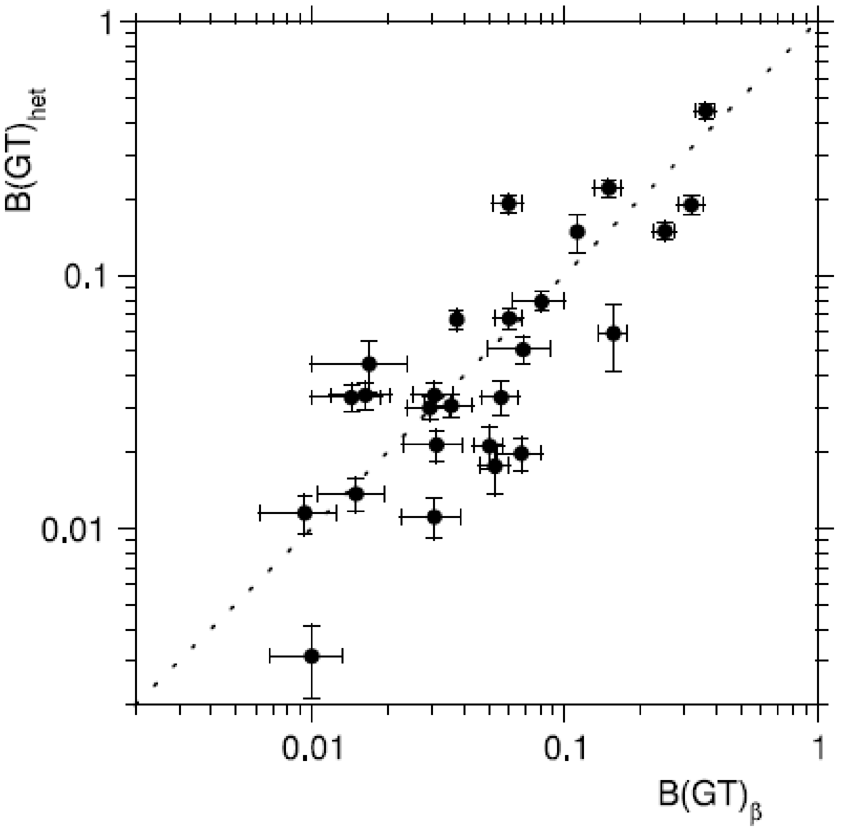


^{41}K→^{41}Ca: Z→Z+1 „β^-" transition

^{41}Ti→^{41}Sc: Z→Z-1 β^+ transition

Y. Fujita, B. Rubio, W. Gelletly, PPNP 66 (2011) 549

Comparison of B(GT) from (^{3}He, ^{3}H) Reactions and β-Decay



….Deviations increase considerably for weak transitions!

Y. Fujita, B. Rubio, W. Gelletly, PPNP 66 (2011) 549

The General Picture:
Coherent Superpostions of Nuclear Multipoles

Natural Parity ($\pi=(-)^J$) „Fermi“ Transition in both Nuclei – L=J, S=0,1 :

$$\frac{d\sigma^{FF}}{d\Omega} \sim \frac{q^{2(J_a+J_A)}}{[(2J_a+1)!!(2J_A+1)!!]^2}|\bar{N}_{\alpha\beta}|^2 \left|V_{01}^{(C)}(0)b^{(AB)}_{J_A0J_A}b^{(ab)}_{J_a0J_a} + e^{i\phi_{aA}}V_{11}^{(C)}(0)b^{(AB)}_{J_A1J_A}b^{(AB)}_{J_A1J_A}\right|^2,$$

Unnatural Parity ($\pi=(-)^{J+1}$) „Gamov-Teller“-type Transition in both Nuclei – L=J±1:

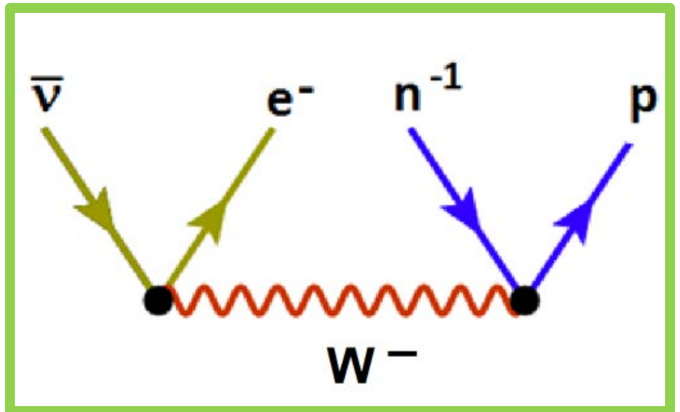
$$\frac{d\sigma^{GG}}{d\Omega} \sim \frac{q^{2(J_a+J_A-2)}}{[(2J_a-1)!!(2J_A-1)!!]^2}\left|V_{11}^{(C)}(0)\right|^2|\bar{N}_{\alpha\beta}|^2 \times\left[\left|b^{(AB)}_{J_A-11J_A}+\frac{q^2}{(2J_A+1)(2J_A+3)}b^{(AB)}_{J_A+11J_A}\right|^2\left|b^{(ab)}_{J_a-11J_a}+\frac{q^2}{(2J_a+1)(2J_a+3)}b^{(ab)}_{J_a+11J_a}\right|^2\right]$$

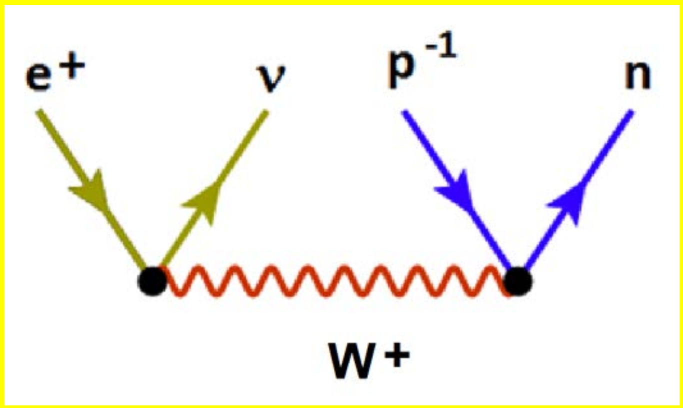
…and mixed σ^{FG} and σ^{GF} , e.g. σ^{FG} spin-flip Fermi in a→b and GT in A → B

PHYSICAL REVIEW C **98**, 044620 (2018)

Lecture 8:
The Gamow-Teller Quenching Mystery

Nuclear beta-Decay:
Weak Charged-Current Interactions and Gamov-Teller strength





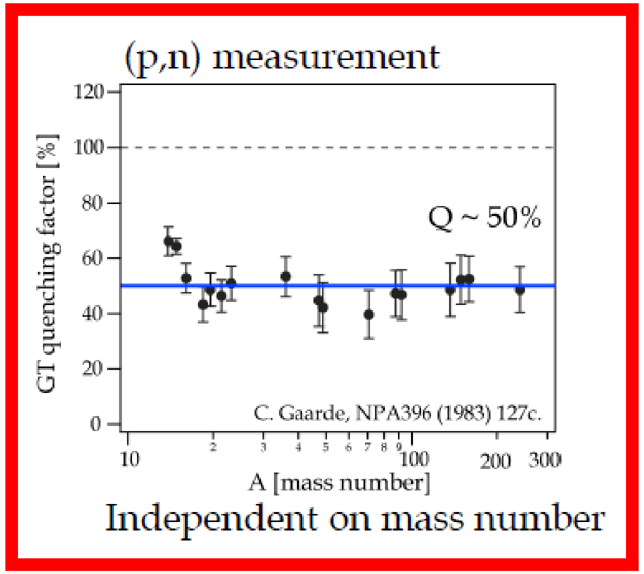
$$M(GT^{\pm}) \propto \left| \left\langle f \left\| \sigma \cdot \tau_{\pm} \right\| i \right\rangle \right|^2$$

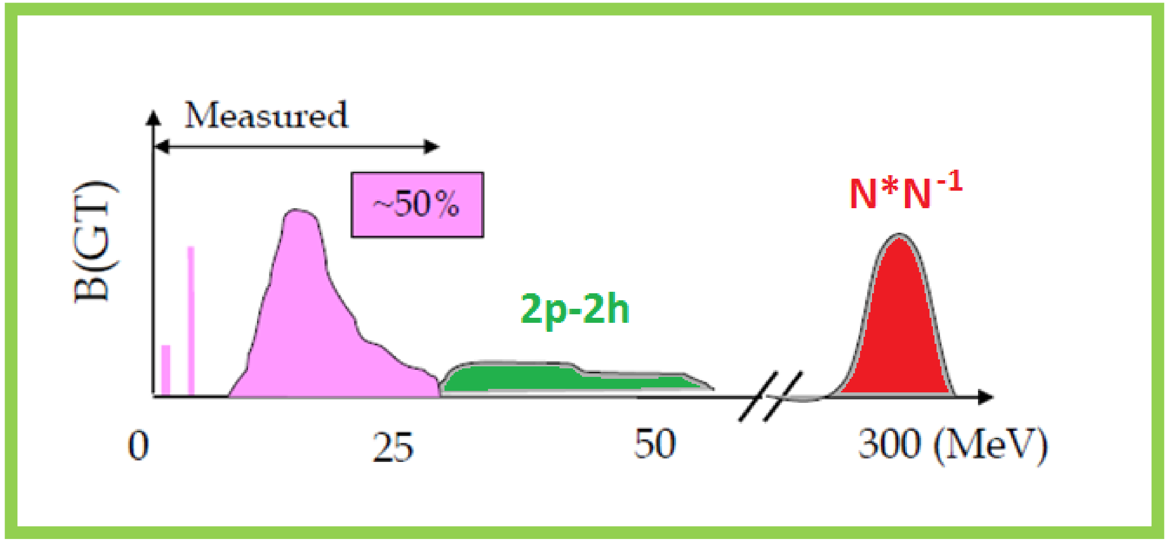
GT sum rule (model independent)

K. Ikeda PL 3, 271 (1963)

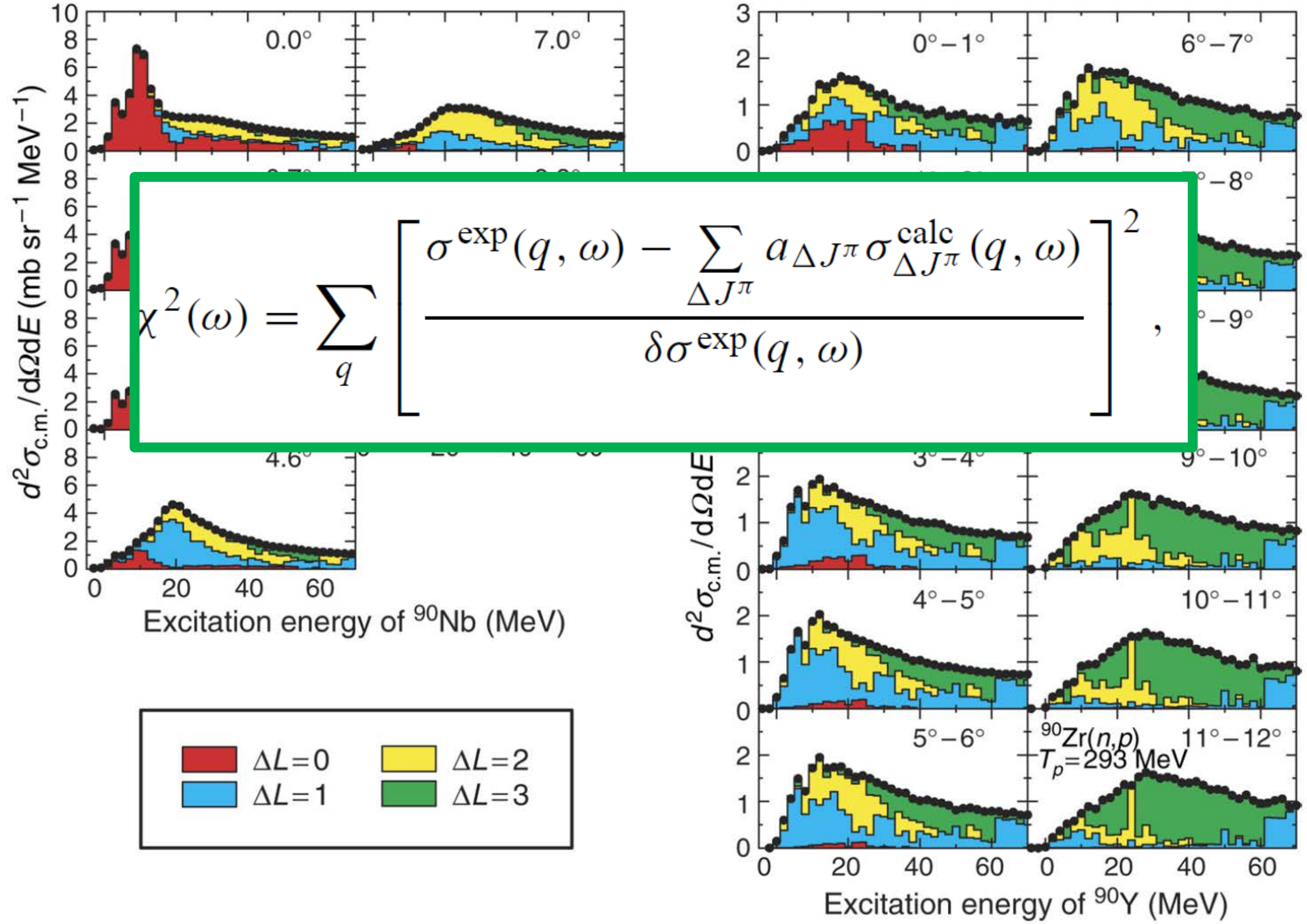
$$S_{\beta^-} - S_{\beta^+} = 3(N - Z)$$

The GT-Quenching Mystery:
~50% of the *Ikeda Sum Rule Strength* is missing!





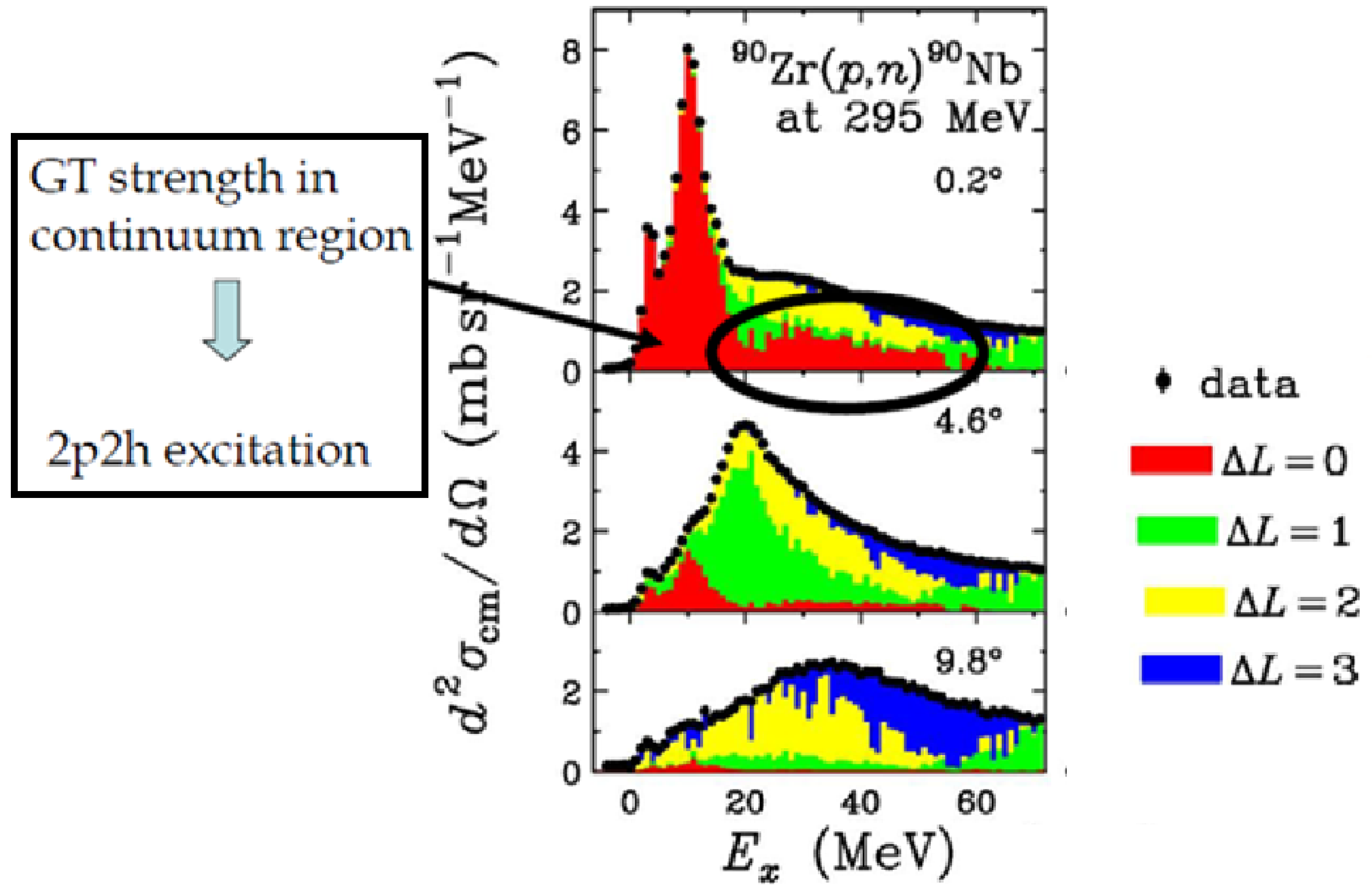
Multipole Decomposition of (n,p) and (p,n) Reactions on ^{90}Zr
T_{lab}≈295 MeV



$$\chi^2(\omega) = \sum_q \left[\frac{\sigma^{\text{exp}}(q,\omega) - \sum_{\Delta J^\pi} a_{\Delta J^\pi} \sigma^{\text{calc}}_{\Delta J^\pi}(q,\omega)}{\delta\sigma^{\text{exp}}(q,\omega)} \right]^2 ,$$

M. Ichimura et al. / Progress in Particle and Nuclear Physics 56 (2006) 446–531

Search for Missing SCE Strength at RCNP@Osaka



M. Ichimura, H. Sakai and T. Wakasa, PPNP. 56, 446 (2006).

Comparision to 2p2h Nuclear Response Functions

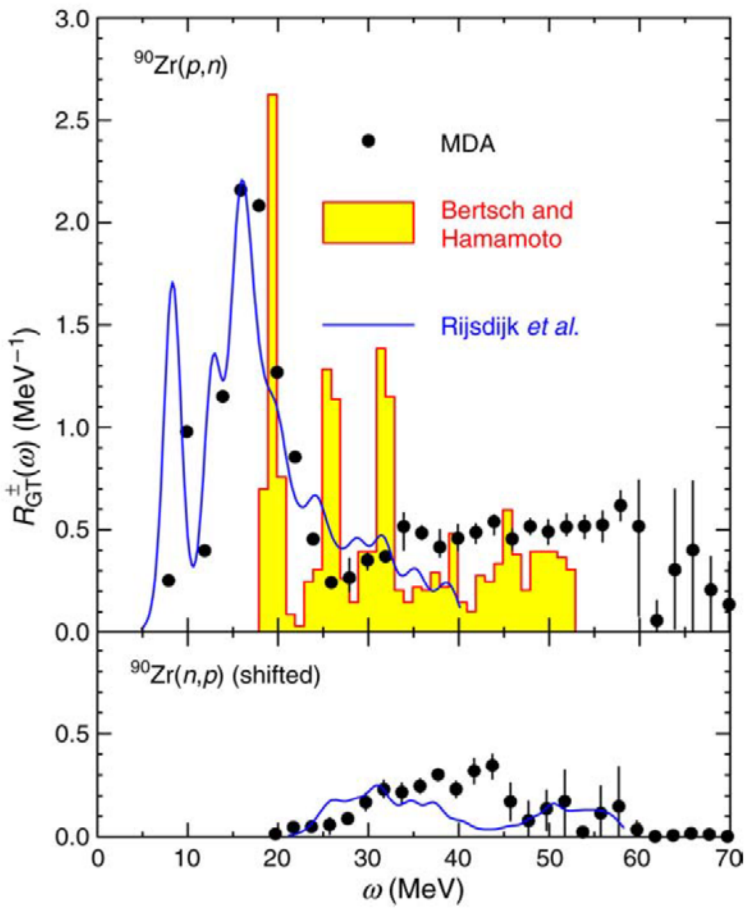


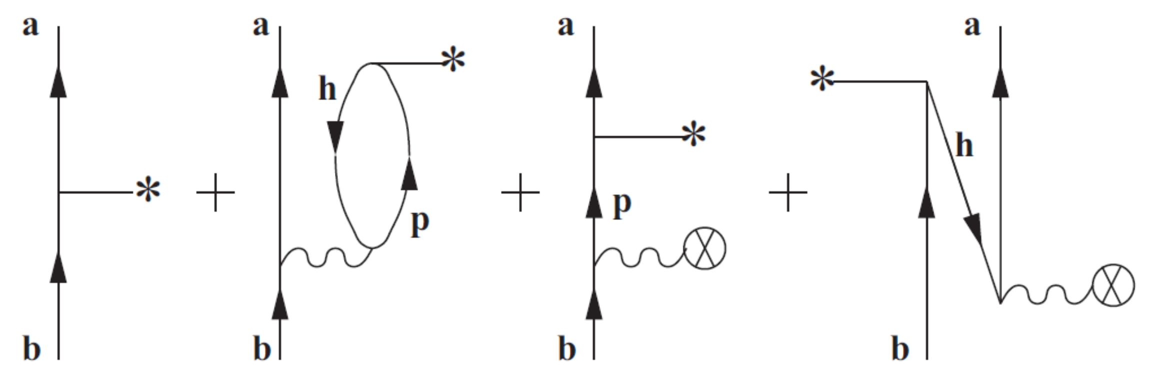
Fig. 23. GT$^-$ (top panel) and GT$^+$ (bottom panel) strength distributions (filled circles) obtained from the $\Delta L = 0$ cross sections deduced from the MDA. The histogram and solid curves represent the perturbative calculation by Bertsch and Hamamoto [132] and the DRPA calculation by Rijsdijk et al. [134].

M. Ichimura et al. / Progress in Particle and Nuclear Physics 56 (2006) 446–531

Effective Operators incorporating Many-Body Dynamics

$$\hat{Q}(\epsilon) = PH_1P + PH_1Q\frac{1}{\epsilon - QHQ}QH_1P.$$

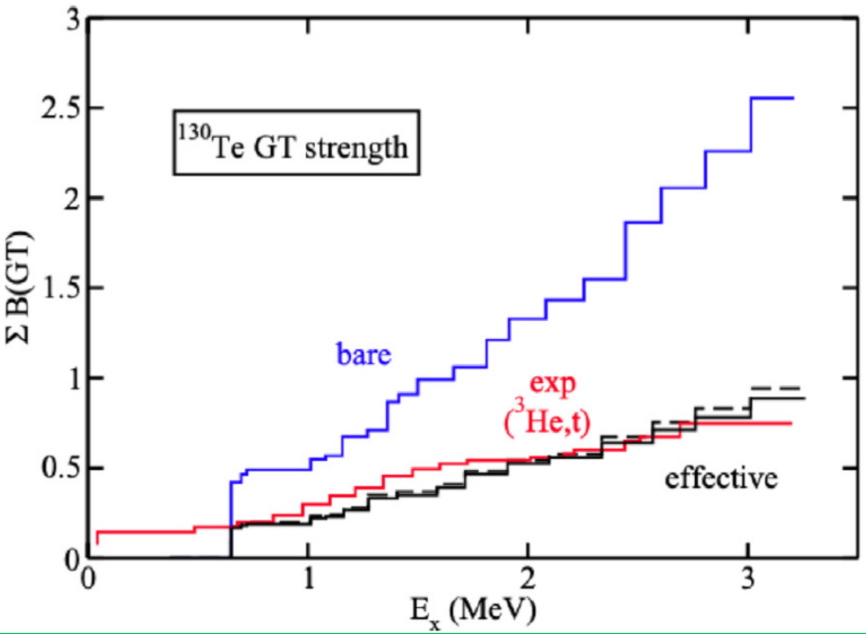
$$Q^{GT}_{free} = g_A\vec{\sigma}\vec{\tau} \quad \rightarrow \quad Q^{GT}_{eff}(\rho,V) \equiv g^*_A(\rho,V)\vec{\sigma}\vec{\tau}$$



* = Q_{free}

Quenching Factors from the Many-Body Shell Model

$$q_{ab} = \frac{\langle b | O_{eff} | a \rangle}{\langle b | O_{free} | a \rangle}$$



State dependent!

$q_{ab} \sim 0.5 \ldots 0.6$

GT^- Single particle matrix elements

$n_a l_a j_a$ $n_b l_b j_c$	*quenching*
$0g_{7/2}$ $0g_{7/2}$	0.521
$1d_{5/2}$ $1d_{5/2}$	0.647
$1d_{5/2}$ $1d_{3/2}$	0.577
$1d_{3/2}$ $1d_{5/2}$	0.613
$1d_{3/2}$ $1d_{3/2}$	0.638
$2s_{1/2}$ $2s_{1/2}$	0.638
$0h_{11/2}$ $0h_{11/2}$	0.570

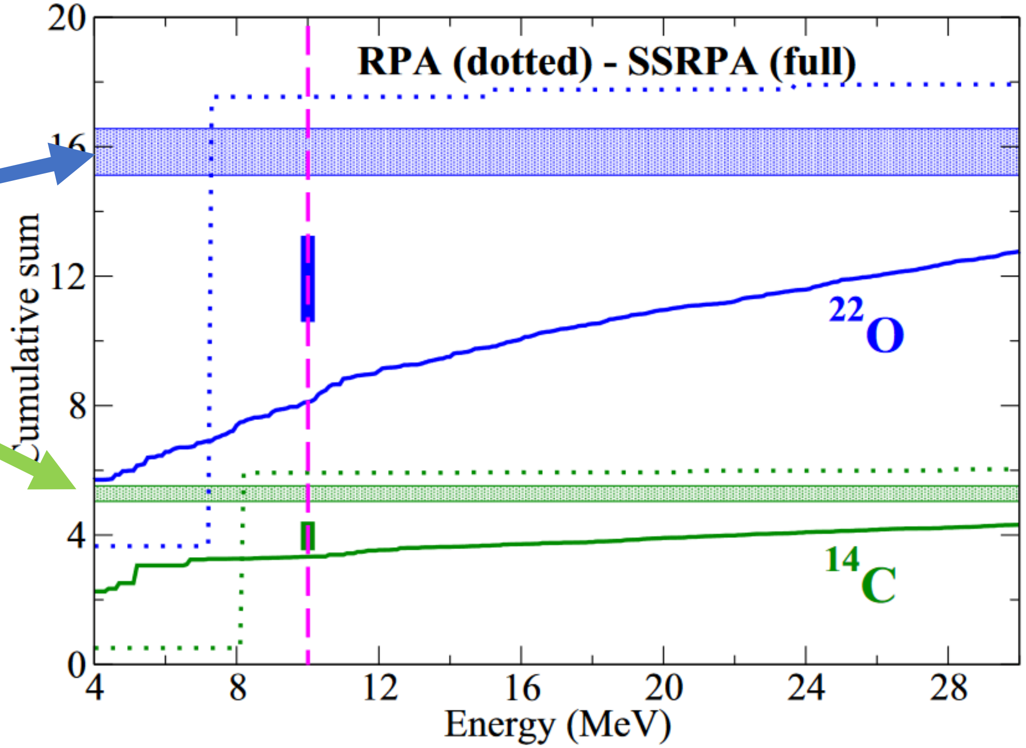
CORAGGIO et al. , PHYS. REV. C **95**, 064324 (2017)

see also: Capuzzello, H.L. et al. Prog.Part.Nucl.Phys. 128 (2023) 103999

Studies of the GT-Quenching Problem by Higher Order RPA Theory

(D. Gambacurta, LNS Catania)

Ekström et al.,
PRL 113, 262504 (2014)
χEFT N^{2}LO+2-body currents



$$S_{GT^-} - S_{GT^+}$$

Quenching of GT Ikeda sum rules by coupling to 2p2h configurations
(Skyrme EDF SGIII)

• *Phys.Rev.C* 105 (2022) 1, 014321

Lecture 9: Nuclear Matrix Elements for Astrophysics from (^{3}He,^{3}H) Reactions

Physics Letters B 706 (2011) 134–138

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The ^{71}Ga(^{3}He, t) reaction and the low-energy neutrino response

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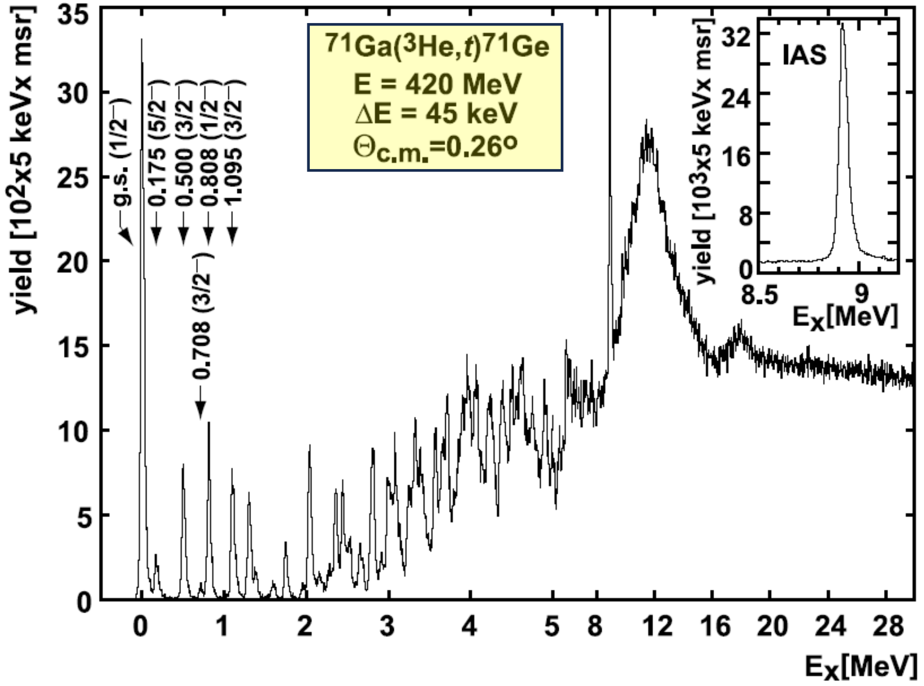


A B S T R A C T

A ^{71}Ga(^{3}He, t)^{71}Ge charge-exchange experiment was performed to extract with high precision the Gamow–Teller (GT) transition strengths to the three lowest-lying states in ^{71}Ge, i.e., the ground state ($1/2^-$), the 175 keV ($5/2^-$) and the 500 keV ($3/2^-$) excited states. These are the relevant states, which are populated via a charged-current reaction induced by neutrinos from reactor-produced ^{51}Cr and ^{37}Ar sources. A precise measurement of the GT transition strengths is an important input into the calibration of the SAGE and GALLEX solar neutrino detectors and addresses a long-standing discrepancy between the measured and evaluated capture rates from the ^{51}Cr and ^{37}Ar neutrino calibration sources, which has recently spawned new ideas about unconventional neutrino properties.

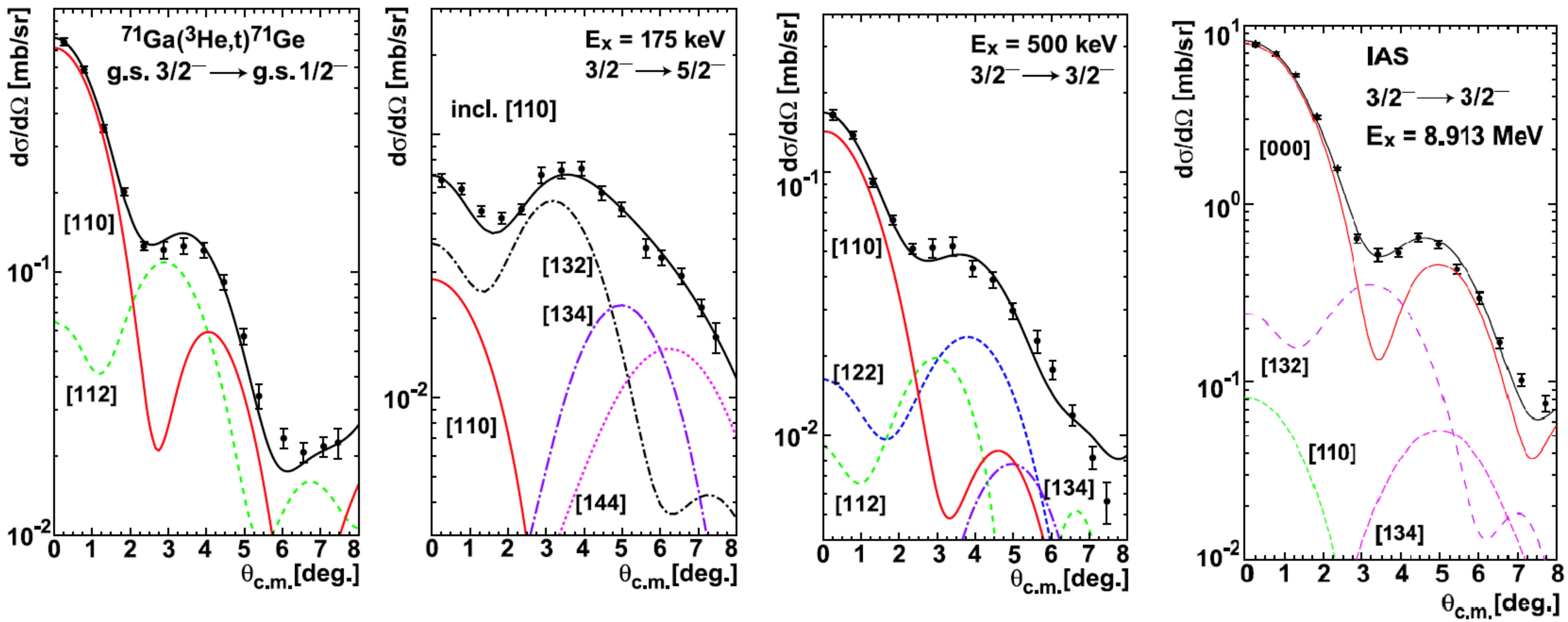
Exp.: RCNP@Osaka and Grand Raiden Spectrometer

Purpose:

Provide data for the calibration of the GALLEX/GNO and SAGE underground neutrino detectors using

$$^{71}\mathrm{Ga}+\nu_e \rightarrow e^- + ^{71}\mathrm{Ge}.$$

DWBA Analysis and Search for the lowest $J^{\pi}=1^{+}$ GT-Transitions and the $J^{\pi}=0^{+}$ IAS



Angular Momentum Transfers

$[ABC] = [J_{proj} J_{targ} J_{tot}]$

GT-strength ↔ $J_{targ}=1$

D. Frekers et al. / Physics Letters B 706 (2011) 134

B(GT) and B(F) derived from the DWBA Analysis

Table 3
Various low-energy cross sections and $B(\mathrm{GT})$ values for the $^{71}\mathrm{Ga}(^3\mathrm{He},t)^{71}\mathrm{Ge}$ reaction. The values for the Fermi transition to the IAS have been included. The errors are statistical errors only, whereby we conservatively added 50% of the non-GT, resp. non-F component of the calculated $q=0$ cross section into the error calculations for the $B(\mathrm{GT})$, resp. $B(\mathrm{F})$ values. The g.s. $B(\mathrm{GT})$ value and the $B(\mathrm{F})$ value are, however, reference values, whose error numbers (given in curly brackets) enter into the evaluation of the effective interaction volume integrals (see text).

^{71}Ge E_x [keV]	J^π of level	Data point $\theta=0.26°$ [mb/sr]	$d\sigma/d\Omega$ ($\theta=0°$) [mb/sr]	$d\sigma/d\Omega$ ($q=0$) [mb/sr]	% GT	$B(\mathrm{GT})$ ($\times 10^{-2}$)
g.s.	$1/2^-$	0.746(23)	0.777(9)	0.786(9)	92%	8.52{40}
175	$5/2^-$	0.067(5)	0.070(4)	0.071(4)	40%	0.34(26)
500	$3/2^-$	0.165(9)	0.169(4)	0.171(4)	87%	1.76(14)
					% F	$B(\mathrm{F})$
8913 $\Gamma \approx 50$	IAS	7.89(40)	8.35(11)	9.04(12)	96%	9.00{22}

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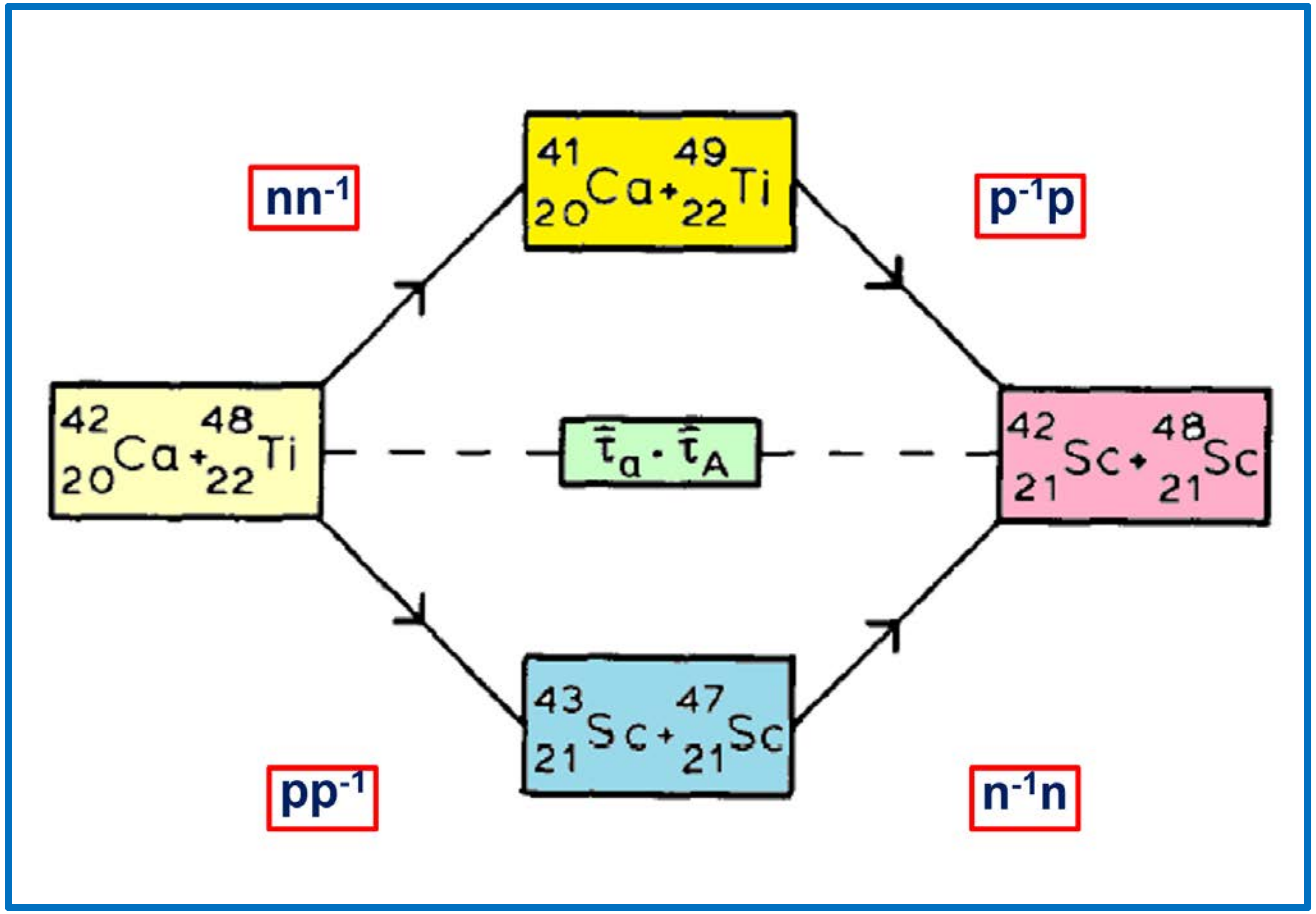
Lecture 10:
Heavy Ion SCE Reactions

Scheme of a Heavy Ion SCE Reaction
$^{42}Ca(^{48}Ti,^{48}Sc)^{42}Sc$

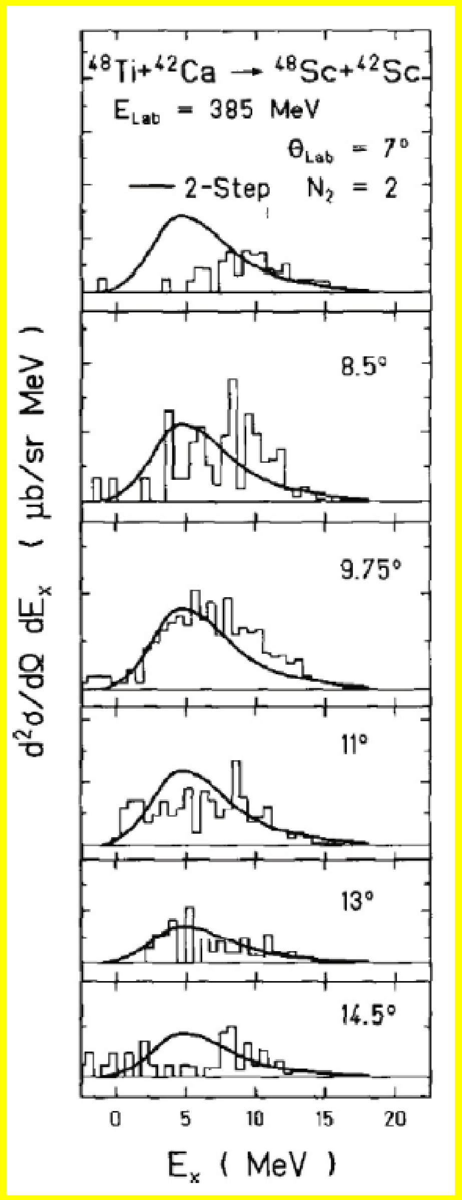


Heavy ion Single Charge Exchange Reaction at the Coulomb-Barrier

Pioneering work

- **The first measurement of SCE reactions with heavy target and heavy ion beam**
- **The first fully microscopic description including 2-step transfer and 1-step mesonic SCE**

C. Brendel, H.L. et al., Nucl. Phys. A 477 (1988) 162

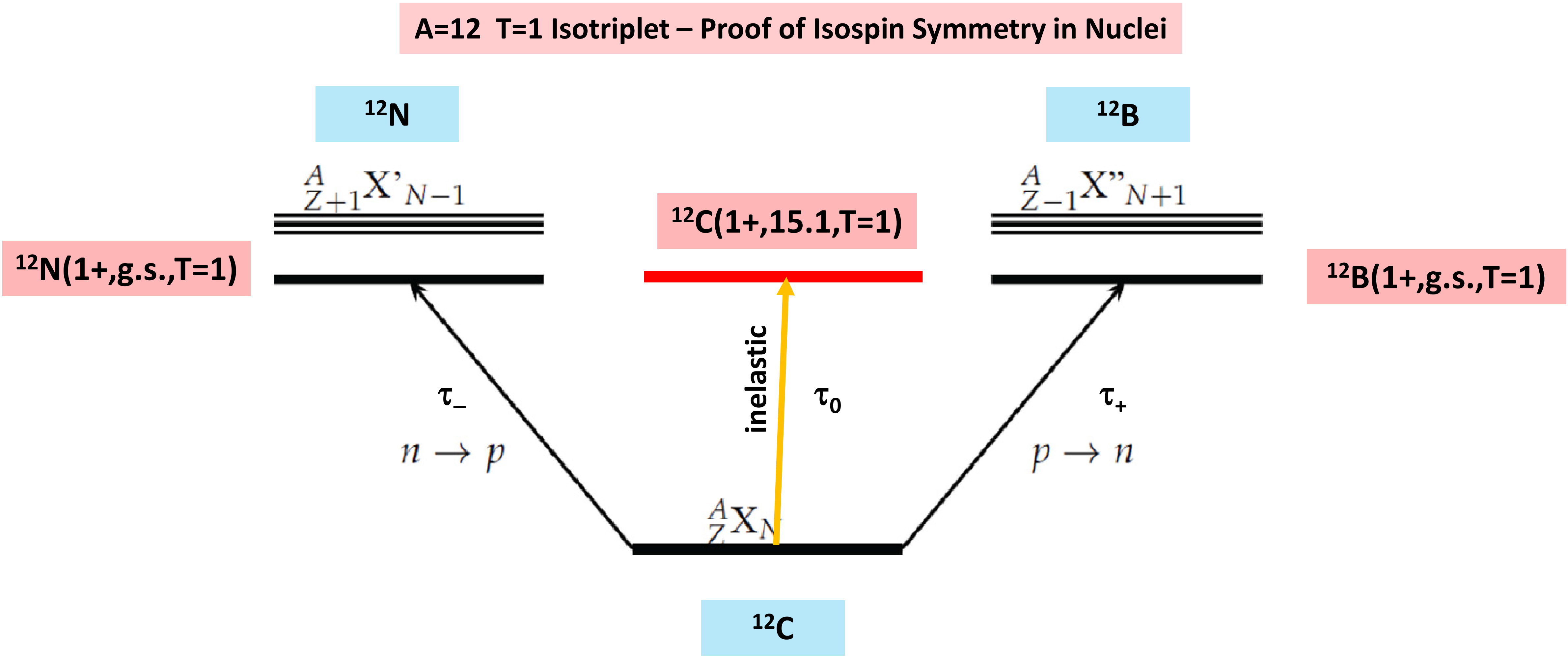


$^{48}Ti + ^{42}Ca \rightarrow ^{48}Sc + ^{42}Sc$
T_{lab}= 8AMeV = 385 MeV
LINAC@GSI Darmstadt

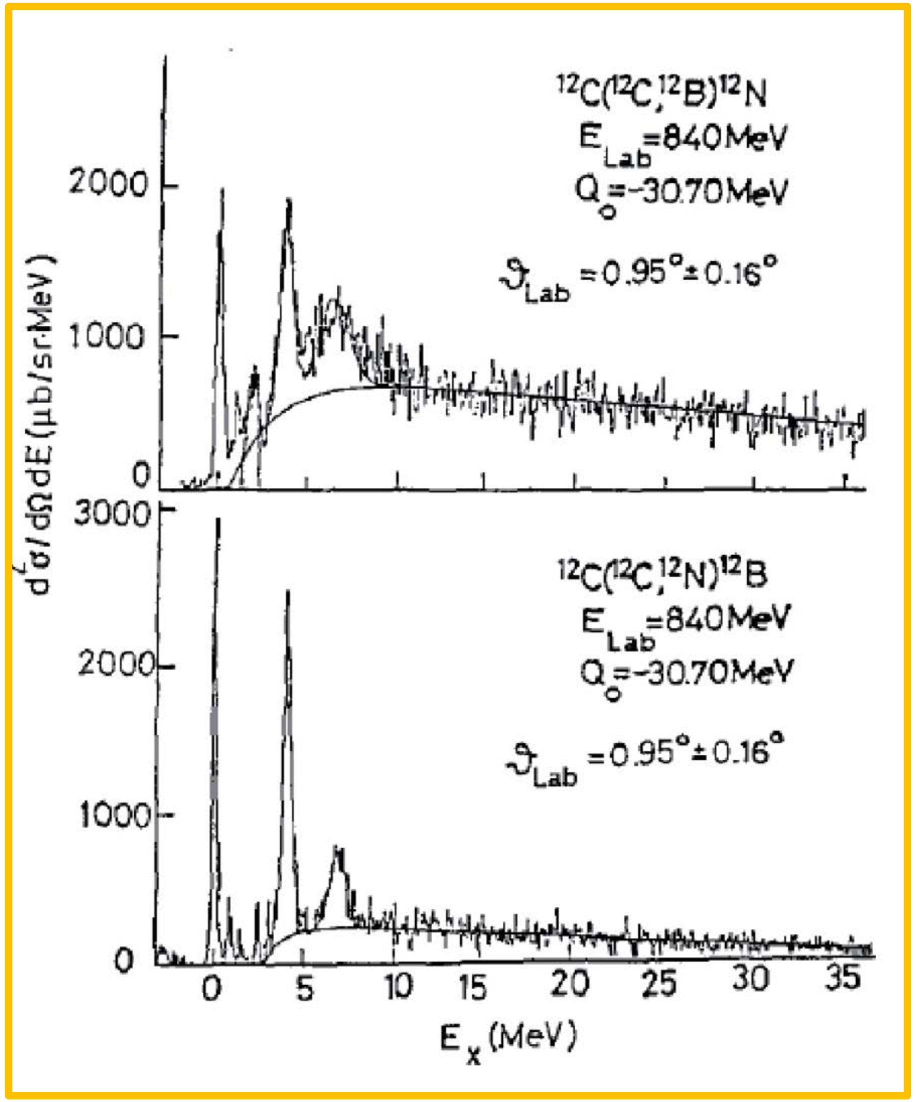
- Dominance of 2-Step Transfer-SCE

- Supression of 1-Step Mesonic SCE

Versatility of Heavy Ion Reactions



$\tau_{\pm}$ Spectral Distributions of the reaction $^{12}C + ^{12}C \rightarrow ^{12}B + ^{12}N$ at 70 AMeV



12C+12C→ 12B(1+.g.s.)+12N(Jπ , Ex)

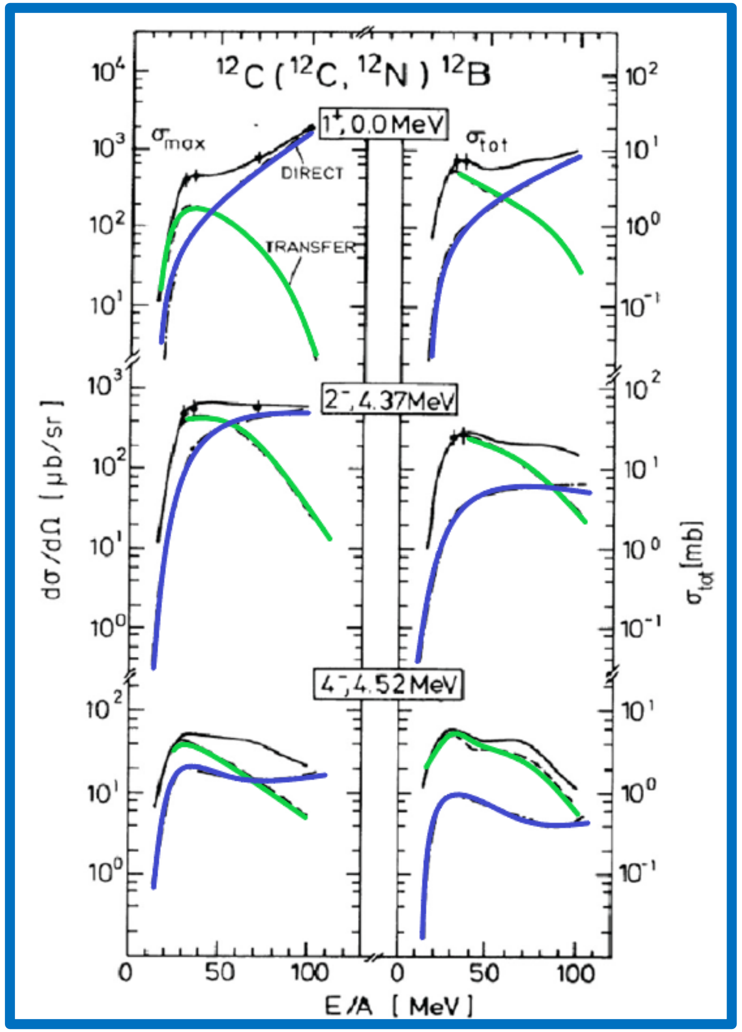
12C+12C→ 12N(1+.g.s.)+12B(Jπ , Ex)

W. von Oertzen, Nuclear Phys. A482 (1988) 357

Reaction Mechanism of a Heavy Ions SCE Reaction

Heavy Ion SCE Reaction Dynamics

- Transfer SCE (TSCE) is a mean-field 2-step process
- Best results if donor and acceptor nuclei are of similar shell-structure
- Symmetric systems like $^{12}C + {}^{12}C$ are most favorable for TSCE
- High angular momentum states favor TSCE
- $A_P \approx A_T$ and low incident energies (Tlab < 30AMeV) favor TSCE

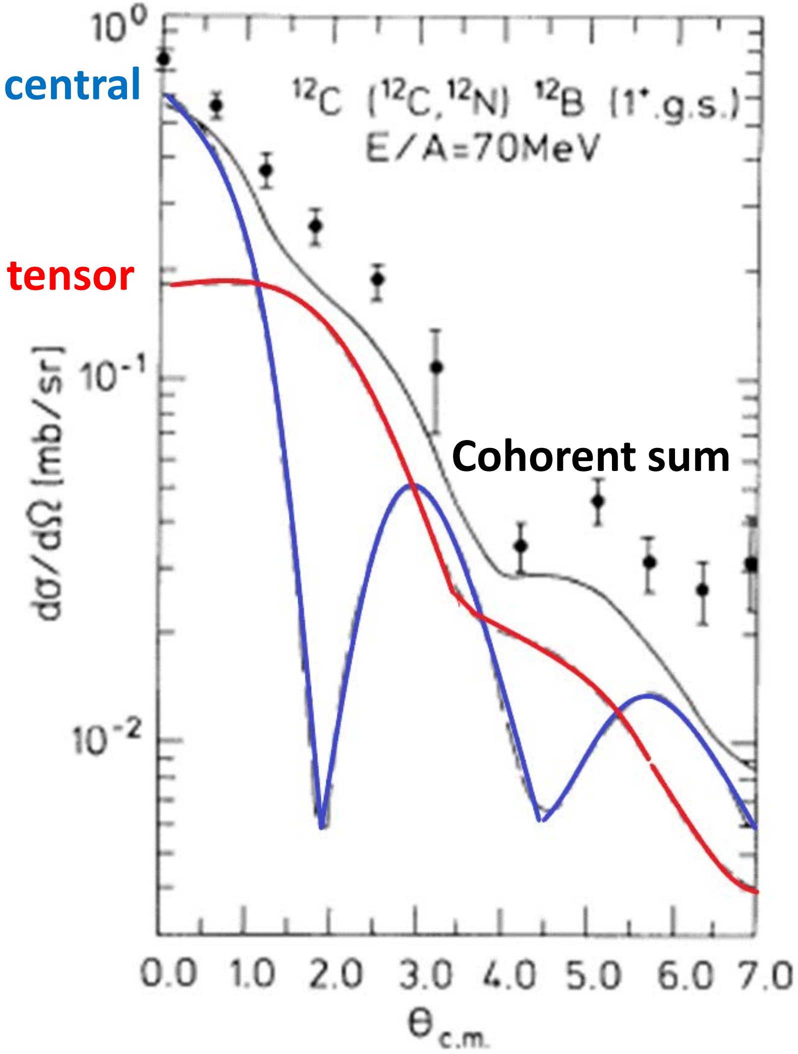


Transfer SCE

Mesonic SCE

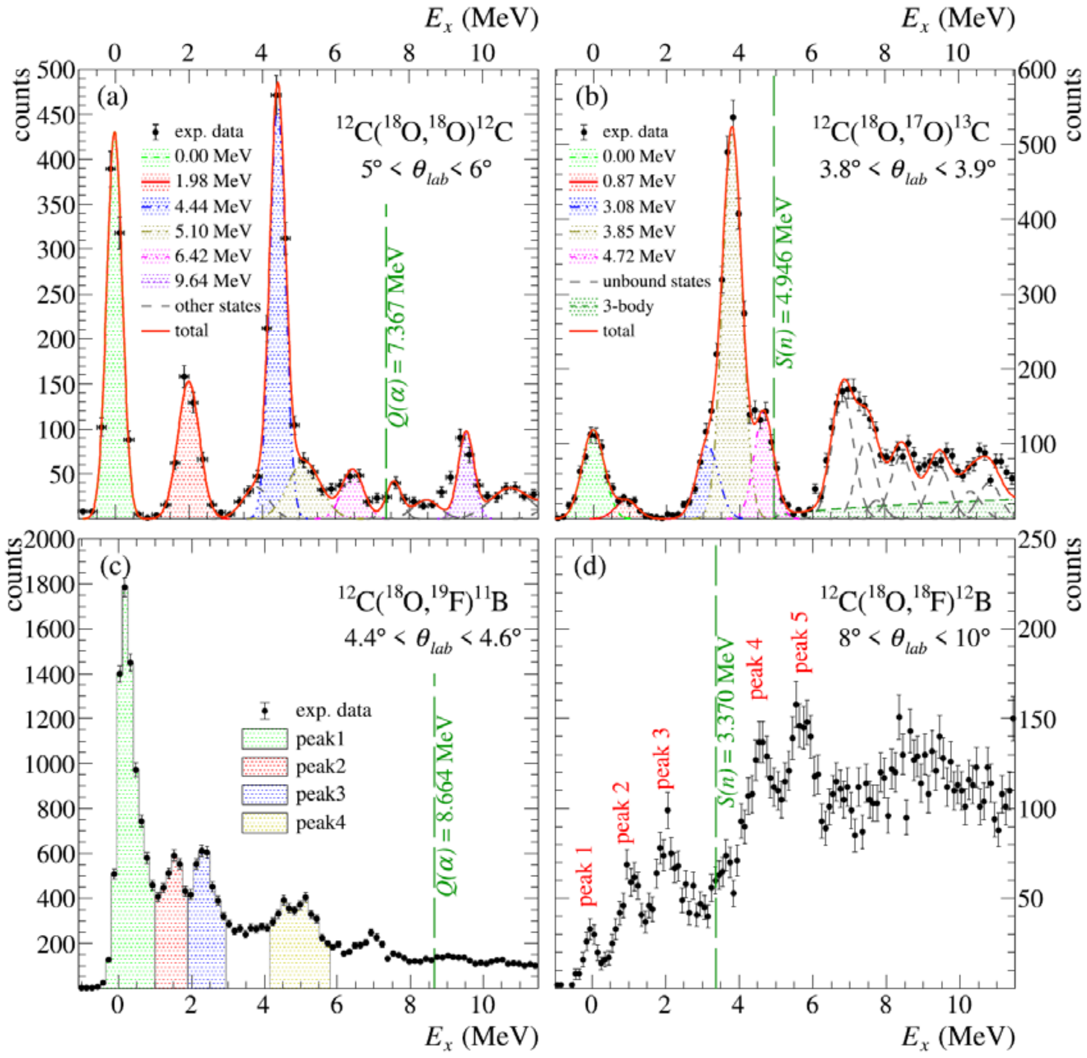
Fig. 3.10. Angular distributions for the reaction $^{12}C + {}^{12}C \rightarrow {}^{12}B(1^+.g.s.) + {}^{12}N(1^+, g.s.)$ (left) and the energy dependence of the peak cross sections for several final states in $^{12}N(J^\pi, E_x)$ and the angle integrated total cross section (right). Theoretical results for the direct (dashed-dotted lines) and transfer (dashed lines) charge-exchange contributions and also of the coherent sum of both (full lines) are shown and compared to data (circles) at E/A = 30 and 70 MeV. Note in the left panel the change in abscissa for E/A = 70 MeV and E/A = 100 MeV.

Heavy Ion SCE Reactions:
Central and Tensor Interaction



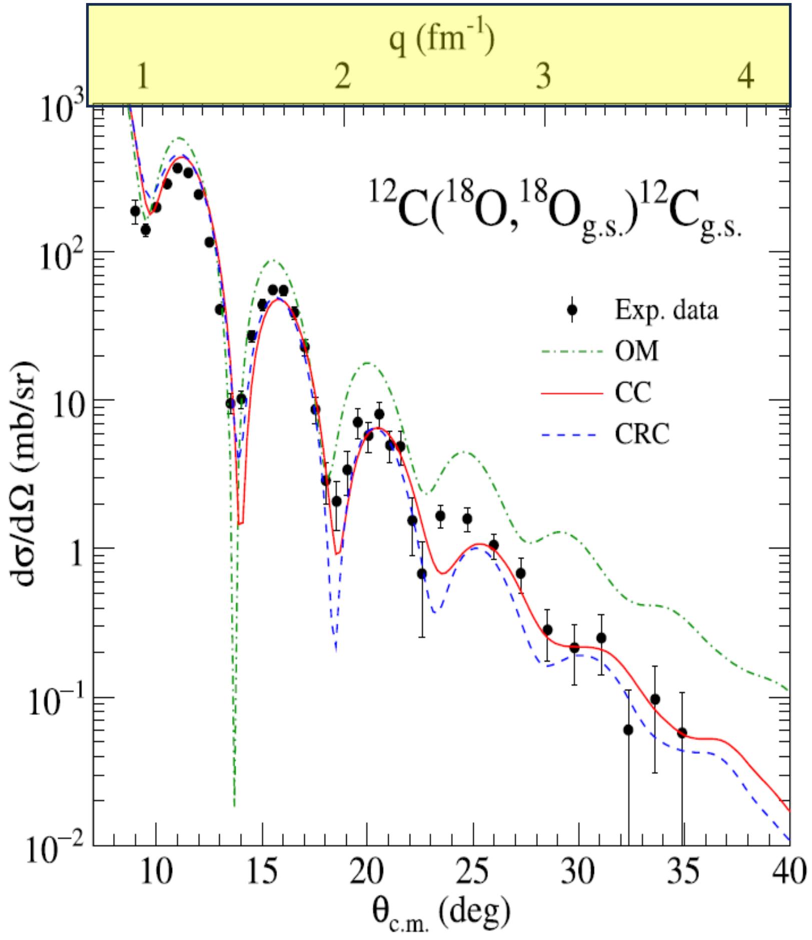
H. Lenske at al.,
Phys. Rev. Lett. 62, 1457 (1989)

Recent Ineastic and Transfer Studies on ^{12}C-Targets at LNS Catania



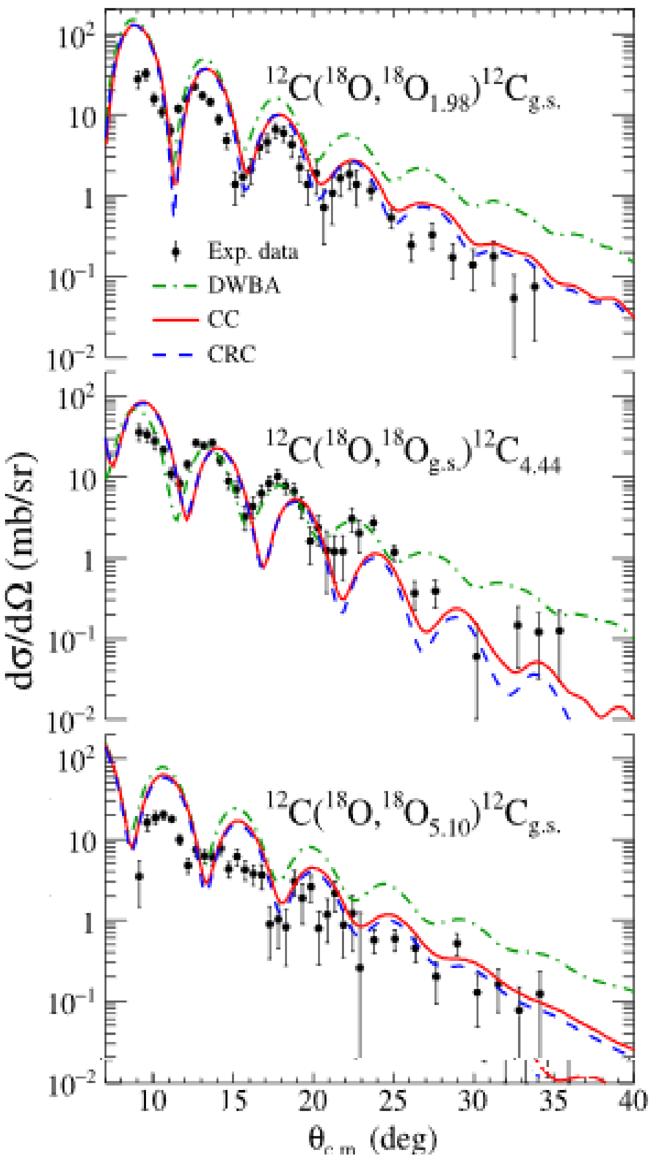
A. SPATAFORA *et al.* PHYS. REV. C 107, 024605 (2023)

Heavy Ion Elastic and Inelastic Scattering Reactions DWBA and CC Description



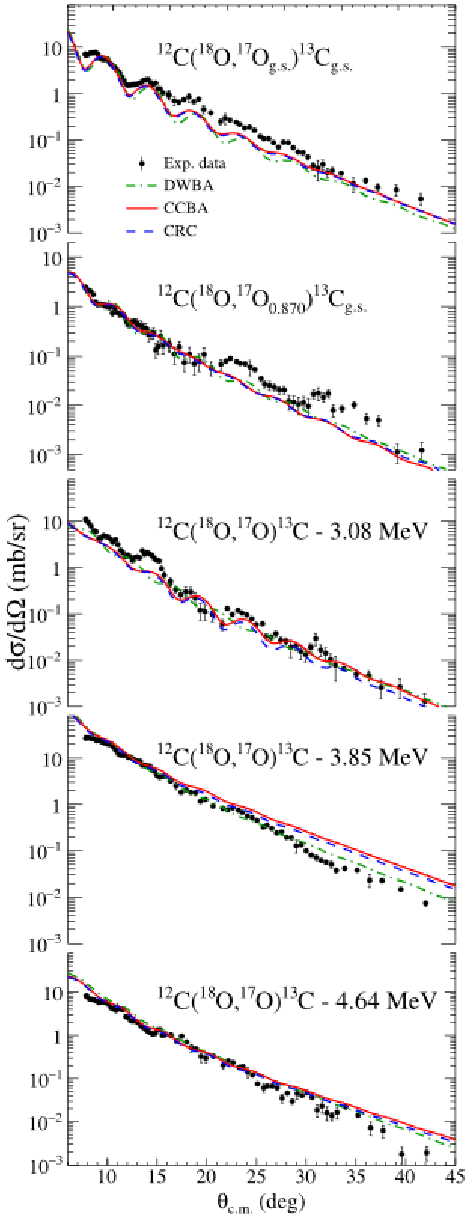
Heavy Ion Scattering
≡
Nuclear Physics at Large Momentum Transfer
(here: q≤ 800 MeV/c (!))

Experimental angular distribution of 12C(18O,18O)12C elastic scattering at 275 MeV incident energy. Results of OM, CC, and CRC approaches are shown



Experimental angular distribution of the 12C(18O, 18O) 12C inelastic scattering at 275 MeV incident energy associated with the peaks at 1.98, 4.44, and 5.10 MeV. Theoretical calculations for the inelastic transitions in DWBA, CC, and CRC approaches are shown

A. SPATAFORA *et al.* PHYS. REV. C 107, 024605 (2023)

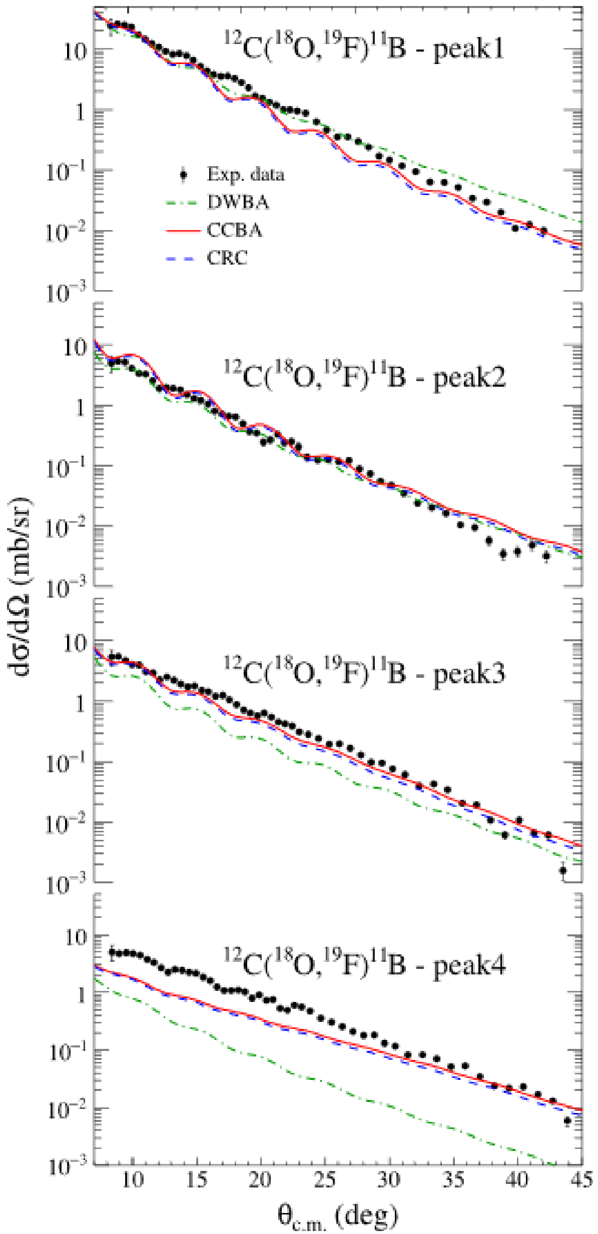


$^{18}O+^{12}C$ @ T_{lab}=275 MeV Single Nucleon Transfer Reactions

Microscopic Description by DWBA and CC Methods

Projectile Neutron Stripping Reactions
$^{18}O \rightarrow ^{17}O$
$\leftrightarrow$
Target Neutron Pickup Reactions
$^{12}C \rightarrow ^{13}C$

Projectile proton Pickup Reactions
$^{18}O \rightarrow ^{19}F$
$\leftrightarrow$
Target Proton Stripping Reaction
$^{12}C \rightarrow ^{11}B$



A. SPATAFORA *et al.* PHYS. REV. C 107, 024605 (2023)

Remarks on Nuclear Response Functions for beta-Decay and Charge Exchange Reactions

$$\boxed{\begin{gathered} E(N,Z)/A = -16\mathrm{MeV} + E_{surf}/A^{1/3} + E_{pair} + E_{shell} + E_{coul} \\ + \left[(N-Z)/A\right]^2 \left(a_4 + C_{sym}/A^{1/3}\right) \end{gathered}}$$

Function(al) of neutron (N) and proton (Z) numbers

→ Generalize to a functional of neutron (q=n) and proton (q=p) densities

$$\boxed{E\left(\tau_q, \rho_q, \kappa_q \ldots\right) = T(\tau_q) + \frac{1}{2} E_{int}\left(\rho_q, \kappa_q \ldots\right)}$$

Nuclear Energy Density Functional (EDF)

Elements of Density Functional Theory

$$E(\rho,\kappa) \approx E(\rho_0,\kappa_0) + \sum_{q=p,n}\Big(\big(T_q + U_q(\rho_0)\big)\delta\rho_q + \Delta_q\delta\kappa_q\Big) + \sum_{q,q'=p,n} f_{qq'}(\rho_0)\delta\rho_q\delta\rho_{q'} + \ldots$$

$$\delta\rho_q \sim \varphi_k^\dagger\varphi_n \sim a_k^\dagger\ a_k\ ;\ \ \delta\kappa_q \sim \varphi_k^\dagger\varphi_n^\dagger \sim a_k^\dagger\ a_k^\dagger\ \ \&\ \ \mathrm{h.c.}$$

Single Particle Self-Energy:

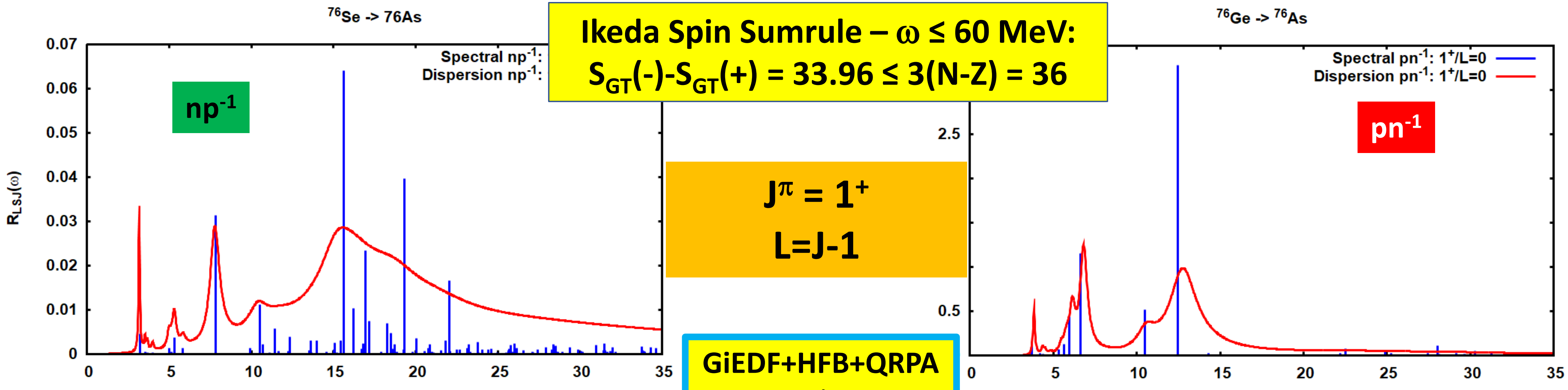
$$U_q = \frac{\delta}{\delta\rho_q}\frac{1}{2}\langle V\rangle = \sum_{q'} V_{qq'}(\rho)\rho_{q'} + \frac{1}{2}\sum_{q'q''}\rho_{q'}\rho_{q''}\frac{\delta}{\delta\rho_q}V_{q'q''}(\rho)$$

Self-consistent Residual Interaction :

$$f_{qq'} = V_{qq'}(\rho) + 2\sum_{q''}\rho_{q''}\frac{\delta}{\delta\rho_q}V_{q'q''}(\rho) + \frac{1}{2}\sum_{k'k''}\rho_{k'}\rho_{k''}\frac{\delta^2}{\delta\rho_q\delta\rho_{q'}}V_{k'k''}(\rho)$$

H.L., N. Tsoneva, Eur.Phys.J.A 57 (2021) 3, 89

Ikeda Spin Sumrule – $\omega \le 60$ MeV: $S_{GT}(-)-S_{GT}(+) = 33.96 \le 3(N-Z) = 36$



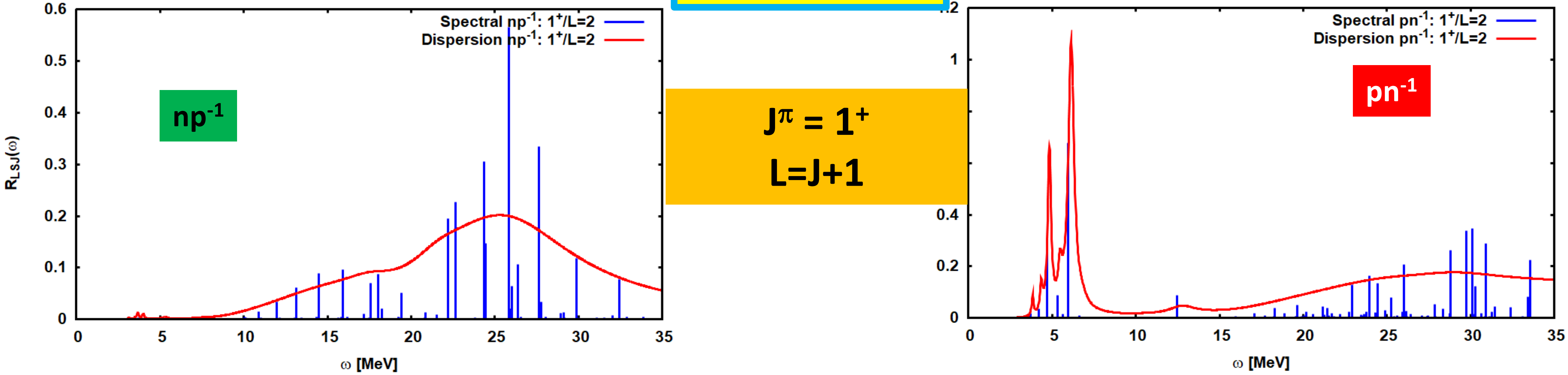
^{76}Se → ^{76}As

$J^\pi = 1^+$
L=J-1

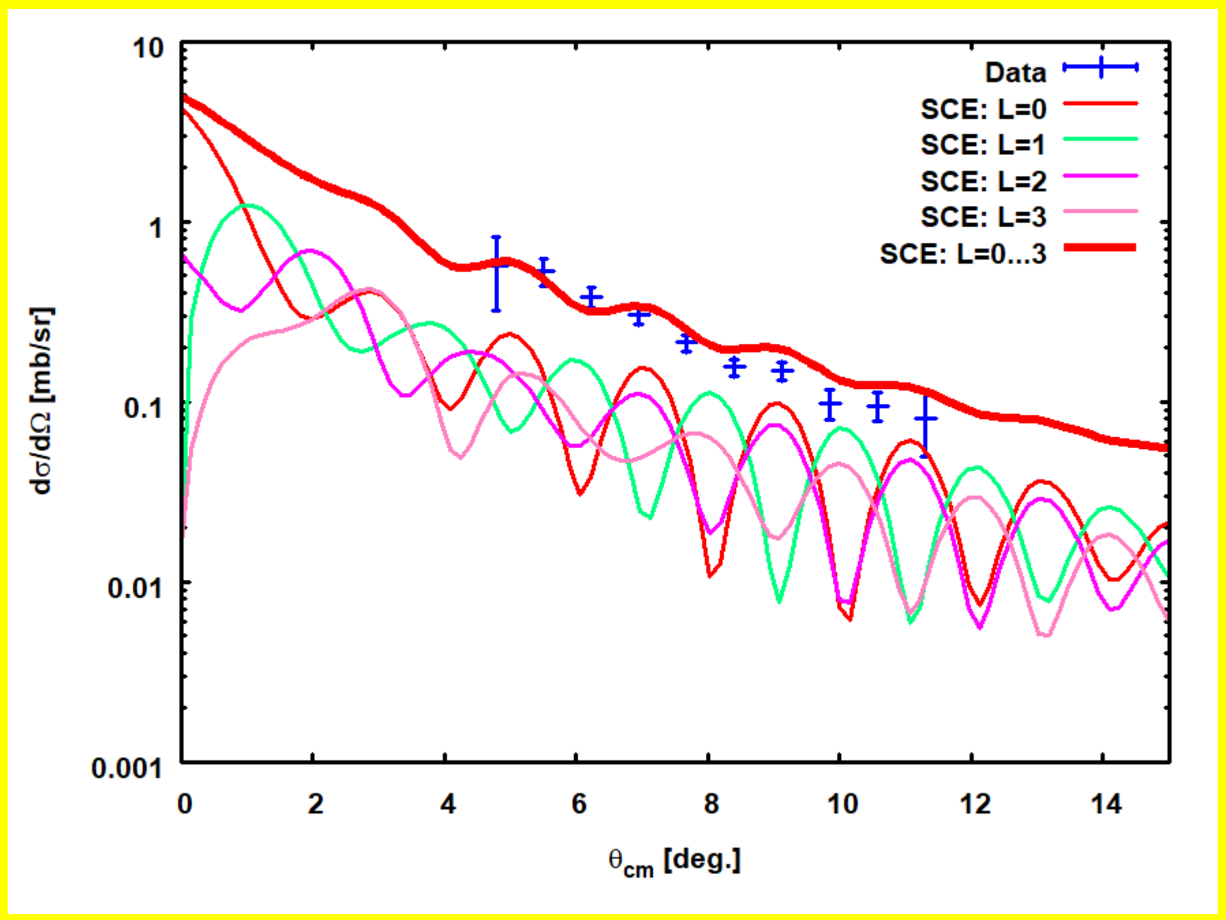
GiEDF+HFB+QRPA
e.g. *Eur.Phys.J.A* 57 (2021) 3, 89

^{76}Ge → ^{76}As

$J^\pi = 1^+$
L=J+1



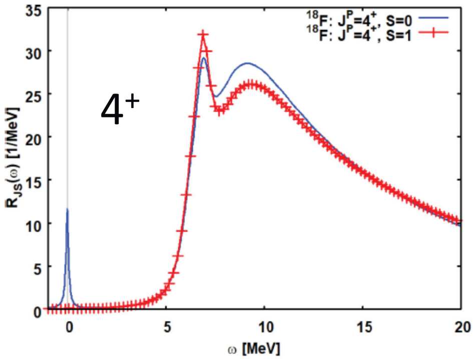
$^{18}O+^{40}Ca \rightarrow ^{18}F+^{40}K$ @ T_{Lab}=15AMeV

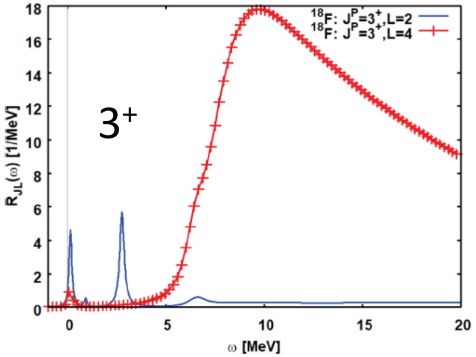


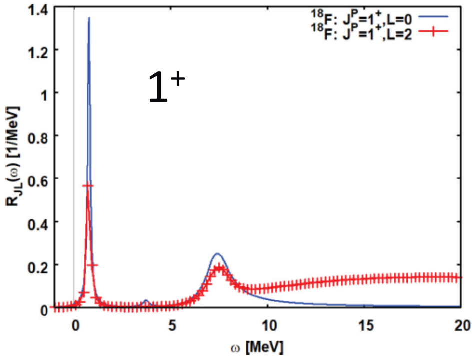
F. Cappuzzello et al., PPNP (108) (2023)

SCE Spectroscopy: ccQRPA Response Functions

$^{18}O \rightarrow {}^{18}F$







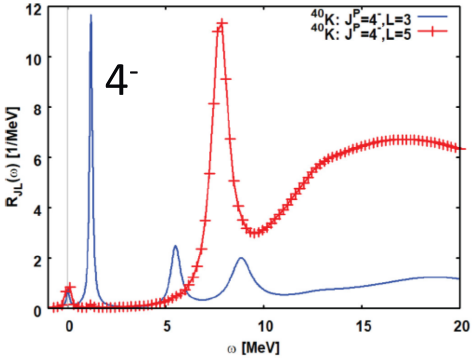
PHYSICAL REVIEW C **98**, 044620 (2018)

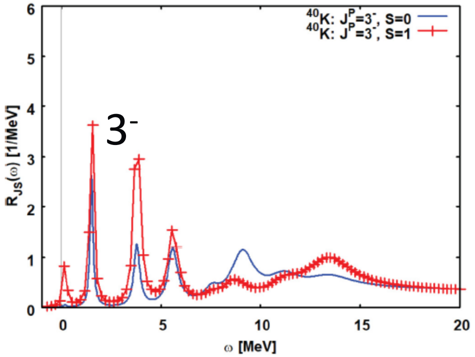
Fermi (S=0) and Gamow-Teller (S=1) Transition Operators:

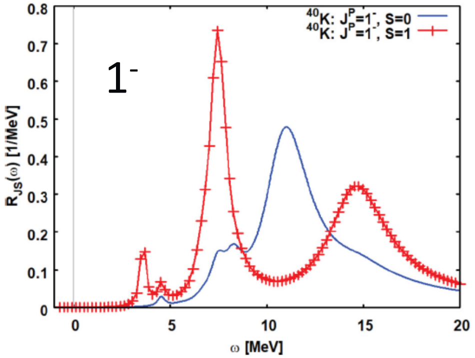
$$T_{\mathrm{LSJM}} = \left(\frac{r}{R_d}\right)^L [\sigma^S \otimes Y_L]_{JM} \tau_\pm$$

…including DCP self-energies and continuum effects

$^{40}Ca \rightarrow {}^{40}K$

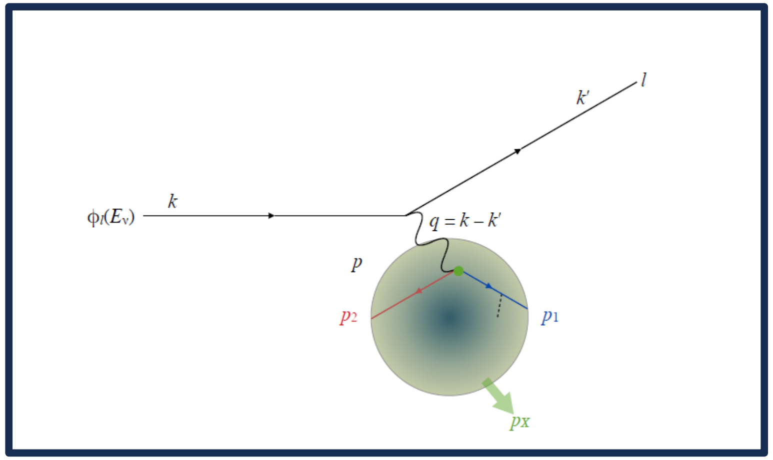






Lecture 11:
Heavy Ion SCE Reactions at Relativistic Energies

Motivation 1: Neutrino Oscillation and Neutrino-Matter Interactions

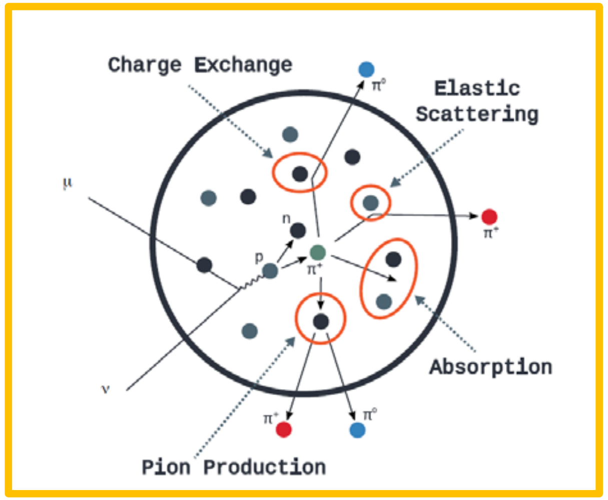


NuSTEC[a] White Paper: Status and Challenges of Neutrino-Nucleus Scattering

L. Alvarez-Ruso,[1] M. Sajjad Athar,[2] M. B. Barbaro,[3] D. Cherdack,[4] M. E. Christy,[5] P. Coloma,[6] T. W. Donnelly,[7] S. Dytman,[8] A. de Gouvêa,[9] R. J. Hill,[10,6] P. Huber,[11] N. Jachowicz,[12] T. Katori,[13] A. S. Kronfeld,[6] K. Mahn,[14] M. Martini,[15] J. G. Morfín,[6] J. Nieves,[1] G. Perdue,[6] R. Petti,[16] D. G. Richards,[17] F. Sánchez,[18] T. Sato,[19,20] J. T. Sobczyk,[21] and G. P. Zeller[6]

NuSTEC = Neutrino Scattering Theory Experiment Collaboration

(PPNP 106 (2018) 1-68)



F. Challenges: The Resonance Region (Section VII)

The resonance region is characterized by transfers of energy larger than in QE peak region corresponding to larger hadronic invariant mass. The most important contribution is from the $\Delta(1232)$ resonance:

$$\nu_\mu p \to \mu^- \Delta^{++}, \quad \Delta^{++} \to p\pi^+$$

and

$$\bar{\nu}_\mu n \to \mu^+ \Delta^-, \quad \Delta^- \to n\pi^-,$$

Motivation 2: QCD Aspects of Resonances

- hadronic (*soft scale*) *molecular-type* components $|N_s>$
- QCD (*hard scale*) confined components $|N_h>$
- N* in compact stellar objects → neutron stars

$$\boxed{\left|N^*\right\rangle = \left|N_s^*\right\rangle + \left|N_h^*\right\rangle = x_1\left|mB\right\rangle + x_2\left|qqq\right\rangle + x_3\left|qqq\right\rangle \otimes \left|q\bar{q}\right\rangle + \ldots}$$

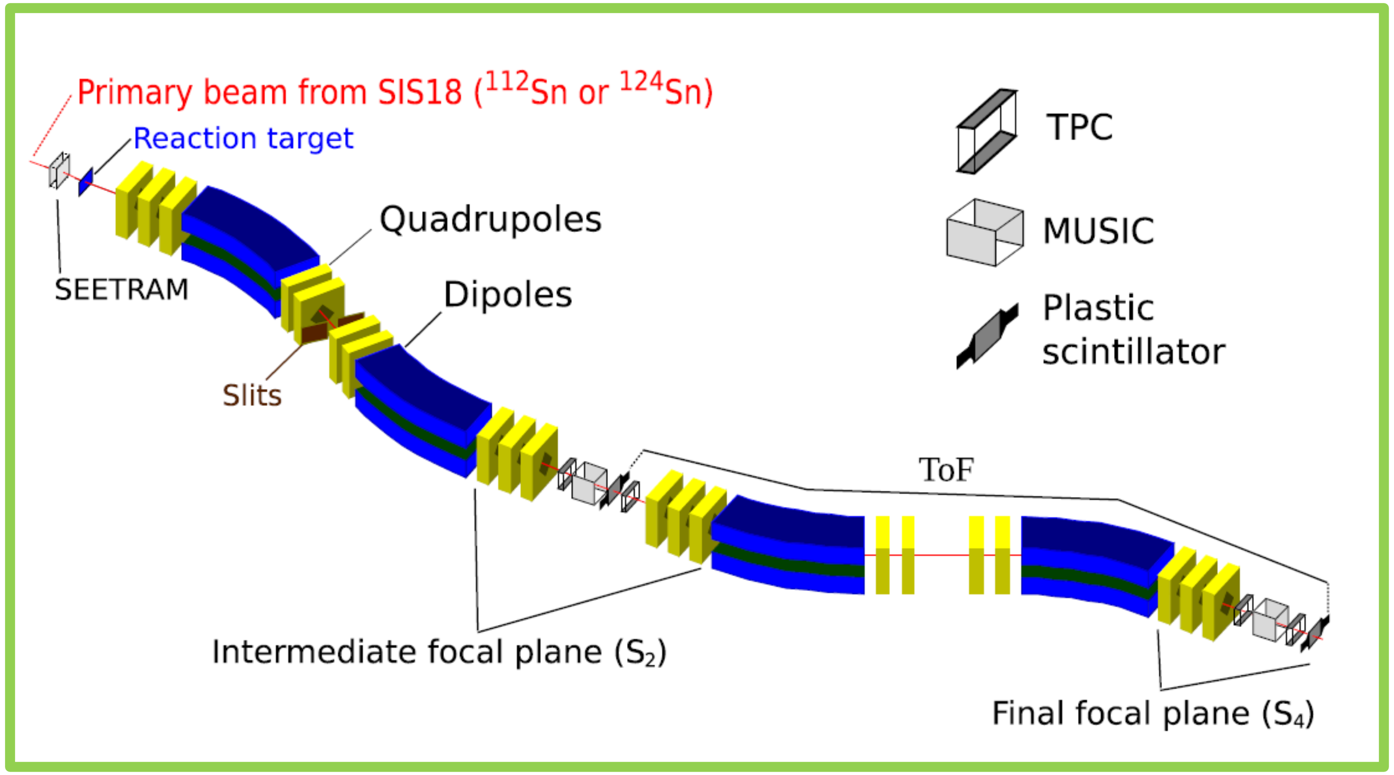
$$|N_s^*\rangle = |MB\rangle$$

$$|N_h^*\rangle = |qqq\rangle + |\mathrm{m.c.}\rangle$$

Strong Medium Dependence

Weak Medium Dependence

Relativistic Heavy Ion Reactions at the FRS@GSI



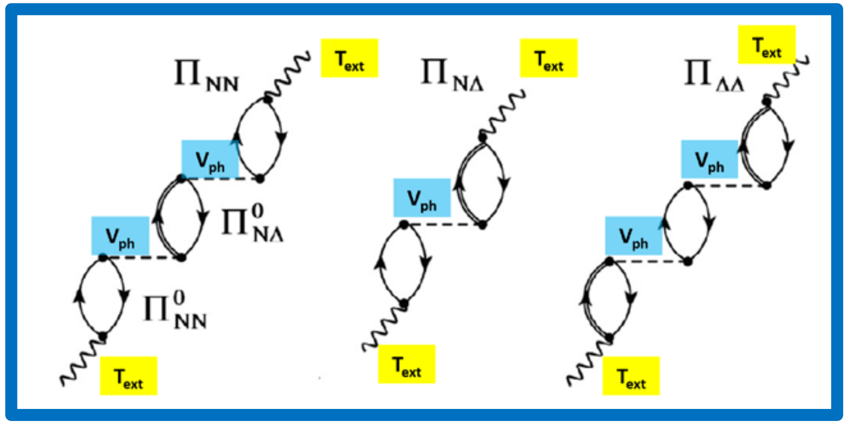
H.L. et al, PPNP 98 (2018)
J. L. Rodríguez-Sánchez, H.L., et al., PHYS. REV. C 106, 014618 (2022)

Substitutional N^*N^{-1} Excitations in Nuclei - „N*RPA"

$$\Pi = \Pi^0 + \Pi^0 \hat{V} \Pi$$

$$\begin{pmatrix} \Pi_{NN} & \Pi_{N\Delta} \\ \Pi_{\Delta N} & \Pi_{\Delta\Delta} \end{pmatrix} = \begin{pmatrix} \Pi^0_{NN} & 0 \\ 0 & \Pi^0_{\Delta\Delta} \end{pmatrix} + \begin{pmatrix} \Pi^0_{NN} & 0 \\ 0 & \Pi^0_{\Delta\Delta} \end{pmatrix} \begin{pmatrix} V_{NN} & V_{N\Delta} \\ V_{\Delta N} & V_{\Delta\Delta} \end{pmatrix} \begin{pmatrix} \Pi_{NN} & \Pi_{N\Delta} \\ \Pi_{\Delta N} & \Pi_{\Delta\Delta} \end{pmatrix}$$

Coupled $N'N^{-1} \leftrightarrow \Delta N^{-1}$ Dyson Equation including N and N* self-energies



V_{ph} by pion, rho, delta/a_0 - meson exchange and „short range" g'

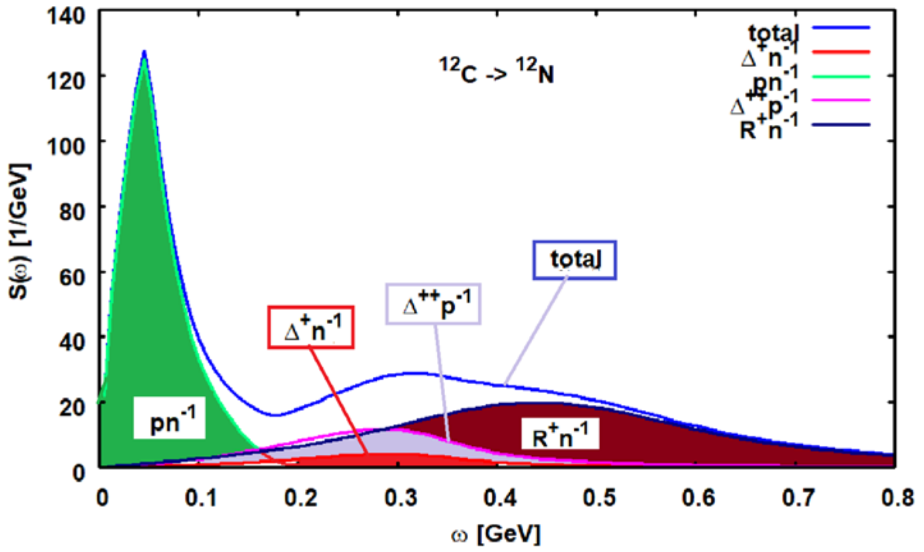
Polarization Tensor and Nuclear Response (H=N,Δ…)

$$R_H^{(X)\mu\nu}(\omega,\vec{q}) = -\frac{1}{\pi}\mathrm{Im}\left(\Pi_{HH}^{(X)\mu\nu}(\omega,\vec{q})\right) = -\frac{1}{\pi}\mathrm{Im}\left(\left\langle X \left| T_{ext}^{\dagger\mu}(\vec{q}) G_{HH}(\omega) T_{ext}^{\nu}(\vec{q}) \right| X \right\rangle\right)$$

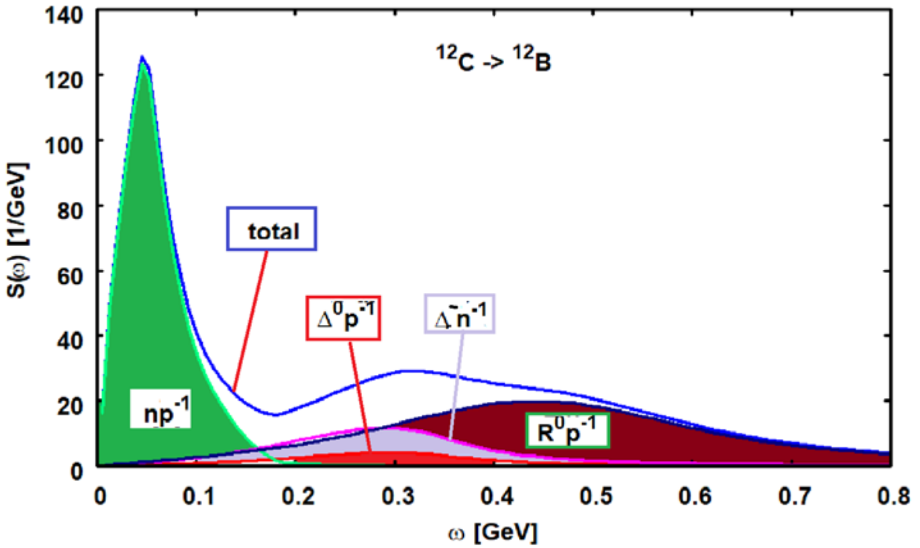
N^*N^{-1} Spectral Distributions for ^{12}C – $P_{33}(1232)$ and $P_{11}(1440)$

Longitudinal Response@q=300 MeV/c

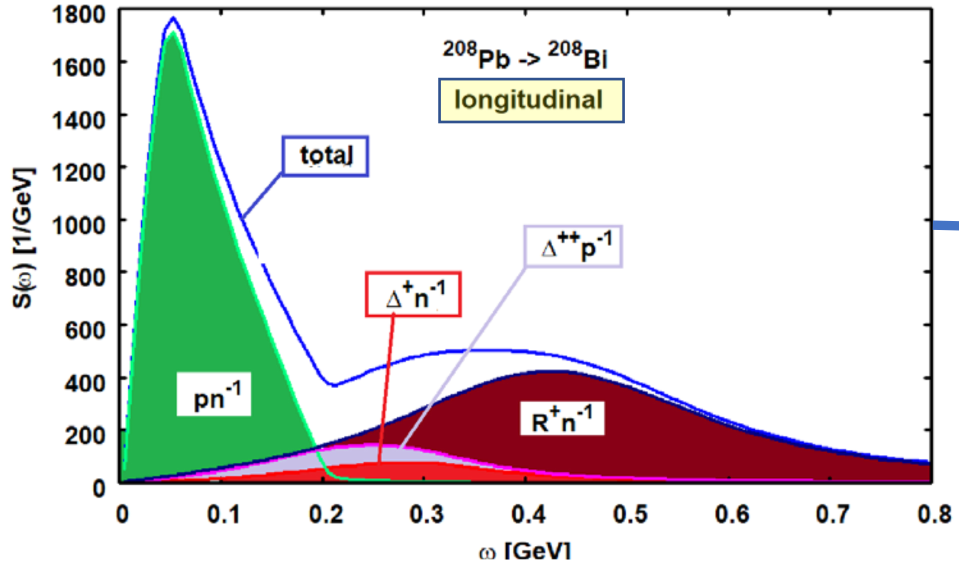
τ_+ operator : pn^{-1} – type
external probe: $\vec{\sigma} \bullet \vec{p}$

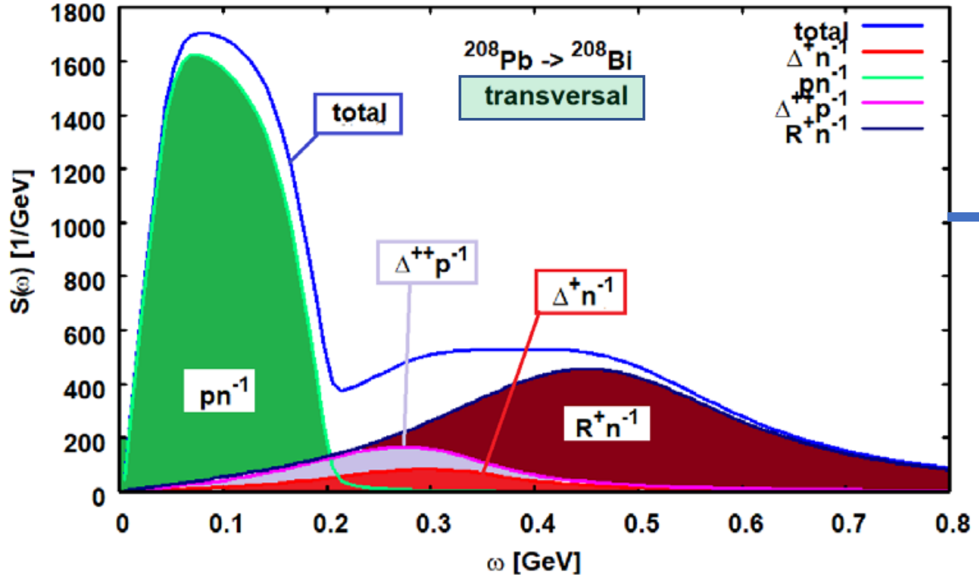


τ_- modes : np^{-1} – type
external probe: $\vec{\sigma} \bullet \vec{p}$



N^*N^{-1} Spectral Distributions for $^{208}Pb \rightarrow {}^{208}Bi$
Longitudinal and Transversal Response@q=300 MeV/c

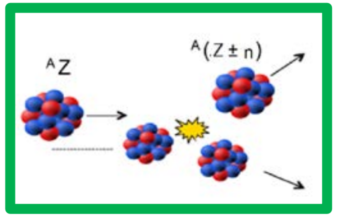




Operator:

$\sigma \bullet q$ (π-meson)

$\sigma \times q$ (ρ-meson)



Comparison to the FRS@GSI Data

Theoretical Results for ^{112}Sn on Pb, Cu, ^{12}C, and Proton Targets

^{112}Sn(0+)→ ^{112}Sb(3+)

S(p)= 2.948 MeV

States E_x<S(p):

$1^+ \leq J^\pi \leq 12^-$

$[p_P n_P^{-1}]$

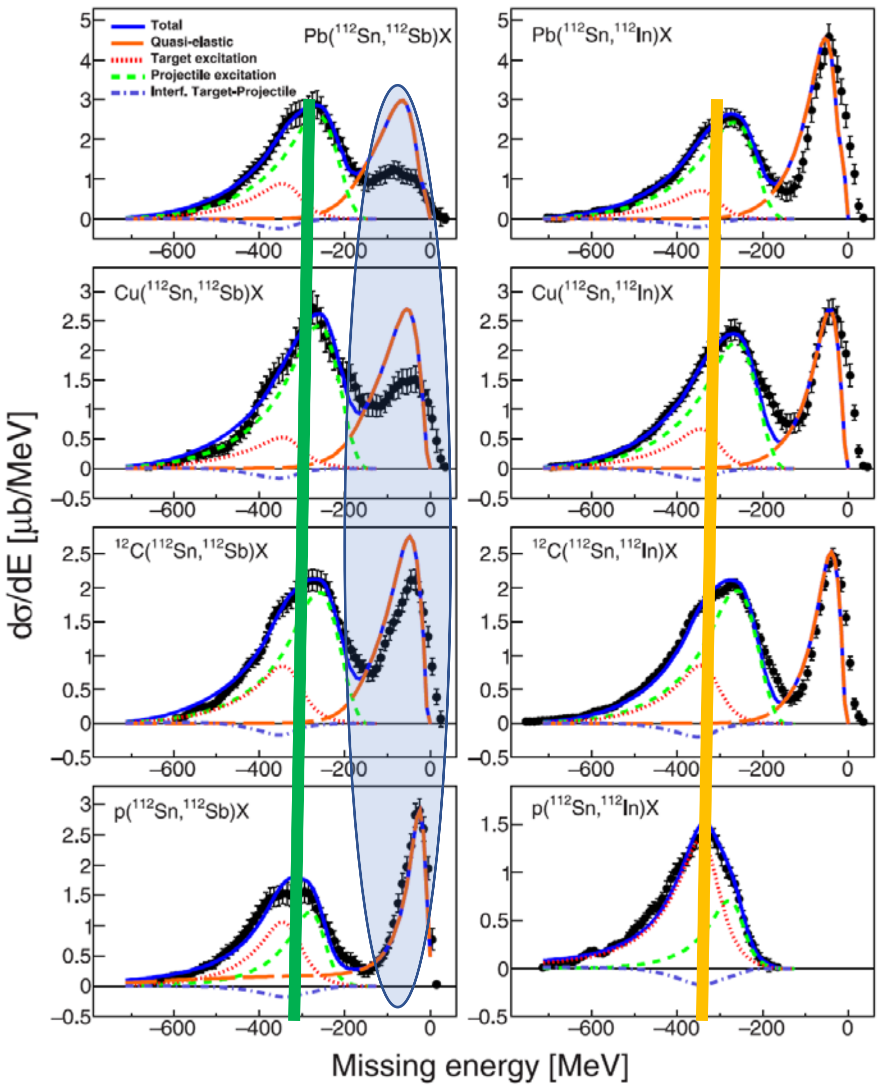
$Z_P \rightarrow Z_P+1$

↔

$Z_T \rightarrow Z_T-1$

$[n_T p_T^{-1}]$

Rodriguez et al., PHYSICAL REVIEW C 106, 014618 (2022)



^{112}Sn(0+)→ ^{112}In(1+)

S(p)= 6.027 MeV

States E_x<S(p):

$0^+ \leq J^\pi \leq 18^\pm$

$[n_P p_P^{-1}]$

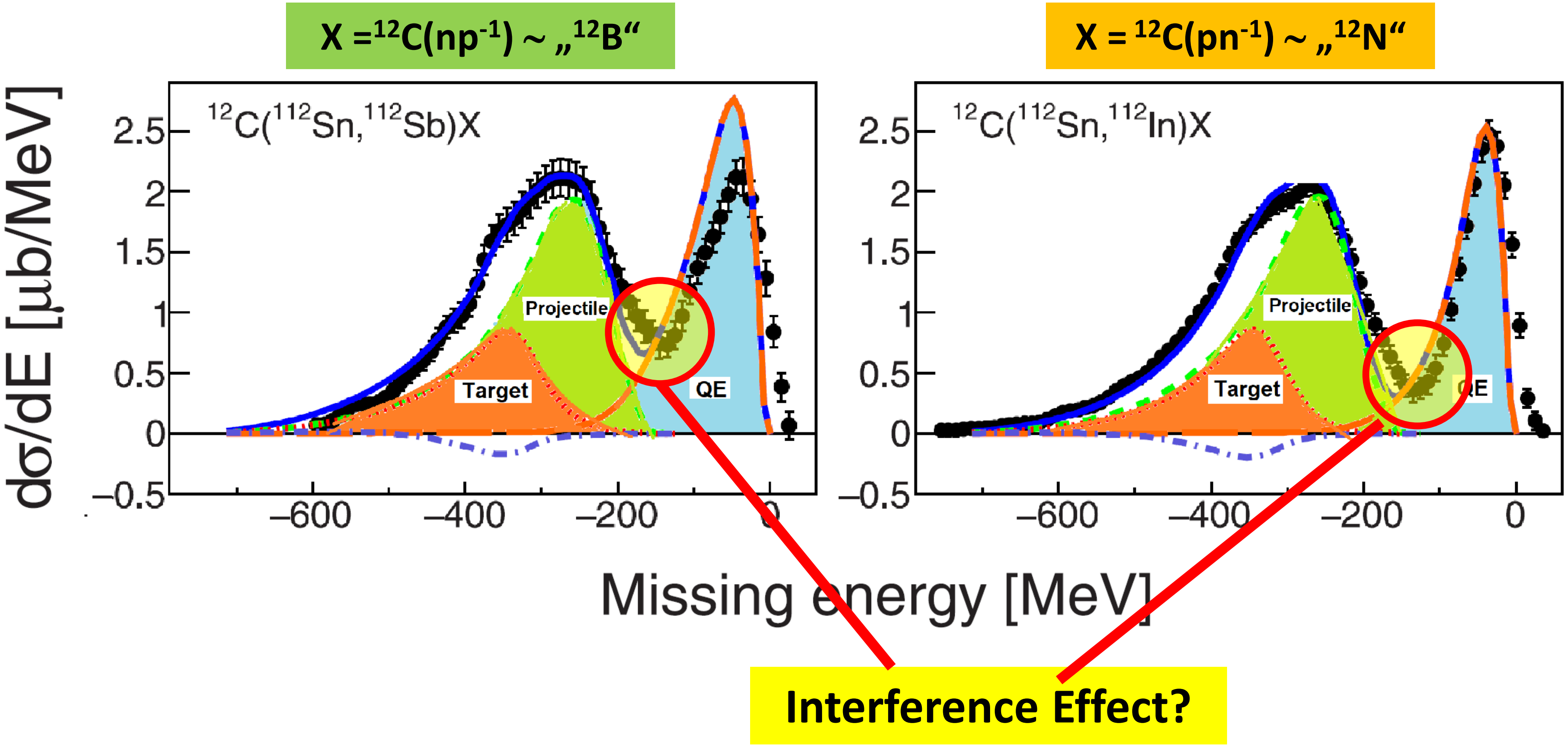
$Z_P \rightarrow Z_P-1$

↔

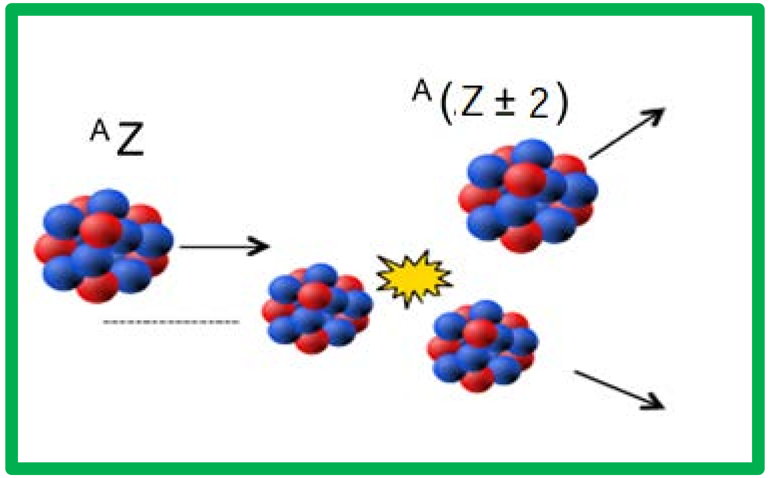
$Z_T \rightarrow Z_T+1$

$[p_T n_T^{-1}]$

^{112}Sn@1AGeV on ^{12}C
Quasi-elastic, Projectile, and Target Excitations



Lecture 12:
Double Charge Exchange Reactions




Physics Letters B 807 (2020) 135528

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Two-step description of heavy ion double charge exchange reactions

Jessica I. Bellone [a], Stefano Burrello [a], Maria Colonna [a,*], José-Antonio Lay [b,c], Horst Lenske [d]

NUMEN Collaboration


universe *Universe* 7 (2021) 4, 98

Article

Nuclear Matrix Elements for Heavy Ion Sequential Double Charge Exchange Reactions

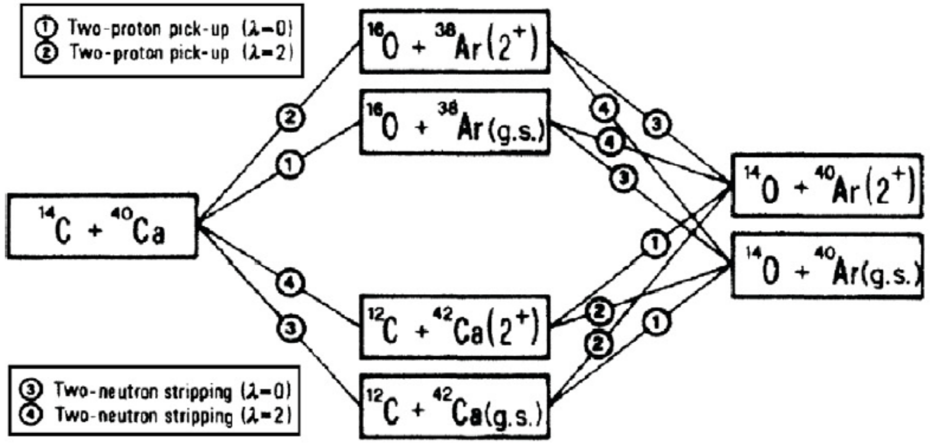
Horst Lenske [1,*,†], Jessica Bellone [2,†], Maria Colonna [2,†] and Danilo Gambacurta [2,†]

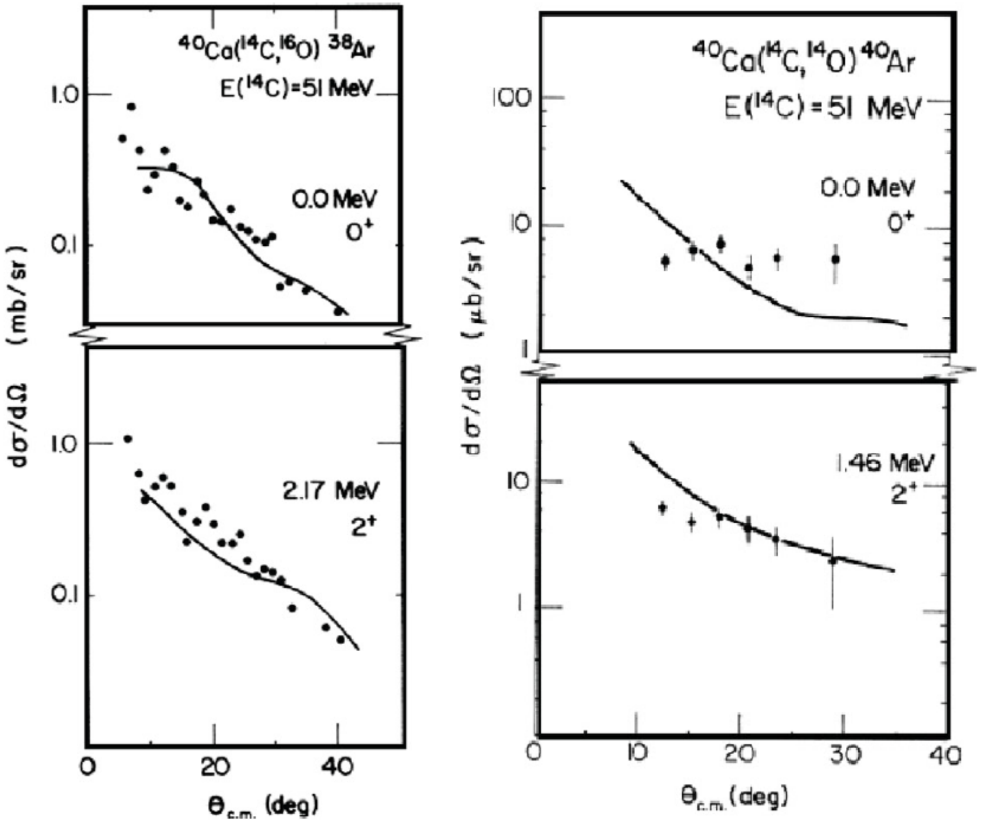
Status of the GERDA experiment for the $0\nu 2\beta$ decay of ^{76}Ge: (2011-2019 campaign)

$$T_{1/2}^{0\nu\beta\beta} > 1.8 \cdot 10^{26}\ \text{yr at } 90\%\ \text{C.I}$$

…bad news for theory: no experimental check visible for DBD-NME

Early Double Charge Exchange (DCE) Reaction Studies at Barrier Energies
Sequential Two-Particle Transfer

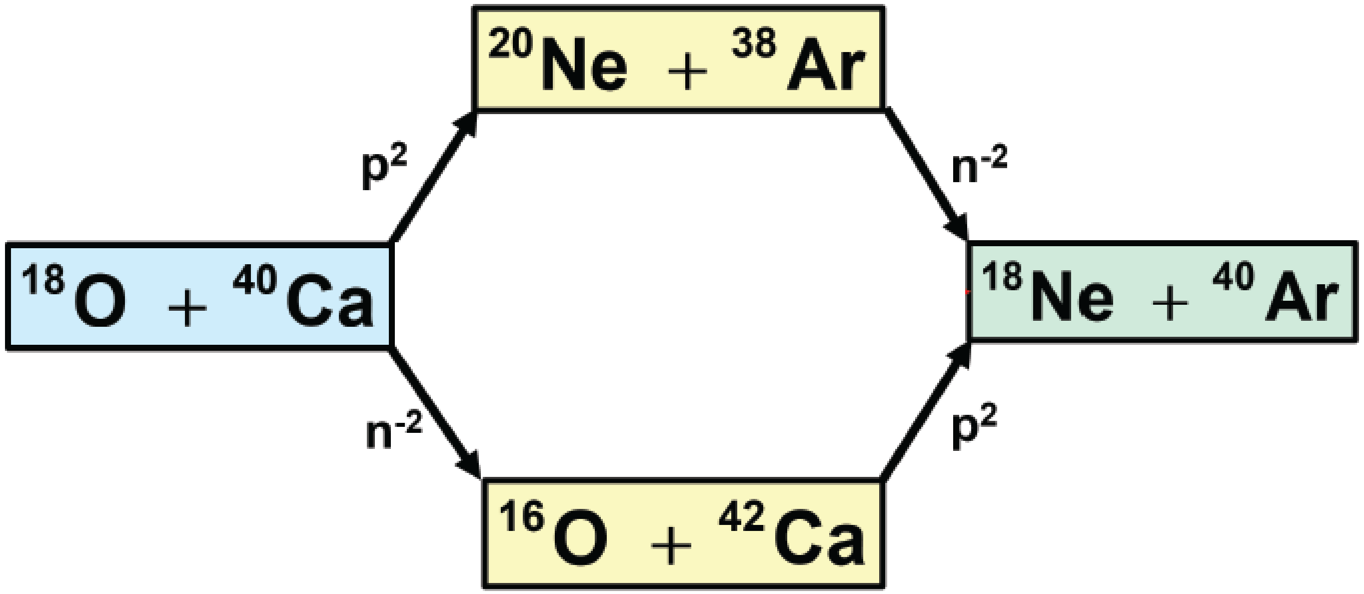




The proton and neutron pair transfer scheme used in the Dasso–Pollarolo–Vitturi approach to the DCE reaction 40Ca(14C,14O)40Ar.

Heavy Ion DCE Reaction Mechanism

New Mechanism: Mesonic DCE!



Transfer DCE (TDCE):

- **Dasso-Pollarolo-Vitturi (1980+): sequential Collective Pair Transfer,** PRC 34:743 (1986)
- **Microscopic TDCE: D. Carbone, H.L., et al. ,** DOI: 10.1103/PhysRevC.102.044606

Mulit-Methods Reaction Network for DCE Reactions
$^{76}Se(^{20}Ne,^{20}O)^{76}Ge$

F. Cappuzzello, H. Lenske, M. Cavallaro et al. — Progress in Particle and Nuclear Physics xxx (xxxx) xxx

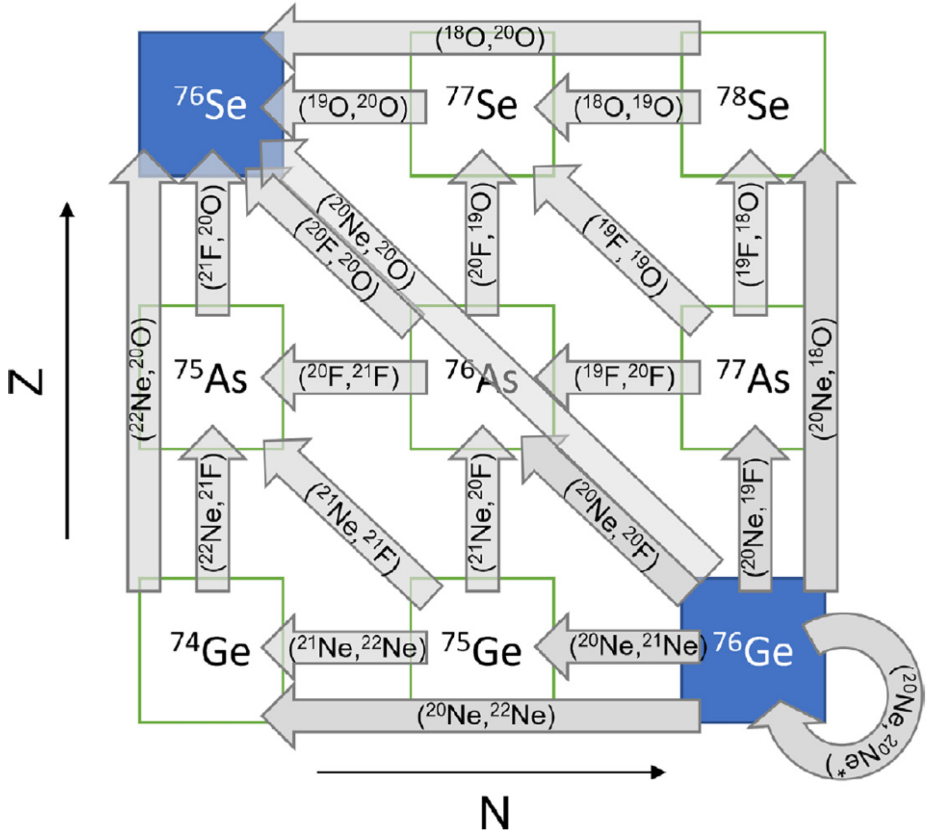
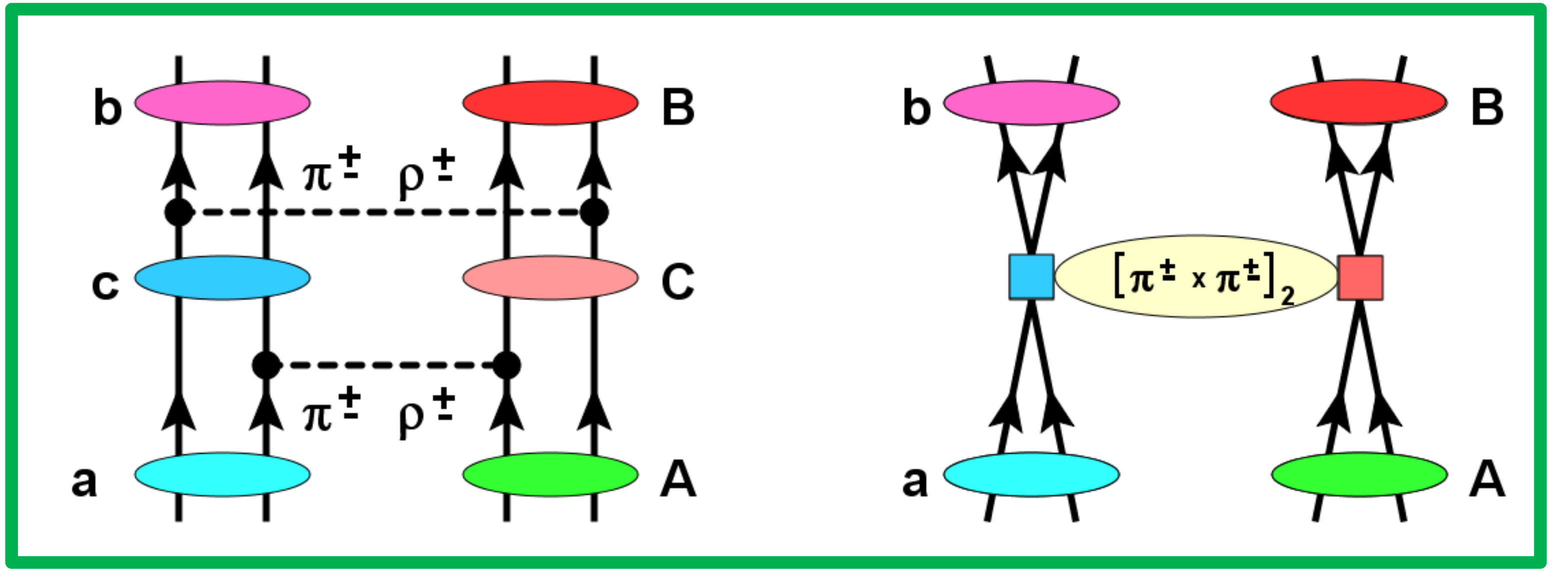


Fig. 2.1. Network of possible reaction routes connecting initial and final states in the $^{76}Ge(^{20}Ne,\ ^{20}O)^{76}Se$ DCE reaction. The scheme is limited to the 4th order in the single nucleon transfer process. The projectile–ejectile pairs are indicated inside the arrows.

Mesonic Nuclear Double Charge Exchange (DCE) Reactions

Sequential „two-step“ SCE | **Correlated Majorana DCE**



DSCE

→ $2\nu2\beta$

MDCE

→ $0\nu2\beta$

Physics Letters B 807 (2020) 135528

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Two-step description of heavy ion double charge exchange reactions

Jessica I. Bellone[a], Stefano Burrello[a], Maria Colonna[a,*], José-Antonio Lay[b,c], Horst Lenske[d]

NUMEN Collaboration

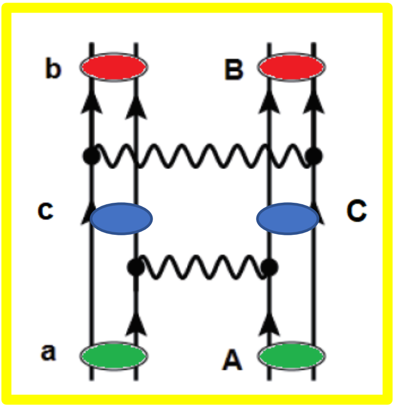
universe — *Universe* 7 (2021) 4, 98

Article

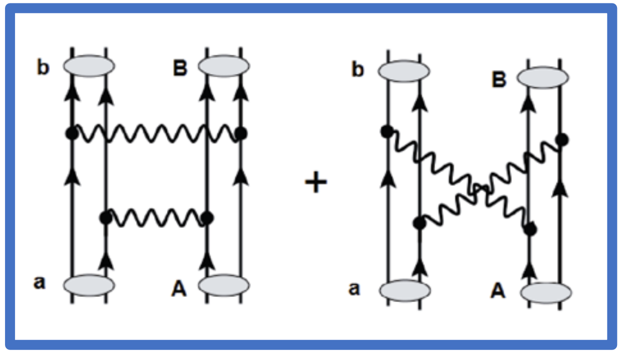
Nuclear Matrix Elements for Heavy Ion Sequential Double Charge Exchange Reactions

Horst Lenske[1,*,†], Jessica Bellone[2,†], Maria Colonna[2,†] and Danilo Gambacurta[2,†]

The DSCE Reaction Mechanism



The DSCE Reaction Amplitude



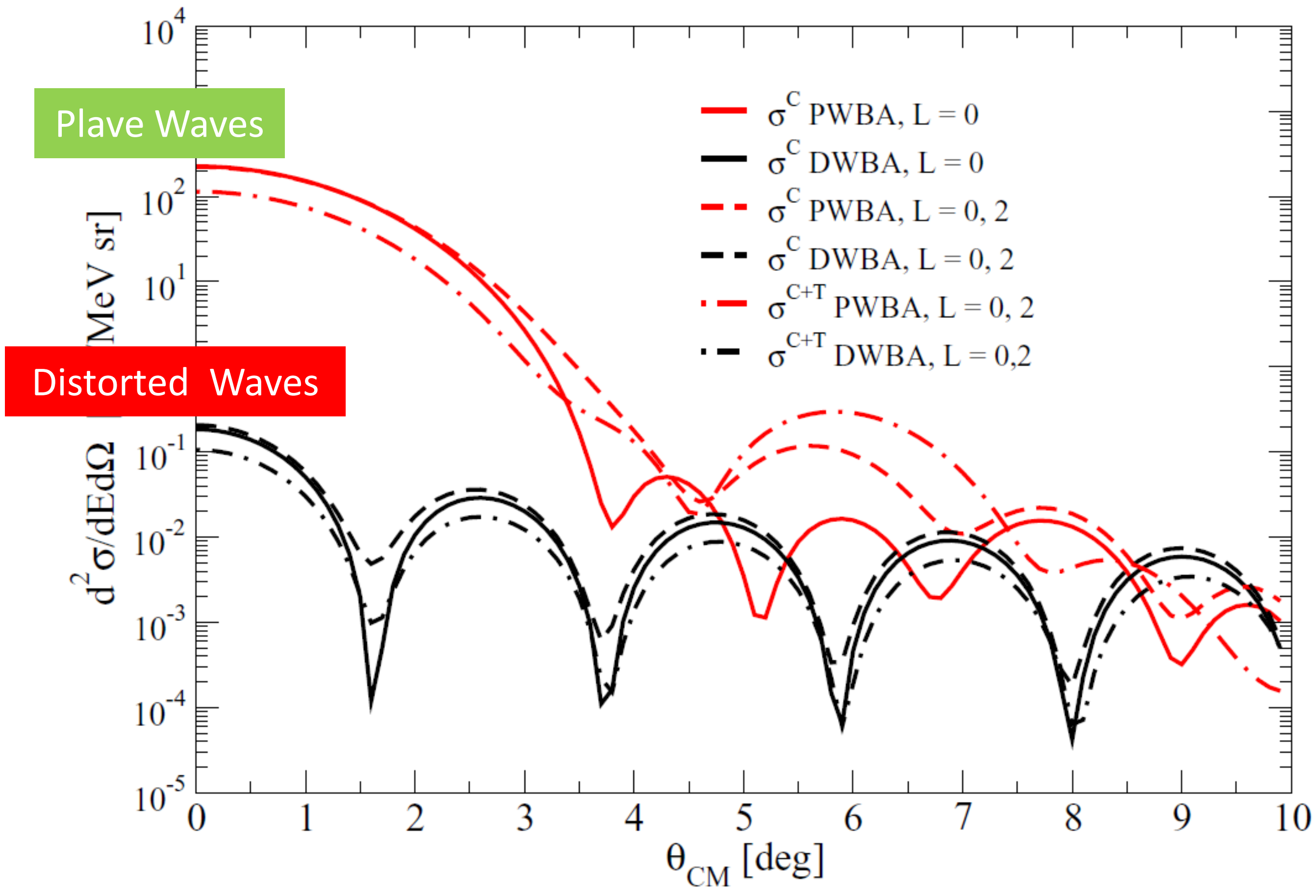
$$\mathcal{M}^{(DSCE)}_{\alpha\beta}(\mathbf{k}_\alpha,\mathbf{k}_\beta) = \langle \chi^{(-)}_\beta, bB|\mathcal{T}_{NN}\mathcal{G}^{(+)}_{aA}(\omega_\alpha)\mathcal{T}_{NN}|aA,\chi^{(+)}_\alpha\rangle.$$

2nd order DSCE ME $\leftrightarrow$ Folding of two (half off-shell) 1st order SCE amplitudes

$$\mathcal{M}^{(2)}_{\alpha\beta}(\mathbf{k}_\alpha,\mathbf{k}_\beta) = \sum_{\gamma=\{c,C\}}\int\frac{d^3k_\gamma}{(2\pi)^3}M^{(1)}_{\gamma\beta}(\mathbf{k}_\gamma,\mathbf{k}_\beta)\frac{\tilde{S}^\dagger_\gamma}{\omega_\alpha - E_c - E_C - T_\gamma + i\eta}M^{(1)}_{\alpha\gamma}(\mathbf{k}_\alpha,\mathbf{k}_\gamma)$$

J. Bellone, M. Colonna, J.-A. Lay, H.L., PLB 807 (2020); H.L. et al. *Universe* 7 (2021) 4, 98

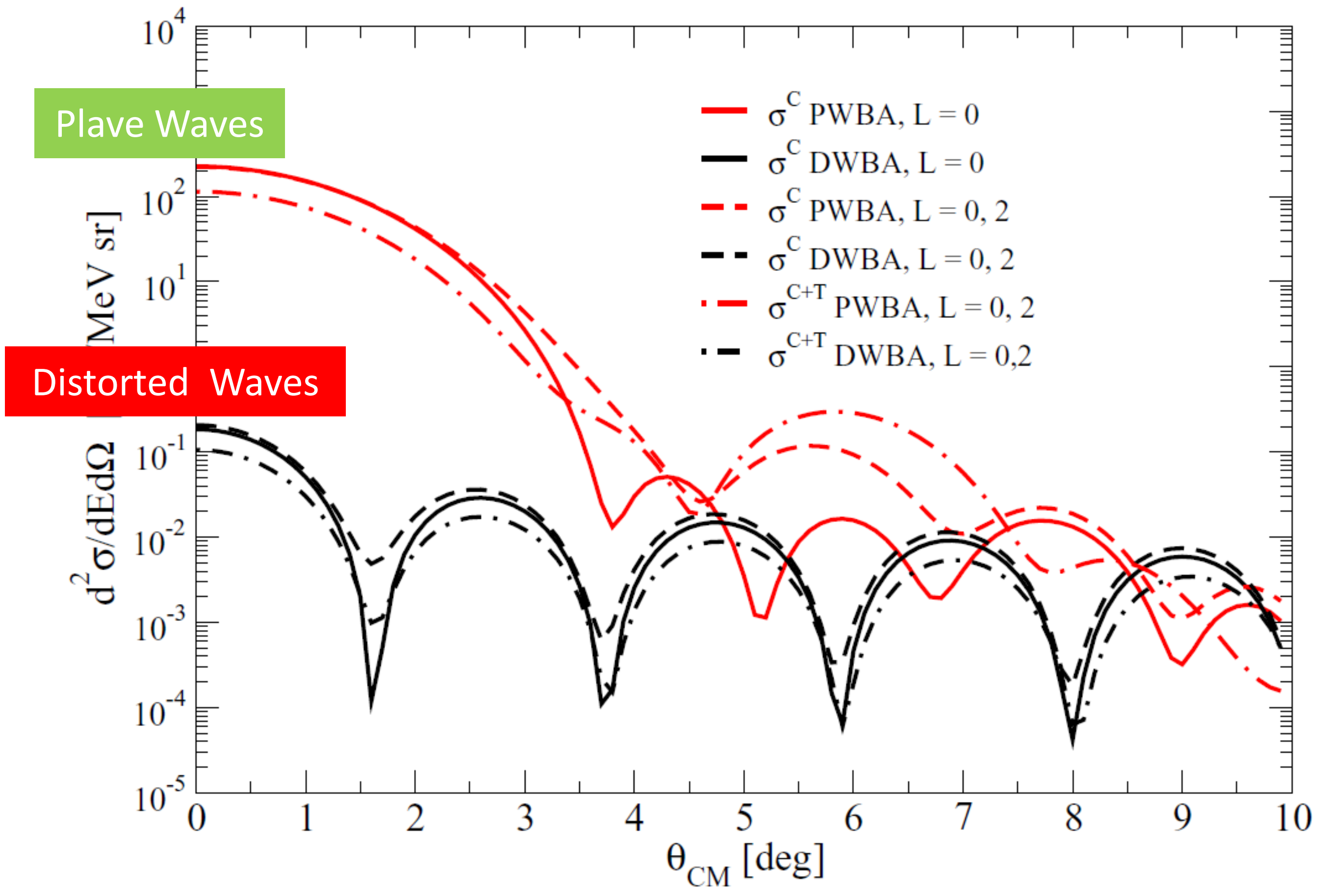
Ion-Ion Interaction Effects in Differential Cross sections
$^{18}O+^{40}Ca$@15AMeV



Strong Absorption → scaling of the cross section by $N_{\alpha\beta} \sim e^{-\alpha\sigma(reac)}$

PHYs. REV.C **98**, 044620 (2018)

Ion-Ion Interaction Effects in Differential Cross sections
$^{18}O+^{40}Ca$@15AMeV

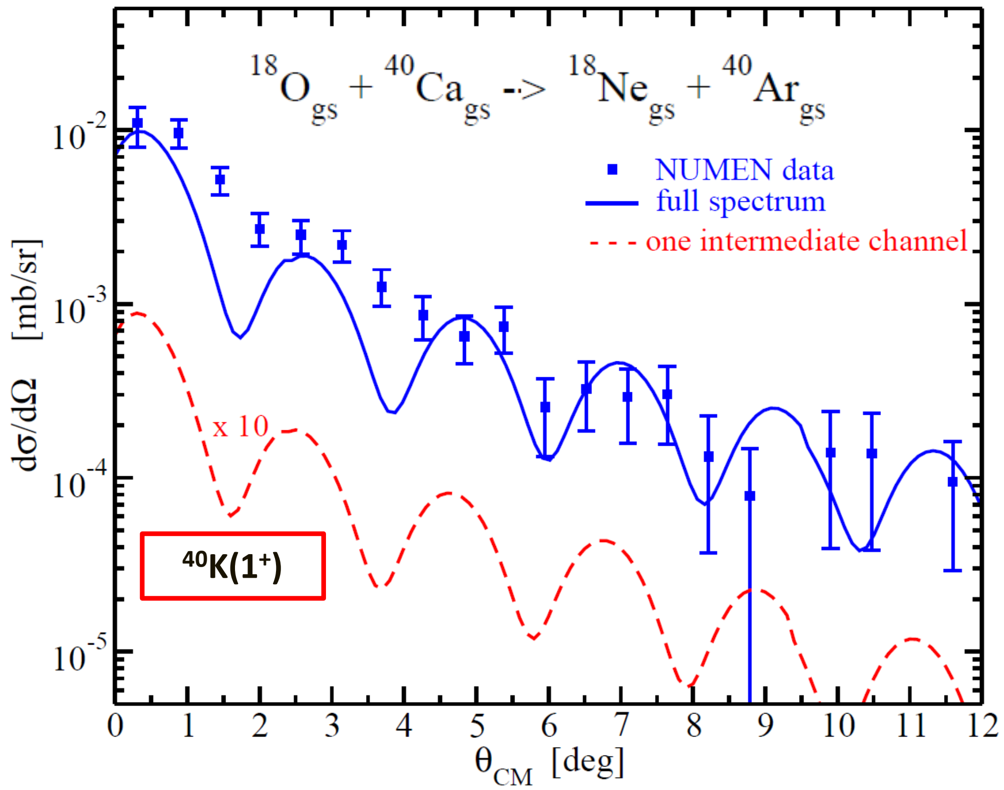


Strong Absorption → scaling of the cross section by $N_{\alpha\beta}$~$e^{-\alpha\sigma(reac)}$

PHYs. REV.C **98**, 044620 (2018)

Microscopic DSCE Theory: 2nd Order DWA and QRPA Nuclear Spectroscopy

$^{18}O + ^{40}Ca \rightarrow ^{18}Ne + ^{40}Ar$ @ T_{lab}=15 MeV/A



- **Optical potentials:**
  HFB g.s. densities and NN T-matrix
- **Nuclear Response:**
  QRPA Theory: $J^\pi \leq 5^\pm$
- **Cross section reproduced In magnitude**

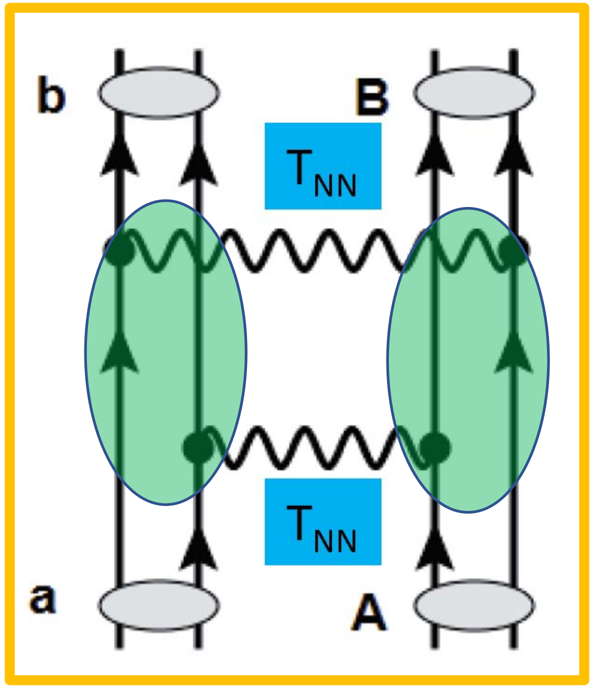
Theory: Jessica Bellone et al., PLB 807 (2020), Data: F. Cappuzzello et al., EPJ A51 (2015)

DSCE Reaction and 2ν2β—type Nuclear Matrix Elements

$$\mathcal{M}^{(2)}_{\alpha\beta}(\mathbf{k}_\alpha,\mathbf{k}_\beta) = \sum_{\gamma=\{c,C\}} \int \frac{d^3k_\gamma}{(2\pi)^3} M^{(1)}_{\gamma\beta}(\mathbf{k}_\gamma,\mathbf{k}_\beta) \frac{\tilde{S}^\dagger_\gamma}{\omega_\alpha - E_c - E_C - T_\gamma + i\eta} M^{(1)}_{\alpha\gamma}(\mathbf{k}_\alpha,\mathbf{k}_\gamma)$$

s-channel
→ propagation

wanted NME



ALWAYS
Projectile and Targét NME

t-channel
→ Reaction Form Factors

H.L. et al. *Universe* 7 (2021) 4, 98

The Merits of Contour Integration and Orthogonal Transformations: From Reaction Amplitudes to 2ν2β-type NME's

$$M^{(2)}_{\beta\alpha}(\mathbf{k}_\beta, \mathbf{k}_\alpha) = \int d^3p_1 d^3p_2 \oint_{C_+} \frac{d\nu}{2i\pi} \sum_{S_1,S_2} \Pi^{(S_2S_1)}_{\alpha\beta}(\mathbf{p}_2, \mathbf{p}_1; \nu)\, I^{(DSCE)}_{S_1S_2}(\mathbf{p}_2, \mathbf{p}_1; \nu\omega_\alpha)$$

$$\Pi^{S_1S_2}_{\alpha\beta}(\mathbf{p}_2, \mathbf{p}_1; \nu) = \sum_{SM_S} (-)^{S_1+S_2-S} \oint_{C+} \frac{d\omega}{2i\pi} (-)^{M_S} \Pi^{(AB)}_{(S_1S_2)SM_S}(\mathbf{p}_2, \mathbf{p}_1; \omega) \cdot \Pi^{(ab)}_{(S_1S_2)S-M_S}(\mathbf{p}_2, \mathbf{p}_1; \nu-\omega)$$

Nuclear polarization tensors

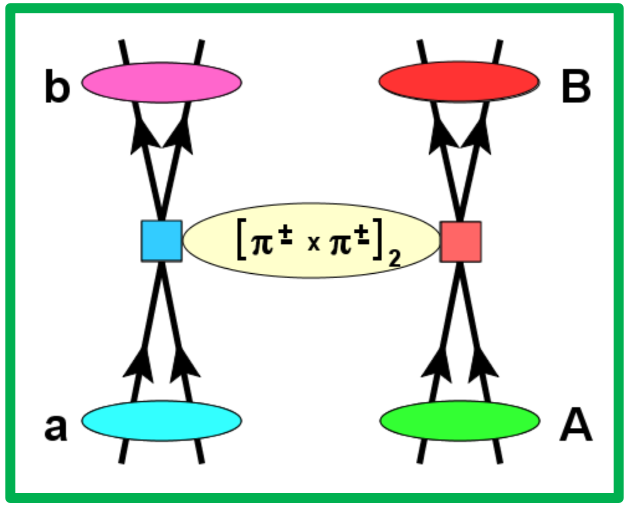
$$\Pi^{(AB)}_{(S_1S_2)SM}(\mathbf{p}_2, \mathbf{p}_1; \omega) = \sum_C \frac{\left[F^{(BC)}_{S_2T}(\mathbf{p}_2) \otimes F^{(CA)}_{S_1T}(\mathbf{p}_1)\right]_{SM}}{\omega - (E_A - E_C)}.$$

2ν2β-type NME for p →0 and ω → E_{2e}

$$F^{(DE)}_{ST}(\mathbf{p}) \;=\; \langle J_E M_E | e^{i\mathbf{p}\cdot r} \boldsymbol{\sigma}^S \tau_\pm | J_D M_D \rangle$$

H.L. et al. *Universe* 7 (2021) 4, 98

Mesonic Majorana DCE (MDCE) Scenario

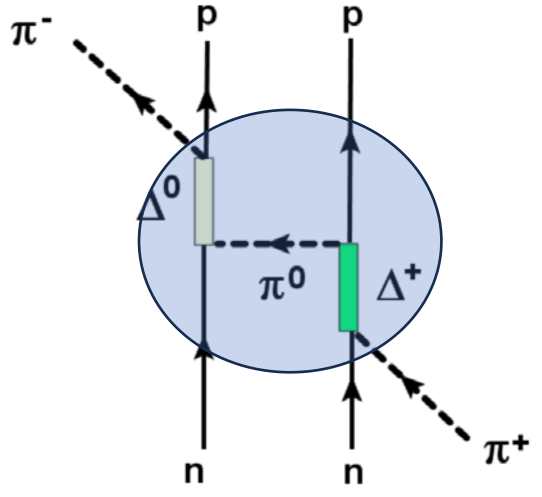


Quest for an Isotensor interaction ~ $[\tau_1 \times \tau_2]_2$:
Conversion of two units of charge in a single step?

Effevtive Rank-2 Isotensor Interaction

Searches for elementary Isotensor I=2 mesons were unsuccessful

Dynamically generated Isotensor Interactions!

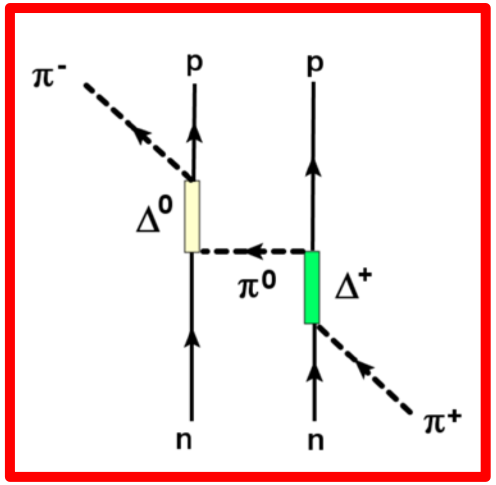


(π+,π-) DCE reactions

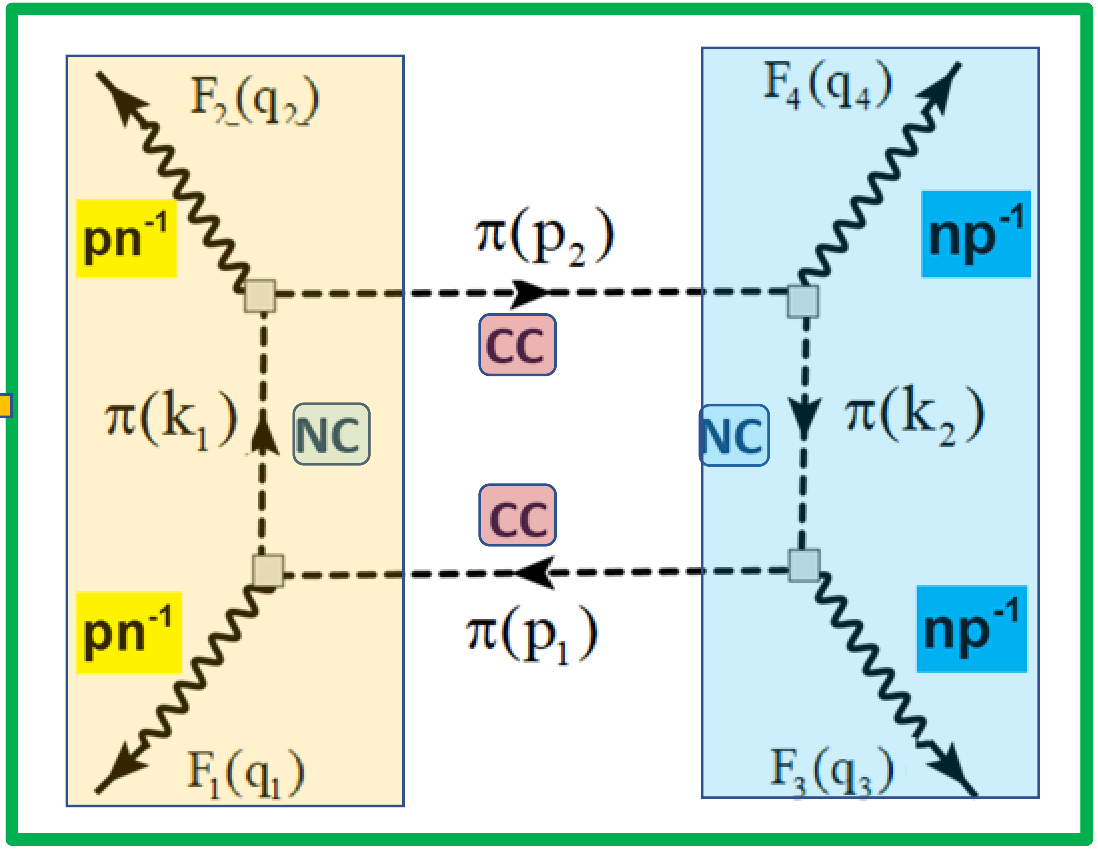
$$V^{(I=2)}_{\pi^{\pm}N,\pi^{\mp}N'} \sim T_{\pi^{\pm}N\pi^{0}N''} G_{\pi^{0}N''} T_{\pi^{0}N''\pi^{\mp}N'}$$

H.L. et al., PPNP 98 (2019) 103716

DCE by Double Meson Exchange
*** Cooperation of Charged (CC) and Neutral (NC) Hadronic Currents ***



On-shell counterpart:
(π^+,π^-) reaction

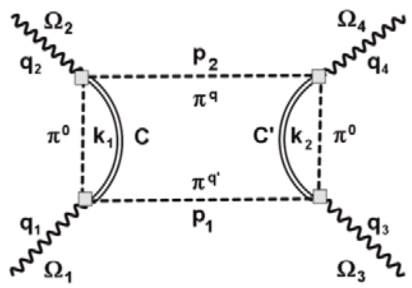


MDCE
Box Diagrams

(☐ = $T_{\pi N}$)

Virtual rank-2 Isotensor Interaction by Correlated Meson Exchange

MDCE Isotensor Transition Kernel



$$\mathcal{U}_{\alpha\beta}(\mathbf{p}_1,\mathbf{p}_2) = \mathcal{W}_{12}(\mathbf{p}_1,\mathbf{p}_2) D_{\pi^q\pi^{q'}}(\mathbf{p}_1,\mathbf{p}_2)\mathcal{W}_{34}(\mathbf{p}_2,\mathbf{p}_1),$$

$$\mathcal{W}_{ij}(\mathbf{p},\mathbf{p}') = \sum_n \int \frac{d^3k}{(2\pi)^3} F_{jn}(\mathbf{p}',\mathbf{k}) D_{\pi^0}(k^2,\omega_n) F_{ni}(\mathbf{p},\mathbf{k}).$$

$$F_{dc}(\mathbf{p},\mathbf{k}) = \langle d|e^{\pm i\mathbf{q}_d\cdot\mathbf{r}}\tilde{T}_{\pi N}(\mathbf{p},\mathbf{k})|c\rangle \qquad (\mathbf{q}_d = \mathbf{p}\pm\mathbf{k})$$

MDCE-NME in projectile and target: two SCE-vertices connected by π^0 exchange

2-Nucleon Mechanism fostered by 2-body correlations

MDCE Vertices in Closure Approximation

$$\boxed{D_{\pi^0 C}(k^2) \sim -\frac{1}{k^2 + m^2_{\pi^0}} \left(1 + \frac{(\omega_A - \omega_C)^2}{m^2_{\pi^0}} + \dots \right)}$$

Pion Mass → NATURAL SEPARATION SCALE! → keep the 1st term only → Closure

MDCE Transition Potential in Closure Approximation
„MDCE Pion Potential“

$$\boxed{\mathcal{U}_\pi(\mathbf{x}) = \int \frac{d^3k}{(2\pi)^3} T_{\pi N}(\mathbf{p}_2, \mathbf{k}) \frac{e^{i\mathbf{k}\cdot\mathbf{x}}}{k^2 + m^2_{\pi^0}} T_{\pi N}(\mathbf{p}_1, \mathbf{k}).}$$

The MDCE Nuclear Transition Amplitude in Closure Approximation

$$\boxed{\mathcal{W}_{AB}(\mathbf{p}_1, \mathbf{p}_2) = -\langle B | e^{-i\mathbf{p}_2 \cdot \mathbf{r}_2} \mathcal{U}_\pi(\mathbf{x}) e^{i\mathbf{p}_1 \cdot \mathbf{r}_1} \mathcal{T}_{2\pm 2} | A \rangle,}$$

Rank-2 Isotensor Operator

$$\boxed{\mathcal{T}_{2\pm 2} = [\boldsymbol{\tau}_1 \otimes \boldsymbol{\tau}_2]_{2\pm 2}}$$

Pion-Potential

$$\boxed{\mathcal{U}_\pi(\mathbf{x}) = \int \frac{d^3k}{(2\pi)^3} T_{\pi N}(\mathbf{p}_2, \mathbf{k}) \frac{e^{i\mathbf{k}\cdot\mathbf{x}}}{k^2 + m_{\pi^0}^2} T_{\pi N}(\mathbf{p}_1, \mathbf{k}).}$$

$$\boxed{T_{\pi N}(p_j, k) = T_0(w) + \frac{1}{m_\pi^2}\left(T_1(w) p_j \bullet k + i T_2(w) \sigma_j \bullet (p_j \times k)\right)}$$

→ **6 potentials: $U_{00}(x)$, $U_{11}(x)$, $U_{22}(x)$, $U_{01}(x)$, $U_{02}(x)$ and $U_{12}(x)$**

Pion-Nucleon Interactions

$$T_{\pi N}(p_j,k)=T_0(w)+\frac{1}{m_\pi^2}\Big(T_1(w)\,p_j\bullet k+iT_2(w)\,\sigma_j\bullet(p_j\times k)\Big)$$

ELSEVIER
Progress in Particle and Nuclear Physics
Volume 98, January 2018, Pages 119-206

Review

Baryons and baryon resonances in nuclear matter

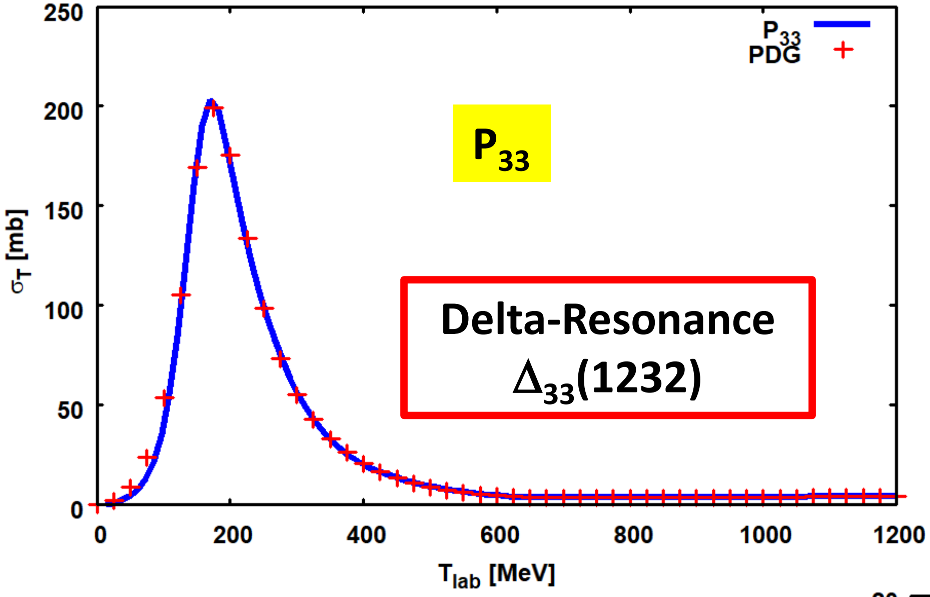
Horst Lenske [a], Madhumita Dhar [a,b], Theodoros Gaitanos [a,c], Xu Cao [a,d,e]

PHYSICAL REVIEW C **106**, 014618 (2022)

Systematic study of Δ(1232) resonance excitations using single isobaric charge-exchange reactions induced by medium-mass projectiles of Sn

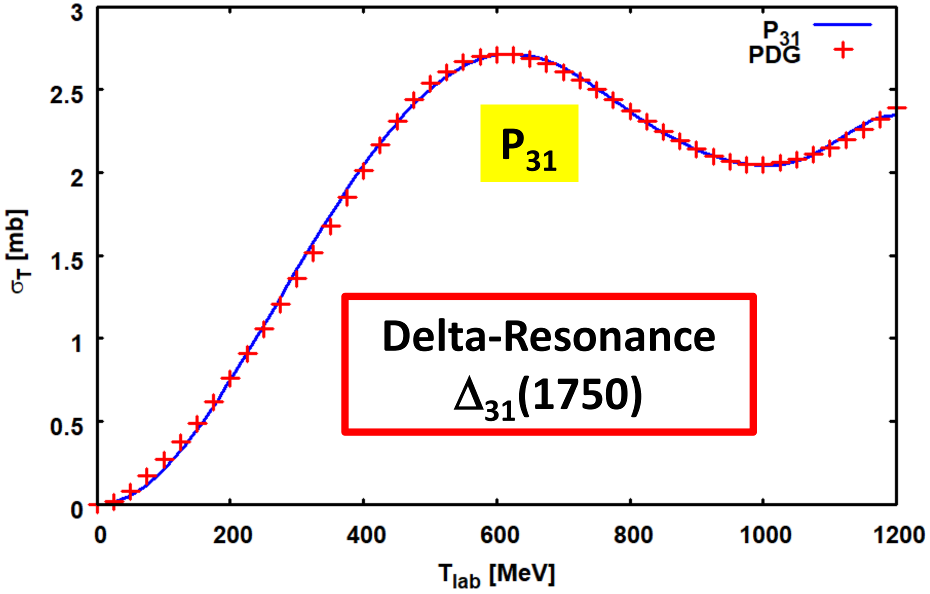
J. L. Rodríguez-Sánchez,[1,2,*] J. Benlliure,[1] I. Vidaña,[3] H. Lenske,[4] J. Vargas,[5,†] C. Scheidenberger,[2] H. Alvarez-Pol,[1] J. Atkinson,[2] T. Aumann,[2,6] Y. Ayyad,[1] S. Beceiro-Novo,[5,‡] K. Boretzky,[2] M. Caamaño,[1] E. Casarejos,[7] D. Cortina-Gil,[1] P. Díaz Fernández,[5,§] A. Estrade,[2,8,∥] H. Geissel,[2] E. Haettner,[2] A. Kelić-Heil,[2] Yu. A. Litvinov,[2] C. Paradela,[5,¶] D. Pérez-Loureiro,[5,#] S. Pietri,[2] A. Prochazka,[2,**] M. Takechi,[2,**] Y. K. Tanaka,[2,9] H. Weick,[2] and J. S. Winfield[2,††]

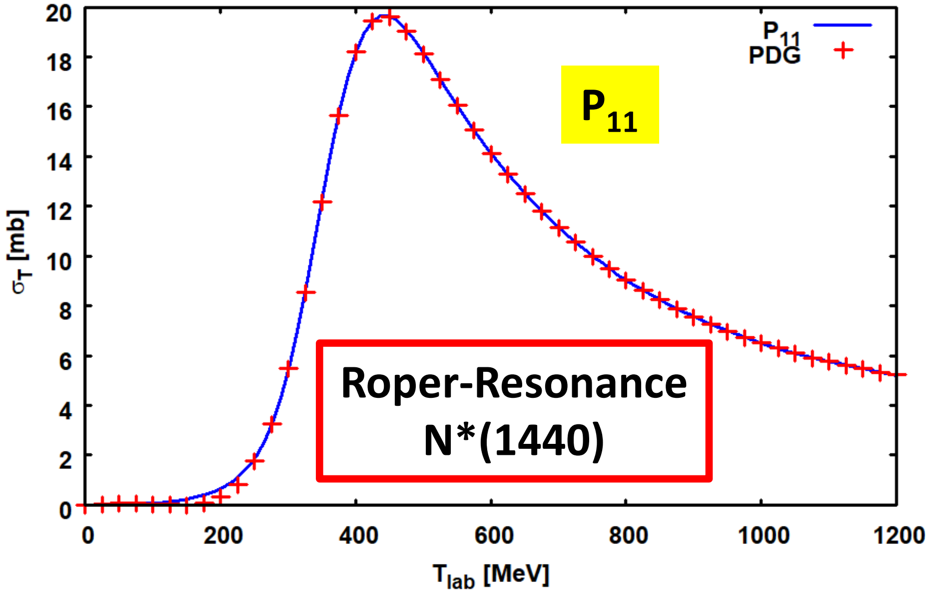
$\pi^{-}N$ total Cross Sections



Relevant partial waves:
P_{33},P_{11},P_{31},S_{11},S_{31}
(Notation: L_{2I2J})

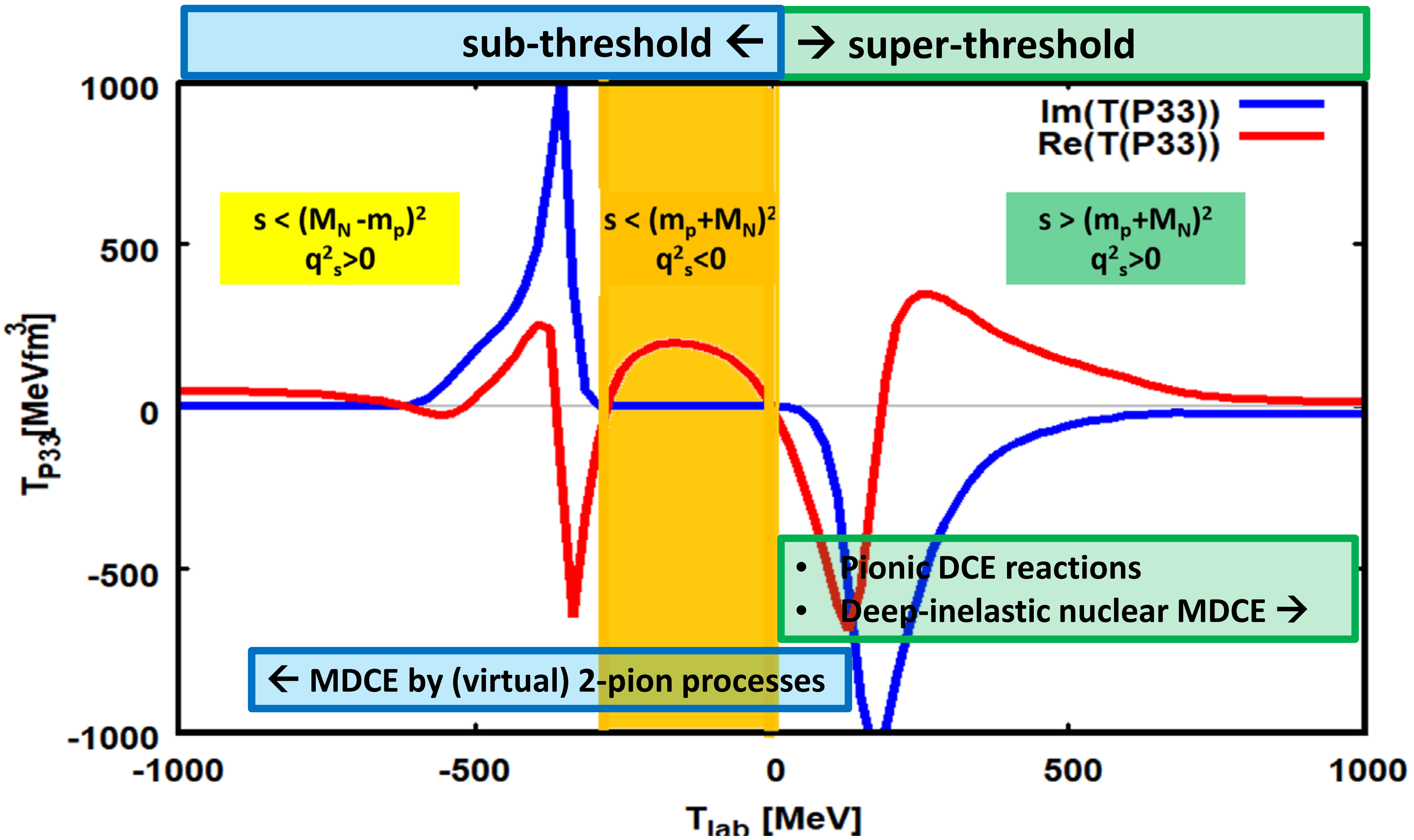
$T_0 \sim \{S_{11},S_{31}\}$
$T_{1,2} \sim \{P_{11},P_{31},P_{31}\}$



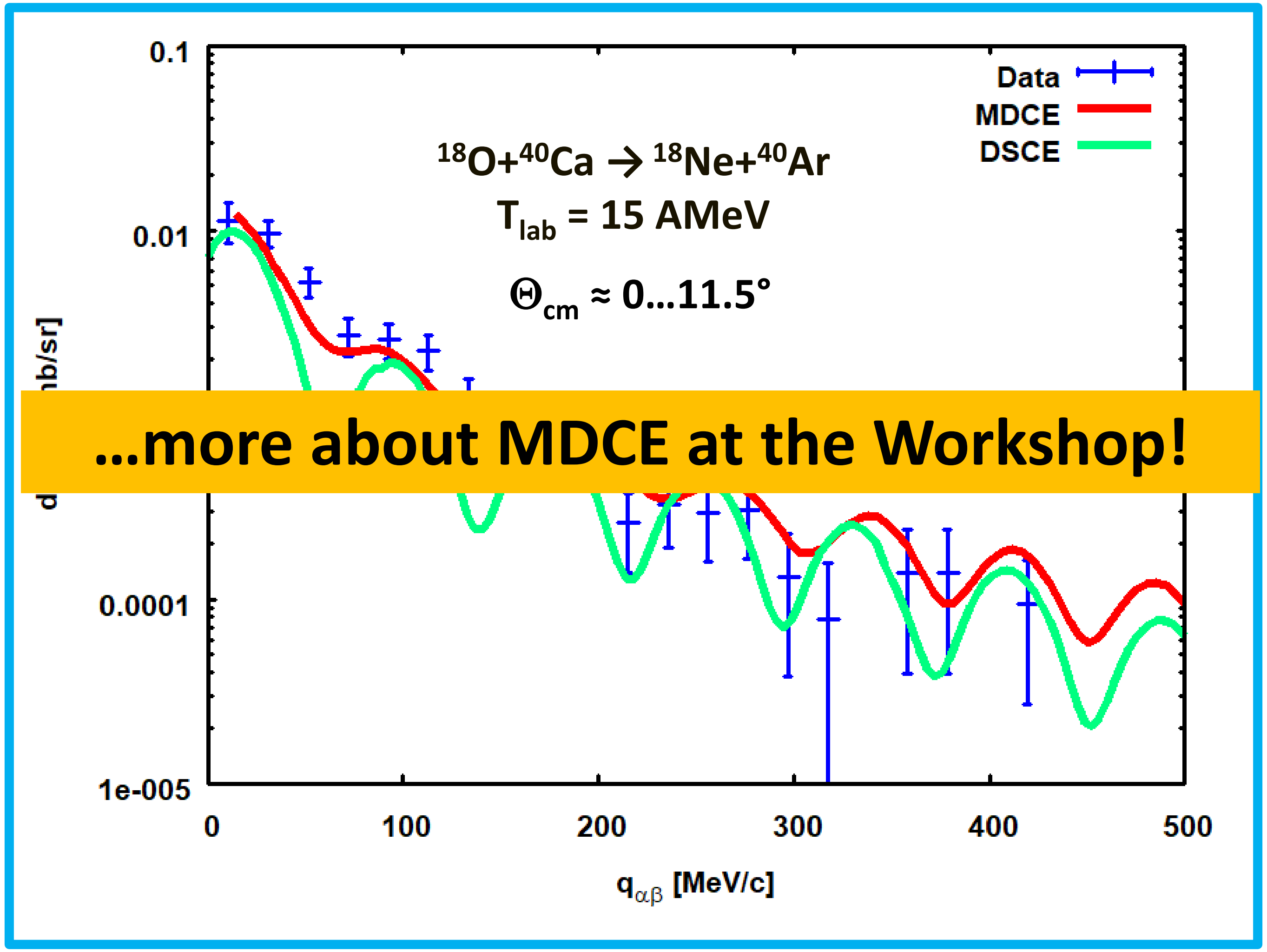


Review on the Giessen π+N CC approach:
H.L. et al., PPNP 98 (2018) 119

Kinematical Regions of Importance for Hadronic DCE Reactions



DCE Reactions: Probing Nuclear Physics @ High Momentum Transfer



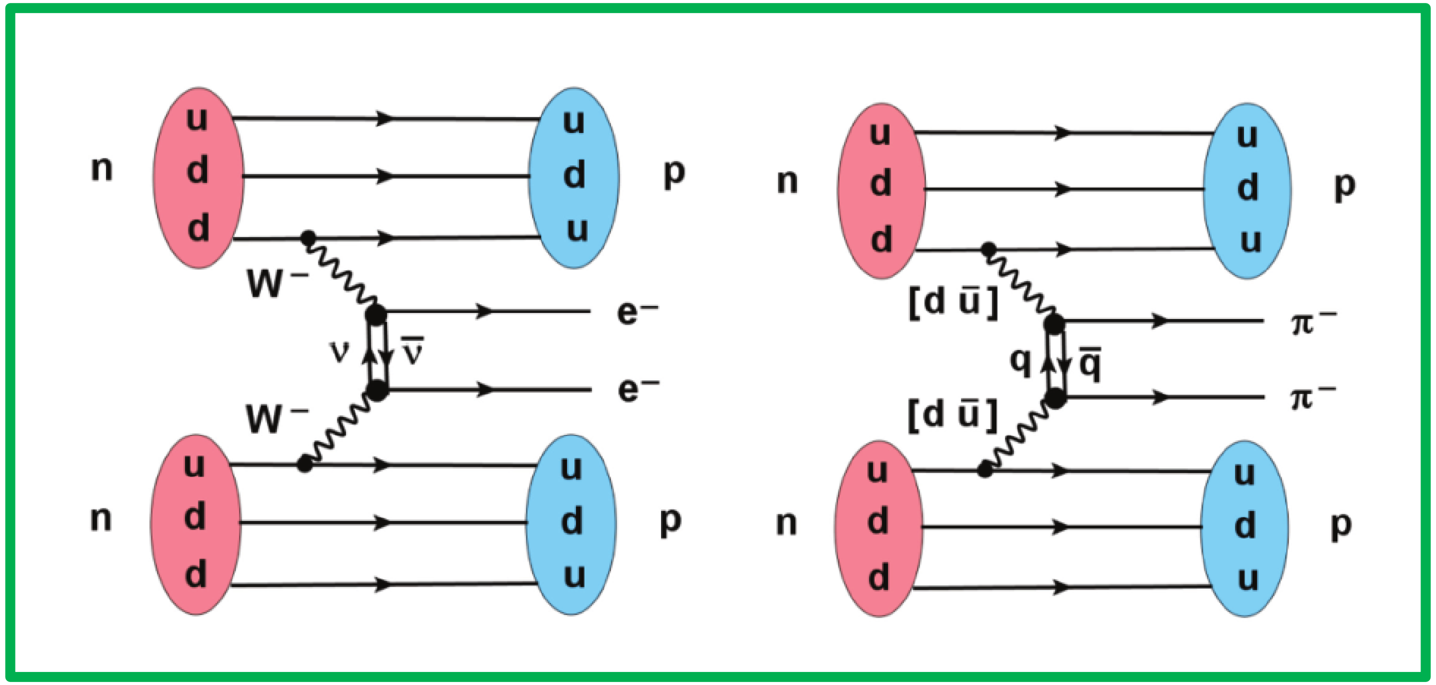
MDCE Preliminary!

2-step DSCE: intermediate states with $J^{\pi} \leq 5^{\pm}$

1-step MDCE: $^{40}Ca(0^{+}) \rightarrow ^{40}Ar([n^{-2}p^{2}]0^{+})$: J=0+ with L=S=0 & $[L=2 \times S=2]_{0+}$

Data: F. Cappuzzello et al., EPJ A51 (2015)

MDCE and 0ν2β Double Beta-Decay
The Majorana Aspect



Ettore Majoran
*Aug., 5, 1906, at Catania
disappeared, Mar, 1938
+ 1959 in Venezuela?
+ in a Sicilian monastery?

- Topological Correspondence of MDCE on the Diagrammatic Level
- MDCE as a Surrogate Reaction for 0ν2β-NME

That‘s what we learnt

Lecture 1: Basic Concepts of Quantum-Mechanical Scattering Theory
Lecture 2: Nuclear Reaction Theory in a Nutshell
Lecture 3: Theory of Nuclear Direct Reactions
Lec ture 4: Optical Potentials and Elastic Scattering
Lecture 5: Perturbative Approach to Non-Elastic Reactions
Lecture 6: Single Charge Exchange (SCE) Reactions
Lecture 7: Light Ion SCE Reactions and beta-Decay
Lecture 8: The Gamow-Teller Quenching Mystery
Lecture 9: Nuclear Matrix Elements for Astrophysics
Lecture 10: Heavy Ion SCE Reactions
Lecture 11: Heavy Ion SCE Reactions at Relativistic Energies
Lecture 12: Double Charge Exchange (DCE) Reactions