Challenges in nuclear structure theory in connection with neutrinoless $\beta\beta$ decay

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Creation of matter in nuclei: $0\nu\beta\beta$ decay

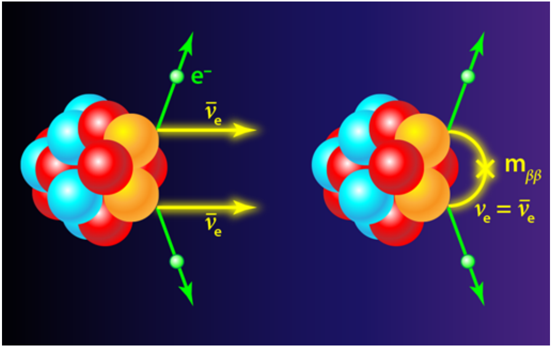
Lepton number is conserved in all processes observed:

single β decay,
$\beta\beta$ decay with ν emission...

Neutral massive particles (Majorana ν's) allow lepton number violation:

neutrinoless $\beta\beta$ decay
creates two matter particles (electrons)

Agostini, Benato, Detwiler, JM, Vissani, Rev. Mod. Phys. 95, 025002 (2023)



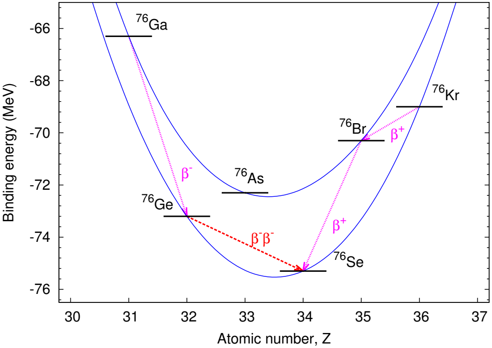
$\beta\beta$ decays

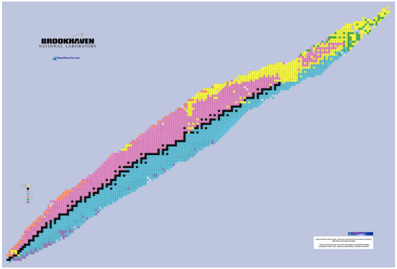
Second order process in the weak interaction

Only observable in nuclei where (much faster) β-decay is forbidden energetically due to nuclear pairing interaction

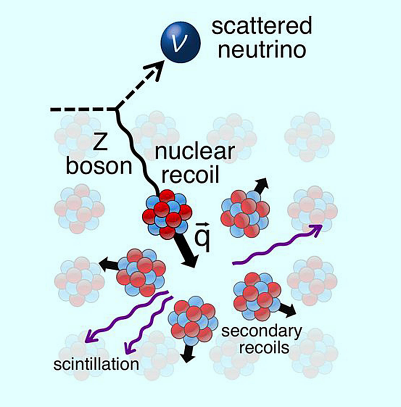
$$BE(A) = -a_v A + a_s A^{2/3} + a_c \frac{Z(Z-1)}{A^{1/3}} + \frac{(A-2Z)^2}{A} + \begin{cases} -\delta_{pairing} & \text{N,Z even} \\ 0 & \text{A odd} \\ \delta_{pairing} & \text{N,Z odd} \end{cases}$$

or where β-decay is very suppressed by ΔJ (total angular momentum) difference between mother and daughter nuclei





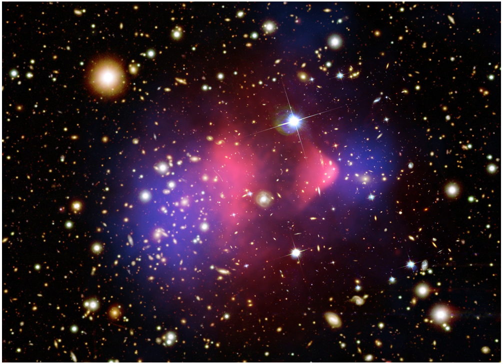
Coherent ν-nucleus scattering, dark matter detection



Coherent ν-nucleus scattering

Neutral current process, tiny cross-section

Neutrinos couple to neutrons, complements EM interactions



Dark matter scattering off nuclei

What is dark matter made of?

Nuclear matrix elements for new-physics searches

Neutrinos, dark matter studied in experiments using nuclei

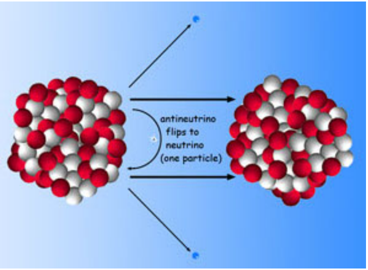
Nuclear structure physics
encoded in nuclear matrix elements
key to plan, fully exploit experiments

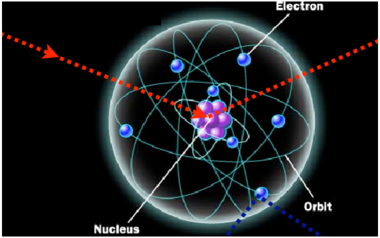
$$0\nu\beta\beta\text{:}\ \left(T_{1/2}^{0\nu\beta\beta}\right)^{-1} \propto g_A^4 \left|M^{0\nu\beta\beta}\right|^2 m_{\beta\beta}^2$$

$$\text{Dark matter:}\ \frac{d\sigma_{\chi\mathcal{N}}}{d\boldsymbol{q}^2} \propto \left|\sum_i c_i \zeta_i \mathcal{F}_i\right|^2$$

$$\text{CE}\nu\text{NS:}\ \frac{d\sigma_{\nu\mathcal{N}}}{d\boldsymbol{q}^2} \propto \left|\sum_i c_i \zeta_i \mathcal{F}_i\right|^2$$

$M^{0\nu\beta\beta}$: Nuclear matrix element
$\mathcal{F}_i$: Nuclear structure factor





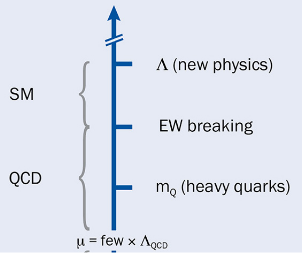
Different scales in new-physics searches using nuclei

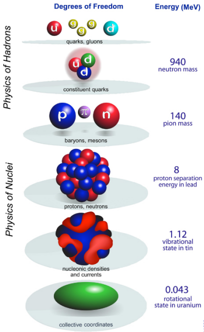
New physics scale: $\Lambda \gg 250$ GeV

Electroweak scale:
$$v = \left(\sqrt{2}G_F\right)^{-1/2} \sim 250 \text{ GeV}$$

QCD (hadron) scale: $m_N \sim$ GeV

Nuclear scale: $k_F \sim m_\pi \sim 200$ MeV

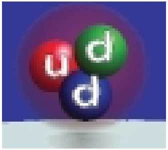


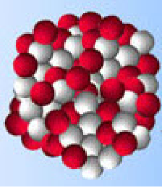


Particle, hadronic and nuclear physics

ν scattering off nuclei
interplay of particle, hadronic and nuclear physics:

ν's: interaction with quarks and gluons
Quarks and gluons: embedded in the nucleon
Nucleons: form complex, many-nucleon nuclei





General ν-nucleus scattering cross-section:

$$\frac{d\sigma_{\nu\mathcal{N}}}{d\boldsymbol{q}^2} \propto \left| \sum_i c_i \, \zeta_i \, \mathcal{F}_i \right|^2$$

ζ: kinematics $(q^2, \cdots)$

c coefficients:
ν couplings to quark, gluons (Wilson coefficients), particle physics
convoluted with hadronic matrix elements, hadronic physics

$\mathcal{F}$ functions: $\mathcal{F}^2 \sim$ structure factor, nuclear structure physics

$0\nu\beta\beta$ decay half-life

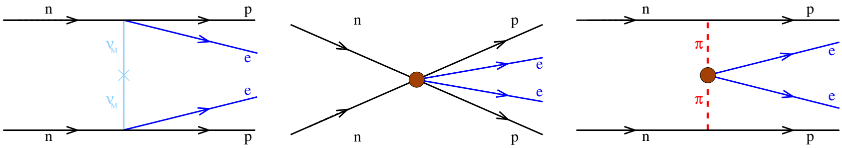
Half-life of $0\nu\beta\beta$ decay sensitive to
$m_{\beta\beta} \sim 1/\Lambda$ (dim-5 operator), new-physics scales $\widetilde{\Lambda}$ (dim-7) or $\widetilde{\Lambda}'$ (dim-9)

$$T_{1/2}^{-1} = G_{01}\, g_A^4 \left(M^{0\nu}_{\text{light}}\right)^2 m_{\beta\beta}^2 + m_N^2 \tilde{G}\, \tilde{g}^4\, \tilde{M}^2 \left(\frac{v}{\widetilde{\Lambda}}\right)^6 + \frac{m_N^4}{v^2} \widetilde{G}'\, \tilde{g}'^4\, \widetilde{M}'^2 \left(\frac{v}{\widetilde{\Lambda}'}\right)^{10}$$

G_{01}, $\widetilde{G}$, $\widetilde{G}'$: phase-space factors (electrons), very well known

g_A, g_ν^{NN}, $\widetilde{g}$, $\widetilde{g}'$: coupling to hadron(s), experiment or calculate with QCD

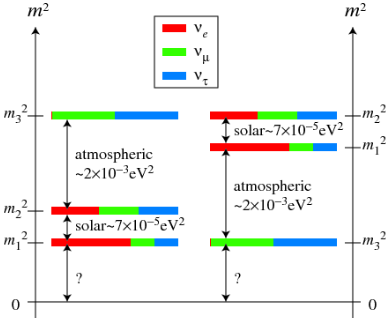
$M^{0\nu}_{\text{long}}$, $M^{0\nu}_{\text{short}}$, $\widetilde{M}$, $\widetilde{M}'$: nuclear matrix elements, many-body challenge

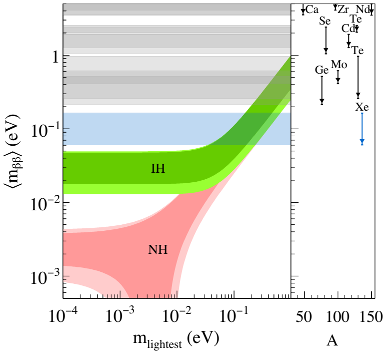


Next generation experiments: inverted hierarchy

Decay rate sensitive to neutrino masses, hierarchy
$m_{\beta\beta} = \left|\sum U_{ek}^2 m_k\right|$

$$T_{1/2}^{0\nu\beta\beta}\left(0^+ \to 0^+\right)^{-1} = G_{0\nu}\, g_A^4 \left|M_{\rm light}^{0\nu}\right|^2 \left(\frac{m_{\beta\beta}}{m_e}\right)^2$$

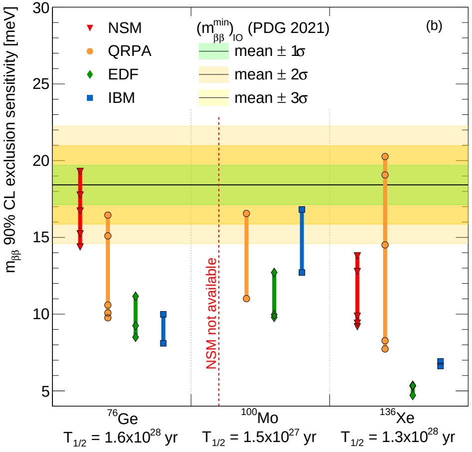




Matrix elements assess if next generation experiments fully explore "inverted hierarchy"

KamLAND-Zen, PRL117 082503(2016)

Uncertainty in physics reach of $0\nu\beta\beta$ experiments



Agostini, Benato, Detwiler, JM, Vissani
Phys. Rev. C 104 L042501 (2021)

Nuclear matrix element theoretical uncertainty critical to anticipate $m_{\beta\beta}$ sensitivity of future experiments

Current uncertainty in $m_{\beta\beta}$ prevents to foresee if next-generation experiments will fully cover parameter space of “inverted” neutrino mass hierarchy

Uncertainty needs to be reduced!

$0\nu\beta\beta$ mediated by new-physics heavy particles

Stantard Model extensions trigger $0\nu\beta\beta$ decay (heavy ν, M_R...)

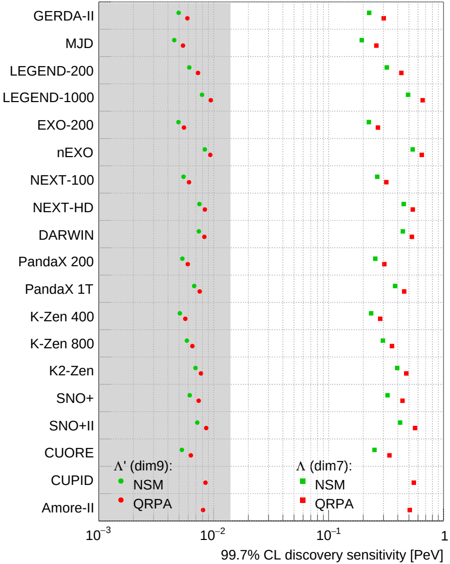
Phase-space, hadronic/nuclear matrix elements, known or calculated

Effective field theory

Cirigliano et al JHEP 12 097 (2018)

dimension-7 ($\sim 1/\Lambda^3$), dimension-9 ($\sim 1/\Lambda^5$) operators

constrained by current searches $\Lambda \gtrsim$ 250 TeV (dim-7) $\Lambda \gtrsim$ 5 TeV (dim-9)

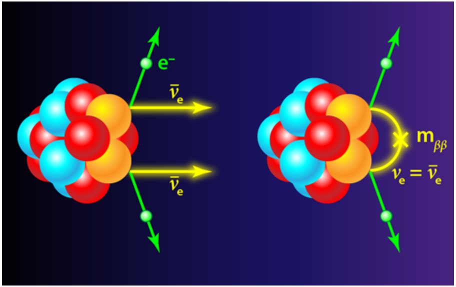


Calculating nuclear matrix elements

Nuclear matrix elements needed in low-energy new physics searches

$$\langle \text{Final} \,|\mathcal{L}_{\text{leptons}-\text{nucleons}}|\, \text{Initial} \rangle = \langle \text{Final} \,| \int dx\, j^{\mu}(x) J_{\mu}(x) \,|\, \text{Initial} \rangle$$

- Nuclear structure calculation of the initial and final states:
  Shell model, QRPA, IBM,
  Energy-density functional
  Ab initio many-body theory
  GFMC, Coupled-cluster, IM-SRG...
- Lepton-nucleus interaction:
  Hadronic current in nucleus:
  phenomenological,
  effective theory of QCD



Double-beta decay emitters

Only decay candidates with $Q_{\beta\beta} > 2$ MeV
experimentally interesting due to extremely long lifetimes

ECEC, ECβ^+ and $\beta^+\beta^+$ also more suppressed

Transition	$T^{2\nu\beta\beta}$ (y)	$Q_{\beta\beta}$ (MeV)	Ab. (%)	
^{48}Ca $\rightarrow$ ^{48}Ti	$4.4 \cdot 10^{19}$	4.274	0.2	CANDLES
^{76}Ge $\rightarrow$ ^{76}Se	$1.7 \cdot 10^{21}$	2.039	8	GERDA, MAJORANA, LEGEND
^{82}Se $\rightarrow$ ^{82}Kr	$9.2 \cdot 10^{19}$	2.996	9	SuperNEMO
^{96}Zr $\rightarrow$ ^{96}Mo	$2.3 \cdot 10^{19}$	3.350	3	
^{100}Mo $\rightarrow$ ^{100}Ru	$7.1 \cdot 10^{18}$	3.034	10	AMoRE, CUPID
^{110}Pd $\rightarrow$ ^{110}Cd		2.013	12	
^{116}Cd $\rightarrow$ ^{116}Sn	$2.9 \cdot 10^{19}$	2.802	7	COBRA
^{124}Sn $\rightarrow$ ^{124}Te		2.288	6	
^{130}Te $\rightarrow$ ^{130}Xe	$6.9 \cdot 10^{20}$	2.530	34	CUORE, SNO+
^{136}Xe $\rightarrow$ ^{136}Ba	$2.2 \cdot 10^{21}$	2.462	9	nEXO, KamLAND-Zen, NEXT, DARWIN
^{150}Nd $\rightarrow$ ^{150}Sm	$8.2 \cdot 10^{18}$	3.667	6	

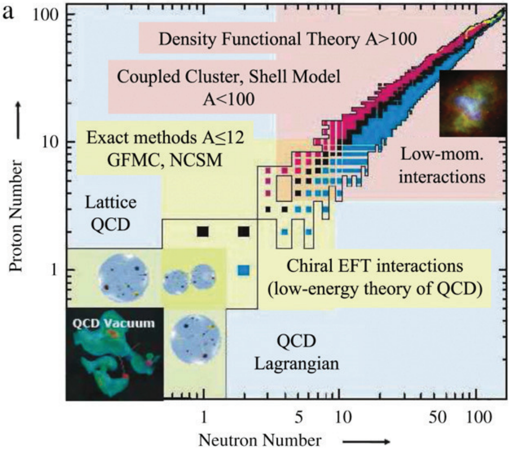
Worldwide running and planned experiments on different isotopes

Outline

Outline

Nuclear Structure from First Principles

All nuclear structure calculations are, to some extent, phenomenological



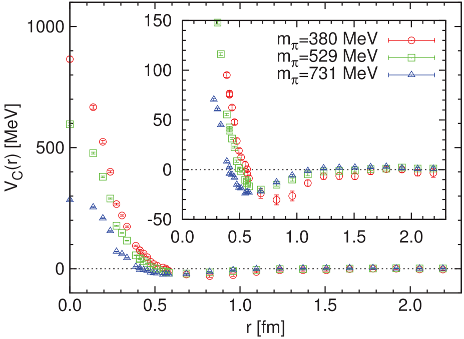
Relevant degrees of freedom: protons and neutrons
Many-body problem too hard in general, approximations are needed

Nuclear force at low (nuclear structure) energies: adjustments to reproduce finite nuclei needed

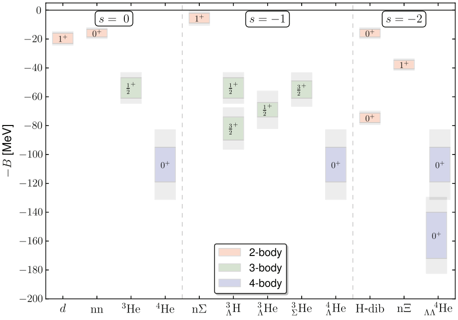
Can we connect nuclear structure calculations to quantum chromodynamics (QCD)?

Lattice QCD

QCD non-perturbative at low energies relevant for nuclear structure
Lattice QCD solves the QCD Lagrangian in discretized space-time Lattice



HALQCD Collaboration



NPLQCD Collaboration

Nuclear potentials, and lightest nuclei and hypernuclei solved
at non-physical pion mass $m_\pi \sim 400 - 800$ MeV, ongoing improvements

Theory for nuclear forces

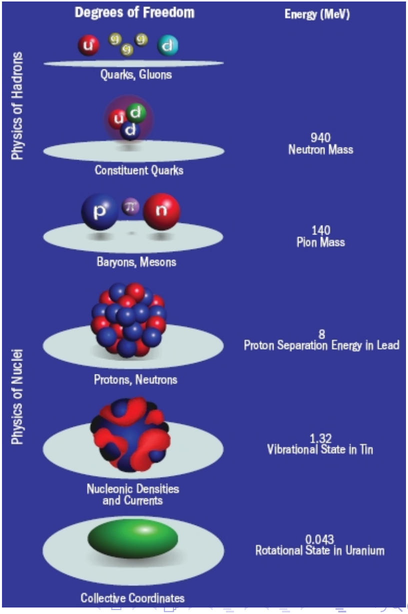
Difficult to find NN potential with consistent NNN forces and connected to QCD...

Use concept of separation of scales!

The energy scale relevant determines the degrees of freedom

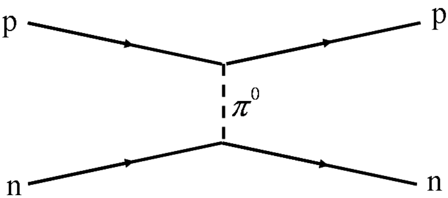
For nuclear structure,
typical energies of interest
point to nucleons and pions
(pions are particularly light mesons!)

Effective theory with nucleons and pions
as degrees of freedom,
with connection to QCD



Nuclear interaction: Yukawa

The foundation for a theory of the nuclear force given by Yukawa (1935)



Predicted new particle (pion!) responsible for attractive nuclear force $r \sim 1$ fm $\Rightarrow m \sim 1/r$

Pion discovered in 1947!

Spin dependent

Isospin dependent

Non-central interaction,
Tensor component
$S_{12} = \{3\,(\sigma_1 \cdot \hat{r})\,(\sigma_2 \cdot \hat{r}) - \sigma_1 \cdot \sigma_2\}$

$$V(r) = -\frac{m_\pi^2\, g_A^2}{f_\pi^2}\,(\tau_1 \cdot \tau_2)\left[\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + S_{12}\left(1 + \frac{3}{mr} + \frac{3}{(mr)^2}\right)\right]\frac{e^{-mr}}{mr}$$

Effective theories

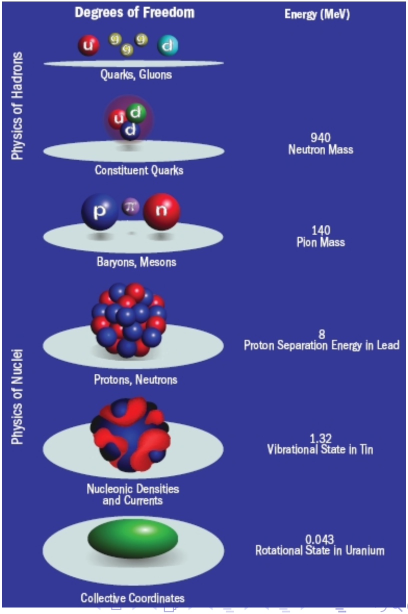
Effective theory:
approximation of the full theory
valid at relevant scales

Expansion in terms of small parameter:
typical scale / breakdown scale

In an effective theory
the physics resolved
at relevant energies is explicit

Terms at different orders given by
symmetries of the full theory

Unresolved physics
encoded in Low Energy Couplings

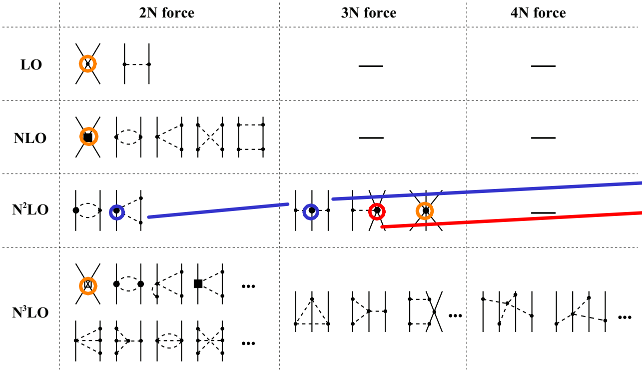


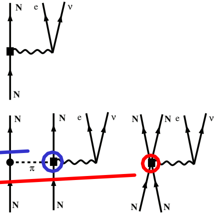
Chiral Effective Field Theory (EFT)

Chiral EFT: low energy approach to QCD, nuclear structure energies

Approximate chiral symmetry: pion exchanges, contact interactions

Systematic expansion: nuclear forces and electroweak currents





Weinberg, van Kolck, Kaplan, Savage, Wise, Meißner, Epelbaum...

Park, Krebs, Baroni...

Short-range couplings fitted to experiment once

How does chiral EFT work?

The chiral EFT Lagrangian is an expansion, in different orders of pion-pion, pion-nucleon and nucleon-nucleon parts

$$\begin{aligned}\mathcal{L}_{\chi EFT} =& \mathcal{L}^{(0)} + \mathcal{L}^{(1)} + \mathcal{L}^{(2)} + \cdots \\ =& \mathcal{L}^{(0)}_{\pi\pi} + \mathcal{L}^{(0)}_{\pi N} + \mathcal{L}^{(0)}_{NN} + \mathcal{L}^{(1)}_{\pi\pi} + \mathcal{L}^{(1)}_{\pi N} + \mathcal{L}^{(1)}_{NN} + \cdots\end{aligned}$$

For example:

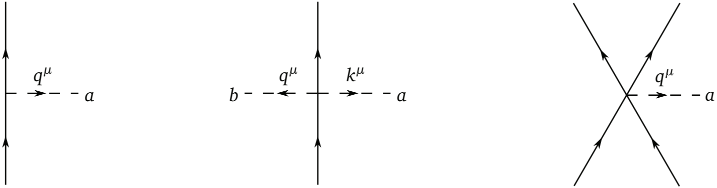
$$\begin{aligned}\mathcal{L}^{(0)}_{\pi\pi} &= \frac{f_\pi^2}{4} Tr\left[\partial^\mu U \partial_\mu U^\dagger + m_\pi^2\left(U + U^\dagger\right)\right], \quad U = exp\left[i\frac{\boldsymbol{\pi}\cdot\boldsymbol{\tau}}{f_\pi}\right] \\ \mathcal{L}^{(0)}_{\pi N} &= \bar{N}\left(i\gamma_\mu \mathcal{D}^\mu + \frac{g_A}{2}\gamma^\mu\gamma_5 u(\pi)_\mu - M\right) N \\ &\cdots\end{aligned}$$

Evaluate these expressions to lowest orders in pion fields obtain Feynman diagrams for each vertex λ_i

The chiral order of a diagram is $\nu = 2(N - C) + 2L + \sum_i \lambda_i$ with N nucleons, C disconnected parts and L loops

Examples of chiral EFT diagrams

Feynman diagrams are read off from the Lagrangian:



(a) (b) (c)

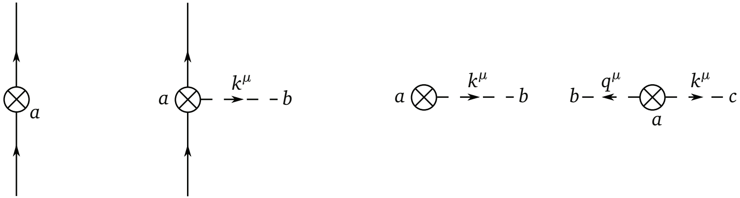
	(a)	(b)	(c)
LO	$-\frac{g_A}{2F_\pi}\gamma_5 \not{q}\tau^a$	$\frac{1}{4F_\pi^2}\epsilon^{abc}\not{q}\tau^c$	—
NLO	—	$\frac{2i}{F_\pi^2}\left(c_4\epsilon^{abc}\frac{\tau^c}{2}k_\mu q_\nu \sigma^{\mu\nu} - c_3 k_\mu q^\mu \delta^{ab} - 2c_1 m_\pi^2 \delta_{ab}\right)$	$\frac{d_1}{F_\pi}\tau^a \boldsymbol{\sigma}_1\cdot\mathbf{q} + (1\longleftrightarrow 2)$ $+\frac{d_2}{F_\pi}(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)^a \mathbf{q}\cdot(\boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_2)$

for the lowest order pion-nucleon diagrams from $\mathcal{L}_{\pi\pi}^{(0)} + \mathcal{L}_{\pi N}^{(0)} + \mathcal{L}_{NN}^{(1)}$

Chiral EFT currents

In addition, from the Chiral EFT Lagrangian we obtain the currents on how nucleons (and pions) couple to different probes of scalar, vector, axial... character

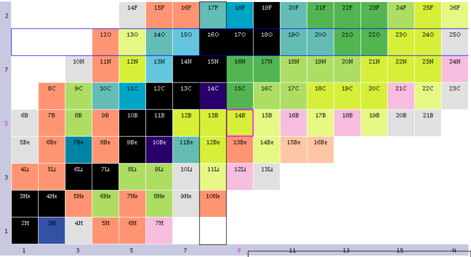
This is consistent with nuclear forces (same couplings)!



(a) (b) (c) (d)

	(a)	(b)	(c)	(d)
LO axial	$ig_A\gamma^\mu\gamma_5\frac{\tau^a}{2}$	$-\frac{i}{F_\pi}\epsilon^{abc}\gamma^\mu\frac{\tau^c}{2}$	$F_\pi k^\mu\delta^{ab}$	—
LO vector	$i\gamma^\mu\frac{\tau^a}{2}$	$-i\frac{g_A}{F_\pi}\epsilon^{abc}\gamma^\mu\gamma_5\frac{\tau^c}{2}$	—	$-\epsilon^{abc}k^\mu$
NLO axial	—	$\frac{2}{F_\pi}\left(-c_4\epsilon^{abc}\frac{\tau^c}{2}k_\nu\sigma^{\mu\nu}+c_3k^\mu\delta^{ab}\right)$	—	—
NLO vector	—	—	—	—

Calculations with NN, NN+3N forces

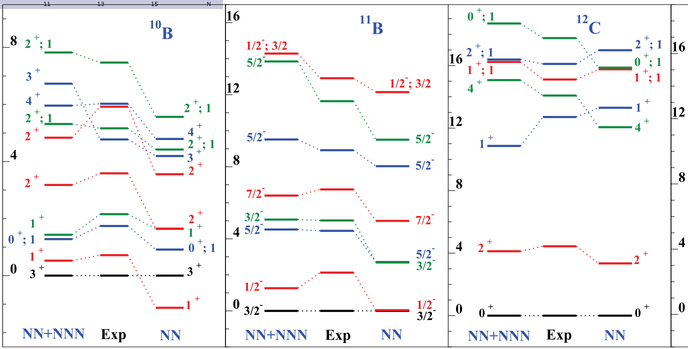


Ab initio many-body calculations feasible in light nuclei

No Core Shell Model
Green's Function Monte Carlo

NN forces
do not reproduce
binding energies
and spectra:
need 3N forces

Agrees with
experience
from Shell Model
in heavier systems

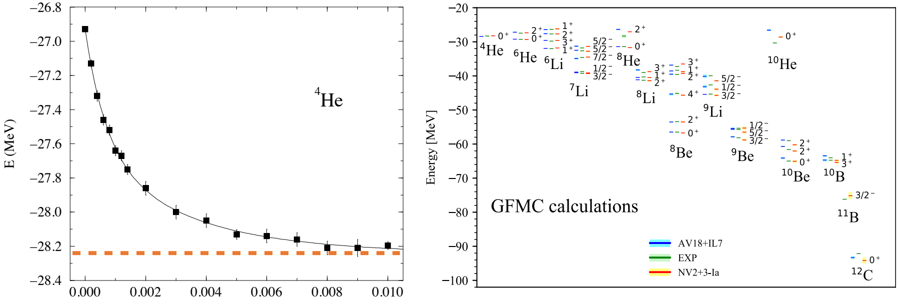


Calculations with NN, NN+3N forces

QUANTUM MONTE CARLO

The Green's function Monte Carlo uses imaginary-time projection techniques to extract the ground-state of the system from the trial wave function

$$\lim_{\tau\to\infty} e^{-(H-E_0)\tau}|\Psi_T\rangle = \lim_{\tau\to\infty}\sum_n c_n e^{-(E_n-E_0)\tau}|\Psi_n\rangle = c_0|\Psi_0\rangle$$



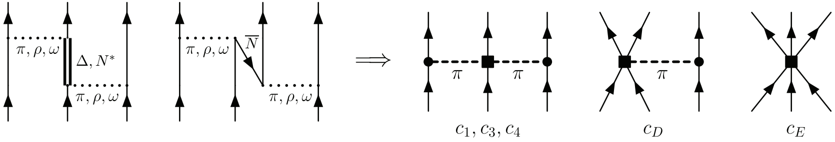
from Alessandro Lovato

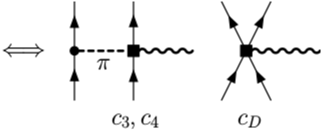
Three-nucleon forces, meson-exchange currents

Forces between 3 nucleons, external probe couplings to 2 nucleons known in nuclear theory for a long time

Fujita and Miyazawa PTP17 (1957), Towner Phys. Rep. 155 (1987)...

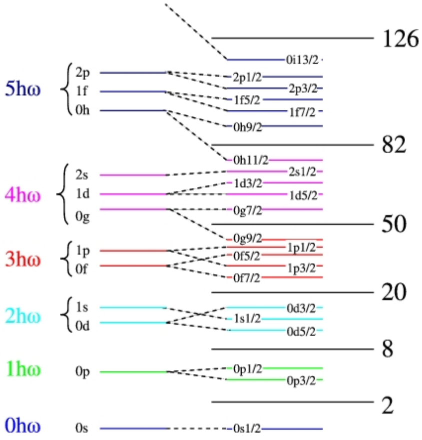
3N forces, 2b currents needed because of missing degrees of freedom (N-body forces appear in any effective theory)





The Δ isobar, with $M_\Delta = 1232$ MeV
relatively low excitation of the nucleon, $M_N = 939$ MeV

The (no core) shell model



The (no core) Shell Model

Many-body wave function
linear combination of
Slater Determinants
from single particle states in the basis
(3D harmonic oscillator)

$$|i\rangle = |n_i l_i j_i m_{j_i} m_{t_i}\rangle$$

$$|\phi_\alpha\rangle = a^+_{i1} a^+_{j2} \dots a^+_{kA} |0\rangle$$

$$|\Psi\rangle = \sum_\alpha c_\alpha |\phi_\alpha\rangle$$

$$H |\Psi\rangle = E |\Psi\rangle$$

$$\text{Dim} \sim \begin{pmatrix} (p+1)(p+2)_\nu \\ N \end{pmatrix} \begin{pmatrix} (p+1)(p+2)_\pi \\ Z \end{pmatrix}$$

Dimensions increase
combinatorially...

Coupled Cluster, In-Medium SRG

Coupled Cluster method

Hagen, Papenbrock, Hjorth-Jensen, Dean, Rep. Prog. Phys. 77, 096302 (2014)

based on a reference state
and acting particle-hole excitation operators (not in the reference state)

$$|\Psi\rangle = e^{-(T_1+T_2+T_3\cdots)}\,|\Phi\rangle$$

with $\quad T_1 = \sum_{\alpha,\bar\alpha} t_{\alpha}^{\bar\alpha}\left\{a^\dagger_{\bar\alpha}, a_\alpha\right\}\,, T_2 = \sum_{\alpha\beta,\bar\alpha\bar\beta} t_{\alpha\beta}^{\bar\alpha\bar\beta}\left\{a^\dagger_{\bar\alpha}a^\dagger_{\bar\beta}, a_\alpha a_\beta\right\}\,,\cdots$

solve $\quad \langle\Phi_{\alpha}^{\bar\alpha}|\, e^{\sum T_i} H e^{-\sum T_i}\,|\Phi\rangle = 0\,, \left\langle\Phi_{\alpha\beta}^{\bar\alpha\bar\beta}\right|\, e^{\sum T_i} H e^{-\sum T_i}\,|\Phi\rangle = 0$

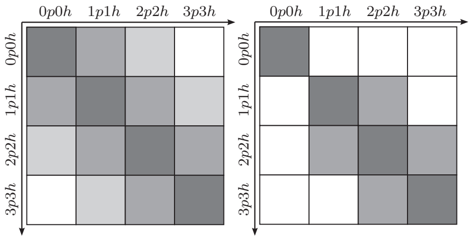
In-medium similarity renormalization group method

Hergert et al. Phys. Rep. 621,165 (2016)

use similarity (unitary) transform
to decouple reference state
from particle-hole excitations

$$H = T + V \to H(s) = U(s) H U^\dagger(s)$$

$$\frac{dH}{ds} = [\eta(s), H(s)] \quad \text{with } \eta(s) = [G(s), H(s)]$$



$\langle i|\,H(0)\,|j\rangle$ $\quad\quad$ $\langle i|\,H(\infty)\,|j\rangle$

Ab initio many-body methods: oxygen

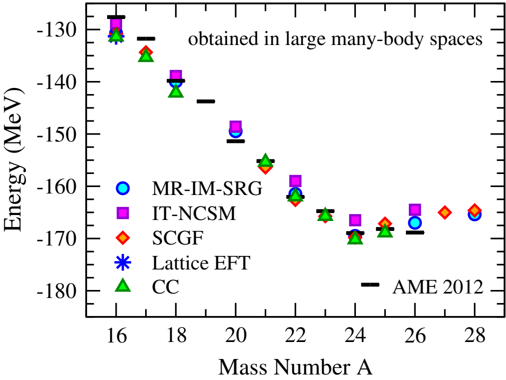
Oxygen dripline using chiral NN+3N forces correctly reproduced
ab-initio calculations treating explicitly all nucleons
excellent agreement between different approaches

No-core shell model
(Importance-truncated)

In-medium SRG
Hergert et al. PRL110 242501(2013)

Self-consistent Green's function
Cipollone et al. PRL111 062501(2013)

Coupled-clusters
Jansen et al. PRL113 142502(2014)



Recent application to ^{208}Pb
Hu, Jiang, Miyagi et al. Nature Phys. 18, 1196 (2022)

Effective shell-model interactions

Coupled Cluster:

Solve coupled-cluster equations for
core (reference state $|\Phi\rangle$), $A+1$ and $A+2$ systems

Project the coupled-cluster solution into valence space
(Okubo-Lee-Suzuki transformation)

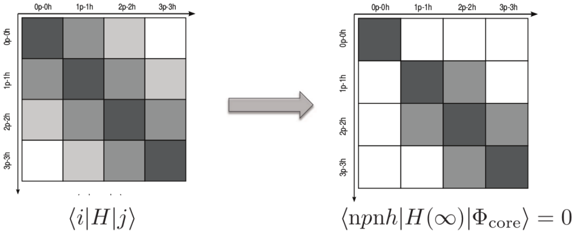
Jansen et al. Phys. Rev. Lett. 113, 142502 (2014)

In-medium similarity
renormalization group

decouple
core from excitations

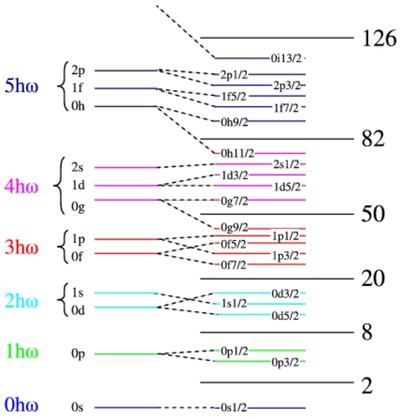
decouple A particles in
valence space from rest

Stroberg et al.
Annu. Rev. Nucl. Part. Sci. 69, 307 (2019)



In addition to H_{eff}, these non-perturbative methods provide the core energy

Nuclear shell model (with core)



The Shell Model solves the many-body problem by direct diagonalization in a relatively small configuration space

The total space is separated into

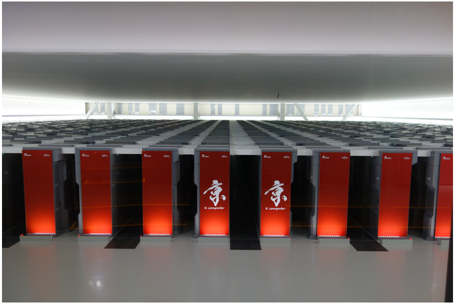
- Outer orbits:
  orbits that are always empty
- Valence space: the space in which we explicitly solve the problem
- Inner core:
  orbits that are always filled

Diagonalization in valence space: $H|\Psi\rangle = E|\Psi\rangle \rightarrow H_{eff}|\Psi\rangle_{eff} = E|\Psi\rangle_{eff}$
where H_{eff} includes the effect of inner core and outer orbits

Nuclear shell model: computational power

Computational power critical for
size of nuclear shell model configuration space to be considered

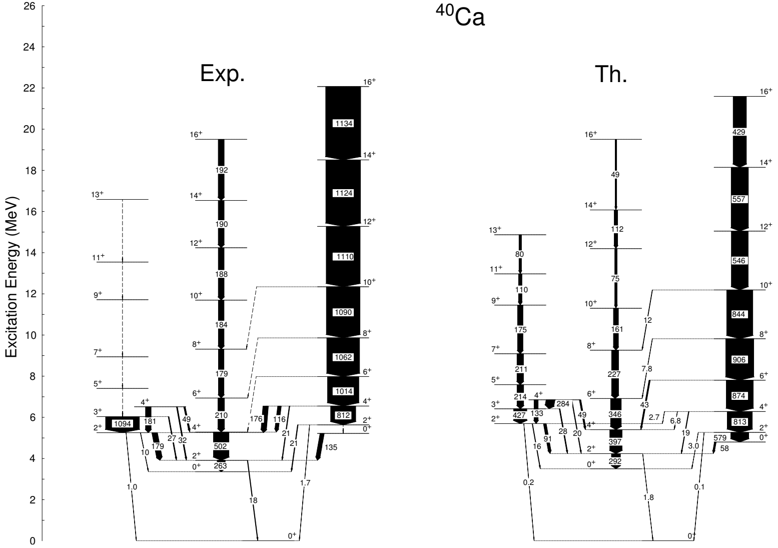
1 major oscillator shell
$\sim 10^9$ Slater dets.
Caurier et al. RMP77 (2005)



>1 major oscillator shells
$\sim 10^{11}$ Slater dets.
Caurier et al. RMP77 (2005)

$\gtrsim 10^{24}$ Slater dets. with Monte Carlo SM
Otsuka, Shimizu, Tsunoda

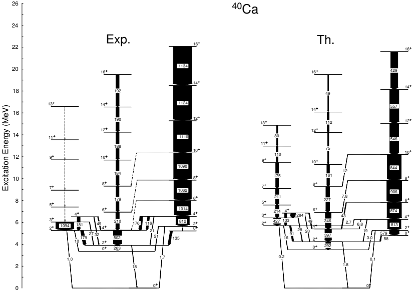
Shell-model calculations, ^{40}Ca

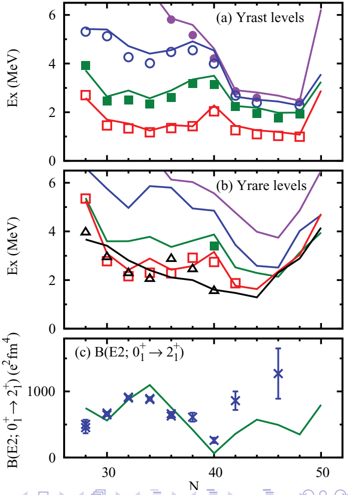


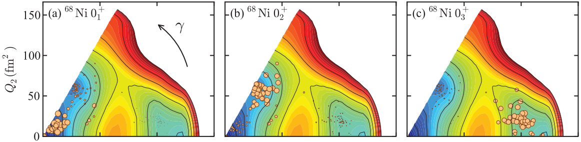
Caurier, JM, Nowacki, Poves, PRC 75, 054317 (2007)

Nuclear shell model: examples

The Shell Model is the method of choice for shell model nuclei:
energies, deformation, electromagnetic and beta transition rates...



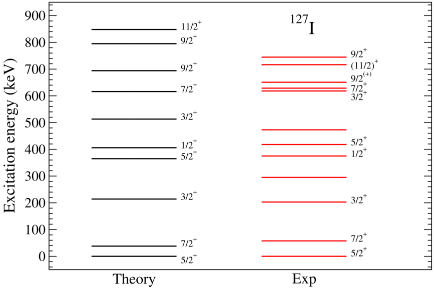


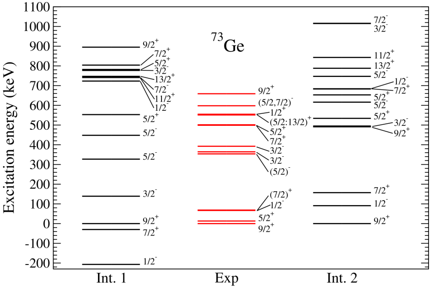


Shell-model spectra for heavy nuclei

Very good general agreement
between the properties of low-energy nuclear states
and nuclear shell-model calculations

However, some nuclei present challenging features
such as ^{73}Ge ground and first-excited state, likely related to deformation

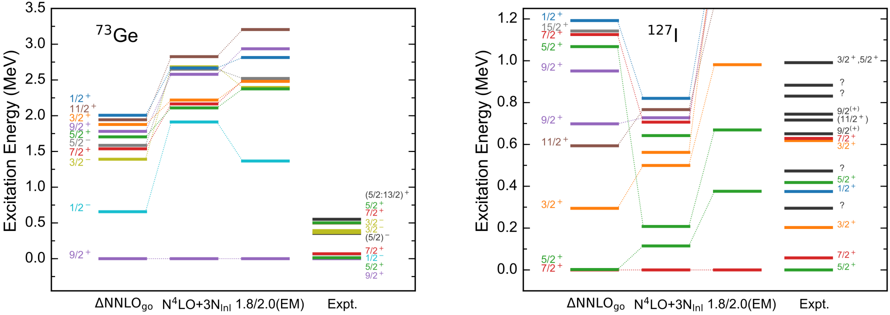




Klos, JM, Gazit, Schwenk, PRD 88, 083516 (2013)

While VS-IMSRG calculations high quality in light nuclei (eg Na) challenges remain in heavier systems, such as ^{73}Ge

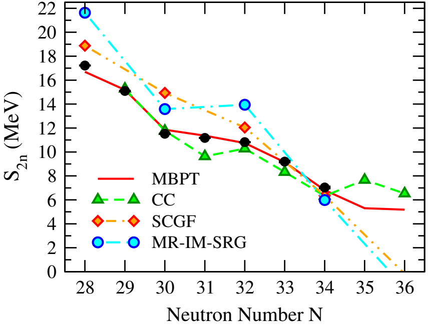
Interesting sensitivity to the chiral nuclear Hamiltonian used for ^{127}I

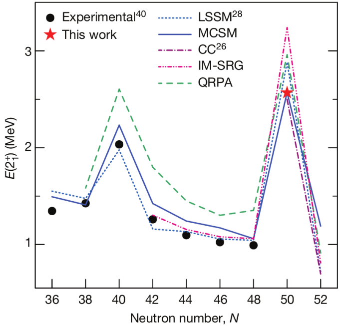


Hu et al. PRL 128, 072502 (2022)

Shell evolution in medium-mass nuclei

Calculations with NN+3N forces predict doubly-magic nuclei ^{52}Ca, ^{54}Ca, ^{78}Ni
groundbreaking mass / 2^+ measurements at ISOLDE / RIBF





LETTER

doi:10.1038/nature12226

Masses of exotic calcium isotopes pin down nuclear forces

F. Wienholtz[1], D. Beck[2], K. Blaum[3], Ch. Borgmann[3], M. Breitenfeldt[4], R. B. Cakirli[3,5], S. George[1], F. Herfurth[2], J. D. Holt[6,7], M. Kowalska[8], S. Kreim[3,8], D. Lunney[9], V. Manea[9], J. Menéndez[6,7], D. Neidherr[2], M. Rosenbusch[1], L. Schweikhard[1], A. Schwenk[7,6], J. Simonis[6,7], J. Stanja[10], R. N. Wolf[1] & K. Zuber[10]

ARTICLE

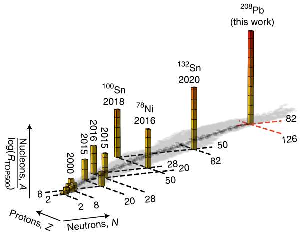
https://doi.org/10.1038/s41586-019-1155-x

^{78}Ni revealed as a doubly magic stronghold against nuclear deformation

R. Taniuchi[1,2], C. Santamaria[2,3], P. Doornenbal[2*], A. Obertelli[2,3,4], K. Yoneda[2], G. Authelet[3], H. Baba[2], D. Calvet[3], F. Château[3], A. Corsi[3], A. Delbart[3], J.-M. Gheller[3], A. Gillibert[3], J. D. Holt[5], T. Isobe[2], V. Lapoux[3], M. Matsushita[6], J. Menéndez[6], S. Momiyama[1,2], T. Motobayashi[2], M. Niikura[1], F. Nowacki[7], K. Ogata[8,9], H. Otsu[2], T. Otsuka[1,2,6], C. Péron[3], S. Péru[10], A. Peyaud[3], E. C. Pollacco[3], A. Poves[11], J.-Y. Roussé[3], H. Sakurai[1,2], A. Schwenk[4,12,13], Y. Shiga[2,14], J. Simonis[4,12,15], S. R. Stroberg[5,16], S. Takeuchi[2], Y. Tsunoda[6], T. Uesaka[2], H. Wang[2], F. Browne[17], L. X. Chung[18], Z. Dombradi[19], S. Franchoo[20], F. Giacoppo[21], A. Gottardo[20], K. Hadyńska-Klęk[21], Z. Korkulu[19], S. Koyama[1,2], Y. Kubota[2,6], J. Lee[22], M. Lettmann[4], C. Louchart[4], R. Lozeva[7,23], K. Matsui[1,2], T. Miyazaki[1,2], S. Nishimura[2], L. Olivier[20], S. Ota[6], Z. Patel[24], E. Şahin[21], C. Shand[24], P.-A. Söderström[2], I. Stefan[20], D. Steppenbeck[6], T. Sumikama[25], D. Suzuki[20], Z. Vajta[19], V. Werner[4], J. Wu[2,26] & Z. Y. Xu[22]

Ab initio predictions for heavy nuclei

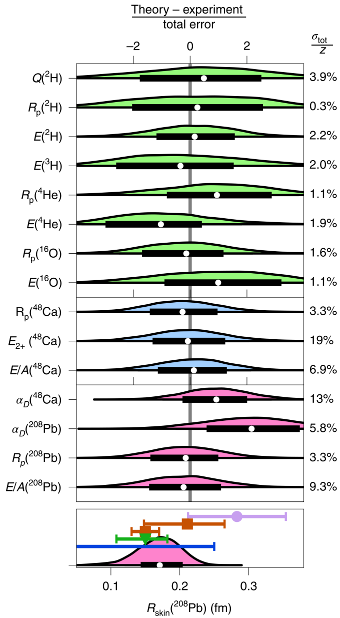
Very notable progress
ab initio calculations of
(relatively uncorrelated) heavy nuclei
reaching ^{208}Pb



Determine ^{208}Pb neutron skin
using Bayesian approach
based on sampling of 10^9
(parameters of) nuclear Hamiltonians

Hu, Jiang, Miyagi et al.

Nature Phys. 18, 1196 (2022)



Many-body methods for $0\nu\beta\beta$ decay

Different many-body methods are used in $0\nu\beta\beta$ decay

- Nuclear shell Model
  Madrid-Strasbourg, Michigan, Bucharest, Tokyo
  Relatively small valence spaces (one shell), all correlations included
- Quasiparticle random-phase approximation (QRPA) method
  Tübingen, Bratislava, Jyvaskyla, Chapel Hill, Prague...
  Several shells, only simple correlations included
- Interacting Boson Model
  Yale-Concepción
  Small space, important proton-neutron pairing correlations missing
- Energy Density Functional theory
  Madrid, Beijing
  $>$ 10 shells, important proton-neutron pairing correlations missing

Ab initio many-body methods:
No Core Shell Model, Green's Function Monte Carlo, Coupled Cluster...

Outline

1. Nuclear structure: initial and final states
2. β decay: operator and nuclear matrix elements
3. $\beta\beta$ decay operators
4. $0\nu\beta\beta$ decay nuclear matrix elements

Weak interactions in nuclei

β and $\beta\beta$ decay processes are driven by the Weak interaction

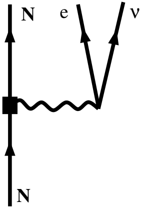
$$H_W = \frac{G_F}{\sqrt{2}} \left(j_{L\mu} J_L^{\mu\dagger} \right) + H.c.$$

$j_{L\mu}$ is the leptonic current (electron, neutrino): $j_{L\mu} = \overline{e}\gamma_\mu \left(1 - \gamma_5\right) \nu_{eL}$

The Lorentz structure is Vector $-$ Axial-Vector ($V - A$) current,
as indicated by the Standard Model of Particle Physics

For neutrinos,
interaction eigenstates are not mass eigenstates:
$\nu_{eL} = \sum_i U_{ei} \nu_{iL}$,
with U the PMNS neutrino-mixing matrix

The treatment of electrons and neutrinos
is relatively easy because
they are elementary particles



Weak interactions: hadronic current

β and $\beta\beta$ decay processes are driven by the Weak interaction

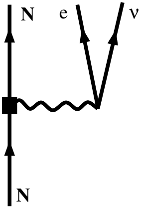
$$H_W = \frac{G_F}{\sqrt{2}} \left(j_{L\mu} J_L^{\mu\dagger} \right) + H.c.$$

$J_L^{\mu\dagger}$ is the hadronic current:
it is not so straightforward because the Standard Model predicts $J_L^{\mu\dagger}$ at the level of quarks and we need $J_L^{\mu\dagger}$ at the level of nucleons:

- Obtain $J_L^{\mu\dagger}$ phenomenologically
- Obtain $J_L^{\mu\dagger}$ using an effective theory: Chiral EFT!

In nuclei (non-relativistic), β decay is simply

$$\langle F| \sum_i g_V\, \tau_i^- + g_A\, \boldsymbol{\sigma}_i \tau_i^- \, |I\rangle$$



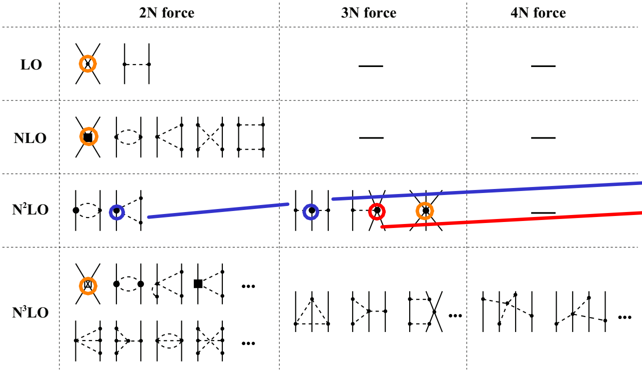
corresponding to Fermi and Gamow-Teller transitions,
corrections (forbidden transitions)
involve an expansion of the lepton wavefunctions

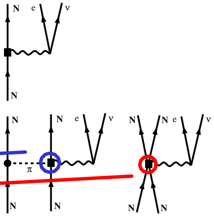
Chiral Effective Field Theory

Chiral EFT: low energy approach to QCD, nuclear structure energies

Approximate chiral symmetry: pion exchanges, contact interactions

Systematic expansion: nuclear forces and electroweak currents





Weinberg, van Kolck, Kaplan, Savage, Wise, Meißner, Epelbaum...

Park, Krebs, Baroni...

Short-range couplings fitted to experiment once

Chiral EFT weak currents

Remember, the weak interaction is V-A: vector-axial
Chiral EFT currents: calculate systematically at Q^0, Q^2 ... order

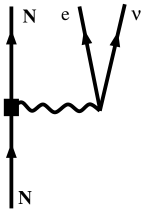
At order Q^0 standard Fermi, Gamow-Teller operators

$$J_i^0(p) = g_V \tau^-, \qquad \boldsymbol{J}_i(p) = g_A \boldsymbol{\sigma}$$

At oder Q^2 loop and pion-pole corrections

$$J_i^0(p) = g_V(p^2)\tau^-,$$
$$\boldsymbol{J}_i(p) = \left[g_A(p^2)\boldsymbol{\sigma} - g_P(p^2)\frac{(\boldsymbol{p}\cdot\boldsymbol{\sigma}_i)\boldsymbol{p}}{2m} + i\,(g_M + g_V)\,\frac{\boldsymbol{\sigma}_i \times \boldsymbol{p}}{2m} \right]\tau^-,$$

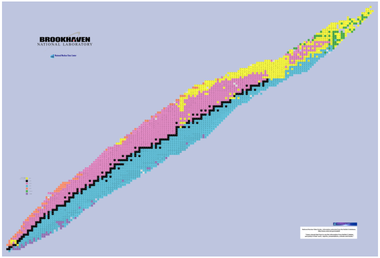
$$g_V(p^2) = g_V\,(1 - 2\frac{p^2}{\Lambda_V^2}), \qquad g_A(p^2) = g_A\,(1 - 2\frac{p^2}{\Lambda_A^2}),$$
$$g_P(p^2) = \frac{2g_{\pi pn}F_\pi}{m_\pi^2 + p^2} - 4\,g_A(p^2)\frac{m}{\Lambda_A^2}, \qquad g_M = \kappa_p - \kappa_n = 3.70,$$

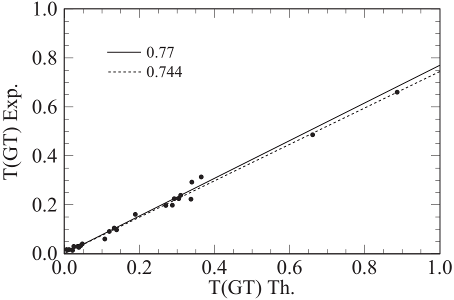


Order Q^2 corrections are not relevant for single-β decay, $2\nu\beta\beta$ decay, because in these processes $\mathbf{p} \sim 0$

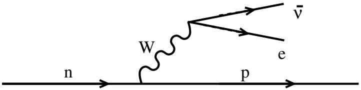
β decay: theory vs experiment

β decays (e^- capture) main decay model along nuclear chart
In general well described by nuclear structure theory: shell model...





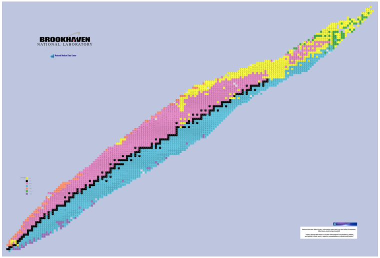
Martinez-Pinedo et al. PRC53 2602(1996)

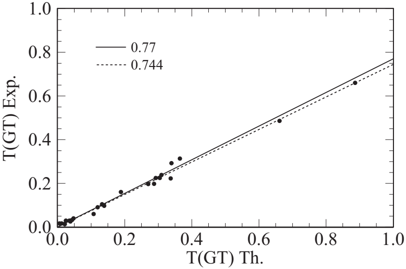


$$\langle F | \sum_i [g_A \, \boldsymbol{\sigma}_i \tau_i^-] \, | I \rangle$$

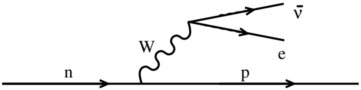
β decay: "quenching"

β decays (e^- capture) main decay model along nuclear chart
In general well described by nuclear structure theory: shell model...





Martinez-Pinedo et al. PRC53 2602(1996)



$$\langle F | \sum_i [g_A \, \boldsymbol{\sigma}_i \tau_i^-]^{\text{eff}} \, | I \rangle \, , \quad [\boldsymbol{\sigma}_i \tau]^{\text{eff}} \approx 0.7 \boldsymbol{\sigma}_i \tau$$

Gamow-Teller transitions:
theory needs $\boldsymbol{\sigma}_i \tau$ "quenching"

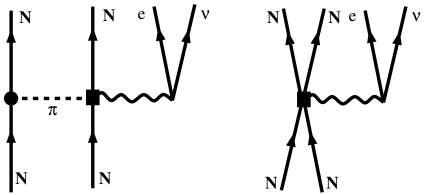
Deficient many-body approach,
or transition operator?

Two-body currents currents

At order Q^3 chiral EFT predicts contributions from two-body (2b) currents

Reflect interactions between nucleons

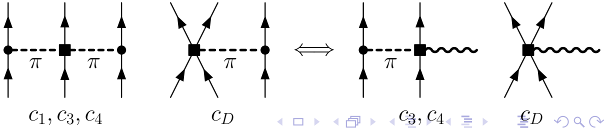
Long-range currents dominate



The expression for the leading Q^3 2b currents is

$$\mathbf{J}_{12}^3 = -\frac{g_A}{4F_\pi^2}\frac{1}{m_\pi^2+k^2}\left[2\left(c_4+\frac{1}{4m}\right)\mathbf{k}\times(\sigma_\times\times\mathbf{k})\tau_\times^3 + 4c_3\mathbf{k}\cdot\left(\sigma_1\tau_1^3+\sigma_2\tau_2^3\right)\mathbf{k} - \frac{i}{m}\mathbf{k}\cdot(\sigma_1-\sigma_2)\mathbf{q}\tau_\times^3\right]$$

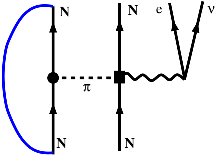
Long-range currents depend on c_3, c_4 couplings of nuclear forces

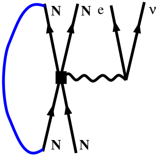


2b currents: normal-ordering

Approximate in medium-mass nuclei:
2b currents imply that the $\beta\beta$ decay operator is 4-body...
normal-ordered 1b part with respect to spin/isospin symmetric Fermi gas

Sum over one nucleon, direct and the exchange terms





$\Rightarrow \mathbf{J}^{\text{eff}}_{n,2b}$ normal-ordered 1b current

Corrections $\sim (n_{\text{valence}}/n_{\text{core}})$
in Fermi systems

The normal-ordered two-body currents modify GT operator

$$\mathbf{J}^{\text{eff}}_{n,2b} = \sum_{\boldsymbol{\sigma}_m}^{FG} \sum_{\tau_m}^{FG} \int \frac{p_m^2 dp_m}{(2\pi)^3} \, \mathbf{J}_{m,n,2b} \, (1 - P_{mn})$$

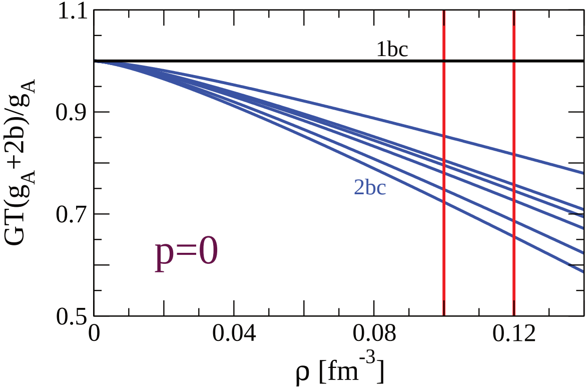
$$= -\frac{g_A \rho}{f_\pi^2} \tau_n^- \sigma_n \left[\frac{2}{3} c_3 \frac{\mathbf{p}^2}{4m_\pi^2 + \mathbf{p}^2} + I(\rho, P) \left(\frac{1}{3}(2c_4 - c_3) + \frac{1}{6m_N} \right) \right],$$

long-range p dependent   long-range p independent

2b currents at zero momentum-transfer

2b currents at $p = 0$: relevant for Gamow-Teller decays, $2\nu\beta\beta$ decay

$$\mathbf{J}^{\text{eff}}_{n,2b} = -\frac{g_A \rho}{f_\pi^2} \tau_n^- \sigma_n \left[I(\rho, P) \left(\frac{1}{3} (2c_4 - c_3) + \frac{1}{6 m_N} \right) \right],$$



Black horizontal line represents 1b current

Difference horizontal line and blue lines 2b effect

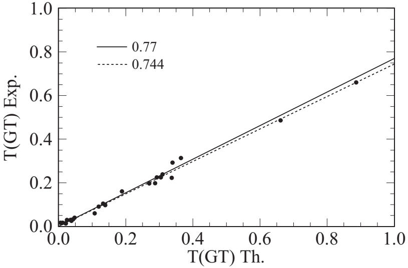
Nuclear density range $\rho = 0.10 \ldots 0.12$ fm^{-3}

Couplings c_3, c_4 taken from NN potentials

2b currents, in normal-ordered approximation predict g_A quenching

Gamow-Teller β decay with IM-SRG

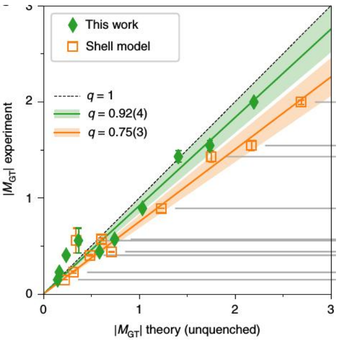
β decays (e^- capture) challenge for nuclear theory



Martinez-Pinedo et al. PRC53 2602(1996)

$$\langle F | \sum_i [g_A \, \boldsymbol{\sigma}_i \tau_i^-]^{\text{eff}} | I \rangle \,, \quad [\boldsymbol{\sigma}_i \tau]^{\text{eff}} \approx 0.7 \boldsymbol{\sigma}_i \tau$$

Phenomenological models
need $\boldsymbol{\sigma}_i \tau$ "quenching"

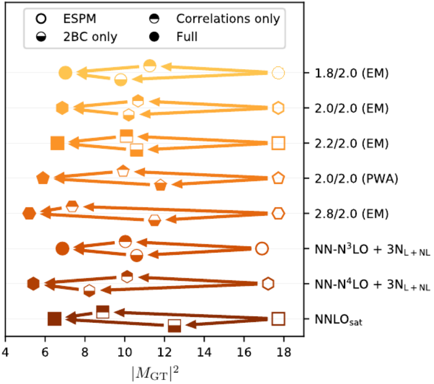


Gysbers et al. Nature Phys. 15 428 (2019)

Ab initio calculations including
meson-exchange currents
do not need any "quenching"

Origin of β-decay "quenching"

Complementary, similar impact of nuclear correlations and meson-exchange currents

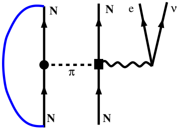


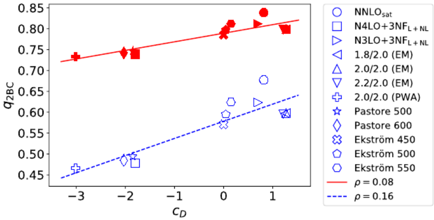
Gysbers et al. Nature Phys. 15 428 (2019)

2b currents modify GT operator

JM, Gazit, Schwenk PRL107 062501 (2011)

$$\boldsymbol{J}^{\text{eff}}_{n,2b} \simeq -\frac{g_A \rho}{f_\pi^2} \tau_n^- \boldsymbol{\sigma}_n \times \left[I(\rho) \frac{2c_4 - c_3}{3} - \frac{c_D}{4 g_A \Lambda_\chi} - \frac{2 g_A \rho}{3 f_\pi^2} c_3 \frac{p^2}{m_\pi^2 + p^2} \right]$$





2b currents in $0\nu\beta\beta$ decay

In $0\nu\beta\beta$ decay, two weak currents lead to four-body operator when including the product of two 2b currents: computational challenge

Approximate 2b current as effective 1b current normal ordering with respect to a Fermi gas

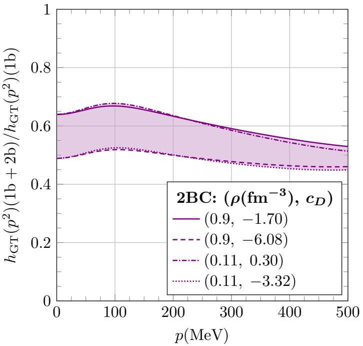
JM, Gazit, Schwenk, PRL107 062501(2011)

Normal-odering approximation works remarkably well for β decay ($q = 0$)

Gysbers et al. Nature Phys. 15 428 (2019)

Some reduction of quenching due to 2b currents at $p \sim m_\pi$ relevant for $0\nu\beta\beta$ decay

Hoferichter, JM, Schwenk PRD102 074018 (2020)



Jokiniemi, Romeo, Soriano, JM, PRC 107 044305 (2023)

Outline

Two-neutrino $\beta\beta$ decay matrix elements

Two-neutrino double-beta decay matrix element, second order process

$$M^{2\nu\beta\beta} = \sum_k \frac{\langle 0_f^+ | \sum_n \tau_n^- + \sigma_n\tau_n^- | J_k^+ \rangle \langle J_k^+ | \sum_m \tau_m^- + \sigma_m\tau_m^- | 0_i^+ \rangle}{E_k - (M_i + M_f)/2}$$
$$= \sum_k \frac{\langle 0_f^+ | \sum_n \sigma_n\tau_n^- | J_k^+ \rangle \langle J_k^+ | \sum_m \sigma_m\tau_m^- | 0_i^+ \rangle}{E_k - (M_i + M_f)/2}$$
$$= \sum_k \frac{\langle 0_f^+ | \sum_n \sigma_n\tau_n^- | 1_k^+ \rangle \langle 1_k^+ | \sum_m \sigma_m\tau_m^- | 0_i^+ \rangle}{E_k - (M_i + M_f)/2}$$

- $\tau_n^- \tau_m^-$ transform two neutrons into two protons
- Only Gamow-Teller spin operator contributes:
  Fermi contribution vanishes due to isospin conservation:

$$\langle 0_f^+ | \sum_m \tau_m^- | J_k^+ \rangle = \langle 0_f^+ | T^- | J_k^+ \rangle \sim 0$$

- Neutrinos are emitted, do not appear in the transition operator

$\Rightarrow$ Only intermediate nucleus $|1_k^+\rangle$ states contribute

Two-neutrino $\beta\beta$ decay calculations

$$M^{2\nu\beta\beta} = \sum_k \frac{\langle 0_f^+ | \sum_n \sigma_n \tau_n^- | 1_k^+ \rangle \langle 1_k^+ | \sum_m \sigma_m \tau_m^- | 0_i^+ \rangle}{E_k - (M_i + M_f)/2}$$

Shell Model $2\nu\beta\beta$ decay calculations
in good agreement to experiment

GT quenching is needed

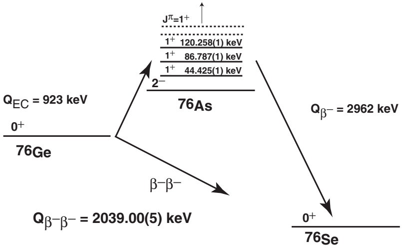
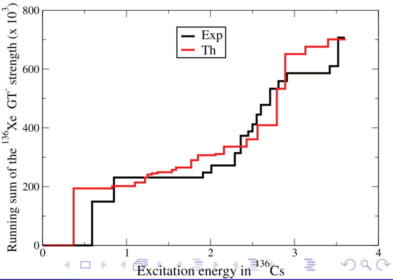


Table 2
The ISM predictions for the matrix element of several 2ν double beta decays (in MeV^{-1}). See text for the definitions of the valence spaces and interactions.

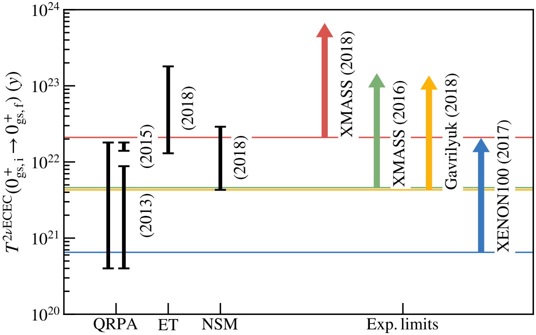
	$M^{2\nu}$(exp)	q	$M^{2\nu}$(th)	INT
$^{48}Ca \rightarrow ^{48}Ti$	0.047 ± 0.003	0.74	0.047	kb3
$^{48}Ca \rightarrow ^{48}Ti$	0.047 ± 0.003	0.74	0.048	kb3g
$^{48}Ca \rightarrow ^{48}Ti$	0.047 ± 0.003	0.74	0.065	gxpf1
$^{76}Ge \rightarrow ^{76}Se$	0.140 ± 0.005	0.60	0.116	gcn28:50
$^{76}Ge \rightarrow ^{76}Se$	0.140 ± 0.005	0.60	0.120	jun45
$^{82}Se \rightarrow ^{82}Kr$	0.098 ± 0.004	0.60	0.126	gcn28:50
$^{82}Se \rightarrow ^{82}Kr$	0.098 ± 0.004	0.60	0.124	jun45
$^{128}Te \rightarrow ^{128}Xe$	0.049 ± 0.006	0.57	0.059	gcn50:82
$^{130}Te \rightarrow ^{130}Xe$	0.034 ± 0.003	0.57	0.043	gcn50:82
$^{136}Xe \rightarrow ^{136}Ba$	0.019 ± 0.002	0.45	0.025	gcn50:82



Gamow-Teller Strengths
(each leg of the $\beta\beta$ decay) are well reproduced

Two-neutrino ECEC of ^{124}Xe

Predicted 2νECEC half-life:
shell model error bar largely dominated by “quenching" uncertainty



Suhonen
JPG 40 075102 (2013)

Pirinen, Suhonen
PRC 91, 054309 (2015)

Coello Pérez, JM, Schwenk
PLB 797 134885 (2019)

Shell model, QRPA and Effective theory (ET) predictions
suggest experimental detection close to XMASS 2018 limit

Two-neutrino ECEC of ^{124}Xe

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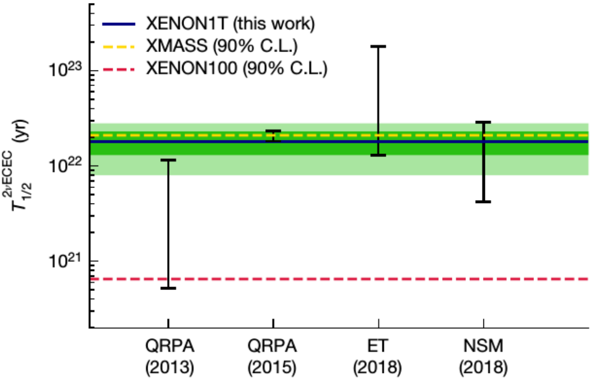


Suhonen
JPG 40 075102 (2013)

Pirinen, Suhonen
PRC 91, 054309 (2015)

Coello Pérez, JM, Schwenk
PLB 797 134885 (2019)

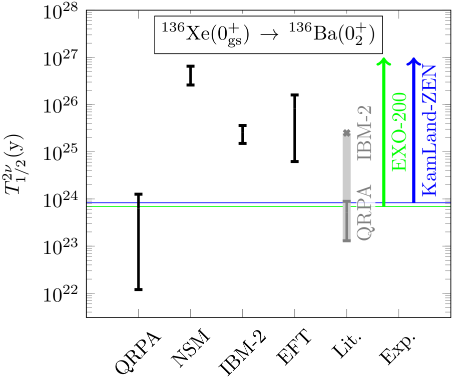
XENON1T
Nature 568 532 (2019)

Shell model, QRPA and Effective theory (ET) predictions
suggest experimental detection close to XMASS 2018 limit

$2\nu\beta\beta$ decay of ^{136}Xe to ^{136}Ba 0_2^+

Current experiments sensitive to two-neutrino $\beta\beta$ of ^{136}Xe to ^{136}Ba 0_2^+

EXO-200, KamLAND-Zen



Nuclear shell model
QRPA, EFT and IBM
very different predictions!

Barea et al.
PRC 91 034304 (2015)

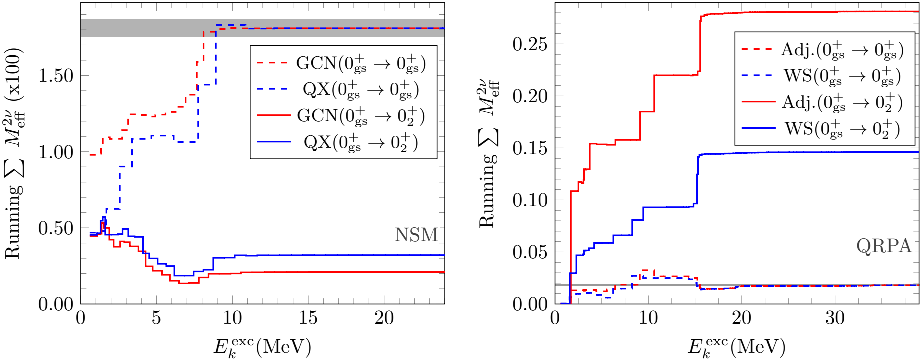
Pirinen, Suhonen
PRC 91, 054309 (2015)

Jokiniemi, Romeo, Brase, Kotila et al.
PLB 838 137689 (2023)

Very good test of theoretical calculations!

^{136}Xe $\longrightarrow$ ^{136}Ba 0_2^+ running sums

Subtle cancellation NME running sum, depends on many-body method



Jokiniemi, Romeo, Brase, Kotila et al. PLB 838 137689 (2023)

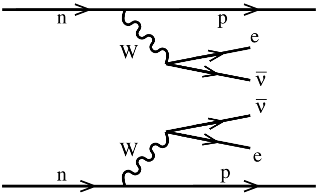
Shell-model running sum shows cancellations in decay to ground state

QRPA running sum shows cancellations in decay to excited state

Since ground-state decay fitted to data, very different decay to excited state

$0\nu\beta\beta$ decay vs $2\nu\beta\beta$ decays

From the theoretical point of view, $0\nu\beta\beta$ and $2\nu\beta\beta$ decays are also different



- In $2\nu\beta\beta$ decay, the momentum transfer to the leptons is limited by $Q_{\beta\beta}$, while for $0\nu\beta\beta$ decay larger momentum transfers are permitted
- In $0\nu\beta\beta$ decay the Majorana neutrinos annihilate each other which is only possible if neutrinos have mass
- In $0\nu\beta\beta$ decay the Majorana neutrinos are part of the transition operator, via the so-called neutrino potential

$0\nu\beta\beta$: closure approximation

The neutrinos can carry large momentum, $p \sim 100$ MeV,
much larger than the excitation energies in the intermediate states $|N_a\rangle$

The closure approximation can be used (good to 90%)

$$\sum_a \frac{\langle N_f| J_L^{\mu\dagger}(\mathbf{x}) |N_a\rangle \langle N_a| J_L^{\rho\dagger}(\mathbf{y}) |N_i\rangle}{p + E_a - \frac{1}{2}(E_i + E_f)}$$
$$\simeq \frac{1}{p + \langle E\rangle - \frac{1}{2}(E_i + E_f))} \sum_a \langle N_f| J_L^{\mu\dagger}(\mathbf{x}) |N_a\rangle \langle N_a| J_L^{\rho\dagger}(\mathbf{y}) |N_i\rangle$$
$$= \frac{1}{p + \langle E\rangle - \frac{1}{2}(E_i + E_f))} \langle N_f| J_L^{\mu\dagger}(\mathbf{x}) J_L^{\rho\dagger}(\mathbf{y}) |N_i\rangle\,.$$

This simplifies the calculation, only initial and final states are needed

We still need the transition operator!

From currents to transition operator

The transition operator originates from the nuclear currents and the neutrinos

$$M^{0\nu\beta\beta}(m_j) = \langle 0_f^+ | \sum_{n,m} \tau_n^- \tau_m^- \frac{R}{g_A^2} \int \frac{d\mathbf{p}}{2\pi^2} e^{i\mathbf{p}\cdot(\mathbf{r}_n - \mathbf{r}_m)} \frac{\mathbf{J}_n \mathbf{J}_m(p^2)}{p\,(p+\mu)} |0_i^+\rangle$$

$$= \langle 0_f^+ | \sum_{n,m} \tau_n^- \tau_m^- \frac{R}{g_A^2} \int \frac{d\mathbf{p}}{2\pi^2} e^{i\mathbf{p}\cdot(\mathbf{r}_n - \mathbf{r}_m)} \frac{\Omega_{nm}(p^2)}{p\,(p+\mu)} |0_i^+\rangle$$

$$= \langle 0_f^+ | \sum_{n,m} \tau_n^- \tau_m^- \left(-H^F(r) + H^{GT}(r)\boldsymbol{\sigma}_n\boldsymbol{\sigma}_m - H^T(r)\mathbf{S}^r_{nm}\right) |0_i^+\rangle$$

The integral over $\mathbf{p}$ is performed and
$r = |\mathbf{r}_n - \mathbf{r}_m|$ distance between decaying neutrons, $H(r)$ neutrino potentials.

There is three spin structures contributing to $0\nu\beta\beta$ decay:
Fermi ($\mathbb{1}$), Gamow-Teller ($\sigma_1\sigma_2$), Tensor (S_{12})

The Gamow-Teller term is dominant ($\sim 85\%$)

$0\nu\beta\beta$ decay nuclear matrix elements

$0\nu\beta\beta$ process needs massive Majorana neutrinos ($\nu = \bar{\nu}$)
$\Rightarrow$ detection would proof Majorana nature of neutrinos

$$\left(T_{1/2}^{0\nu\beta\beta}\left(0^+ \to 0^+\right)\right)^{-1} = G_{01}\, g_A^4 \left|M^{0\nu\beta\beta}\right|^2 m_{\beta\beta}^2$$

G_{01} is the phase space factor
includes information of $Q_{\beta\beta}$, electrons...

g_A is the axial coupling (hadronic matrix element)

$M^{0\nu\beta\beta}$ is the nuclear matrix element

$m_{\beta\beta} = |\sum U_{ek}^2 m_k|$, represents physics beyond the Standard Model

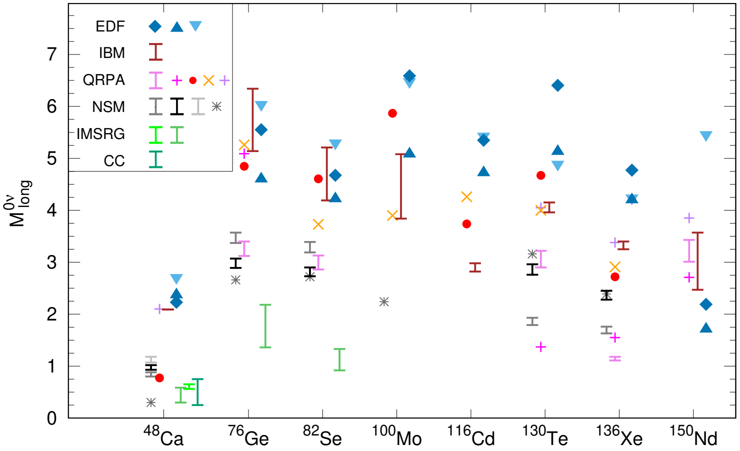
Sensitive to absolute neutrino masses, compete with other determinations:
single-β decay ($\sqrt{\sum |U_{ek}|^2 m_k^2}$) and cosmology ($\sum m_k$)

Outline

1. Nuclear structure: initial and final states
2. β decay: operator and nuclear matrix elements
3. $\beta\beta$ decay operators
4. $0\nu\beta\beta$ decay nuclear matrix elements

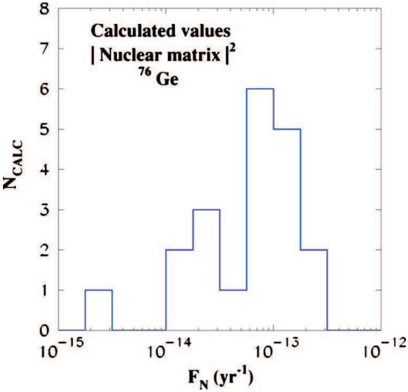
$0\nu\beta\beta$ decay nuclear matrix elements

Large difference in nuclear matrix element calculations: factor ~ 3



Agostini, Benato, Detwiler, JM, Vissani, Rev. Mod. Phys. 95, 025002 (2023)

Spread about factor two – three in nuclear matrix element calculations



But this means a big improvement!

The uncertainty in the calculated nuclear matrix elements for neutrinoless double beta decay will constitute the principal obstacle to answering some basic questions about neutrinos. The essential problem is that the correct theory of nuclei

Bahcall, Murayama, Peña-Garay
PRD70 033012 (2004)

IM-GCM $0\nu\beta\beta$ NME for ^{48}Ca

Multi-reference calculation:
correlations systematically built on collective reference state
Generator coordinate method: deformation, isoscalar pairing

$$\langle 0_f^+ | \sum_{n,m} \tau_n^- \tau_m^- \sum_x H^x(r)\, \Omega^x \, | 0_i^+ \rangle$$

Best IM-GCM calculation reproduces EM transitions in ^{48}Ti

NME $\sim 0.4/30\%$ smaller than nuclear shell model

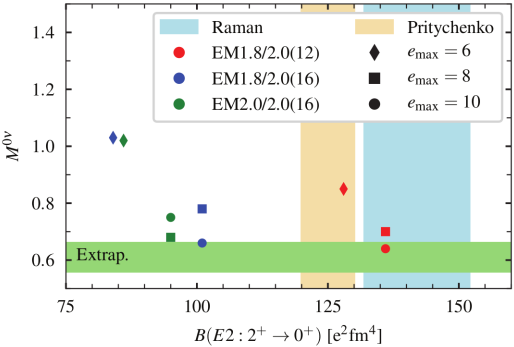
Yao et al.

PRL 124 232501 (2020)

Consistent with coupled cluster NME

Novario et al.

PRL 126 182502 (2021)



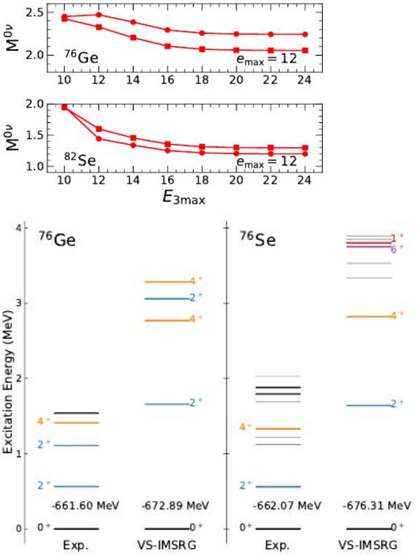
VS-IMSRG $0\nu\beta\beta$ NME for ^{76}Ge, ^{82}Se

VS-IMSRG reaches ^{76}Ge
one of the targets used in
most advanced experiments
(GERDA, MAJORANA)

VS-IMSRG NME converged
in 3N matrix elements included
Miyagi et al.
PRC105 014302 (2022)

Excitation spectra too spread
quadrupole correlations
not properly captured?

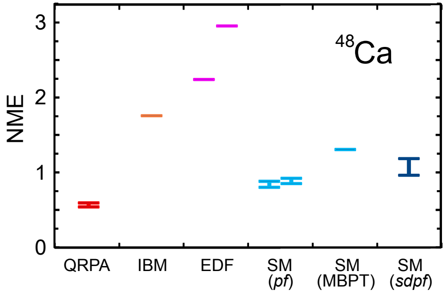
NME $\sim 20\%/50\%$ smaller
than nuclear shell model
Belley et al.
PRL126 042502 (2021)

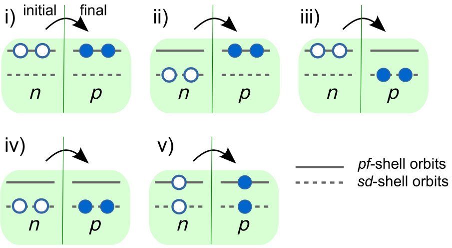


Shell model configuration space: two shells

^{48}Ca extended configuration space from *pf* to *sdpf*, 4 to 7 orbitals
dimension 10^5 to 10^9
^{48}Ca 0_2^+ state lowered by 1.3 MeV
nuclear matrix elements enhanced only moderately 30%

Iwata et al. PRL116 112502 (2016)



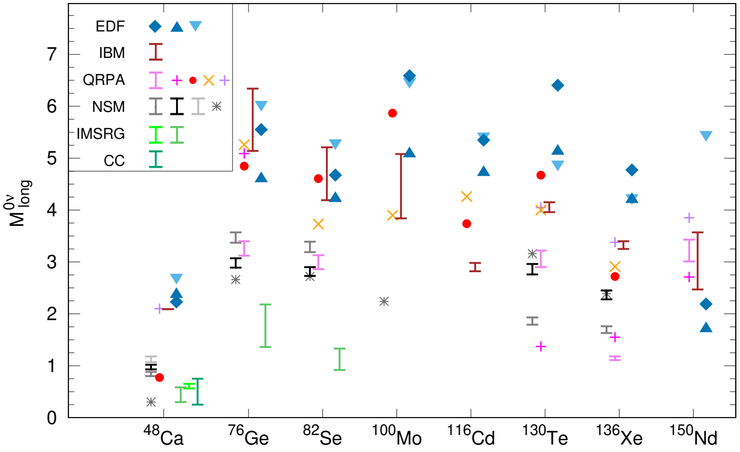


Terms dominated by pairing
2 particle – 2 hole excitations
enhance the $\beta\beta$ matrix element

Terms dominated by
1 particle – 1 hole excitations
suppress the $\beta\beta$ matrix element

$0\nu\beta\beta$ decay nuclear matrix elements

Large difference in nuclear matrix element calculations: factor ~ 3

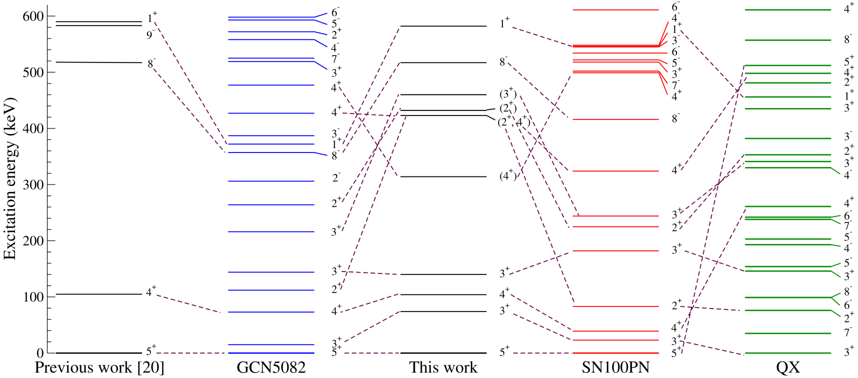


Agostini, Benato, Detwiler, JM, Vissani, Rev. Mod. Phys. 95, 025002 (2023)

^{136}Cs experimental spectrum

While all these interactions are well, tested recent data on ^{136}Cs suggests GCN5082 results agree better with experiment than QX

Rebeiro, Triambak et al. arXiv:2301.11371

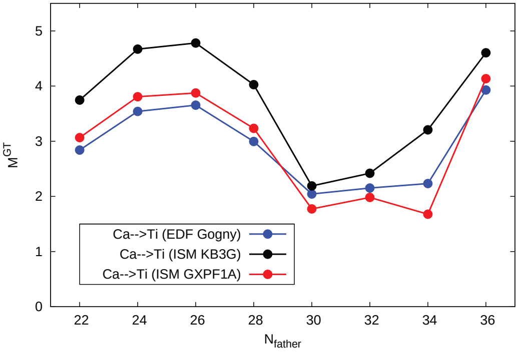


QX gives systematically smaller ^{136}Xe $0\nu\beta\beta$-decay nuclear matrix elements

$0\nu\beta\beta$ decay without correlations

Non-realistic spherical (uncorrelated) mother and daughter nuclei:

- Shell model (SM): zero seniority, neutron and proton $J = 0$ pairs
- Energy density functional (EDF): only spherical contributions



In contrast to full (correlated) calculation SM and EDF NMEs agree!

NME scale set by pairing interaction

JM, Rodríguez, Martínez-Pinedo, Poves PRC90 024311(2014)

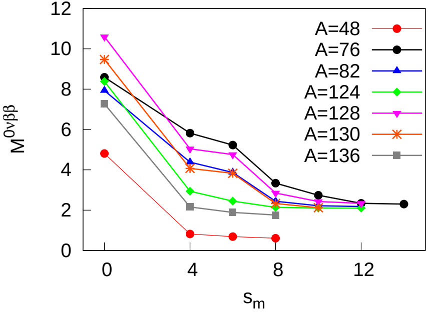
NME follows generalized seniority model:

$$M_{GT}^{0\nu\beta\beta} \simeq \alpha_\pi \alpha_\nu \sqrt{N_\pi + 1}\sqrt{\Omega_\pi - N_\pi}\sqrt{N_\nu}\sqrt{\Omega_\nu - N_\nu + 1},$$ Barea, Iachello PRC79 044301(2009)

Pairing correlations and $0\nu\beta\beta$ decay

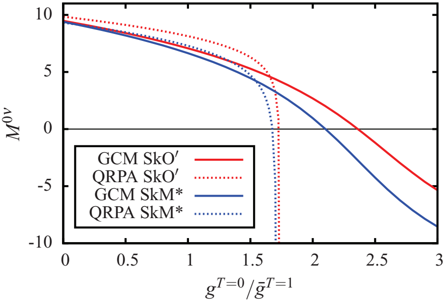
$0\nu\beta\beta$ decay favoured by proton-proton, neutron-neutron pairing, but it is disfavored by proton-neutron pairing

Ideal case: superfluid nuclei reduced with high-seniorities



Caurier et al. PRL100 052503 (2008)

Addition of isoscalar pairing reduces matrix element value



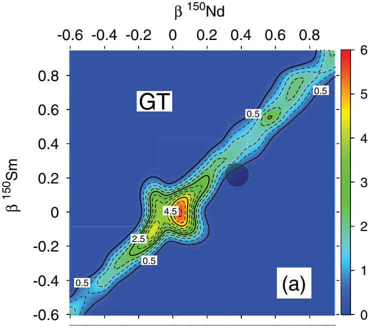
Hinohara, Engel PRC90 031301 (2014)

Related to approximate $SU(4)$ symmetry of the $\sum H(r)\boldsymbol{\sigma}_i\boldsymbol{\sigma}_j\tau_i\tau_j$ operator

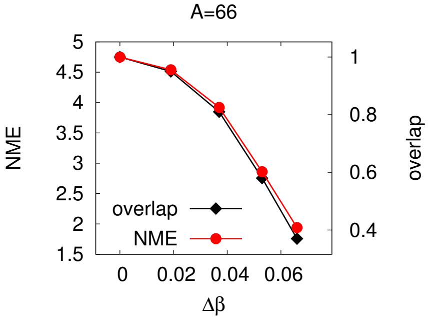
Deformation and $0\nu\beta\beta$ decay

$0\nu\beta\beta$ decay is disfavoured by quadrupole correlations

$0\nu\beta\beta$ decay very suppressed when nuclei have different structure



Rodríguez, Martínez-Pinedo
PRL105 252503 (2010)



JM, Caurier, Nowacki, Poves
JPCS267 012058 (2011)

Suppression also observed with QRPA Fang et al. PRC83 034320 (2011)

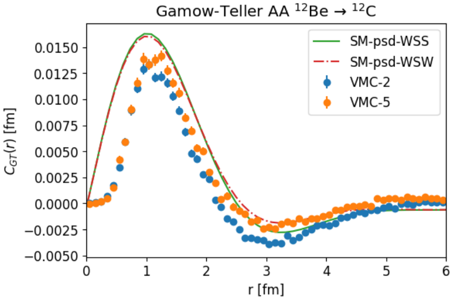
Shell model vs quantum Monte Carlo: correlations

Compare $\beta\beta$ transition densities in
nuclear shell model and quantum Monte Carlo calculations in light nuclei

$$4\pi r^2 \rho_{GT}(r) = \langle \Psi_f | \sum_{a<b} \delta(r - r_{ab}) \sigma_{ab} \tau_a^+ \tau_b^+ | \Psi_i \rangle \,,$$

$$M_{GT}^{0\nu} = \int_0^\infty dr\, C_{GT}^{0\nu} \,,$$

Agreement at long distances, missing short-range correlations in shell model



Weiss, Soriano, Lovato, JM, Wiringa, PRC106 065501 (2022)

Similar findings in Wang et al. PLB 798 134974 (2019)

Generalized contact formalism (GCF)

Generalized contact formalism Weiss, Bazak, Barnea PRL 114 012501 (2015)

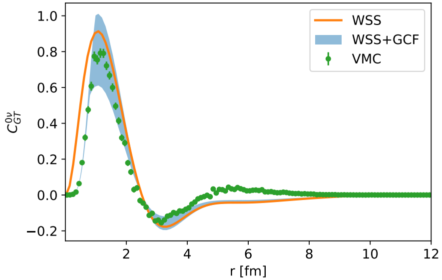
Separation of scales: wf, transition density factorize for two nearby nucleons

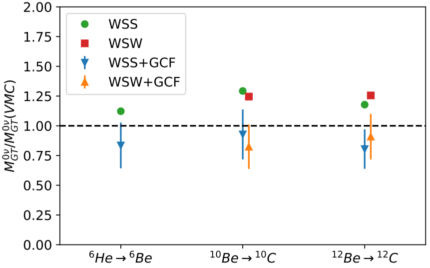
$$\Psi \xrightarrow[r_{ij}\to 0]{} \sum_\alpha \varphi^\alpha(\boldsymbol{r}_{ij}) A^\alpha(\boldsymbol{R}_{ij}, \{\boldsymbol{r}_k\}_{k\neq i,j}), \quad \rho_{GT}(r) \xrightarrow[r\to 0]{} -3|\varphi^0(r)|^2 C^0_{pp,nn}(f,i)$$

with $\varphi(r)$ the solution of the two-nucleon Schrödinger equation

The contact $C^0(f,i) = \frac{A(A-1)}{2}\langle A^\alpha(f)|A^\beta(i)\rangle$ is model dependent

Replace shell-model by QMC contact
to improve transition density and nuclear matrix element

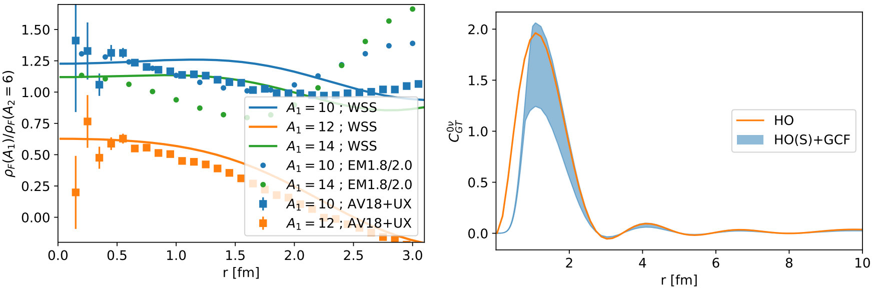




GCF: model independence of ratios

Generalized contact formalism Weiss, Bazak, Barnea PRL 114 012501 (2015)

The contact $C^0(f,i) = \frac{A(A-1)}{2}\langle A^\alpha(f)|A^\beta(i)\rangle$ is model dependent
(shell model, quantum Monte Carlo, no-core shell model...)
but for two nuclei the ratio $C^0_{pp,nn}(X)/C^0_{pp,nn}(Y)$ relatively model independent:
combine QMC calculation in light nuclei with two shell model calculations:

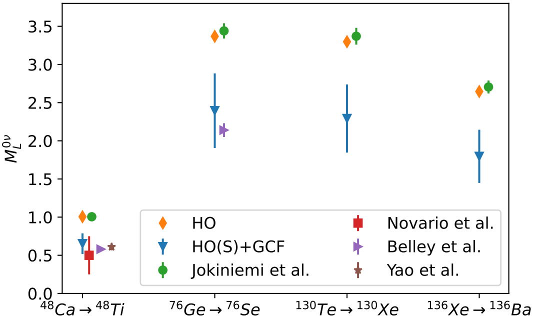


Weiss, Soriano, Lovato, JM, Wiringa, PRC106 065501 (2022)
Yao, Belley et al. PRC 103, 014315 (2021)

Shell model + Generalized contact formalism: NMEs

GCF builds QMC short-range correlations to shell model transitions densities can be extended to heavy nuclei where shell model calculations are possible

Weiss, Soriano, Lovato, JM, Wiringa, PRC106 065501 (2022)



Short-range correlations included by GCF reduce $0\nu\beta\beta$ NMEs moderately

$\sim$ 30% reduction in general consistent with ab initio NMEs in ^{48}Ca, ^{76}Ge

Good agreement in benchmark NMEs in light nuclei with ab initio calculations

Light-neutrino exchange: contact operator

Short-range operator contributes to light-neutrino exchange
for RG invariance of two-nucleon decay amplitude: high-energy ν's

$$T_{1/2}^{-1} = G_{01}\, g_A^4 \left(M_{\rm long}^{0\nu} + M_{\rm short}^{0\nu}\right)^2 \frac{m_{\beta\beta}^2}{m_e^2},$$

Cirigliano et al. PRL120 202001(2018)

$$M_{\rm short}^{0\nu} \equiv \frac{1.2A^{1/3}\,{\rm fm}}{g_A^2} \langle 0_f^+ | \sum_{n.m} \tau_m^- \tau_n^- \, \mathbb{1} \left[\frac{2}{\pi}\int j_0(qr)\, 2g_\nu^{\rm NN}\, g(p/\Lambda)\, p^2 dp\right] |0_i^+\rangle,$$

$$M_{\rm GT}^{0\nu} \simeq \frac{1.2A^{1/3}\,{\rm fm}}{g_A^2} \langle 0_f^+ | \sum_{n.m} \tau_m^- \tau_n^- \, \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \left[\frac{2}{\pi}\int j_0(qr)\, \frac{1}{p^2}\, g_A^2 f^2(p/\Lambda_A)\, p^2 dp\right] |0_i^+\rangle$$

Unknown value (and sign) of the hadronic coupling $g_\nu^{\rm NN}$!

Lattice QCD calculations can obtain value of $g_\nu^{\rm NN}$

Davoudi, Kadam, Phys. Rev. Lett. 126, 152003 (2021), PRD105 094502('22)

match $nn \to pp + ee$ amplitude calculated with dispersion QCD methods

Cirigliano et al. PRL126 172002 (2021), JHEP 05 289 (2021)

charge-independence breaking of nuclear Hamiltonians

Cirigliano et al. PRC100, 055504 (2019)

Contact matrix element: relative impact

Modified decay rate: $T_{1/2}^{-1} = G_{01}\, g_A^4 \left(M_{\rm long}^{0\nu} + M_{\rm short}^{0\nu}\right)^2 \frac{m_{\beta\beta}^2}{m_e^2}$

Assume $g_\nu^{\rm NN} \sim 1 {\rm fm}^2$

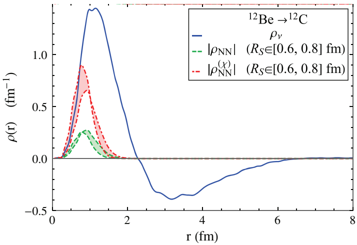
Cirigliano et al.
PRC100 055504 (2019)

TABLE II. Values of $\mathcal{C}_1 + \mathcal{C}_2$ obtained from the CIB contact interactions in various chiral potentials.

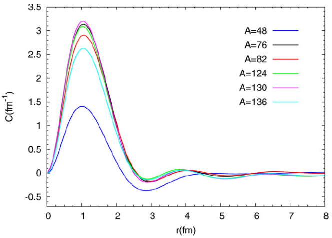
Model	Ref.	R_S (fm)	$C_0^{\rm IT}$ (fm^2)	$(\mathcal{C}_1+\mathcal{C}_2)/2$ (fm^2)	Model	Ref.	Λ (MeV)	$(\mathcal{C}_1+\mathcal{C}_2)/2$ (fm^2)
NV-Ia*	[38]	0.8	0.0158	−1.03	Entem-Machleidt	[34]	500	−0.47
NV-IIa*	[38]	0.8	0.0219	−1.44	Entem-Machleidt	[34]	600	−0.14
NV-Ic	[38]	0.6	0.0219	−1.44	Reinert *et al.*	[39]	450	−0.67
NV-IIc	[38]	0.6	0.0139	−0.91	Reinert *et al.*	[39]	550	−1.01
					NNLO$_{\rm sat}$	[37]	450	−0.39

$\sim$ 75% correction for QMC ^{12}Be NME

What about heavy nuclei?



Cirigliano et al. PRL120 202001(2018)



JM et al. NPA818 139 (2009)

Short-range NME calculations in heavy nuclei

Calculate $M^{0\nu}_{\text{short}}$ in heavy nuclei to see impact in $0\nu\beta\beta$ searches

Use g_ν^{NN} and Λ values from
charge independence breaking (CIB) contact term, chiral EFT potentials
assume same value for two CIB couplings $\mathcal{C}_1 = \mathcal{C}_2$

$g_\nu^{\text{NN}}(\text{fm}^2)$	Λ (MeV)	
-0.67	450	Reiner et al. Eur. Phys. J. A 54 86 (2018)
-1.01	550	"
-1.44	465	Piarulli et al. Phys. Rev. C 94 054007 (2016)
-0.91	465	"
-1.44	349	"
-1.03	349	"

Consider Gaussian regulators: $h_s = 2g_\nu^{\text{NN}} g(p/\Lambda)$

Perform calculations with the nuclear shell model:
^{48}Ca, ^{76}Ge, ^{82}Se, ^{124}Sn, ^{128}Te, ^{130}Te and ^{136}Xe

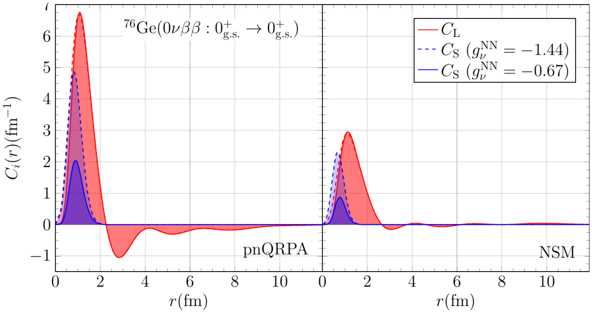
and the quasiparticle random-phase approximation method (QRPA):
^{76}Ge, ^{82}Se, ^{96}Zr, ^{100}Mo, ^{116}Cd, ^{124}Sn, ^{128}Te, ^{130}Te and ^{136}Xe

Long and short-range NME in heavy nuclei

Relatively stable contribution of new term M_S/M_L:

$20\% - 50\%$ impact of short-range NME in shell model
$30\% - 70\%$ impact of short-range NME in QRPA

consistent with 43% effect in IM-GCM for ^{48}Ca
using result from $nn \to pp + ee$ decay Wirth et al. PRL127 242502 (2021)



Jokiniemi, Soriano, JM, Phys. Lett. B 823 136720 (2021)

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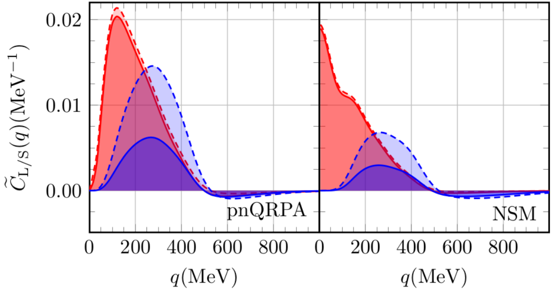


Jokiniemi, Soriano, JM, Phys. Lett. B 823 136720 (2021)

Relative impact of new short-range contribution

In transitions with larger cancellation from tail in NME distribution new short-range term becomes relatively more important

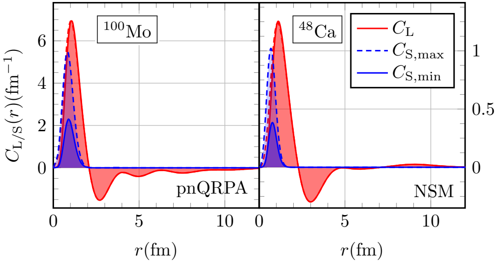
Nuclear shell model: ^{48}Ca with 25% – 65% contribution

consistent with Wirth et al. PRL127 242502 (2021)

QRPA: ^{100}Mo with 50% – 100% contribution

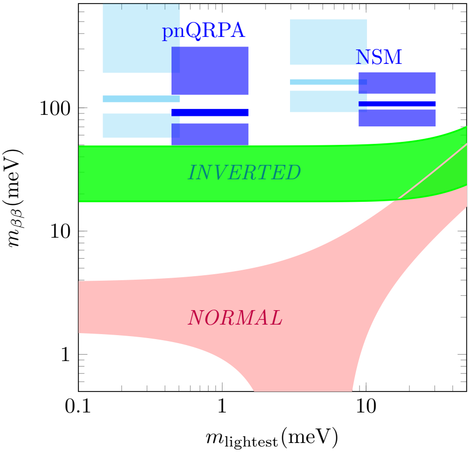
due to negative contributions of 1^+ intermediate states

explains larger QRPA than shell model impact, but less than QMC



Jokiniemi, Soriano, JM, Phys. Lett. B 823 136720 (2021)

Impact on tests of inverted hierarchy of ν mass



Jokiniemi, Soriano, JM

Phys. Lett. B 823 136720 (2021)

With these g_ν^{NN} values significant impact on current $0\nu\beta\beta$ limits on neutrino mass, $m_{\beta\beta}$

Ab initio determination based on $nn \to pp + ee$ result suggests constructive sign between M_L and M_S

Wirth et al. PRL127 242502 (2021)

Short-range matrix element may roughly compensate effect of missing correlations meson-exchange currents in shell model, QRPA NME calculations

Double Gamow-Teller strengths and $\beta\beta$ decay

Measurement of Double Gamow-Teller (DGT) resonance
in double charge-exchange reactions ^{48}Ca(pp,nn)^{48}Ti proposed in 80's

Auerbach, Muto, Vogel... 1980's, 90's

Recent experimental plans in RCNP, RIKEN (^{48}Ca), INFN Catania

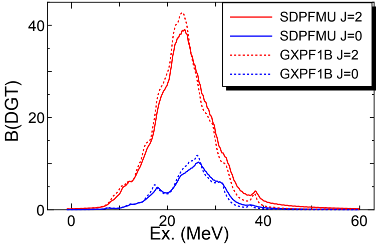
Takaki et al. JPS Conf. Proc. 6 020038 (2015)

Capuzzello et al. EPJA 51 145 (2015), Takahisa, Ejiri et al. arXiv:1703.08264

Promising connection to $\beta\beta$ decay,
two-particle-exchange process,
especially the (tiny) transition
to ground state of final state

Shell model calculation

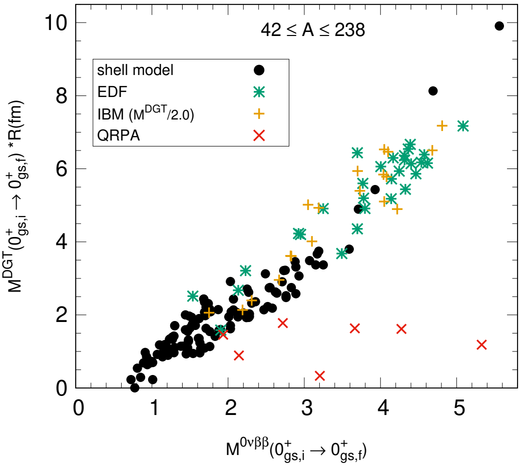
Shimizu, JM, Yako, PRL120 142502 (2018)



$$B(DGT^-;\lambda;i\to f)=\frac{1}{2J_i+1}\left|\left\langle^{48}\text{Ti}\left|\left|\left[\sum_i \boldsymbol{\sigma}_i\tau_i^- \times \sum_j \boldsymbol{\sigma}_j\tau_j^-\right]^{(\lambda)}\right|\right|^{48}\text{Ca}_{gs}\right\rangle\right|^2$$

Correlation of $0\nu\beta\beta$ decay to DGT transitions

Double GT transition to ground state
good linear correlation with $0\nu\beta\beta$ decay NMEs



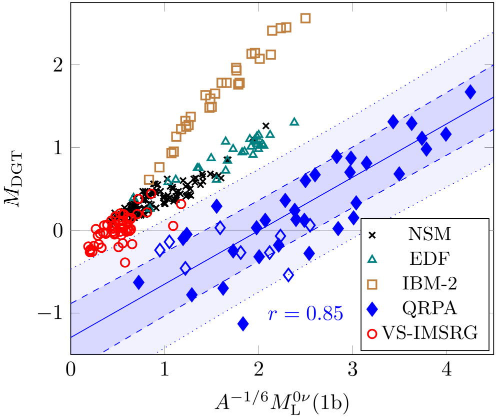
Double Gamow-Teller correlation with $0\nu\beta\beta$ decay holds across nuclear chart
Shimizu, JM, Yako
PRL120 142502 (2018)

Common to shell model energy-density functionals interacting boson model, disagreement to QRPA
Also correlation in VS-IMSRG (but weaker)
Yao et al. PRC106 014315(2022)

Experiments at RIKEN, INFN, RCNP? access DGT transitions

Correlation of $0\nu\beta\beta$ decay to DGT in QRPA

In QRPA, g_{pp} parameter
typically fitted to reproduce $2\nu\beta\beta$ half-life of measured transitions
but actually some tension between g_{pp} values to reproduce single-β decays
Faessler et al., J. Phys. G 35, 075104 (2008)



Jokiniemi, JM, PRC 107 044316 (2023)

Perform QRPA calculations with range of $g_{pp} = (0.6 - 0.9)$

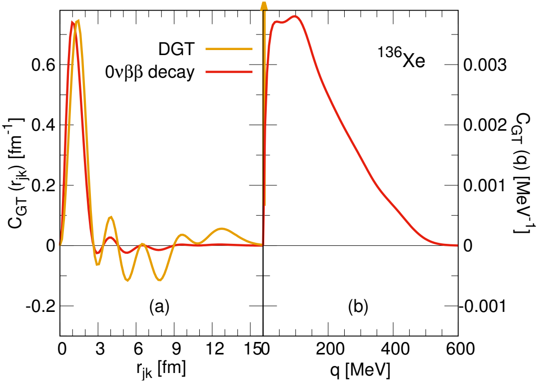
Correlation between DGT and $0\nu\beta\beta$ NMEs!
but different than for other many-body methods

Partially caused by relevance of $J > 1$ intermediate states in QRPA compared to eg shell model
Ejiri et al. Phys. Rept. 797 1 (2019)
Horoi et al, PRC 93, 044334 (2016)

Short-range character of DGT, $0\nu\beta\beta$ decay

Correlation between DGT and $0\nu\beta\beta$ decay matrix elements explained by transition involving low-energy states combined with dominance of short distances between exchanged/decaying neutrons

Bogner et al. PRC86 064304 (2012)



$0\nu\beta\beta$ decay matrix element limited to shorter range

Short-range part dominant in double GT matrix element due to partial cancellation of mid- and long-range parts

Long-range part dominant in QRPA DGT matrix elements

Shimizu, JM, Yako, PRL120 142502 (2018)

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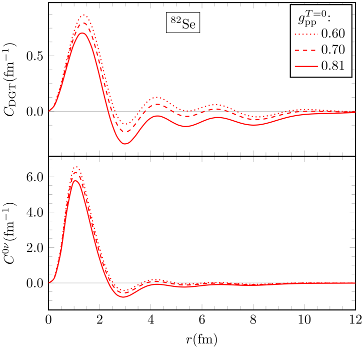

Jokiniemi, JM, PRC 107 044316 (2023)

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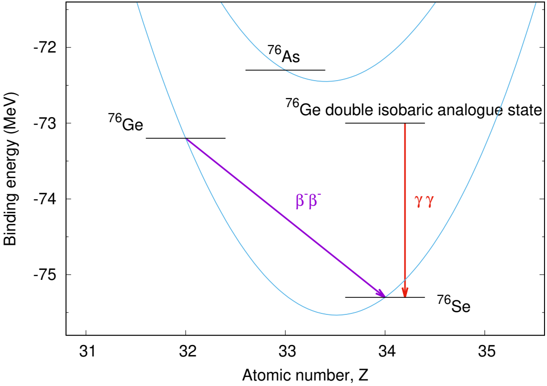
Long-range part dominant in QRPA DGT matrix elements

Shimizu, JM, Yako, PRL120 142502 (2018)

$\gamma\gamma$ decay of the DIAS of the initial $\beta\beta$ nucleus

Explore correlation between $0\nu\beta\beta$ and $\gamma\gamma$ decays,
focused on double-M1 transitions

$$M^{\gamma\gamma}_{M1\,M1} = \sum_k \frac{\langle 0_f^+ | \sum_n (g_n^l \boldsymbol{l}_n + g_n^s \boldsymbol{\sigma}_n)^{IV} | 1_k^+(\text{IAS}) \rangle \langle 1_k^+(\text{IAS}) | \sum_m (g_m^l \boldsymbol{l}_m + g_m^s \boldsymbol{\sigma}_m)^{IV} | 0_i^+(\text{DIAS}) \rangle}{E_k - (E_i + E_f)/2}$$



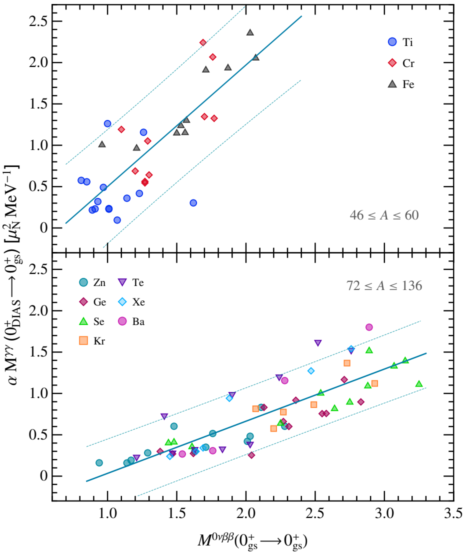
Similar initial and final states
but both in same nucleus
for electromagnetic transition

M1 and GT operators similar,
physics of spin operator
M1 also angular momentum

Different energy denominator

Romeo, JM, Peña-Garay
PLB 827 136965 (2022)

Correlation between $M1M1$ and $0\nu\beta\beta$ NMEs



Good correlation between $M1M1$ same-energy photons and shell-model $0\nu\beta\beta$ NMEs

A dependence:
energy denominator
dominant states at higher energy in heavier nuclei

Overall, study $\sim$ 50 transitions
several nuclear interactions for each of them

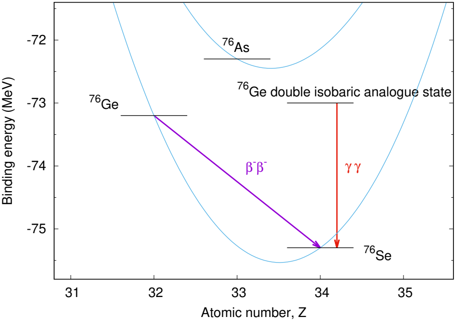
Romeo, JM, Peña-Garay
PLB 827 136965 (2022)

Experimental feasibility of $\gamma\gamma$ decay?

$\gamma\gamma$ decays are very suppressed with respect to γ decays
just like $\beta\beta$ decays are much slower than β decays

$\gamma\gamma$ decays have been observed recently
in competition with γ decays

Waltz et al. Nature 526, 406 (2015), Soderstrom et al. Nat. Comm. 11, 3242 (2020)



Outlook:

Study in detail leading
decay channels for $M1M1$ decay
in DIAS of $\beta\beta$ nuclei

Particle emission $M1$, $E1$ decay:
BR $\sim 10^{-7} - 10^{-8}$

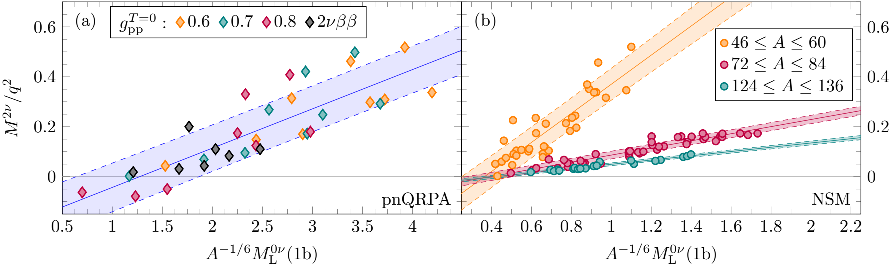
Experimental proposal for ^{48}Ti
by Valiente-Dobón et al.

Valiente-Dobón, Romeo et al., in prep

Correlation of $0\nu\beta\beta$ decay and $2\nu\beta\beta$ decay

Good correlation between 2ν and 0ν modes of $\beta\beta$ decay
in nuclear shell model (systematic calculations of different nuclei)
and QRPA calculations (decays of $\beta\beta$ emitters with different g_{pp} values)

Similar but not common correlation, depends on mass for shell model
$0\nu\beta\beta - 2\nu\beta\beta$ correlation also observed in ^{48}Ca, ^{136}Xe
Horoi et al. PRC 106, 054302 (2022), PRC 107, 045501 (2023)



Jokiniemi, Romeo, Soriano, JM, PRC 107 044305 (2023)

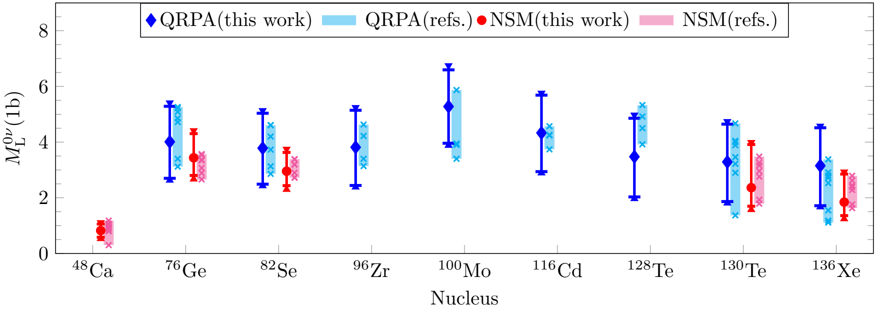
Use $2\nu\beta\beta$ data to predict $0\nu\beta\beta$ NMEs!

$0\nu\beta\beta$ NMEs from $2\nu\beta\beta - 0\nu\beta\beta$ correlation

NMEs consistent with previous nuclear shell model, QRPA results

Theoretical uncertainty involves
systematic calculations covering dozens of nuclei and interactions
error of each calculation (eg quenching) and experimental $2\nu\beta\beta$ error

Previous theoretical uncertainty mostly ignored: collection of calculations



Jokiniemi, Romeo, Soriano, JM, PRC 107 044305 (2023)

2b currents in $0\nu\beta\beta$ decay

In $0\nu\beta\beta$ decay, two weak currents lead to four-body operator
when including the product of two 2b currents: computational challenge

Approximate 2b current as
effective 1b current normal ordering
with respect to a Fermi gas

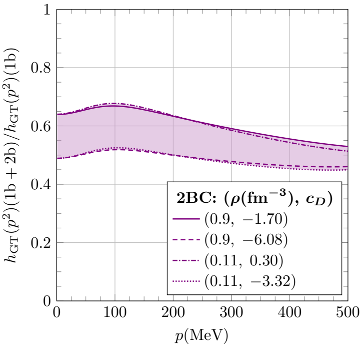
JM, Gazit, Schwenk, PRL107 062501(2011)

Normal-odering approximation works
remarkably well for β decay ($q = 0$)

Gysbers et al. Nature Phys. 15 428 (2019)

Some reduction of quenching
due to 2b currents at $p \sim m_\pi$
relevant for $0\nu\beta\beta$ decay

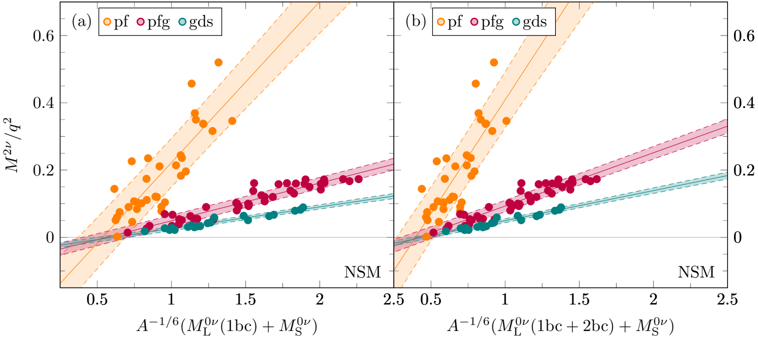
Hoferichter, JM, Schwenk
PRD102 074018 (2020)



Jokiniemi, Romeo, Soriano, JM, PRC 107
044305 (2023)

Correlation of $0\nu\beta\beta$ decay to $2\nu\beta\beta$: general case

A good correlation between $2\nu\beta\beta$ and $0\nu\beta\beta$
also appears when we include to the calculation of $0\nu\beta\beta$ NMEs
2b currents and the short-range nuclear matrix element



Jokiniemi, Romeo, Soriano, JM, PRC 107 044305 (2023)

Use $2\nu\beta\beta$ data to predict $0\nu\beta\beta$ NMEs with 2b currents, short-range NME

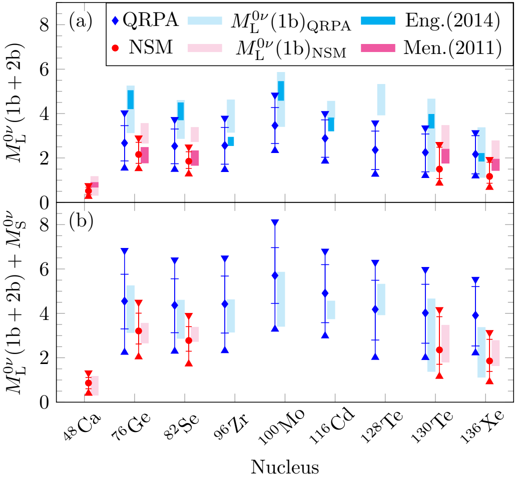
$0\nu\beta\beta$ NMEs from correlation: 2bc, short-range

$0\nu\beta\beta$ NMEs including 2b currents and short-range NME obtained from $0\nu\beta\beta - 2\nu\beta\beta$ correlation and $2\nu\beta\beta$ data

Theoretical uncertainty due to correlation,
calculation uncertainties: quenching, 2bc, short-range NME coupling (dominant uncertainty)

First complete estimation of $0\nu\beta\beta$ nuclear matrix elements with theoretical uncertainties

Jokiniemi, Romeo, Soriano, JM, PRC 107 044305 (2023)



$0\nu\beta\beta$ decay light- and heavy-particle exchange

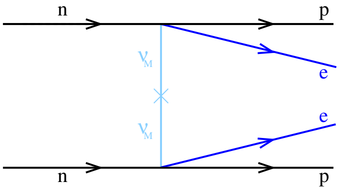
Neutrinoless $\beta\beta$ decay mediated by light or heavy particles

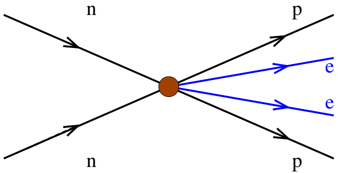
Barea, Horoi, JM, Šimkovic, Suhonen...

$$M^{0\nu\beta\beta} = \langle 0_f^+ | \sum_{n,m} \tau_n^- \tau_m^- \sum_X H^X(r)\, \Omega^X \, | 0_i^+ \rangle$$

$$H^X(r) = \frac{2}{\pi} \frac{R}{g_A^2} \int_0^\infty f^X(pr) \frac{h^X(p^2)}{\left(\sqrt{p^2+m_\nu^2}\right)\left(\sqrt{p^2+m_\nu^2} + \langle E^m \rangle - \frac{1}{2}(E_i - E_f)\right)} p^2\, dp$$

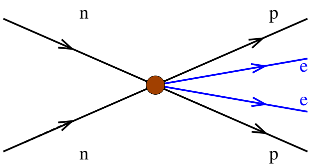
Same contributions in both channels
but in heavy-neutrino exchange the standard term becomes shorter range
$p \sim 100 - 200$ MeV, set by typical distance between decaying nucleons

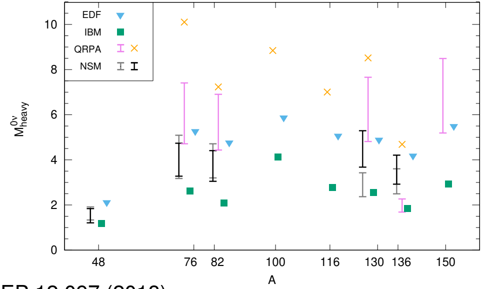




$0\nu\beta\beta$ mediated by BSM heavy particles

Extensions of the Stantard Model can also trigger $0\nu\beta\beta$ decay
typically mediated by exchange of heavy particle (heavy ν, M_R...)





Effective field theory Cirigliano et al JHEP 12 097 (2018)
dimension-7 ($\sim 1/\Lambda^3$), dimension-9 ($\sim 1/\Lambda^5$) operators can lead to $0\nu\beta\beta$

$$T^{-1}_{1/2} = G_{01}\left(g_A^2\, M^{0\nu} + g_\nu^{\rm NN}\, m_\pi^2\, M^{0\nu}_{\rm cont}\right)^2 \frac{m^2_{\beta\beta}}{m_e^2} + \frac{m_N^2}{m_e^2}\tilde{G}\,\tilde{g}^4\,\tilde{M}^2\left(\frac{v}{\Lambda}\right)^6 + \frac{m_N^4}{m_e^2\,v^2}\tilde{G}'\,\tilde{g}'^4\,\tilde{M}'^2\left(\frac{v}{\Lambda'}\right)^{10} + \cdots,$$

Phase-space, hadronic/nuclear matrix elements, known or calculated
Present experiments constrain dim-7 / dim-9 operators $\Lambda \gtrsim 250$ / 5 TeV

Thank you very much for your attention!

Feel free to ask any questions!
I will be around until the end of the school