

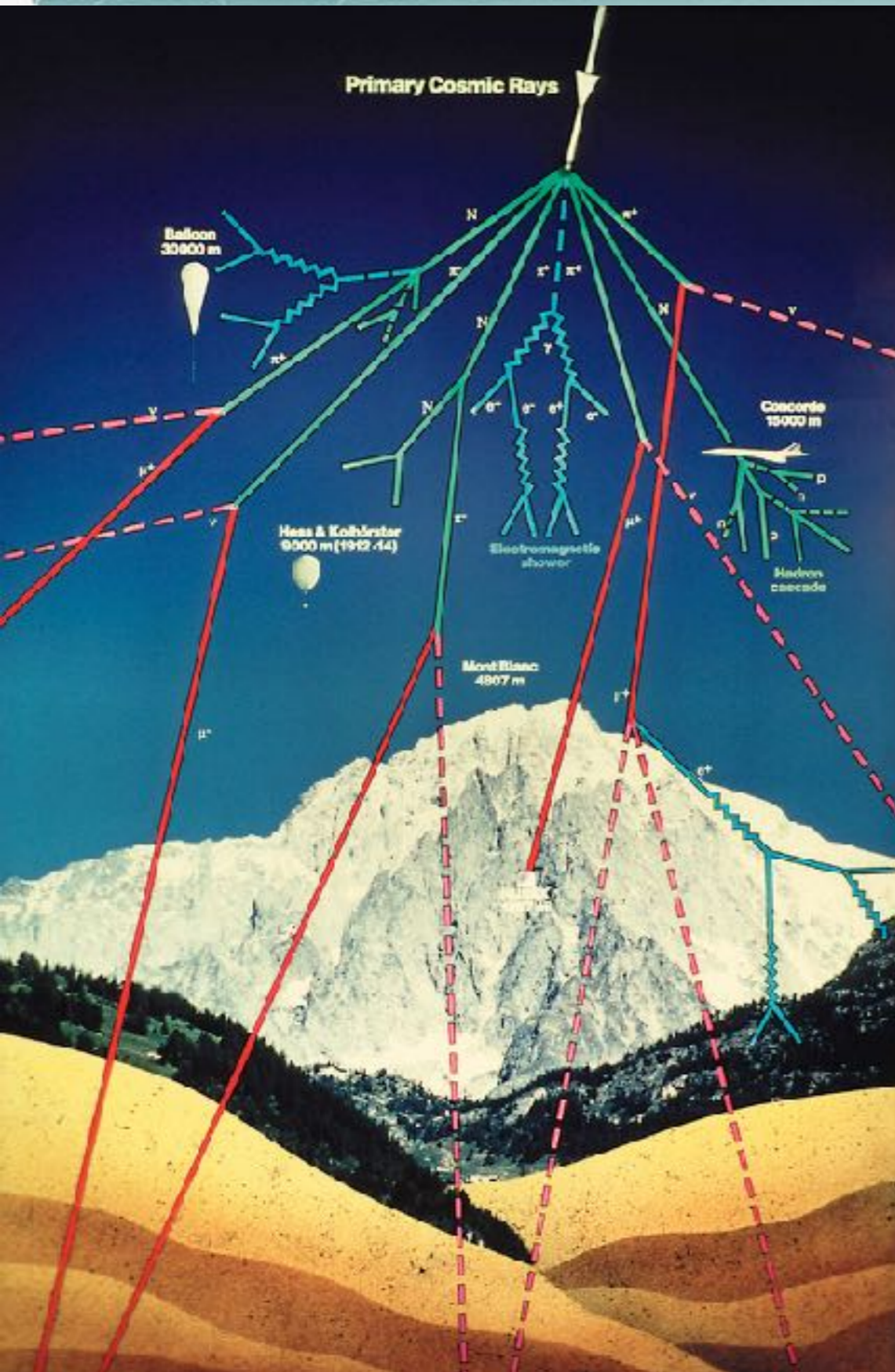
From Exotic Charmonia to \mathcal{T}_{cc} and \mathcal{T}_{bb}
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For an extended exposition, see the Lectures at Galileo Galilei Institute, Firenze 2022:

GGI Lectures on Exotic Hadrons

L. Maiani and A. Pilloni, arXiv:2207.05141 [hep-ph]

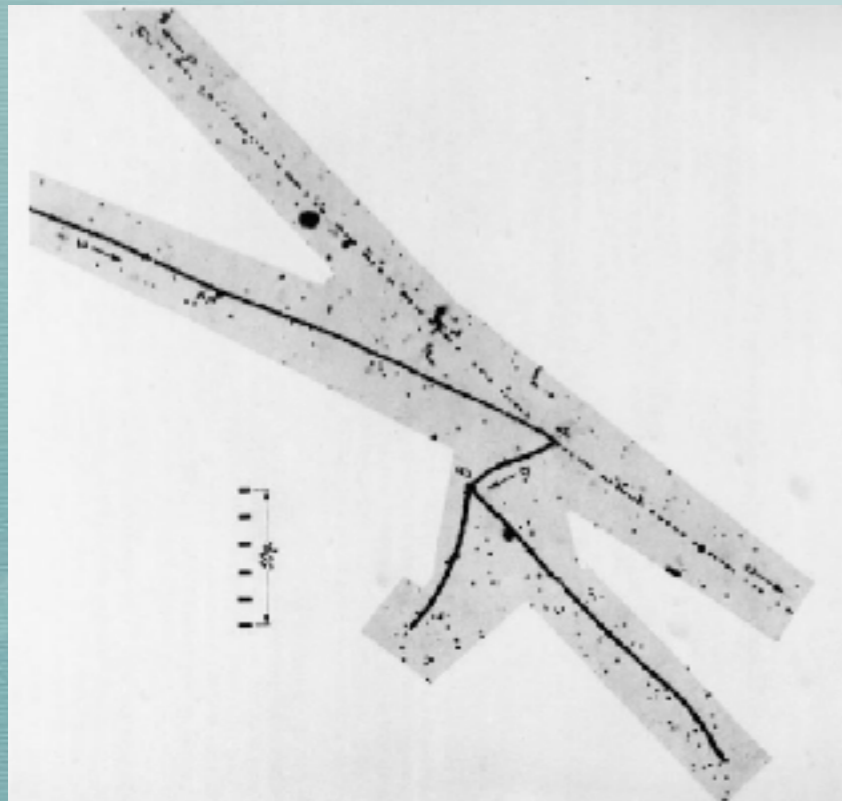
1. Where all started



- C. Anderson, S. Neddermeyer (1937) discover a new particle produced in the upper atmosphere at high altitude by the collisions of Cosmic Rays.
- 1946 (Roma): M. Conversi, E. Pancini e O. Piccioni, prove that the mesotron (μ particle, today) **is not** the particle responsible for the nuclear forces, proposed by H. Yukawa;
- Many consider this discovery the birth of modern Elementary Particle physics

THE MESOTRON AND ITS SIBLINGS

- Everybody (Fermi, Marshak, etc.) was worried: *where is the pion ?*
- Pontecorvo asked a deeper question: *what is the mesotron ?*
- and proposed a surprising answer: it is a second generation electron
- who ordered that ?* (I. Rabi)
- 1940-1950: a *particle zoo* emerges from the study of cosmic ray interactions;
 - The new particles do not arise from further subdivision of normal matter (atoms, nuclei, nucleons, atomic and nuclear forces)
 - A new quantum number: Strangeness. Particles with $S \neq 0$, K mesons, Λ baryon have long lifetimes



The first τ ($K_{\pi 3}$) decay: the primary heavy meson (called τ in the picture) comes from left to right and stops. A slow π comes down and makes a two-pronged star. Two other lightly ionizing particles are emitted from the first stopping point.

Elementary Particles, what ?

THE PHYSICAL REVIEW

A journal of experimental and theoretical physics established by E. L. Nichols in 1893

SECOND SERIES, VOL. 76, No. 12

DECEMBER 15, 1949

Are Mesons Elementary Particles?

E. FERMI AND C. N. YANG*

Institute for Nuclear Studies, University of Chicago, Chicago, Illinois

(Received August 24, 1949)

I. INTRODUCTION

IN recent years several new particles have been discovered which are currently assumed to be “elementary,” that is, essentially, structureless. The probability that all such particles should be really elementary becomes less and less as their number increases.

Fermi&Yang’s proposal:

$$\pi^+ = p\bar{n}$$

π has to have negative parity !!!

muon

strange particles

Δ^{++}

.....

composite by
“constituents” which are
more elementary ?

related by a large symmetry?
possibly including spin ?

2. The constituent way, first attempts

- Fermi&Yang (1949): only $F=(p, n)$ are elementary,

$$\pi - \text{mesons} = F\bar{F}$$

- Sakata (1956): one new constituent to account for strange particles:
Sakata triplet = (p, n, Λ)
- A new quantum number: strangeness, S .
- p, n : strangeness=0, Λ : strangeness=-1

$$\text{mesons} = S\bar{S}; \text{ baryons} = SSS$$

- a clear prediction: there must exist baryons with strangeness $S=+1$.
Unfortunately it is a wrong prediction, no such particle has been seen until today !
- basic symmetry of Sakata model: $SU(2)$ = isotopic spin symmetry \Rightarrow $SU(3)$, unitary transformation of the Sakata triplet

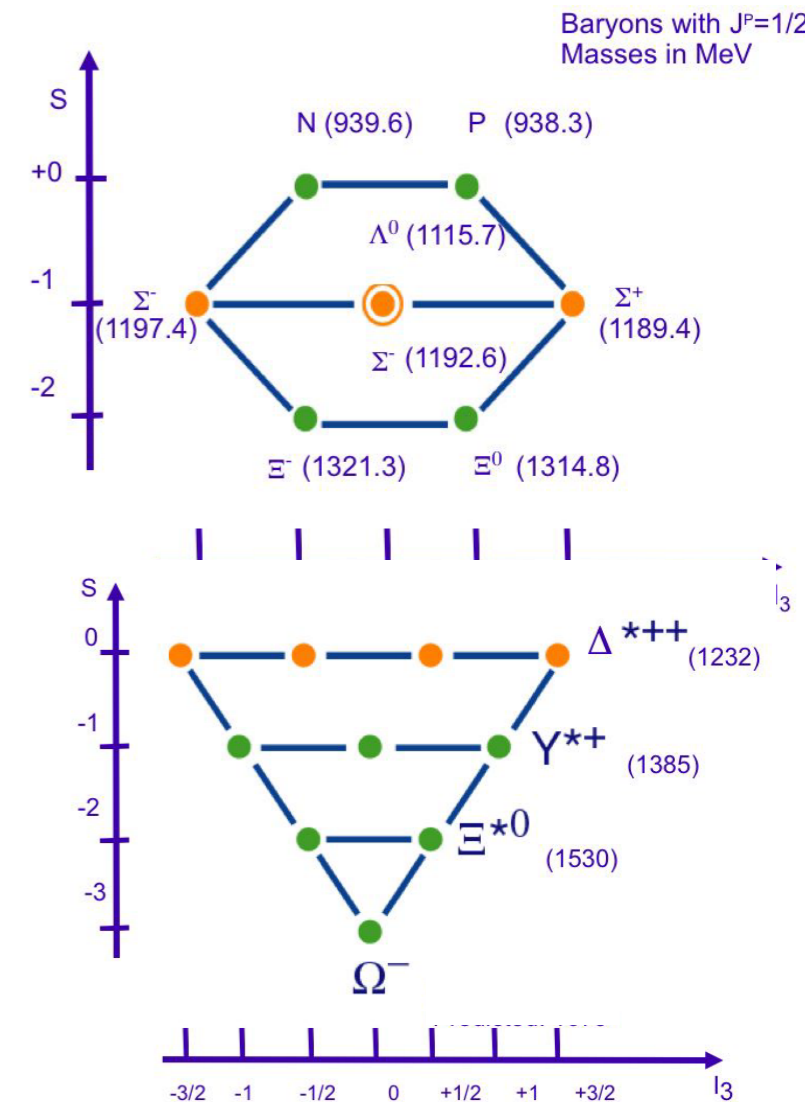
Eightfold Way (Gell-Mann, Ne'eman, 1962)

- Symmetry: SU(3)
- Mesons in octet, as in Sakata model
- Baryons in octet and decuplet, forget Sakata!
- assuming SU(3) broken by octet interaction, Gell-Mann and Okubo derived mass-formulae for octet and decuplet

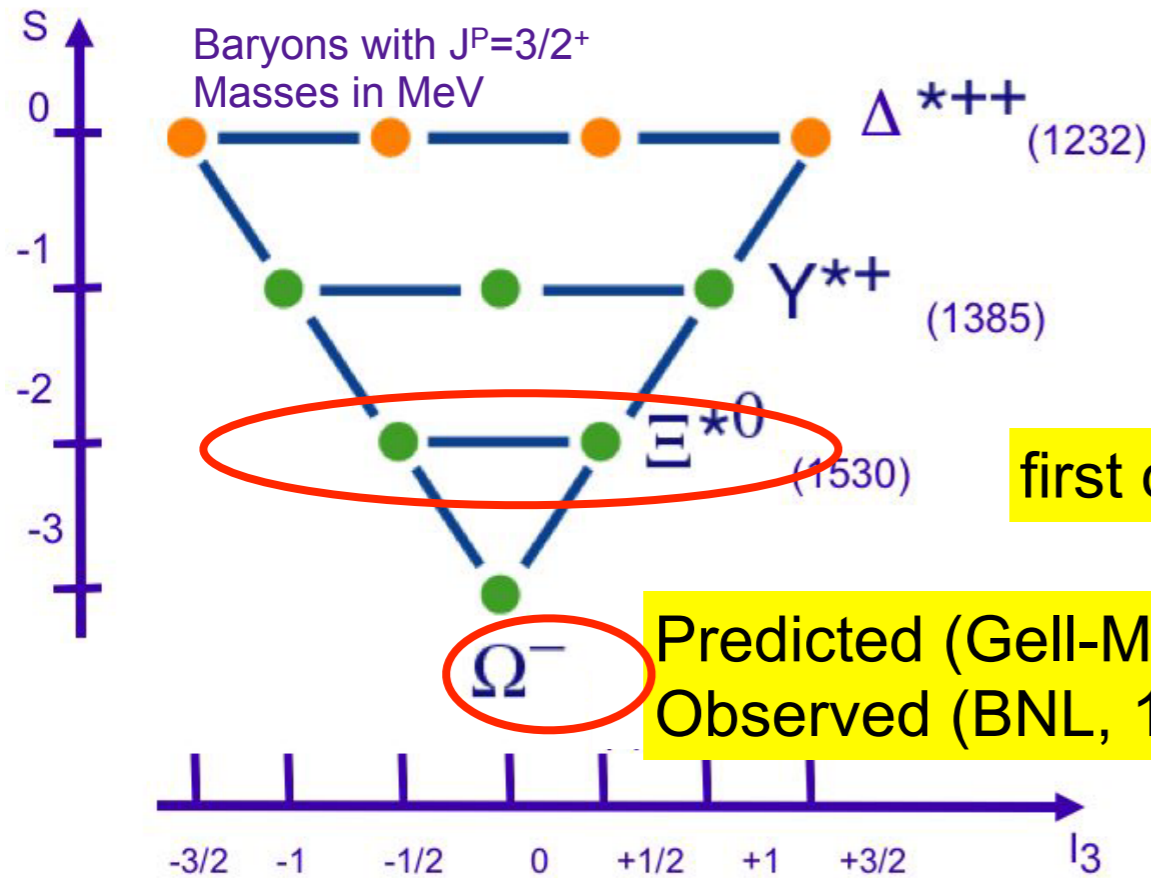
- octet baryons, a formula very well obeyed:

$$\frac{N + \Xi}{2} (1128 \text{ MeV}) = \frac{3\Lambda + \Sigma}{4} (1136 \text{ MeV})$$

- decuplet masses equally spaced: from Δ and Σ^* masses one could predict Ξ^* and Ω masses
- the discovery of two Ξ^* particles was presented at the Ginevra Conference, 1962, and Gell-Mann observed there that their mass checked with equal spacing
- Ω discovered in 1964 with the expected mass



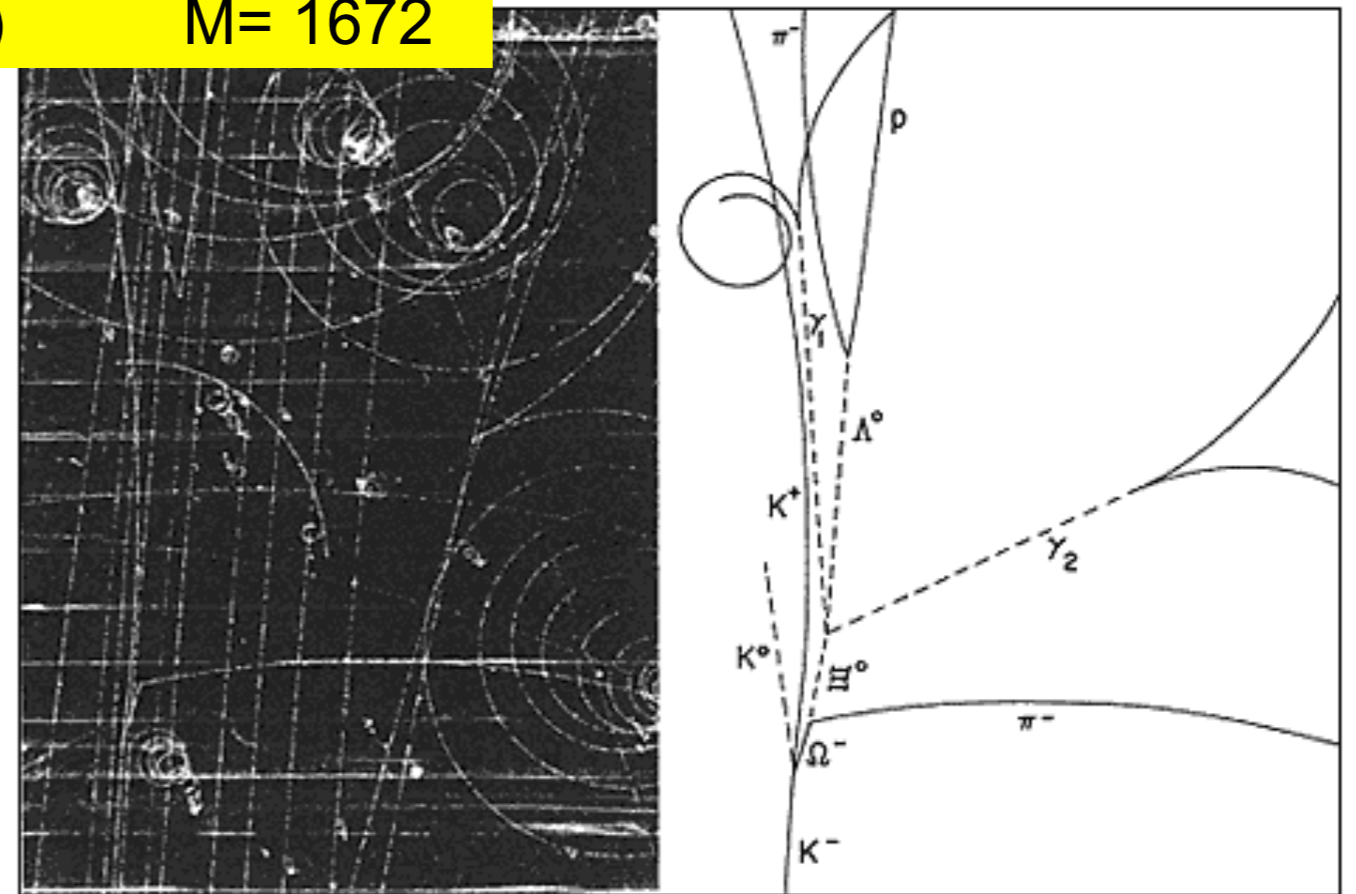
The Ω^-



first confirmation

Predicted (Gell-Mann, 1962) $M=1679$
Observed (BNL, 1964) $M=1672$

- first mass and quantum number predictions in particle physics !
- SU(3) symmetry was established.



Bubble chamber picture of the first Omega-minus (N. Samios and coworkers)

Quarks !

- SU(3) representations and symmetry breaking can be studied by pure group theory
- but quarks are much simpler to handle!
- Quarks are the basic SU(3) triplet, first fundamental representation
- antiquarks \bar{q} : antitriplet, second fundamental representation (3)

	I_3	Y	(S)	Q
$q = \begin{bmatrix} u \\ d \\ s \end{bmatrix} = \mathbf{3}$	$\frac{1}{2}$	$\frac{1}{3}$	(0)	$+\frac{2}{3}$
Quantum numbers :	$-\frac{1}{2}$	$\frac{1}{3}$	(0)	$-\frac{1}{3}$
	0	$-\frac{2}{3}$	(-1)	$-\frac{1}{3}$

- if spin 1/2, we should be able to construct *all hadrons* \vee *quark and/or antiquark* bound states (forget Fermi statistics for a while, we'll come back!)
- how do we make mesons and baryons?

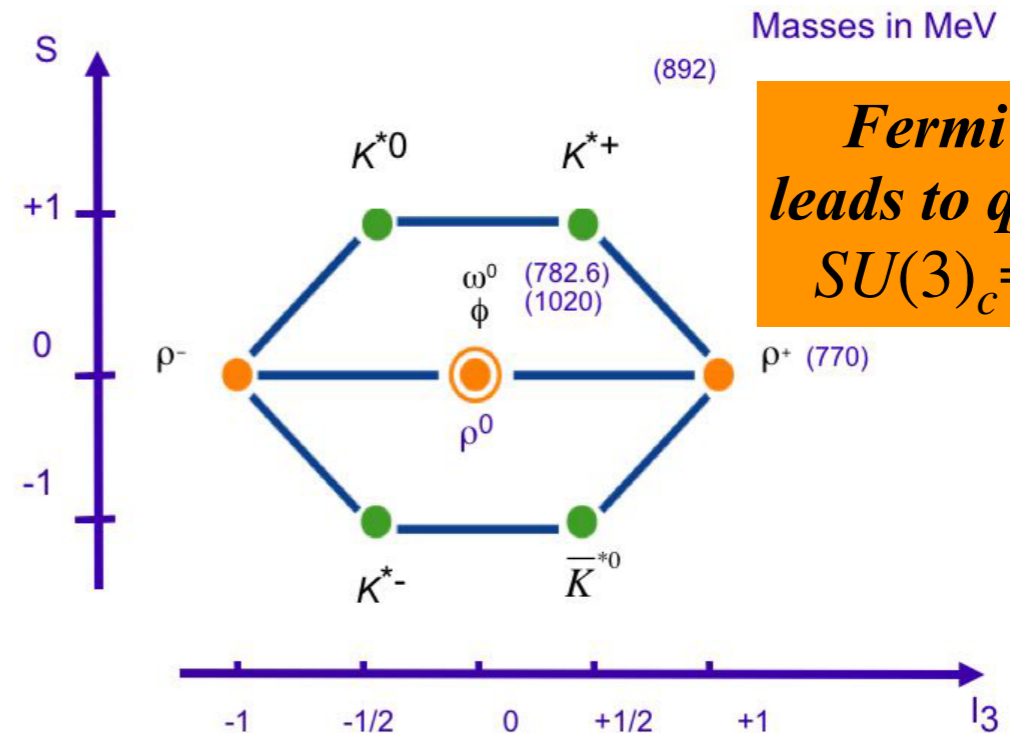
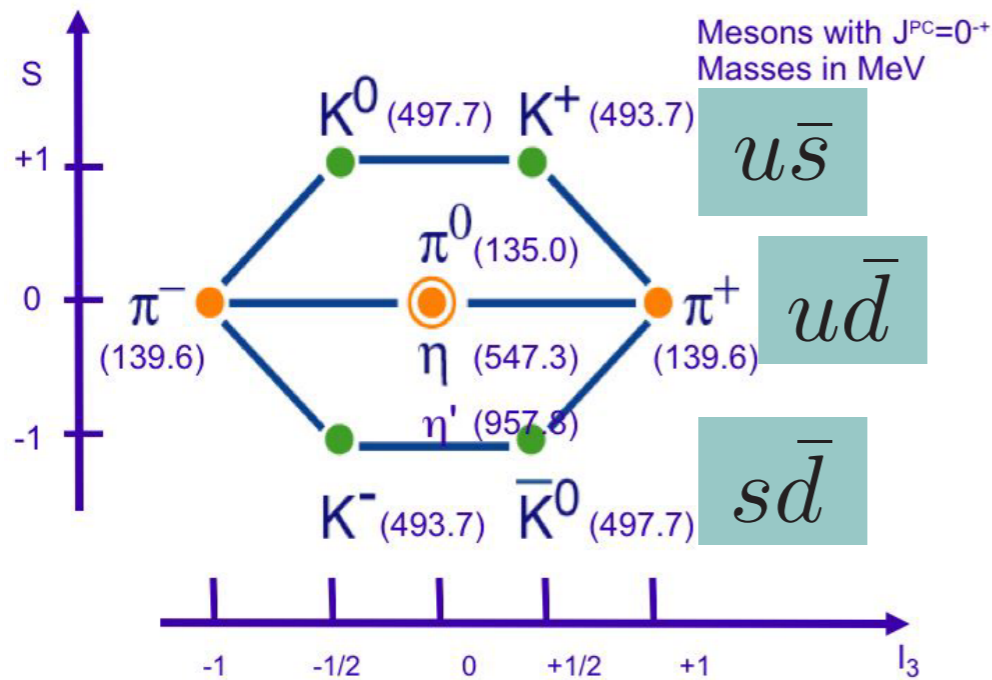
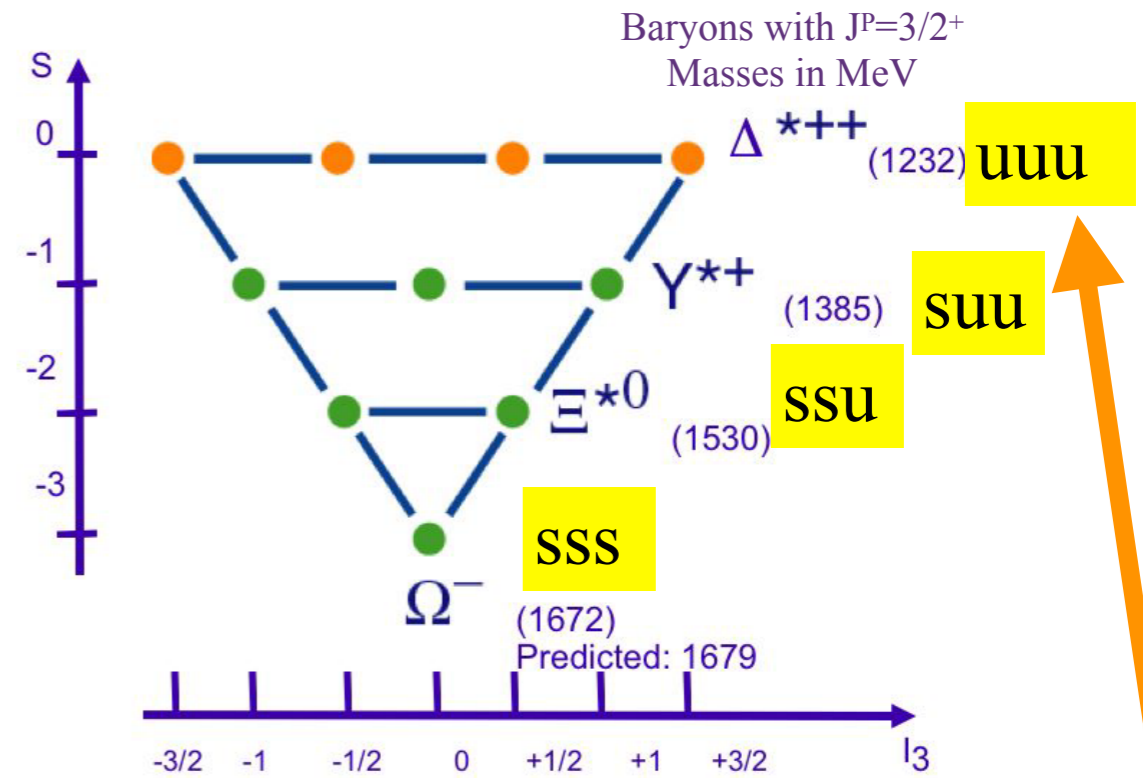
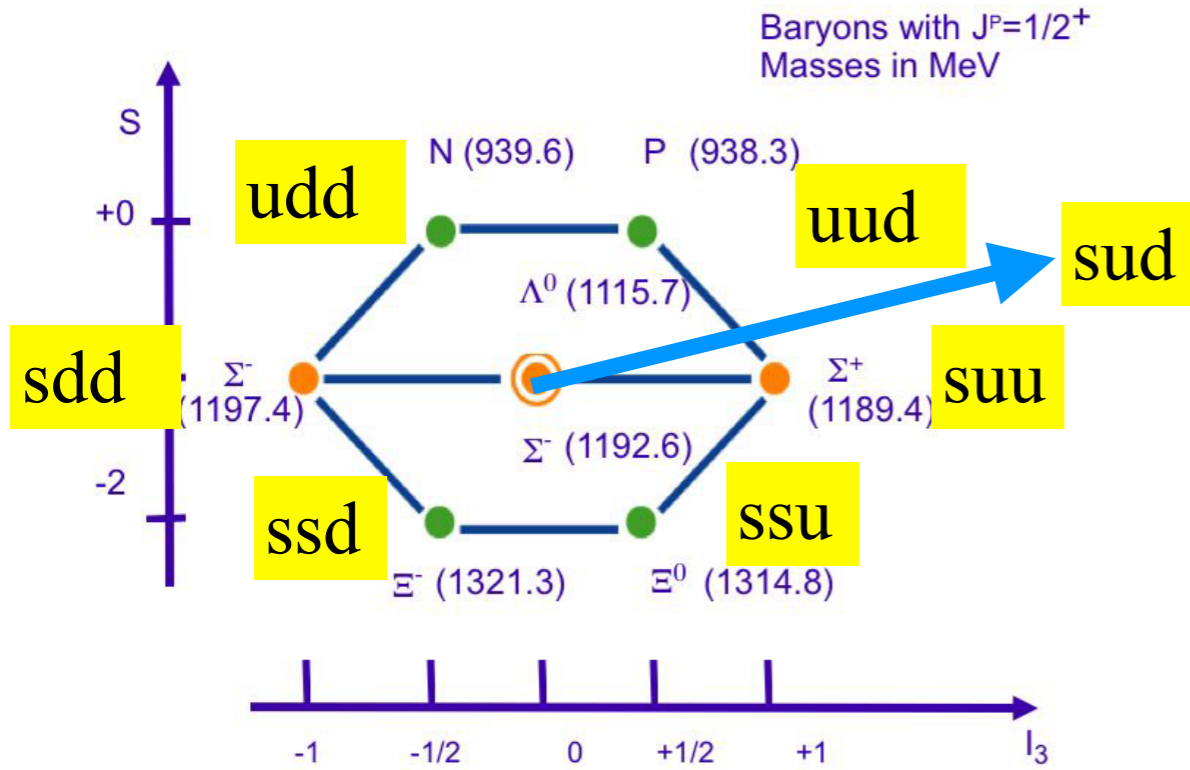
Baryons can now be constructed from quarks by using the combinations

(qqq), (qqqq \bar{q}), etc., while mesons are made out of (q \bar{q}), (qq $\bar{q}\bar{q}$), etc.

1964

M. Gell-Mann, A Schematic Model of Baryons and Mesons, PL **8**, 214, 1964

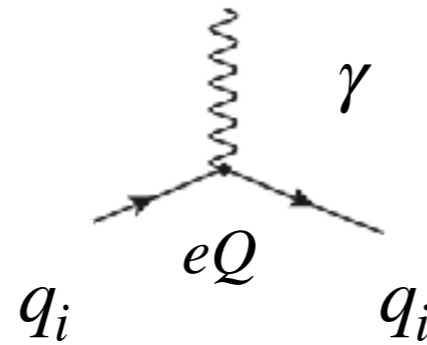
Quark composition of the lowest lying Baryons and Mesons



Fermi Statistics leads to quark colour $SU(3)_c=QCD$!!!!

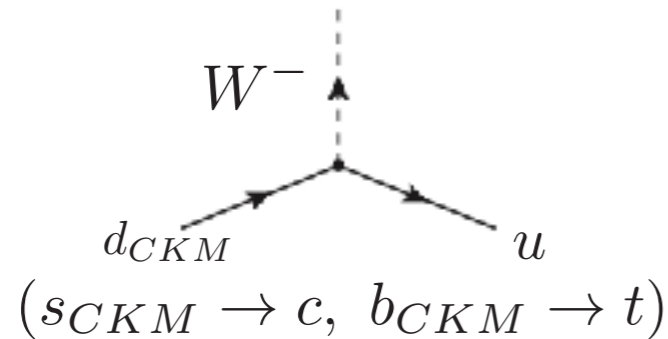
3. The Standard Theory of Particle Interactions (circa 1972)

E. M. INTERACTIONS= QED



WEAK INTERACTIONS (after GIM and KM)

Quark-charged currents

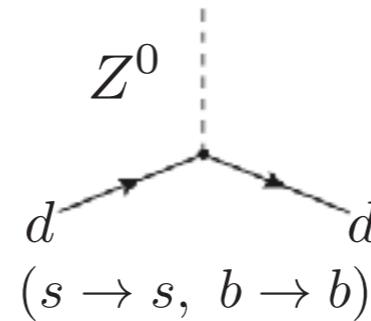


$$\begin{bmatrix} d_{CKM} \\ s_{CKM} \\ b_{CKM} \end{bmatrix} = U_{CKM} \begin{bmatrix} d \\ s \\ b \end{bmatrix}$$

$$d_C = \cos \theta d + \sin \theta s,$$

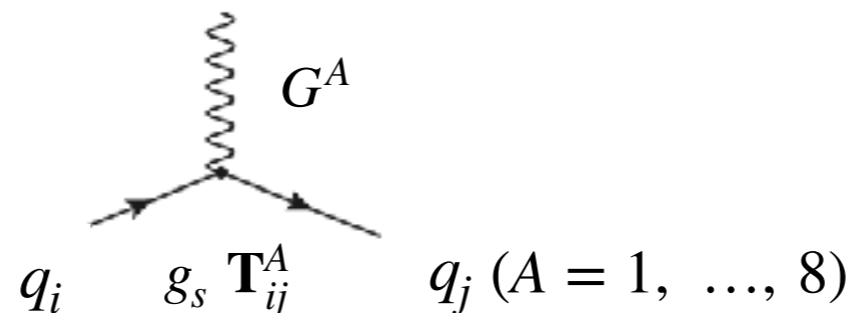
$$\theta = \text{Cabibbo angle}$$

Quark-neutral currents



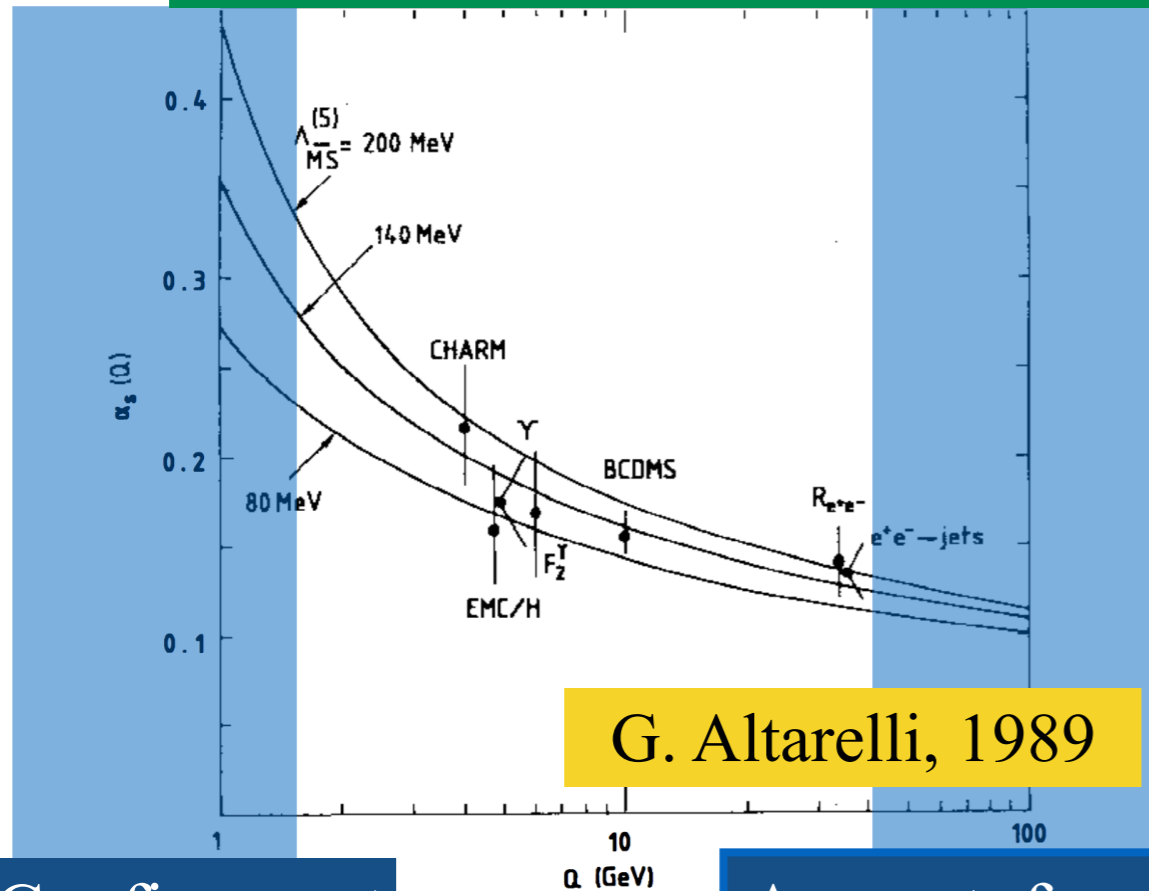
*with dc only, $d + Z^0 \rightarrow s$:
strangeness changing neutral
currents would be allowed*

STRONG INTERACTIONS= COLOUR SU(3)



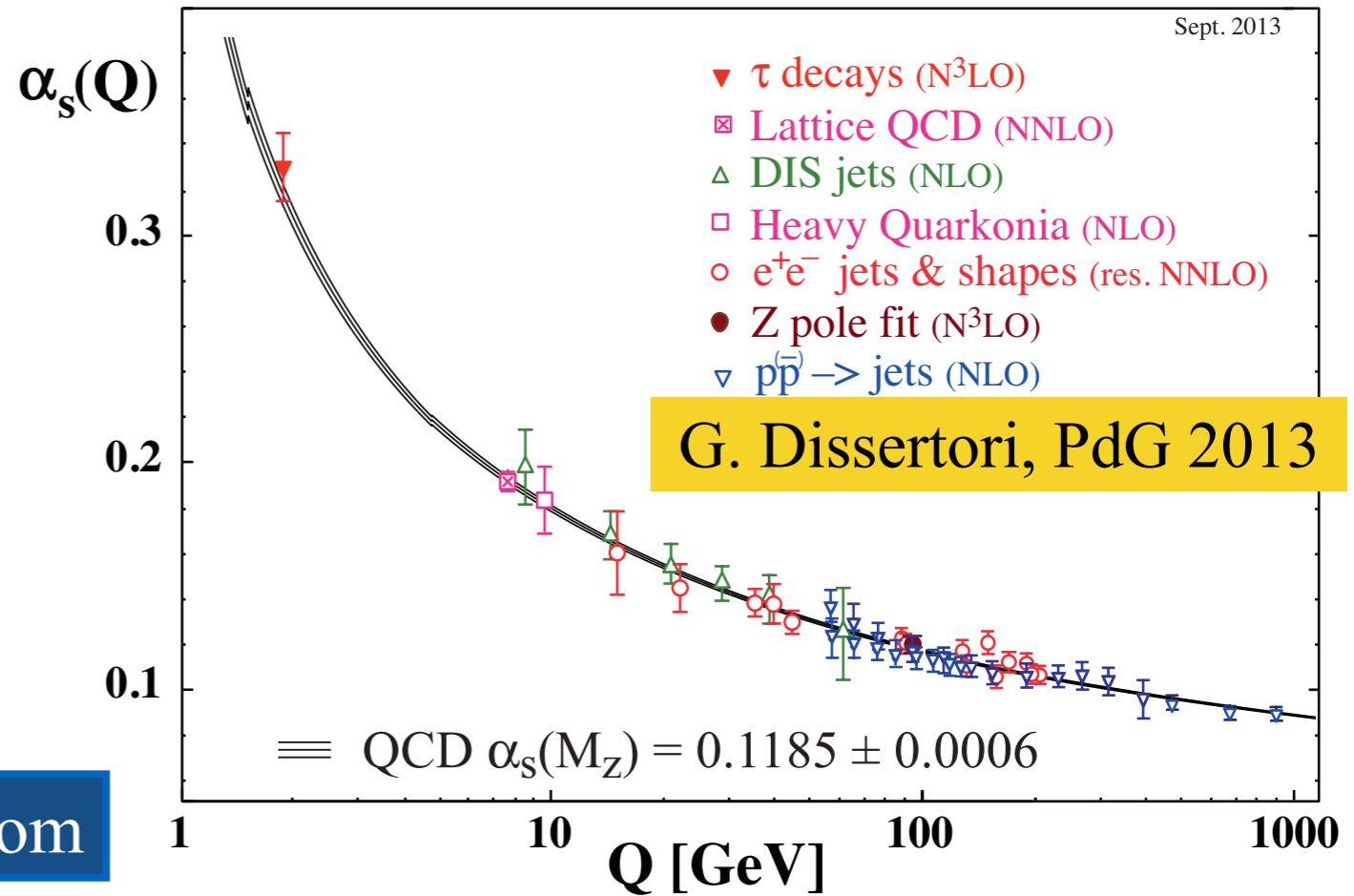
*Quantum Chromo
Dynamics is
asymptotically free.
Critical parameter:
 $\Lambda_{QCD} = 0.25 \text{ GeV}$*

QCD is the answer to (almost) any question



Confinement

Asympt. freedom



- QCD is asymptotically free
- quarks carry **color symmetry**, $SU(3)_{\text{col}}$, and are confined inside **color singlet hadrons**,
- $\Delta^{(++)} = \epsilon^{\alpha\beta\gamma} u_{\alpha}^{\uparrow} u_{\beta}^{\uparrow} u_{\gamma}^{\uparrow}$: Fermi statistics is obeyed
- increasing q^2 , quarks radiate gluons (the Altarelli-Parisi picture of scaling violations)
- at large q^2 , we see quarks and neutral gluons as almost free partons.

Constituent Quarks

QCD Partons

Heavy quarks ($m_Q \gg \Lambda_{QCD}$):

- inclusive decays are calculable like deep inelastic processes;
- $c\bar{c}$ or $b\bar{b}$ bound states involve short distance forces: a calculable spectrum of charmonia/bottomonia;
- inside hadrons, $c\bar{c}$ or $b\bar{b}$ pairs are not easily created or destroyed:
- a hadron decaying into J/Ψ or $\Upsilon + \dots$ indicates a valence $c\bar{c}$ or $b\bar{b}$ pair
- **heavy-quark counting is possible.**

4. Unanticipated charmonia X, Y, Z.. and more

- Hidden charm/beauty resonances not fitting in prepredicted charmonium/bottomonium spectrum because of mass/decay properties or because charged.

- X, e.g. X(3872): neutral, typically seen in $\Psi + \text{pions}$, positive parity, $J^{PC} = 0^{++}, 1^+, 2^{++}$

- Y, e.g. Y(4260): neutral, seen in e^+e^- annihilation with *Initial State Radiation* (ISR) ($e^+e^- \rightarrow e^+e^- + \gamma_{ISR} \rightarrow Y + \gamma_{ISR}$), therefore $J^{PC} = 1^{--}$,

- Z, e.g. Z(4430): charged/neutral, typically positive parity, 4 valence quarks manifest, mostly seen to decay in $\Psi^+ \pi$ and some in $h_c(1P) + \pi$ (valence quarks: $c\bar{c}u\bar{d}$);

- Z_b observed ($b\bar{b}u\bar{d}$).

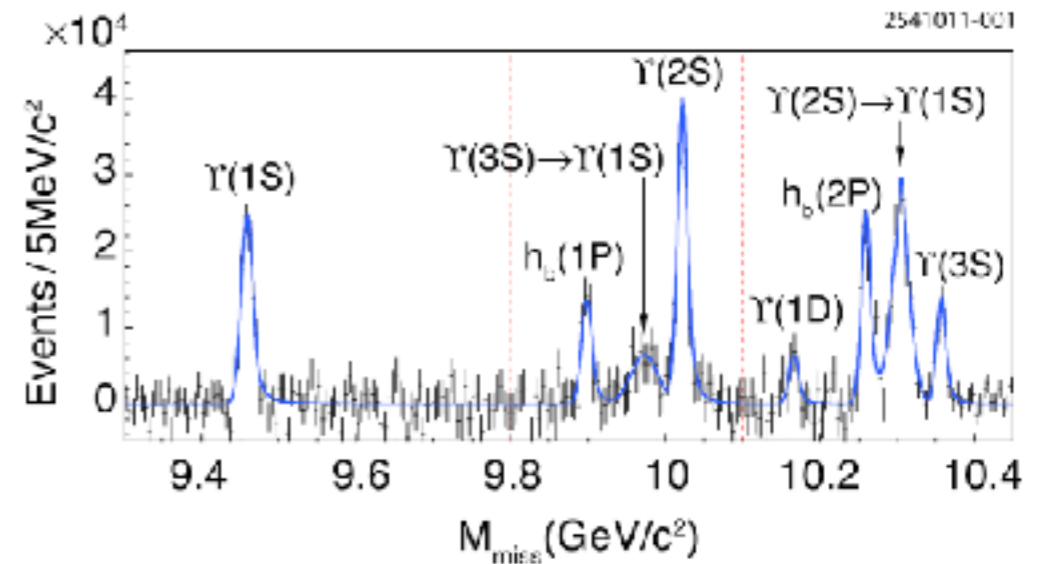


Figure 1: From Belle [31], the mass recoiling against $\pi^+\pi^-$ pairs, M_{miss} , in e^+e^- collision

A new wave of Exotic Hadrons started in 2016:

- Hidden charm and Hidden strangeness seen, e.g. $X(4140) \rightarrow \Psi + \phi$, $J^{PC}=1^{++}$
- 4 charm tetraquarks seen as di- Ψ resonances by LHCb, e.g. $X(6900) \rightarrow \Psi + \Psi \rightarrow (\mu^+\mu^-)^2$
- Hidden charm- Open strangeness ($c\bar{c}u\bar{s}$), seen (last year!) by BES III: $Z_{cs}^+(3985) \rightarrow \Psi + K^+$ and by LHCb: $Z_{cs}^+(4003) \rightarrow \Psi + K^+$.
- Double charm tetraquark seen by LHCb: $\mathcal{T}_{cc}^+(3875) \rightarrow D^0 D^0 \pi^+$ (valence quarks: $cc\bar{u}\bar{d}$)

The saga of $Z^\pm(4430)$

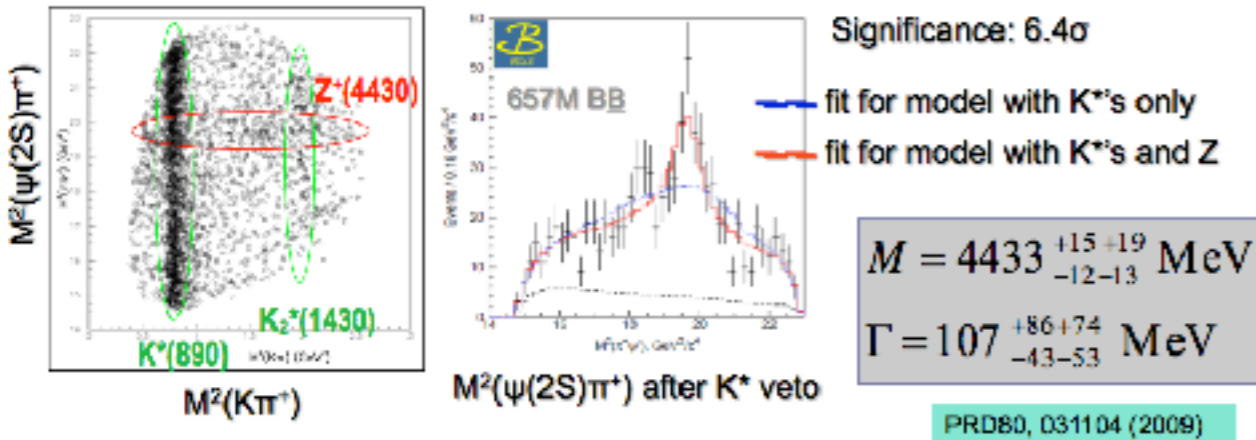


Belle observed $Z(4430)^\pm \rightarrow \psi(2S)\pi^\pm$

PRL100, 142001 (2008)

- Found in $\psi(2S)\pi^+$ from $B \rightarrow \psi(2S)\pi^+K$. Z parameters from fit to $M(\psi(2S)\pi^+)$
- Confirmed through Dalitz-plot analysis of $B \rightarrow \psi(2S)\pi^+K$
- $B \rightarrow \psi(2S)\pi^+K$ amplitude: coherent sum of Breit-Wigner contributions
- Models: all known $K^* \rightarrow K\pi^+$ resonances only**

all known $K^* \rightarrow K\pi^+$ and $Z^+ \rightarrow \psi(2S)\pi^+ \Rightarrow$ favored by data



PRD80, 031104 (2009)

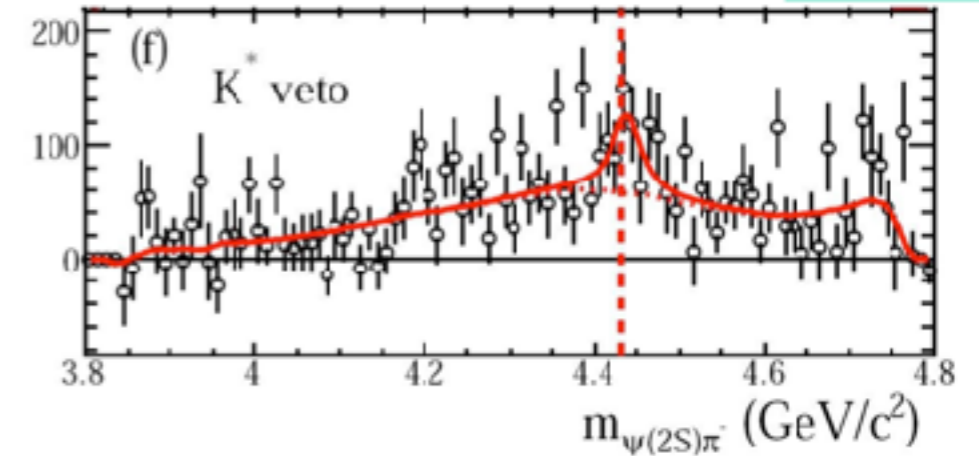
- [cu][cd] tetraquark? neutral partner in $\psi^+\pi^0$ expected**
- $D^*D_1(2420)$ molecule? should decay to $D^*D^*\pi$**

42



BaBar doesn't see a significant $Z(4430)^+$

PRD79, 112001 (2009)



"For the fit ... equivalent to the Belle analysis... we obtain mass & width values that are consistent with theirs,... but only $\sim 1.9\sigma$ from zero; fixing mass and width increases this to only $\sim 3.1\sigma$."

$$BF(B^0 \rightarrow Z^+K) \times BF(Z^+ \rightarrow \psi(2S)\pi^+) < 3.1 \times 10^{-5}$$

Belle PRL: $(4.1 \pm 1.0 \pm 1.4) \times 10^{-5}$

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- Babar inserts in the fit all K^* resonances
- is Belle effect due to K^* reflections ???

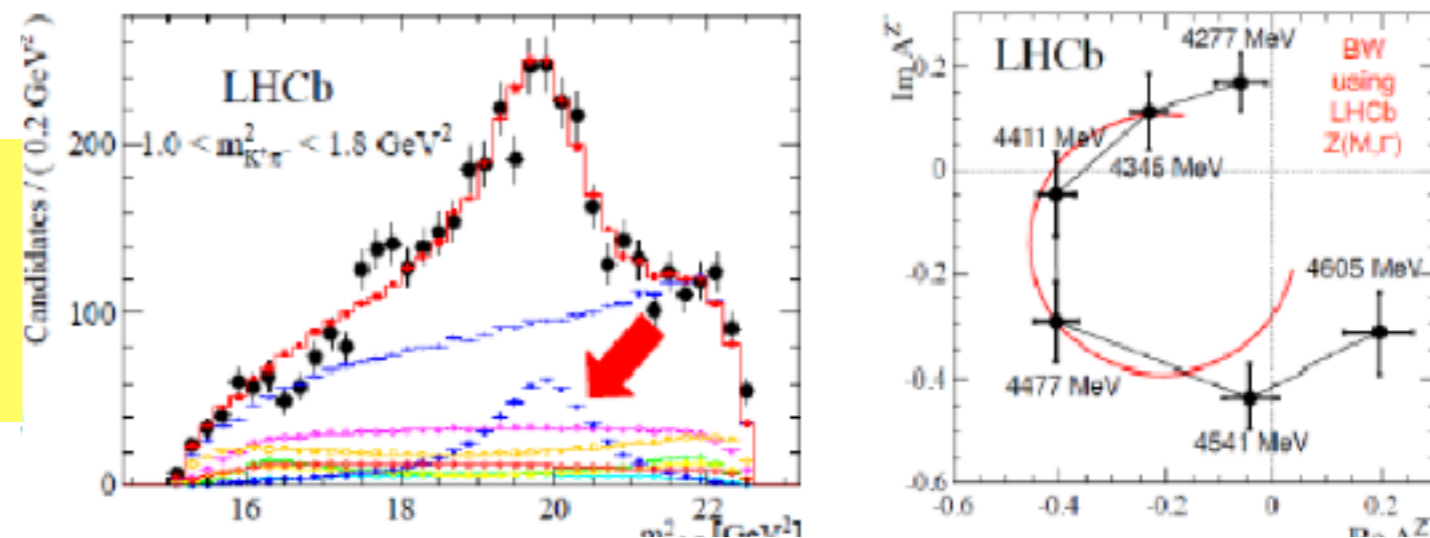
LHCb:

- confirms BELLE's observation of a bump

- CANNOT be built as a molecule of standard states: $D^*D_1 =$ in S-wave may have $J=1$ but has negative parity

- Argand Plot shows 90° phase: Z is a genuine resonance

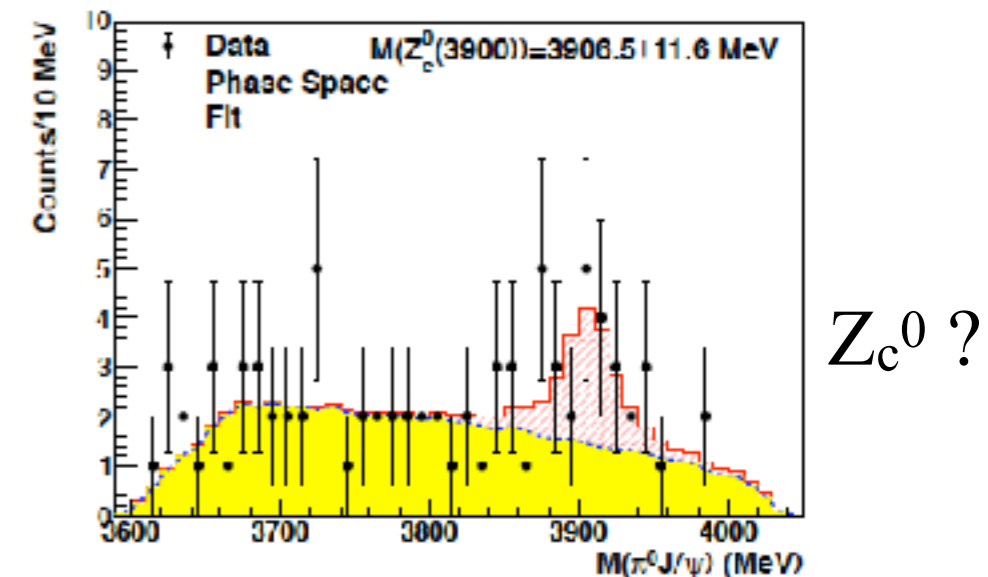
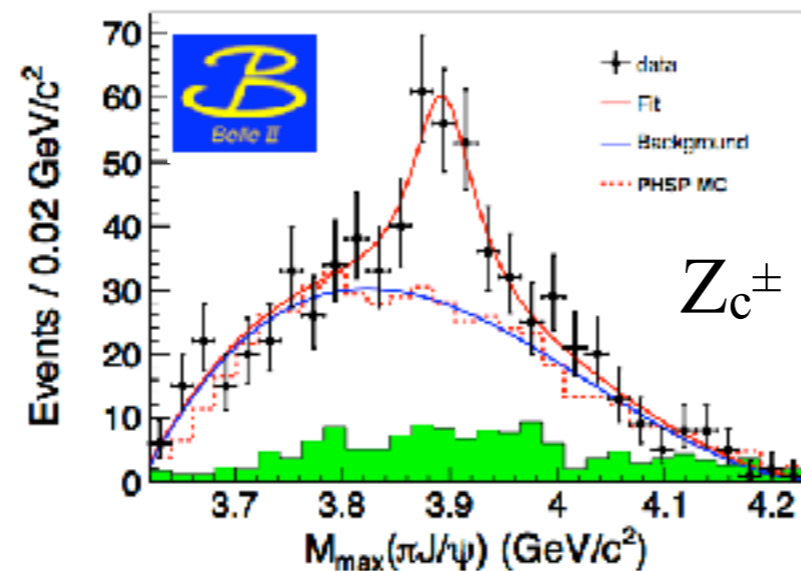
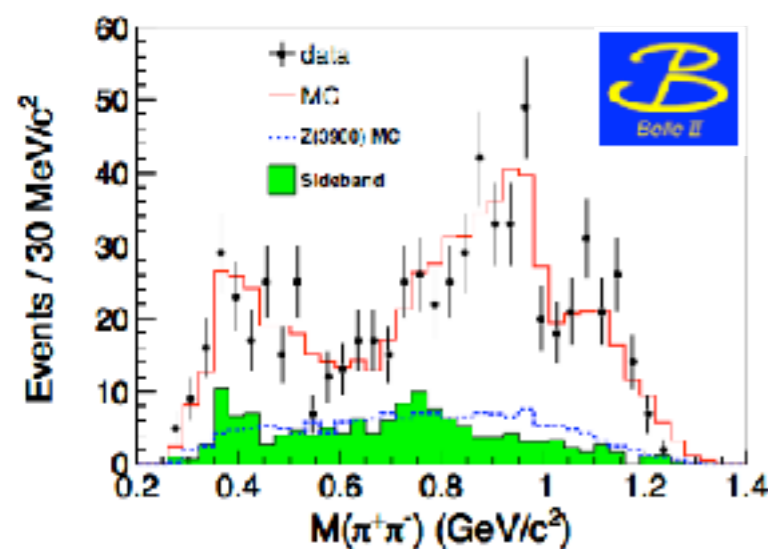
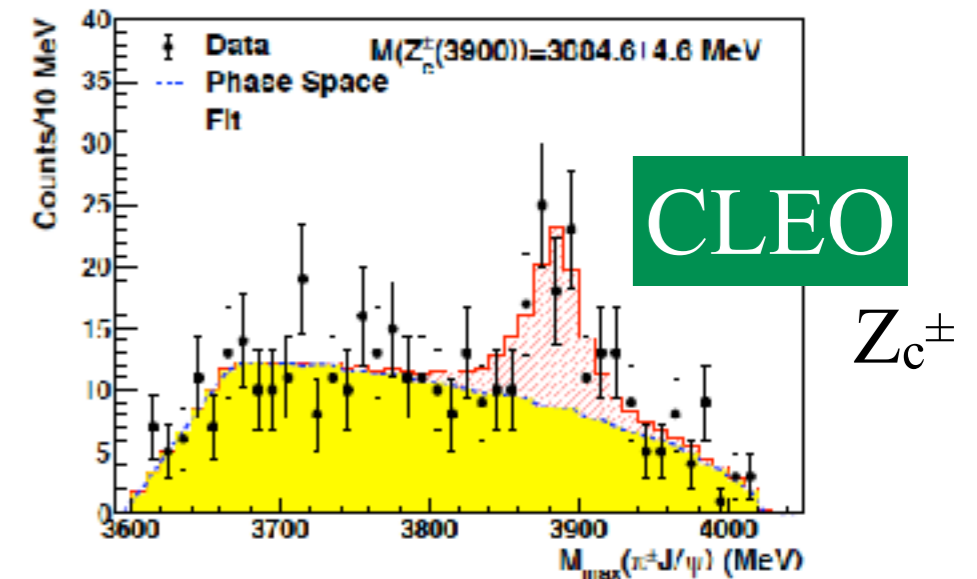
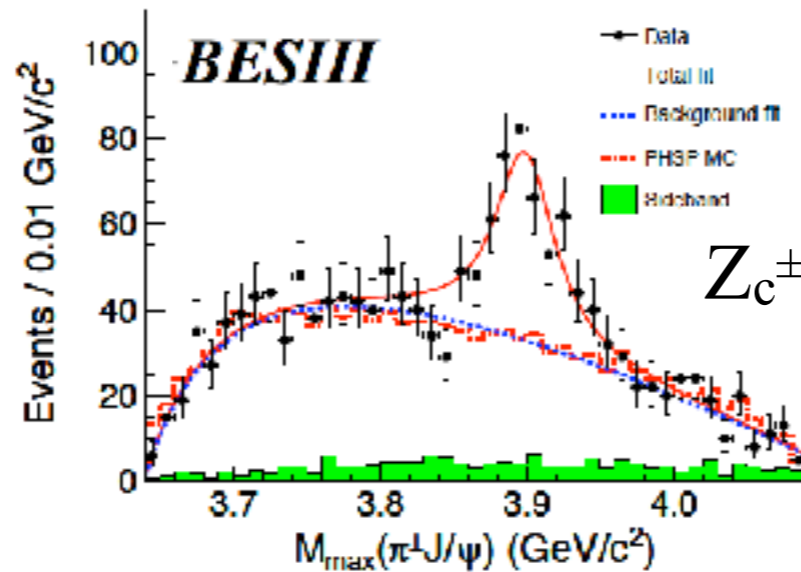
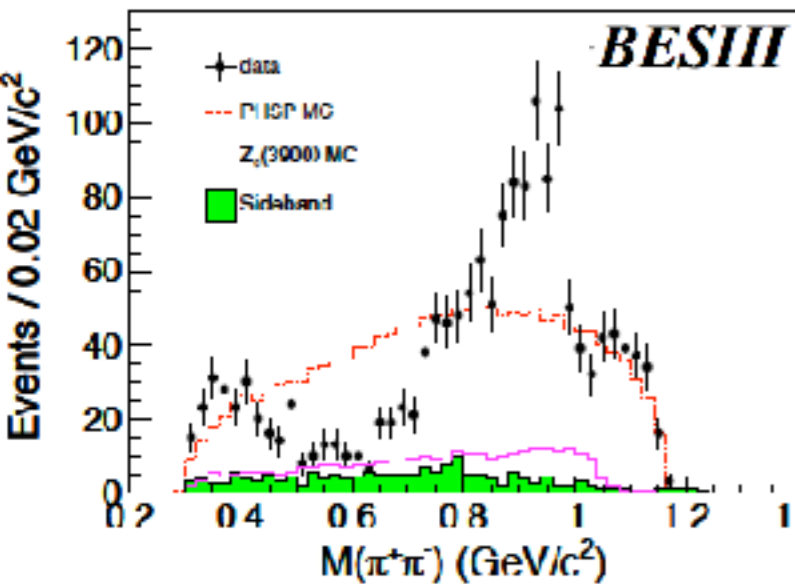
[PRL 112 (2014) 222002]



The unexpected $J^{PC}=1--$ resonance, $Y(4260)$, and its unexpected $J^{PG}=1^{++}$ descendants (2013)

Observed by BES III, BELLE and CLEO:

$Y(4260) \rightarrow \pi^\pm Z_c^\mp(3900) \rightarrow \pi^+ \pi^- \Psi$ tetraquark de-excitation ?



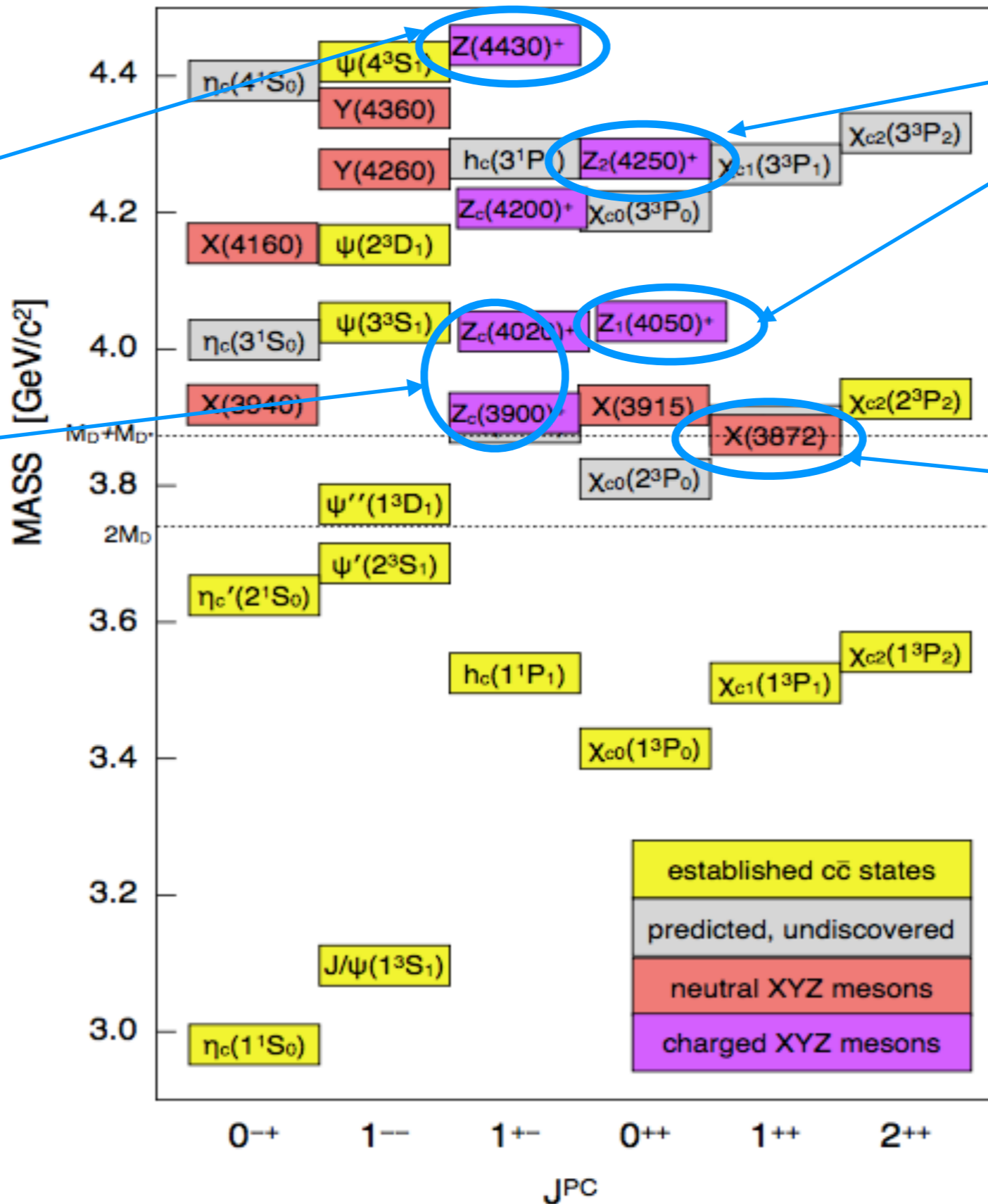
Expected and Unexpected Charmonia

figure by:
S. L. Olsen (2015)
arXiv:1511.01589

2nd Unexpected (2007)
a radial excitation?

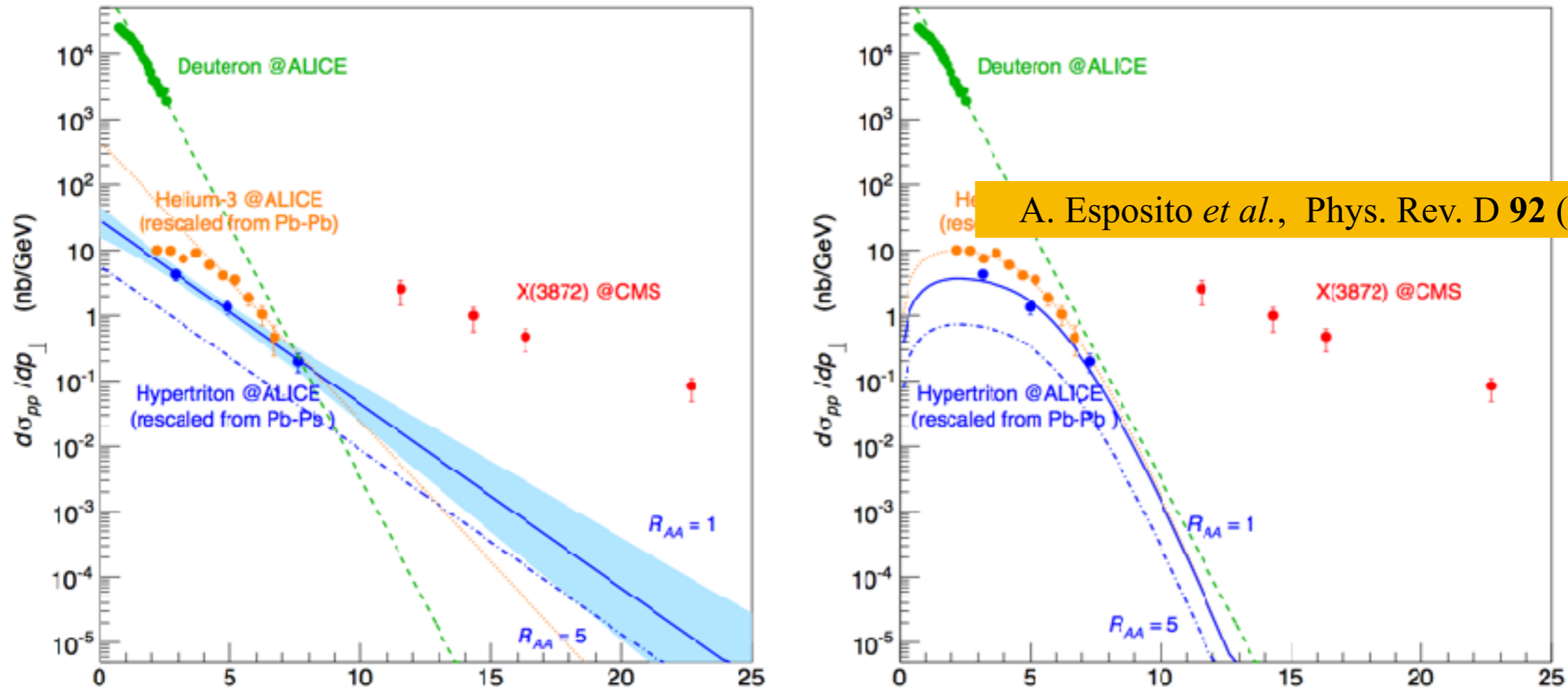
3rd Unexpected (2013):
a multiplet ground state Tetraquarks?

recent additions:
more than coincidence?
or
an almost filled multiplet?



1st Unexpected (2003)

In most cases Exotic Hadrons are produced in the decay of Beauty Mesons or Baryons but, for X(3872), production from unelastic reactions at large p_T has been observed (by CMS). This gives an important clue about its structure



Rescaling from Pb-Pb ALICE cross sections to p-p CMS cross section is done with: Glauber model (**left panel**) and blast-wave function (**right panel**) (R_{AA} or $R_{CP} = 1$)

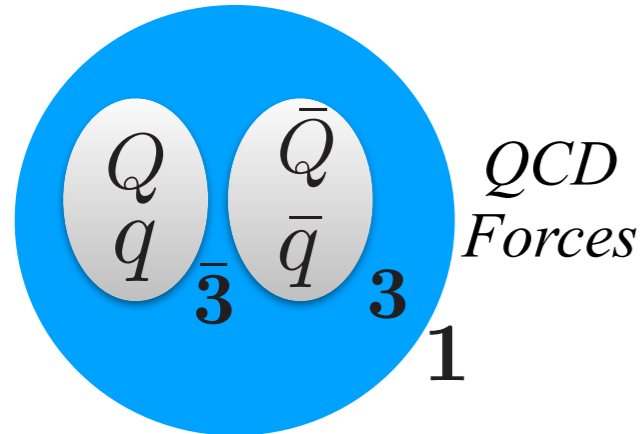
- There is a vast difference in the probability of producing X(3872) and that of producing light nuclei, true “hadronic molecules”, in high energy collisions
- high energy production of suspected exotic hadrons from quark-gluon plasma in HI collisions at colliders can be very effective tool to discriminate different models
- a long list of suspects: $f_0(980)$, X(3872), $Z^\pm(3900)$, $Z^\pm(4020)$, $Z^\pm(4430)$, X(4140)....

No consensus, yet



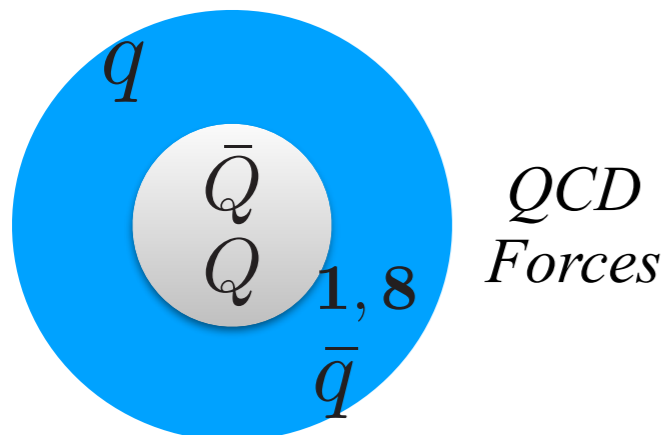
Hadron Molecule

F-K. Guo, C. Hanhart, U-G Meißner, Q. Wang, Q. Zhao, and B-S Zou, arXiv 1705.00141 (2017)



Compact Diquark-Antidiquark

L. Maiani, F. Piccinini, A. D. Polosa and V. Riquer, Phys. Rev. D 71 (2005) 014028; D 89 (2014) 114010.



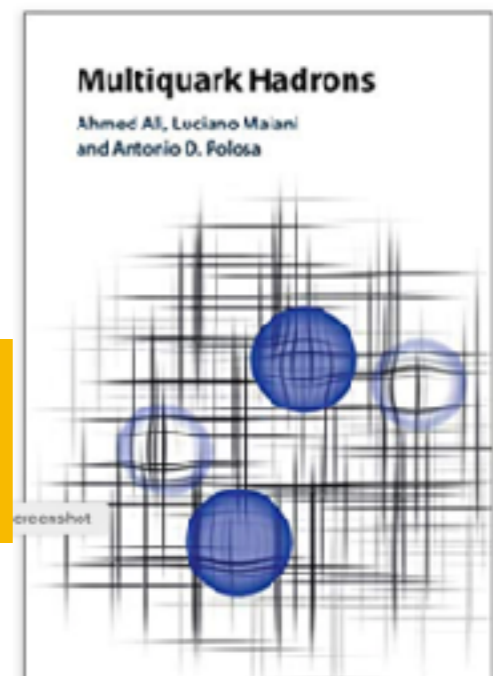
HadroCharmonium (1)
Quarkonium Adjoint Meson (8)

S. Dubynskiy, S. and M. B. Voloshin, Phys. Lett. B 666, (2008) 344.

E. Braaten, C. Langmack and D. H. Smith, Phys. Rev. D 90 (2014) 01404

For a review, see:

A. Ali, L. Maiani and A.D. Polosa, *Multiquark Hadrons*, Cambridge University Press (2019)

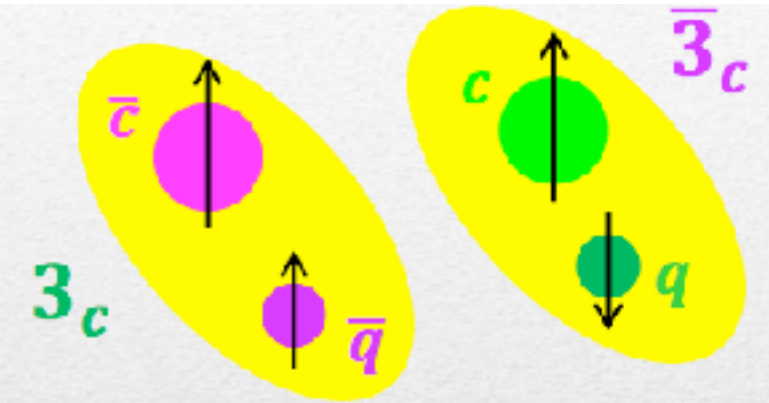


5. Tetraquark constituent picture of unexpected quarkonia

L.Maiani, F.Piccinini, A.D.Polosa and V.Riquer, Phys. Rev. D 71 (2005) 014028

$$[cq]_{S=0,1} [\bar{c}\bar{q}']_{\bar{S}=0,1}$$

- $I=1, 0$
- S-wave: positive parity
- total spin of each diquark: $S=1, 0$
- heavy-light bad diquarks admitted!
- neutral states may be mixtures of isotriplet and isosinglet
- mass splitting described by the Hamiltonian:



$$\mathbf{H} = 2m_{(diquark)} + \sum_{i < j} 2\kappa_{ij} (\mathbf{s}_i \cdot \mathbf{s}_j)$$

The S-wave, $J^P=1^+$ charmonium tetraquarks

in the $|S, \bar{S}\rangle_J$ basis we have the following states

$$J^P = 0^+ \quad C = + \quad X_0 = |0, 0\rangle_0, \quad X'_0 = |1, 1\rangle_0$$

$$J^P = 1^+ \quad C = + \quad X_1 = \frac{1}{\sqrt{2}} (|1, 0\rangle_1 + |0, 1\rangle_1)$$

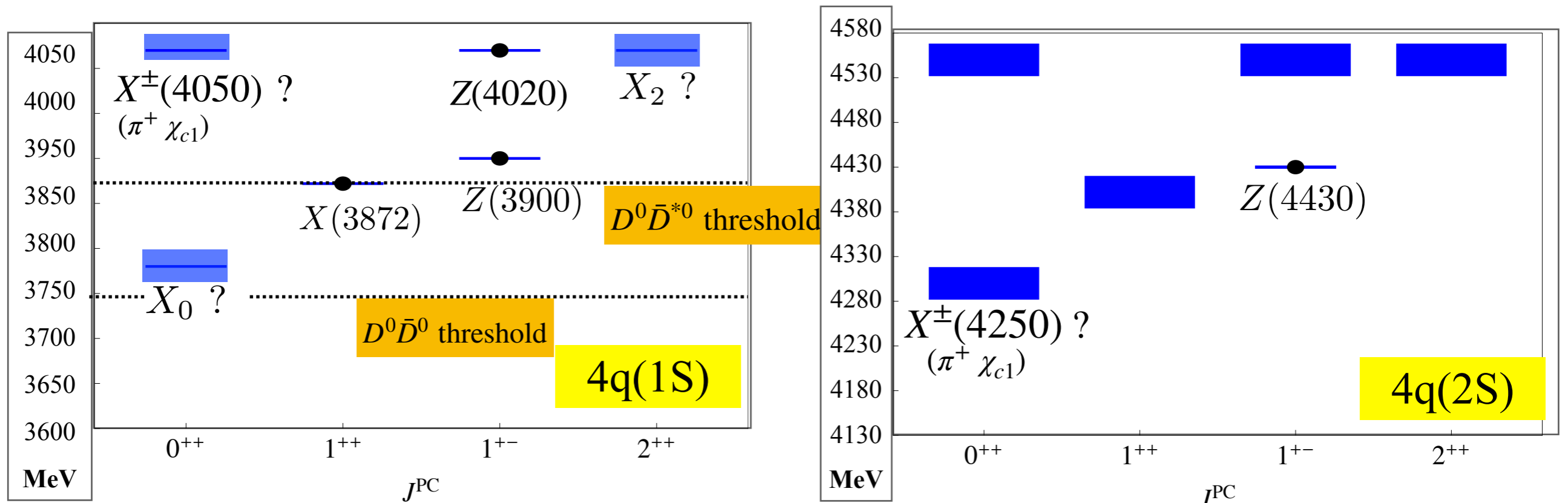
$$J^P = 1^+ \quad C = - \quad Z = \frac{1}{\sqrt{2}} (|1, 0\rangle_1 - |0, 1\rangle_1), \quad Z' = |1, 1\rangle_1$$

$$J^P = 2^+ \quad C = + \quad X_2 = |1, 1\rangle_2$$

$X(3872)=X_1$; $Z^\pm(3900), Z^\pm(4020)=$ lin. combs. of Z & Z' that diagonalize H ;
 $X^\pm(4250) = X'_0$??

Compact tetraquarks $[cq][\bar{c}\bar{q}']$: the first multiplets

- The spectrum of 1S ground states is characterised by two quantities:
 - the diquark mass, $m_{[cq]}$
 - the spin-spin interaction inside the diquark or the antidiquark, κ_{cq} .
- The first radially excited, 2S, states are shifted up by a common quantity, the radial excitation energy, ΔE_r expected to be mildly dependent on the diquark mass: $E_r(cq) \sim E_r(cs)$

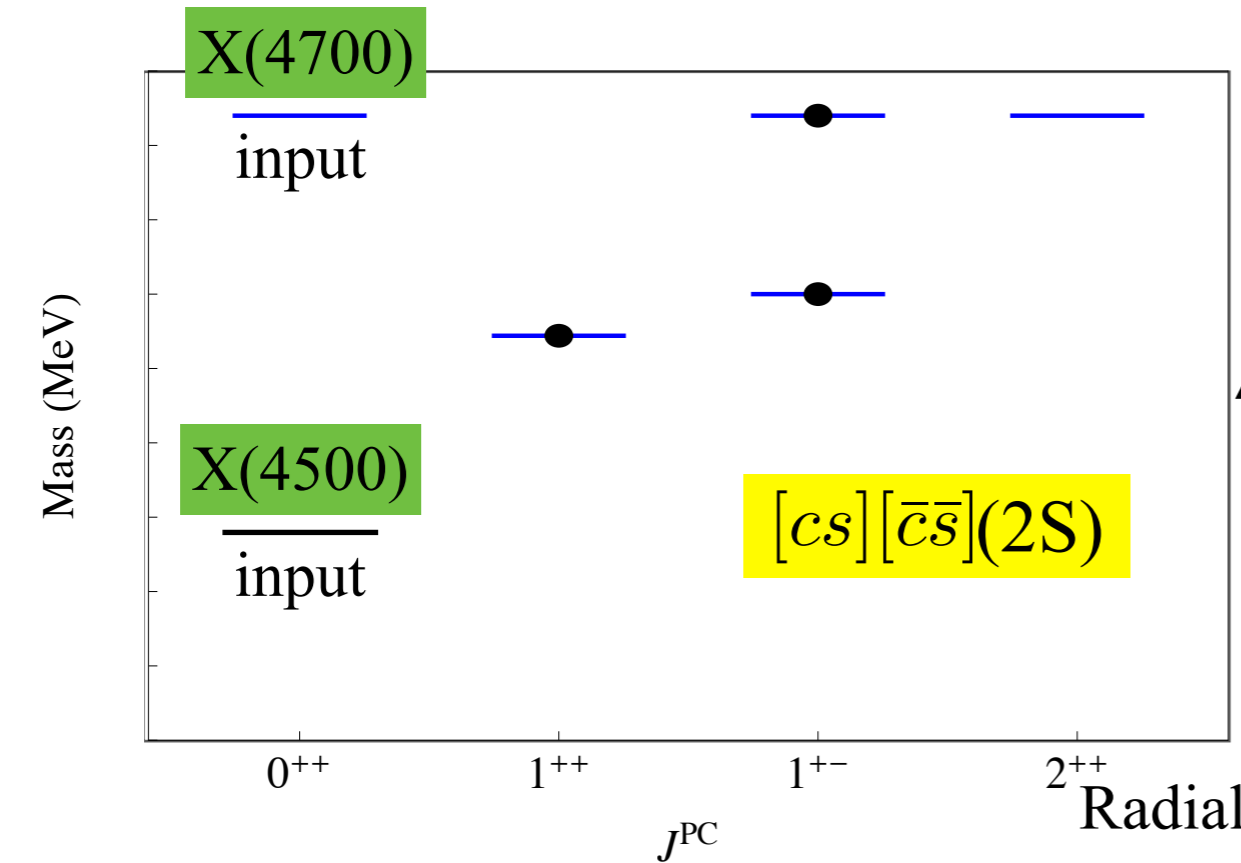


$$m_{[cq]} = 1980 \text{ MeV}$$

$$\kappa_{cq} = 67 \text{ MeV}$$

$$\Delta E_r(cq) = 530 \text{ MeV}$$

J/ψ-φ structures and S-wave tetraquarks (2016)

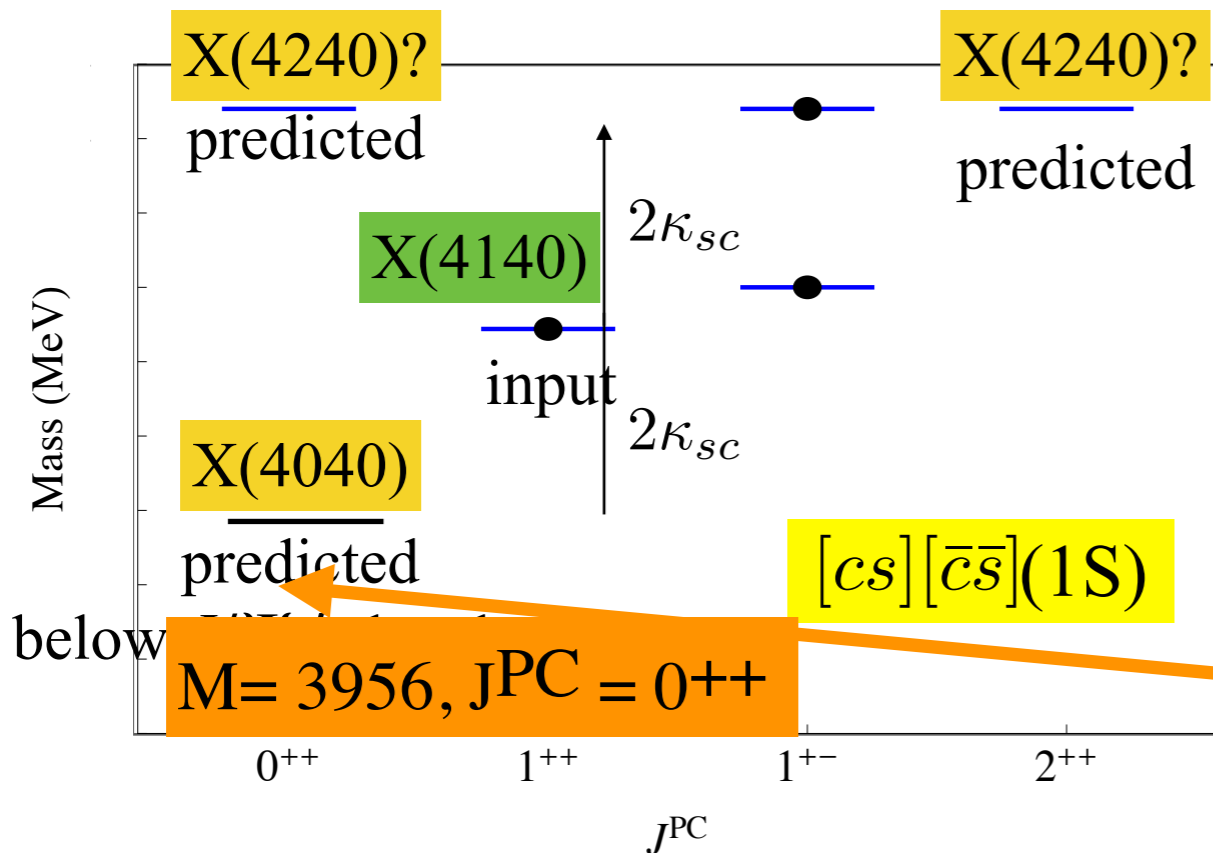


$$\Delta m = m_{cs} - m_{cq} = 129 \text{ MeV};$$

$$\kappa_{sc} = 50 \text{ MeV} \quad (\kappa_{qc} = 67 \text{ MeV})$$

radial excit. = 460 MeV

$$[Z(4430) - Z(3900) = 530 \text{ MeV}]$$



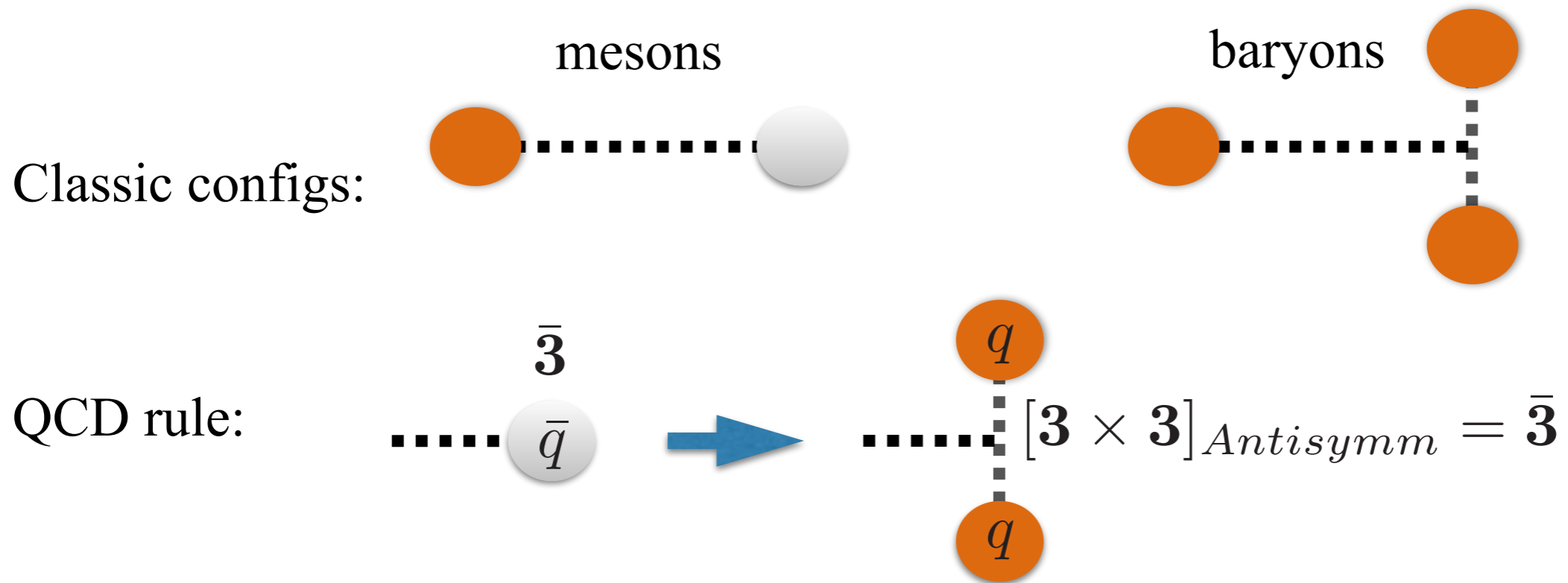
NOTE :

$$X(4140) - X(3872) \sim 270 \text{ MeV};$$

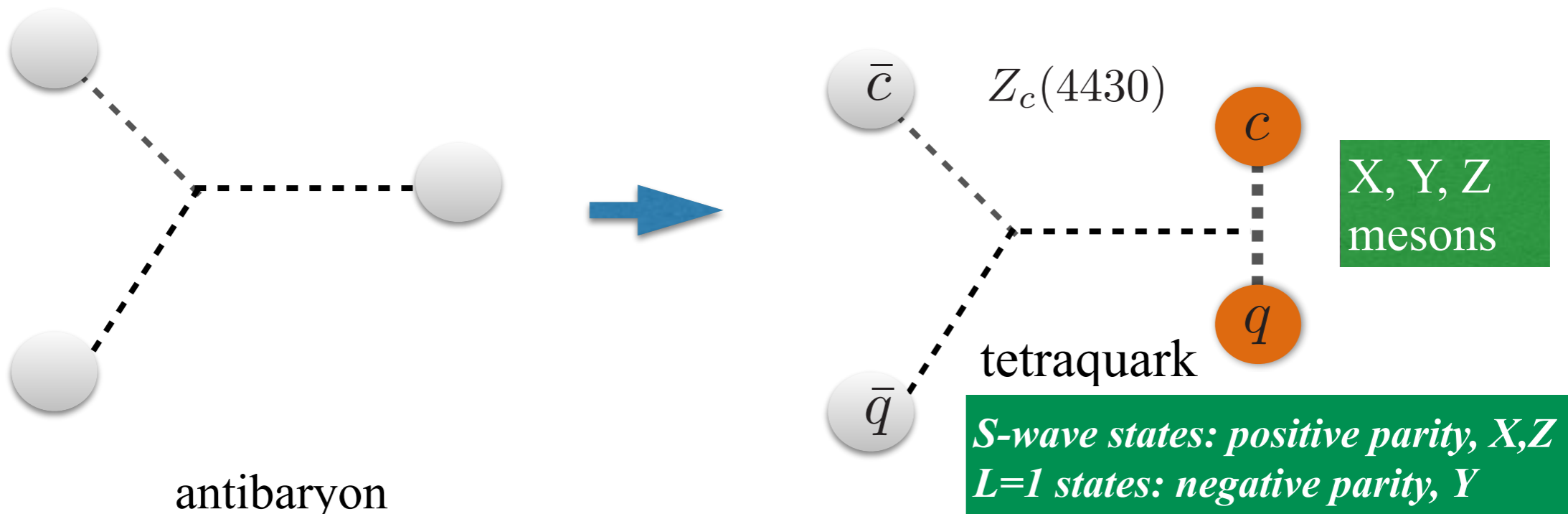
$$\phi(1020) - \rho(770) \sim 244 \text{ MeV}$$

LHCb Sept. 2022, arXiv:2210.15153
 Evidence for an additional structure is found around 4140 MeV in the $D_s^+ D_s^-$ invariant mass, which might be caused either by a new resonance with the 0^{++} assignment or by a $J/\psi\phi \leftrightarrow D_s^+ D_s^-$ coupled-channel effect.

6. Generating multiquark hadrons in QCD



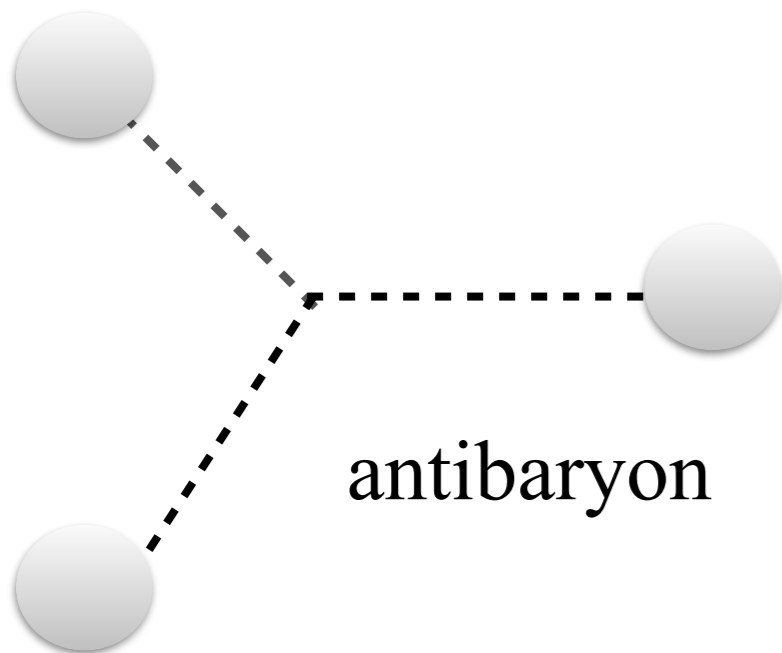
New objects can be made by the substitution: antiquark \rightarrow diquark



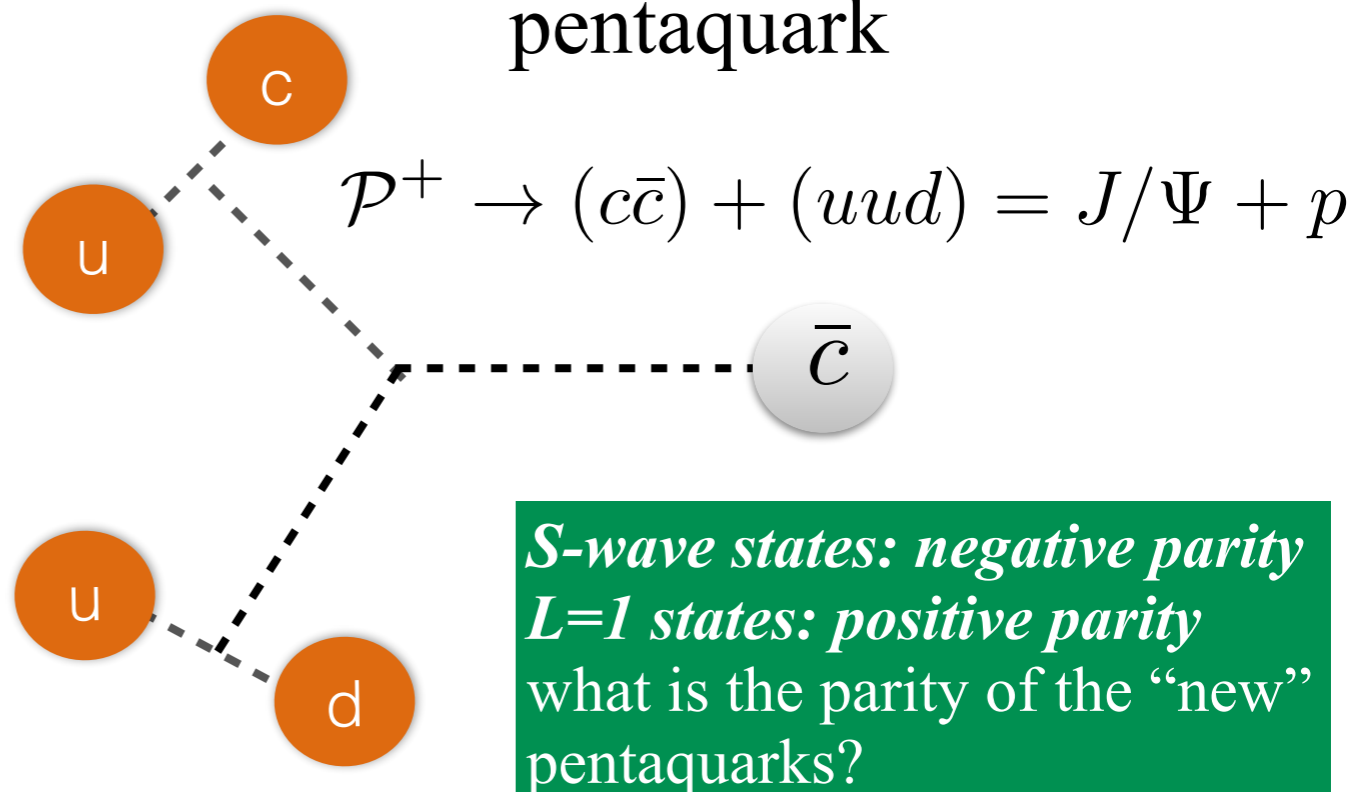
A second spectroscopic sequence

L. Maiani, A. D. Polosa and V. Riquer, PLB 749 (2015) 289

Two substitutions



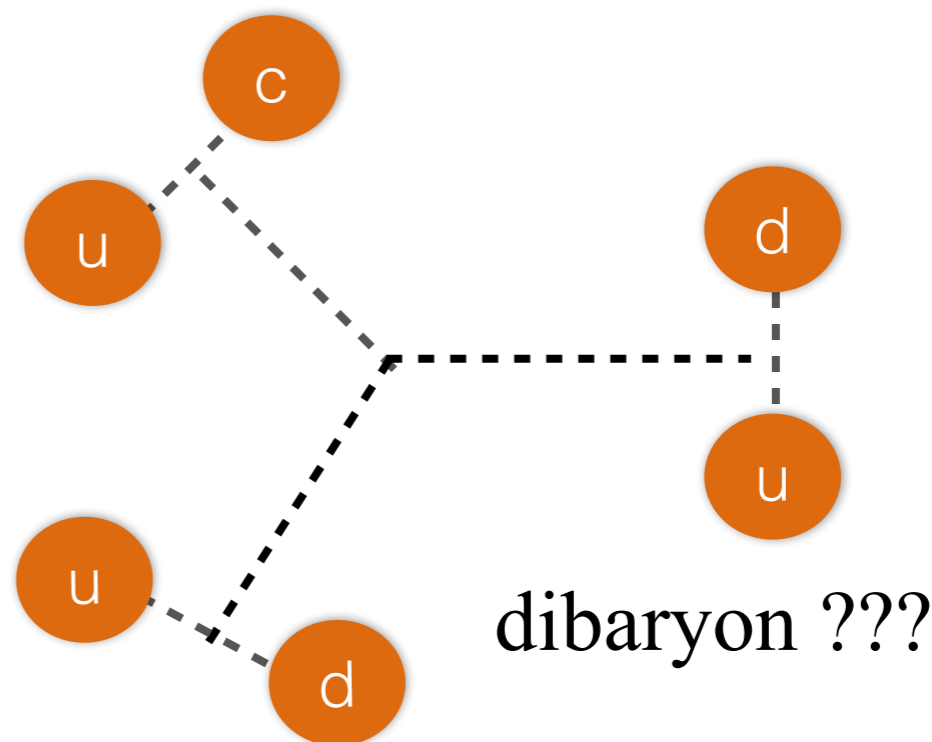
pentaquark



S-wave states: negative parity
L=1 states: positive parity
 what is the parity of the “new” pentaquarks?

S-wave states: positive parity
L=1 states: negative parity

Three substitutions



dibaryon ???

Substitution with diquark (antidiquark) reproduces qualitatively *all we have seen in Exotic Hadrons until now...* and more

Lowest lying Spectroscopic series (color singlets) in QCD with 3 colours

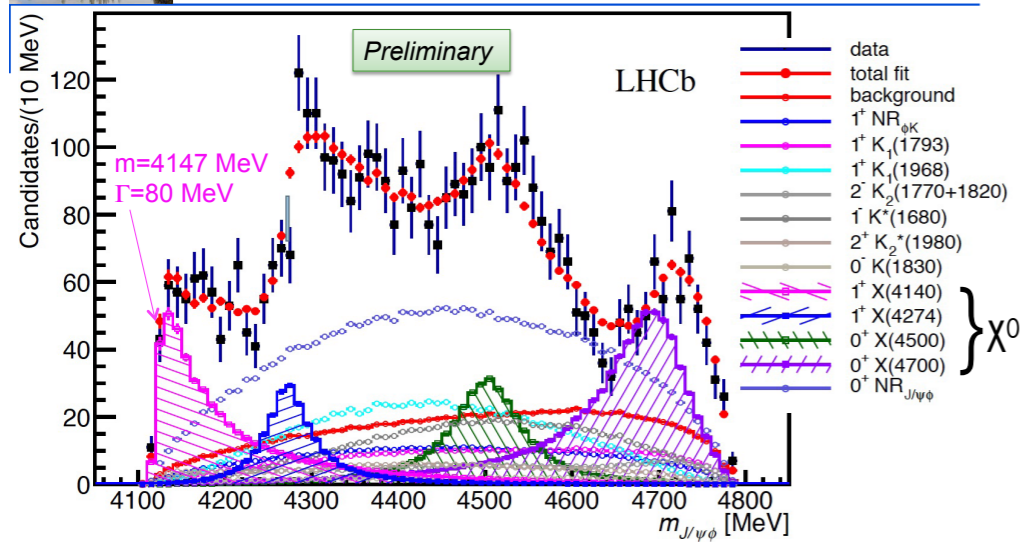
- Spectroscopic series depend upon the number of colors: *with 2 colours* we have only $q\bar{q}$ mesons and qq “baryons” : all extra states must be molecules.
- Multiquark states can be formulated in arbitrary number of colours, N,
- *With 3 colours*, from baryons qqq we generate *three new spectroscopic series*: tetraquarks, pentaquarks and dibaryons, all in different flavours.
- BELLE, BESIII, LHCb have produced well established examples of (valence) tetra- and pentaquarks, no dibaryon has been seen thus far.
- A recent lattice QCD calculation shows a deeply bound 6 b-quarks dibaryon:
 $\Omega_{bbb}\Omega_{bbb}, E_B = -89^{+16}_{-12} \text{ MeV}$

N. Mathur et al. ArXiv:2205.02862

7. Exotics: the New Wave

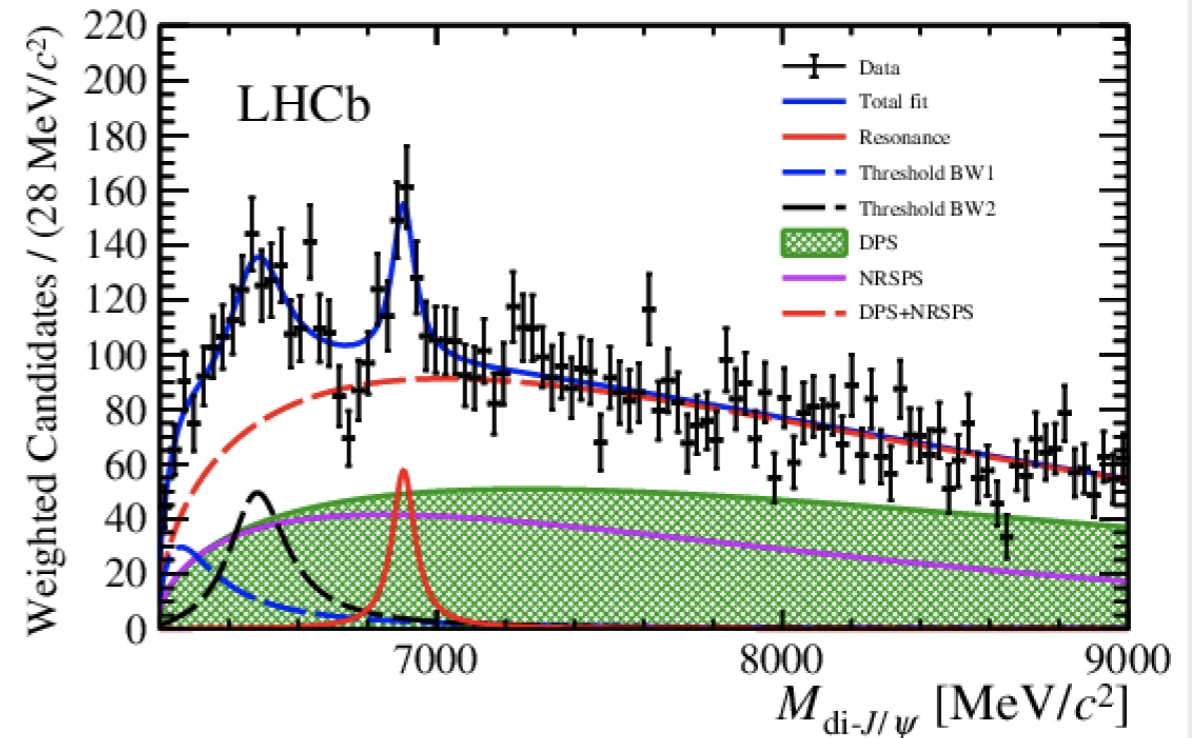
$$B^+ \rightarrow K^+ + X(4140) \rightarrow K^+ + \phi \Psi, \text{ etc.}$$

Results of fit: $m(J/\psi\phi)$



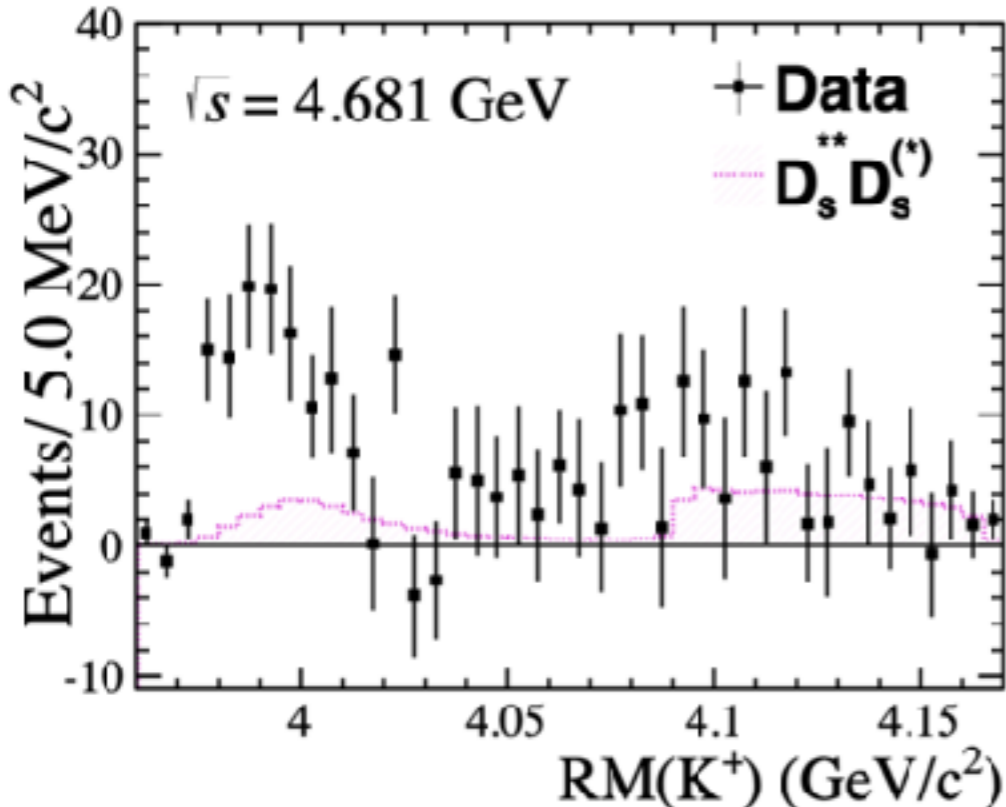
4 visible structures fit with BW amplitudes

LHCb (2016): $\Psi \phi$ resonances (2016)

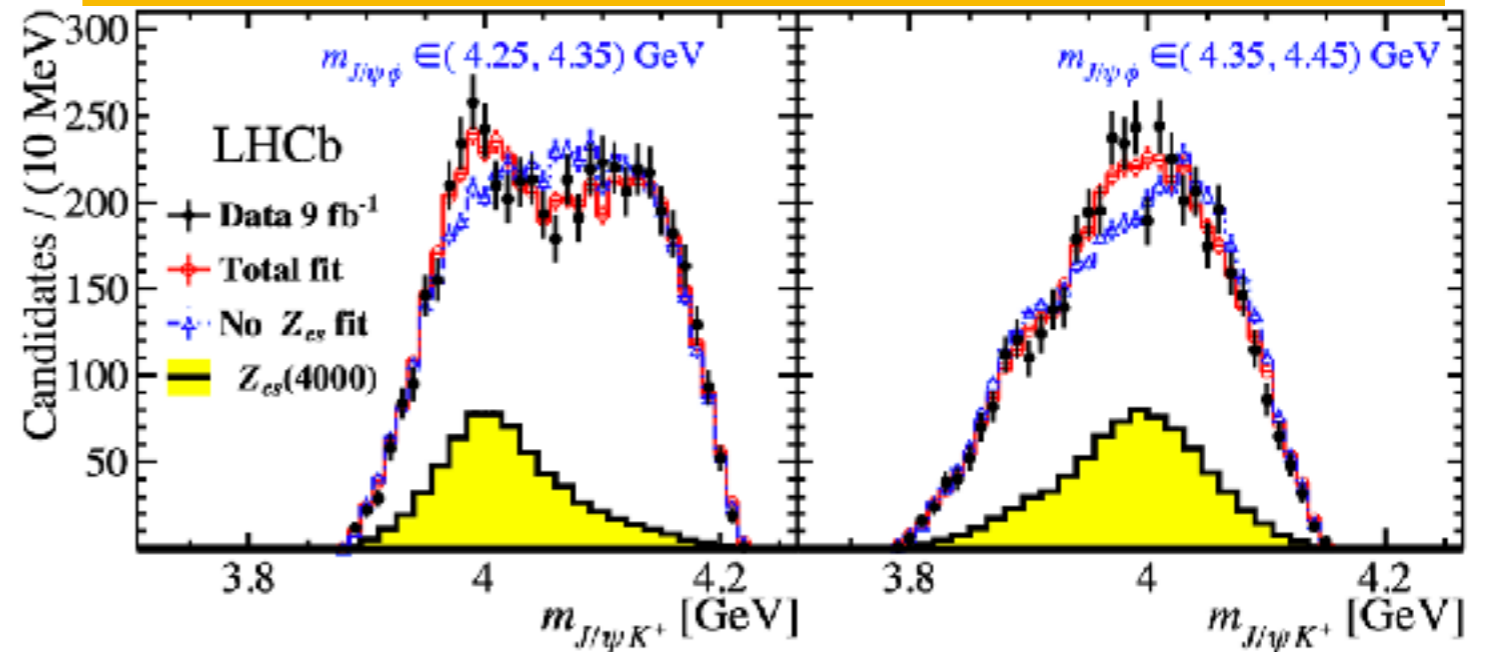


LHCb (2020): di- Ψ resonance(s) spectrum

$$\text{BES III(2021): } e^+e^- \rightarrow K^+ + Z_{cs}^-(3985) \rightarrow K^+(D_s^* D^0 + D_s^- D^{*0})$$



$$\text{LHCb (2021): } B \rightarrow \Psi + K^+ + \phi \rightarrow Z_{cs}(4003) + \phi$$



new wave (cont'd)

- Starting from 2016, new kinds of exotic hadrons have been discovered:
 - $J/\Psi \phi$ resonances, $di - J/\Psi$ resonances,
 - open strangeness Exotics: $Z_{cs}(3082)$ and $Z_{cs}(4003)$
- No pion exchange forces could bind them as hadron molecules made by color singlet mesons
- molecular models applied to the new hadrons have to stand on the existence of “phenomenological forces” with undetermined parameters
- The New Exotics arise very naturally as $([cq]^3[\bar{c}\bar{q}']^3)_1$ bound in color singlet.

The compact tetraquark model makes a firm prediction:
hidden charm tetraquarks must form *complete multiplets of flavor*
 $SU(3)$ with mass differences determined by the quark mass difference

$$m_s - m_{u,d}$$

with $Z_{cs}(3082)$ and $Z_{cs}(4003)$ we can almost fill two tetraquark nonets with the expected scale of mass differences

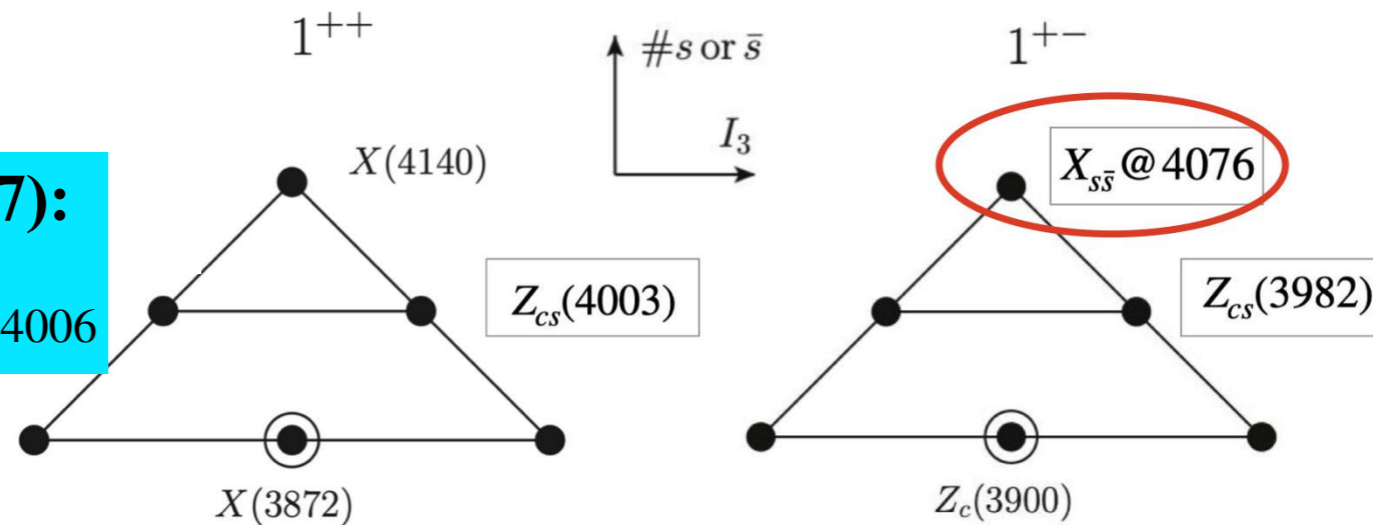
Two nonets: Solution 1 (preferred)

L. Maiani, A. D. Polosa and V. Riquer, Sci. Bull. **66** (2021), 1616, arXiv:2103.08331

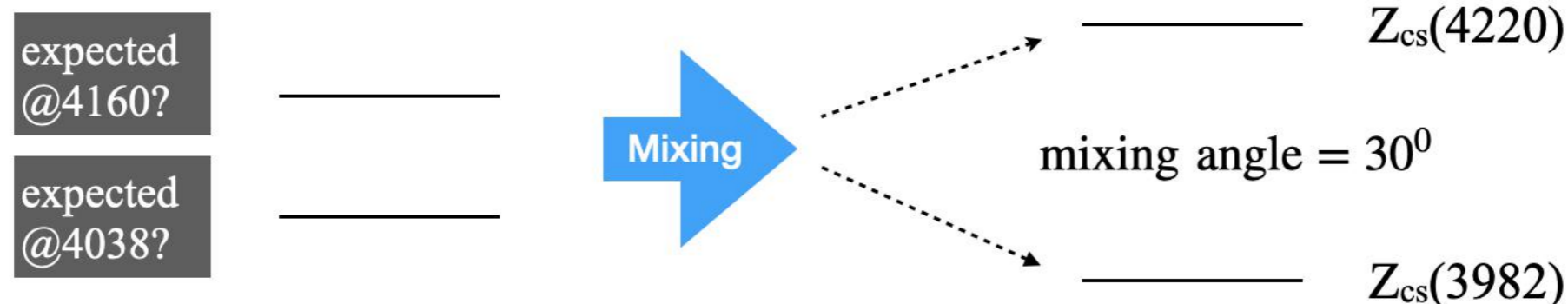
Solution 1

Predicted (2017):

$$\frac{X(4140) + X(3872)}{2} = 4006$$



- There is a *third nonet* associated to $Z_c(4020)$, $J^{PC} = 1^{+-}$: a third Z_{cs} is required, Mass=4150 - 4170
- *LHCb sees a $Z_{cs}(4220)$, $J^P = 1^+$ or 1^- : is it too heavy?* A bold proposal:



A well defined shopping list towards completion:

- $X_{s\bar{s}}$, $M = 4076$ (Sol. 2 : 4121), decays: $\eta\psi$, $\eta_c \phi$, $D_s^* \bar{D}_s$ (if phase space allows)
- the $I=1$ partner of $X(3872)$, decays: $X^+ \rightarrow J/\psi \rho^\pm \rightarrow J/\psi \pi^+ \pi^0$
- the $I=0$ partners of $Z_c(3900)$ and $Z_c(4020)$, possibly decaying into: $J/\psi + f_0(500)$ (aka $\sigma(500)$)

8. Molecule or compact ? the QCD framework (following Weinberg's argument about deuteron, PRL 1965)

- We know that QCD produces hidden charm, confined hadron states: charmonia, $D^*\bar{D} + \bar{D}^*D$.

??? Do confined tetraquarks exist??

- Suppose we switch off the interactions between confined hadrons. The space of possible hidden charm states is made by two components

- discrete energy states: charmonia and possibly tetraquarks: $|C\rangle \langle C| + |T\rangle \langle T|$
- continuum charmed meson pairs: $|D^*\bar{D}(\alpha)\rangle \langle D^*\bar{D}(\alpha)|$

Completeness relation: $\langle X|X\rangle = 1 = Z + \int d\alpha |\langle X|D^*\bar{D}(\alpha)\rangle|^2$,

$$Z = |\langle X|C\rangle|^2 + |\langle X|T\rangle|^2$$

There are *two regimes*

- $Z=0$: corresponds to a pure molecular state: X results from $D^* - \bar{D}$ interactions only (like the deuteron = pn)
- $Z \neq 0$: some compact, discrete state *must* exist: could it be charmonium only?
- unlike charmonium states, X decays violate isospin: $\Gamma(\Psi\rho) \sim \Gamma(\Psi\omega)$

so that:

- $Z \neq 0 \rightarrow$ *Tetraquark with X quantum numbers exists.*

How can we know?

S. Weinberg, Phys.Rev. **137**, (1965) B672

- The key is the $D^*\bar{D}$ scattering amplitude, f , that near threshold (k =center of mass momentum ~ 0) can be parametrised as $f^{-1} = k \cot \delta(k) - ik = -\kappa_0 + \frac{1}{2}r_0k^2 - ik + \dots$
- in presence of a shallow, below-threshold, resonance Weinberg finds:

$$\kappa_0^{-1} = 2\frac{1-Z}{2-Z}\kappa^{-1} + O(1/m_\pi); \quad r_0 = -\frac{Z}{1-Z}\kappa^{-1} + O(1/m_\pi)$$

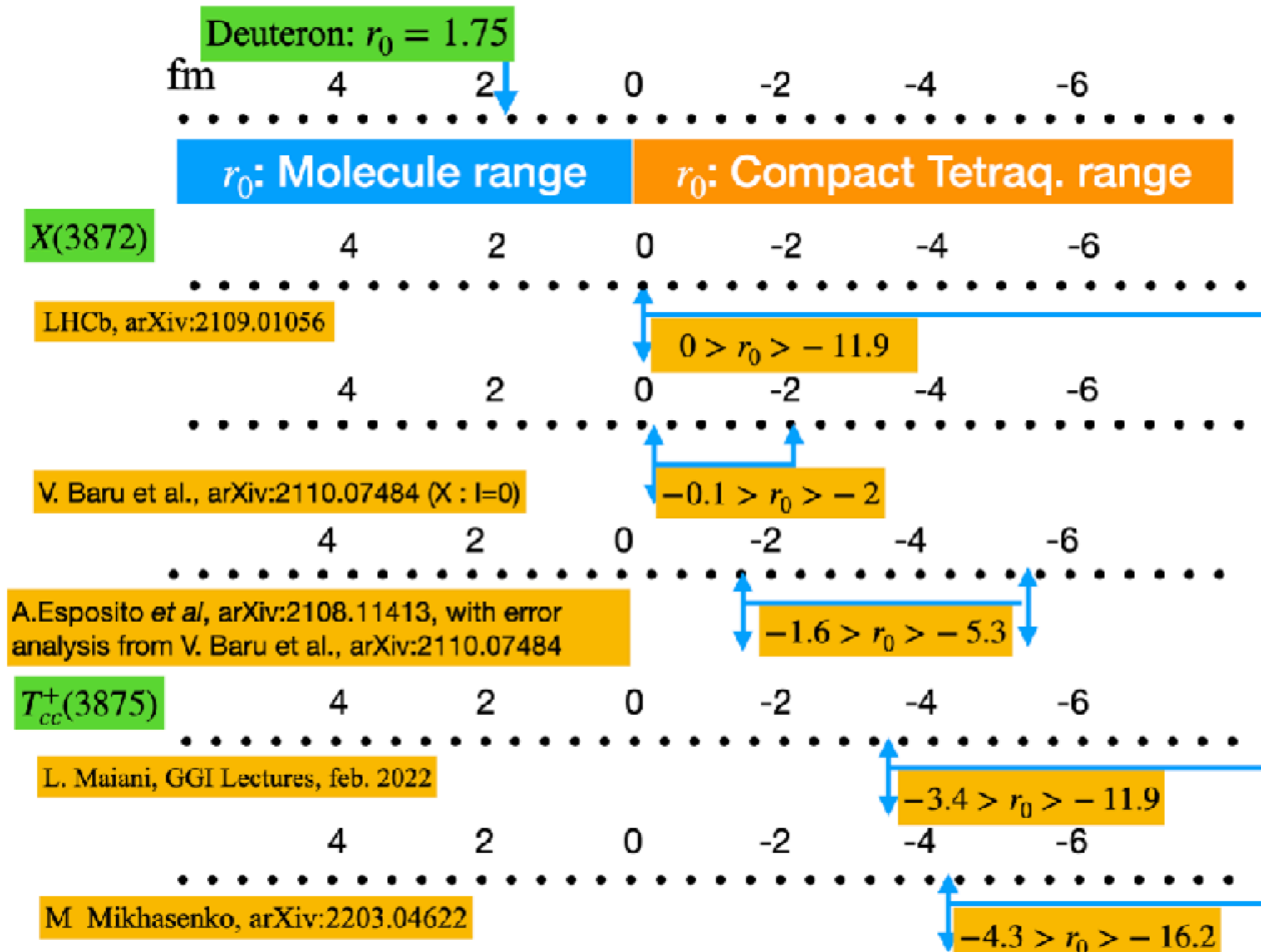
$$\kappa^{-1} = \sqrt{2\mu B}, \quad B = M(D^*) + M(D) - M(X) \text{ (the "binding energy")}$$

- in the molecular case ($Z=0$) one has $r_0 = O(1/m_\pi)$
-and, for attractive potentials, one can show that ***the unspecified part $O(1/m_\pi)$ is positive:***

$$r_0 > 0$$

This result is stated as solution to a problem given in the book of Landau and Lifshitz, *Quantum Mechanics*;
H.Bethe, 1949, gives a concise demonstration, see A. Esposito *et al.* Phys. Rev. **D 105** (2022), L031503

My Summary about r_0

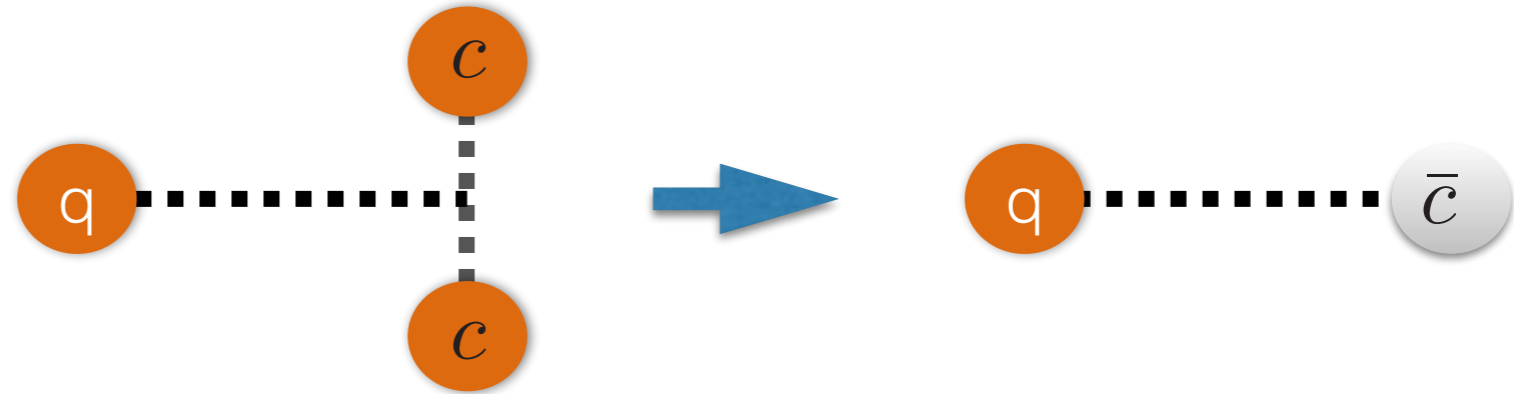


No consensus yet, but we are on a very promising road.
Stay tuned!!

9. The new sensation: doubly heavy baryons and doubly heavy tetraquarks

Single heavy-doubly heavy quark symmetry: M. Savage, M. B. Wise, PLB 248,1990; N. Brambilla, A. Vairo and T. Rosch, PRD 72, 2005; T. Mehen, arXiv:1708.05020v3

- Doubly heavy baryons are related to single quark heavy mesons:



- QCD forces are mainly spin independent, so there is an approximate symmetry relating masses of DH baryons to SH mesons: e.g. $M(\Xi_{cc}^*) - M(\Xi) = \frac{3}{4}(M(D^*) - M(D))$

- Doubly heavy tetraquarks have been anticipated long ago.

Esposito, M. Papinutto, A. Pilloni, A. D. Polosa, and N. Tantalo, Phys. Rev. D88, 054029 (2013)

- The possibility has been raised that the $I = 0$, $J^P = 1^+$, $\mathcal{T}_{cc}^+ = bb\bar{u}\bar{d}$ be stable under strong and e.m. decays

M. Karliner and J. L. Rosner, PRL 119 (2017) 202001. E. J. Eichten and C. Quigg, PRL 119 (2017) 202002.; S. Q. Luo et al. Eur. Phys. J. C 77 (2017) 709.

- extended calculations of \mathcal{T}_{cc} and \mathcal{T}_{bb} mass have been presented, in the Born-Oppenheimer approximation, analytical

L. Maiani, A. D. Polosa and V. Riquer, Phys. Rev. D100 (2019) 074002,

- and in Lattice QCD see later for Refs.

Radiography of the double beauty tetraquark

Born-Oppenheimer approximation in 3 steps:

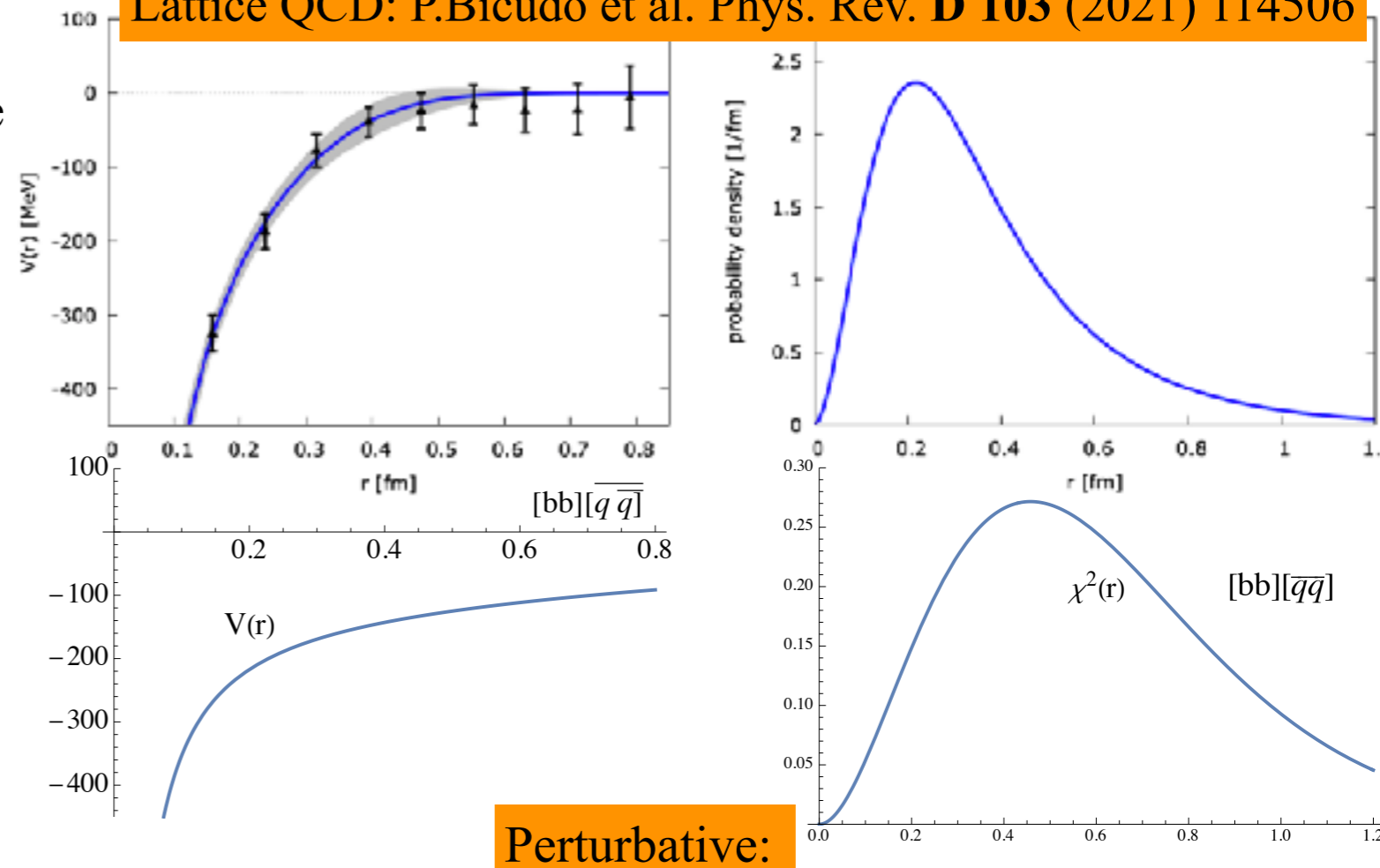
- take the b quarks as fixed sources at distance R;
- find V_{BO} = energy of the bound state of light quarks in presence of the b quarks;
- solve Schroed. equation of b quarks with potential $V_{BO}(R)+V_{bb}(R)$.

For $R \rightarrow +\infty$ $V_{BO}(R)$ vanishes in both cases: why not confined?

- at ∞ , each b quark is screened by a \bar{q} : the state is a superposition of $B_{color\ 8} - B_{color\ 8}$ and $B_{color\ 1} - B_{color\ 1}$
- but in $B_{color\ 8} - B_{color\ 8}$ color can be screened by soft gluons from vacuum
- **the asymptotic state is $B_{color\ 1} - B_{color\ 1}$ with vanishing interaction at $R \rightarrow \infty$**

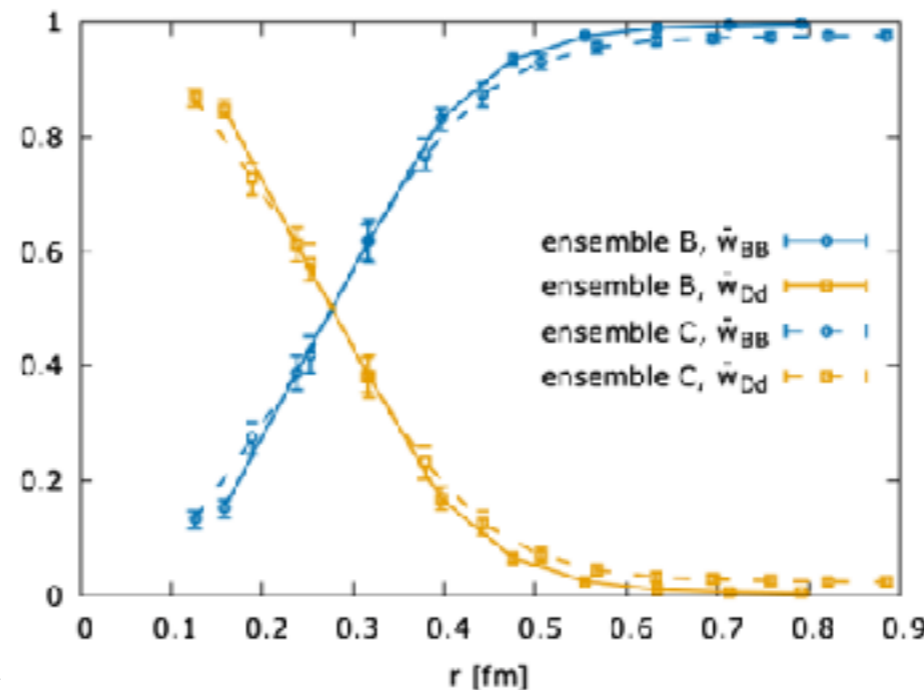
- Projection of the lattice $\bar{q}\bar{q}$ result over Dd (yellow) or BB (blue) gives a picture of the space arrangement of light quarks at a given bb distance, R
- Mainly $[bb]_3[\bar{q}\bar{q}]_3$ at the peak of the bb wave function ~ 0.2 fm.

Lattice QCD: P.Bicudo et al. Phys. Rev. D 103 (2021) 114506



Perturbative:

L. Maiani, A. D. Polosa and V. Riquer, PR D100 (2019) 074002



← $B_1 B_1 = \text{Meson-Meson}$

Lattice QCD

← $Dd = [bb][\bar{q}\bar{q}]$

The mass of the lightest double heavy tetraquarks can be computed!

- There are recent estimates of the mass of the lightest, double heavy tetraquarks, that may indicate that the $I=0$, $bb\bar{u}\bar{d}$ tetraquark may be stable. We give in the table below a comparison of different theoretical results.
- ***Q-value is taken with respect to PS-PS threshold (not V-PS!) $M(D^*) - M(D) = 140$ MeV***
- ***BO revised:*** L. Maiani, A. Pilloni, A. D. Polosa and V. Riquer, PLB to appear [arXiv:2208.02730 [hep-ph]]

$QQ\bar{u}\bar{d}$	BO revised [1]	K. and R.[2]	E. and Q.[3]	L.[4]	Lattice QCD
$cc\bar{u}\bar{d}$	+136(+119)	+140	+102	+39	-23 ± 11 Junn. <i>et al.</i> [5]
$bb\bar{u}\bar{d}$	-1.8(-9.7)	-170	-121	-75	-143 ± 34 Junn.[5] <i>et al.</i> $-143(1)(3)$ Francis <i>et al.</i> [6] $-82 + 24 + 10$ Leskovec <i>et al.</i> [7] -13^{+38}_{-30} P.Bicudo <i>et al.</i> [8]

[1] L. Maiani, et al. (2022) [2] M. Karliner and J. L. Rosner (2017). [3] E. J. Eichten and C. Quigg, (2017); [4] S. Q. Luo et al. (2017)

Lattice QCD:

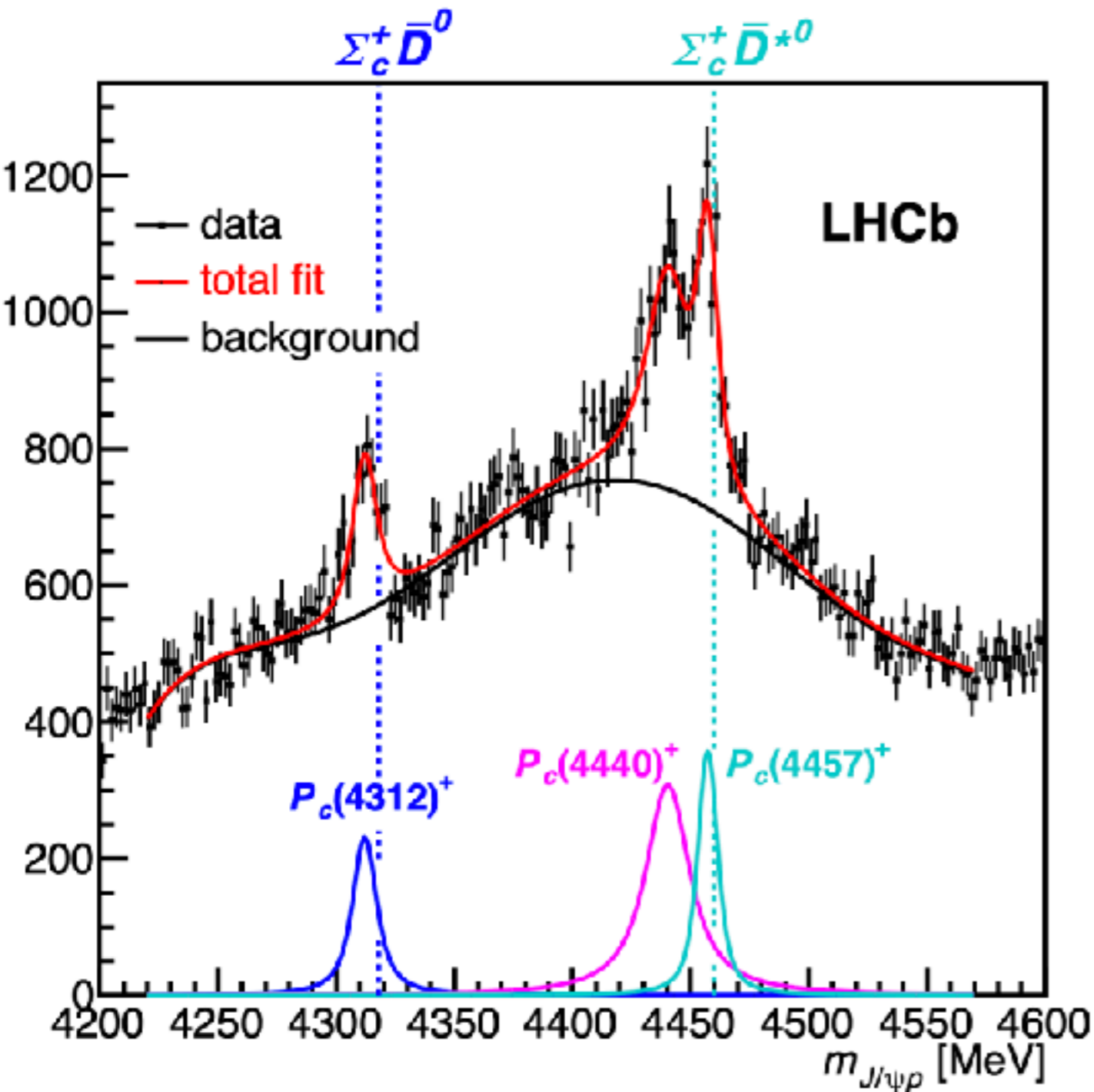
[5] Junnarkar *et al.* Phys. Rev. **D 99** (2019) 034507; [6] Francis *et al.* Phys. Rev. Lett. **118** (2017), Phys. Rev. **D 99** (2019); [7] L. Leskovec *et al.* Phys. Rev. **D 100** (2019) 014503; [8] P. Bicudo *et al.* (BO in lattice QCD) Phys. Rev. **D 103** (2021) 114506.

Born-Oppenheimer approximation:

- Estimate of \mathcal{T}_{cc} mass close to the observed mass:

$$M(\mathcal{T}_{cc}^{BO}) = 3871(3854) \leftrightarrow \text{LHCb} : \mathcal{T}_{cc}^+(3875)$$
- Stable Double Beauty tetraquarks ? still possible but not so clear
- Searching for $\mathcal{T}_{cc}, I = 1$ around DD/D^*D thresholds *is essential!!*

10. Pentaquarks



Three non strange pentaquark lines:

$P_c^N(4312)$: $M = 4311.9 \pm 0.7 + 6.8$, $\Gamma = 9.8 \pm 2.7$

$P_c^N(4440)$: $M = 4440.3 \pm 1.3 + 4.1$, $\Gamma = 20.6 \pm 4.9$

$P_c^N(4457)$: $M = 4457.3 \pm 0.6 + 4.1$, $\Gamma = 6.4 \pm 2.0$

$\Lambda_b \rightarrow J/\Psi + p + K^-$, R.~Aaij et al. [LHCb],
 Phys. Rev. Lett. **122** (2019) 222001,
 arXiv:1904.03947 [hep-ex]

In addition two (three ?) strange pentaquark lines seen

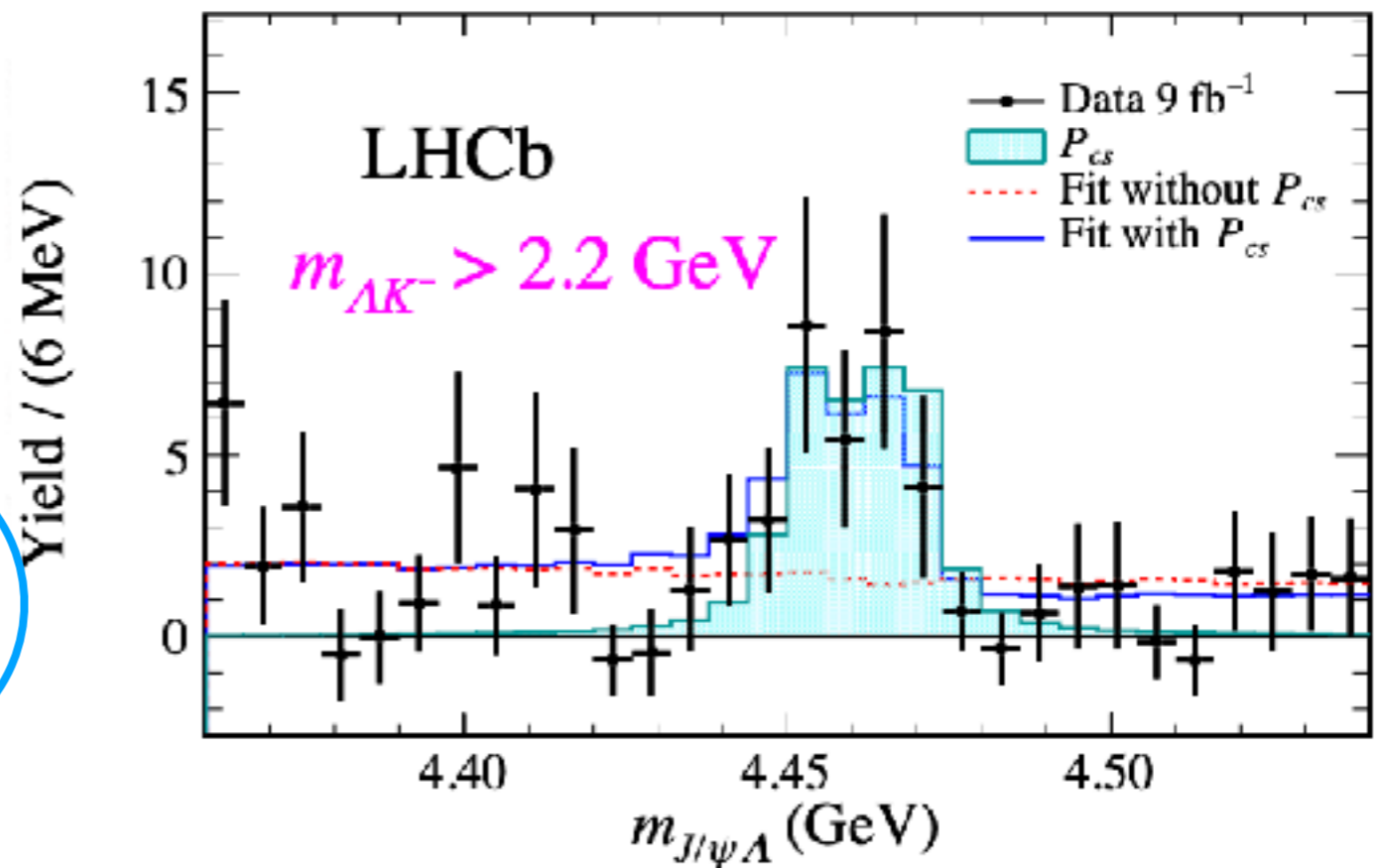
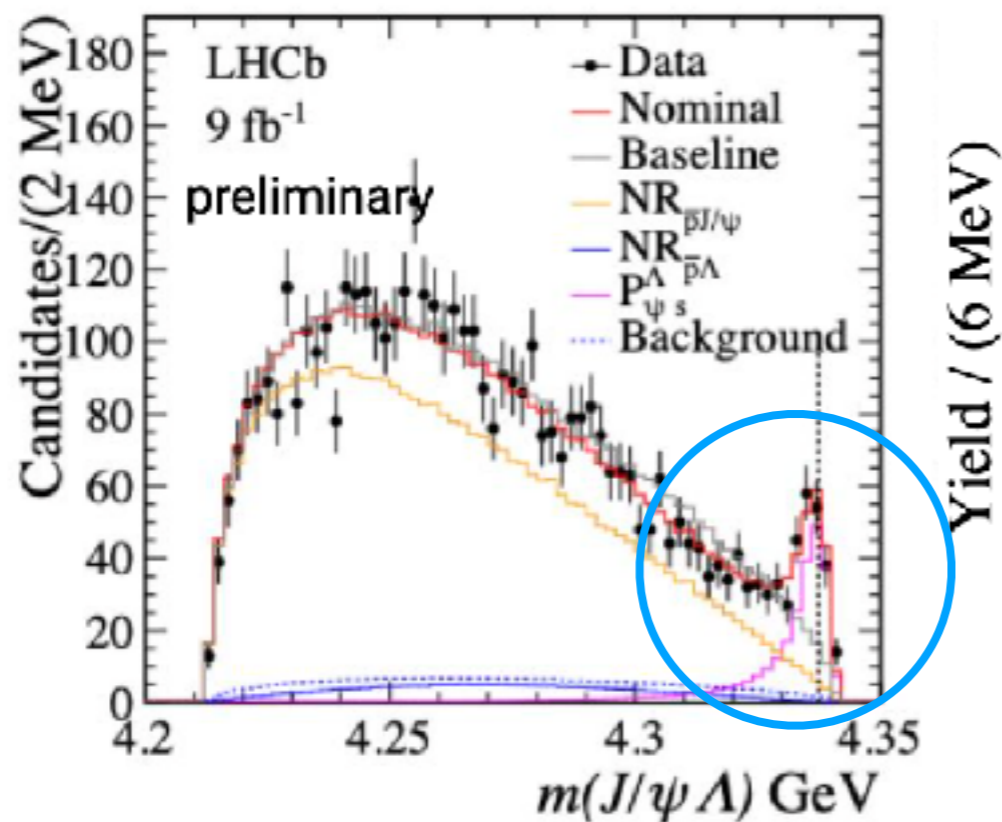
$B^- \rightarrow J/\Psi + \Lambda + \bar{p}$, LHCb Paper 2022-031,

R.Aaij *et al.* [LHCb], Sci. Bull. **66** (2021),
1278; arXiv:2012.10380

P_{Λ}^{Δ} (4338) : $M = 4338.2 \pm 0.7$ MeV, $\Gamma = 7.0 \pm 1.2$ MeV

P_{Λ}^{Δ} (4455) : $M = 4454.9 \pm 2.7$ MeV, $\Gamma = 7.5 \pm 9.7$ MeV

P_{Λ}^{Δ} (4468) : $M = 4467.8 \pm 3.7$ MeV, $\Gamma = 5.2 \pm 5.3$ MeV



1. Molecules?

- Non strange Pentaquarks : $\Sigma_c - \bar{D}^0$ (spin 1/2), $\Sigma_c - \bar{D}^{*0}$ (spin 1/2, 3/2)
- Strange Pentaquarks : $\Xi_c - \bar{D}^0$ (spin 1/2), $\Xi_c - \bar{D}^{*0}$ (spin 1/2, 3/2)

Karliner & Rosner
arXiv:2207.07581

2. Compact Pentaquarks ?

Maiani, Polosa, Riquer, arXiv: 2303.04056, and...

- We consider non strange Pentaquark in the Born-Oppenheimer approximation, with $\bar{c} [cu] [ud] \rightarrow [(\bar{c}c)_8 \times (uud)_8]_1$,
- To extend to a full flavour SU(3) multiplet one must consider the restrictions due to Fermi Statistics to the configurations of *three light quarks in colour octet*
- We consider the exchange of colour, coordinates and flavour \times spin (summarised in the $SU(6) \supset SU(3)_f \times SU(2)_{spin}$ symmetry)
- We find that full quark antisymmetry is reached for *two* flavour-spin configurations:
 $\mathbf{8}_{1/2} + \mathbf{10}_{3/2} = \mathbf{56}$ or $\mathbf{1}_{3/2} + \mathbf{8}_{1/2} = \mathbf{20}$.
- In both cases, the ground state is in $\mathbf{8}_{1/2}$. Combining with with $c\bar{c}$ spin= 0,1, this gives rise to three Pentaquark octets: $2 \times \mathbf{8}_{1/2} + \mathbf{8}_{3/2}$



- *We predict three lines as observed*, corresponding to pentaquark decays:

$$\mathcal{P}^{(S=0)} \rightarrow J/\Psi + p, \quad \mathcal{P}^{(S=-1)} \rightarrow J/\Psi + \Lambda$$

- decays $\mathcal{P}^{(S=-1)} \rightarrow J/\Psi + \Sigma$ and $\mathcal{P}^{(S=-2)} \rightarrow J/\Psi + \Xi$ are also predicted,

- The two alternatives (**56** or **20**) would be distinguished by presence or absence of Pentaquarks decaying into spin 3/2 resonances, e.g. : $\mathcal{P}^* \rightarrow J/\Psi + \Delta^{++} \rightarrow J/\Psi + p + \pi^+$

In conclusion....

- *First class results on Exotic Hadrons have been obtained in the last decade by BELLE, BES III and LHCb*
- *The nex decade will see substantial progress by the same collaborations, with upgraded detectores, and with the start of Panda@ GSI Darmstadt*
- The existence of exotic SU(3) flavour multiplets, with a characteristic scale of symmetry breaking is a distinctive prediction of compact tetraquarks.
- The newly found strange exotics are close in mass, like X(3872) and Z_c(3900), and fit into their nonets: a clear score in favour.
- Lineshape analyses of both X(3872) and $\mathcal{T}_{cc}^+(3875)$ seem to indicate negative values of r_0 .
- Much remains to be done, to produce more precise data and to search for still missing particles, to complete the flavour multiples required by QCD bound, multiquark Exotics.
- Among the missing particles:
 - X(3872)⁺, isX(3872) split into two lines: $X(3872) \rightarrow X_u + X_d$?
 - $\mathcal{T}_{cc}^{++}(?)$, $\mathcal{T}_{bb}^-(?)$
- many other states are still missing, with well defined mass and decay modes as discussed before.
- Quantum numbers of Pentaquarks and of $di - J/\Psi$
- Exotic hadrons produced in hadron collisions at large p_T : are there other, besides X(3872)?
- ***Tough orders***: more luminosity, better energy definition, detectors with exceptional qualities... a lot of work...
- ***Close exchange between theory and experiments*** is essential and it has to continue.

so much accomplished, and so much more left to do (Winston Churchill)