MAGO: A SRF Cavity for Gravitational Wave Observation

Detection Theory and MAGO Prototype

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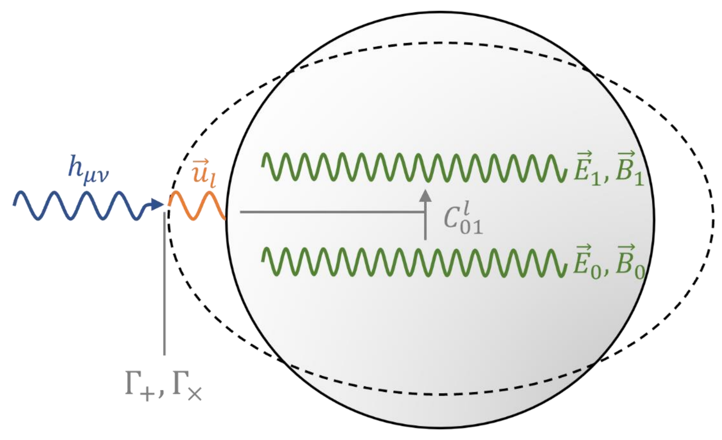
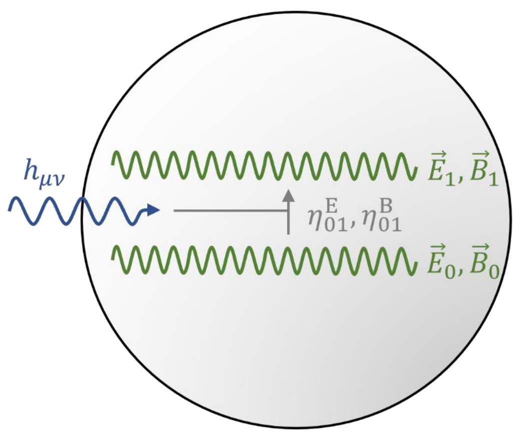
MAGO Prototype Cavity

- Built in 2005 at INFN Genoa
- FNAL, INFN Genoa, UHH and DESY collaborate on the project
- Prototype designed for gravitational wave (GW) frequencies 5-10 kHz
- Radio frequency mode pumped by external oscillator
- **Indirect coupling** via mechanical modes
- **Direct coupling** of GW to RF mode
- Measure RF power in higher mode



Indirect interaction



Direct interaction

Direct coupling (Gertsenshtein effect)

- GW induces current in Maxwells equations
- The Lagrangian for the EM field

$$\mathcal{L} = -\tfrac{1}{4}F^{\mu\nu}F_{\mu\nu} - j^{\mu}_{eff}A_\mu$$

- Current for GW strain $h_{\mu\nu}$

$$j^{\mu}_{eff} = \partial_\nu\left(\tfrac{h}{2}F^{\mu\nu} + h^{\nu}{}_{\alpha}F^{\alpha\mu} - h^{\mu}{}_{\alpha}F^{\alpha\nu}\right)$$

- For monochromatic GW propagating in z direction the overlap factors are

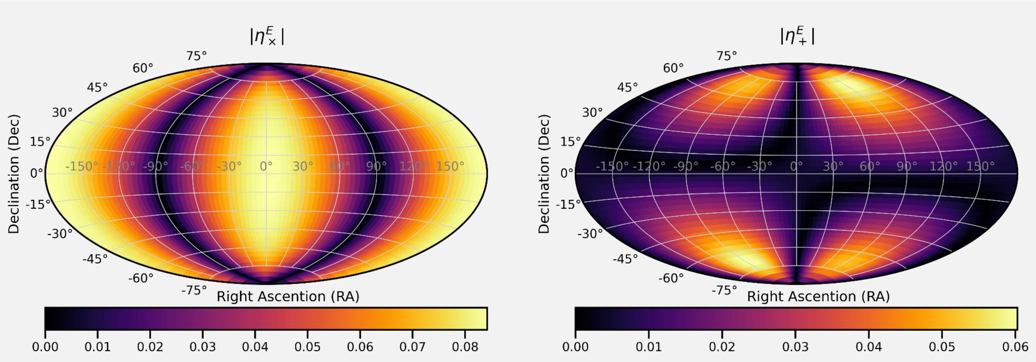
$$\eta^{E}_{0n} := \frac{1}{H\sqrt{U_0U_n}}\int_{V_{cav}} d^3x\, H_0(\vec{x})\epsilon_0\vec{E}_0(\vec{x})\vec{E}_n(\vec{x})$$

$$\eta^{E}_{0n} := \frac{1}{H\sqrt{U_0U_n}}\int_{V_{cav}} d^3x\, H_0(\vec{x})\frac{1}{\mu_0}\vec{B}_0(\vec{x})\vec{B}_n(\vec{x})$$

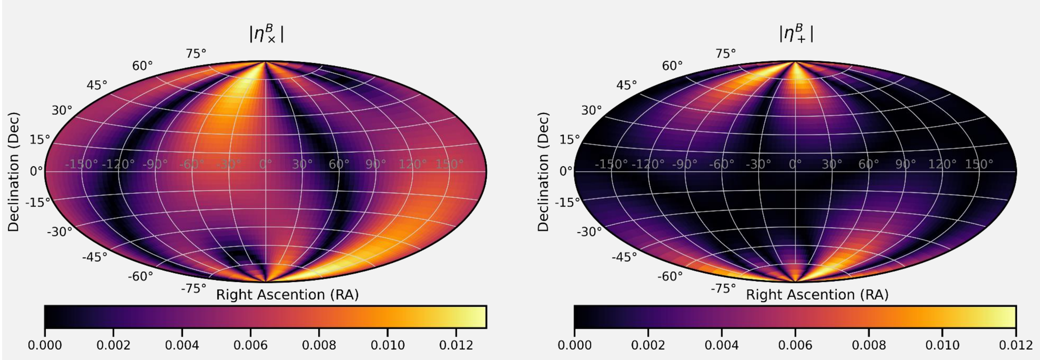
- Where H is the normalization of

$$H_0(\vec{x}) = (h_+(x^2-y^2) + 2h_\times xy)$$

- Gertsenshtein effect has small effect at low frequencies



Purely polarized GW couplings to the E field of the TE_{011} mode



Purely polarized GW couplings to the B field of the TE_{011} mode

Indirect coupling

- GW deform the cavity and excite mechanical modes
- Tidal force density from equation of geodesic deviation

$$\vec{f}\,(t,\vec{x}) = -\rho(\vec{x})R_{0i0j}(t)x_j\vec{e_i}$$

- Generalized force density on one mode

$$f_l(t,k_z) = -\frac{\omega_g^2}{2}MV_{cav}^{1/3}\left[h_+\Gamma_+^l - h_\times\Gamma_\times^l\right]e^{i(\omega_g t - k_z z)}$$

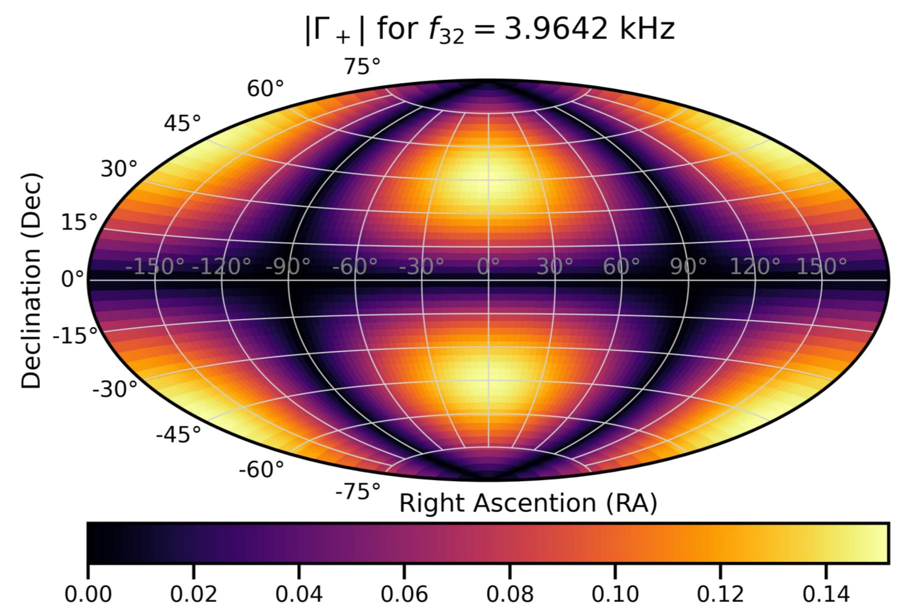
- Separation of displacement $\vec{u}(t,\vec{x}) = \sum_l \vec{\xi}_l(\vec{x})q_l(t)$

$$\Gamma_+^l = \frac{V_{cav}^{-1/3}}{M}\int_{V_{cav}} d^3x\rho(\vec{x})(x\xi_{l,x}(\vec{x}) - y\xi_{l,y}(\vec{x}))$$

$$\Gamma_\times^l = \frac{V_{cav}^{-1/3}}{M}\int_{V_{cav}} d^3x\rho(\vec{x})(x\xi_{l,y}(\vec{x}) + y\xi_{l,x}(\vec{x}))$$

- Coupling coefficient for GW from arbitrary directions

$$h_{ij}^{TT}(\alpha,\beta) = \mathcal{R}(\alpha,\beta)h_{ij,z}^{TT}\mathcal{R}^T(\alpha,\beta)$$



Plus polarized GW coupling to vibrational mode

RF mode overlap factor

- RF mode overlap in cavity perturbation theory

$$\vec{E}_n' = \vec{E}_n + \sigma\vec{E}_n^{(1)} + \mathcal{O}(\sigma^2)$$

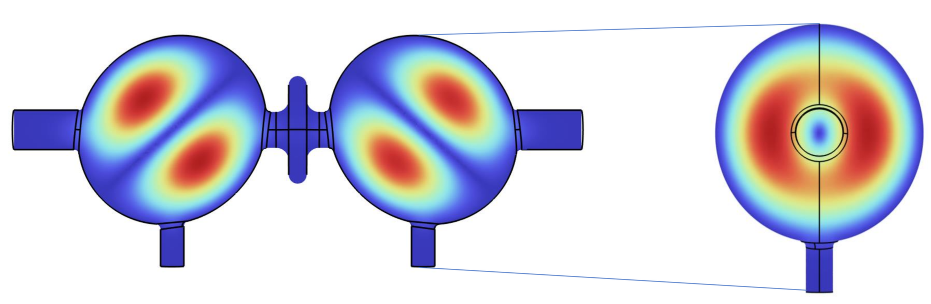
- Displacement field for isotropic elastic solids

$$\vec{f}(t,\vec{x}) = \rho(\vec{x})\frac{\partial^2\vec{u}}{\partial t^2} - (\lambda+\mu)\nabla(\nabla\cdot\vec{u}) - \mu\Delta\vec{u}$$

- Overlap factor of RF modes for one mechanical mode

$$C_{01}^l = \frac{V_{cav}^{1/3}}{2\sqrt{U_0U_1}}\int_{\partial V_{cav}} d\vec{S}\cdot\vec{\xi}_l(\vec{x})\left[\frac{1}{\mu_0}\vec{B}_0(\vec{x})\vec{B}_1(\vec{x}) - \epsilon_0\vec{E}_0(\vec{x})\vec{E}_1(\vec{x})\right]$$

Detector simulation with COMSOL



Absolute E-field distribution of the TE_{011} mode, proposed by the MAGO collaboration[2]

Lorentz Force Detuning[1]

- Deriving the equations of motion from

$$\mathcal{L}_{mech} = \frac{1}{2}\sum_n 2U_n(e_n'^2 - b_n'^2) + \sum_l\left(\frac{1}{2}M\dot{q}_l^2 - \frac{1}{2}M\omega^2q_l^2 + q\right)$$

- Additional term appears in the mechanical EoMs

$$\ddot{q}_l(t) + \frac{\omega_l}{Q_l}\dot{q}_l(t) + \omega_l^2q_l(t) = \frac{1}{M}\left(f_l(t) + f_l^{ba}(t)\right)$$

- Back-action of the EM fields on the cavity shell damps the signal strength

$$f_l^{ba}(t) = 2V_{cav}^{-1/3}\sqrt{U_0U_1}C_{01}^l b_0(t)b_1(t)$$

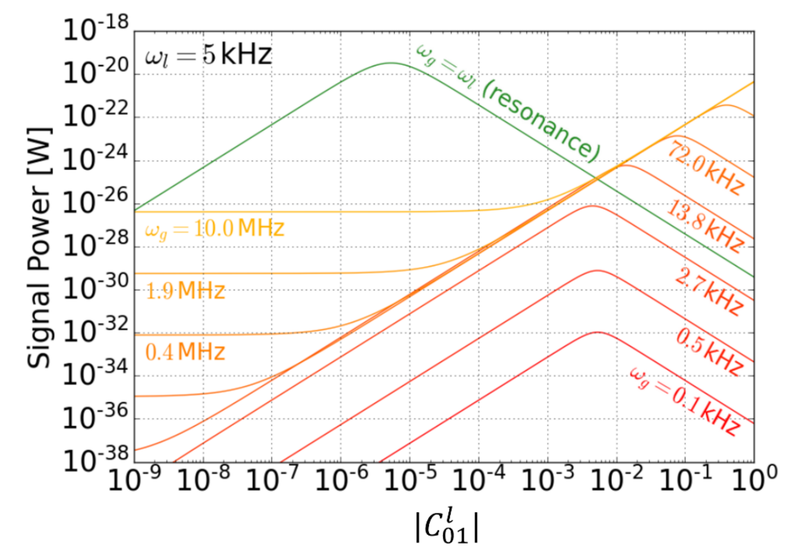
Signal power

- Signal power with damping of the EM field back-action

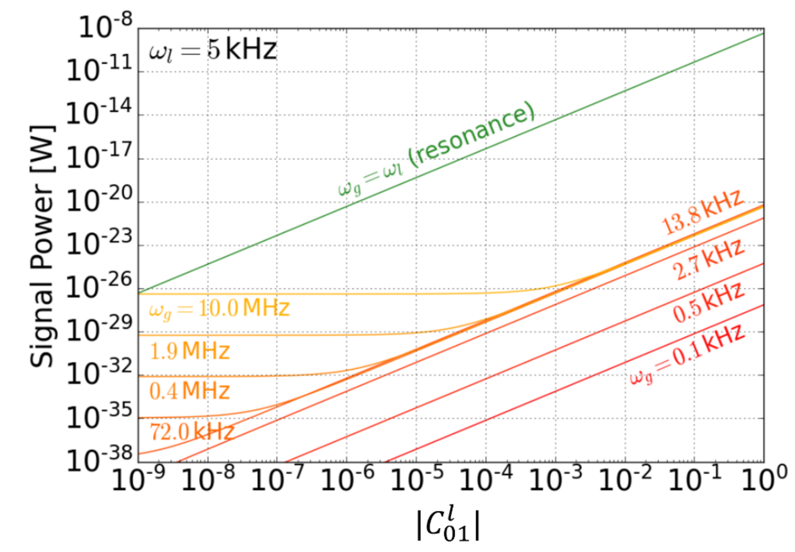
$$P_{sig} = \frac{\omega_1}{Q_{cpl}}\omega_g^4U_0\left|\frac{1}{2}\frac{\omega_l^2C_{01}^l(h_+\Gamma_+^l + h_\times\Gamma_\times^l)}{\beta_l\beta_l - \gamma_1\gamma_l} - \frac{\beta_lH(\kappa_1\eta_{01}^E + \lambda_1\eta_{01}^B)}{\beta_l\beta_l - \gamma_1\gamma_l}\right|^2$$

- With the damping term

$$\gamma_1\gamma_l = \frac{1}{M}V_{cav}^{-2}U_0\left(\omega_1C_{01}^l\right)^2$$



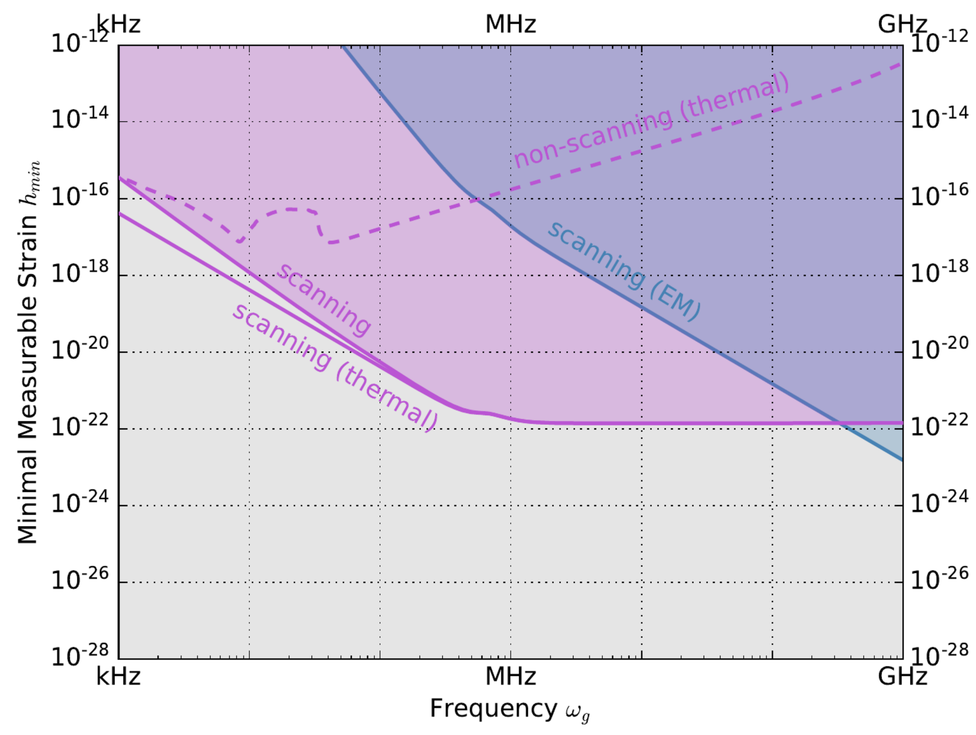
Signal power with damping



Signal power without damping

MAGO Sensitivity

- **Scanning mode**, GW is on resonance with RF mode difference $\omega_g = \omega_1 - \omega_0$
- **Broadband mode**, GW of all frequencies couple to one mechanical mode ω_l with $\omega_l = \omega_1 - \omega_0$.
- High sensitivity in high frequency regime $h_{min} \sim \mathcal{O}(10^{-21})$



Sensitivity estimate for the MAGO cavity, R. Löwenberg

[1] Löwenberg et al., arXiv/2307.14379 (2023)
[2] Bernard et al., arXiv:gr-qc/0203024v1 (2002)

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