A protocol for global multiphase estimation

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Global estimation strategies allow one to extract information on a phase or a set of phases without any prior knowledge. Here we introduce a global multiphase protocol based on Holevo's estimation theory and apply it to the case of digital estimation, where the figure of merit is the mutual information. We test the protocol in the cases of single- and double-phase estimation. Moreover, we retrieve the ultimate digital bound on precision when a generic number of phases is simultaneously estimated.

Protocol

- Fix a set of N *commuting* generators H_j defining unitary representations U_{ϕ_j} of the phase-shift group for the set of phases ϕ_j to be estimated
- Use the Holevo POVM

$$\mu(m_j) \equiv \frac{1}{N+1} |e(m_j)\rangle\langle e(m_j)|$$

with $|e(m_j)\rangle \equiv \sum_{n=0}^{N} e^{in\cdot 2\pi m_j/(N+1)} |n\rangle$ and $m_j \in [0, N]$ the discrete estimators of the phases ϕ_j

- Choose a ***cost functional***
- Optimize the cost functional over the set of input states

Digital Estimation

As a cost functional, here we choose the *conditional entropy*

$$H(m|\phi) = -\int_0^{2\pi} \frac{d\phi}{2\pi} \sum_{m=0}^{N} p(m|\phi) \log_2 p(m|\phi)$$

i.e. our figure of merit is the ***mutual information***

$$I(m:\phi) = H(\phi) - H(m|\phi)$$

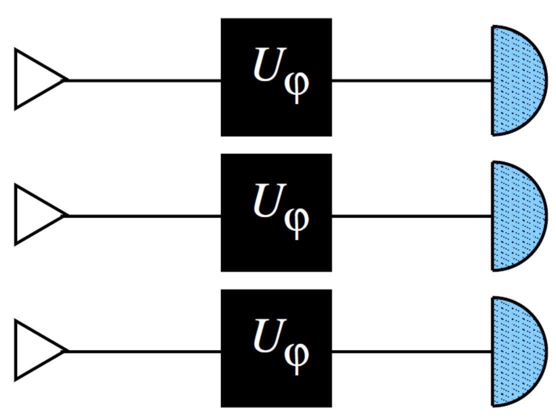
Single-phase estimation [1]

$$\left|\Psi_0^{(1)}\right\rangle = \frac{1}{\sqrt{2^N}} \sum_{n=0}^{N} \binom{N}{n} |n\rangle \quad \rightarrow \quad U_\phi^{\otimes N}$$

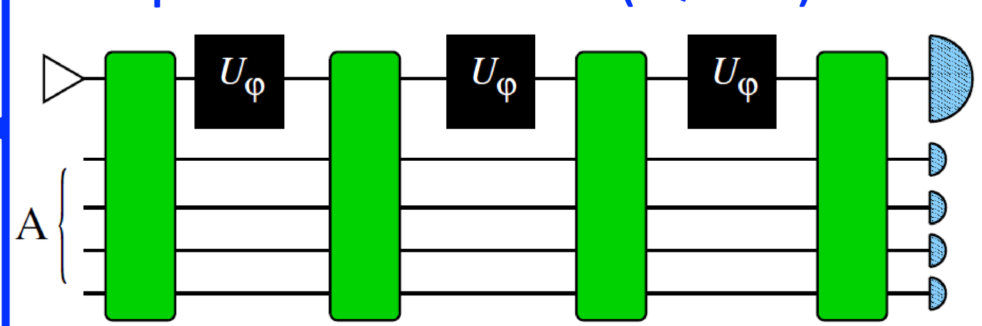
$$\left|\Psi_0^{(2)}\right\rangle = \frac{1}{\sqrt{N+1}} \sum_{n=0}^{N} |n\rangle \quad \rightarrow \quad U_\phi^{\otimes N}$$

$$\rightarrow \mu(m)$$

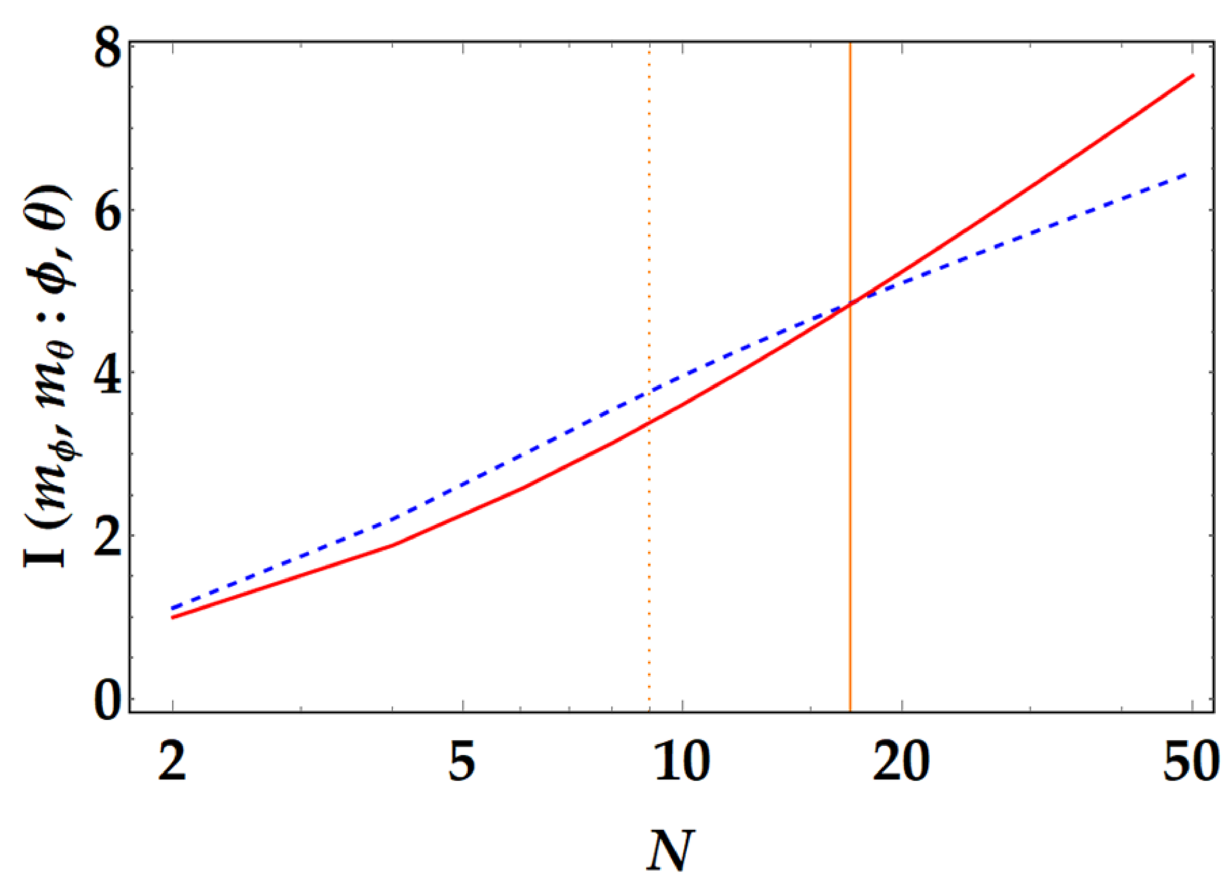
Parallel - separable scheme



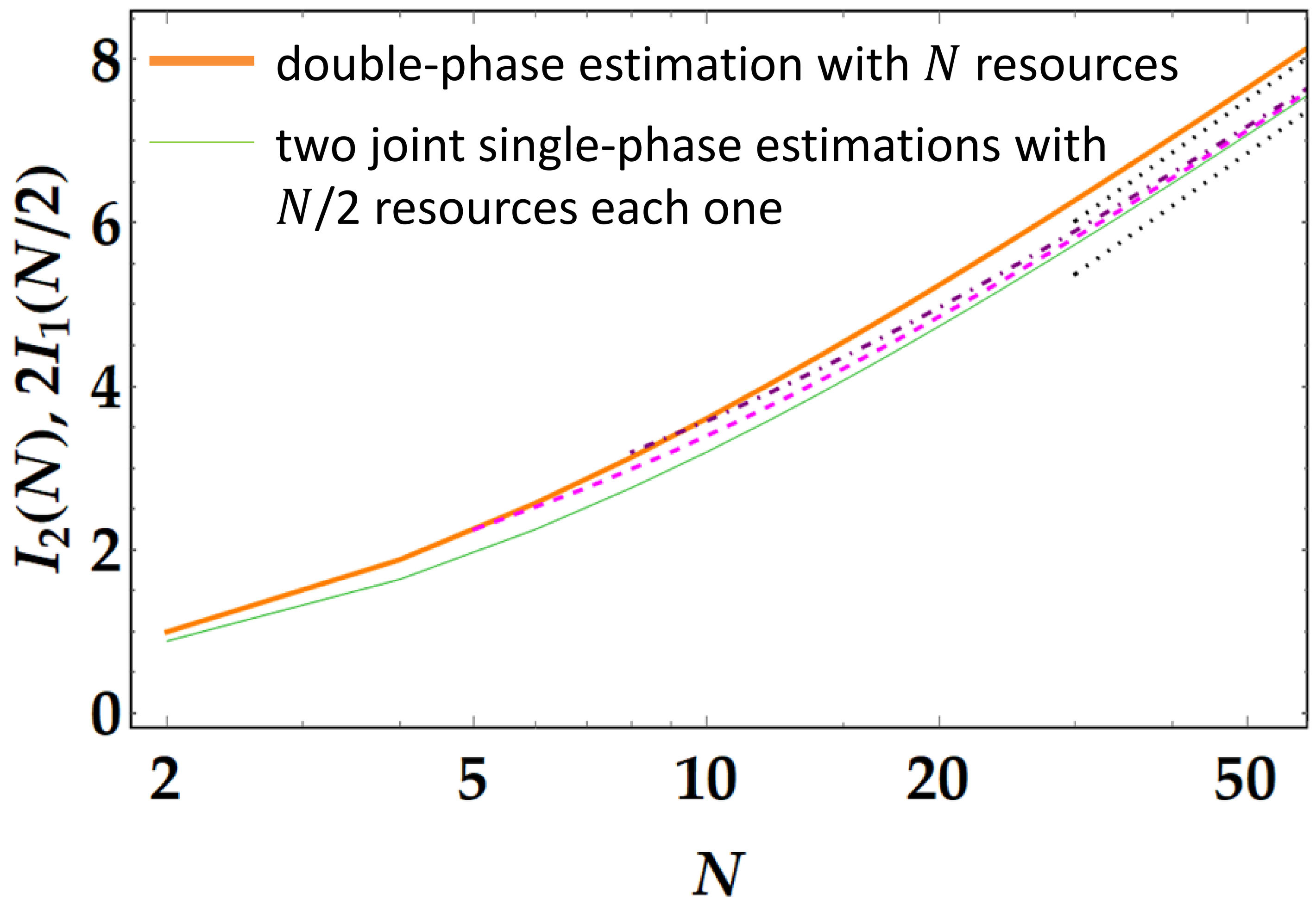
Sequential scheme (QPEA)



Double-phase estimation

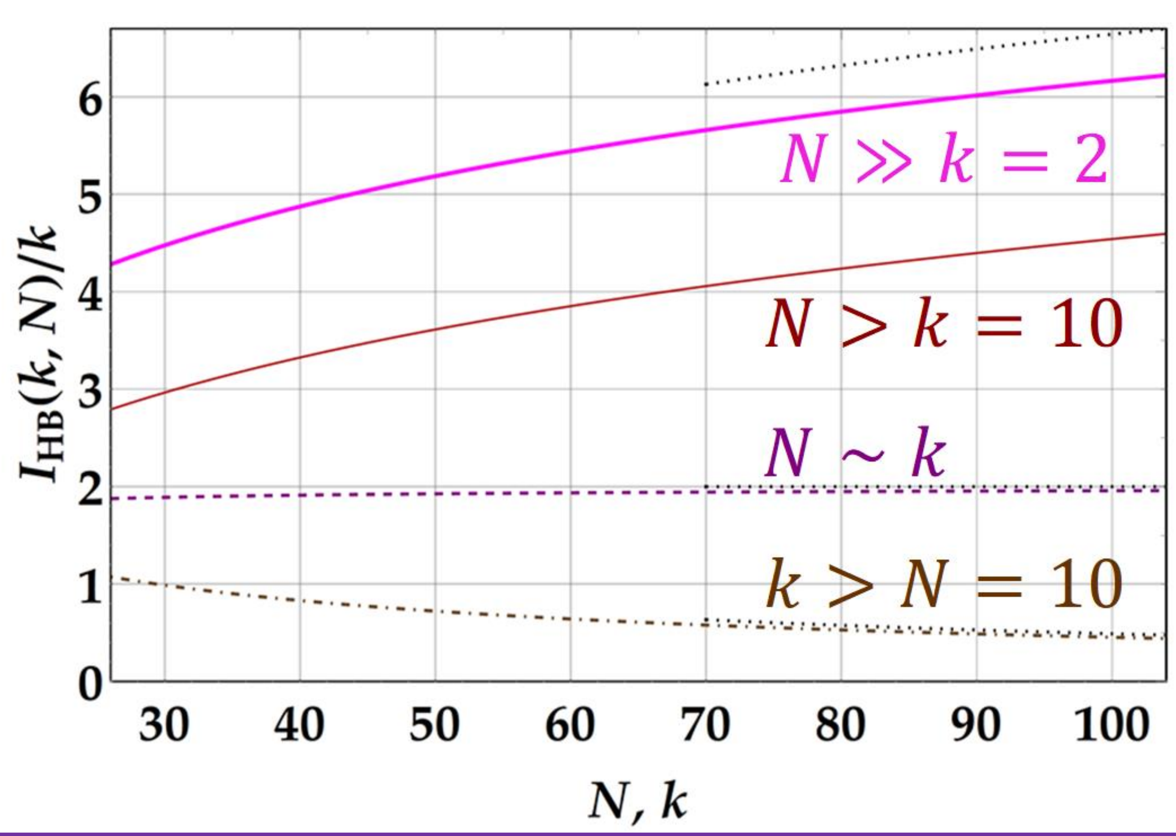


Double- *vs* single-phase estimation



Digital Heisenberg bound

$$I(m_{\phi_1}, m_{\phi_2}, \dots, m_{\phi_k} : \phi_1, \phi_2, \dots, \phi_k) \leq \log_2 \binom{N+k}{N} \equiv I_{HB}(k, N)$$



Multiphase estimation advantage:

$$\Delta I_k \equiv I_{HB}(k, N) - k I_{HB}(1, N/k)$$

$$\frac{\Delta I_k}{k} \rightarrow \log_2 e$$

amounts to a *constant factor* for each phase!

For more details, see G. Chesi, R. Rubboli, A. Riccardi, L. Maccone and C. Macchiavello, *Phys. Rev. A* **108** (2023)

[1] V. Giovannetti, S. Lloyd, and L. Maccone, *Phys. Rev. Lett.* **96**, *103* (2006)