Making the case for QC4QG

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Quantum Technologies for Fundamental Physics

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Plan

* Why to bet on quantum gravity

* Testing A with experiments on B

* Two cases:
  BTZ black hole analogy
  SYK/JT correspondence

* One quantum computer to rule that all

* Why to bet on quantum gravity

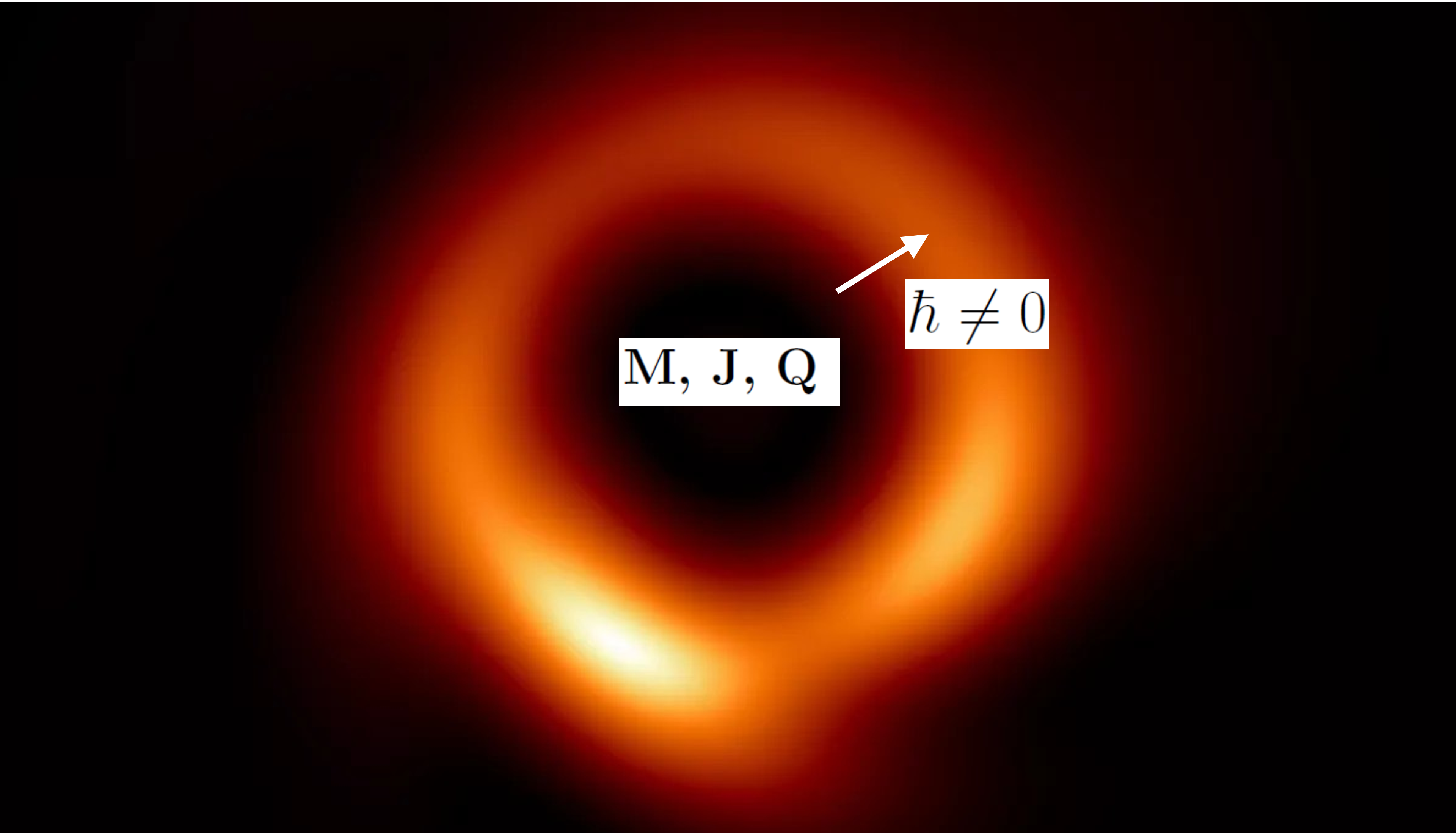
Open issues in fundamental physics:

- dark sector
- beyond SM
- SUSY (SUGRA)
- extra dimensions
- noncommutativity
- info loss
- ...

- **quantum gravity**

1. **Despite the experimental successes of QM and GR, a QG is not here.**
2. **QG will change the way we think about space, time and matter as profoundly as QM and GR did.**
3. **QG will solve issues that we do not even think to be related to it, besides some from the list.**
4. **Theory cannot make it on its own.**
5. **The times are mature for experiments.**

BHs $R_S = \frac{2MG}{c^2}$ **and QM** $\ell_P = \sqrt{\frac{\hbar G}{c^3}}$



$$ds^2 = (1 - R_S/r)c^2dt^2 - (1 - R_S/r)^{-1}dr^2 - r^2d\Omega^2$$

- **Its divergence $R_{\lambda\mu\nu\rho}R^{\lambda\mu\nu\rho} \sim 1/r^6$ saved by QM $R \to 1/\ell_P^2$?**
- **Its thermodynamics is of quantum origin, $T_{Haw} = \hbar f(M, J, Q)$?**
- **If $\lambda^\gamma \sim \ell_P$ then $R_S^\gamma \sim \ell_P$?**
- **The only system that saturates $S \leq 2\pi RE/(\hbar c) \leq S_{BH}$?**
- **What quantum microstates make the classical macrostate, $S_{BH} = k_B \ln W_n$?**

This is a tale of three entropies.

The BH entropy:

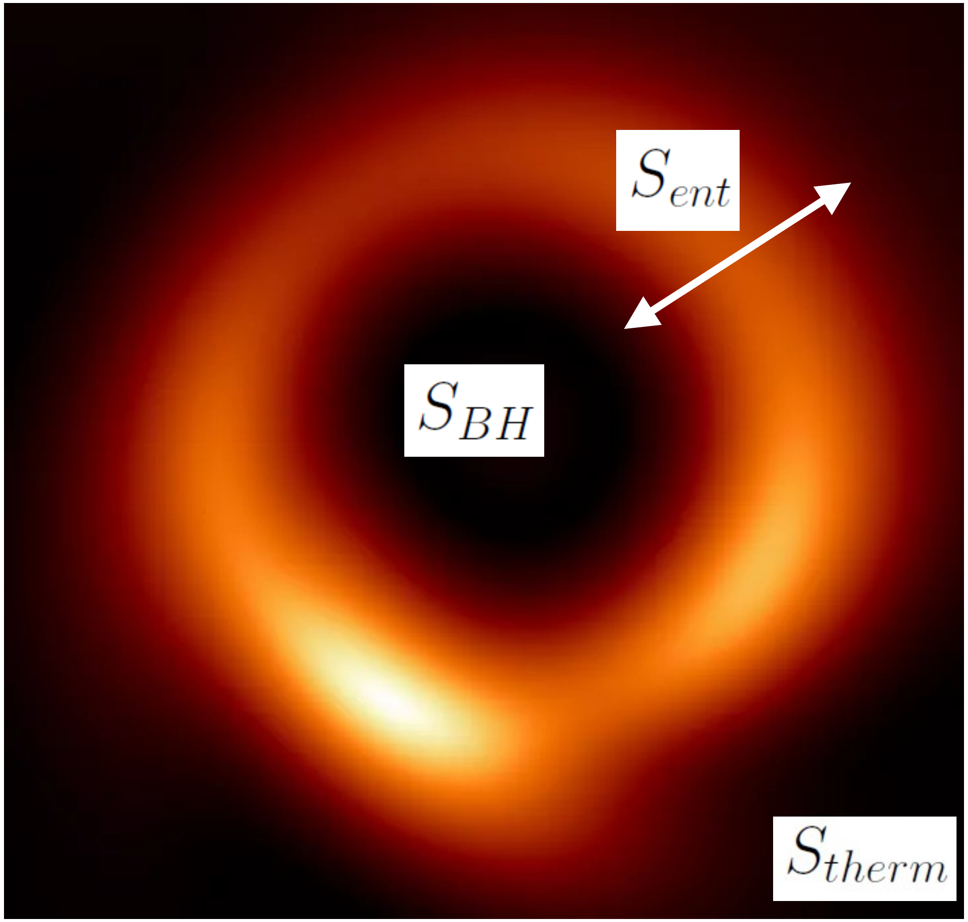
$$S_{BH} = \frac{1}{4} \frac{A}{\ell_P^2} \left(+\kappa \ln(A/\ell_P^2) + \cdots \right)$$

The entanglement entropy:

$$S_{ent} = -\mathrm{Tr}_I \rho_{II} \ln \rho_{II}$$

The environment or thermodynamics entropy:

$$S_{therm}$$


S_{ent}
S_{BH}
S_{therm}

Stellar-mass BH (X-ray emission), e.g., V404 Cygni

$$M \approx 9\, M_{\odot} \quad S_{BH} \approx 8.7 \times 10^{78} k_B$$

that is 10^{20} more than the entropy of V404 Cygni

BH at the galactic center M87* (Event Horizont Telescope)

$$M \approx 6.5 \times 10^{9}\, M_{\odot} \quad S_{BH} \approx 4.5 \times 10^{96} k_B$$

that is 10^{7} more than the whole non-gravitational entropy of the observable Universe...

Many secrets of QG are hiding in S_{BH}.

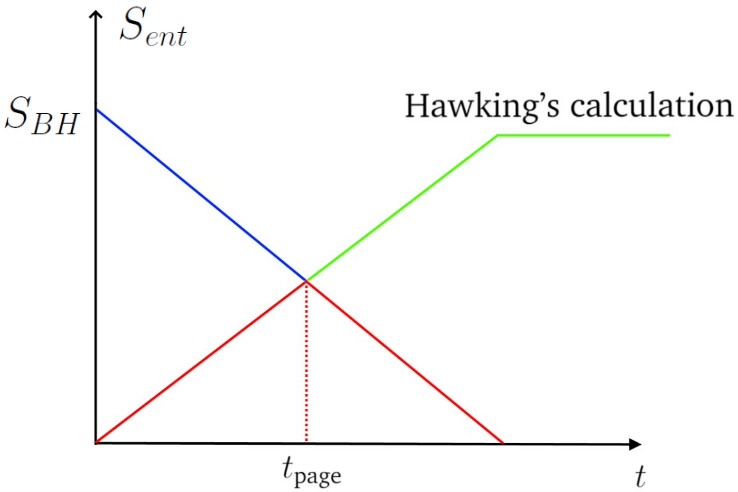
It counts the number of internal states of matter and gravity (Xons)? or the number of horizon gravitational states? or the number of states of a fundamental string?

Is it $S_{BH} = S_{ent}$? or $S_{BH} = S_{Wald}$, the Noether conserved charge of diffeo invariance of $\mathcal{L}$? or $S_{BH} = S_{therm}$ of the gas of quanta of the thermal BH atmosphere ('t Hooft brick wall)? or $S_{BH} = S_{therm}$ of the radiation on the boundary of the BH (Ads/CFT)?

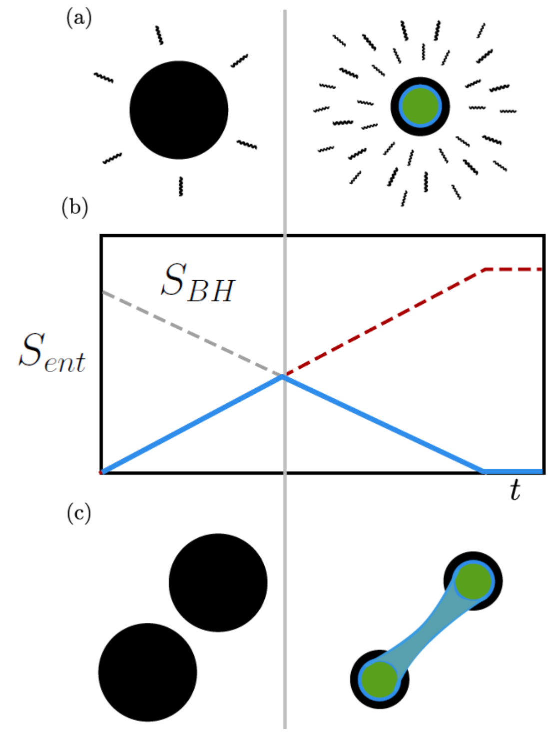
Or is S_{BH} something else?

$$(*)\; S_{Wald} = -2\pi \int_{\Sigma} d^2x\sqrt{h}\epsilon_{\lambda\mu}\epsilon_{\nu\rho}\delta\mathcal{L}/\delta R_{\lambda\mu\nu\rho}|_{\mathcal{L}=R} = \frac{1}{4G}A_{\Sigma}$$

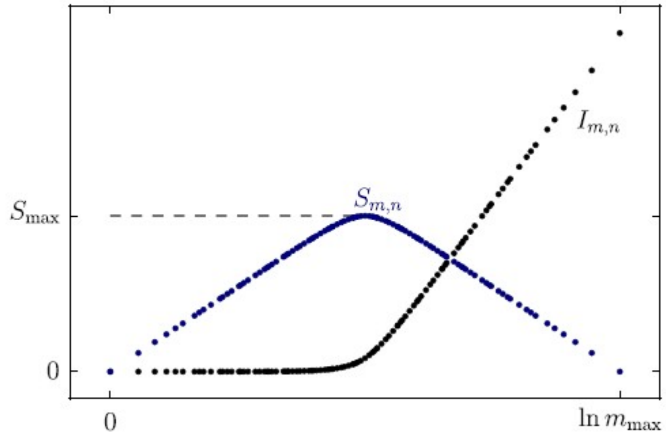
And, when it comes to evolution, who is right?



J Røsok Eskilt, Recovering the Page Curve
from the AdS/CFT Correspondence,
Master of Science, Imperial College London, 2020



R Bousso *et al*, Snowmass White Paper:
Quantum Aspects of Black Holes
and the Emergence of Spacetime
arXiv:2201.03096v2 [hep-th]



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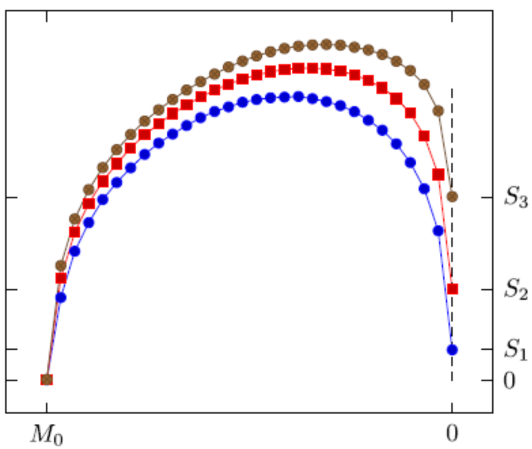
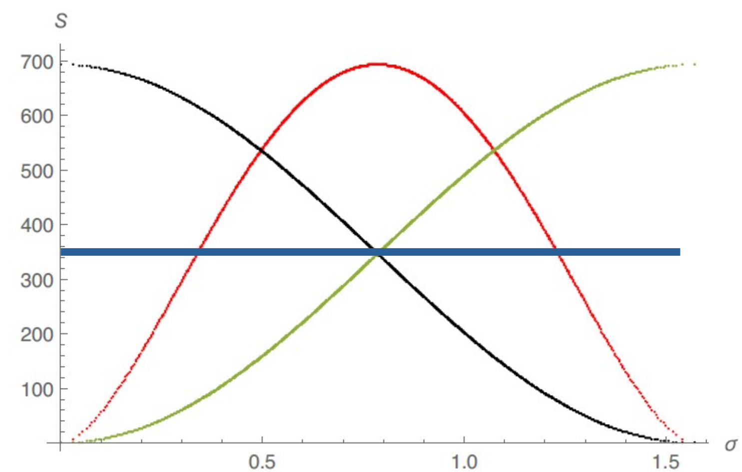


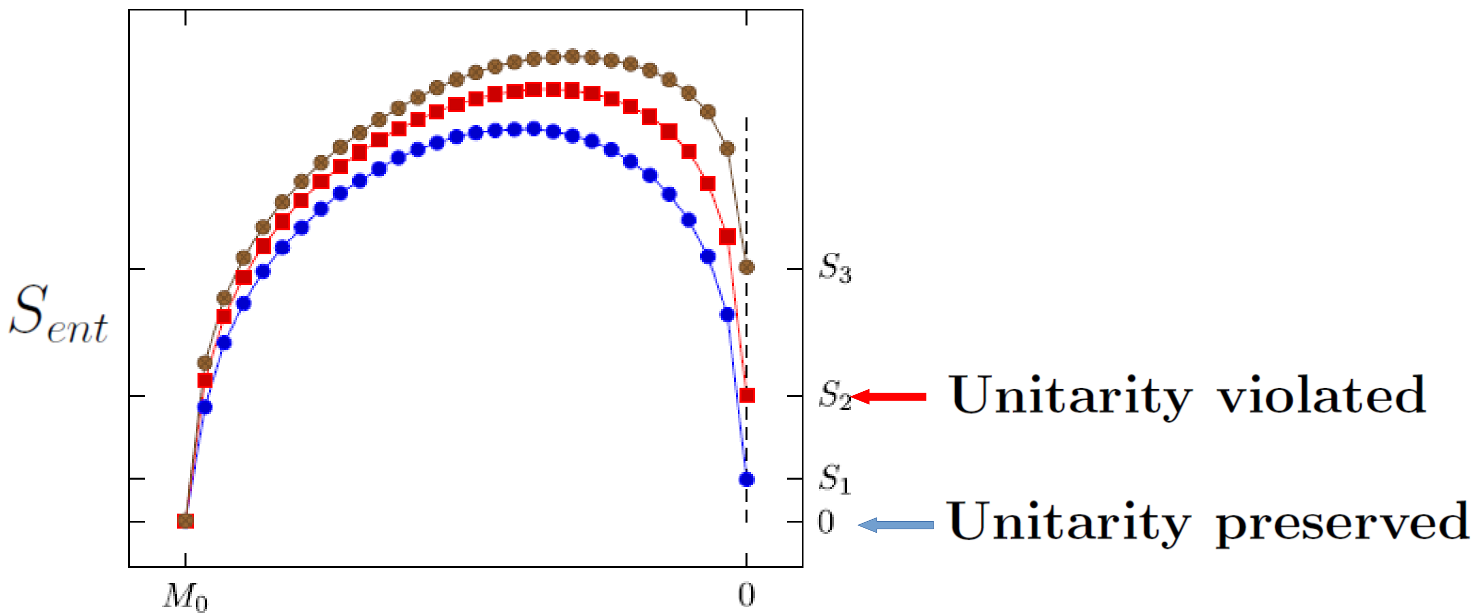
Modified Page?



Grumiller?

Else ?

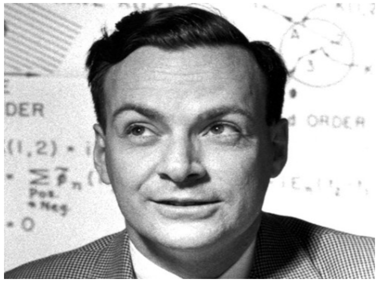
Measure $S_{ent}(\sigma)$



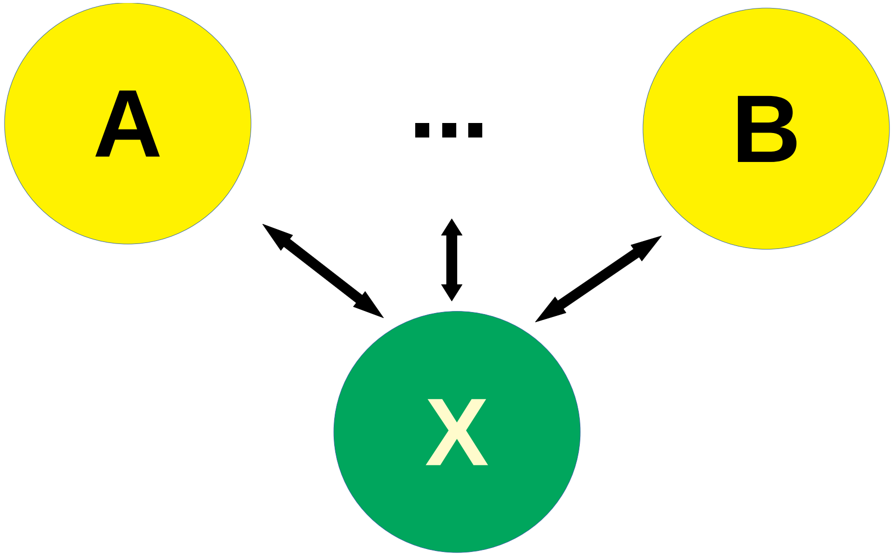
G Acquaviva, A I, M Scholtz, On the implications of the Bekenstein bound for black hole evaporation arXiv:1704.00345 [gr-qc]

* Testing A with experiments on B

QG is difficult to reach. We can test it on other systems. Feynman ignited the investigation of A by probing B.

1. 

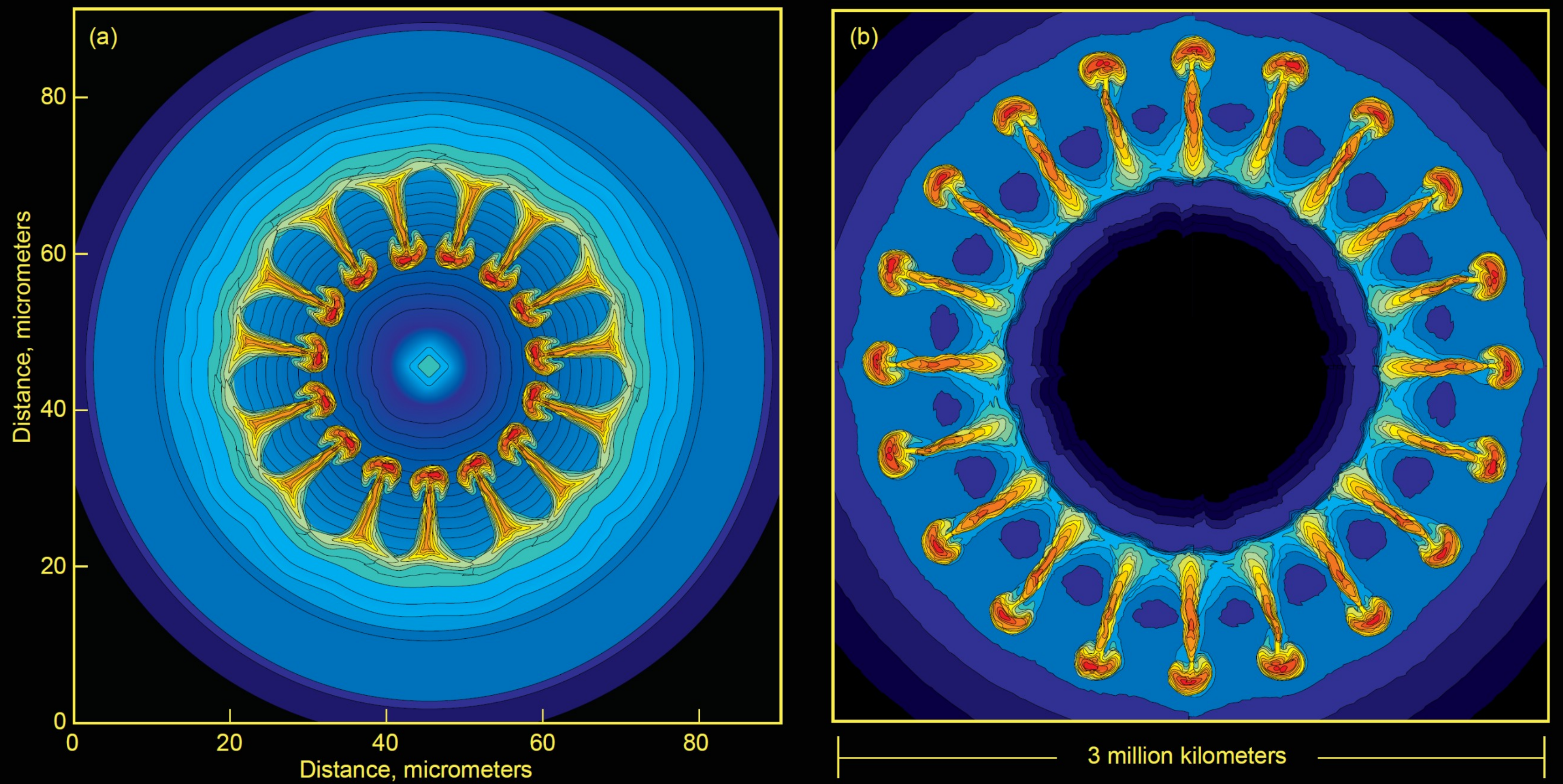
Why different physical systems respond to the same physical laws? The role of spacetime.



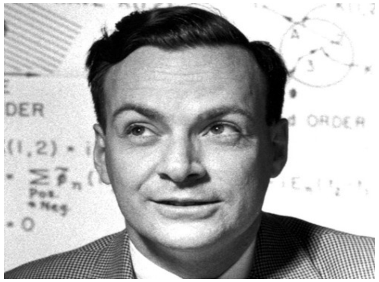
R Feynman, R Leighton, M Sands, Electrostatic analogs, Feynman Lectures on Physics (1964, 1966), Vol II, Chp 12

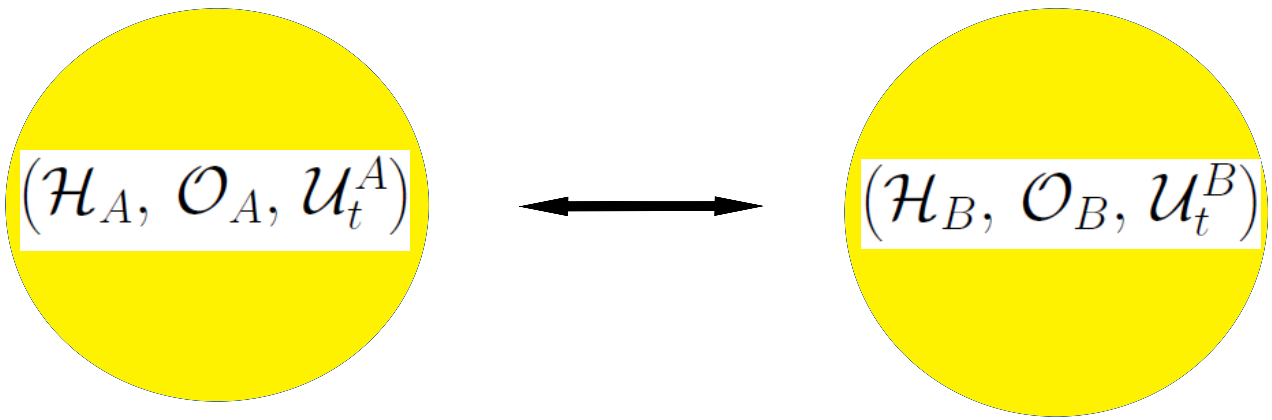
Analogy works when all systems $A, ..., B$ have the same underlying structure.

E.g., the fluid dynamics of a fusion capsule and of a supernova.



Striking similarities exist between hydrodynamic instabilities in (a) inertial confinement fusion capsule implosions and (b) core-collapse supernova explosions. [Image (a) is from Sakagami and Nishihara, *Physics of Fluids B* **2**, 2715 (1990); image (b) is from Hachisu et al., *Astrophysical Journal* **368**, L27 (1991).]

2.  **Can we simulate exactly A with B? The role of quantum.**



Quantum system A **Quantum system B**

R Feynman, Simulating physics with computers, Int J Theor Phys 21 (1982) 467–488

Two quantum systems are equivalent

$$(\mathcal{H}_A, \mathcal{O}_A, \mathcal{U}_t^A) \simeq (\mathcal{H}_B, \mathcal{O}_B, \mathcal{U}_t^B)$$

when their von Neumann algebras $\mathcal{M}_A$ and $\mathcal{M}_B$ are isomorphic.

When the isomorphism is given by $\mathcal{U}$ unitary

$$\mathcal{M}_B \rightarrow_{\mathcal{U}} \mathcal{M}_A$$

e.g., a symmetry, we have the strongest case.

Example: $|0\rangle_{Minkowski} = \mathcal{U}^{\dagger} |0\rangle_{HO} \mathcal{U}$

3. 

Gravity can be quantized the old way only when

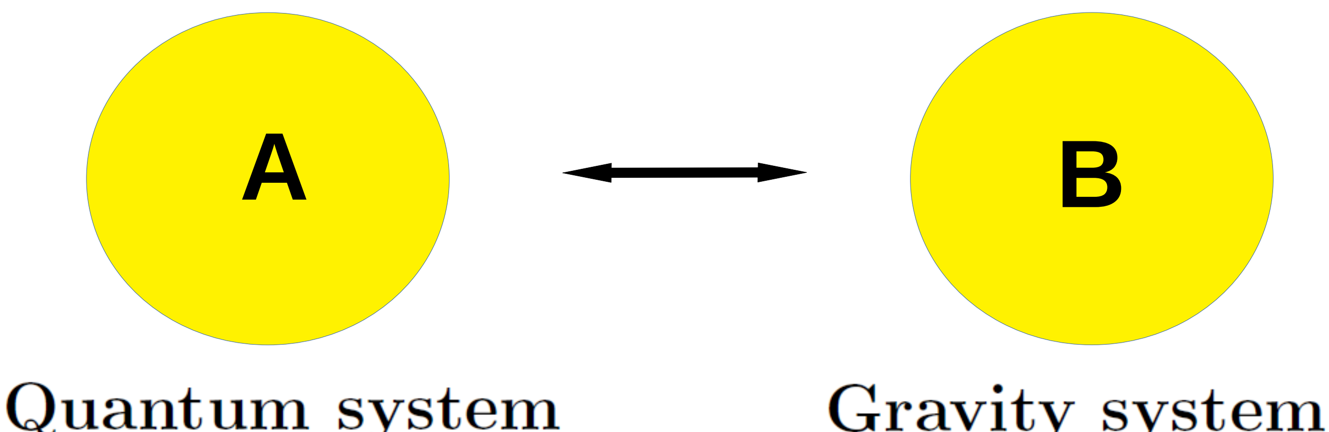
$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

The current understanding is that gravity is *emergent*:

from an underlying spin foam

from a QFT one dimension less

This gave a powerful toolbox of correspondences, dualities, symmetries, that map



Today all these fields are mature:

1. Analogs are at a climax of precision.

2. Quantum computing and quantum simulations are very advanced.

3. The correspondences between quantum matter and gravity became predictive.

Proposal: push 1, 2 and 3 to establish a precise correspondence, beyond analogy and simulation, to:

$$\textbf{measure } S_{ent}(\sigma) \textbf{ on a QC}$$

* Two cases: BTZ analogy and SYK/JT correspondence

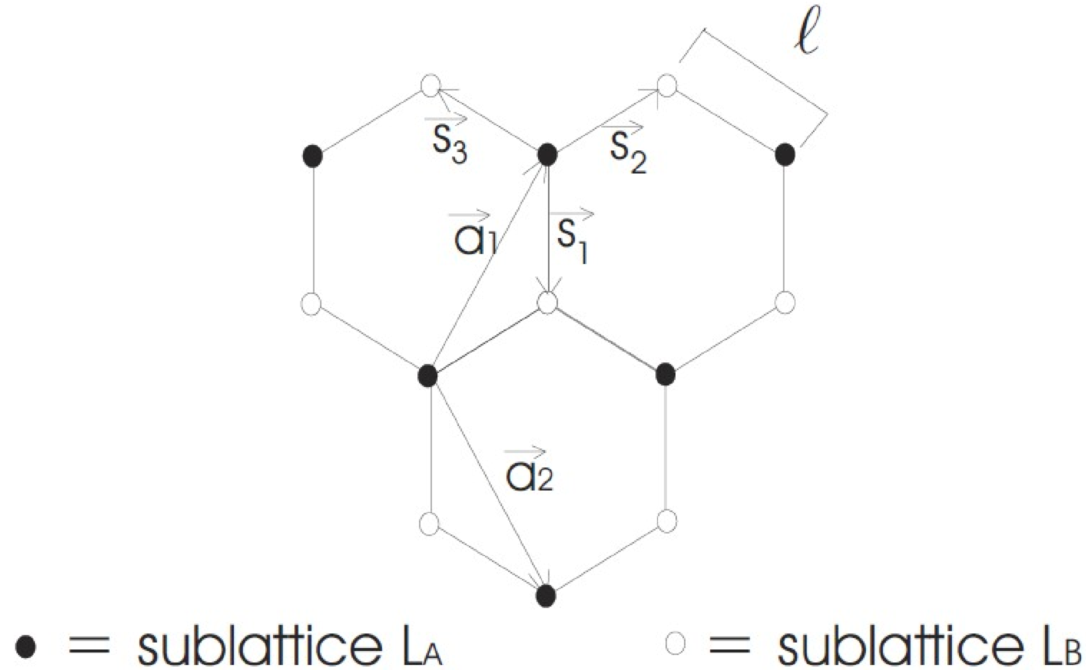
In the first case, we are in 3 dimensions, use an *analogy*, based on Weyl symmetry, between the quantum quasi-particles of specifically shaped graphene and a Fermi field on fixed background spacetime

$$\psi \quad \text{on} \quad g_{\mu\nu}^{BTZ}$$

In the second case, we are in 2 dimensions, use a *correspondence*, based on $SL(2,R)$ symmetry, between a 1 dimensional quantum many body Fermi system, without quasi-particles, and a dynamical theory of spacetime

$$\psi^{SYK} \to g_{\mu\nu}^{JT}$$

BTZ analogy



The electronic properties of graphene (Dirac materials) are customarily described by ($\hbar = 1$)

$$H = -\eta_1 \sum_{\vec{r} \in L_A} \sum_{i=1}^{3} \left(a^\dagger(\vec{r}) b(\vec{r} + \vec{s}_i) + b^\dagger(\vec{r} + \vec{s}_i) a(\vec{r}) \right)$$

with $\eta_1 \simeq 2.8$ eV the hopping parameter (usually called t)

BTZ

Fourier transform, $a(\vec{r}) = \sum_{\vec{k}} a(\vec{k}) e^{i\vec{k}\cdot\vec{r}}$, etc.

$$H = -\eta_1 \sum_{\vec{k}} \left(\mathcal{F}_1(\vec{k}) a^\dagger(\vec{k}) b(\vec{k}) + \text{h.c.} \right)$$

with

$$\mathcal{F}_1(\vec{k}) \equiv \sum_{i=1}^{3} e^{i\vec{k}\cdot\vec{s}_i} = e^{-i\ell k_y} \left(1 + 2\, e^{i3\ell k_y/2} \cos(\sqrt{3}\ell k_x/2) \right)$$

At $\vec{k}^D_\pm = \left(\pm \frac{4\pi}{3\sqrt{3}\ell}, 0 \right)$, $E(\vec{k}) = \pm|\mathcal{F}_1(\vec{k})| = 0$

There ($v_F \equiv 3\eta_1\ell/2 \simeq c/300$) $E_\pm \simeq \pm v_F |\vec{p}|$ and

$$H \simeq v_F \sum_{\vec{p}} \left(\psi_+^\dagger \vec{\sigma}\cdot\vec{p}\, \psi_+ - \psi_-^\dagger \vec{\sigma}^* \cdot \vec{p}\, \psi_- \right)$$

where $\psi_\pm = (a_\pm, b_\pm)^T$

BTZ

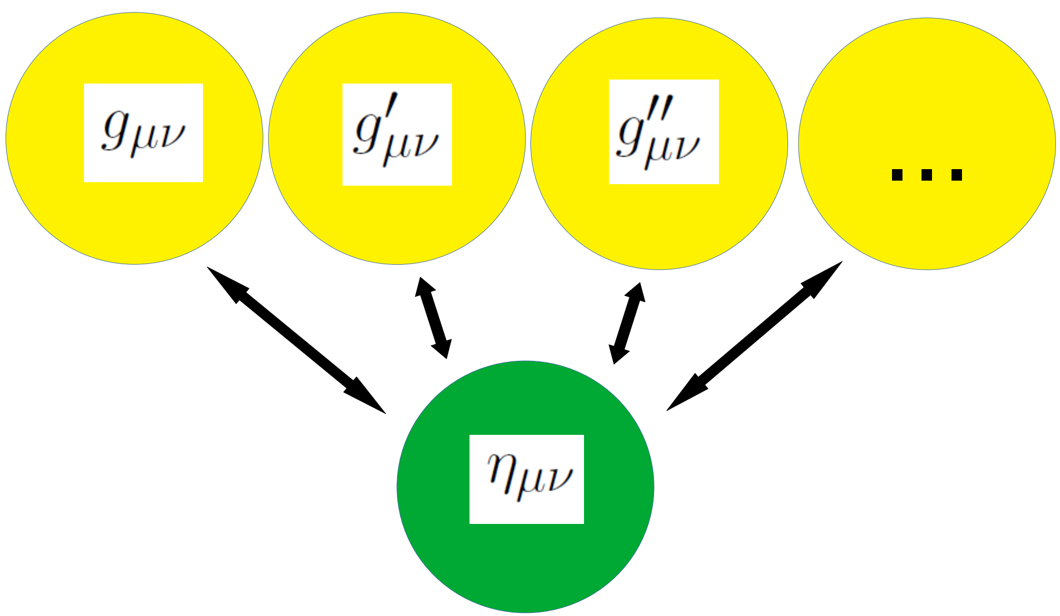
Include time as $x^0 = v_F t$

$$\mathcal{A}(\psi, \eta_{\mu\nu}) = i v_F \int d^3x \overline{\psi} \gamma^a \partial_a \psi$$

$\psi = (\psi_+, \psi_-)^T$, and then

$$\mathcal{A}(\psi, g_{\mu\nu}) = i v_F \int d^3x \, |e| \, \overline{\psi} \gamma^\mu \nabla_\mu \psi$$

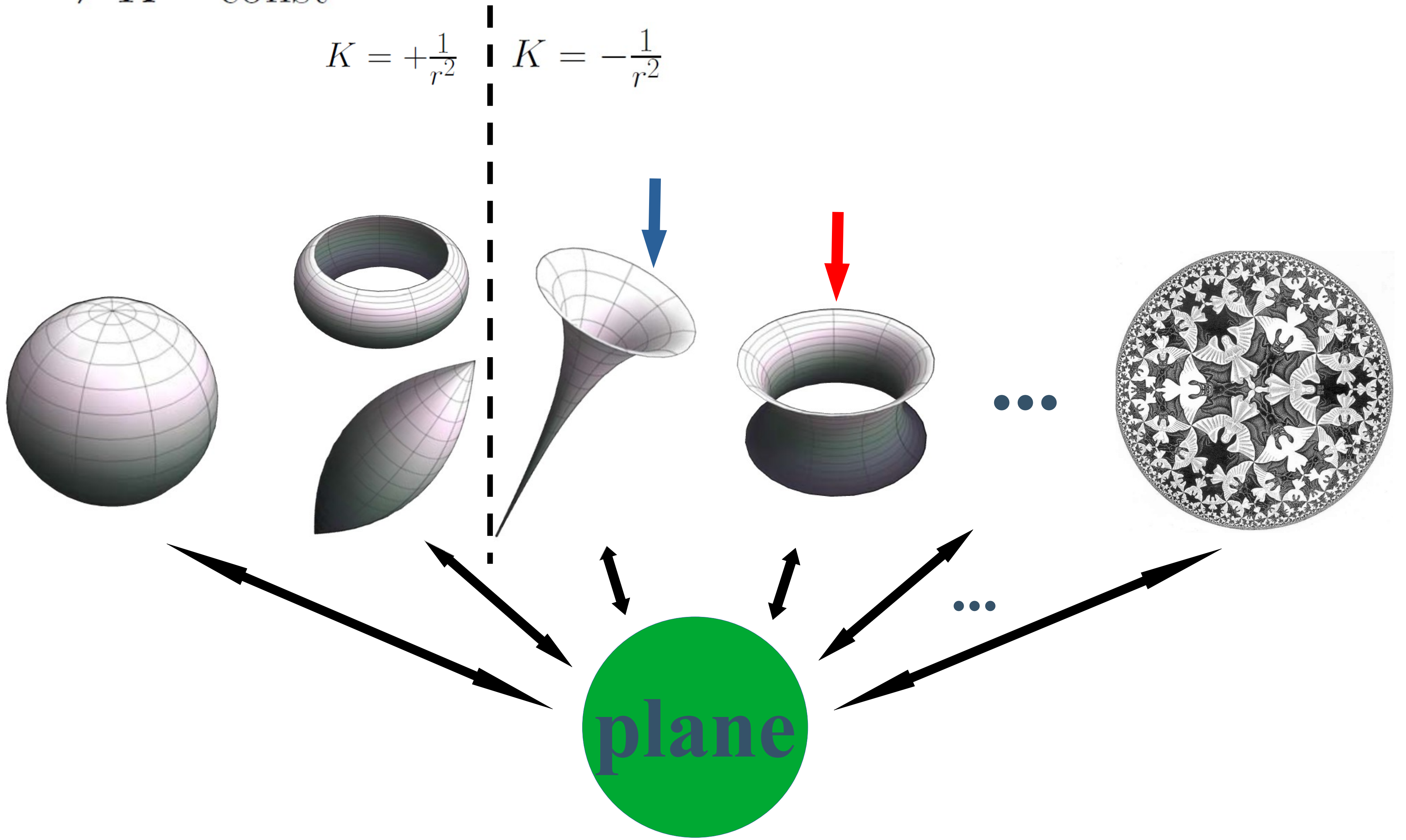
When $g_{\mu\nu} = e^{2\sigma} \eta_{\mu\nu}$ and $\psi' = e^{d_\psi \sigma} \psi$, Weyl symmetry gives meaning to a strong analogy $\mathcal{A}(\psi', g_{\mu\nu}) \to \mathcal{A}(\psi, \eta_{\mu\nu})$



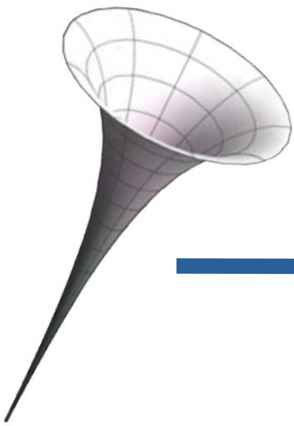
BTZ

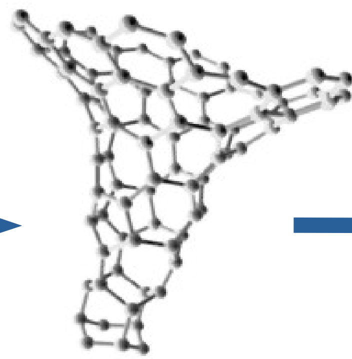
$$n = 3 \text{ and } g^{(3)}_{\mu\nu}(x,y) = \begin{pmatrix} 1 & 0 \\ 0 & g^{(2)}_{ij}(x,y) \end{pmatrix} \text{ and CF}$$

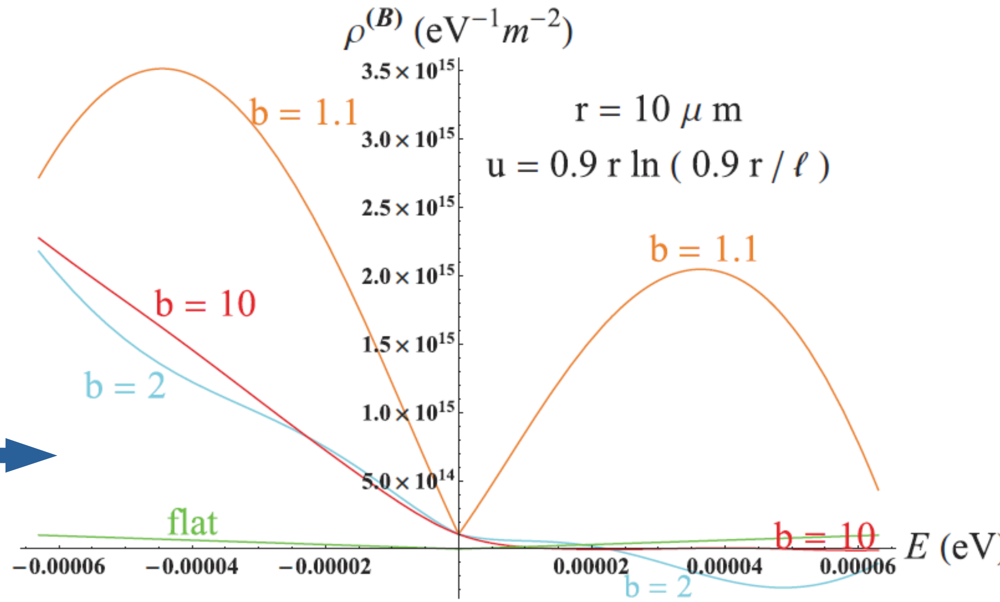
$$\Rightarrow K = \text{const}$$



BTZ





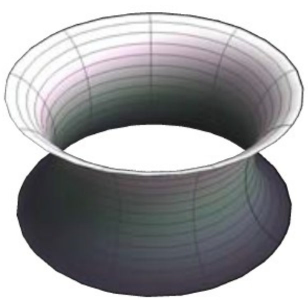


$$\mathcal{T}(r \ln(r/\ell)) = \frac{\hbar v_F}{k_B} \frac{1}{2\pi r}$$

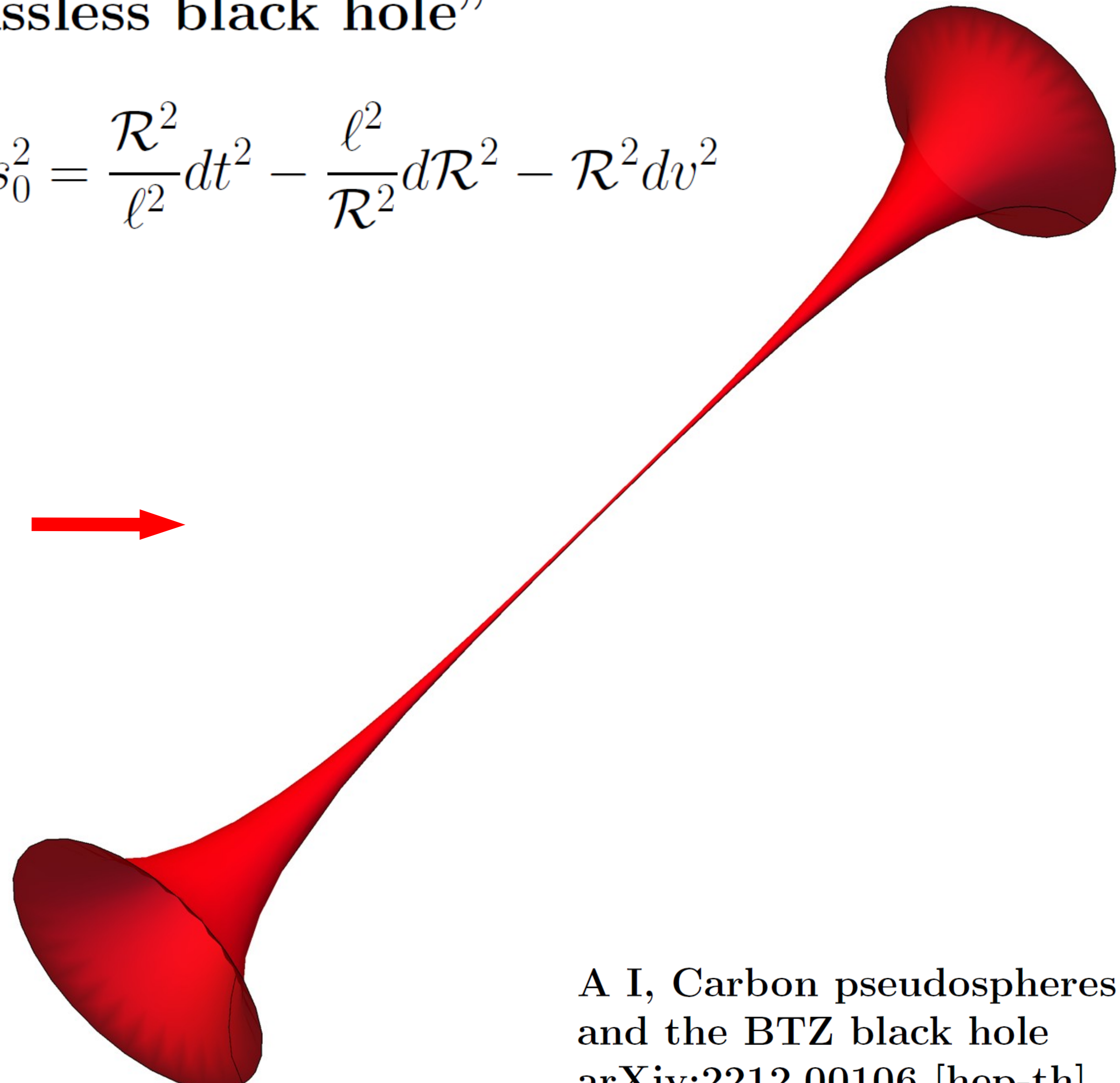
A I, G Lambiase, Quantum field theory in curved graphene spacetimes, Lobachevsky geometry, Weyl symmetry, Hawking effect, and all that, arXiv:1308.0265 [hep-th]

The extreme BTZ case $M \to 0$ $(J \to 0)$ is a peculiar "massless black hole"

$$ds_0^2 = \frac{\mathcal{R}^2}{\ell^2}dt^2 - \frac{\ell^2}{\mathcal{R}^2}d\mathcal{R}^2 - \mathcal{R}^2 dv^2$$



$\times \mathbf{R}$ →



A I, Carbon pseudospheres and the BTZ black hole arXiv:2212.00106 [hep-th]

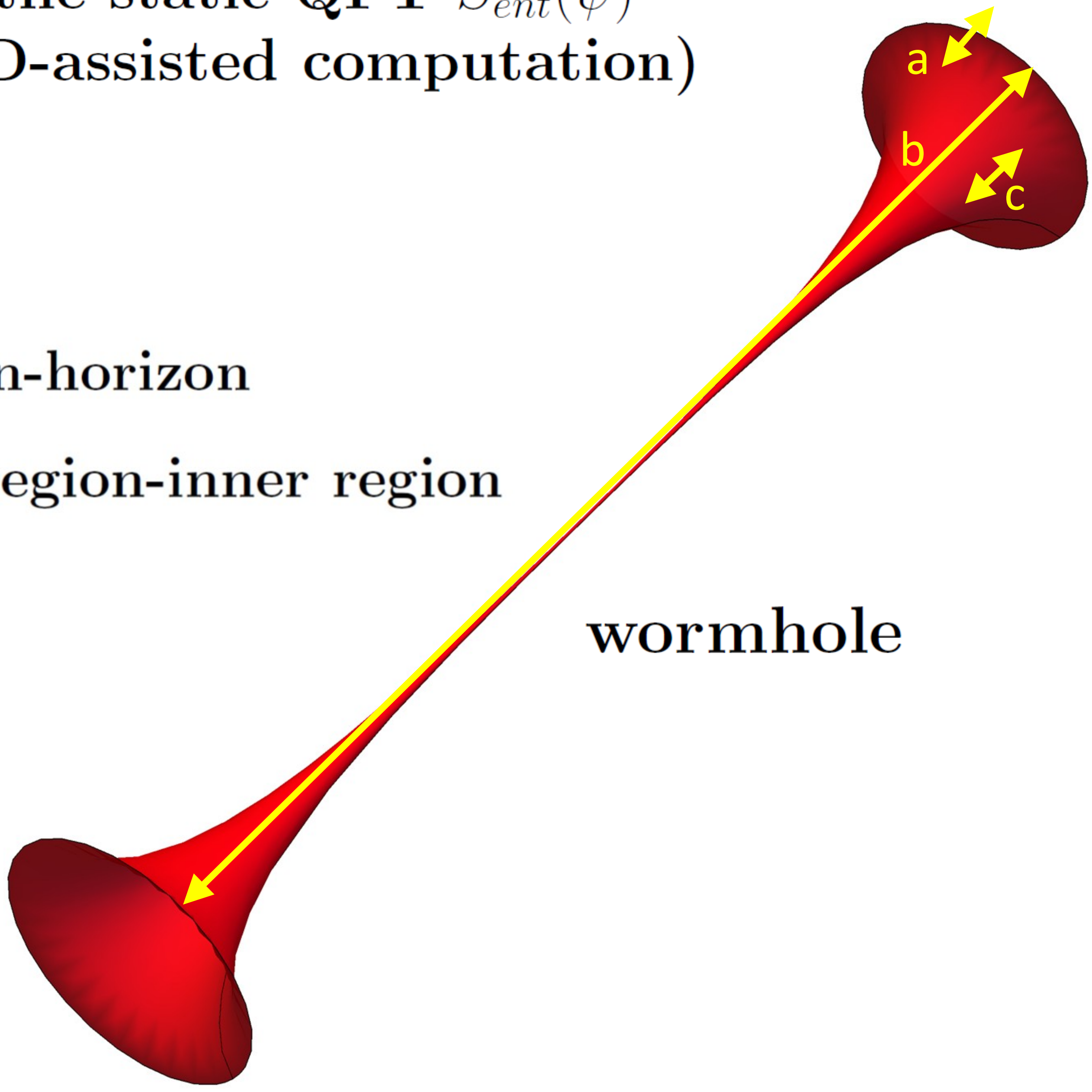
BTZ

Measure the static QFT $S_{ent}(\psi)$ (e.g., TFD-assisted computation)

(a) in-out

(b) horizon-horizon

(c) inner region-inner region



wormhole

SYK/JT correspondence

The Sachdev-Ye-Kitaev model of N spin-less electrons, with quartic interaction

$$H_4 = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,l=1}^{N} U_{ij;kl}\, c_i^\dagger c_j^\dagger c_k c_l - \mu \sum_{i=1}^{N} c_i^\dagger c_i$$

with

$$\{c_i, c_j^\dagger\} = \delta_{ij}\,, \quad \{c_i, c_j\} = 0$$

and $U_{ij;kl}$ Gaussian distributed random variables

$$U_{ij;kl} = -U_{ji;kl} = -U_{ij;lk} = U^*_{kl;ij}$$

$$\overline{U}_{ij;kl} = 0\,, \quad \overline{|U_{ij;kl}|^2} = U^2$$

There is a popular Majorana, $c_i^\dagger = c_i$, version with no $U(1)$ symmetry.

Jackiw-Teitelboim gravity lives in two dimensions

$$\mathcal{A}_{JT}(\Phi, g_{\mu\nu}) = -\frac{1}{16\pi G_N}\int_{\mathcal{M}} d^2x\sqrt{g}\,\Phi\,(R-\Lambda)$$

with $\Lambda = \pm 1/\ell^2$**. Note that**

$$\int_{\mathcal{M}} d^2x\sqrt{g}R = 2\pi\chi(\mathcal{M})$$

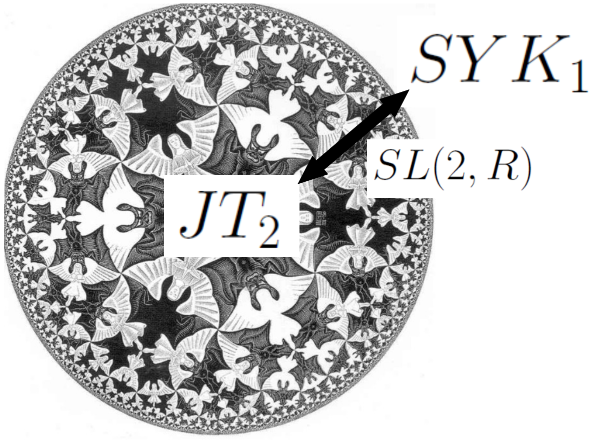
while $\mathcal{A}_{JT}$ **admits BH solutions (take** $\Lambda = -1/\ell^2$**)**

$$ds^2 = V(r)dt^2 - \frac{1}{V(r)}dr^2$$

$$V(r) = \frac{r^2}{\ell^2} + 1 - \frac{M}{r} + \frac{\Theta}{r^2}$$

with $V(r_h) = 0$ **and** $T = V'(r_h)/(4\pi)$**.**
($\mathcal{A}_{JT}$ **can be seen as a dim reduction of** $\mathcal{A}_{EM}$ **from** $d+2$**)**

This is an example of AdS_2/CFT_1 correspondence



In the large N limit ($G_N \sim 1/N$), for $T \to 0$

Same (Schwarzian) action
Same correlators (enjoying $SL(2,R)$ symmetry)
Same thermodynamics, e.g.

$$\frac{\partial \mathcal{S}}{\partial \mathcal{Q}} = 2\pi\mathcal{E}$$

(important role of quantum chaos, $\lambda_{Lyap} \le 2\pi T/\hbar$)

S Sachdev *et al*, Sachdev-Ye-Kitaev models and beyond: Window into non-Fermi liquids arXiv:2109.05037 [cond-mat.str-el]

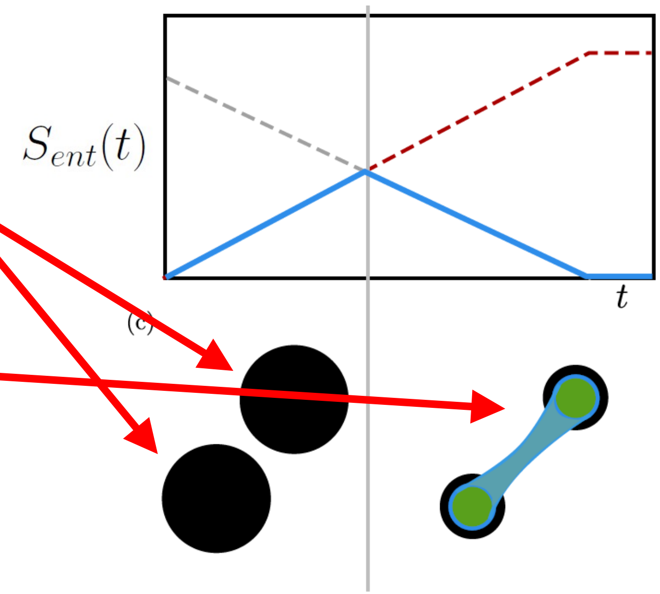
SYK/JT

An evaporating BH corresponds to

$$\sum_{i,j,k,l=1}^{N} U_{ij;kl}\,\psi_i\psi_j\psi_k\psi_l + i\sum_{a,b=1}^{M} V_{ab}\chi_a\chi_b + i\sum_{a,i} W_{ai}\chi_a\psi_i$$

thus $S_{ent}(t)$ can be computed

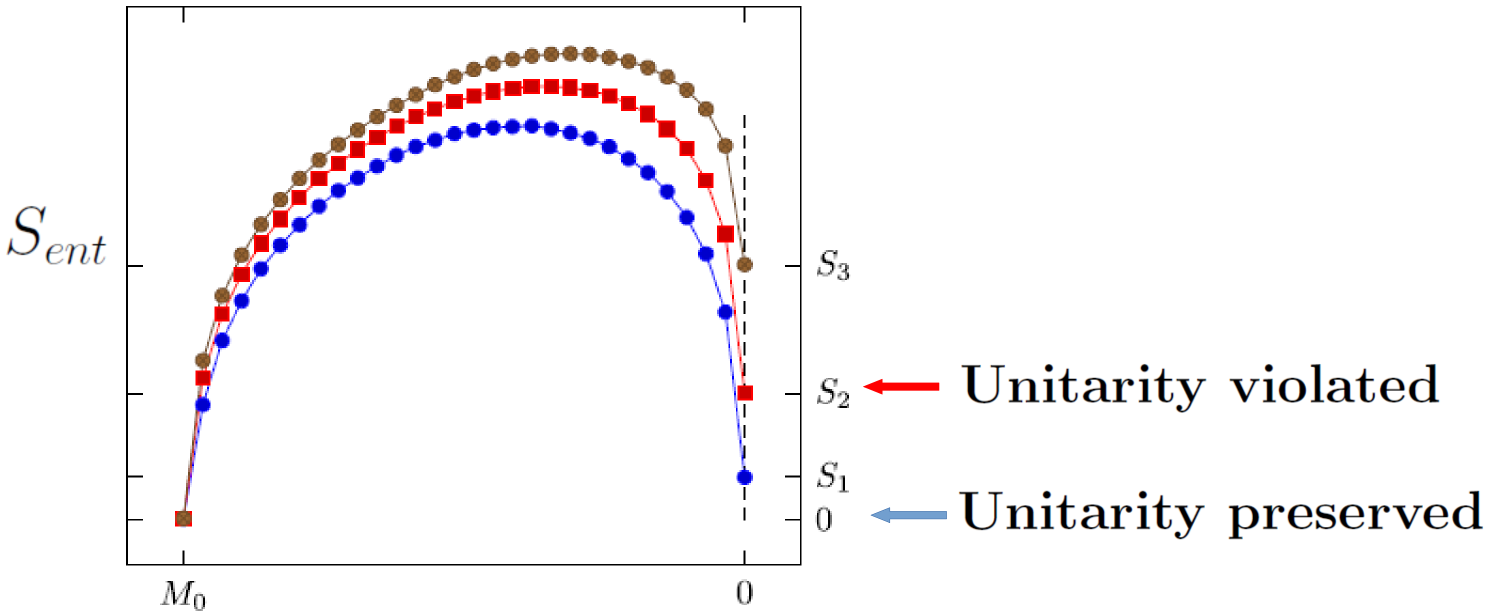
(importance of two copies (TFD) of the BH and of the associated wormhole)



D Nedel, Time dependent entanglement entropy in SYK models and Page curve arXiv:2007.06770 [hep-th]

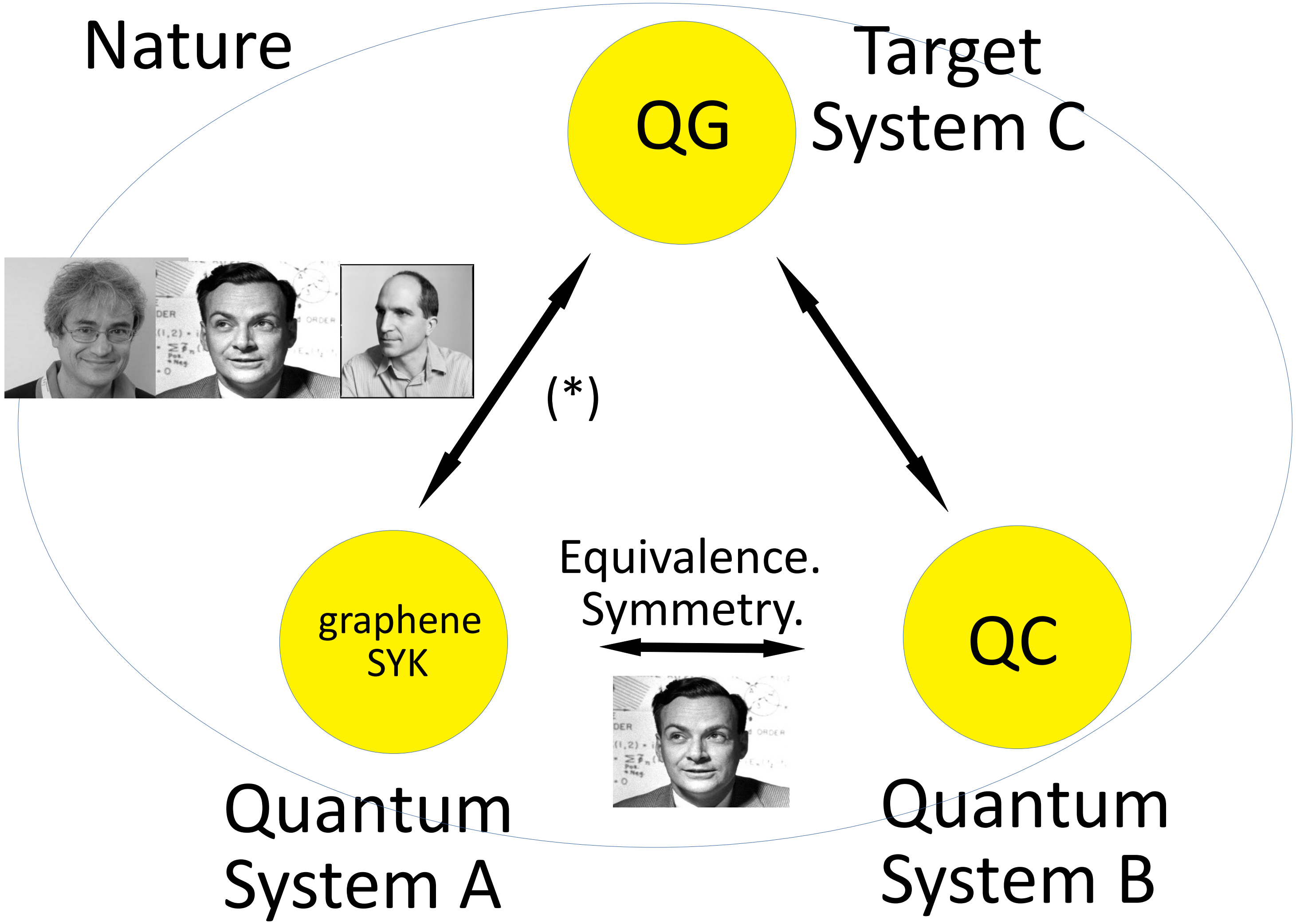
* One quantum computer to rule that all

Sharp goal (the new 'ATLAS'): **measure $S_{ent}(\sigma)$ by building a dedicated QC.**



Broad goal (facility): **have the QC as versatile as possible (graphene geometries; SYK models; λ_{Lyap}, diffeo invariance, ...)**

This must work because:



(*) Analogy. Correspondence. Duality. Emergence.
Symmetry.