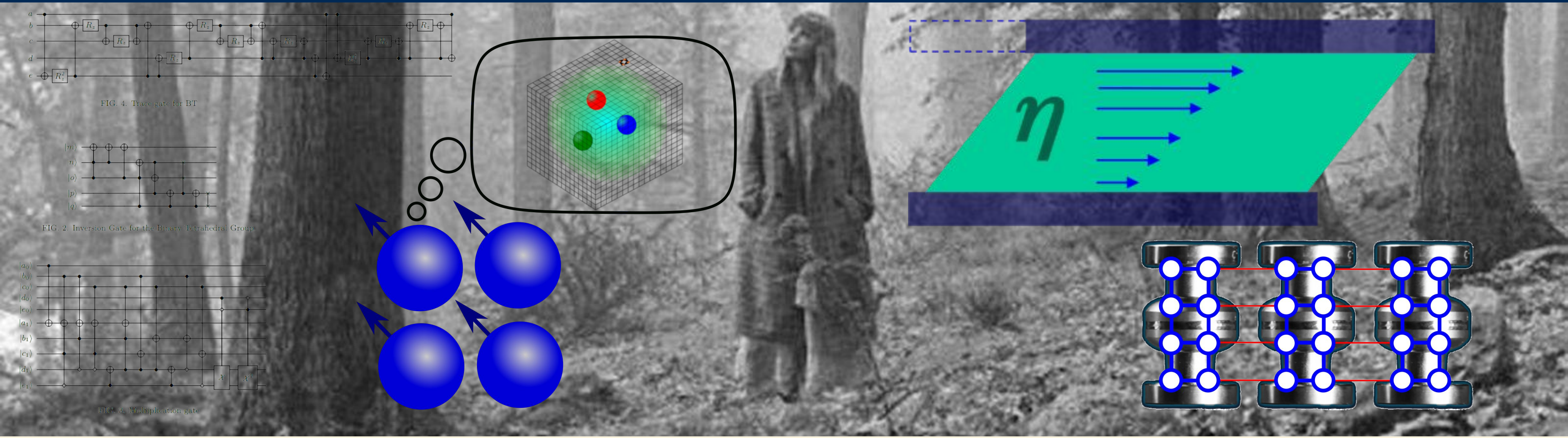


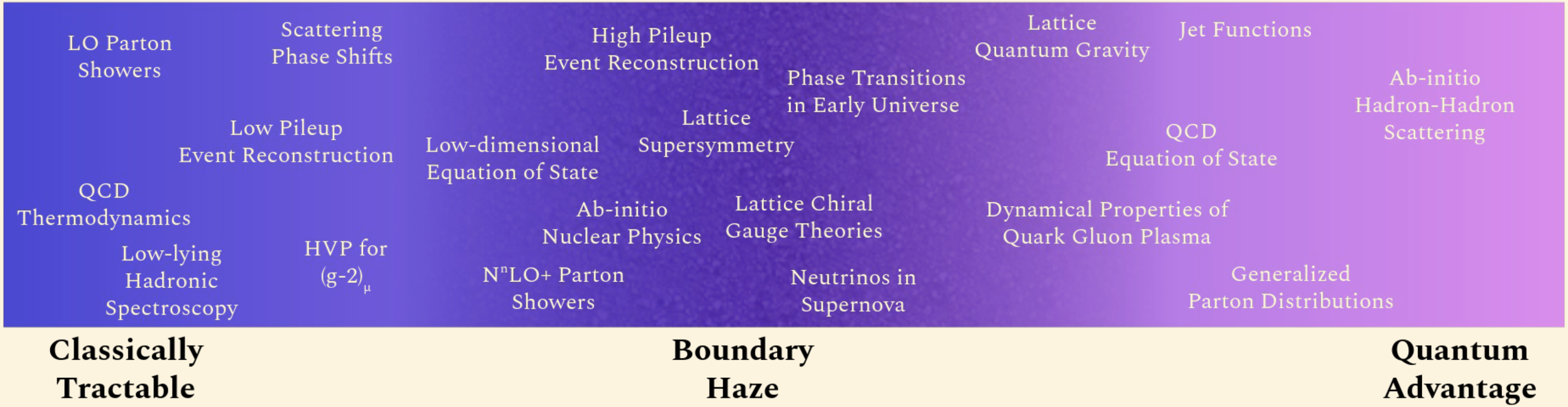
The Quantum Price of Particle Physics

Hank Lamm

6 September, 2023

Theoretical High Energy Physics demands Quantum Computing

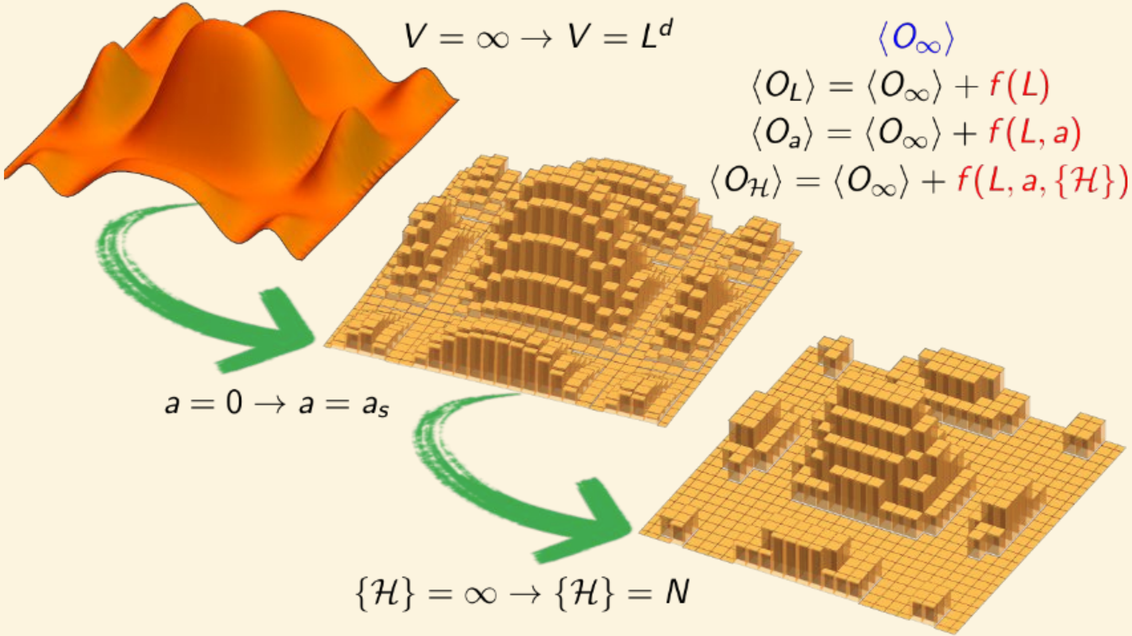
- **The world is quantum, and we are lucky anything is amenable to classical computers**
  - Large-scale quantum computers can tackle computations in HEP otherwise **inaccessible**
  - This opens up new frontiers & extends the reach of LHC, LIGO, EIC & DUNE



While broad, these topics often are formulated as **lattice field theories**

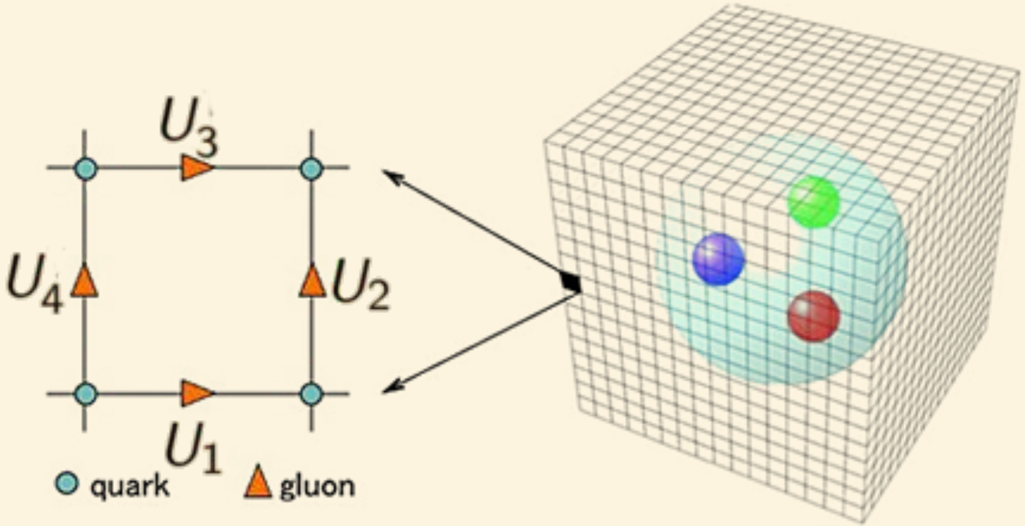
Quantum Simulation for High-Energy Physics
Bauer, Davoudi *et al.* - *PRX Quantum 4 (2023) 2, 027001*
Wonderful survey of physics questions, methods, and outstanding problems in field

Take it to the limit



- O(L,a,$\mathcal{H}$) is an **approximation** for HEP
- Truncations leads to **systematic errors**
- **Extrapolating** is done on results, reducing computational resources

3+1d Lattice QCD



- Put the **4-component, 3 color spinor** for quark on each site in lattice
- Put the **3x3 matrix of complex numbers** for each gluon on each link of lattice
- Perform a Monte Carlo by sampling field configurations
- Modern simulations performed on **100^4 lattices** w/ **yrs** of supercomputing time

Sign problems stymie lattice field theory because….

$|\psi\rangle$ is a **complex-valued** probability amplitude

All I need is…(the industrial workforce of a small country)

$$\langle \psi_0 | e^{-iHt} \mathcal{O} e^{iHt} | \psi_0 \rangle$$

- **Prepare a state**
- **Time evolve the state**
- **Perform a measurement**

As a "near-term" target, consider the viscosity of QCD

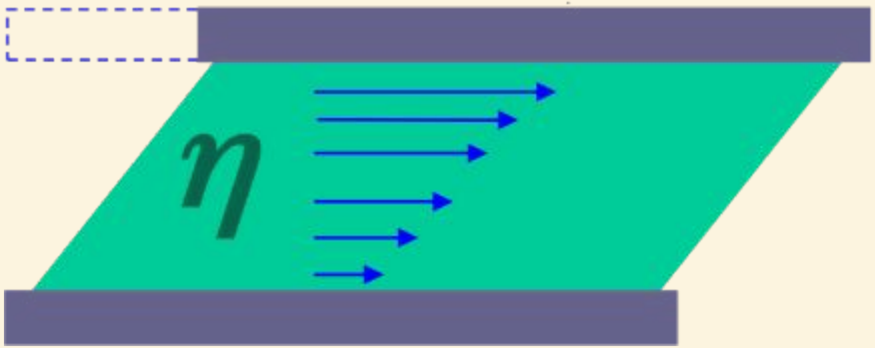
- $\eta = \frac{V}{T}\int_0^\infty \langle T_{12}(t)T_{12}(0)\rangle$

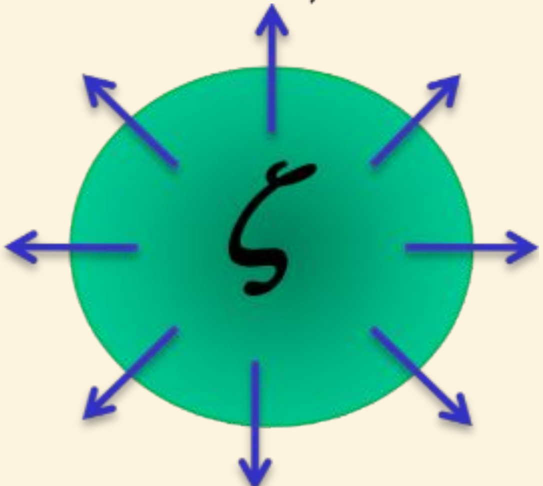
Quantum algorithms for transport coefficients in gauge theories
NuQS Collaboration - *Phys.Rev.D 104 (2021) 9, 094514*
Formulates lattice operators and propose correlators

- A **good** goal allows for focus while introducing **all** the necessary pieces

Viscosity of pure-glue QCD from the lattice
Altenkort *et al.* - *2211.08230 [hep-lat]*
State of the art lattice results, but massive uncertainties persist

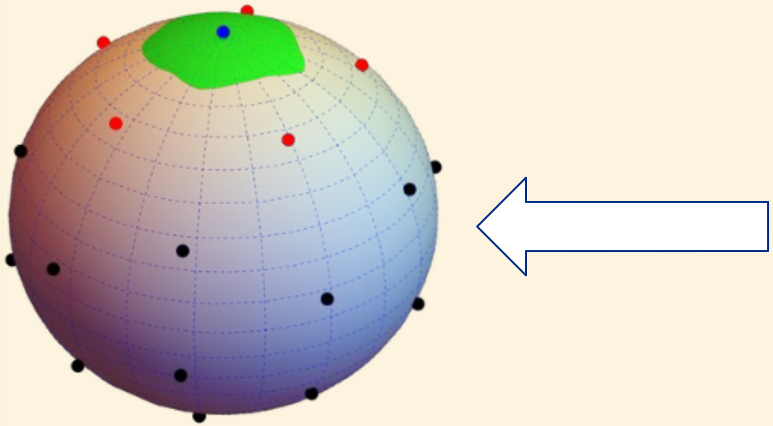
$$\eta/s = 0.15 - 0.48, T = 1.5T_c$$
$$\zeta/s = 0.017 - 0.059, T = 1.5T_c$$

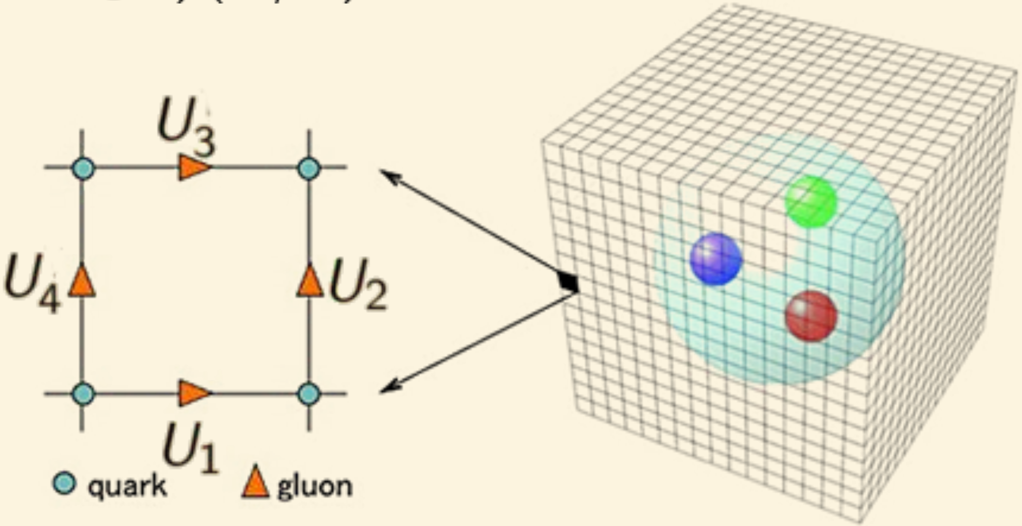




Qubit Costs for Lattice Field Theory

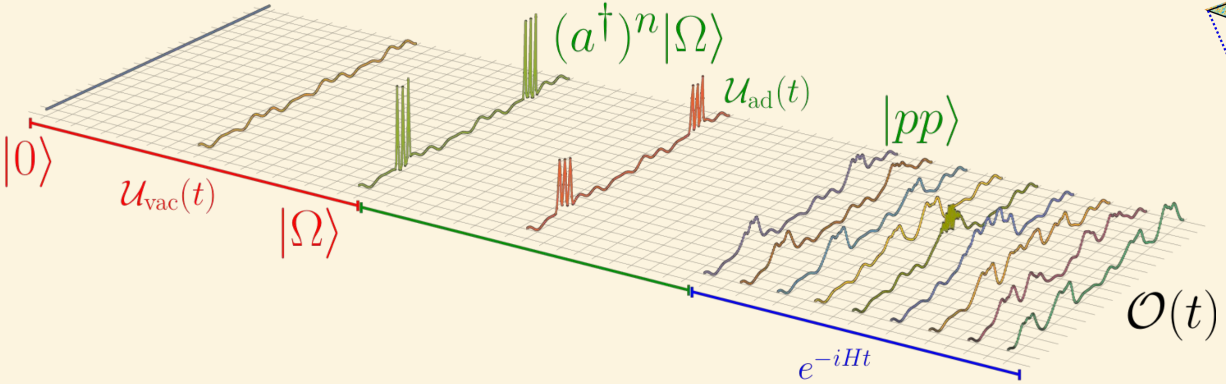
- Lattice field theory discretizes spacetime into a lattice of $(L/a)^d$ sites
  - $L\to\infty$ and $a\to 0$ must be taken
- Matter fields are placed on sites, gauge fields on links
  - Fermionic matter need **$\mathcal{F}$ qubits per site**
  - Gauge links are bosonic and need efficient truncation **Λ qubits per link**
  - So **logical** qubit cost is: $(d\Lambda + \mathcal{F})(L/a)^d$

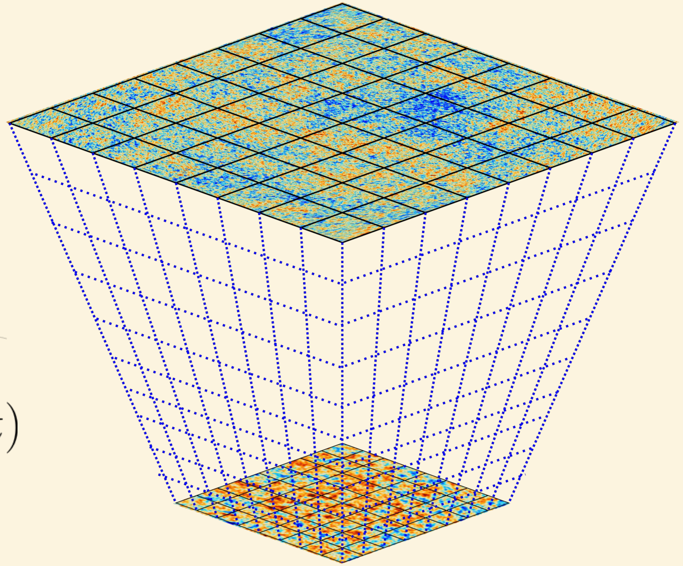




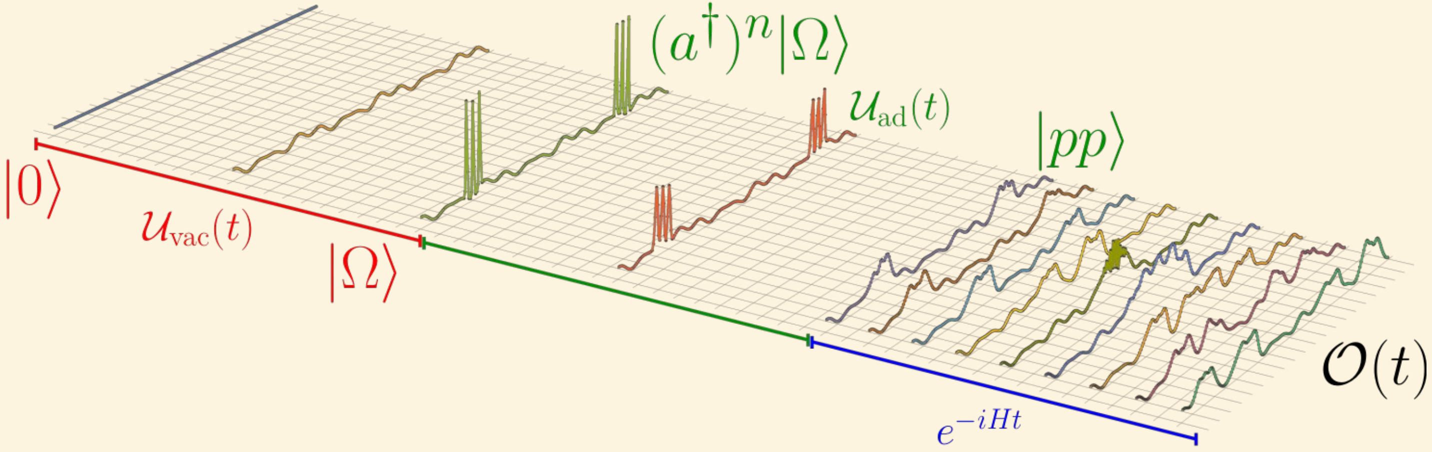
Gate Costs for Lattice Field Theory

- Approximating $U(T) = e^{-iHT}$ *can* corresponds to a lattice of size Ta_t
  - $a_t \rightarrow 0$ or equivalent limit must be taken
- **Logical** gate cost is heuristically: $\frac{T}{a_t} \times [\mathcal{O}(1)(d\Lambda + F)(L/a)^d]^{\mathcal{O}(1)}$





It's one calculation, Hank. What could it cost?



O(10^{11}) q and **O(10^{55})** T-gates which is **< 3 yrs** on an **exascale** QC

Lattice Quantum Chromodynamics and Electrodynamics on a Universal Quantum Computer
Kan and Nam - 2107.12769 [quant-ph]
Rough, conservative, model- and algorithm-dependent estimates for viscosity and heavy-ion collisions

Compare to **O(10^7) q** and **O(10^{20})** T-gates for RSA Cracking and Chemistry

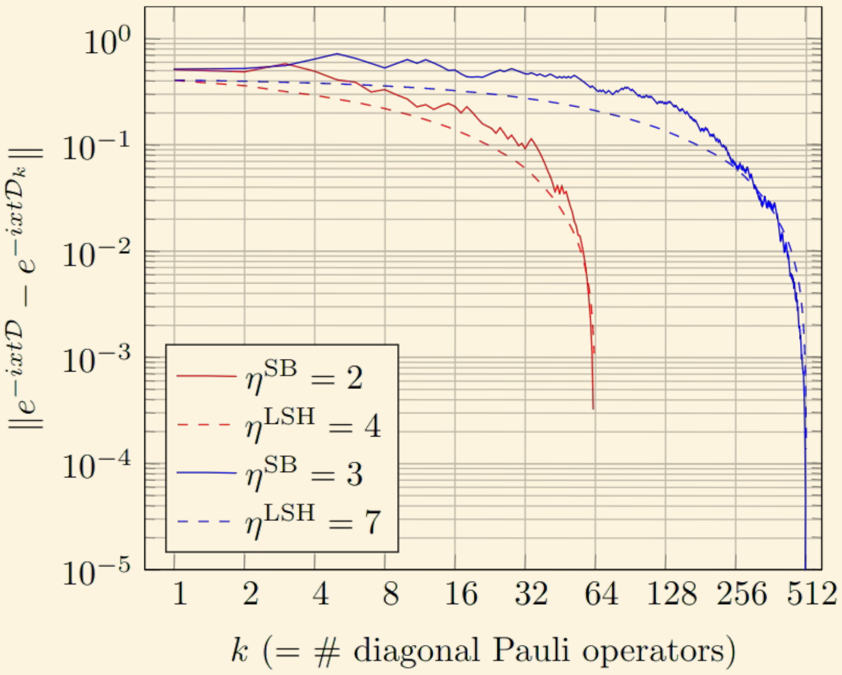
What will QCD viscosity take?

Qubits: $\mathcal{E}(d\Lambda + \mathcal{F})(L/a)^d$ Gates: $\frac{T}{a_t} \times [G_q\, \mathcal{E}(d\Lambda + \mathcal{F})(L/a)^d]^{\mathcal{G}}$

- What dimension, d? (3)
  - Note: **universality errors**
- How will you truncate Λ? (9 64-bit $\mathbb{C}$~ 10^3)
  - Note: **truncation errors**
- How large will you take L? (r_{proton}~1fm)
  - Note: **finite volume errors**
- What QEC is needed, $\mathcal{E}$? (1-10^8)
  - Note: **quantum noise errors**
- How small will you take a_t?
  - Note: **Trotter errors**
- What is Ƒ? (Staggered=12; Wilson=24)
  - Note: **a_t scaling of errors**
- How small will you take a? (1fm^{-1}~ 200 MeV)
  - Note: **discretization errors**
- How efficient is your algorithm G_q?
  - Note: ***shrug* errors**
- How well approximated are your gates $\mathcal{G}$?
  - Note: **gate synthesis errors**
- How long do you need to run for (T)?
  - Note: **Signal resolution errors**

What didja get?

- **Qubit costs: 10^3-10^{11}**
  - **10q** for SU(3) might be reasonable
  - a~**0.5 fm**, L~**3 fm**
  - Perhaps we **drop** fermions
  - Perhaps **lower** dimensions
- **Gate costs: 10^7-10^{60}**
  - a_t~**0.1 fm**, T~**1 fm**
  - Perhaps **sloppy** synthesis
  - Perhaps **improved** algorithms



General quantum algorithms for Hamiltonian simulation with applications to a non-Abelian lattice gauge theory
Davoudi, Shaw, Stryker - 2212.14030 [hep-lat]
Understanding the synthesis and Trotter errors, along with algorithmic choices in 1+1 SU(2)

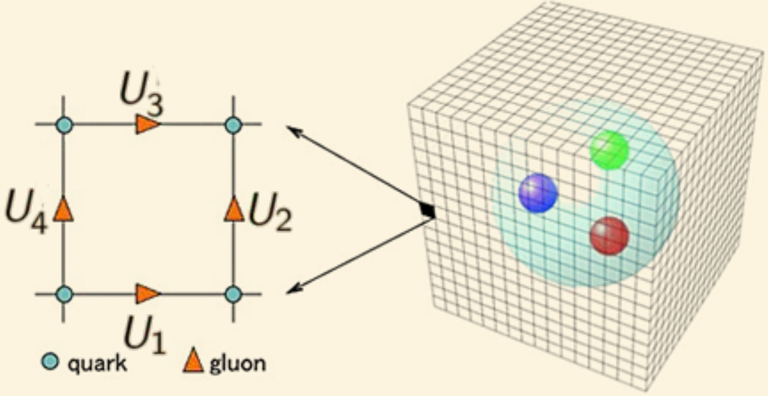
But we don't today have a good sense of **theoretical** errors…

Does considering the NISQ era make sense?

…if not, the questions we need to address **change**

- **Scalable, networked qudits**
  - **Considerations of bandwidth**
- **Quantum error correction**
  - **Potentially huge overhead**
- **Gate set limitations**
  - **Must synthesize**
  - **Count nontransverse T-gates**

Kogut-Susskind Hamiltonian



$$H_{KS} = \sum_n m_n \psi_n^\dagger \psi + \sum_{n,k} [\psi_n^\dagger U_{nk} \psi_{n+k} + h.c.]$$

Fermionic mass term   Fermionic kinetic or *hopping* term

$$+ \sum_n E_n^2 + \sum_{n,k} \mathrm{ReTr}\, U_p$$

Gauge **E** field   Gauge **B** or *plaquette* term

Hamiltonian Formulation of Wilson's Lattice Gauge Theories
Kogut & Susskind *Phys.Rev.D 11 (1975) 395-408*
Formulated $O(a^2)$ lattice Hamiltonian for LGT with staggered matter

Symanzik-Improved Hamiltonian

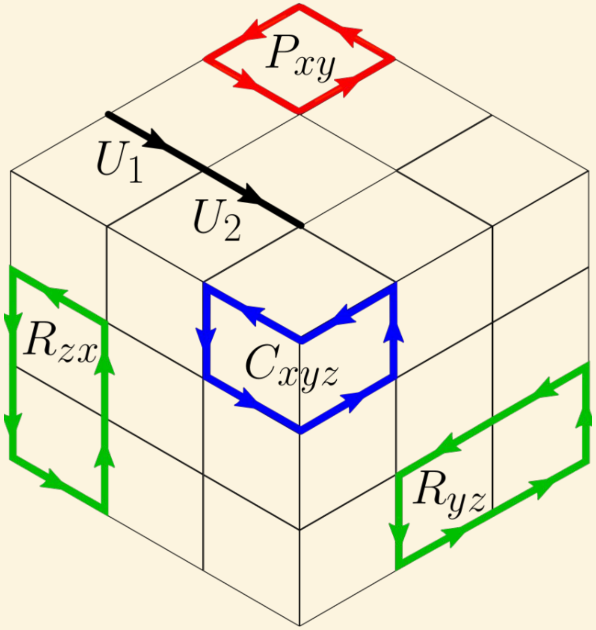
$$\mathrm{Tr}[\mathbf{E}^2(\mathbf{x})] \approx \frac{g^2}{2a}\mathrm{Tr}[X\mathcal{E}_i(\mathbf{x})\mathcal{E}_i(\mathbf{x}) + Y\mathcal{E}_i(\mathbf{x})U_i(\mathbf{x})\mathcal{E}_i(\mathbf{x}+a\hat{i})U_i^\dagger(\mathbf{x})]$$

$$\mathrm{Tr}[\mathbf{B}^2(\mathbf{x})] \approx \frac{2N}{ag^2}[XP_{ij}(\mathbf{x}) + \frac{Y}{2}(R_{ij}(\mathbf{x}) + R_{ji}(\mathbf{x}))]$$

Which is related to the continuum:

$$K = \frac{X+Y}{2}E_i^2 + \frac{5Y-X}{12}E_i\partial_i^2 E + \mathcal{O}(ea^2, a^4)$$

$$V \approx a^d[(X+4Y)\mathrm{Tr}(F_{ij}^2) + \frac{a^2}{12}(X+10Y)\mathrm{Tr}(F_{ij}\{D_i^2 + D_j^2\}F_{ij}) + \mathcal{O}(e^2a^2, a^4)]$$

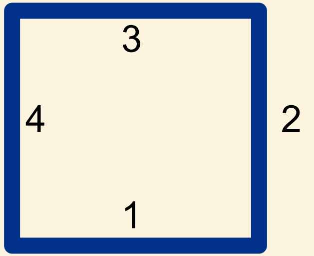


H_I should require **>2^d fewer** qubits with **comparable** gate counts to H_{KS}

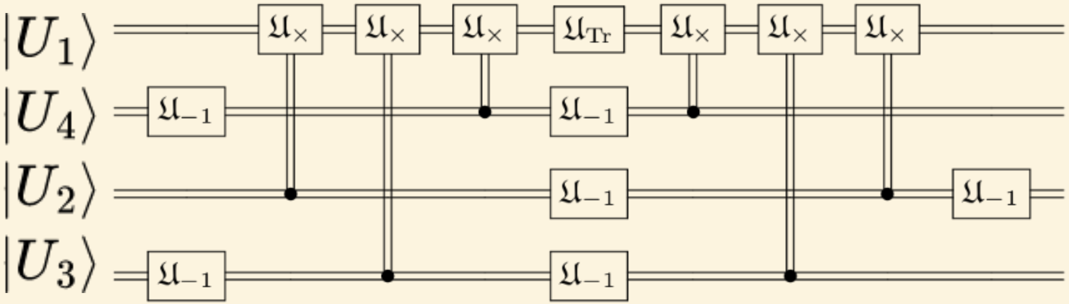
Improved Hamiltonians for Quantum Simulation of Gauge Theories
Carena, Lamm, Li, Liu *PRL 129 (2022) 5*
Developed quantum circuits for O(a^4) pure-gauge Hamiltonian

Group Primitives as subroutines

$$H_{KS,1} = \sum_{i=\text{color}} E^2_{1,i}(n) + \sum_{k=direction} \text{ReTr} U_1 U_2 U_3^\dagger U_4^\dagger$$



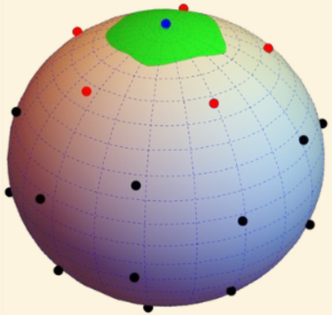
- Inversion gate: $\mathfrak{U}_{-1} |g\rangle = |g^{-1}\rangle$
- Multiplication gate: $\mathfrak{U}_\times |g\rangle |h\rangle = |g\rangle |gh\rangle$
- Trace gate $\mathfrak{U}_{\mathrm{Tr}}(\theta) |g\rangle = e^{i\theta \,\mathrm{Re\,Tr}\, g} |g\rangle$
- Fourier Transform gate: $\mathfrak{U}_F \sum_{g\in G} f(g) |g\rangle = \sum_{\rho \in \hat{G}} \hat{f}(\rho)_{ij} |\rho, i, j\rangle$



General Methods for Digital Quantum Simulations of Gauge Theories
Lamm, Lawrence, Yamauchi - *Phys.Rev.D* 100 (2019) 3, 034518
Constructed this general formalism for group independent implementation

How to choose a digitization?

- Must map **∞-dimensional** Hilbert space of bosons to **finite** quantum register



$$H_{KS,1} = \sum_{i=\text{color}} E^2_{1,i}(n) + \sum_{k=direction} \text{ReTr} U_1 U_2 U_3^\dagger U_4^\dagger$$

E (irrep) basis

Trailhead for quantum simulation of SU(3) Yang-Mills lattice gauge theory in the local multiplet basis
Ciavarella, Klco, Savage *Phys.Rev.D 103 (2021) 9, 094501*
Qubit implementation of SU(3) with irrep truncations

Mixed basis

A new basis for Hamiltonian SU(2) simulations
Bauer, D'Andrea, Freytsis, Grabowska
Formulated an alternative basis that contains parts of E & B basis

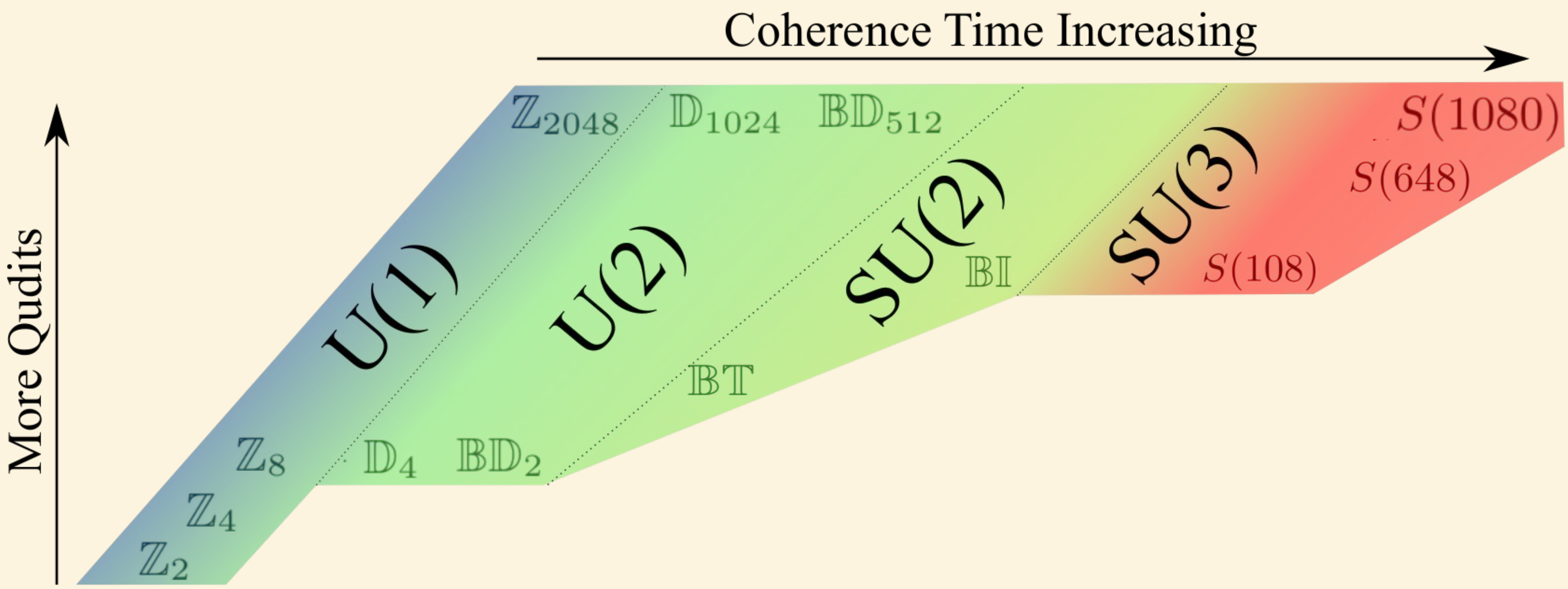
B (group element) basis

Primitive Quantum Gates for an SU(2) Discrete Subgroup: BT
Gustafson, Lamm, Lovelace, Musk - *Phys.Rev.D* 106 (2022) 11, 114501
Qubit and Qudit gates for approximating SU(2) with subgroups

Well, what keeps **you** up at night?

arbitrary precision, gauge fixing, quantum noise, error correction, gate costs, classical simulatability

The ladder of discrete gauge theories in HEP calculations



Gluon Field Digitization for Quantum Computers
NuQS collaboration - *Phys.Rev.D 100 (2019) 11, 114501*
Demonstrated that S(1080) approximates certain 3+1d SU(3) observables

How do we represent discrete groups?

- Ordered product of generators

$$h_{\{o_k\}} = \prod_k \lambda_k^{o_k} = h_d$$

$$\mathbb{D}_4: \quad h_d = s^a r^b$$

$$\mathbb{Q}_8: \quad h_d = (-1)^a \mathbf{i}^b \mathbf{j}^c$$

$$\mathbb{BT}: \quad h_d = (-1)^a \mathbf{i}^b \mathbf{j}^c \mathbf{l}^d$$

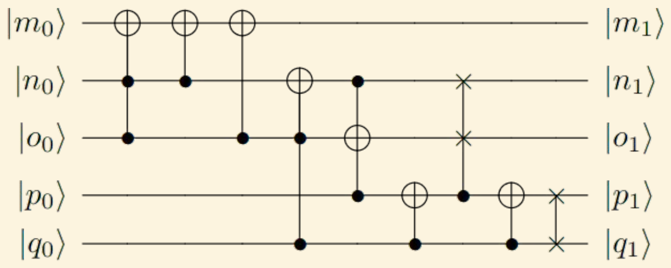
$$\Sigma(36 \times 3): \quad h_d = \omega_3^a \mathbf{C}^b \mathbf{E}^c \mathbf{V}^d$$

$$\rightarrow |abcd \cdots\rangle$$

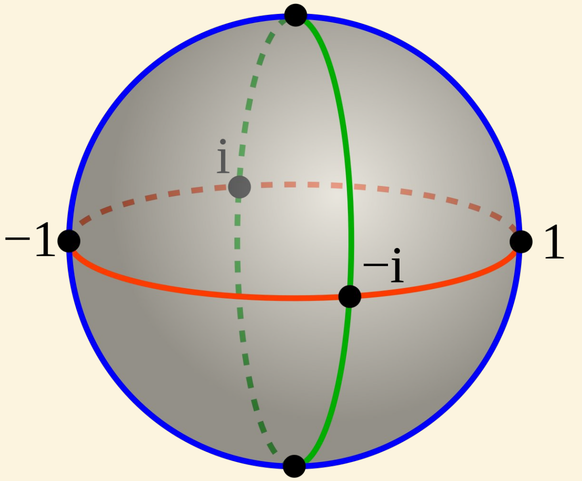
These integers are not all binary, so naturally more robust and easier on **qudits!**

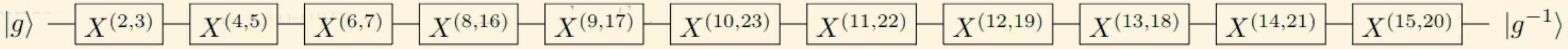
Robustness of Gauge Digitization to Quantum Noise
Gustafson, Lamm - 2301.10207 [hep-lat]
Discusses quantum registers with qubits, qudits for U(1), SU(2), SU(3)

Group Primitives for Different Hardware



A qubit implementation of $\mathfrak{U}_{-1}$





An quicosotetrit implementation of $\mathfrak{U}_{-1}$ using the $X^{(a,b)}$ gate.

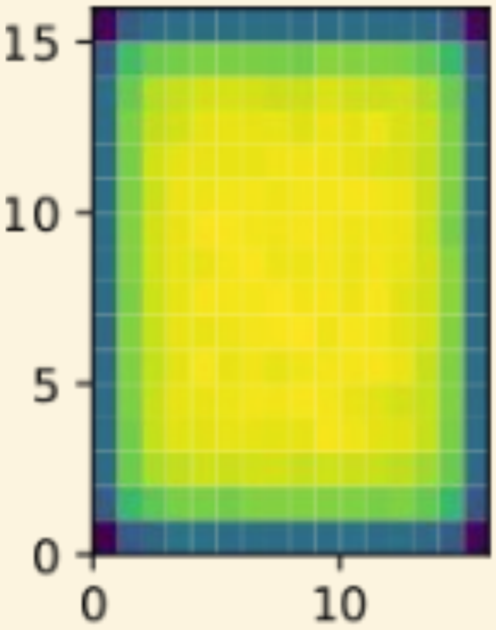
$$\mathcal{F}_2^{N_q(N_q-1)} = \mathcal{F}_{2_q^N}$$

Can multimode qudits provide **nonabelian** QEC and virtual gates?

Primitive Quantum Gates for an SU(2) Discrete Subgroup: BT
Gustafson, Lamm, Lovelace, Musk - *Phys.Rev.D* 106 (2022) 11, 114501
Derived and implemented using custom QEM necessary primitives for HEP simulations

The 2T-qutrit, a two-mode bosonic qutrit
Denys & Leverrier - Quantum 7, 1032 (2023)
Created logical qutrit from nonabelian code constellation

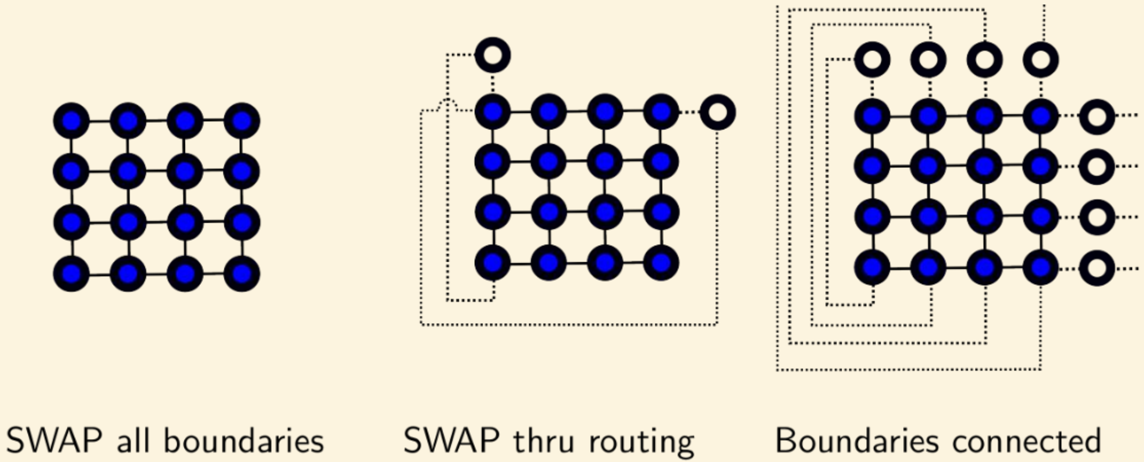
Periodic Boundary Conditions are *HIGHLY* desirable



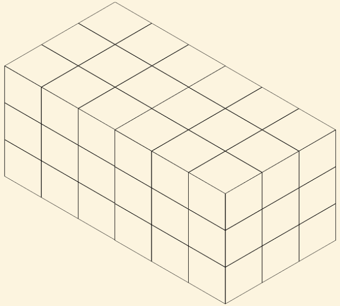
$$N^{OBC}_{qubits} \approx x(a)^d \times N^{PBC}_{qubits}$$

Typical $x(a)$ can be **2-5**

Is **125x more qudits** easier that **quantum networking?**



Multigrid and Circuit Knitting



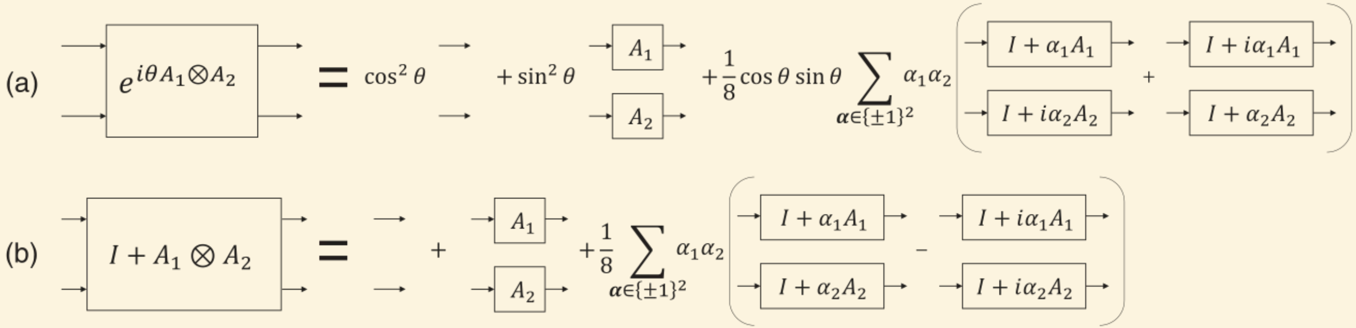
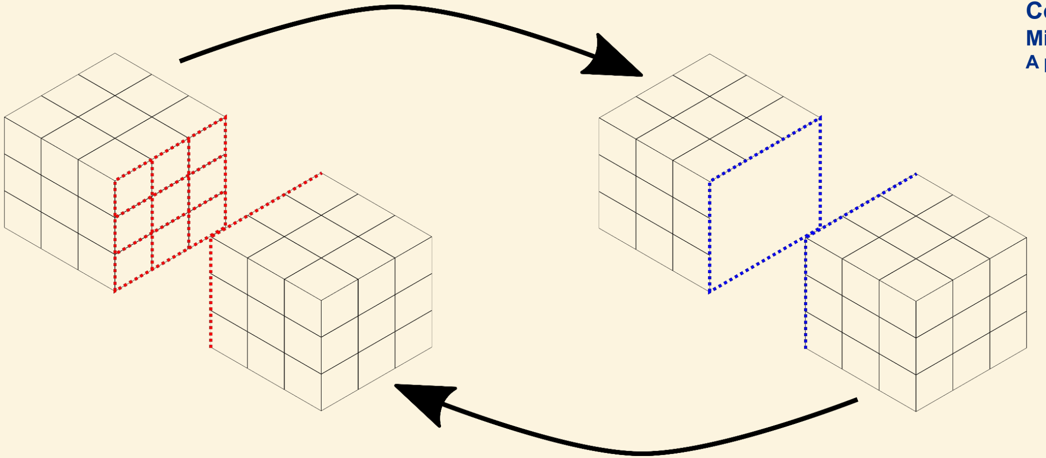


Figure 1. Decomposition of (a) a non-local gate and (b) a non-local non-destructive measurement into a sequence of local operations. A_1 and A_2 are operators such that $A_1^2 = I$ and $A_2^2 = I$.

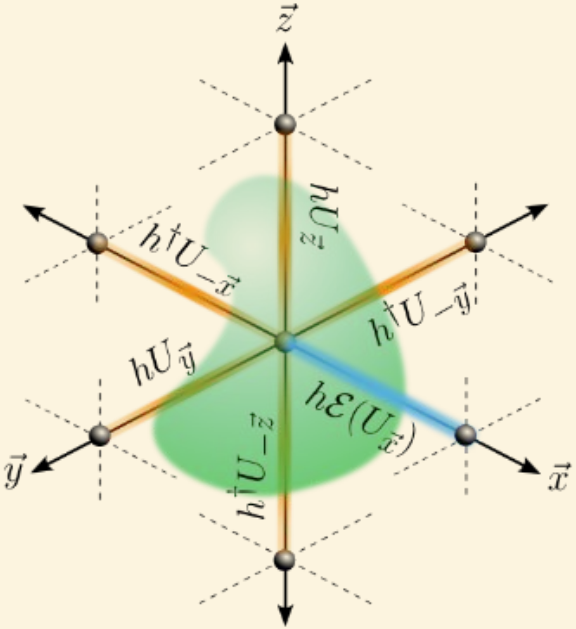
Constructing a virtual two-qubit gate by sampling single-qubit operations
Mitarai, Fujii - New J. Phys. 23 023021 2021
A particularly good explanation and lit review of topic



- Circuit Knitting has **<O(9^N) scaling**
- **Quasiprobabilities** increase costs
  - Sign problem!
- Reduce this for LFT through **multigrid** techniques?

QEC for LFT

- Given a register, **prioritize error channels** for mitigation and correction
- Reduction of **large theoretical error** at **lower cost**



Robustness of Gauge Digitization to Quantum Noise
Gustafson, Lamm - 2301.10207 [hep-lat]
Classification of Gauge Violating noise for qubits, qudits for U(1), SU(2), SU(3)

TABLE I. $\mathcal{N}_i$ vs. $\mathbb{G}$ for $U(1)$ subgroups: $\mathbb{Z}_N$ where $N = 2^n$

Binary	Gray	Qudits	$\mathbb{G}$
$\hat{Y}_0$	$\hat{Y}_0$	$\hat{B}^{(i,j)}$, $\hat{Z}^{(i)}$	—
—	$\hat{B}_{a\neg 0}$, $\hat{Z}_a$	$\hat{\mathcal{V}}^m$	$\mathbb{Z}_2$
$\hat{X}_{a\neg 0}$	—	—	$\mathbb{Z}_{2^{n-a}}$
$\hat{Z}_a$, $\hat{Y}_{a\neg 0}$	$\hat{X}_0$	—	$\mathbb{Z}_{2^{n-1}}$
$\hat{X}_0$	—	$\hat{\chi}^m$	$\mathbb{Z}_N$

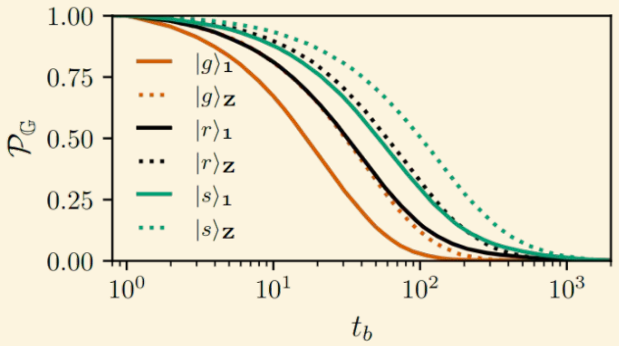


FIG. 2. $\mathcal{P}_{\mathbb{G}}(t_b)$ for $\mathbb{Z}_8$ versus t_b using $|g\rangle$, $|r\rangle$, and $|s\rangle$ for depolarizing and dephasing channels.

Endgame

- The road to **quantum practicality in HEP** will be **long** and **winding**
- We **do not have** anything close to realistic game plan
- Material fabrication, cryogenics, hardware design, quantum software stack, and classical communication **all profoundly affect** the questions we can ask in HEP

But also can be affected by HEP