Quantum metrology of displacements

Lorenzo Maccone

Dip. Fisica, INFN Sez. Pavia,
Universita' di Pavia

Wojchiech Gorecki

University of Warsaw

Alberto Riccardi

Universita' di Pavia

quantum information theory group

www.qubit.it

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Quantum metrology?

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Metrology: estimation of a parameter, through measurements.

The estimation is always performed by averaging over N measurements, so that (central limit theorem), the error of the average goes as $1/\sqrt{N}$

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Usually: $\sqrt{N}$ enhancement: the error goes as $1/N$

Measurements

Want to estimate a parameter φ written onto a probe by a transformation U_φ

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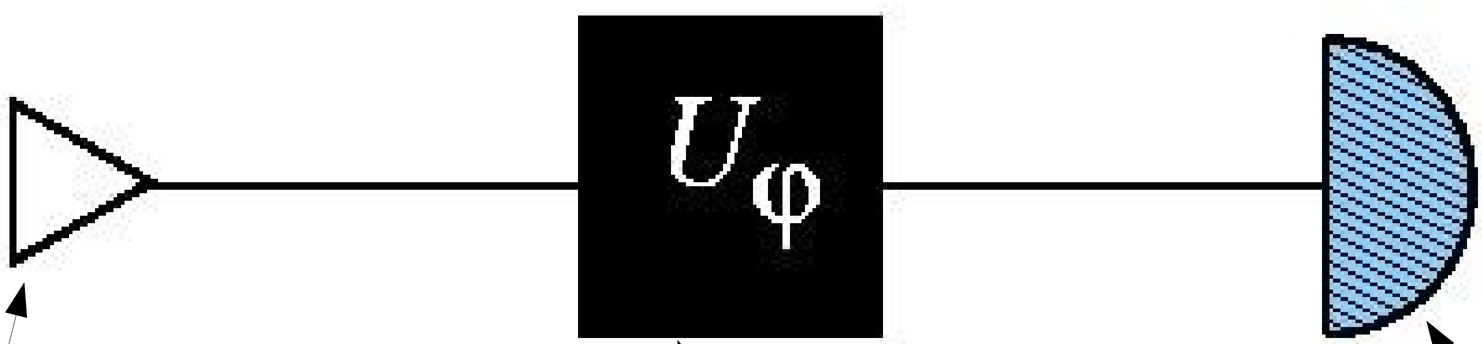
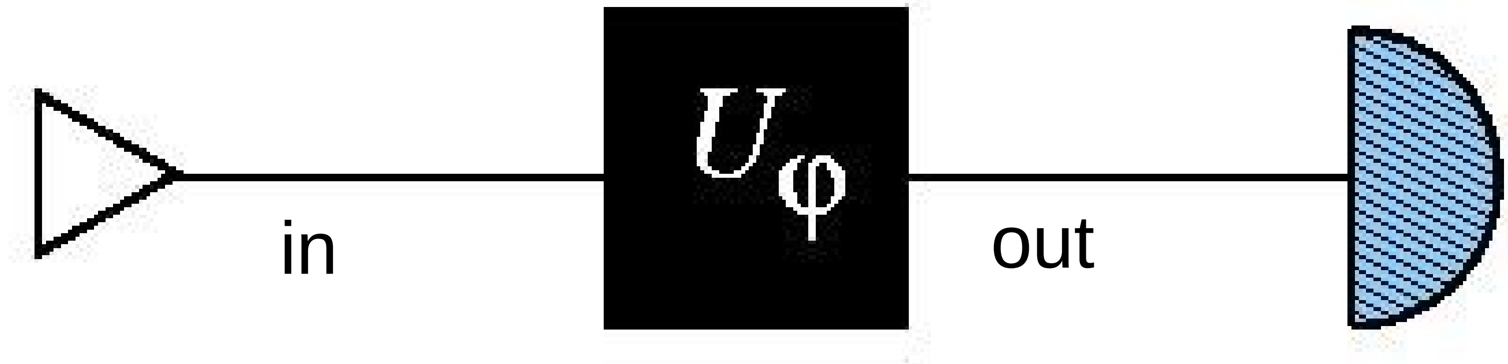


Probe preparation

interaction

Probe measurem and data analysis

How?



Main idea: minimize the overlap between the input and the output state!

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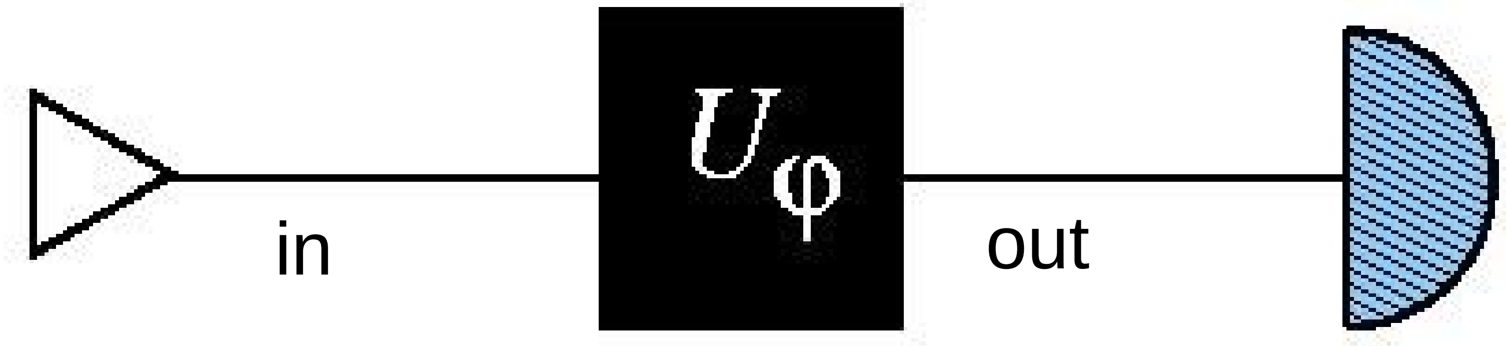

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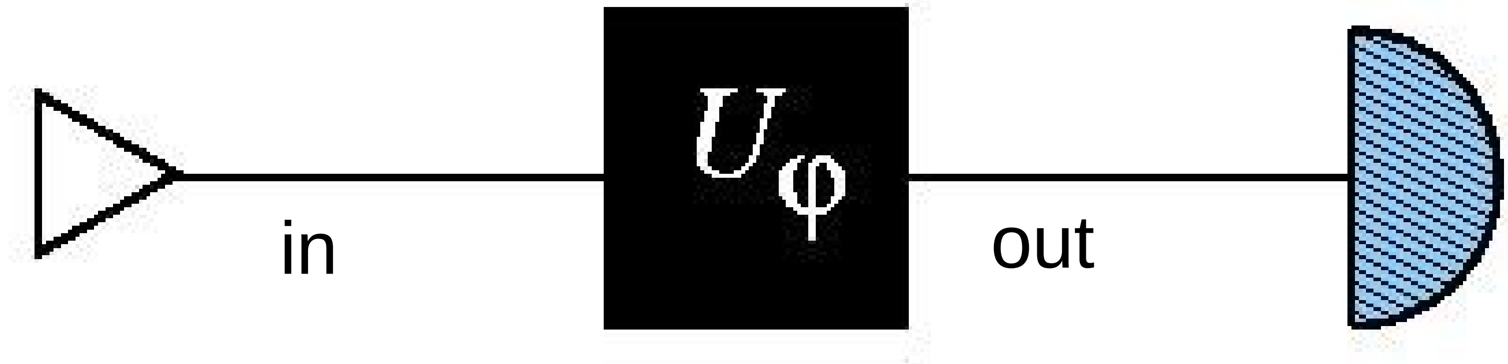

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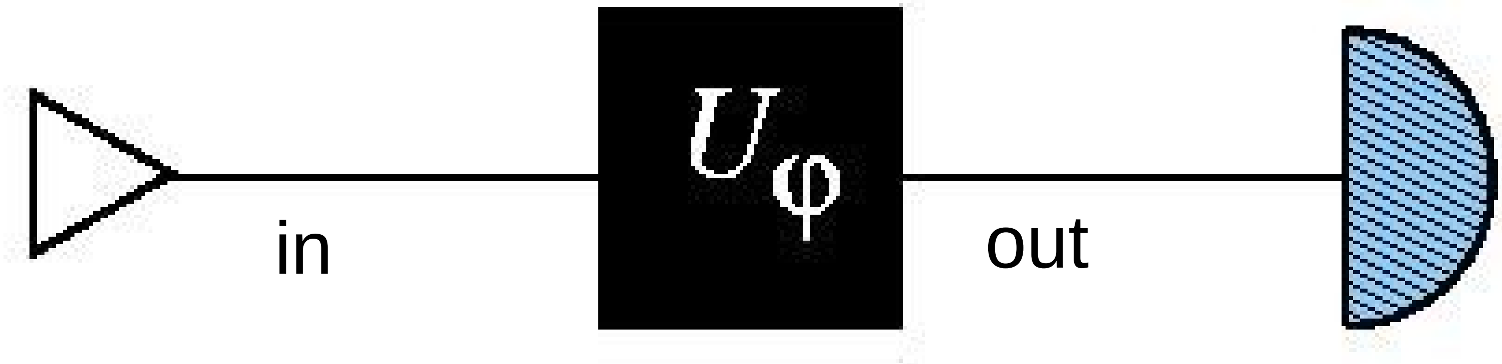

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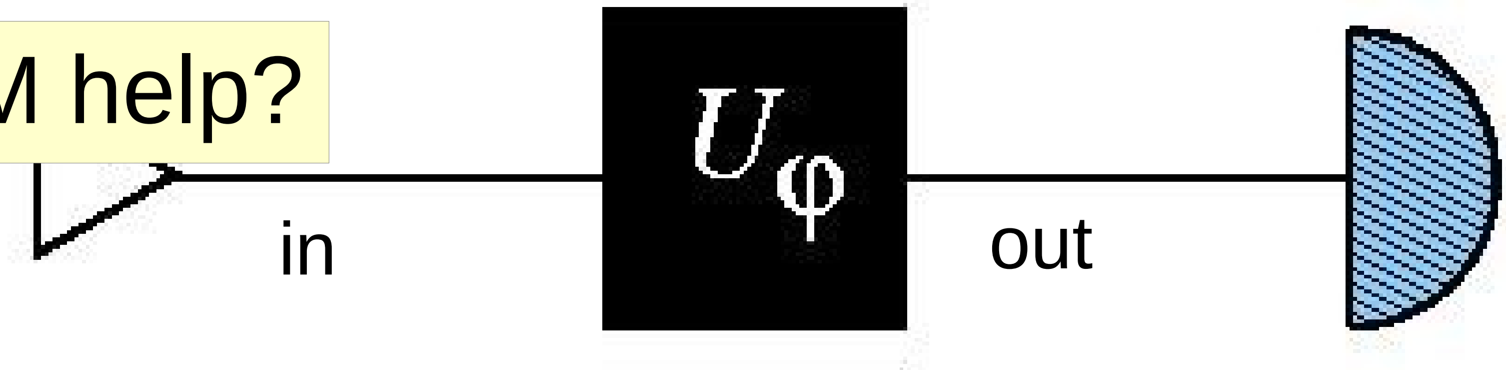
the metric in Hilbert space (to measure distances)

Does QM help?



Compare quantum strategies to the $\boxed{\textbf{corresponding}}$ classical strategy

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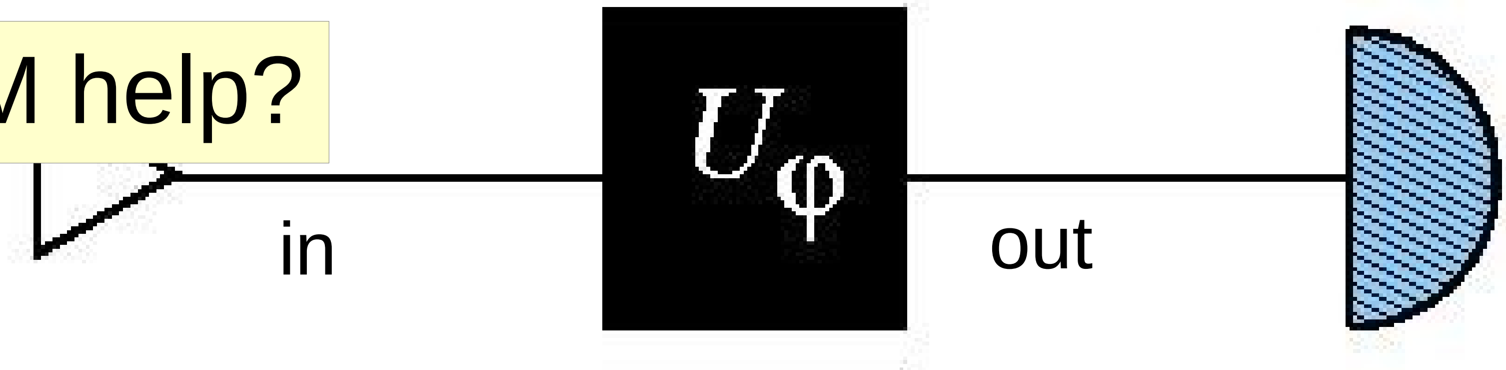

Compare quantum strategies to the **corresponding** classical strategy

Same number of uses of U_φ

Same employed energy

Parallel strategies

PRL 96,010401

Parallel strategies

use $-\boxed{U_\varphi}-$ in parallel:

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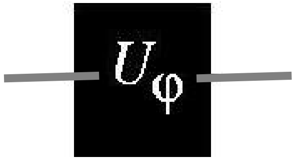
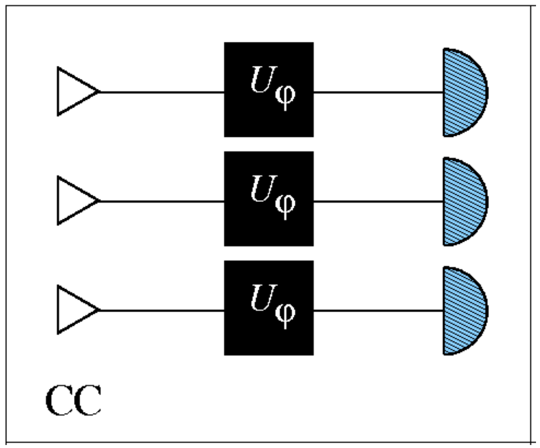
Parallel strategies

use  in parallel:

Classical strategies:



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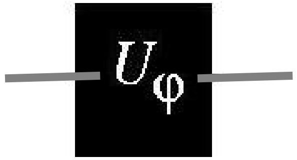
use  in parallel:

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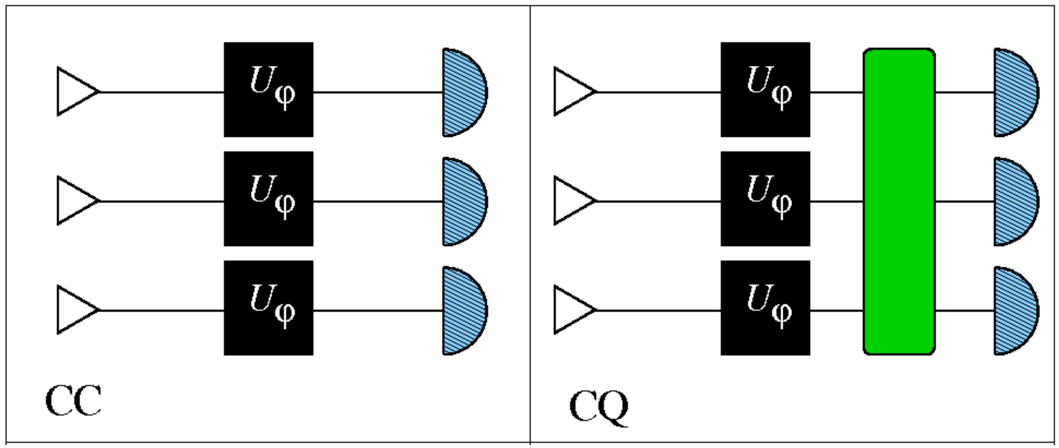

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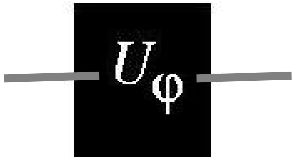
use  in parallel:

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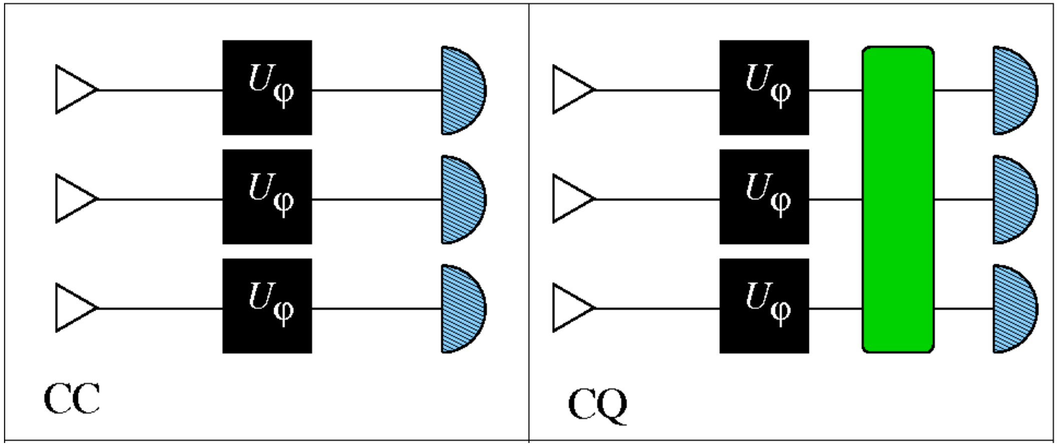

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(shot noise)

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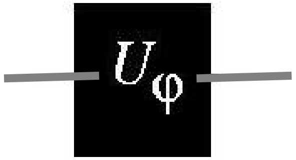
use  in parallel:

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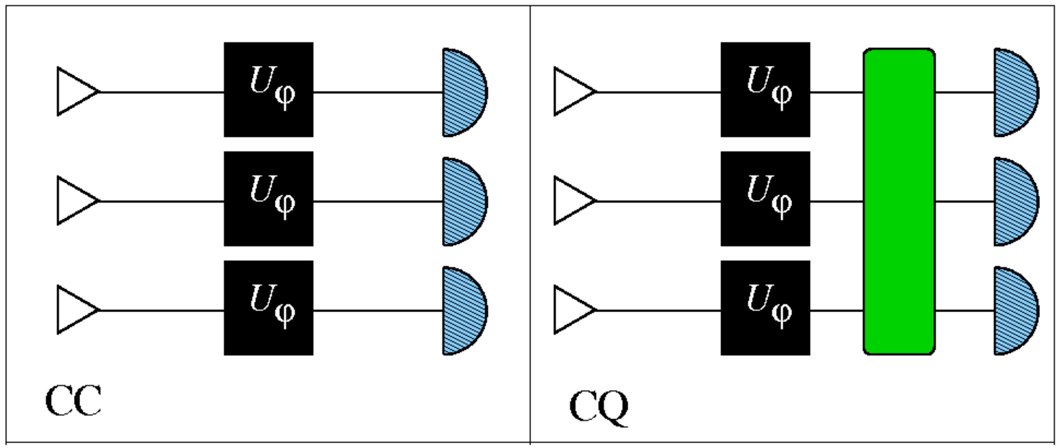

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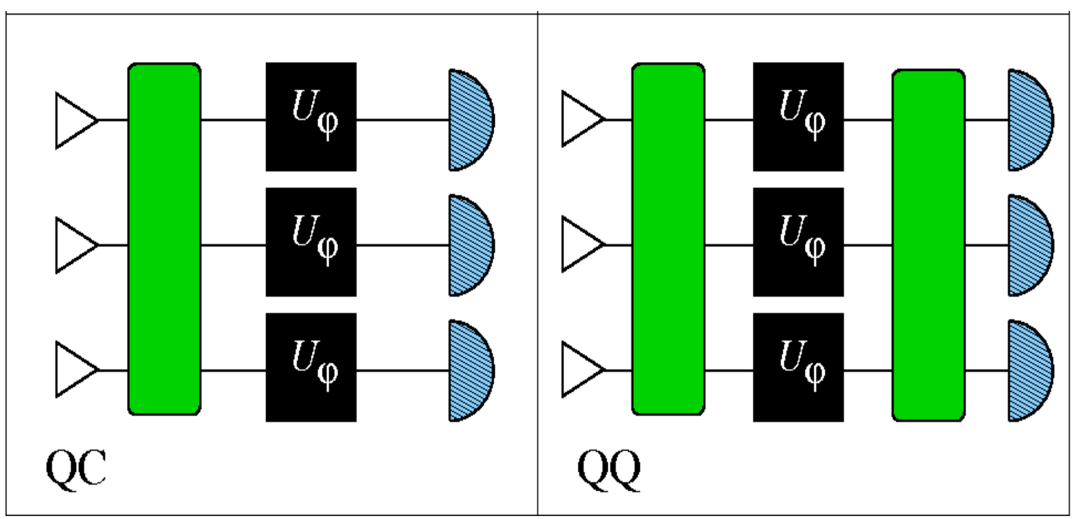

the N transformations act on an **entangled state**

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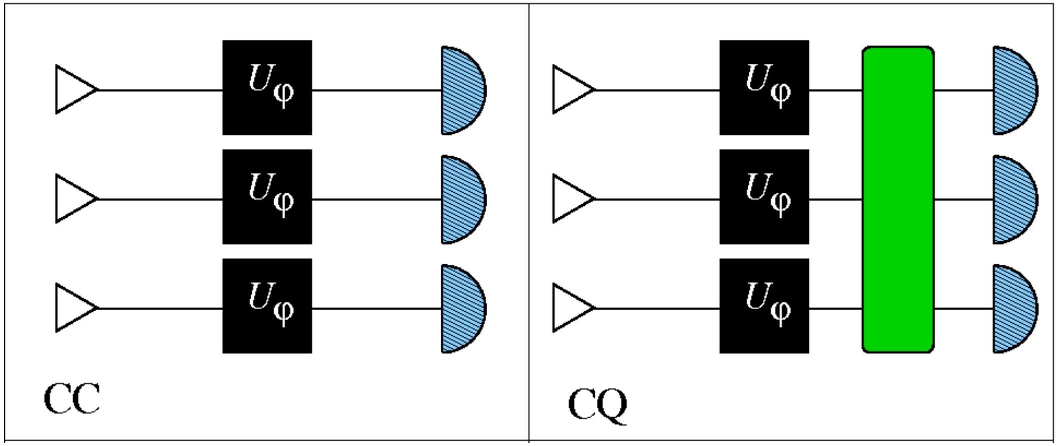

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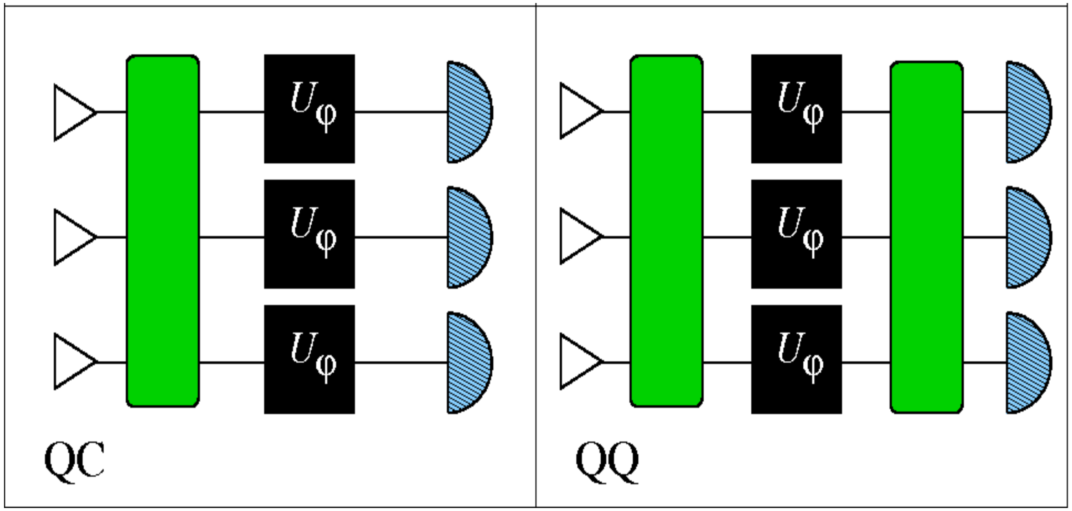

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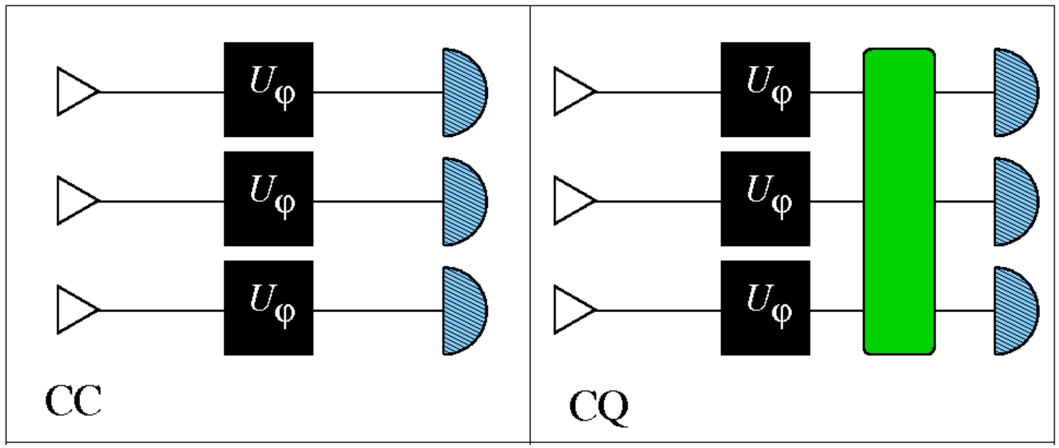

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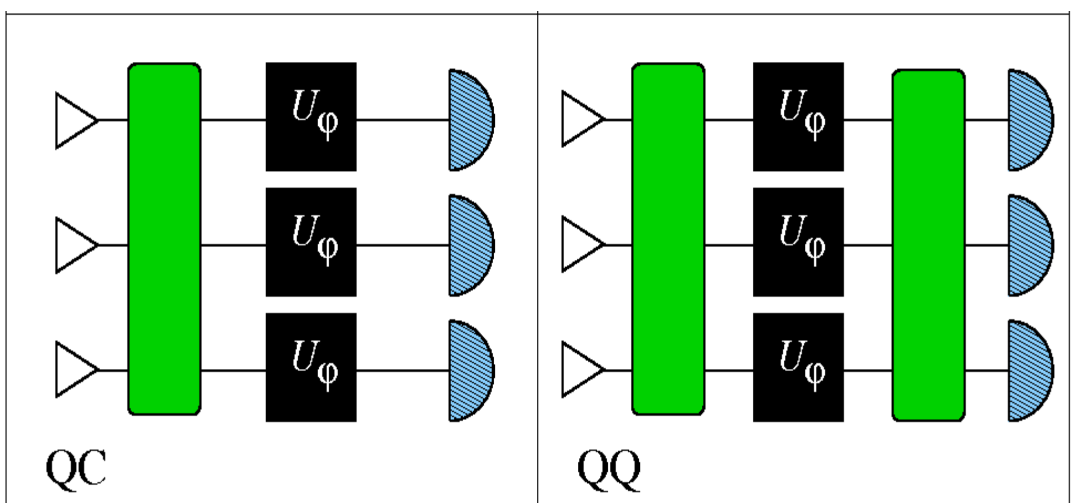

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Note: entanglement at the measurement stage is useless!

u

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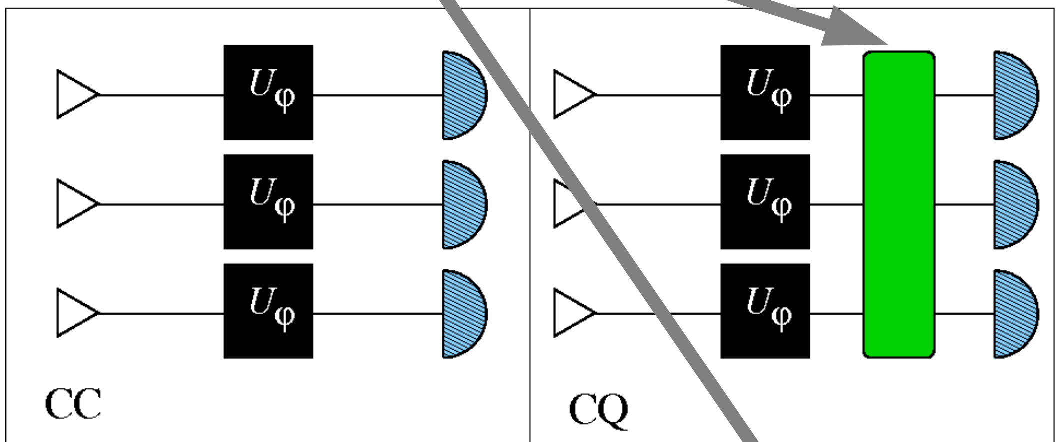

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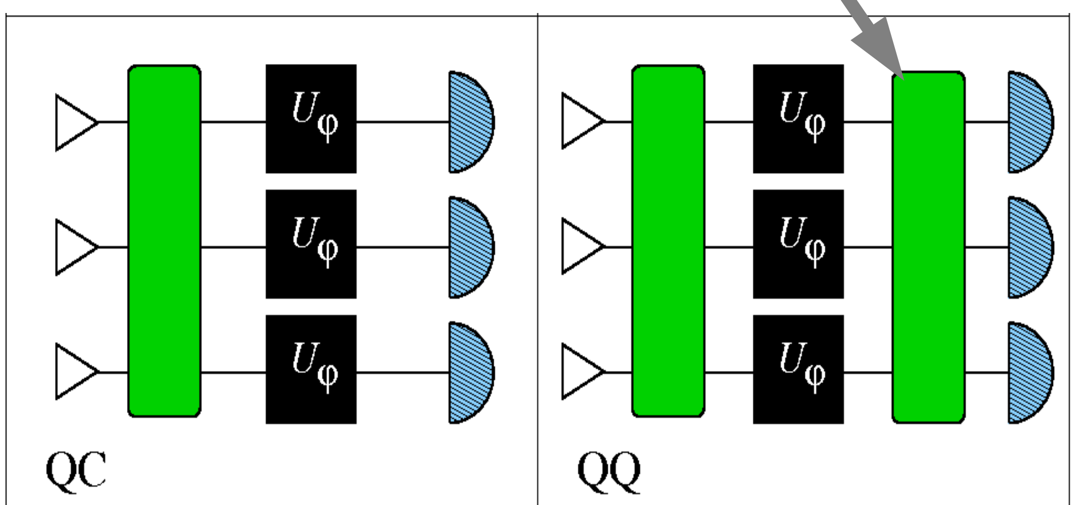

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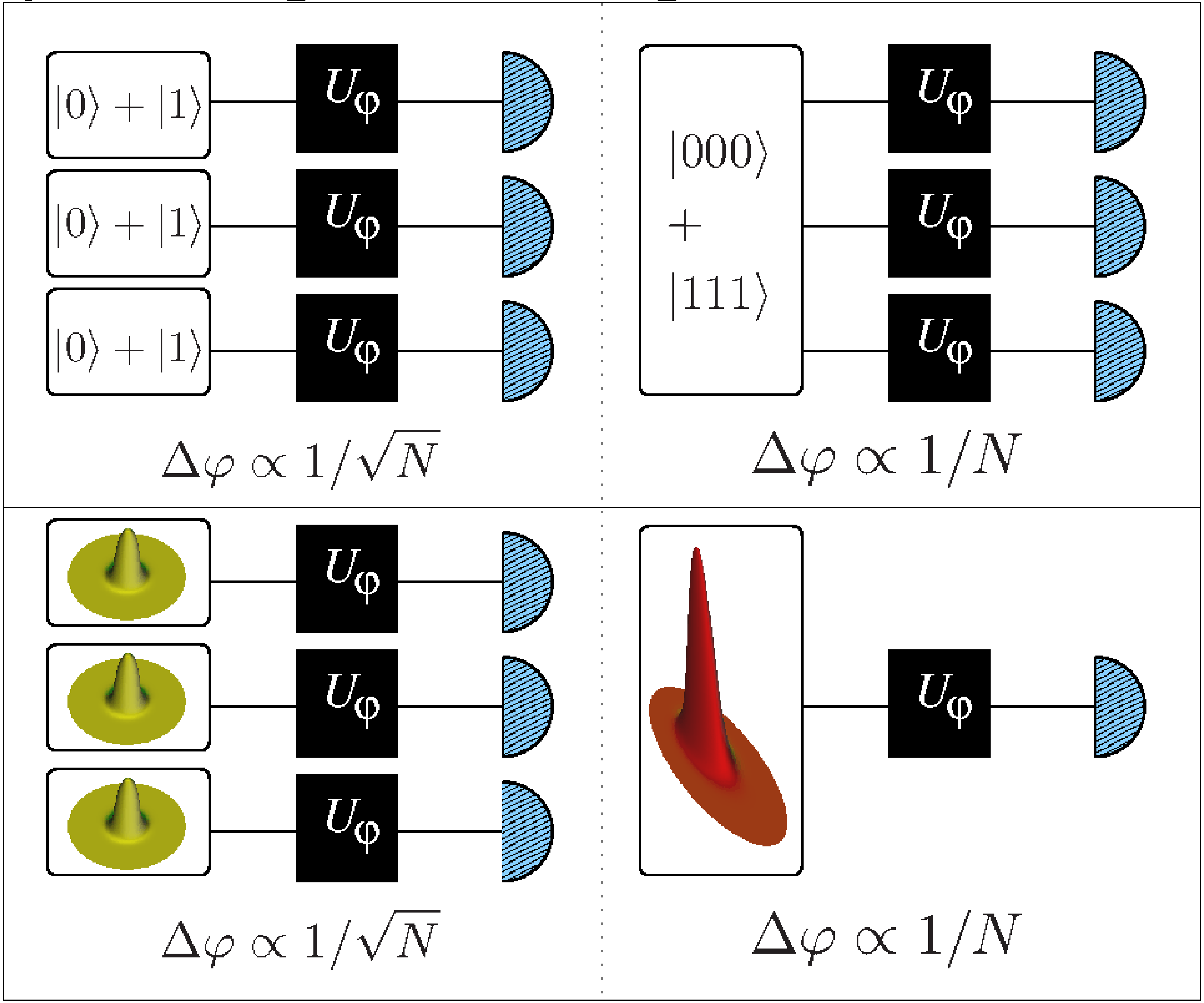
Squeezing vs entanglement

arXiv:1901.07482



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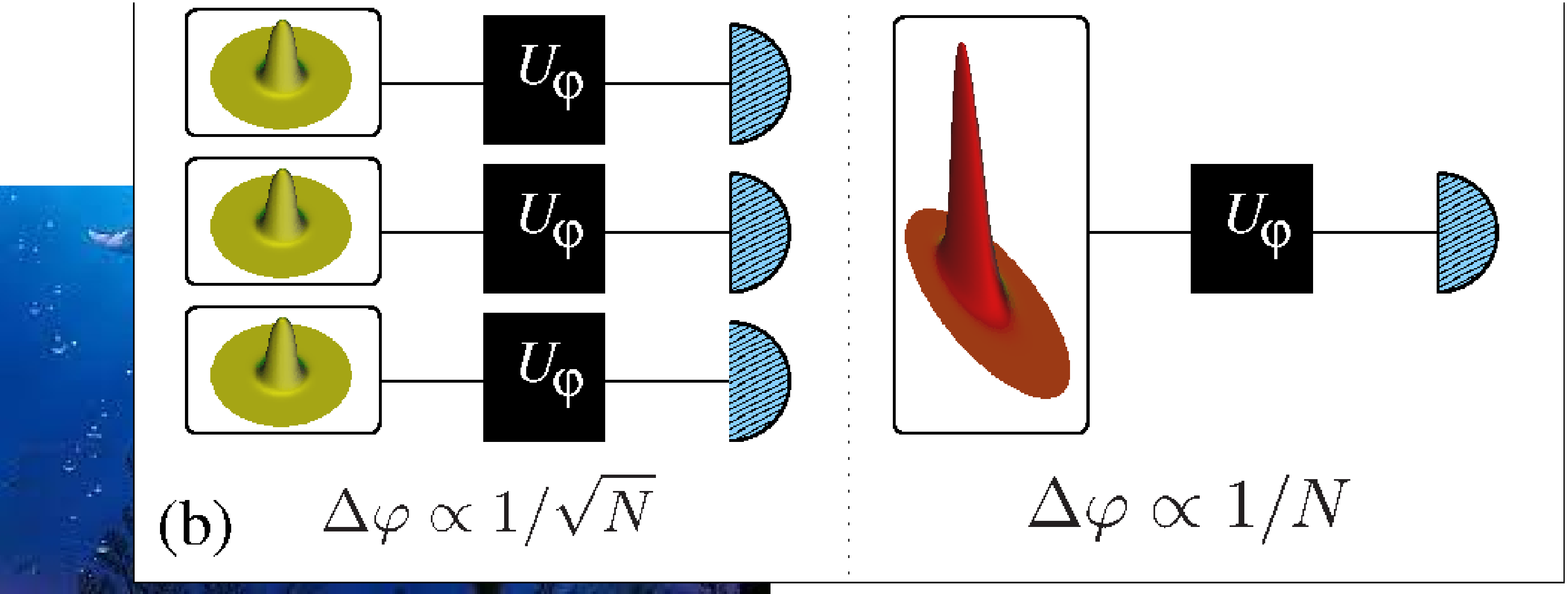

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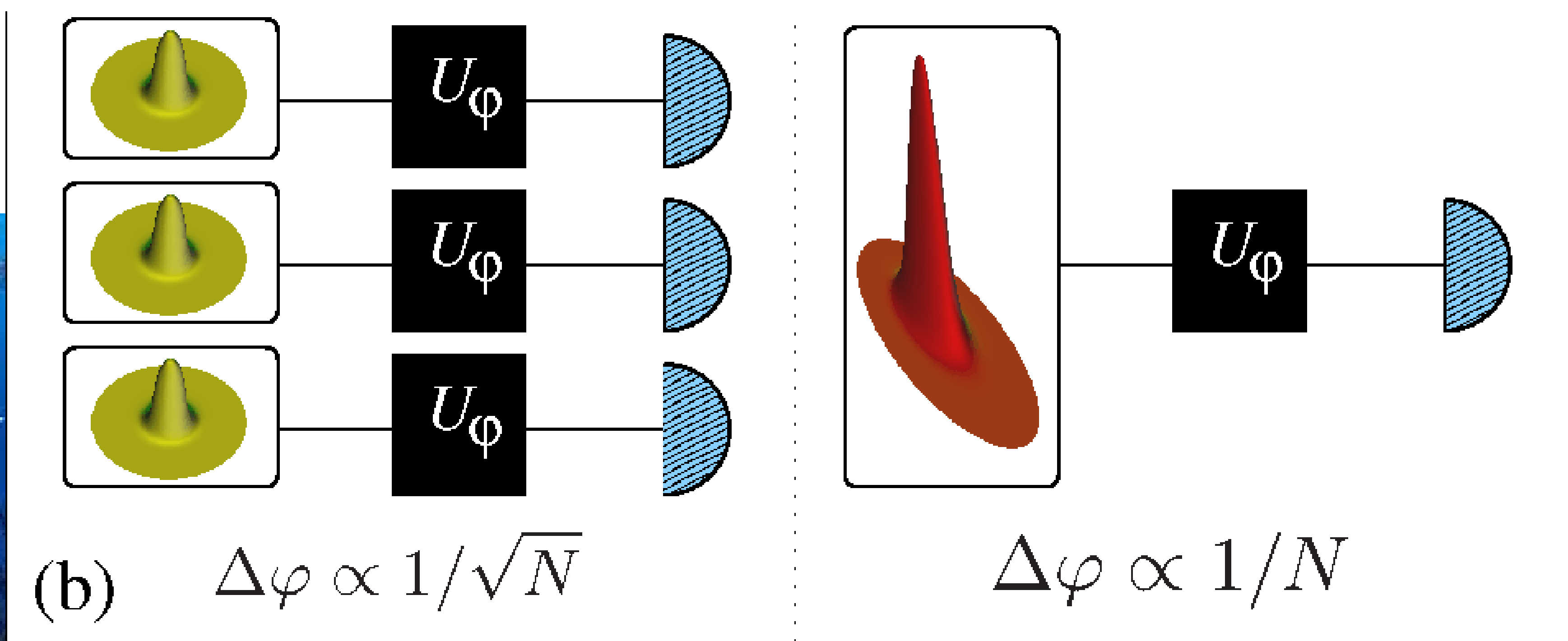

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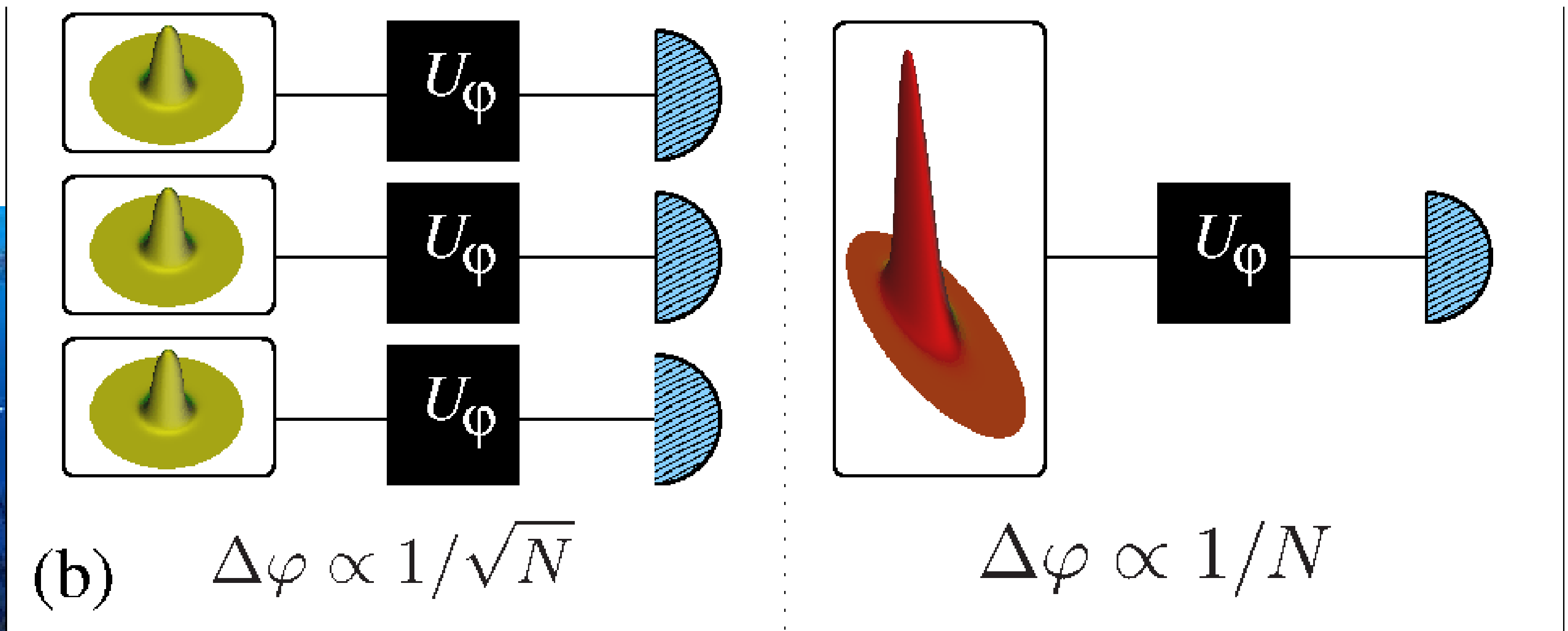

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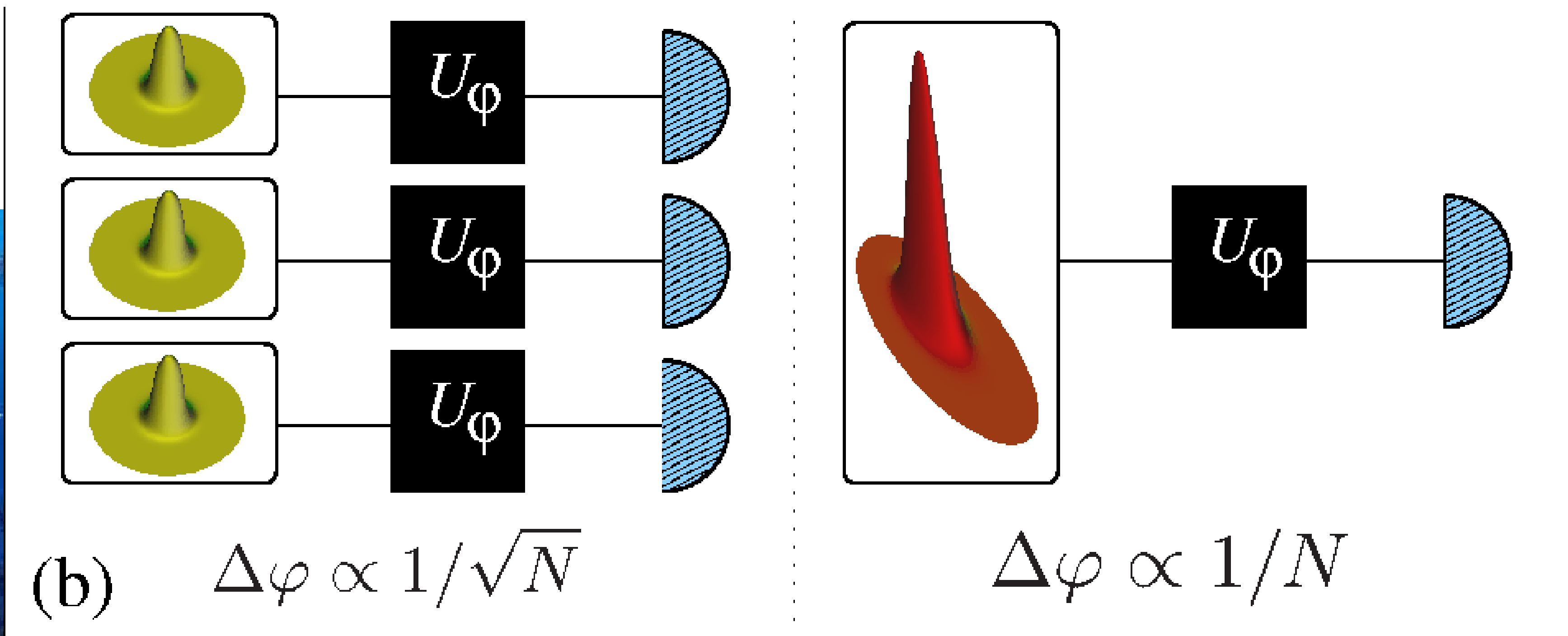

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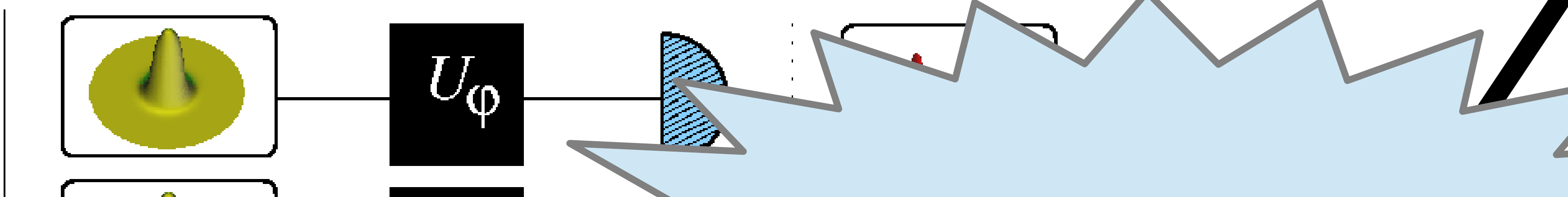

The Heisenberg bound

Noiseless case is simple and that's where the $\sqrt{N}$ quantum advantage is

The noisy case is a **mess**, but the noiseless case is an upper bound.

This paper: **spreading channels!**

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Example: displacement (nonzero α
if the axion and em field interact)

$$D(\alpha, \varphi) = e^{\alpha(e^{i\varphi}a^\dagger - e^{-i\varphi}a)}$$

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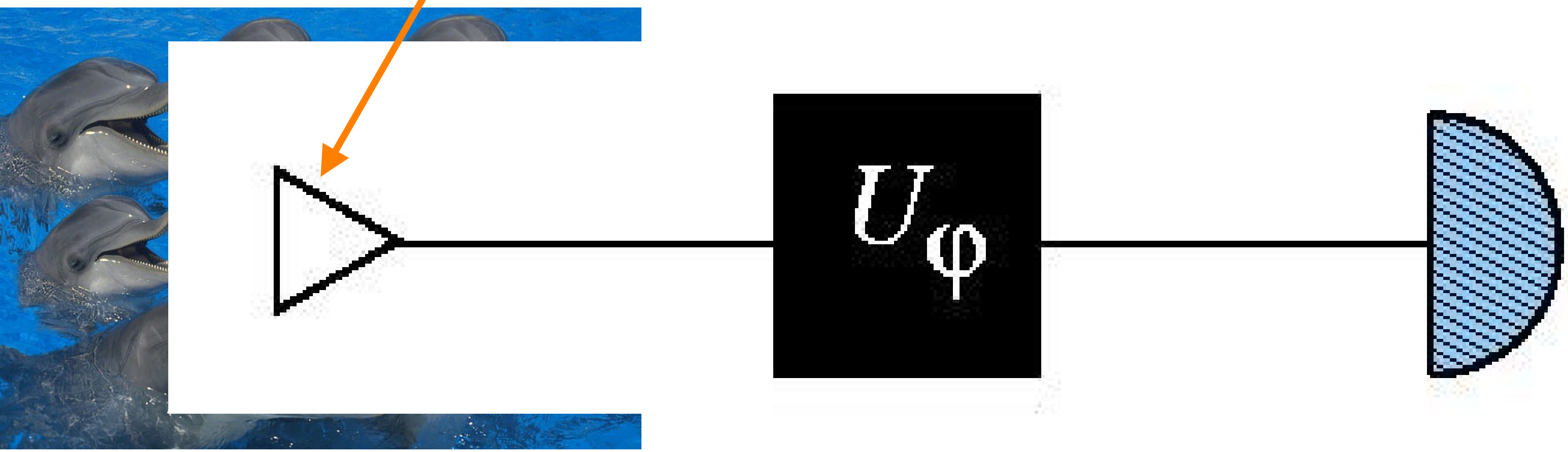

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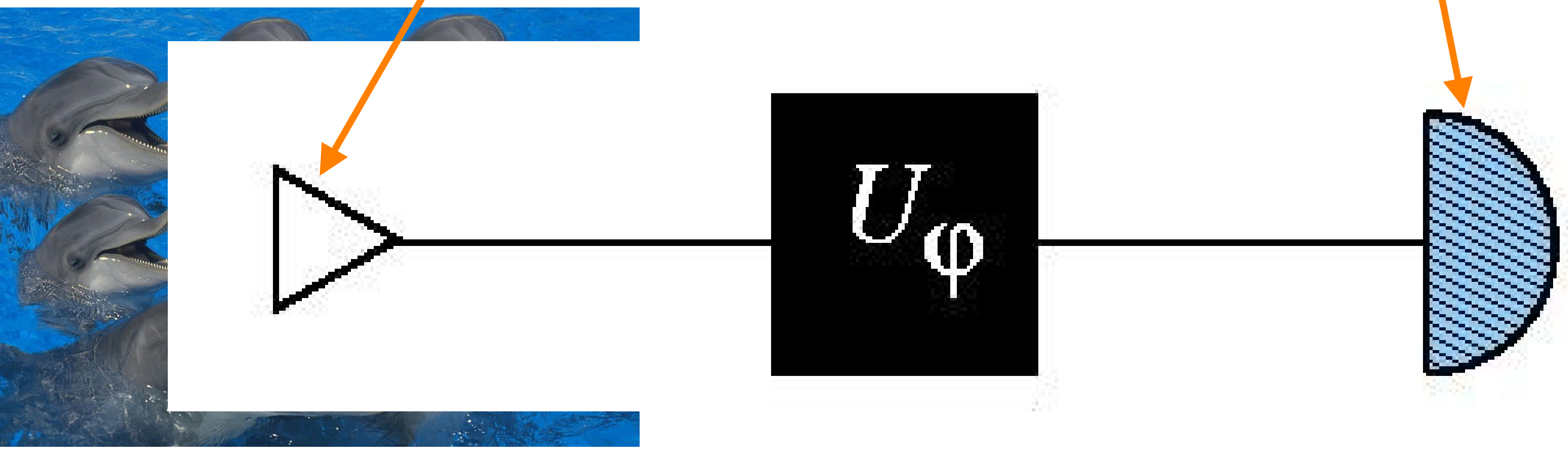

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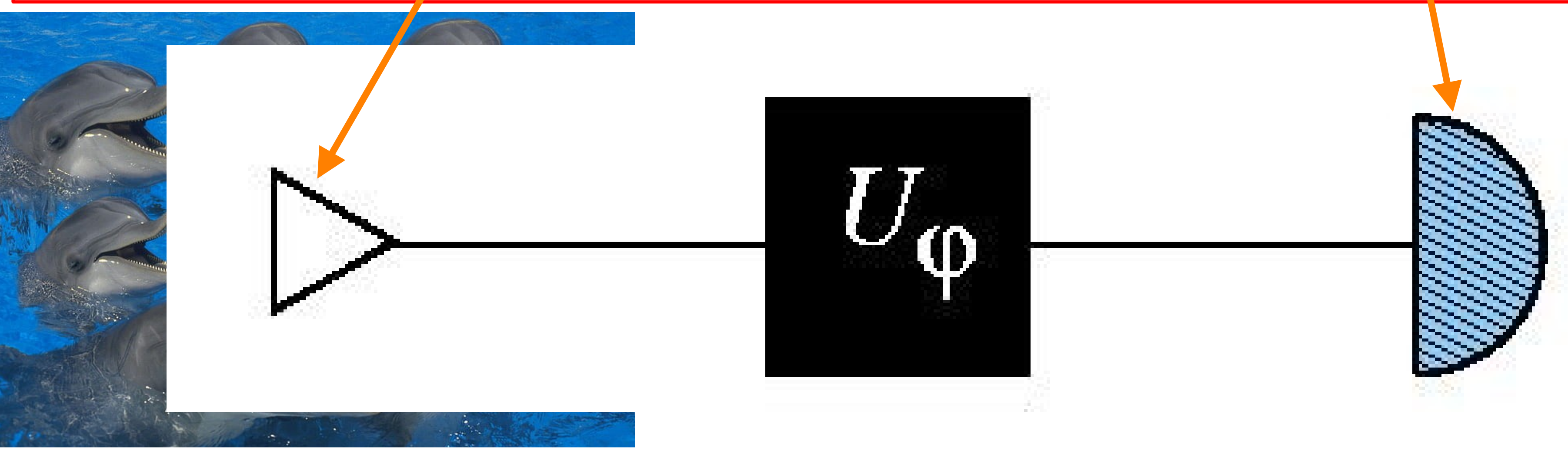
3) we retain the $\sqrt{N}$ advantage over the classical strategies! Just as the noiseless case!



Homodyne detection does not work!

A surprising connection to Rayleigh's curse in imaging, which is a particular case of the class of channels we consider

Results

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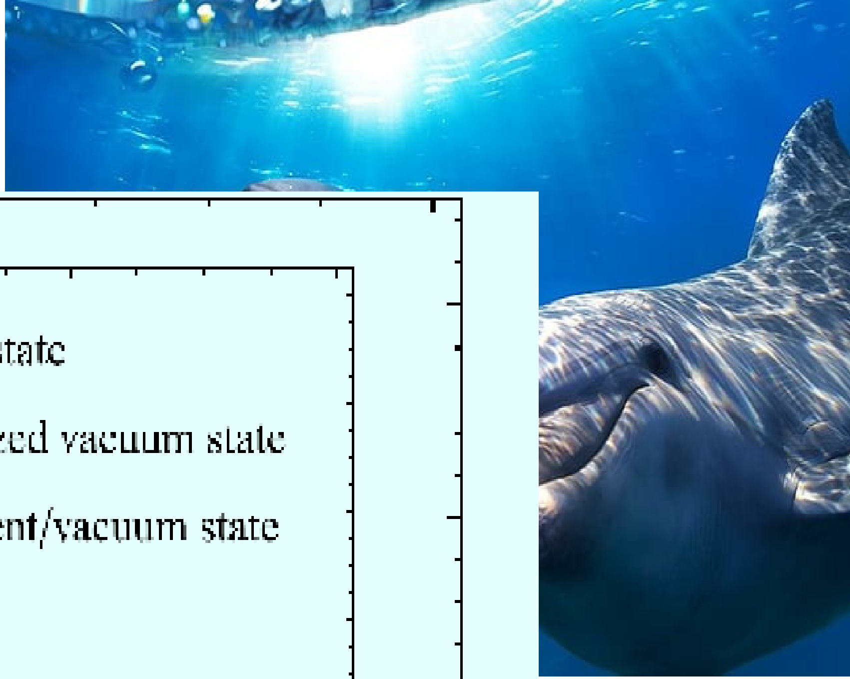
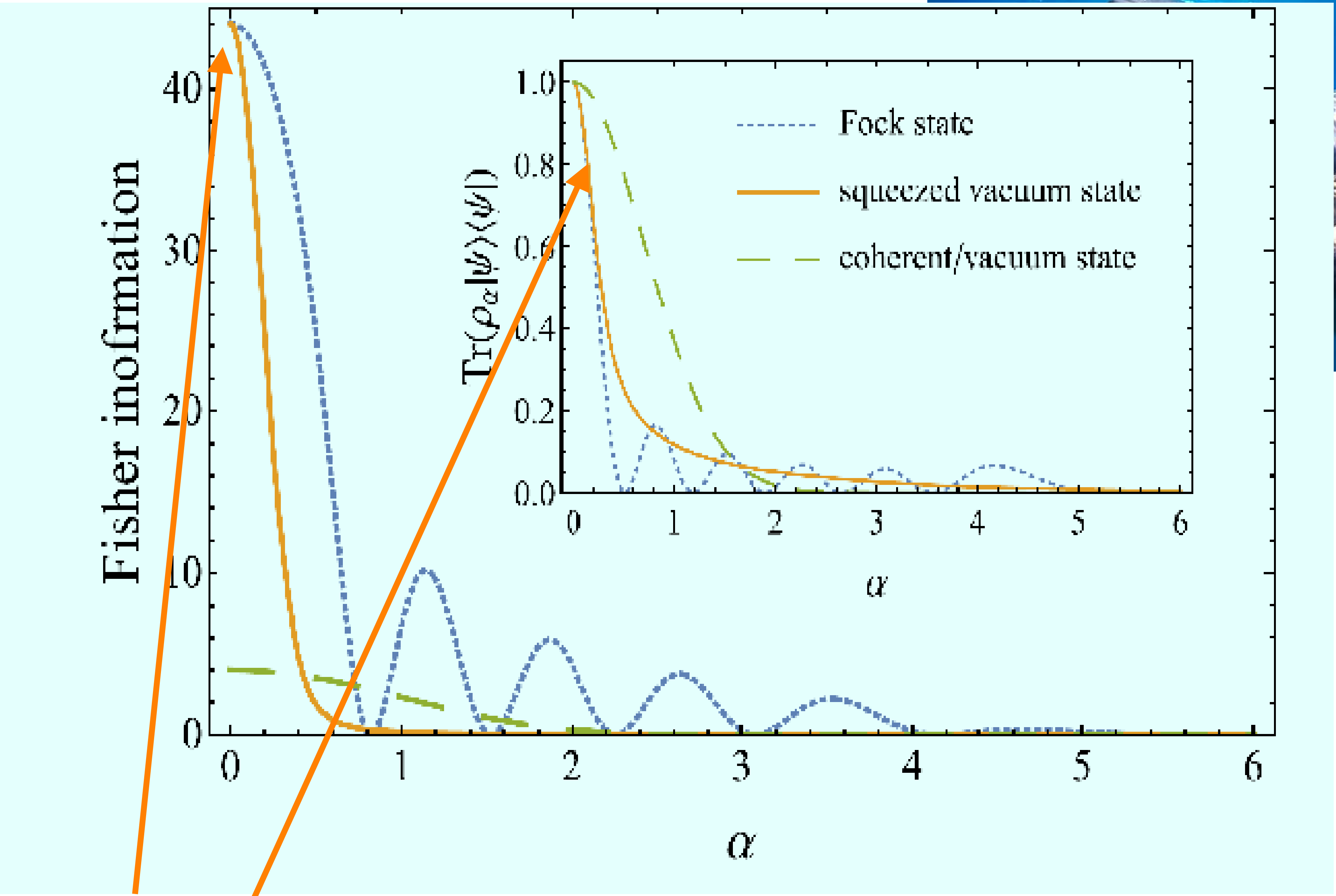
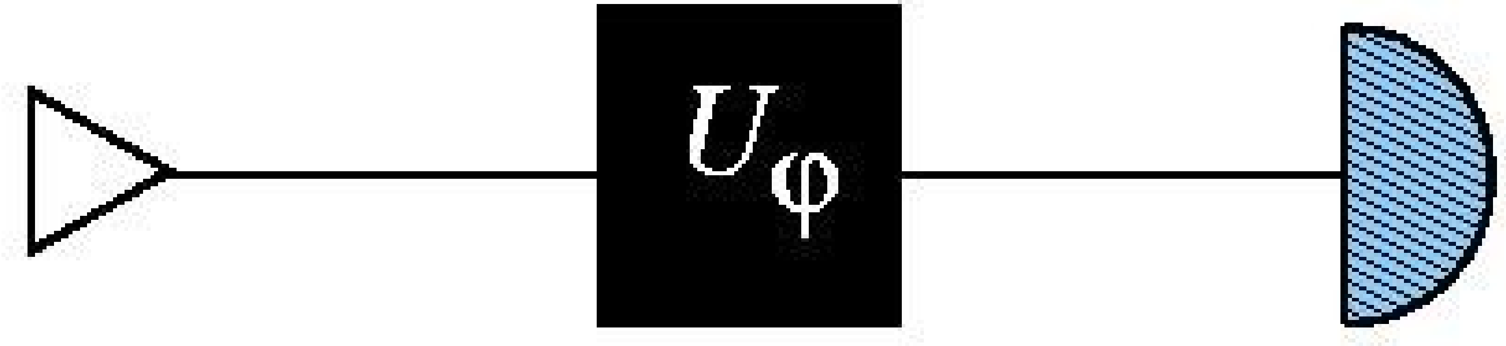




Large Fisher info=good estimation

Small fidelity between initial and final state=good discrimination

Experiment ideally



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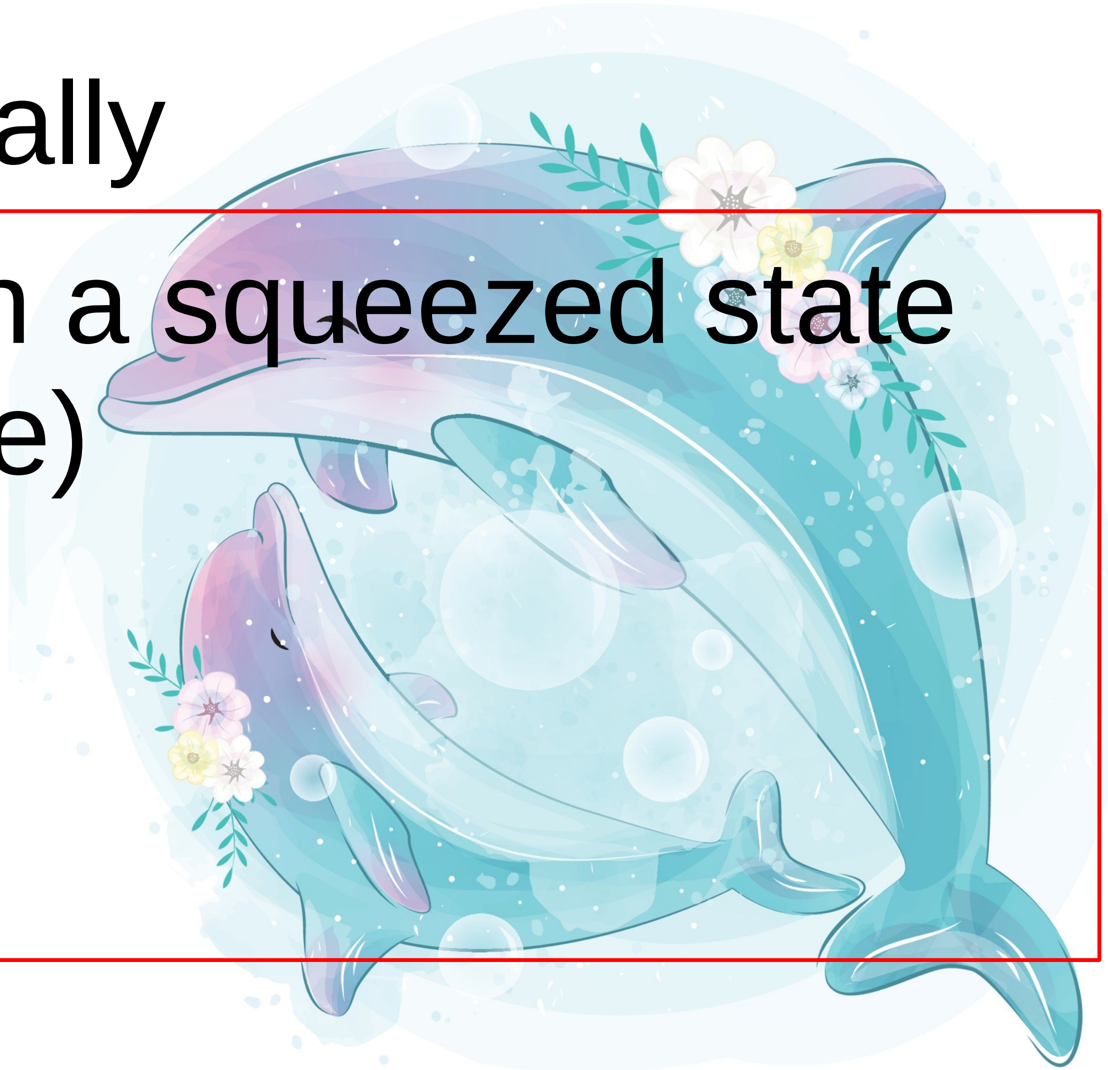

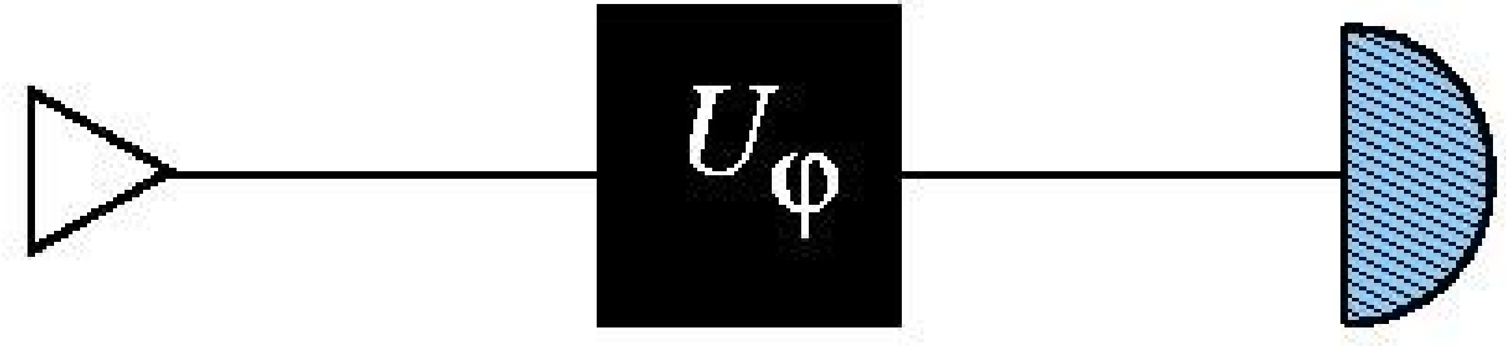

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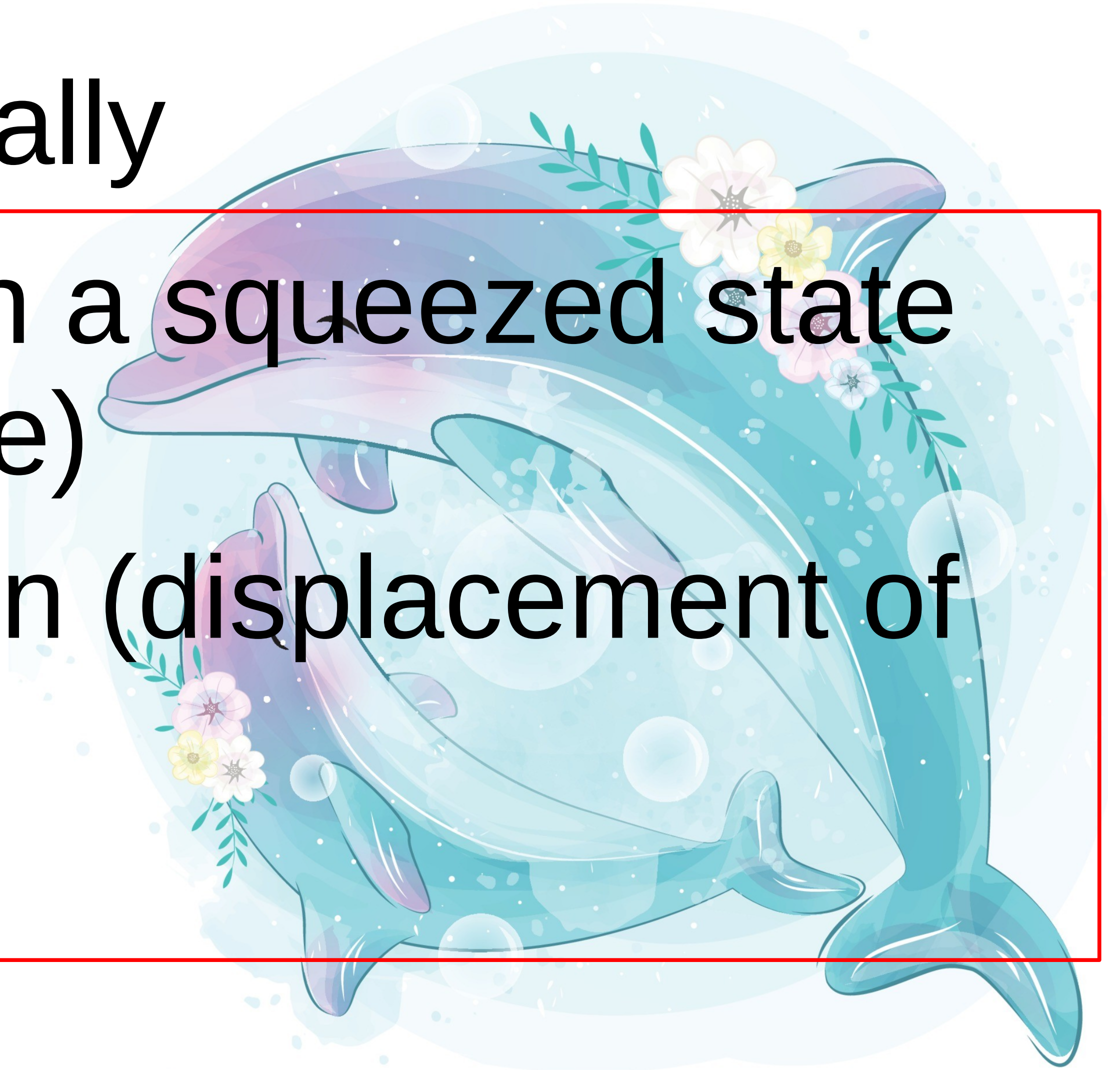

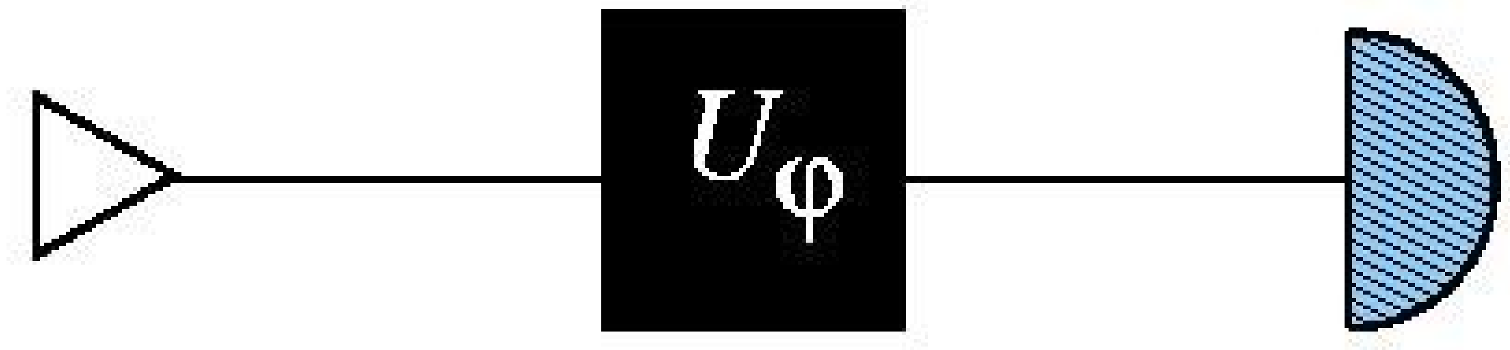

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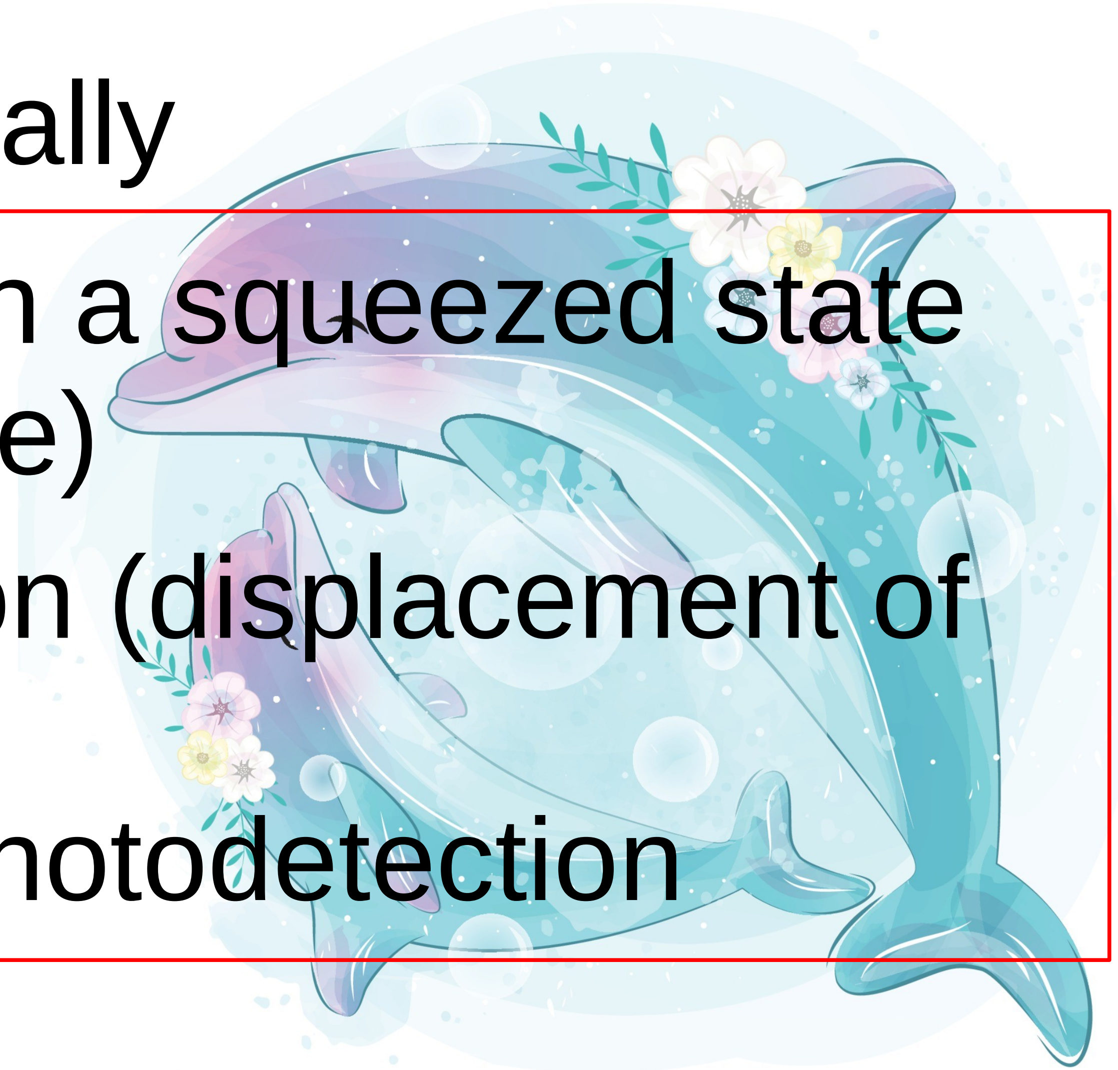

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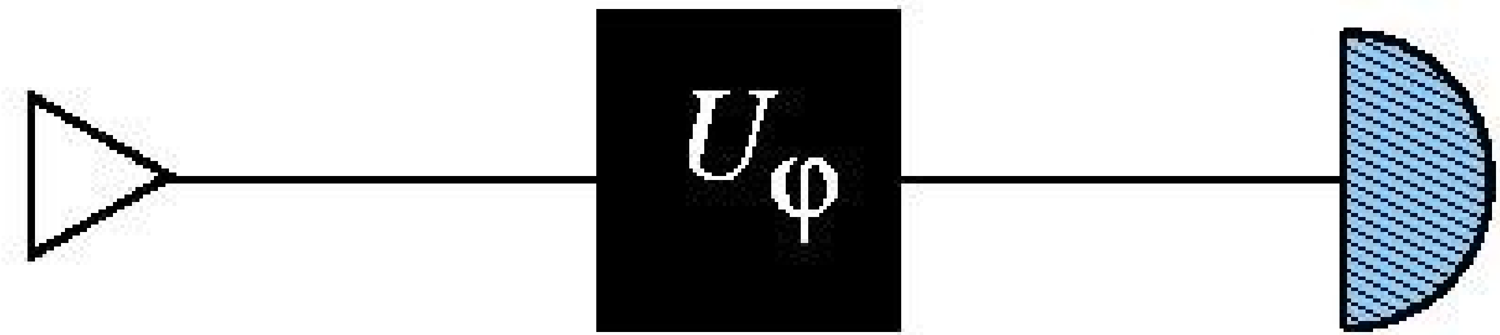

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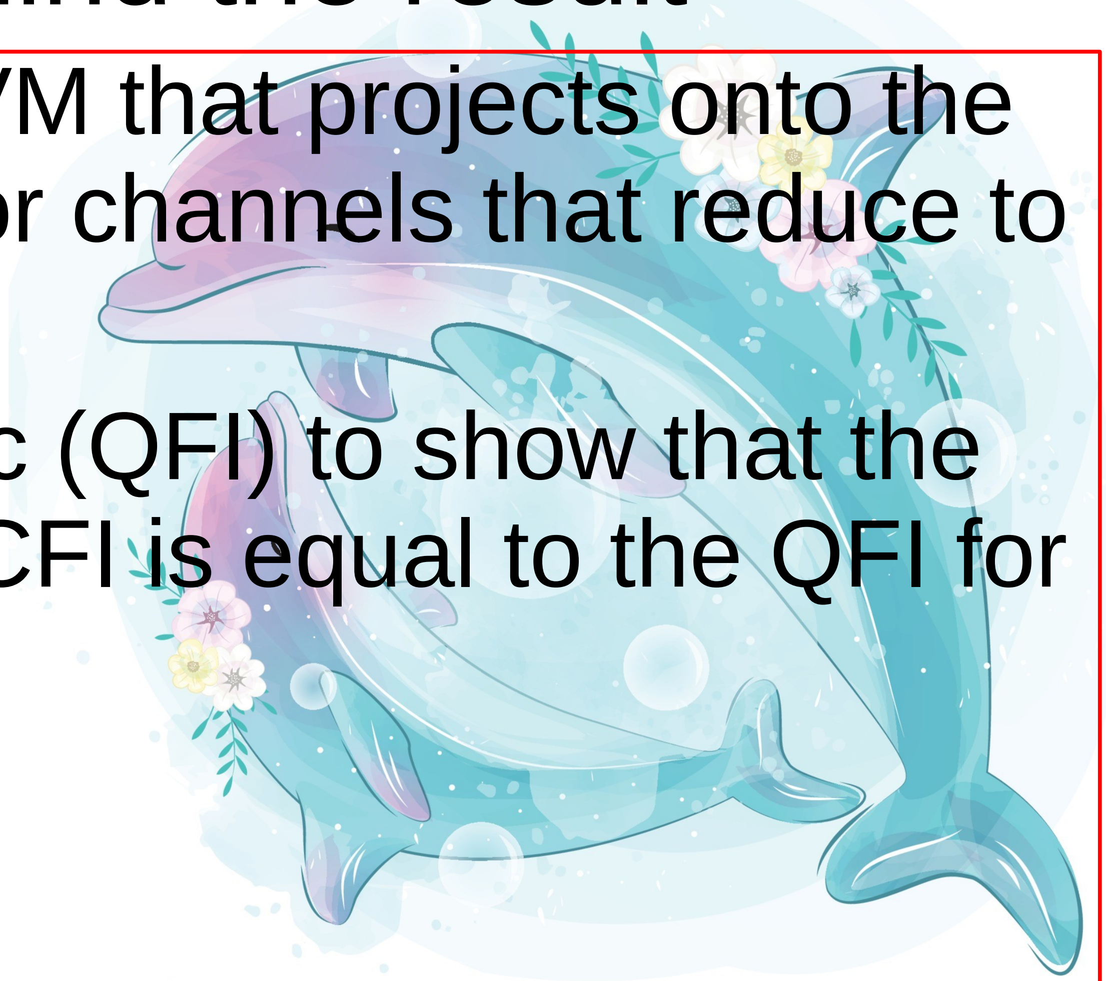

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4) for optical displacements show that the QFI averaged over noise is bounded by the average energy of the state: both Fock and sq. vacuum saturate the bound

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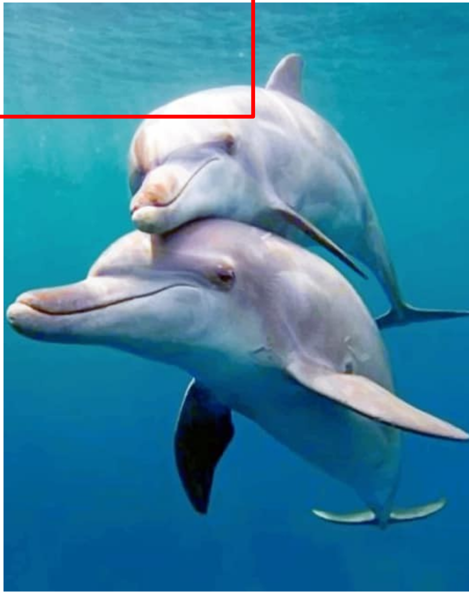

Open problems! (see Quntao’s paper)

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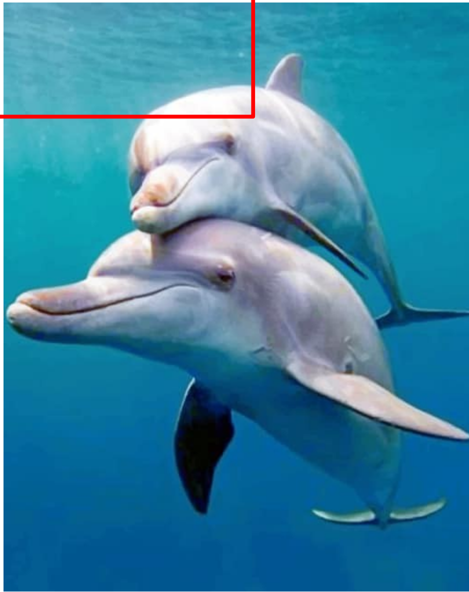
Not all required transformations can be easily implemented in the lab, the single mode analysis may not be



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Typically we compare quantum and classical strategies that use the **same resources** (energy or uses of the channel)

In practice → other figures of merit may be more relevant (scan rate!)

What did I say?

1. Quantum metrology noiseless case $\sqrt{N}$

2. QFI vs fidelity: overlap between initial and final state

3. Spreading channels

4. Squeezing input and antisqueezing+photodetection optimal.

Take home message

Lorenzo Maccone
maccone@unipv.it

Squeezing is optimal to estimate displacements with random (and irrelevant) phase in the noiseless case

L 129, 240503 (2022)

Squeezing metrology: arXiv:1901.0748